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ORIGINAL ARTICLE

A generic method for multi-criterion decision-making problems in design for additive manufacturing

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Abstract
In this paper, a generic method based on fuzzy aggregation operator for multi-criterion decision-making problems in design for additive manufacturing is proposed. Firstly, a fuzzy power weighted Maclaurin symmetric mean operator based on Hamacher T-norm and T-conorm is constructed via a combination of fuzzy numbers, power average operator, weights, Maclaurin symmetric mean operator, and operational rules of fuzzy numbers based on Hamacher T-norm and T-conorm. Based on the constructed operator, a generic method for solving the multi-criterion decision-making problems in design for additive manufacturing is then developed. After that, an example of additive manufacturing machine and material selection and an example of optimal build direction selection are introduced to illustrate the developed method. Finally, a set of numerical experiments are reported to demonstrate the effectiveness and capabilities of the method. The demonstration results suggest that the method can effectively solve a multi-criterion decision-making problem in design for additive manufacturing and has the characteristics in considering the interactions of criteria, reducing the effect of noise criterion values, and capturing the risk attitude of decision-makers.

Keywords Design for additive manufacturing · Additive manufacturing machine · Additive manufacturing material · Build direction · Multi-criterion decision-making · Fuzzy aggregation operator

1 Introduction

Multi-criterion decision-making (MCDM) problems, also known as multi-attribute decision-making problems, refer to a set of problems in which the preference decisions are made via evaluating and sorting a limited number of alternatives on the basis of multiple criteria. In design for conventional manufacturing, there are many problems belonging to MCDM problems [1–8]. This is also the case in design for additive manufacturing (AM). Design for AM is an activity of designing an AM part and determining appropriate process variables to build the part, in which the quality and other key factors such as property, cost, and manufacturability of the part are optimised simultaneously subjected to the capabilities of the used AM technique [9–12]. This activity includes a set of successive tasks, which are conceptual design, detailed design, build direction determination, support structure generation, slicing, and path planning.

Conceptual design is the very first task in design for AM. It serves to give a description of the outline of functionality and the form of an AM part [13]. Detailed design is the task in which the design is refined. It mainly involves the design of the geometry and specifications of an AM part and the selection of an AM machine and certain materials to build the part [10]. Build direction determination, support structure generation, slicing, and path planning are four process planning tasks [14–16], which aim to design proper direction, support structure, slices, and tool path and process parameters, respectively, to build an AM part.

In these successive tasks, there are a number of problems that can be regarded as MCDM problems. As one example, in the detailed design of an AM part, a designer needs

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to select an AM machine and certain materials from a certain number of available AM machines and materials based on certain multiple considerations, such as material property, part quality, part property, surface quality, build time, and part cost [17]. An AM machine or material selection problem like this is a typical MCDM problem. As another example, in the build direction determination of an AM part, a designer needs to determine a proper build direction from an infinite number of theoretical directions or a certain number of alternative directions on the basis of certain multiple factors, such as part property, part accuracy, surface quality, support structure, manufacturing time, and manufacturing cost [14]. A build direction determination problem also belongs to an MCDM problem.

Since the problems above are essentially MCDM problems, they can naturally be solved via MCDM methods. In the literature, a number of researchers either developed some specialised MCDM methods or applied some existing MCDM methods to address the problems. To be specific, aiming at the AM machine or material selection problem, [18–25] introduced the analytic hierarchy process (AHP); [26] and [27] applied the technique of knowledge value measuring; [28–30] used the technique for order of preference by similarity to ideal solution (TOPSIS); [31] presented an MCDM method based on graph theory and matrix; [32] proposed an improved preference ranking organisation method for enrichment of evaluations; [33] applied the multi-objective optimisation method by ratio analysis; [34] and [35] respectively introduced the theories of fuzzy logic and fuzzy set; [36] developed an MCDM method based on grey relational analysis; [37] constructed a specialised ranking model; [38] established an integrated MCDM model based on deviation and similarity; [39] presented a fuzzy axiomatic design method based on the rough set theory; [40] and [41] adopted a combined AHP-TOPSIS method; [42] applied the best-worst method; [43] presented an MCDM method based on a linear combination of a fuzzy power weighted Bonferroni mean (FPWBM) operator and a fuzzy power weighted geometric Bonferroni mean (FPWGBM) operator. To slove the build direction determination problem, [44] and [45] determined the alternative directions via a set of feature-based rules and selected the optimal direction via an MCDM method based on the weighted average (WA) operator; [46] developed a decision support system where the MCDM module was implemented using the WA operator; [47] generated the alternative directions via a feature-based approach and selected the best direction using an MCDM method based on deviation function; [48, 49] determined the alternative directions according to the surfaces of convex hull and selected the most desirable direction via an MCDM method based on the WA operator; [50] introduced the concept of AM feature and determined the alternative directions and the optimal direction using an AM feature-based approach and the integrated MCDM model in [38], respectively; [51] determined the alternative directions via quaternion rotation and selected the best direction using an MCDM method based on the ordered weighted average operator; [52] presented an MCDM method based on a fuzzy power weighted partitioned Muirhead mean (FPWPMM) operator and a fuzzy power weighted prioritised average (FPWPA) operator; [53] determined the alternative directions through quaternion rotation and selected the optimal direction via a negative feedback MCDM model; [54] and [55, 56] generated the alternative directions via facet clustering and selected the best direction using the WA operator.

In practical MCDM problems in design for AM, the considered multiple criteria are usually interacted with each other. For example, the material property may affect the part quality, part property, and surface quality and the build time may influence the part cost in the AM machine or material selection problem; The surface quality may be affected by the support structure and the manufacturing cost may be influenced by the support structure and manufacturing time in the build direction determination problem. To produce reasonable decision-making results in this case, the used MCDM methods should be generic and flexible enough to capture the interactions of the considered criteria [57]. Further, the values of the considered criteria are generally obtained from evaluation of domain experts, estimation of theoretical models, or prediction of simulations or experiments. These approaches may produce a few extreme values. To obtain consistent decision-making results under this situation, the used MCDM methods should also have the capability to reduce the influence of noise criterion values [58]. Apart from the interactions of criteria and the noise of criterion values, the risk attitude of decision-makers is also a critical factor that should be taken into consideration in the used MCDM methods, as different risk attitudes could affect the decision-making results significantly [59].

Among the MCDM methods reviewed above, the methods of [31] and [39] express the interactions among criteria via digraph and matrix. The method of [30] represents them by regression coefficients. The methods of [43] capture the interactions between any two criteria using the FPWBM and FPWGBM operators. The method of [52] capture the interactions between any two criteria or among any multiple criteria using the FPWPMM and FPWPA operators. In addition to these methods, the remaining methods assume that all criteria are independent of each other. That is, they do not consider the interactions of criteria. Further, apart from the methods of [43, 52], the remaining methods do not take into account the reduction of the effect of noise criterion values on the decision-making results. As for the risk attitude of decision-makers, only the method of [43] takes it into consideration. To sum up, the methods of [43, 52] could be more desirable in terms of the consideration of the interactions of criteria, the noise of criterion values, and the risk attitude of decision-makers. However, the method of [43] can only capture the interactions between any two criteria and cannot express the interactions among more than two criteria. The method of [52] cannot capture the risk attitude of decision-makers.

