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Optimum Tolerance Allocation in Assembly

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Dimensioning and tolerancing are both important phases of product design. Although dimensions can usually be assigned based on design constraints and aestheticism without much difficulty, tolerancing is often a major problem to designers. Traditionally, tolerances are assigned intuitively followed by an analysis to check for any violation of the assembly requirements. Based on the analysis results, modifications are made manually by a "trial and error" method. This method relies on the experience of the designers and the results may not be optimal.

This paper presents a new approach to optimum tolerancing of components in an assembly such that all interaction requirements are met. The requirements may be for unilateral tolerance for control of clearance and interference or they may be for bilateral tolerance control. A model showing the relationship between components is constructed directly from the design. Using the model, coupled with a unique algorithm, a set of linear equations is formulated based on the design constraints and assembly requirements. The linear equations are then solved to determine the optimum tolerances of the assembly.

Keywords: Assembly requirement; Dimensioning; Tolerancing

1. Introduction

All machining processes involve some form of variation and owing to this inherent variability, a part cannot be produced exactly to the dimensions specified in the blueprint. Therefore, in designing a part or an assembly, some variation must be catered for. Tolerance is the maximum amount of variation allowed. Tolerances have a direct impact on the cost and manufacturability of the part or assembly. Unnecessarily tight tolerances will lead to higher costs whereas excessively loose tolerances may result in deteriorating functionality of the assembly, or rejection of the parts. The continuous effort to increase part interchangeability, and reduce scrap and cost has prompted much research in the area of tolerance allocation.

Smathers and Ostwald [1] applied Bellman's principle of optimality to select the most cost-efficient set of processes that meet the assembly requirements. The limitation is that the approach requires different sets of processes with different cost–tolerance relationships to choose from. Kim and Knott [2] presented a pseudo-Boolean approach in determining the lowest-cost assembly tolerances. However, the approach is limited to tolerances with a standard normal distribution. Lee and Woo [3] made use of a reliability index to formulate tolerance selection as an integer programming problem. A "branch and bound" algorithm was developed for optimum selection, but the method lacks emphasis on the assembly's interaction requirements. Manivannan et al. [4] used a knowledge-based specification system to assign dimensions and tolerances according to the ISO Standard for limits and fits. Although different types of interaction requirements (clearance, interference and transition) can be accommodated, the method is limited to two cylindrical mating parts and is not applicable to complex assemblies. Kalajdzic et al. [5] developed a feature-based design system which generates multiple representations of the product model. The process plan is then generated using this product data and multiple rule-bases. The disadvantages are that the method is iterative in nature and it requires extensive process data to establish the knowledge base. Ngoi and Ong [6] proposed a method to dimension the components of an assembly which allows real control over the size of clearance or interference in an assembly.

The purpose of this paper is to introduce another approach for tolerance allocation. This approach ensures that the allocated tolerances of all components in an assembly are an optimum while satisfying all the assembly requirements. The requirements may be unilateral tolerance for control of clearance and interference, or bilateral tolerance control. The method makes use of a model showing the relationships between components which can be constructed directly from the design.

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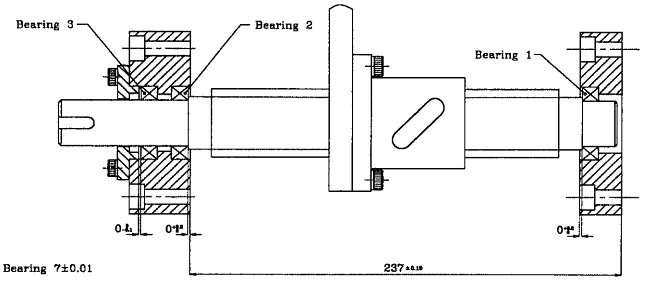

Fig. 1. Ball screw assembly showing the assembly requirements.

