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ORIGINAL ARTICLE

The optimal layout design for minimizing operating costs in a picker-to-part warehousing system

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Abstract
The decision problems in the design and control of warehousing order picking process include layout design, storage assignment, routing, and order batching policies. Previous researches on layout design for pick-and-pass systems are not abundant. However, a pick-and-pass system is a commonly used approach because of the growth of e-commerce and the faster delivery of small and frequent orders. This paper presents an algorithm to determine the number of zones, and the number of picking bins and racks in each zones for the pick-and-pass systems such that both investment and operational costs can be minimized. A cycle time evaluation model is developed and incorporated into the proposed algorithm to aid a warehouse designer to quickly evaluate design alternatives for warehouse configurations. The result indicates that the cycle time evaluation model appears to be sufficiently accurate for practical purpose and the optimal layout can be determined by the proposed algorithm.

Keywords Order picking · Warehouse management · Optimal layout · Pick-and-pass system

1 Introduction

Order picking is the most costly part of the warehouse operations. According to Coyle et al. [1], 50% to 75% of the total operating cost of a warehouse can be attributed to the order picking operation. Generally speaking, the decision problems in the design and control of warehouse order picking process include layout design, storage assignment, routing, and order batching [2, 3].

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The warehouse layout design can be divided two subproblems [2]: *facility layout* and *internal layout design*. The first problem is to decide where to locate various departments such as receiving, storage, picking, sorting, and shipping for minimizing the handling cost, while the other problem concerns the determination of the number of blocks and the number, length, and width of aisles in each block of a picking area for minimizing the travel distance or operational cost. Gu et al. [4] indicated that the warehouse layout problems affect its performance including construction and maintenance costs, material handling cost, storage capacity, space utilization, and equipment utilization. Bassan et al. [5] compared two configurations of shelves in a homogeneous or a zoned warehouse for handling and layout costs. Rosenblatt and Roll [6] presented a search procedure for finding a global optimal solution for a specific formulation of the warehouse design problem in which the proposed procedure enables one to obtain parameters for the design of warehouse, so that storage needs are met in an optimal way costwise. Rosenblatt and Roll [7] developed a simulation model to measure the relationship between warehouse capacity and various pertinent parameters. Park and Webster [8] proposed a storage structure layout method for minimizing the travel time of selected handling equipments in a three-dimensional palletized storage system. Pandit and Palekar [9] presented a queueing model that captures the complex tradeoffs among different costs of a forward-reserve warehouse in order to achieve a global optimal design satisfying throughput requirement. Larson et al. [10] presented a class-based storage procedure for warehouse layout to increase floor space utilization and decrease material handling. The three phases of the proposed procedure are (1) determination of aisle layout and storage zone dimensions, (2) assignment of material to a storage locations, and (3) allocation of floor space. Caron et al. [11] presented an analytical approach to the layout design of the picking area in low-level picker-to-part systems using cube-per-order index and random storage policies and developed a formula to determine the optimal number of aisles. Roodbergen [12] developed a number of routing methods and an approach to determine a layout for the order picking area for minimizing the average travel distance needed to pick an order or a batch of orders. Petersen [13] examined the configurations or shapes of picking zones by simulating a bin-shelving warehouse to measure a picker's travel where stock keeping units are assigned storage locations either using random or volume-based storage policies. The research shows that the size or storage capacity of a zone, number of items on the pick list, and the storage policy have a significant effect on picking zone configurations. Le-Duc and De Koster [14] elaborated the problem of choosing the right number of zones in a manual pick-and-pack order picking system and solved the problem by optimally assigning items to pick routes in each zone. Heragu et al. [15] presented a mathematical model and a heuristic algorithm to solve the product allocation and functional area size determination problem in the design of a warehouse so that annual handling and storage costs can be minimized. Parikh and Meller [16] proposed a cost-based optimization model to identify the optimal height to store a given number of pallets for a person-onboard order picking system. Kuo et al. [17] proposed a metaheuristic method to assign the place of the items in the synchronized zoning system by reducing the idle time. Petersen and Aase [18] examined the effect and placement of cross aisles in order picking operations. Cano et al. [19] presented a model to calculate the traveled distance and travel time and evaluate the picker routing in different warehouse layouts. Bortolini et al. [20] presented a model based on the class-based storage strategy to design the best layout for non-traditional warehouse with diagonal cross-aisles. Hsieh and Tsai [21] investigated a BOM-oriented class-based storage assignment system to improve the performance of the operation in the production process.

Moreover, a performance evaluation model is important for both warehouse design and operation purposes since it can aid a designer to quickly evaluate design alternatives and narrow down the design space during the early design stage [4]. According to Tompkins et al. [22], the order picking time mainly consists of travel time (50%) and search time (20%) in a manual warehouse. Most related studies also show that travel time is the dominant component of the picking time. Bartholdi and Hackman [23] indicated that travel time is waste since it costs labor hours but doesn't add value. Thus, the development of a model to analyze accurately the travel distance is crucial before any decision can be made for operating policies. For a pick-and-pass system, a picker in a zone finishes picking one portion of an order and then hands to the picker in the next zone until all the items are picked in that order. De Koster [24] developed an approximation method to evaluate the effect of changing the layout of the system. Yu and De Koster [25] presented an approximation method based on $G/G/m$ queueing network for a pick-and-pass system, and the proposed method appears to be sufficiently accurate in practical situations. Pan and Wu [26] developed an analytical model for a low-level pick-and-pass system in which only the horizontal movement of a picker is considered by modeling the operation of a picker as a Markov Chain for the estimation of the expected distance traveled of a picker in a picking line. Pan et al. [27] proposed an analytical approximation model to analyze the mean order throughput time in a picker-to-part order picking system. They also modelled the order picking system as a 2-stage tandem queueing network with Er/G/1 queues on each node. Hong [28] proposed an analytical model to quantify the operating time of the pickers in the picking line and to suggest how to deal with the hand-off delay efficiently. Hong [29] developed a closed-form expression to investigate the performances of flow time, work-in-process, and throughput in a two-picker order picking system. Tappia et al. [30] developed an analytical model to study integrated remote order picking systems.

While most of warehouse layout related researches focused on the picker-to-part systems or automatic storage and retrieval systems (AS/RS), only a few researches considered the pick-and-pass systems. However, due to the growth of e-commerce and emergence of the time-based competition, global supply chain management has been focusing on the faster delivery of small and frequent orders of inventory at a lower total cost [13]. A pick-and-pass system, also called a progressive zoning system, is a commonly used approach for small-to-medium size items such as health and beauty, household, office, or food products where items can be stored in relatively small and accessible pick locations along the picking line [31]. As the operation volumes increase, many companies also use the system to improve the efficiency of their warehouse operations.

This paper presents a non-linear programming model for the determination of the optimal number of zones, racks, and bins so as to minimize the operational and facility costs in the pick-and-pass systems under the preconditions of satisfying demand needs. A performance analytical approach is developed to evaluate the cycle time of a picking line and aid the development of a heuristic algorithm for the purpose of solving the non-linear programming problem efficiently. Moreover, this study analyzes the effect of storage assignment polices and demand distribution of items.

2 Description of the warehouse system and picking operations

The pick-and-pass system considered in this study consists of a roller conveyor connecting all pick stations (zones) located along a conveyor line, as illustrated in Fig. 1. There are Z pickers in a picking line, and each one picks the items of an order located in the vertical shelves (bins) in his/her zone. Each bin has R racks in it, and a rack only stores one item type. A well-known logistic company headquartered in Taiwan was selected as a reference model for the warehouse and picking operation system. In this selected model company, their warehouse applied a computer-aided picking system (CAPS) for information transfer in a pick-and-pass system.