In this paper, a generic method to solve the MCDM problems in design for AM is proposed. This method is based on a fuzzy power weighted Maclaurin symmetric mean (FPWMSM) operator based on Hamacher T-norm and T-conorm (HTT), which is constructed via a combination of fuzzy numbers (FNs) [60], power average (PA) operator [61], weights, Maclaurin symmetric mean (MSM) operator [62], and operational rules (ORs) of FNs based on HTT. The FNs are effective mathematical tools for quantifying the values of different criteria in a unified range. The PA operator is a function for aggregating two or more positive numbers to obtain a single summary positive number. It has the capability to reduce the influence of those extreme positive numbers to be aggregated on the aggregation results. The weights are general means for quantifying the degree of importance of criteria. The MSM operator is also a function for aggregating two or more positive numbers to obtain a single summary positive number. It can produce consistent aggregation results when all of the positive numbers to be aggregated are independent, when any two of them are interacted, and when any multiple of them are interacted. The ORs of FNs based on HTT are a set of rules for performing the operations between two FNs and the operations between an FN and a positive number. They were found to have the capability to capture different risk attitudes in MCDM [63]. Benefiting from the combination, the proposed method can effectively solve an MCDM problem where all criteria are independent, any two criteria are interacted, or any multiple criteria are interacted. It also has the capabilities to reduce the effect of extreme criterion values and to capture the risk attitude of decision-makers.

The remainder of the paper is organised as follows. Section 2 gives a brief introduction of some basic concepts involved in the construction of the FPWMSM operator based on HTT. Section 3 explains the details of the proposed method. Two MCDM examples in design for AM are reported to illustrate the application of the method in Section 4. A set of experiments for demonstrating the effectiveness and capabilities of the method are documented in Section 5. Section 6 ends the paper with a conclusion.

2 Preliminaries

In this section, some basic concepts involved in the construction of an FPWMSM operator based on HTT are introduced.

2.1 Fuzzy set theory

The concept of fuzzy sets was introduced by Zadeh [60]. A fuzzy set is somewhat like a set whose elements have degrees of membership. Its definition is as follow:

Definition 1 A fuzzy set A over a universe of discourse X is defined as $\{\langle x,\ \mu(x)\rangle \mid x \in X\}$, where $\mu : X \to [0,\ 1]$ is a membership function whose value indicates the degree to which x belongs to A.

In general, the value of the membership function $\mu(x)$ (denoted as μ) is called a degree of membership or an FN. An FN β is usually denoted as $\langle\mu\rangle$. Any two FNs can be compared by the following rule:

Definition 2 Let $\beta_1 = \langle\mu_1\rangle$ and $\beta_2 = \langle\mu_2\rangle$ be two arbitrary FNs. Then: if $\mu_1 < \mu_2$, then $\beta_1 < \beta_2$; if $\mu_1 = \mu_2$, then $\beta_1 = \beta_2$; if $\mu_1 > \mu_2$, then $\beta_1 > \beta_2$.

The distance between any two FNs can be calculated using an Euclidean distance measure of FNs:

Definition 3 Let $\beta_1 = \langle\mu_1\rangle$ and $\beta_2 = \langle\mu_2\rangle$ be two arbitrary FNs. Then the Euclidean distance between β_1 and β_2 is given by

$$d(\beta_1, \beta_2) = \sqrt{(\mu_1 - \mu_2)^2} \tag{1}$$

2.2 ORs of FNs based on HTT

HTT is a type of Archimedean t-norm and t-conorm. It has a flexible parameter that can be used to capture different risk attitudes in MCDM [63]. ORs of FNs are rules for performing the operations between two FNs and the operations between an FN and a positive number. The ORs of FNs based on HTT, which can be regarded as the special cases of the ORs of intuitionistic fuzzy numbers based on HTT in [64] and the ORs of picture fuzzy numbers based on HTT in [65], are defined as follows:

Definition 4 Let $\beta_1 = \langle\mu_1\rangle$, $\beta_2 = \langle\mu_2\rangle$, and $\beta = \langle\mu\rangle$ be three arbitrary FNs and λ and δ be two arbitrary positive numbers. Then the sum operation of β_1 and β_2, the product operation of β_1 and β_2, the operation of β multiplied by λ, and the operation of β to the power of λ can be respectively performed using

$$\beta_1 \oplus \beta_2 = \left\langle \frac{\mu_1 + \mu_2 - \mu_1\mu_2 - (1-\delta)\mu_1\mu_2}{1-(1-\delta)\mu_1\mu_2} \right\rangle \tag{2}$$

$$\beta_1 \otimes \beta_2 = \left\langle \frac{\mu_1\mu_2}{\delta + (1-\delta)(\mu_1 + \mu_2 - \mu_1\mu_2)} \right\rangle \tag{3}$$

$$\lambda\beta = \left\langle \frac{(1+(\delta-1)\mu)^\lambda - (1-\mu)^\lambda}{(1+(\delta-1)\mu)^\lambda + (\delta-1)(1-\mu)^\lambda} \right\rangle \tag{4}$$

$$\beta^\lambda = \left\langle \frac{\delta\mu^\lambda}{(\delta-1)\mu^\lambda + (1+(\delta-1)(1-\mu))^\lambda} \right\rangle \tag{5}$$

2.3 PA operator

The PA operator was introduced by Yager [61]. It has the capability to reduce the effect of those unduly large or unduly small positive numbers to be aggregated on the aggregation results. This capability is achieved via assigning dynamic weights to the positive numbers to be aggregated according to the degrees of support between them. The definition of the PA operator is as follow:

Definition 5 Let $y_1, y_2, ..., y_n$ be n positive numbers to be aggregated, $d(y_i, y_j)$ $(i, j = 1, 2, ..., n$ and $j \neq i)$ be the distance between y_i and y_j, and $s(y_i, y_j) = 1 - d(y_i, y_j)$ be the degree of support for y_i from y_j that satisfies the following conditions: $0 \leq s(y_i, y_j) \leq 1$; $s(y_i, y_j) = s(y_j, y_i)$; if $|y_i - y_j| < |y_{i'} - y_{j'}|$, then $s(y_i, y_j) \geq s(y_{i'}, y_{j'})$. Then the PA operator is given by

$$PA(y_1, y_2, ..., y_n) = \frac{\sum_{i=1}^{n} \left(1 + \sum_{j=1, j\neq i}^{n} s(y_i, y_j)\right) y_i}{\sum_{i=1}^{n} \left(1 + \sum_{j=1, j\neq i}^{n} s(y_i, y_j)\right)} \tag{6}$$

2.4 MSM operator

The MSM operator was introduced by Maclaurin [62]. It is an all-in-one aggregation operator for capturing the interactions of the positive numbers to be aggregated, since it can generate consistent aggregation results when all of the positive numbers to be aggregated are independent of each other, when any two of them have interactions, and when any multiple of them have interactions. The definition of the MSM operator is as follow:

Definition 6 Let $y_1, y_2, ..., y_n$ be n positive numbers to be aggregated and $k = 1, 2, ..., n$. If $(i_1, i_2, ..., i_k)$ traverse all of the k-tuple combinations of $(1, 2, ..., n)$, then the MSM operator is given by

$$MSM^{(k)}(y_1, y_2, ..., y_n) = \left(\frac{k!(n-k)!}{n!} \sum_{1 \leq i_1 < i_2 < ... < i_k \leq n} \prod_{j=1}^{k} y_{i_j} \right)^{\frac{1}{k}} \tag{7}$$

3 Generic MCDM method

In this section, a generic method for solving the MCDM problems in design for AM is presented. The general flow of this method is shown in Fig. 1. The presented method takes as input a decision matrix for a set of alternatives and a set of criteria of alternatives and the weights of criteria. It outputs a sequence of alternatives together with the best alternative. Firstly, the decision matrix is transformed into a fuzzy decision matrix via a ratio model. The fuzzy decision matrix is then normalised and a normalised fuzzy decision matrix is obtained. After that, a constructed FPWMSM operator based on HTT is applied to aggregate the fuzzy information in the obtained matrix. Finally, a sequence of alternatives is generated via sorting the input alternatives according to the aggregation results, and the best alternative is determined from the generated sequence of alternatives.