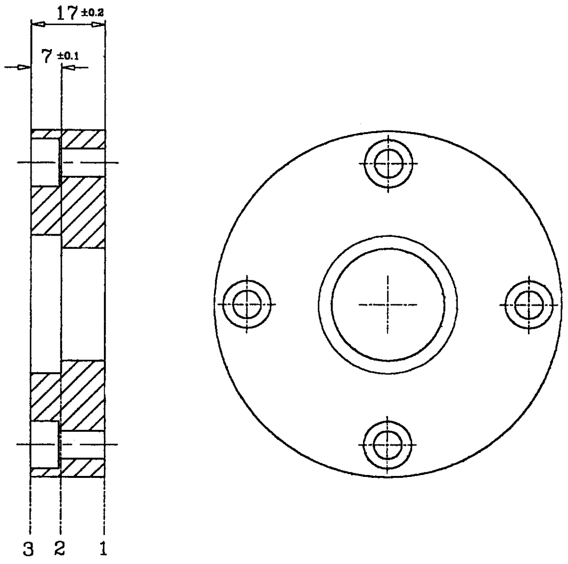

Fig. 2. Part A with initial tolerances allocated.

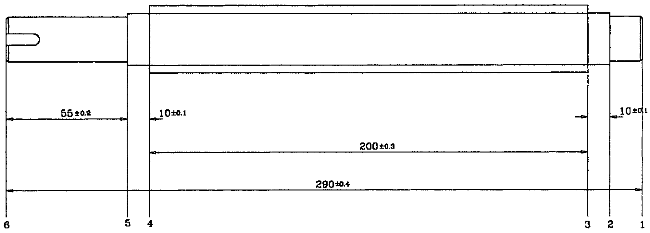

Fig. 3. Part B with initial tolerances allocated.

Coupled with a unique algorithm, a set of linear equations is formulated based on the design constraints and assembly requirements. The optimum tolerances of all components are then determined by solving the linear equation.

2. The Method

A ball screw assembly in Fig. 1 is used to illustrate the method. Part drawings of the assembly are shown in Figs 2–5. This example is only concerned with length dimensions and all diameter dimensions are omitted. Letters are used to denote each part; A for one part, B for another, etc. The surfaces of each part are first numbered, starting from 1 for the rightmost surface ascending towards the left as shown in Figs 2–5. The tolerance for each dimension is divided into two parts, namely the upper and the lower limits. The dimensions, in this paper, are represented by the expression PD*S where P represents the respective part, * refers to the limits (U for upper limit and L for lower limit) and S is a 2-digit number indicating the two surfaces to which the dimension refers. For example, ADU13 refers to the upper limit dimension between surfaces 1 and 3 of part A.

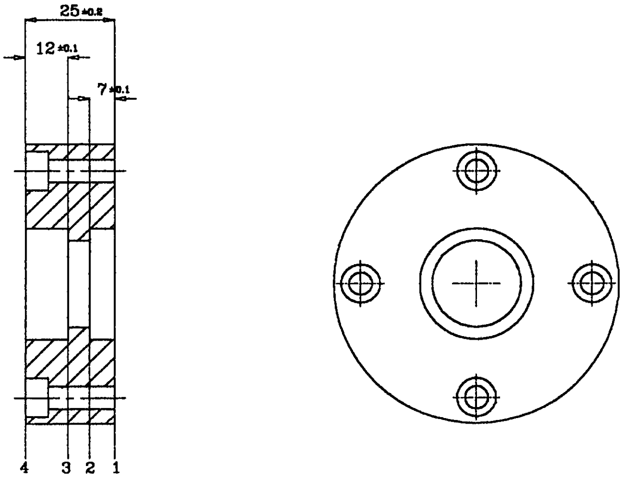

Fig. 4. Part C with initial tolerances allocated.

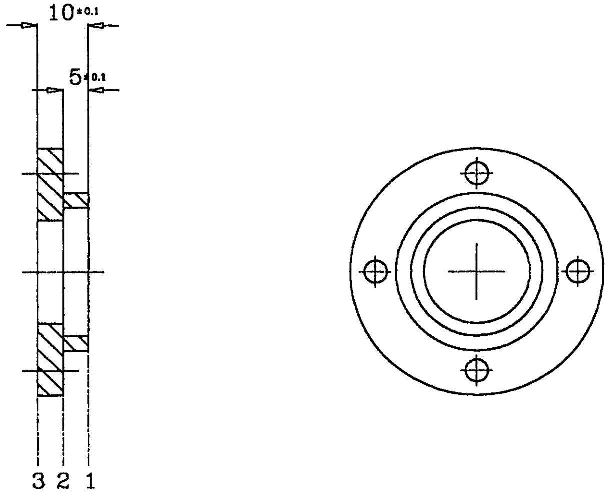

Fig. 5. Part D with initial tolerances allocated.