The CAPS has light indicator module can guide pickers to the correct picking locations and demonstrate the correct quantity of items need to be picked. Jane and Laih [32] had addressed many advantages of CAPS including support the information transfer of picking systems, effectively enhance over 50% of picking productivity and reduce error. Moreover, CAPS can reduce operation cost by simplifying the training or shorten the learning curve of pickers. In CAPS system, by using light indicators, only the needed items in particular zone will light up which can guide and help pickers retrieve the goods efficiently. All pickers have to confirm the pick by pressing the lighted button when retrieved the correct items and quantity of the items. Once a pick has finished, CAPS will send the next picking data, and the light indicator module will notify the picker for next task [26].

In addition, when a picker has to go to the right side and left side from his/her current position for pickings, he/she will always first move towards the side with shorter distance, the shorter one between the starting bin and the first bin or last bin to be picked, and then goes to the bins in the other side for the remaining items. In other words, if the starting bin is relatively close to the right side, the picker will first pick the items in the right side. Otherwise, he/she will go to the left side first.

Since both the horizontal and vertical movements of a picker are considered, it is assumed that the sequence of picking is from the top location down in a picking bin; that is, a picker will first pick the items at the top rack in the next picking bin after picking all items in a certain bin. For example, when a picker starts to pick items at the bin 2 in Fig. 2a, he/she first picks the items in R_3 and R_2 at that bin and then goes toward the left side to pick the items in R_1. After all items in left side are picked, the picker turns to the right side to pick the items in R_4, R_6, and R_5 sequentially. If the picker starts at bin 5 in Fig. 2b, he/she will next go to the right side for R_6 and R_5 and then goes to the bins in the other side for the remaining items.

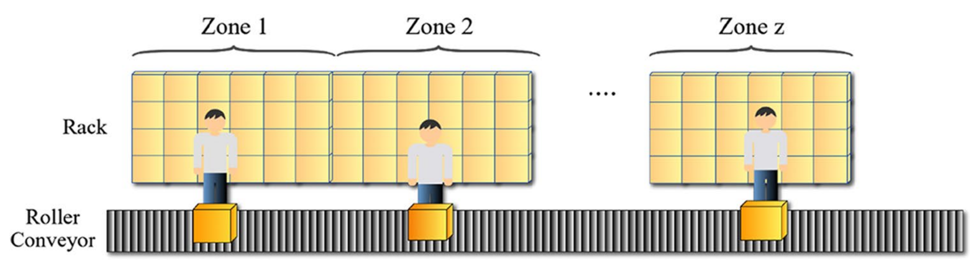


Fig. 1 A pick-and-pass system with multiple pickers

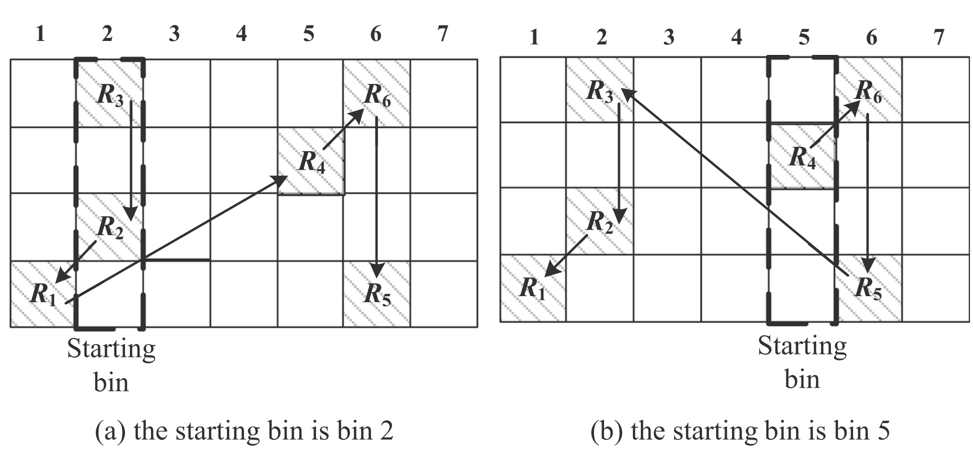


Fig. 2 An example of a picking sequence in a zone. **a** The starting bin is bin 2. **b** The starting bin is bin 5

According to Pan and Wu [26], the following assumptions are made for the warehouse and the picking operation model:

(1) A container is sufficiently large to handle all the items in a picking tour.
(2) The speed of a picker is constant.
(3) The time to pick an item from a rack is constant.
(4) The unit size of all items is smaller than the capacity of the shelf.
(5) No orders can be spread, i.e., strict picking order policy is implemented.
(6) A picker always chooses the shortest travel distance for each order.
(7) Upon the completion of an order, a picker stays at the bin of the last item in the order and starts next order from that location.
(8) Each item is independent of the others within an order.

3 Model formulation and travel time model development

The notations used in the paper are defined as follows:

Parameters

N	The number of items in an order
n_z	The number of items to be picked in zone z
p_{zbr}	The probability that at least one item to be picked is located in rack r of bin b of zone z
D_h	The height of a rack
D_w	The width of a rack
vv_z	The vertical travel speed of the picker in zone z (bins/unit time)
vh_z	The horizontal travel speed of the picker in zone z (bins/unit time)
T_z	The travel time of a picker in zone z
p_t	The time of picking an item
S	The floor space of a warehouse (ft^2)
U	The maximum height of a bin
C_s	Annual cost of storage space (\$/ft^2)
C_r	Hourly labor cost per picker
C_m	Annual maintenance cost per rack (\$/rack)
C_p	Purchase cost per rack (\$/rack)
L	The useful life of a rack
α	The interest rate per time period
W_h	Working hours per day
W_d	Working days per year
P	The designed order capacity of a picking line
Decision variables	The number of types of items

Decision variables

Z	The number of zones in a picking line
B_z	The number of bins in zone z, $z \in Z$.
R	The number of racks in a bin

The total annual system cost consists of annual labor cost, equipment cost, and space cost in an order picking system [26]. Since a zone has only one picker to fulfill orders, the annual labor cost can be express as $C_r \times W_h \times W_d \times Z$. As for the annual equipment cost, both of the annual purchase cost and annual maintenance cost per rack are taken into account. The annual purchase cost of a rack can be estimated by the one-time purchase cost divided by the useful life of the rack. In order to further consider the time value of the money, the one-time purchase cost can be converted into the corresponding cost through α/L, where $\alpha \geq 1$. The cost of space depends on the use and occupation of land in a warehouse. An optimal cost model for pick-and-pass systems is presented to identify the optimal number of zones and bins and the corresponding number of racks in a bin given a limited space in order to minimize the total annual system cost. The model can be formulated as follows:

$$\text{Min } C = C_r W_h W_d Z + \left(C_m + C_p \times \alpha/L\right) \times \sum_{z \in Z} \left(B_z R\right) + C_s \times \sum_{z \in Z} B_z, \tag{1}$$

Subject to

$$\sum_{z \in Z} B_z D_w \leq S, \tag{2}$$

$$R D_h \leq U, \tag{3}$$

$$W_h / \omega[Z, R, \{B_z | z \in Z\}] \geq P, \tag{4}$$

$$\sum_{z \in Z} B_z R \geq I, \tag{5}$$

$$Z, B_z, R \in \{1, 2, 3, \ldots\}, \forall z \in Z \text{ and } S > 0$$

Constraint set (2) ensures that the space of all bins must not be greater than the available floor space, constraint set (3) is to restrict the sum of heights of all racks in a bin does not exceeding the maximum height allowed for a bin. In general, the maximum height of a bin should be at the highest position that a picker can retrieve an item. Constraint set (4) ensures that the daily throughput of the picking system is able to satisfy the customer's needs or design order capacity, where $\omega[Z, R, \{B_z|z \in Z\}]$ denotes the cycle time of a picking line with a layout. Constraint set (5) ensures that the total number of racks is not fewer than the number of types of items so that each item can be stored in at least one rack.