The present section first explains the details of the constructed FPWMSM operator based on HTT. It then describes the specific process of the presented method.

3.1 FPWMSM operator based on HTT

An FPWMSM operator based on HTT is a function for grouping together one or more FNs to obtain a single summary FN. This operator is established via combining the MSM and PA operators of FNs with weights, in which the operations are performed via the ORs of FNs based on HTT in Definition 4. It can be regarded as a special case of the picture fuzzy power weighted MSM operator based on HTT in [65]. The definition of the FPWMSM operator based on HTT is as follow:

Definition 7 Let $\beta_1 = \langle\mu_1\rangle, \beta_2 = \langle\mu_2\rangle, ..., \beta_n = \langle\mu_n\rangle$ be n FNs to be aggregated, $w_1, w_2, ..., w_n$ be respectively the weights of $\beta_1, \beta_2, ..., \beta_n$ such that $0 \leq w_1, w_2, ..., w_n \leq 1$ and $w_1 + w_2 + ... + w_n = 1$, $k = 1, 2, ..., n$, $d(\beta_i, \beta_j)$ $(i, j = 1, 2, ..., n$ and $j \neq i)$ be the distance between β_i and β_j, and $s(\beta_i, \beta_j) = 1 - d(\beta_i, \beta_j)$ be the degree of support for β_i from β_j that satisfies the following conditions: $0 \leq s(\beta_i, \beta_j) \leq 1$; $s(\beta_i, \beta_j) = s(\beta_j, \beta_i)$; if $d(\beta_i, \beta_j) < d(\beta_{i'}, \beta_{j'})$, then $s(\beta_i, \beta_j) \geq s(\beta_{i'}, \beta_{j'})$. If $(i_1, i_2, ..., i_k)$

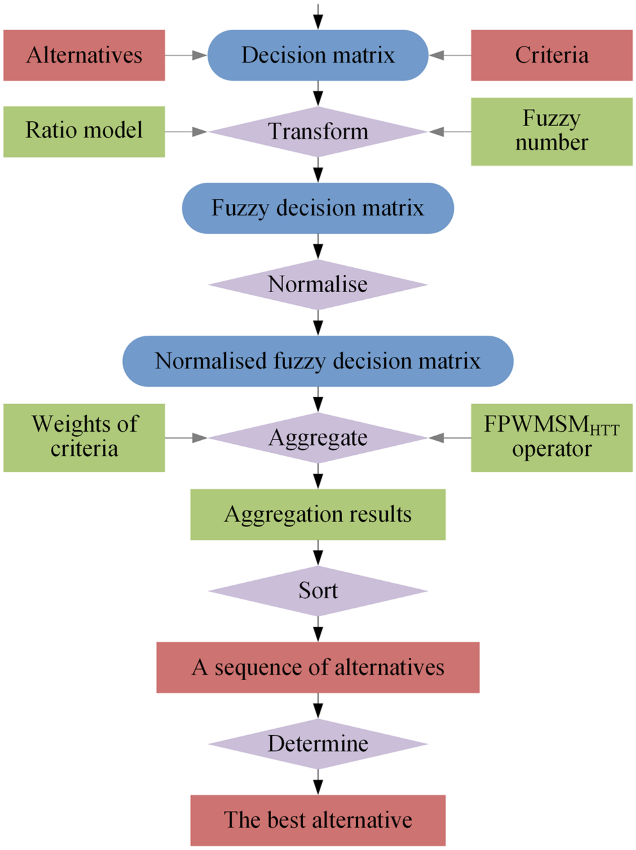

Fig. 1 General flow of the proposed generic MCDM method

traverse all of the k-tuple combinations of $(1, 2, ..., n)$, then the FPWMSM operator based on HTT is given by

$$FPWMSM^{(k)}_{\text{HTT}}(\beta_1, \beta_2, ..., \beta_n) = \left(\frac{k!(n-k)!}{n!} \bigoplus_{1 \le i_1 < i_2 < ... < i_k \le n} \bigotimes_{j=1}^{k} \left(W_{i_j} \beta_{i_j} \right) \right)^{\frac{1}{k}} \quad (8)$$

where

$$W_{i_j} = \frac{n w_{i_j} \left(1 + \sum\limits_{q=1, q \ne i_j}^{n} s(\beta_{i_j}, \beta_q) \right)}{\sum\limits_{p=1}^{n} w_p \left(1 + \sum\limits_{q=1, q \ne p}^{n} s(\beta_p, \beta_q) \right)} \quad (9)$$

and all of the operations related to FNs are performed using the ORs of FNs based on HTT in Definition 4.

Equation (8) is an implicit expression of the FPWMSM operator based on HTT. If the ORs of FNs based on HTT in Definition 4 are applied to this expression, then an explicit expression of the operator can be derived:

Definition 8 The explicit expression of the FPWMSM operator based on HTT is given by

$$FPWMSM^{(k)}_{\text{HTT}}(\beta_1, \beta_2, ..., \beta_n) = \left\langle \frac{\delta(\mu'' - 1)^{\frac{1}{k}}}{(\mu'' + \delta^2 - 1)^{\frac{1}{k}} + (\delta - 1)(\mu'' - 1)^{\frac{1}{k}}} \right\rangle \quad (10)$$

where

$$\mu'' = \prod_{1 \le i_1 < i_2 < ... < i_k \le n} \left(\frac{\mu' + \delta^2 - 1}{\mu' - 1} \right)^{\frac{k!(n-k)!}{n!}} \quad (11)$$

$$\mu' = \prod_{j=1}^{k} \frac{\left(\delta + (1-\delta)(1-\mu_{i_j})\right)^{W_{i_j}} + (\delta^2 - 1)(1-\mu_{i_j})^{W_{i_j}}}{\left(\delta + (1-\delta)(1-\mu_{i_j})\right)^{W_{i_j}} - (1-\mu_{i_j})^{W_{i_j}}} \quad (12)$$

There are three important parameters in the FPWMSM operator based on HTT. They are k, δ, and W_{i_j}. The parameter k determines how the interactions of $\beta_1, \beta_2, ..., \beta_n$ are considered in the operator: If $k = 1$, then the interactions are not considered (i.e. $\beta_1, \beta_2, ..., \beta_n$ are independent of each other); If $k = 2$, then the interactions between any two of $\beta_1, \beta_2, ..., \beta_n$ are considered; If $k = 3, 4, ..., n$, then the interactions among any multiple $(3, 4, ..., n)$ of $\beta_1, \beta_2, ..., \beta_n$ are considered. The parameter δ reflects the risk attitude captured in the operator. The larger the value of this parameter, the more pessimistic the aggregation expectation. The parameter W_{i_j} is the dynamic weight assigned to β_{i_j}. Its value is calculated via the degrees of support between β_{i_j} and the remaining FNs to be aggregated. Because of the use of this parameter, the operator has the capability to reduce the influence of those unduly large or unduly small FNs to be aggregated on the aggregation results.