2.1 Initial Tolerance Allocation

In an assembly, some tolerances may not be constrained by the assembly requirements. Thus, it is necessary to set an upper limit to each tolerance. The limit should be chosen such

Table 1. Initial tolerance allocation.

Dimension	Maximum allowable tolerance
$L \leq 4$	±0.05
$4 < L \leq 16$	±0.1
$16 < L \leq 63$	±0.2
$63 < L \leq 250$	±0.3
$250 < L$	±0.4

that it can be easily attained and it should not violate any other requirements.

Table 1 is used as a reference in this paper. Note that it follows the form of tolerance allocation often used in engineering drawings where a similar table is placed on the title block. In practice, the tolerance may differ depending on the standard adopted. This standard ensures that all uncritical dimensions can be achieved with minimum cost, given the types of machines available.

The purpose of this step is to first allocate the "loosest" tolerance possible to all dimensions and subsequently, it tightens all the necessary tolerances to meet the assembly requirements. For standard parts such as bearings, couplings, etc., the tolerance is obtained from the respective catalogues.

2.2 Construction of a "Numbered Tree" Model

After the initial tolerance is allocated, a "numbered tree" model is constructed from the drawings. The procedure is as follows:

1. Represent each surface with a node, and draw the nodes such that the nodes for each part are arranged vertically.
2. Divide each node into two halves, the letter in the top half shows the part which it represents and the lower half indicates the associated number of the surface as numbered in Figs 2–5,
3. Link the nodes or surfaces. There are two types of links, either within a part or between parts. Within a part, nodes are linked when the dimension between the two surfaces is assigned. A solid line is used. The dimension between the two surfaces is indicated in the form of an upper and lower limit beside the link. Between parts, nodes are linked with a dotted line. Since the surfaces between parts are in contact, the dimensions and tolerances between them are zero.

With the "numbered tree" model constructed as shown in Fig. 6, two sets of equations can be derived directly from the model.

The first set of equations is derived based on the upper and lower limits of each dimension. For example, based on the initial tolerance allocated, the upper and lower limit dimensions between surfaces 1 and 3 of part A are 17.2 and 16.8. Hence the equation formulated are

$$\text{For upper limit,} \quad ADU13 \leq 17.2$$
$$\text{For lower limit,} \quad ADL13 \geq 16.8$$

Another set of equations is derived based on the manufacturability of the assembly. To ensure that the assembly is manufacturable by the machines available, the minimum difference between the upper and lower limits of a dimension is specified. In this paper, the minimum difference between the upper and lower limits of each dimension is taken as 0.04. The equation formulated for the dimension between surfaces 1 and 3 of part A is

$$ADU13 - ADL13 \geq 0.04$$

2.3 Formulation of Assembly Requirements into Linear Equations

With the "numbered tree" model constructed, the dimension link between any two surfaces can be established. Since dimen-