Since a picking line in a pick-and-pass system can be regarded as an assembly line and each zone can be considered as a work station, the cle time of the line is always determined by the zone taking the longest picking time. The expected operational time of a picker fulfilling an order in a zone is composed of the expected picking time and the expected travel time and can be expressed as

$$E\left[\text{The operational time in zone z}\right] = p_t \times \mathrm{E}\left[n_z\right] + \mathrm{E}\left[T_z\right]. \tag{6}$$

Thus, the cycle time of a picking line is

$$\omega[Z, R, \{B_z | z \in Z\}] = \max\{p_t E[n_z] + E[T_z] | z \in Z\}. \tag{7}$$

Moreover, n_z is the number of items to be picked in zone z, and it follows a binomial distribution as

$$f(n_z) = \binom{N}{n_z}\left(\sum_{i \in R}\sum_{j \in B_z} p_{ijz}\right)^{n_z}\left(1 - \sum_{i \in R}\sum_{j \in B_z} p_{ijz}\right)^{N-n_z}, \text{ for } n_z = 0, 1, 2, \ldots, N. \tag{8}$$

Hence, the expected picking time in zone z is

$$E[\text{Picking time in zone z}] = p_t \times N \times \sum_{i \in R}\sum_{j \in B_z} p_{ijz}. \tag{9}$$

Because of the consideration of both the vertical and horizontal movements of a picker in a zone, the expected travel time within zone z, $\mathrm{E}[T_z]$, can be divided into two parts: (i) the expected travel time between picking bins and (ii) the expected vertical travel time in each picking bins. Hence, $\mathrm{E}[T_z]$ can be presented as.

$$\mathrm{E}[T_z] = \mathrm{E}[T_z^{\mathrm{bin}}] + \mathrm{E}[T_z^{\mathrm{ver}}], z \in Z. \tag{10}$$

Tchebychev travel typically can be performed in the pick-and-pass systems. The expected travel time model between picking bins can be obtained by finding the maximum of the horizontal and vertical travel times to reach next pick location from the current location using Tchebychev travel.

Therefore, the expected travel time between picking bins in zone z given the starting location is s can be expressed as

$$\begin{aligned}&\mathrm{E}[T_z^{\mathrm{bin}} | S_z = s] \\ &= \sum_{b_1=1}^{B-1}\sum_{b_2=b_1+1}^{B}\sum_{r_1=1}^{R}\sum_{r_2=1}^{R} \max\{(b_2 - b_1)D_w/vh_z, |r_1 - r_2|D_h/vv_z\} P\{L(r_1, b_1, z) \text{ to } L(r_2, b_2, z) | n_z\},\end{aligned} \tag{11}$$

where $L(r, b, z)$ denotes that the picking location is at rack r of bin b in zone z and x_{rbz} denotes the number of items to be picked at $L(r, b, z)$. If there are n_z items to be picked in zone z, the conditional probability that a picker will pick at rack r_2 of bin b_2 in zone z after he/she picks at rack r_1 of bin b_1 in zone z, $P\{L(r_1, b_1, z) \text{ to } L(r_2, b_2, z) | n_z\}$, can be stated as

$$\begin{aligned}&P\left\{x_{R1z} \geq 0, x_{(R-1)1z} \geq 0, \ldots, x_{(r_1+1)b_1z} \geq 0, x_{r_1b_1z} > 0, x_{(r_1-1)b_1z} = x_{(r_1-2)b_1z} = \cdots = x_{(r_2+1)b_2z} = 0, x_{r_2b_2z} > 0, x_{(r_2-1)b_2z} \geq 0, x_{(r_2-2)b_2z} \geq 0, \ldots x_{1Bz} \geq 0 | n_z\right\} \\ &= \left(\sum_{i=1}^{R}\sum_{j=1}^{b_1-1} p'_{ijz} + \sum_{i=r_1}^{R} p'_{ib_1z} + \sum_{i=1}^{R}\sum_{j=b_2+1}^{B} p'_{ijz} + \sum_{i=1}^{r_2} p'_{ib_2z}\right)^{n_i} - \left(\sum_{i=1}^{R}\sum_{j=1}^{b_1-1} p'_{ijz} + \sum_{i=r_1+1}^{R} p'_{ib_1z} + \sum_{i=1}^{R}\sum_{j=b_2+1}^{B} p'_{ijz} + \sum_{i=1}^{r_2} p'_{ib_2z}\right)^{n_i} \\ &- \left(\sum_{i=1}^{R}\sum_{j=1}^{b_1-1} p'_{ijz} + \sum_{i=r_1}^{R} p'_{ib_1z} + \sum_{i=1}^{R}\sum_{j=b_2+1}^{B} p'_{ijz} + \sum_{i=1}^{r_2-1} p'_{ib_2z}\right)^{n_i} + \left(\sum_{i=1}^{R}\sum_{j=1}^{b_1-1} p'_{ijz} + \sum_{i=r_1+1}^{R} p'_{ib_1z} + \sum_{i=1}^{R}\sum_{j=b_2+1}^{B} p'_{ijz} + \sum_{i=1}^{r_2-1} p'_{ib_2z}\right)^{n_i}\end{aligned} \tag{12}$$

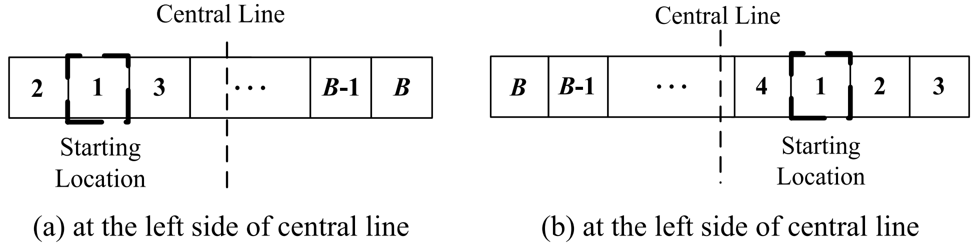


Fig. 3 The numbering of bins for different starting locations. **a** At the left side of central line. **b** At the left side of central line

where $p'_{ijk} = p_{ijz} / \sum_{i \in R} \sum_{j \in B} p_{ijz}$ and $b_2 > b_1$.

Similarly, because nz follows a binomial distribution $B(N, \sum_{i \in R} \sum_{j \in B} p_{ijz})$, the probability $P\{L(r1, b1, z)$ to $L(r2, b2, z)\}$ can be obtained by

$$\sum_{n_z=1}^{N} P\{L(r_1, b_1, a) \text{ to } L(r_2, b_2, b) | n_z\} \binom{N}{n_z} \left(\sum_{i \in R} \sum_{j \in B} p_{ijz} \right)^{n_z} \left(1 - \sum_{i \in R} \sum_{j \in B} p_{ijz} \right)^{N-n_z}$$
$$= \left(1 - \sum_{i=1}^{R} \sum_{j=b_1+1}^{b_2-1} p_{ijz} - \sum_{i=1}^{r_1-1} p_{ib_1z} - \sum_{i=r_2+1}^{R} p_{ib_2z} \right)^N - \left(1 - \sum_{i=1}^{R} \sum_{j=b_1+1}^{b_2-1} p_{ijz} - \sum_{i=1}^{r_1} p_{ib_1z} - \sum_{i=r_2+1}^{R} p_{ib_2z} \right)^N$$
$$- \left(1 - \sum_{i=1}^{R} \sum_{j=b_1+1}^{b_2-1} p_{ijz} - \sum_{i=1}^{r_1-1} p_{ib_1z} - \sum_{i=r_2}^{R} p_{ib_2z} \right)^N + \left(1 - \sum_{i=1}^{R} \sum_{j=b_1+1}^{b_2-1} p_{ijz} - \sum_{i=1}^{r_1} p_{ib_1z} - \sum_{i=r_2}^{R} p_{ib_2z} \right)^N$$

All of the bins in zone z needs to be numbered sequentially based on the starting position for estimating $\mathrm{E}[T_z^{\text{bin}} | S_z = s]$ because of the different directions a picker may pick the items in zone z, as shown in Fig. 3.