3.2 Specific process of the method

The basic components of an MCDM problem in design for AM include m alternatives $A_1, A_2, ..., A_m$, n criteria $C_1, C_2, ..., C_n$, n weights $w_1, w_2, ..., w_n$ $(0 \le w_1, w_2, ..., w_n \le 1$ and $w_1 + w_2 + ... + w_n = 1)$, and a decision matrix $\boldsymbol{M} = [y_{i,j}]_{m \times n}$ $(i = 1, 2, ..., m;\ j = 1, 2, ..., n;\ y_{i,j} > 0)$, where w_j is the weight of C_j and $y_{i,j}$ is the value of C_j of A_i. Based on these components, the objective for the problem can be described as: To determine an alternative from $A_1, A_2, ..., A_m$ that best meets $C_1, C_2, ..., C_n$ on the basis of $w_1, w_2, ..., w_n$ and $\boldsymbol{M}$. Using the constructed FPWMSM operator based on HTT, this objective is achieved through the following steps:

(1) Construct a fuzzy decision matrix. In an MCDM problem in design for AM, the values of criteria of alternatives are generally quantified by positive numbers. To establish a fuzzy decision matrix for the problem, these values need to be converted into FNs. A direct way for such conversion is to use a ratio model. Brauers et al. [66] tested a number of different ratio models and found that the best ratio model is

$$y'_{i,j} = \frac{y_{i,j}}{\sqrt{\sum_{i=1}^{m} y_{i,j}^2}} \tag{13}$$

Using this model, each positive number $y_{i,j}$ in the decision matrix $\boldsymbol{M}$ can be converted into an FN $\langle y'_{i,j} \rangle$. Then $\boldsymbol{M}$ is transformed into a fuzzy decision matrix $\boldsymbol{M}' = [\langle y'_{i,j} \rangle]_{m \times n}$.

(2) Normalise the fuzzy decision matrix. The criteria considered in an MCDM problem in design for AM can be classified into positive criteria and negative criteria, which respectively have positive influence and negative influence on the decision-making result. To unify the influence of different types of criteria, a complement rule is usually used to normalise the FNs representing the values of negative criteria [43, 52, 56]. This rule is also applied to normalise the fuzzy decision matrix $\boldsymbol{M}'$ in the presented method. That is, $\boldsymbol{M}'$ is normalised as $\boldsymbol{M}'' = [\langle y''_{i,j} \rangle]_{m \times n} = [\beta_{i,j}]_{m \times n}$, where

$$y''_{i,j} = \begin{cases} y'_{i,j} & \text{if } C_j \text{ is a positive criterion} \\ 1 - y'_{i,j} & \text{if } C_j \text{ is a negative criterion} \end{cases} \tag{14}$$

(3) Calculate the dynamic weight of each FN in the normalised fuzzy decision matrix. Using Eq. (9), the dynamic weights of the FNs $\beta_{i,j}$ in $\boldsymbol{M}''$ are calculated as $\boldsymbol{W} = [W_{i,j}]_{m \times n}$. It is worth noting that the distance between two FNs in this equation can be measured via Eq. (1).

(4) Aggregate the FNs in each row of the normalised fuzzy decision matrix into a single FN. Using the explicit expression of the constructed FPWMSM operator based on HTT in Eq. (10), the FNs $\beta_{i,1}, \beta_{i,2}, ..., \beta_{i,n}$ in $\boldsymbol{M}''$ are aggregated into m single FNs β_i. It is worth noting that the values of the parameters k and δ are respectively assigned according to the actual interactions among criteria and the actual risk attitude. If the actual interactions are unclear, the value of k can be specified as $\lfloor n/2 \rfloor$ ($\lfloor \rfloor$ is the round down function) [67]. If the actual attitude is uncertain, the value of δ can be assigned as 3 [65, 68].

(5) Sort the alternatives on the basis of the aggregation results. According to the aggregated FNs $\beta_1, \beta_2, ..., \beta_m$ and the comparison rule in Definition 2, the alternatives $A_1, A_2, ..., A_m$ are sorted and a sequence of them is obtained.

(6) Determine an alternative that best meets the criteria. Each of the alternatives that are at the first place of the sequence can be selected as the best alternative.

4 Application examples

In this section, an example of AM machine and material selection and an example of optimal build direction selection are introduced to illustrate the application of the proposed method.

4.1 AM machine and material selection example

This example was developed by [38]. Its objective is to select the best combination of AM machine and material from six alternative combinations. In the example, a frame structure whose three-dimensional model is shown in Fig. 2 is about to be manufactured using an AM machine and an AM material. According to a preliminary assessment of the KARMA platform, there are six desirable combinations of AM machine and material for selection, which are listed in Table 1. The selection criteria of these combinations are surface roughness (C_1), mechanical strength (C_2), part density (C_3), build time (C_4), and build cost (C_5). The values of these criteria of the six combinations are predicted by the KARMA platform and also listed in Table 1. The degrees of importance of the five criteria are quantified by the weights 0.50, 0.10, 0.10, 0.15, and 0.15, respectively.

Using the proposed generic MCDM method, the objective for the example is achieved via the following six steps:

(1) Construct a fuzzy decision matrix. Using the ratio model in Eq. (13), the positive numbers in Table 1 are

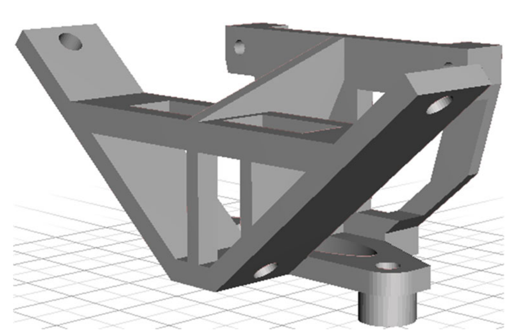

Fig. 2 Three-dimensional model of a frame structure

Table 1 Predicted values of criteria in the first application example

Alternative combination	C_1	C_2	C_3	C_4	C_5
Unit of criterion values	µm	MPa	g/cm^3	h	€
A_1: Viper & ProtoGen 18420	3.49	61.38	1.20	5.40	47.70
A_2: Viper & Somos NeXt	7.80	55.10	1.17	2.32	35.08
A_3: M3 Linear & CL 20 (316L)	3.12	475.00	7.80	6.66	211.42
A_4: EOSINT & PA 2200	19.03	47.22	1.05	3.40	146.14
A_5: EOSINT & PA 3200 GF	19.81	37.92	1.32	3.40	146.14
A_6: APCAMA2 & Ti6Al4V ELI	24.96	936.60	4.42	9.02	481.78

converted into FNs. Based on this, a fuzzy decision matrix is constructed as

$$\boldsymbol{M}' = \begin{bmatrix} \langle 0.0913\rangle & \langle 0.0582\rangle & \langle 0.1294\rangle & \langle 0.3988\rangle & \langle 0.0839\rangle \\ \langle 0.2041\rangle & \langle 0.0522\rangle & \langle 0.1261\rangle & \langle 0.1713\rangle & \langle 0.0617\rangle \\ \langle 0.0816\rangle & \langle 0.4502\rangle & \langle 0.8409\rangle & \langle 0.4918\rangle & \langle 0.3720\rangle \\ \langle 0.4980\rangle & \langle 0.0448\rangle & \langle 0.1132\rangle & \langle 0.2511\rangle & \langle 0.2571\rangle \\ \langle 0.5184\rangle & \langle 0.0359\rangle & \langle 0.1423\rangle & \langle 0.2511\rangle & \langle 0.2571\rangle \\ \langle 0.6532\rangle & \langle 0.8877\rangle & \langle 0.4765\rangle & \langle 0.6661\rangle & \langle 0.8477\rangle \end{bmatrix}$$