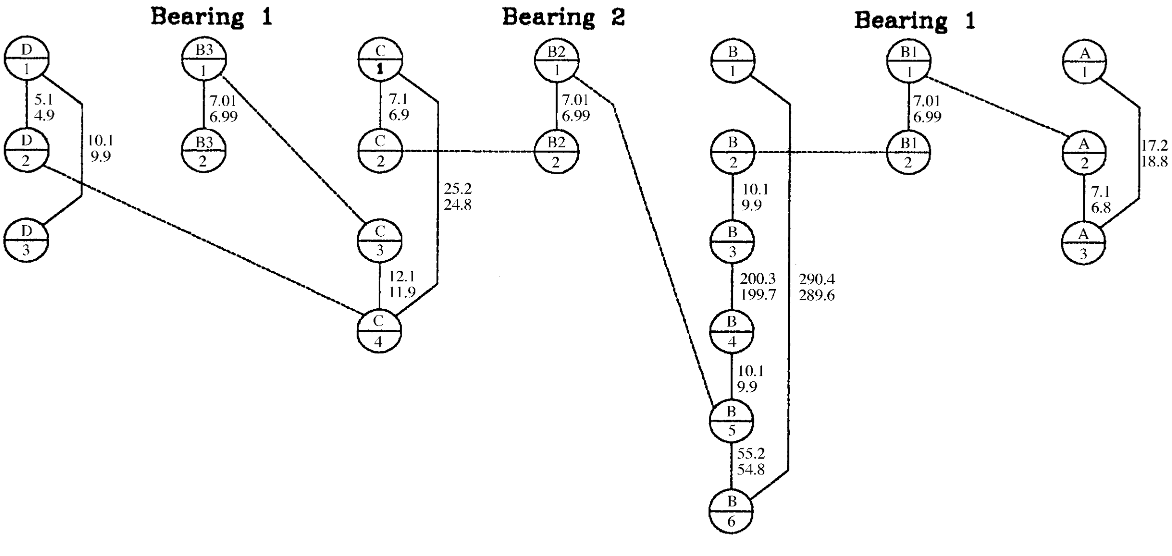

Fig. 6. "Numbered tree" model.

sion is a vector quantity, the sign has to be taken care of. This can be achieved easily by referring to the "numbered tree" model. For the purposes of illustration, different forms of assembly requirements are specified as indicated in Fig. 1. Note that unilateral tolerance requirements are drawn as clearances. Two equations are formulated for each assembly requirement, i.e. the upper and the lower limits. Starting from the righthand surface, the constraint equations for the upper and lower limits can be formulated by applying Eqs (1) and (2), respectively.

The equations automatically assign a (+) sign or a (−) sign to each link. A (+) sign is assigned if the number is from small to big and a (−) sign is assign if the number is from big to small. Since the two nodes linked by a dotted line refer to contacting surfaces, the dimensions and tolerances between them are zero.

For example, to formulate the equation for the tolerance requirement between surface 1 of part D and the lefthand surface of bearing 3, there are 6 nodes (D-1, D-2, C-4, C-3, B3-1 and B3-2) and 5 links involved. With the nodes and links identified, the dimension link can be formed by starting from the righthand surface and tracing to the lefthand surface, i.e. surface 2 of bearing 3, B3-2 to B3-1 to C-3 to C-4 to D-2 to D-1. Note that the dimensions for contacting surfaces (B3-1 to C-3 and C-4 to D-2) are zero.

For the upper limit of the assembly requirement, the equation is

$$\sum_{k=1}^{n-1} \frac{(S_{k+1}-S_k)}{|S_{k+1}-S_k|}(D^*) \le \text{Upper} \quad \text{Limit (dimension)} \tag{1}$$

where D^* = dimension of k^{th} link
Upper limit is used when $S_{k+1} > S_k$
Lower limit is used when $S_{k+1} < S_k$
S_k = associated number of k^{th} surface/node
n = number of surfaces/nodes in the process link

For the lower limit of the assembly requirement, the equation is

$$\sum_{k=1}^{n-1} \frac{(S_{k+1}-S_k)}{|S_{k+1}-S_k|}(D^*) \ge \text{Lower} \quad \text{Limit (dimension)} \tag{2}$$

where D^* = dimension of kth link
Upper limit is used when $S_{k+1} < S_k$
Lower limit is used when $S_{k+1} > S_k$
S_k = associated number of kth surface/node
n = number of surfaces/nodes in the process link

Note that in both equations, different limits of dimension are used when $S_{k+1} > S_k$ and $S_{k+1} < S_k$. When formulating the equation for the upper limit of the interaction requirement, upper limits of the dimension are used when $S_{k+1} > S_k$ and lower limit of the dimension are used when $S_{k+1} < S_k$. When formulating the equation for the lower limit of the interaction requirement, upper limits of dimension are used when $S_{k+1} < S_k$ and lower limits of dimension are used when $S_{k+1} > S_k$. For example, if the dimension and tolerance between surface 1 of part D and the lefthand surface of bearing 3 are to be 0 and (0, −0.1) respectively, the equation formulated are

$$-6.99 + \text{CDU34} - \text{DDL12} \le 0$$
$$-7.01 + \text{CDL34} - \text{DDU12} \ge -0.1$$

2.4 Formulation of Objective Function

The aim of optimum tolerance allocation can be seen as maximising all blueprint tolerances while ensuring that all assembly requirements are not violated. The aim can be formulated into an objective function and represented mathematically as

$$\text{maximise} \sum_{i=1}^{m} (\text{Upper limit dimension} - \text{Lower limit dimension})_i \tag{3}$$

where

m = total number of blueprint dimensions in the assembly

With the upper and lower limits of each dimension known and the assembly requirements formulated into a set of linear equation, the optimum blueprint tolerances can be determined using the Microsoft Excel Solver.