To calculate the stationary probability that the picking starts at a certain bin in the zone, let S_t denote the starting position of the tth order in a zone and define $P_{\text{is}} \equiv P\{S_t = i | S_{t1} = s\}$. Accordingly, the way bins are numbered in Fig. 3; the conditional probability can be obtained as

$$P_{\text{is}} = \left((1 - \sum_{j=i+1}^{B_z} \sum_{r=1}^{R} p_{rjz} \right)^N - \left(1 - \sum_{j=i}^{B_z} \sum_{r=1}^{R} p_{rjz} \right)^N \tag{14}$$

In addition, when the number of bins in a zone is odd and the starting bin is at the middle of the zone, the distance between the right side and the staring bin is equal to the distance between the left side and the starting bin; thus, the picker may move toward either the right or the left side. If the probabilities that the picker first goes to the right or left side are same, then it follows that

$$\mathrm{E}[T_z^{\text{bin}} | S_z = s] = \sum_{b_1=1}^{B-1} \sum_{b_2=b_1+1}^{B} \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \max\{(b_2 - b_1) D_w / vh_z, |r_1 - r_2| D_h / vv_z\} \times \frac{1}{2} \left(P^L\{L(r_1, b_1, z) \text{ to } L(r_2, b_2, z)\} + P^R\{L(r_1, b_1, z) \text{ to } L(r_2, b_2, z)\} \right) \tag{15}$$

where P^L and P^R denote the probabilities that the picker first goes to a certain bin at the right and the left side, respectively. Therefore, the probability that the starting position of pickings at bin i in the zone under this circumstance is

$$P_{\text{is}} = \frac{1}{2} \left(P_{\text{is}}^L + P_{\text{is}}^R \right). \tag{16}$$

Let P represent the matrix of one-step transition probabilities P_{ij}, where $i \in B_z$ and $j \in B_z$, and then the long-term proportion of time that a picker will be at bin s at the completion of an order in zone z, π_s, is the unique nonnegative solution of

$$[\pi_1 \cdots \pi_{B_z}] \mathrm{P} = \begin{bmatrix} \pi_1 \\ \vdots \\ \pi_{B_z} \end{bmatrix}, \sum_{s \in B_z} \pi_s = 1. \tag{17}$$

Finally, the expected travel-time between picking bins in each picking bin in zone z can be obtained as

$$\mathrm{E}[T_z^{\text{bin}}] = \mathrm{E}[\mathrm{E}[T_z^{\text{bin}} | S_z = s]] = \sum_{s \in B_z} \mathrm{E}[T_z^{\text{bin}} | S_z = s] \times \pi_s. \tag{18}$$

The expected vertical travel time in each picking bin in zone $z, \mathrm{E}[T_z^{\text{ver}}]$, can be obtained by

Fig. 4 Flow chart of the proposed algorithm

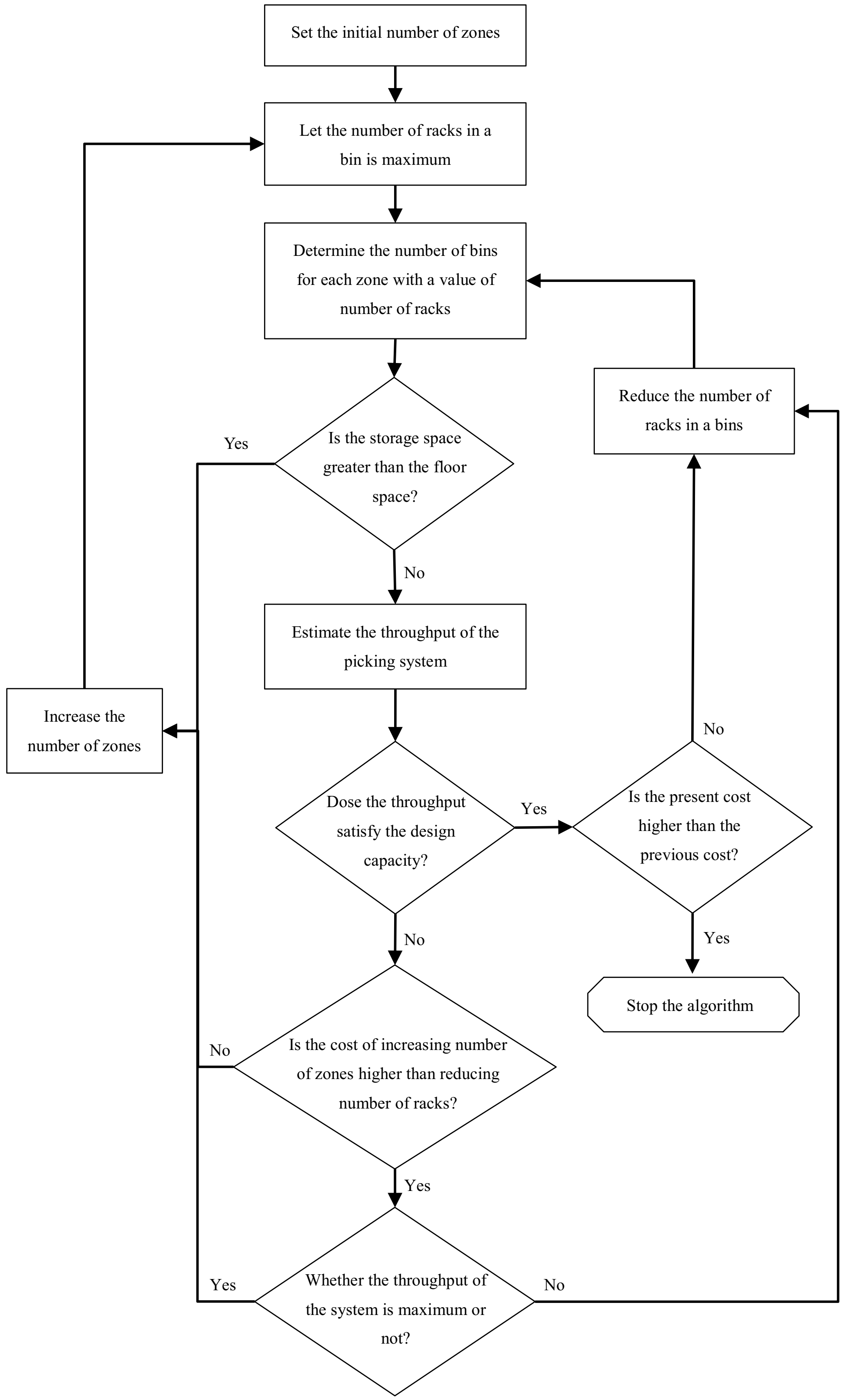
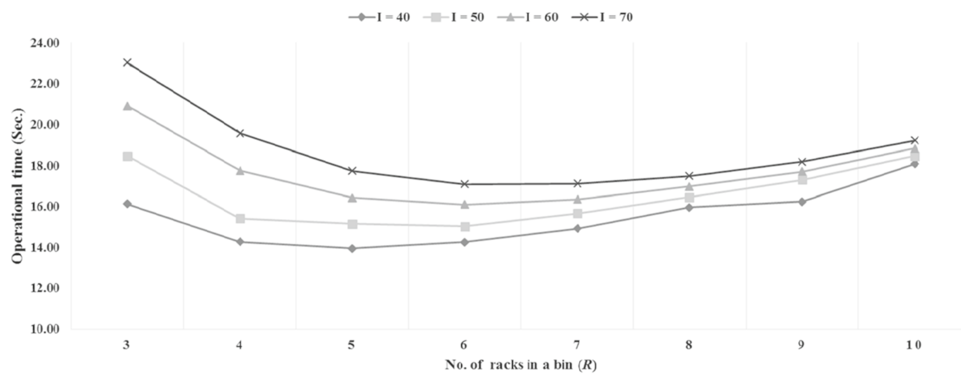
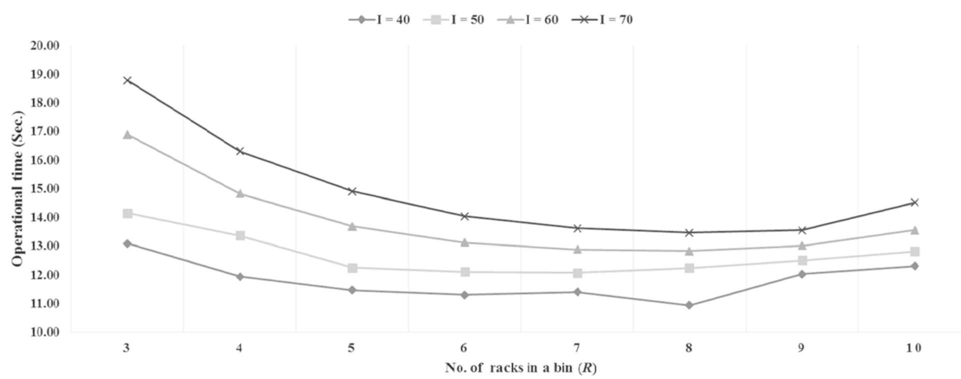