(2) Normalise the fuzzy decision matrix. For the AM machine and material selection problem, surface roughness, part density, build time, and build cost are four negative criteria, and mechanical strength is a positive criterion. According to Eq. (14), the fuzzy decision matrix is normalised as

$$\boldsymbol{M}'' = \begin{bmatrix} \langle 0.9087\rangle & \langle 0.0582\rangle & \langle 0.8706\rangle & \langle 0.6012\rangle & \langle 0.9161\rangle \\ \langle 0.7959\rangle & \langle 0.0522\rangle & \langle 0.8739\rangle & \langle 0.8287\rangle & \langle 0.9383\rangle \\ \langle 0.9184\rangle & \langle 0.4502\rangle & \langle 0.1591\rangle & \langle 0.5082\rangle & \langle 0.6280\rangle \\ \langle 0.5020\rangle & \langle 0.0448\rangle & \langle 0.8868\rangle & \langle 0.7489\rangle & \langle 0.7429\rangle \\ \langle 0.4816\rangle & \langle 0.0359\rangle & \langle 0.8577\rangle & \langle 0.7489\rangle & \langle 0.7429\rangle \\ \langle 0.3468\rangle & \langle 0.8877\rangle & \langle 0.5235\rangle & \langle 0.3339\rangle & \langle 0.1523\rangle \end{bmatrix}$$

(3) Calculate the dynamic weight of each FN in the normalised fuzzy decision matrix. Using Eq. (9), the dynamic weights of the FNs $\beta_{i,j}$ $(i = 1, 2, ..., 6;\ j = 1, 2, ..., 5)$ in $\boldsymbol{M}''$ are calculated as

$$\boldsymbol{W} = \begin{bmatrix} 0.5308 & 0.0541 & 0.1072 & 0.1495 & 0.1584 \\ 0.5333 & 0.0472 & 0.1063 & 0.1613 & 0.1519 \\ 0.4452 & 0.1161 & 0.0908 & 0.1766 & 0.1713 \\ 0.5117 & 0.0641 & 0.0973 & 0.1633 & 0.1636 \\ 0.5088 & 0.0645 & 0.0998 & 0.1633 & 0.1636 \\ 0.5299 & 0.0730 & 0.1014 & 0.1585 & 0.1372 \end{bmatrix}$$

(4) Aggregate the FNs in each row of the normalised fuzzy decision matrix into a single FN. Using the explicit expression of the constructed FPWMSM operator based on HTT in Eq. (10) ($k = 2$ since mechanical strength and part density are interacted and build time and build cost are interacted and $\delta = 3$), the FNs $\beta_{i,1}, \beta_{i,2}, ..., \beta_{i,5}$ in M'' are aggregated into six single FNs $\beta_1 = \langle 0.6247\rangle$, $\beta_2 = \langle 0.6812\rangle$, $\beta_3 = \langle 0.5020\rangle$, $\beta_4 = \langle 0.5823\rangle$, $\beta_5 = \langle 0.5724\rangle$, and $\beta_6 = \langle 0.3841\rangle$.

(5) Sort the alternatives on the basis of the aggregation results. According to the aggregated FNs $\beta_1, \beta_2, ..., \beta_6$ and the comparison rule in Definition 2, the alternative combinations of AM machine and material $A_1, A_2, ..., A_6$ are sorted and a sequence of them is obtained as $A_2 \succ A_1 \succ A_4 \succ A_5 \succ A_3 \succ A_6$.

(6) Determine an alternative that best meets the criteria. Since A_2 is the only alternative that is at the first place of the sequence, it is selected as the best alternative. That is, Viper & Somos NeXt is selected as the most desirable combination of AM machine and material.

For the example above, a sequence of alternatives and the best alternative generated by the method of [38] under the same conditions are respectively $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_6$ and A_1. These results are different from the results of the proposed method. This is mainly because the method of [38] does not consider the interactions of criteria and does not have the capabilities to reduce the effect of noise criterion values and capture the risk attitude of decision-makers, while the proposed method can capture the interactions between mechanical strength and part density and the interactions between build time and build cost and has such capabilities. The difference in results indicates the importance of properly considering the interactions among criteria, reducing the influence of noise criterion values, and capturing the risk attitude in an AM machine and material selection problem.

4.2 Optimal build direction selection example

This example was developed by [50]. Its objective is to select the optimal build direction from seven alternative directions. In the example, seven alternative build directions for the frame structure in Fig. 2 are generated by an AM feature-based approach. The schematic diagram of these directions is shown in Fig. 3. The selection criteria of the directions are feature favourableness (C_1), support volume (C_2), surface roughness (C_3), build time (C_4), and build cost (C_5). The values of these criteria of the seven directions are also predicted by the KARMA platform and listed in Table 2. The degrees of importance of the five criteria are

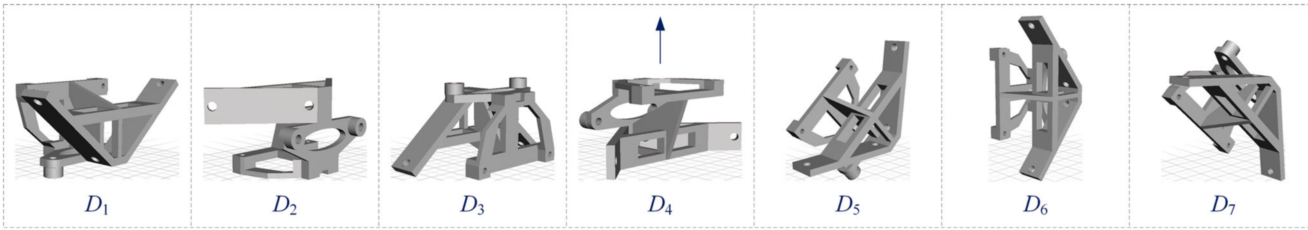

Fig. 3 Schematic diagram of the seven alternative build directions

quantified by the weights 0.125, 0.125, 0.500, 0.125, and 0.125, respectively.

Using the proposed generic MCDM method, the objective for the example is achieved via the following six steps:

(1) Construct a fuzzy decision matrix. Using the ratio model in Eq. (13), the positive numbers in Table 2 are converted into FNs. Based on this, a fuzzy decision matrix is constructed as

$$\boldsymbol{M}' = \begin{bmatrix} \langle 0.3268\rangle & \langle 0.3203\rangle & \langle 0.3370\rangle & \langle 0.2888\rangle & \langle 0.3039\rangle \\ \langle 0.5388\rangle & \langle 0.4082\rangle & \langle 0.2222\rangle & \langle 0.2855\rangle & \langle 0.2986\rangle \\ \langle 0.3268\rangle & \langle 0.4471\rangle & \langle 0.3595\rangle & \langle 0.2888\rangle & \langle 0.3039\rangle \\ \langle 0.5388\rangle & \langle 0.4408\rangle & \langle 0.2403\rangle & \langle 0.2855\rangle & \langle 0.2986\rangle \\ \langle 0.1684\rangle & \langle 0.2480\rangle & \langle 0.5192\rangle & \langle 0.4567\rangle & \langle 0.4479\rangle \\ \langle 0.3862\rangle & \langle 0.4094\rangle & \langle 0.3301\rangle & \langle 0.5055\rangle & \langle 0.4905\rangle \\ \langle 0.1684\rangle & \langle 0.3278\rangle & \langle 0.5211\rangle & \langle 0.4540\rangle & \langle 0.4425\rangle \end{bmatrix}$$