3. An Application

The approach to optimum tolerance allocation described above is applied on the ball screw assembly in Fig. 1. The steps are as follows.

First, the surfaces of each part are numbered, starting from 1 for the rightmost surface ascending towards the left as shown in Figs 2–5. Next, an upper limit is set for each tolerance. In this paper, this initial tolerance allocation follows a standard as shown in Table 1. After that, a "numbered tree" model is constructed and two sets of equations are formulated, one based on the upper and lower limits of each dimension and another based on the minimum difference between the upper and the lower limits.

The equations are

Part A ADU13 ≤ 17.2
ADL13 ≥ 16.8
ADU13 – ADL13 ≥ 0.04
ADU23 ≤ 7.1
ADL23 ≥ 6.9
ADU23 – ADL23 ≥ 0.04
Part B BDU16 ≤ 290.4
BDL16 ≥ 289.6
BDU16 – BDL1 6 ≥ 0.04
BDU23 ≤ 10.1
BDL23 ≥ 9.9
BDU23 – BDL23 ≥ 0.04
BDU34 ≤ 200.3
BDL34 ≥ 199.7
BDU34 – BDL34 ≥ 0.04
BDU45 ≤ 10.1
BDL45 ≥ 9.9
BDU45 – BDL45 ≥ 0.04
BDU56 ≤ 55.2

Table 2. Optimum blueprint tolerances

Part	Surface	Dimension limits	Blueprint dimension	Blueprint tolerances
1	13	16.84	17	–0.16
		16.8		–0.20
	23	7.05	7	+0.05
		7.01		+0.01
2	16	290.4	290	+0.40
		289.6		−0.40
	23	10.05	10	+0.05
		10.01		+0.01
	34	200.3	200	+0.30
		200.26		+0.26
	45	9.96	10	−0.04
		9.9		−0.10
	56	55.2	55	+0.20
		54.8		−0.20
3	12	7.05	7	+0.05
		7.01		+0.01
	14	25.2	25	+0.20
		24.8		−0.20
	34	12.05	12	+0.05
		12.01		+0.01
4	12	5.1	5	+0.10
		5.06		+0.06
	13	10.1	10	+0.10
		9.9		−0.10

$BDL56 \geq 54.8$
$BDU56 - BDL56 \geq 0.04$
Part C $CDU12 \leq 7.1$
$CDL12 \geq 6.9$
$CDU12 - CDL12 \geq 0.04$
$CDU14 \leq 25.2$
$CDL14 \geq 24.8$
$CDU14 - CDL14 \geq 0.04$
$CDU34 \leq 12.1$
$CDL34 \geq 11.9$
$CDU34 - CDL34 \geq 0.04$
Part D $DDU12 \leq 5.1$
$DDL12 \geq 4.9$
$DDU12 - DDL12 \geq 0.04$
$DDU13 \leq 10.1$
$DDL13 \geq 9.9$
$DDU13 - DDL13 \geq 0.04$

Based on the assembly requirements, another set of equation is formulated. Figure 1 shows the different forms of assembly requirements specified. Using Eq. (1) and Eq. (2), the constraint equation formulated are For surface 2 (bearing 3) to surface 1 (part D),

$$-6.99 + CDU34 - DDL12 \leq 0$$
$$-7.01 + CDL34 - DDU12 \geq -0.1$$

For surface 1 (part C) to surface 1 (bearing 2),

$$+CDU12 - 6.99 \leq 0.2$$
$$+CDL12 - 7.01 \geq 0$$

For surface 2 (bearing 1) to surface 3 (part A),

$$-6.99 + ADU23 \leq 0.2$$
$$-7.01 + ADL23 \geq 0$$

For surface 1 (part A) to surface 1 (part C),

$$+ADU13 - ADL23 + 7.01 + BDU23 + BDU34 + BDU45$$
$$+ 7.01 - CDL12 \leq 237.15$$
$$+ADL13 - ADU23 + 6.99 + BDL23 + BDL34 + BDL45$$
$$+ 6.99 - CDU12 \geq 236.85$$

Applying Eq. (3), the objective function is

$$\text{maximise } [(ADU13-ADL13)+(ADU23-ADL23)+(BDU16-BDL16)$$
$$+(BDU23-BDL23)$$
$$+(BDU34-BDL34)+(BDU45-BDL45)$$
$$+(BDU56-BDL56)+(CDU12-CDL12)$$
$$+(CDU14-CDL14)+(CDU34-CDL34)$$
$$+(DDU12-DDL12)+(DDU13-DDL13)]$$

Using the above objective function and the constraints as input to the Microsoft Excel Solver, an optimisation software, the optimum blueprint tolerances are determined. The results obtained are shown in Table 2 and the report from the Microsoft Excel Solver is attached in the Appendix.