Fig. 5 The operational time for different number of racks and various ratios of D_h/v_h to D_w/v_{wz}



(a) 1:1



(b) 2:1

Table 1 Related parameters for Example 1

Parameter	Value	Parameter	Value
D_h	4 inches	D_w	4 inches
vh_z	4 inches/s	vv_z	4 inches/s
C_m	10$	p_t	1
C_r	1$	W_h	8
C_p	10$	W_d	208
C_s	100$	U	60
L	5	S	150 ft^2
α	1		

$$\mathrm{E}[T_z^{\mathrm{ver}}] = \sum_{b\in B}\sum_{u\in R}\sum_{d<U}(u-d)D_h/vv_z P\{u_{bz}, d_{bz}\} \tag{19}$$

where $P\{u_{bz}, d_{bz}\}$ is the joint probability mass function of the location of the bottom-most or top-most to be picked at picking bin b in zone z. Given n_{bz} items to be picked at picking bins b in zone i, the conditional probability $P\{u_{bz}, d_{bz}n_{bz}\}$ can be expressed as

$$\begin{aligned} &P\{x_{1bz} = x_{2bz} = \ldots = x_{(d-1)bz} = 0, x_{dbz} > 0, x_{(d+1)bz} \geq 0, x_{(d+2)bz} \geq 0, \ldots,\\ &x_{(u-1)bz} \geq 0, x_{ubz} > 0, x_{(u+1)bz} = x_{(u+2)bz} = \cdots = x_{Rbz} = 0 | n_{bz}\}\\ &= \left(\sum_{i=d}^{u} p'_{ibz}\right)^{n_{bz}} - \left(\sum_{i=d}^{u-1} p'_{ibz}\right)^{n_{bz}} - \left(\sum_{i=d+1}^{u} p'_{ibz}\right)^{n_{bz}} + \left(\sum_{i=d+1}^{u-1} p'_{ibz}\right)^{n_{bz}} \end{aligned} \tag{20}$$

where $p'_{ibk} = p_{ibz} / \sum_{i\in R} p_{ibz}$.

Therefore, $P\{u_{bz}, d_{bz}|n_{bz}\}$ can be calculated by

$$\begin{aligned} P\{u_{bz}, d_{bz}\} &= \sum_{n_z=1}^{N} P\{u_{bz}, d_{bz}|n_{bz}\} P\{n_{bz}\}\\ &= \sum_{n_z=1}^{N} P\{u_{bz}, d_{bz}|n_{bz}\} \binom{N}{n_{bz}} \left(\sum_{i=1}^{R} p_{ibz}\right)^{n_z} \left(1 - \sum_{i=1}^{R} p_{ibz}\right)^{N-n_{bz}}\\ &= \left(1 - \sum_{i=u_{bz}+1}^{R} p_{ibz} - \sum_{i=1}^{d_{bz}-1} p_{ibz}\right)^{N} - \left(1 - \sum_{i=u_{bz}}^{R} p_{ibz} - \sum_{i=1}^{d_{bz}-1} p_{ibz}\right)^{N}\\ &\quad - \left(1 - \sum_{i=u_{bz}+1}^{R} p_{ibz} - \sum_{i=1}^{d_{bz}} p_{ibz}\right)^{N} + \left(1 - \sum_{i=u_{bz}}^{R} p_{ibz} - \sum_{i=1}^{d_{bz}} p_{ibz}\right)^{N} \end{aligned} \tag{21}$$

Table 2 The results yielded by the proposed algorithm for Example 1

Layout no	No. of zones (Z)	No. of racks in a bin (R)	No. of bins in a line (B)	Through-put (orders)	Cost			
					Labor	Facilities	Space	Total
1	5	15	35	890	8320	6000	3500	17,820
2	5	14	40	904	8320	6000	4000	18,320
3	5	13	40	967	8320	6000	4000	18,320
4	5	12	45	1003	8320	6000	4500	18,820
5	5	11	50	1153	8320	6000	5000	19,320
6	5	10	50	1043	8320	6000	5000	19,320
7	6	15	36	1084	9984	6000	3600	19,584
8	7	15	35	1239	11,648	6000	3500	21,148
9	7	14	42	1308	11,648	6000	4200	21,848
10	7	13	42	1439	11,648	6000	4200	21,848
11	7	12	43	1428	11,648	6000	4300	21,948
12	8	15	40	1360	13,312	6000	4000	23,312
13	9	15	36	1491	14,976	6000	3600	24,576
14	9	14	37	1499	14,976	6000	3700	24,676
15	9	13	45	1751	14,976	6000	4500	25,476*
16	9	12	45	1853	14,976	6000	4500	25,476*
17	9	11	46	1857	14,976	6000	4600	25,576