(2) Normalise the fuzzy decision matrix. For the optimal build direction selection problem, feature favourableness is a positive criterion, and support volume, surface roughness, build time, and build cost are four negative criteria. According to Eq. (14), the fuzzy decision matrix is normalised as

$$\boldsymbol{M}'' = \begin{bmatrix} \langle 0.3268\rangle & \langle 0.6797\rangle & \langle 0.6630\rangle & \langle 0.7112\rangle & \langle 0.6961\rangle \\ \langle 0.5388\rangle & \langle 0.5918\rangle & \langle 0.7778\rangle & \langle 0.7145\rangle & \langle 0.7014\rangle \\ \langle 0.3268\rangle & \langle 0.5529\rangle & \langle 0.6405\rangle & \langle 0.7112\rangle & \langle 0.6961\rangle \\ \langle 0.5388\rangle & \langle 0.5592\rangle & \langle 0.7597\rangle & \langle 0.7145\rangle & \langle 0.7014\rangle \\ \langle 0.1684\rangle & \langle 0.7520\rangle & \langle 0.4808\rangle & \langle 0.5433\rangle & \langle 0.5521\rangle \\ \langle 0.3862\rangle & \langle 0.5906\rangle & \langle 0.6699\rangle & \langle 0.4945\rangle & \langle 0.5095\rangle \\ \langle 0.1684\rangle & \langle 0.6722\rangle & \langle 0.4789\rangle & \langle 0.5460\rangle & \langle 0.5575\rangle \end{bmatrix}$$

(3) Calculate the dynamic weight of each FN in the normalised fuzzy decision matrix. Using Eq. (9), the dynamic weights of the FNs $\beta_{i,j}$ ($i = 1, 2, ..., 7$; $j = 1, 2, ..., 5$) in M'' are calculated as

$$\boldsymbol{W} = \begin{bmatrix} 0.1002 & 0.1291 & 0.5146 & 0.1274 & 0.1287 \\ 0.1217 & 0.1261 & 0.4941 & 0.1288 & 0.1293 \\ 0.1066 & 0.1261 & 0.5145 & 0.1257 & 0.1271 \\ 0.1227 & 0.1244 & 0.4967 & 0.1279 & 0.1283 \\ 0.1017 & 0.1136 & 0.5208 & 0.1321 & 0.1318 \\ 0.1214 & 0.1288 & 0.4881 & 0.1306 & 0.1311 \\ 0.1014 & 0.1205 & 0.5163 & 0.1311 & 0.1307 \end{bmatrix}$$

(4) Aggregate the FNs in each row of the normalised fuzzy decision matrix into a single FN. Using the explicit expression of the constructed FPWMSM operator based on HTT in Eq. (10) ($k = 3$ since support volume, build time, and build cost are interacted and $\delta = 3$), the FNs $\beta_{i,1}, \beta_{i,2}, ..., \beta_{i,5}$ in $\boldsymbol{M}''$ are aggregated into seven single FNs $\beta_1 = \langle 0.5313\rangle$, $\beta_2 = \langle 0.5563\rangle$, $\beta_3 = \langle 0.5063\rangle$, $\beta_4 = \langle 0.5491\rangle$, $\beta_5 = \langle 0.4487\rangle$, $\beta_6 = \langle 0.4640\rangle$, and $\beta_7 = \langle 0.4379\rangle$.

(5) Sort the alternatives on the basis of the aggregation results. According to the aggregated FNs $\beta_1, \beta_2, ..., \beta_7$ and the comparison rule in Definition 2, the alternative build directions $A_1, A_2, ..., A_7$ are sorted and a sequence of them is obtained as $A_2 \succ A_4 \succ A_1 \succ A_3 \succ A_6 \succ A_5 \succ A_7$.

(6) Determine an alternative that best meets the criteria. Since A_2 is the only alternative that is at the first place of the sequence, it is selected as the best alternative. That is, the build direction D_2 is selected as the optimal build direction.

Table 2 Predicted values of criteria in the second application example

Alternative direction	C_1	C_2	C_3	C_4	C_5
Unit of criterion values	N/A	mm^3	μm	h	€
A_1: Build direction D_1	1.65	510.00	5.40	4.32	57.00
A_2: Build direction D_2	2.72	650.00	3.56	4.27	56.00
A_3: Build direction D_3	1.65	712.00	5.76	4.32	57.00
A_4: Build direction D_4	2.72	702.00	3.85	4.27	56.00
A_5: Build direction D_5	0.85	395.00	8.32	6.83	84.00
A_6: Build direction D_6	1.95	652.00	5.29	7.56	92.00
A_7: Build direction D_7	0.85	522.00	8.35	6.79	83.00

For the example above, a sequence of alternatives and the best alternative generated by the method of [50] under the same conditions are respectively $A_5 \succ A_1 \succ A_7 \succ A_2 \succ A_4 \succ A_3 \succ A_6$ and A_5. These results are different from the results of the proposed method. This is mainly because the method of [50] does not consider the interactions of criteria and does not have the capabilities to reduce the effect of noise criterion values and capture the risk attitude of decision-makers, while the proposed method can capture the interactions among support volume, build time, and build cost and has such capabilities. The difference in results indicates the importance of properly considering the interactions among criteria, reducing the influence of noise criterion values, and capturing the risk attitude in an optimal build direction selection problem.

5 Demonstration experiments

In this section, a set of numerical experiments are carried out to demonstrate the effectiveness and capabilities of the proposed method.

5.1 Effectiveness demonstration experiment

In general, the effectiveness of a new MCDM method can be demonstrated using a benchmark consisting of a certain number of MCDM examples where the sequences of alternatives are generated by an existing MCDM method. In the method of [56], twelve optimal build direction selection examples were developed and the sequences of alternatives for these examples were produced on the basis of comprehensive consideration of support volume, volumetric error, surface roughness, build time, and build cost. The twelve examples and their sequences of alternatives can form a benchmark for validating the proposed method.

In this subsection, a numerical experiment based on this benchmark was conducted. The twelve examples were solved by the proposed method in the experiment. The sequences of alternatives for these examples generated by the proposed method ($k = 3$ since support volume, build time, and build cost are interacted according to their estimation models in [56] and $\delta = 3$), as well as those produced by the method of [56], are listed in Table 3. It can be seen from the table that the optimal alternative for each