4. Concluding Remarks

This paper describes a new approach for optimum tolerance allocation in assembly. The method allows all blueprint tolerances to be determined while ensuring that all the assembly requirements are satisfied. The algorithm is simple and hence it is suitable for all users. It reduces the amount of work and "guessing" required in the allocation of blueprint tolerances. Moreover, it is assured that the result obtained is an optimum and none of the assembly requirements are violated. Further work is being carried out to integrate tolerance charting with the above approach to determine the blueprint tolerances, working dimensions and tolerances concurrently.

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Appendix

Target Cell (Max)

Cell	Name	Original Value	Final Value
A3	maximize	4	2.14

Adjustable Cells

Cell	Name	Original Value	Final Value
B10	ADU13	17.2	16.84
B11	ADL13	16.8	16.8
B13	ADU23	7.1	7.05
B14	ADL23	6.9	7.01
B16	BDU16	290.4	290.4
B17	BDL16	289.6	289.6
B19	BDU23	10.1	10.05
B20	BDL23	9.9	10.01
B22	BDU34	200.3	200.3
B23	BDL34	199.7	200.26
B25	BDU45	10.1	9.96
B26	BDL45	9.9	9.9
B28	BDU56	55.2	55.2
B29	BDL56	54.8	54.8
B31	CDU12	7.1	7.05
B32	CDL12	6.9	7.01
B34	CDU14	25.2	25.2
B35	CDL14	24.8	24.8
B37	CDU34	12.1	12.05
B38	CDL34	11.9	12.01
B40	DDU12	5.1	5.1
B41	DDL12	4.9	5.06
B43	DDU13	10.1	10.1
B44	DDL13	9.9	9.9

Objective Function

Cell		Name
A3	maximize	B6-B7+B8-B9+B10-B11+B12-B13+B14-B15+B16-B17+B18-B19+B20-B21+B22-B23+B24-B25+B28-B29+B26-B27

Constraints

Cell	Name	Cell Value
B12	ADU13-ADL13	0.04
B15	ADU23-ADL23	0.04
B18	BDU16-BDL16	0.8
B21	BDU23-BDL23	0.04
B24	BDU34-BDL34	0.04
B27	BDU45-BDL45	0.06
B30	BDU56-BDL56	0.4
B33	CDU12-CDL12	0.04
B36	CDU14-CDL14	0.4
B39	CDU34-CDL34	0.04
B42	DDU12-DDL12	0.04
B45	DDU13-DDL13	0.2
B46	-6.99+CDU34-DDL12	8.88E-16
B47	-7.01+CDL34-DDU12	-0.1
B48	CDU12-6.99	0.06
B49	CDL12-7.01	0
B50	-6.99+ADU23	0.06
B51	-7.01+ADL23	0
B52	ADU13-ADL23+7.01+BDU23+BDU34+BDU45+7.01-CDL12	237.15
B53	ADL13-ADU23+6.99+BDL23+BDL34+BDL45+6.99-CDU12	236.85
B10	ADU13	16.84
B11	ADL13	16.8
B13	ADU23	7.05
B14	ADL23	7.01
B16	BDU16	290.4
B17	BDL16	289.6
B19	BDU23	10.05
B20	BDL23	10.01
B22	BDU34	200.3
B23	BDL34	200.26
B25	BDU45	9.96
B26	BDL45	9.9
B28	BDU56	55.2
B29	BDL56	54.8
B31	CDU12	7.05
B32	CDL12	7.01
B34	CDU14	25.2
B35	CDL14	24.8
B37	CDU34	12.05
B38	CDL34	12.01
B40	DDU12	5.1
B41	DDL12	5.06
B43	DDU13	10.1
B44	DDL13	9.9