*Indicates the minimum cost with corresponding design capacity

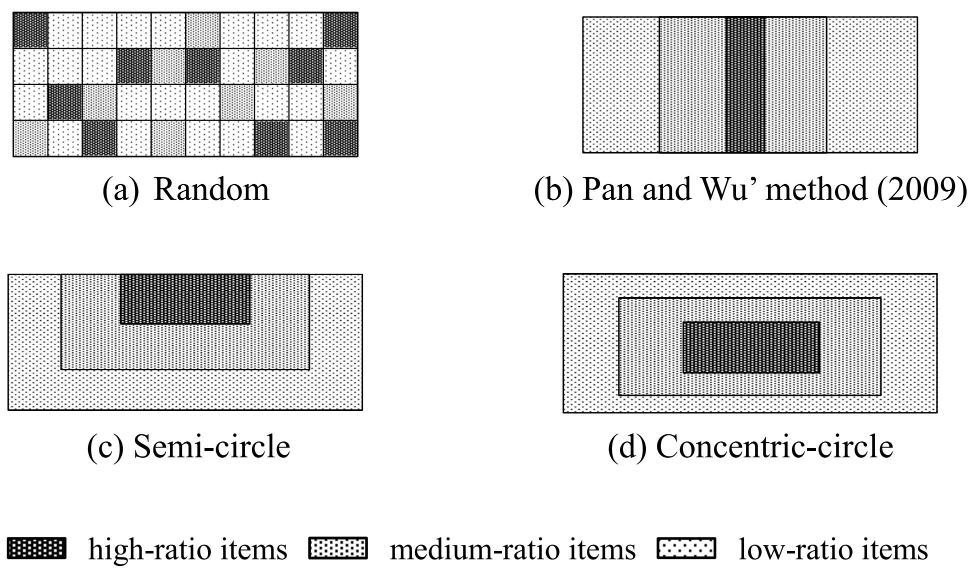
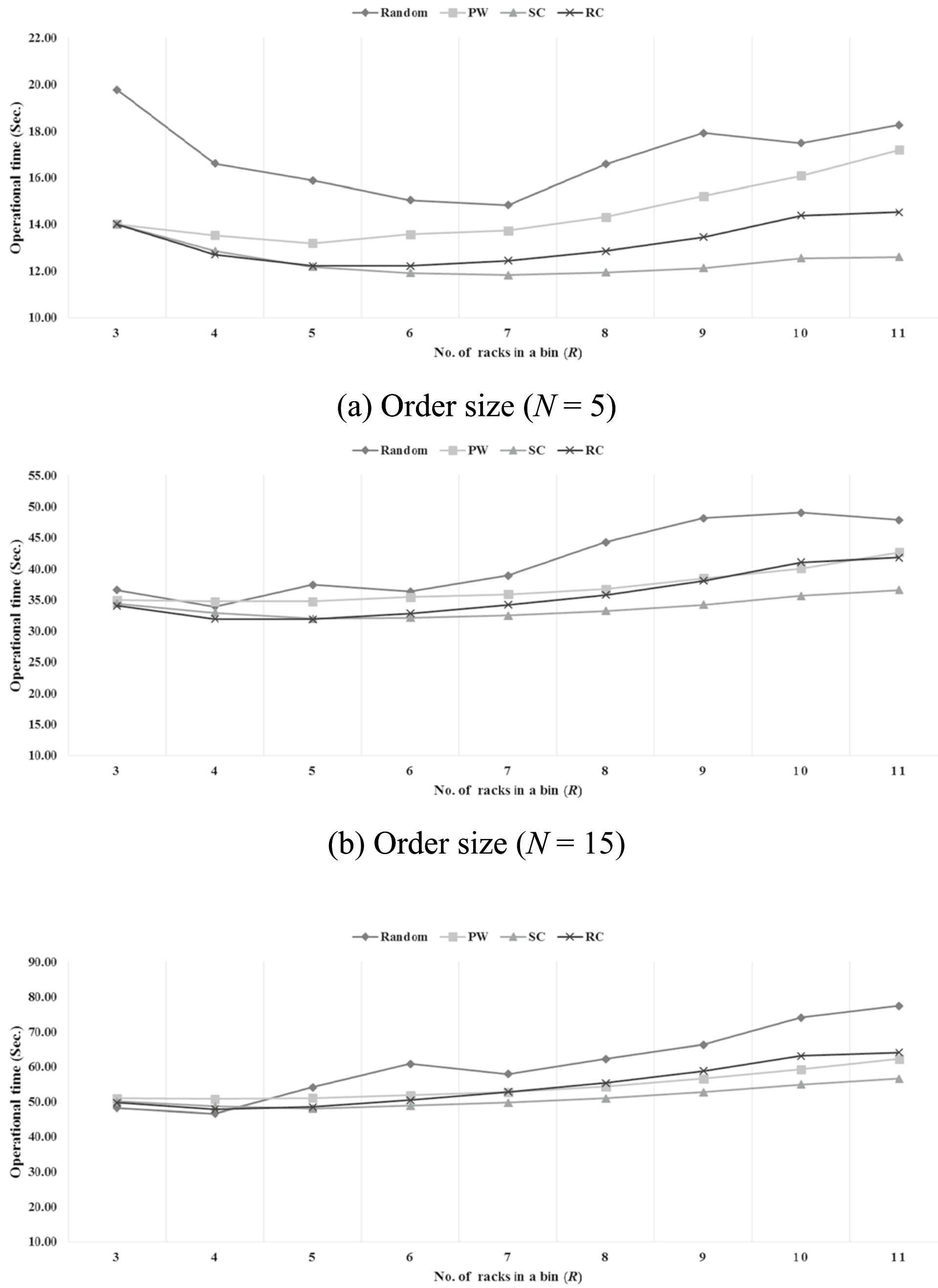


Fig. 6 Illustration of the four different ways to implement the class-based storage policy. **a** Random. **b** Pan and Wu's method [26]. **c** Semi-circle. **d** Concentric circle

4 Optimal layout algorithm and numerical experiments

4.1 Algorithm presentation and case study

Parikh and Meller [16] indicated that the optimal value of storage levels (racks) corresponding to the minimum travel time exists in the high level picker-to-part warehousing systems and there is a negative correlation between the ratio of the vertical travel speed to the horizontal travel speed and the optimal number of storage levels. A pick-and-pass system has a structure similar to a high-level picker-to-part warehousing system with two-dimensional storage architecture; hence, it also possesses an optimal number of racks in a bin. In order to see that this relationship holds, two ratios of the vertical travel rate (D_h/v_h) to the horizontal travel rate (D_w/v_{wz}) with four different numbers of racks in a single zone under a 50/50 demand distribution are investigated. Note that the adoption of a 50/50 demand distribution is to



(a) Order size ($N = 5$)

(b) Order size ($N = 15$)

(c) Order size ($N = 25$)

Fig. 7 The operational time for different number of racks, storage assignment polices and order sizes. **a** Order size ($N=5$). **b** Order size ($N=15$). **c** Order size ($N=25$)

study the issue without the impact of storage assignment polices since it implies that every item has the same demand rate, i.e., a uniform demand rate. As expected, Fig. 5 shows that the association between the operational time and the number of racks in a bin exhibits a convex relationship. This illustrates that the optimal picking zone configuration indeed exists and can be found by the proposed evaluation model. Figure 5 also displays that the optimal number of racks in a bin (R) will increase when the ratio of D_h/v_h to D_w/v_{wz} increases.

In order to solve the underlying mathematical model, an algorithm based on the proposed evaluation model is developed. Figure 4 shows the flow chart of the proposed algorithm. The objective of the mathematical model is to find the optimal configuration of each picking zone with the minimum number of zone that satisfies the customer's needs such that the annual system cost is minimized. Therefore, the initial number of zones is assumed to be one. It can be seen from constraint set (5) that the minimum number of racks in a warehouse is necessarily equal to the number of types

Table 3 The resulting layouts for the four storage assignment polices tested under a 80/20 demand distribution

Storage assignment policy	No. of zones Z	No. of racks in a bin R	No. of bins in a line B	Throughput (orders)	Cost			
					Labor	Facilities	Space	Total
Random	9	10	54	1750	14,976	6000	5400	26,376
PW	5	7	75	1525	8320	6000	7500	21,820
SC*	5	15	35	1550	8320	6000	3500	17,820
CC	6	15	36	1600	9984	6000	3600	19,584

*Indicates the optimal storage assignment policy with corresponding design capacity and the minimum cost

of items. Hence, it is assumed that total I racks are used in a pick-and-pass system such that the annual facility cost of the system with the lowest number of racks is minimized (Fig. 5).