Table 3 Results of the effectiveness demonstration experiment

Example	Sequence of alternatives of the method of [56]	Sequence of alternatives of the proposed method
Example 1	$\boldsymbol{A_{02}} \succ A_{01} \succ A_{06} \succ A_{03} \succ A_{05} \succ A_{04}$	$\boldsymbol{A_{02}} \succ A_{01} \succ A_{06} \succ A_{03} \succ A_{05} \succ A_{04}$
Example 2	$\boldsymbol{A_{02}} \succ A_{04} \succ A_{01} \succ A_{03} \succ A_{06} \succ A_{05}$	$\boldsymbol{A_{02}} \succ A_{04} \succ A_{01} \succ A_{03} \succ A_{06} \succ A_{05}$
Example 3	$\boldsymbol{A_{02}} \succ A_{04} \succ A_{03} \succ A_{05} \succ A_{06} \succ A_{01}$	$\boldsymbol{A_{02}} \succ A_{04} \succ A_{03} \succ A_{05} \succ A_{06} \succ A_{01}$
Example 4	$\boldsymbol{A_{01}} \succ A_{02} \succ A_{05} \succ A_{07} \succ A_{03} \succ A_{08} \succ A_{06} \succ A_{04}$	$\boldsymbol{A_{01}} \succ A_{02} \succ A_{05} \succ A_{07} \succ A_{03} \succ A_{08} \succ A_{06} \succ A_{04}$
Example 5	$\boldsymbol{A_{01}} \succ A_{02} \succ A_{07} \succ A_{08} \succ A_{04} \succ A_{05} \succ A_{06} \succ A_{03}$	$\boldsymbol{A_{01}} \succ A_{02} \succ A_{07} \succ A_{08} \succ A_{04} \succ A_{05} \succ A_{06} \succ A_{03}$
Example 6	$\boldsymbol{A_{02}} \succ A_{01} \succ A_{04} \succ A_{05} \succ A_{08} \succ A_{03} \succ A_{06} \succ A_{07}$	$\boldsymbol{A_{02}} \succ A_{01} \succ A_{04} \succ A_{05} \succ A_{08} \succ A_{03} \succ A_{06} \succ A_{07}$
Example 7	$\boldsymbol{A_{04}} \succ A_{16} \succ A_{21} \succ A_{15} \succ A_{07} \succ A_{08} \succ A_{18} \succ A_{12} \succ A_{11} \succ A_{03} \succ A_{06} \succ A_{17} \succ A_{22} \succ A_{02} \succ A_{14} \succ A_{05} \succ A_{10} \succ A_{20} \succ A_{09} \succ A_{01} \succ A_{19} \succ A_{13} \succ A_{23} \succ A_{24}$	$\boldsymbol{A_{04}} \succ A_{16} \succ A_{21} \succ A_{15} \succ A_{18} \succ A_{07} \succ A_{11} \succ A_{12} \succ A_{08} \succ A_{17} \succ A_{22} \succ A_{06} \succ A_{03} \succ A_{14} \succ A_{02} \succ A_{20} \succ A_{05} \succ A_{10} \succ A_{19} \succ A_{09} \succ A_{01} \succ A_{23} \succ A_{13} \succ A_{24}$
Example 8	$\boldsymbol{A_{09}} \succ A_{03} \succ A_{15} \succ A_{13} \succ A_{07} \succ A_{01} \succ A_{02} \succ A_{22} \succ A_{18} \succ A_{10} \succ A_{08} \succ A_{23} \succ A_{05} \succ A_{11} \succ A_{16} \succ A_{17} \succ A_{21} \succ A_{12} \succ A_{20} \succ A_{06} \succ A_{19} \succ A_{24} \succ A_{14} \succ A_{04}$	$\boldsymbol{A_{09}} \succ A_{03} \succ A_{15} \succ A_{13} \succ A_{07} \succ A_{01} \succ A_{02} \succ A_{22} \succ A_{18} \succ A_{10} \succ A_{08} \succ A_{23} \succ A_{05} \succ A_{16} \succ A_{11} \succ A_{17} \succ A_{21} \succ A_{12} \succ A_{20} \succ A_{06} \succ A_{19} \succ A_{24} \succ A_{14} \succ A_{04}$
Example 9	$\boldsymbol{A_{17}} \succ A_{05} \succ A_{23} \succ A_{06} \succ A_{24} \succ A_{18} \succ A_{21} \succ A_{08} \succ A_{09} \succ A_{22} \succ A_{14} \succ A_{07} \succ A_{10} \succ A_{12} \succ A_{11} \succ A_{16} \succ A_{15} \succ A_{19} \succ A_{13} \succ A_{20} \succ A_{04} \succ A_{03} \succ A_{02} \succ A_{01}$	$\boldsymbol{A_{17}} \succ A_{05} \succ A_{23} \succ A_{06} \succ A_{21} \succ A_{24} \succ A_{18} \succ A_{22} \succ A_{09} \succ A_{08} \succ A_{14} \succ A_{10} \succ A_{07} \succ A_{12} \succ A_{11} \succ A_{19} \succ A_{16} \succ A_{15} \succ A_{13} \succ A_{20} \succ A_{04} \succ A_{03} \succ A_{02} \succ A_{01}$
Example 10	$\boldsymbol{A_{08}} \succ A_{09} \succ A_{10} \succ A_{07} \succ A_{18} \succ A_{17} \succ A_{12} \succ A_{23} \succ A_{24} \succ A_{11} \succ A_{05} \succ A_{14} \succ A_{13} \succ A_{06} \succ A_{20} \succ A_{19} \succ A_{01} \succ A_{02} \succ A_{04} \succ A_{16} \succ A_{03} \succ A_{15} \succ A_{22} \succ A_{21}$	$\boldsymbol{A_{08}} \succ A_{09} \succ A_{10} \succ A_{07} \succ A_{18} \succ A_{17} \succ A_{12} \succ A_{23} \succ A_{24} \succ A_{11} \succ A_{05} \succ A_{14} \succ A_{13} \succ A_{06} \succ A_{20} \succ A_{19} \succ A_{04} \succ A_{01} \succ A_{02} \succ A_{16} \succ A_{15} \succ A_{03} \succ A_{22} \succ A_{21}$
Example 11	$\boldsymbol{A_{01}} \succ A_{02} \succ A_{08} \succ A_{10} \succ A_{07} \succ A_{16} \succ A_{09} \succ A_{15} \succ A_{06} \succ A_{22} \succ A_{12} \succ A_{24} \succ A_{11} \succ A_{21} \succ A_{23} \succ A_{14} \succ A_{18} \succ A_{20} \succ A_{04} \succ A_{05} \succ A_{17} \succ A_{13} \succ A_{03} \succ A_{19}$	$\boldsymbol{A_{01}} \succ A_{02} \succ A_{08} \succ A_{10} \succ A_{07} \succ A_{06} \succ A_{16} \succ A_{09} \succ A_{15} \succ A_{22} \succ A_{24} \succ A_{12} \succ A_{21} \succ A_{23} \succ A_{11} \succ A_{14} \succ A_{18} \succ A_{20} \succ A_{04} \succ A_{05} \succ A_{17} \succ A_{13} \succ A_{03} \succ A_{19}$
Example 12	$\boldsymbol{A_{15}} \succ A_{02} \succ A_{11} \succ A_{23} \succ A_{01} \succ A_{16} \succ A_{24} \succ A_{05} \succ A_{12} \succ A_{08} \succ A_{04} \succ A_{07} \succ A_{06} \succ A_{17} \succ A_{09} \succ A_{03} \succ A_{14} \succ A_{18} \succ A_{10} \succ A_{22} \succ A_{13} \succ A_{19} \succ A_{21} \succ A_{20}$	$\boldsymbol{A_{15}} \succ A_{02} \succ A_{11} \succ A_{23} \succ A_{01} \succ A_{16} \succ A_{05} \succ A_{24} \succ A_{12} \succ A_{08} \succ A_{04} \succ A_{07} \succ A_{09} \succ A_{06} \succ A_{17} \succ A_{03} \succ A_{14} \succ A_{10} \succ A_{18} \succ A_{19} \succ A_{22} \succ A_{20} \succ A_{13} \succ A_{21}$

The optimal alternative for each example is highlighted in bold example generated by the proposed method is the same as that produced by the method of [56]. This demonstrates the feasibility and effectiveness of the proposed method.

5.2 Capability demonstration experiments

In this subsection, three numerical experiments based on the two MCDM examples in Section 4 were conducted to demonstrate the capabilities of the proposed method.