For the purpose of utilizing the warehouse space as efficiently as possible, the initial number of racks in a bin is first set to be $R=U/D_h$ and then adjusted accordingly to satisfy the system throughput. Brynzer and Johansson [33] indicated that the workload in a pick-and-pass system needs to be equally distributed over all the pickers because imbalance can cause serious deterioration of order throughput and order throughput time. Therefore, the total number of racks in a zone is initially determined by $I_z=I/Z$, where $z \in Z$. Since the result of the calculation may not be an integer, the value of I_z is revised as $I_z=I/Z$, where $z=1, 2,\ldots, Z-1$, and the last zone, I_Z, is set to be $I-\sum_{i=1}^{Z-1} I_i$. The number of bins for each zone can be obtained by $B_z=I_z/(Z\times R)$.

After the configuration has been determined, the throughput of the picking system can be estimated accordingly. If the resulting throughput is not enough to meet the design capacity, then the alternative of either increasing number of zones or decreasing number of racks in a bins must be chosen to improve the throughput of the system. If any of the following situations occurs, the addition of a picking zone, i.e., a picker, is required in the system to comply with the design capacity:

(1) The cost of increasing number of bins is higher than that of adding number of zones. In other words, annual cost of storage space is greater than annual labor cost.
(2) No more floor space available in the warehouse to be used to increase the number of bins.
(3) The current configuration already yields the maximum throughput of the system.

After a zone is added, the number of racks in a bin is set to be $R=U/D_h$ and the number of bins for each zone must be recalculated. Otherwise, each bin is decreased by a rack, and the required number of bins for each zone is recalculated.

The steps are performed repeatedly until the system throughput satisfies the design capacity. The algorithmic procedure can be stated as follows.

Step 1 Set $Z=1$, $\omega^*=\infty$ and $C^*=\infty$.
Step 2 Set the initial number of racks in a bin as $R=U/D_h$.
Step 3 Calculate the total number of racks in zone z by $I_z=I/Z$, for $z=1, 2, \ldots, Z-1$, and $I_Z=I-\sum_{i=1}^{Z-1} I_i$
Step 4 Calculate the number of bins in zone z by $B_z=I_z/(Z\times R)$, for $z=1, 2, \ldots, Z$.
Step 5 If $\sum_{z\in Z} B_z \times D_w \leq S$, then go to Step 6; else, $Z=Z+1$ and go to Step 2
Step 6 Calculate the cycle time, $\omega[Z, R, \{B_z z \in Z\}]$.
Step 7 If $Wh/\omega_{[Z, R, \{B_z|_z \in Z\}]} \geq P$, then go to Step 11; else, go to Step 8
Step 8 Let $Z^*=Z+1$, $R_1=U/D_h$, $R_2=R-1$, $C_1=C(Z^*, R_1)$ and $C_2=C(Z, R_2)$
Step 9 If $C_1 < C_2$, then $Z=Z+1$ and go to Step 2; else, go to Step 10.
Step 10 If $\omega[Z, R, \{B_z z\in Z\}] \leq \omega^*$, then let $\omega^*=\omega[Z, R, \{B_z z\in Z\}]$, $C^*=C(Z, R)$, $R=R-1$ and go to Step 4; else, let $Z=Z+1$ and go to Step 2.
Step 11 If $C^* > C(Z, R)$, Let $C^*=C(Z, R)$ and $R=R-1$, then go to step 9; else, stop.

Example 1. Consider a pick-and-pass system having 500 items with a 50/50 demand distribution. The related data are listed in Table 1. The system has a design capacity of 1500 orders with 30 items each per day. Table 2 lists the results yielded by the proposed algorithm for the case of five picking zones. The minimum cost with the corresponding design capacity is $25,476. Layouts 15 and 16 have the same minimum cost; but the throughput of 16 is slightly greater than that of 15. Consequently, layout 16 is chosen to be the optimal layout.

4.2 Sensitivity analysis

Further analysis was conducted to study the sensitivity of the results to various storage assignment policies and levels of

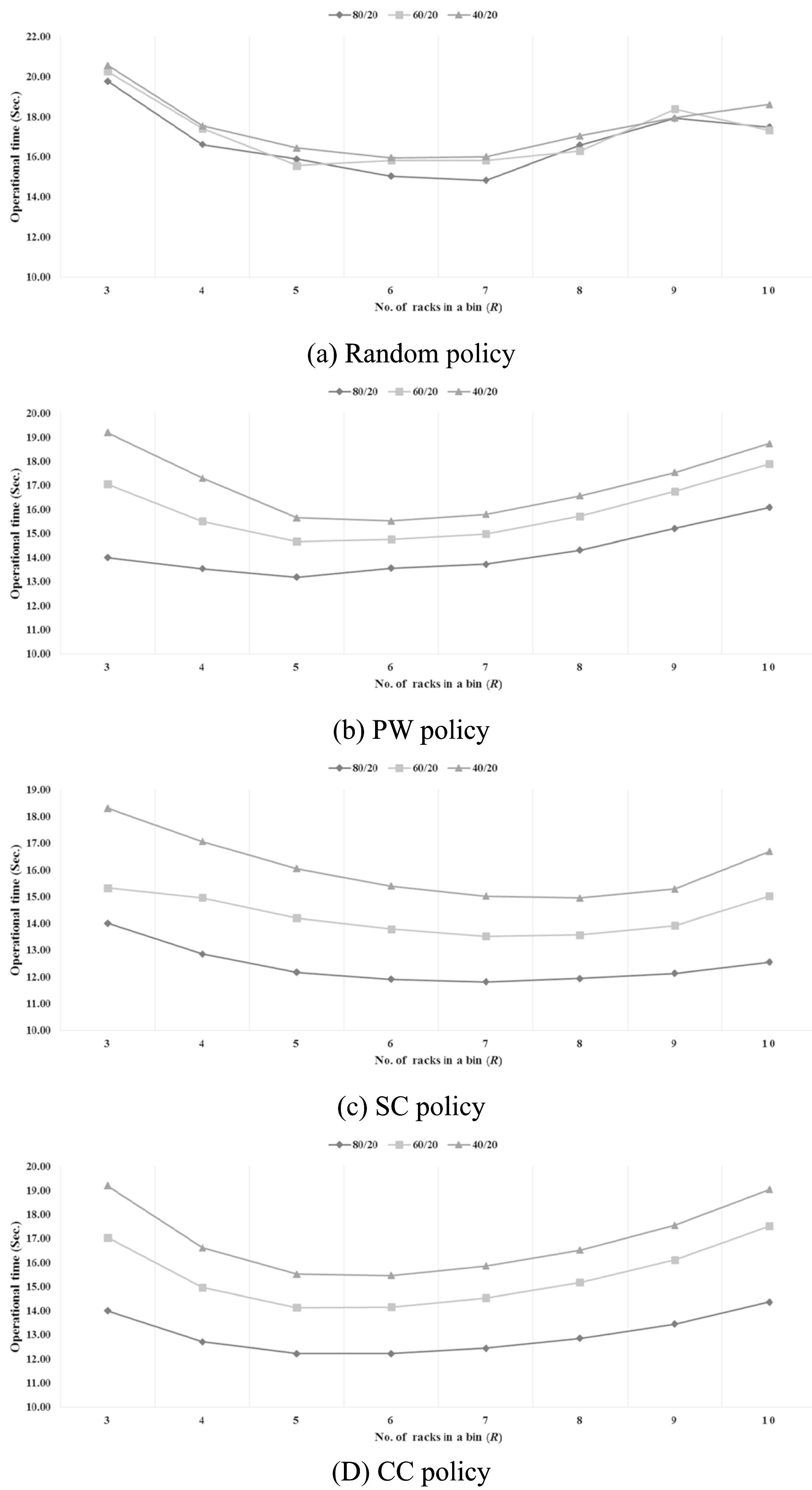


Fig. 8 The operational time for different number of racks, storage assignment polices, and demand distributions. **a** Random policy. **b** PW policy. **c** SC policy. **d** CC policy

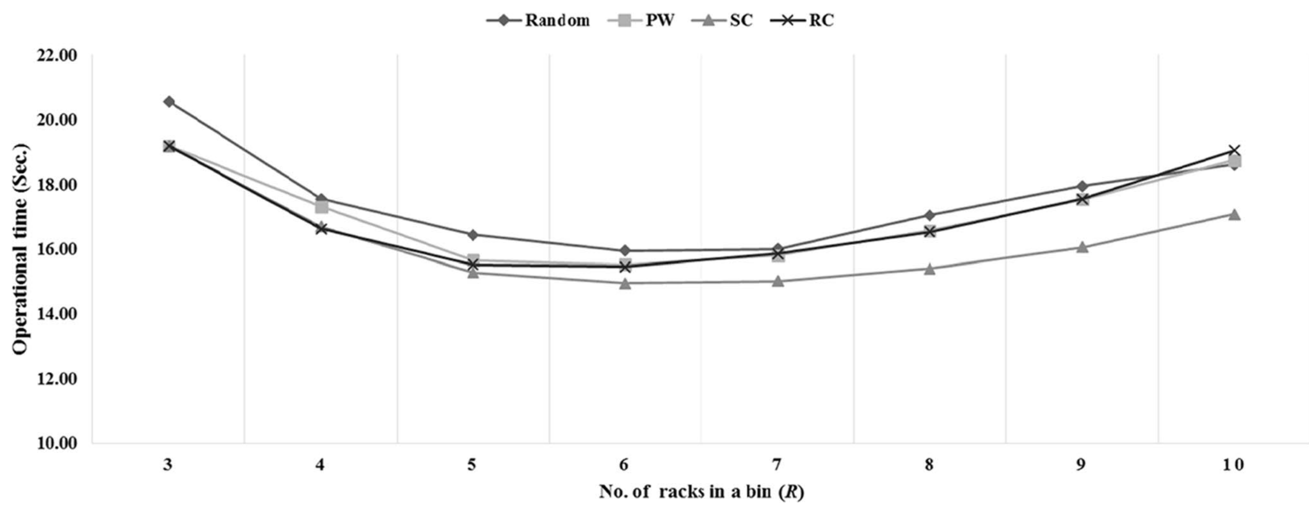


Fig. 9 The operational time for different number of racks and storage assignment polices under a 40/20 demand distribution

Table 4 The resulting layouts for the four storage assignment polices tested under a 40/20 demand distribution

Storage assignment policy	No. of zones Z	No. of racks in a bin R	No. of bins in a line B	Through-put (orders)	Cost			
					Labor	Facilities	Space	Total
Random	7	12	43	1516	11,648	6000	4300*	21,948
PW	7	15	35	1525	11,648	6000	3500	21,148
SC*	6	11	48	1554	9984*	6000	4800	20,784*
CC	6	9	60	1560	9984	6000	6000	21,984

*Indicates the optimal storage assignment policy with corresponding design capacity and the minimum cost

input parameters including the order sizes (N) and demand distributions.

Storage assignment policies attempt to provide an effective way of locating products in a warehouse in order to improve the operational efficiency of order picking, and it influences almost all key performance indicators of a warehouse such as order picking time and cost [34]. Thus, this paper further examines the impact of the following four storage assignment policies based on the picking frequency of items and these policies are described below and illustrated in Fig. 6.

(a) Random policy.

The random storage policy is widely used in many warehouses because it is simple to use, often requires less space than other storage methods, and results in a better level utilization of all picking aisles [35].

(b) Pan and Wu's assignment algorithm (PW).

Pan and Wu [26] presented an algorithm considering only the horizontal movement of a picker to assign items to bins in a zone. This algorithm places the first R highest demand items in the bin located in the middle of the zone the second R highest demand items in a bin next to the bin in the middle and so on, so that the items with the lowest demands are placed in the farthest bin.

(c) Semi-circle (SC).

The top level rack of the bin located at the middle of the zone is considered as the center point. The higher demand frequency item is placed closer to the center point; hence as a result, the items stored in the zone formed a semi-circle pattern.

(d) Concentric-circle (CC).

This method is similar to the previous one, but the center point is changed to be the rack located at the center of the whole shelve instead.

Example 2. Consider a single picking zone in a pick-and-pass system which has 60 items with an 80/20 demand distribution. The ratio of the vertical travel rate (D_h/v_h) to the horizontal travel rate (D_w/v_{wz}) in the zone is 1:1. Figure 7 illustrates the effects of storage assignment polices, order sizes, and the number of racks in a bin. The following points can be observed:

(1) Except for random policy, Fig. 7 demonstrates that respective optimal layout can be found for the other three policies due to the presence of the convex relationship between the operational time and the number of racks in a bin.
(2) While PW is the optimal policy when only vertical travel time is considered [26], SC is the best in most cases among the ones tested if both horizontal and vertical travel times are included.
(3) With the increase of order sizes, the differences on the operational times among these polices are less significant because the majority of racks may be visited. That is, the storage assignment policies do not affect as much the performance of the pick-and-pass system when order size is large.

Example 3. Re-do Example 1 for an order picking line with 80/20 demand distribution for the four storage assignment policies. The minimum cost is $17,820, and an optimal layout is depicted in Table 3. SC policy provides a qualified throughput capacity with the minimum number of zones and bins as expected.

To study the impact of demand distributions, a sensitivity analysis was performed to evaluate 60/20 and 40/20 demand distributions of the items in addition to the 80/20 demand distribution, and the results are shown in Fig. 8. The convex relationship still exists between the operational time and the number of racks in a bin for each scenario. Figure 8 also indicates that the 40/20 demand distribution yields the longest operational time for any storage assignment policy because of the scattered locations of items to be picked. When the picking frequency of each item is close, no storage policy exhibits significant performance advantage as displayed in Fig. 9. Table 4 shows the results of the algorithm implemented for the four storage assignment polices under 40/20 demand distribution. The SC policy still yields the minimum cost of the four.

5 Conclusions

This paper presents a performance evaluation model to estimate the cycle time of a picking line by considering both the horizontal and vertical travel distance of a picker in a zone. Based on the proposed performance evaluation method, a layout algorithm is developed to determine the optimal number of picking zones, bins, and racks in a bin for finding the minimum operational cost with corresponding design capacity. The effects of the storage assignment policies and system parameters such as the order sizes, the number of items, and demand distribution are examined in several scenarios. Results indicate that an optimal layout can be found by the proposed algorithm according to the characteristics of each storage assignment policy and picking environment.

Finally, due to the fact that in the real world, it is very difficult to estimate the actual operating cost accurately. Therefore, this paper assumes that the picking line under study is perfectly smooth with no delay between two consecutive zones [26, 36]. However, congestion may occur in reality, and hence, there is a natural extension to put the possible delay and waiting times into consideration between pickers for future study.

Author contribution Conceptualization: JCHP Formal analysis: MFY, P-HS Investigation: M-FY, P-HS, P-HS Methodology: M-FY Project administration: JC-HP. Software:P-HS Supervision: JC-HP Writing—original draft: M-FY and M-CL Writing—review and editing: JC-HP

Data availability Not applicable.

Declarations

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Consent to participate Not applicable.

Consent for publication Not applicable.

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