The first experiment was carried out to illustrate that the proposed method has the capability to consider the interactions of criteria. In this experiment, five different values of the parameter k and the two MCDM examples in Section 4 were taken as the input of the proposed method ($\delta = 3$). The results of the experiment are the calculated degrees of membership of alternatives and the generated sequences of alternatives for each example, which are shown in Fig. 4. As can be seen from the figure, the proposed method works well under different k values and the best alternative and sequences of alternatives could vary with respect to different k values, which indicates that the proposed method is suitable for the situation where all criteria are independent or any two or multiple of them have interactions. This also reflects the importance of properly considering the interactions of criteria in an MCDM method. In general, the value of k in the proposed method is specified according to the actual interactions of criteria in the MCDM problem to be solved. For example, the value of k can be assigned 2 in the first MCDM example, since there are interactions between mechanical strength and part density and there are interactions between build time and build cost. This value can be assigned 3 in the second MCDM example, because there are interactions among support volume, build time, and build cost.

The second experiment was conducted to demonstrate that the proposed method has the capability to reduce the influence of noise criterion values. In this experiment, the two MCDM examples in Section 4 were used as the input of the proposed method ($k = 1$ since all criteria are assumed to be independent of each other to facilitate the demonstration and $\delta = 3$). The value of $\beta_{1,1}$ in $\boldsymbol{M}''$ for the AM machine and material selection example and the value of $\beta_{2,3}$ in $\boldsymbol{M}''$ for the optimal build direction selection example are constantly decreased according to the two abscissas in Fig. 5. After decreasing to a certain value, the decreased value for each example will be automatically considered as a noise value by the method, and thus the weight of this value (i.e. $W_{1,2}$ for the former example and $W_{2,3}$ for the latter example), as depicted in Fig. 5, will be reduced dynamically. As the weight of a noise criterion value decreases, the contribution of this value to the aggregation result for the corresponding alternative also decreases accordingly. Because of this, the effect of the noise criterion value on the aggregation result is reduced.

The third experiment was carried out to show that the proposed method has the capability to capture the risk attitude of decision-makers. In this experiment, the values from 0.01 to 6 of the parameter δ and the two MCDM examples in Section 4 were used as the input of the proposed method ($k = 1$ since all criteria are assumed to be independent of each other to facilitate the demonstration). The results of the experiment are the calculated degrees of membership of alternatives for each example, which are depicted in Fig. 6. It can be seen from the figure

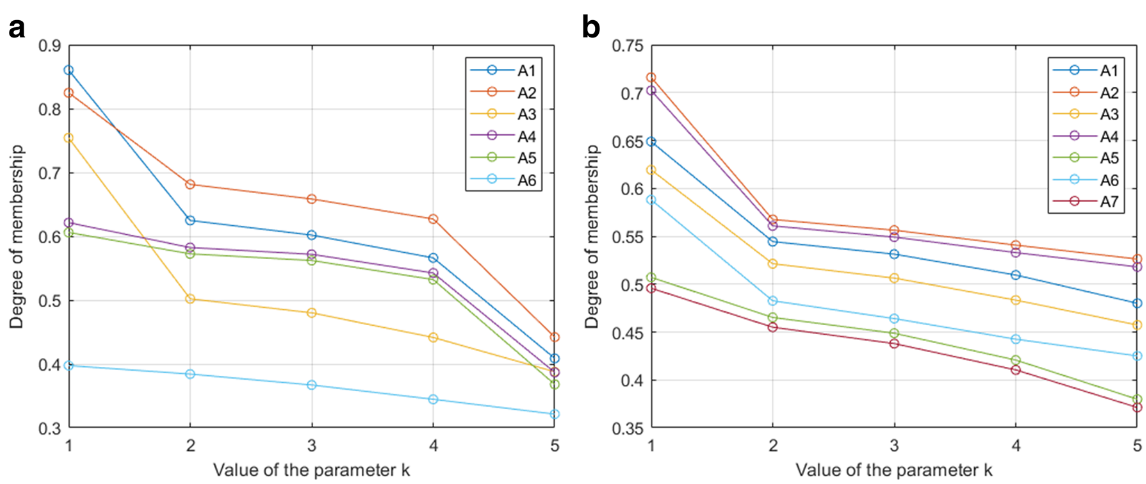

Fig. 4 Results of the first capability demonstration experiment. **a** AM machine and material selection example. **b** Optimal build direction selection example

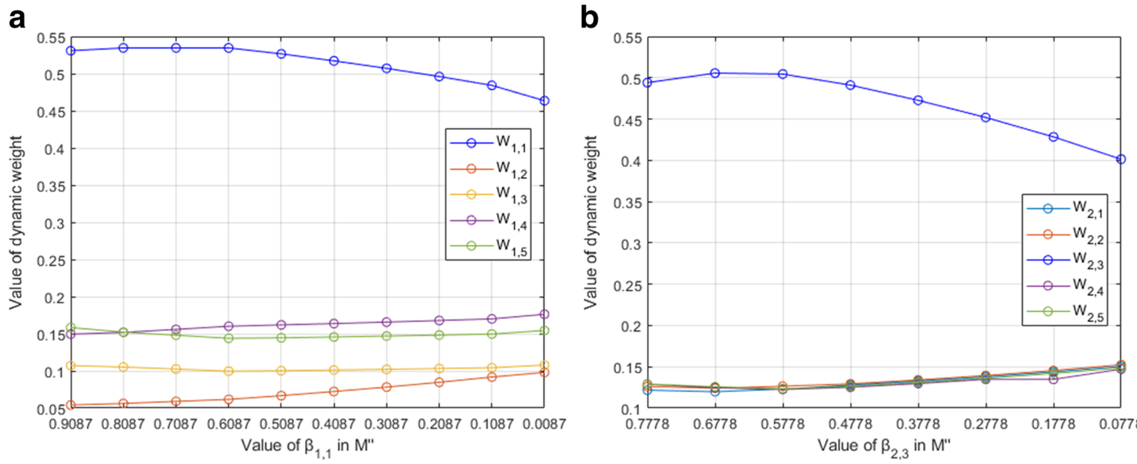

Fig. 5 Results of the second capability demonstration experiment. **a** AM machine and material selection example. **b** Optimal build direction selection example

that the degrees of membership of all alternatives for each example continue to decrease as the value of δ increases. This indicates that the risk attitude becomes more and more pessimistic. If $\delta = 0.01$ and $\delta = 6$ are used to denote an optimistic risk attitude and a pessimistic risk attitude, respectively, then $\delta = 3$ can be leveraged to express a neutral risk attitude [65, 68]. Although different risk attitudes do not have obvious influence on the generated sequences of alternatives for the two examples, this does not mean that the risk attitude is not important for an MCDM method. As investigated by [59], the risk attitude is a critical factor that should be captured in an MCDM method, as it could affect the decision-making results significantly. To this end, the value of δ in the proposed method should be assigned according to the actual risk attitude in the MCDM problem to be solved.

6 Conclusion

In this paper, an FPWMSM operator based on HTT has been constructed and a generic method based on this operator for solving the MCDM problems in design for AM has been presented. The presented method mainly consists of fuzzification, normalisation, and aggregation of criterion values and generation of alternative sequence. In the fuzzification of criterion values, the values of criteria of alternatives are converted into FNs via a ratio model. The converted FNs are normalised according to a complement rule in the normalisation of criterion values. In the aggregation of criterion values, the normalised FNs quantifying the values of criteria of each alternative are aggregated into a summary FN using the constructed FPWMSM operator based on HTT. A sequence of

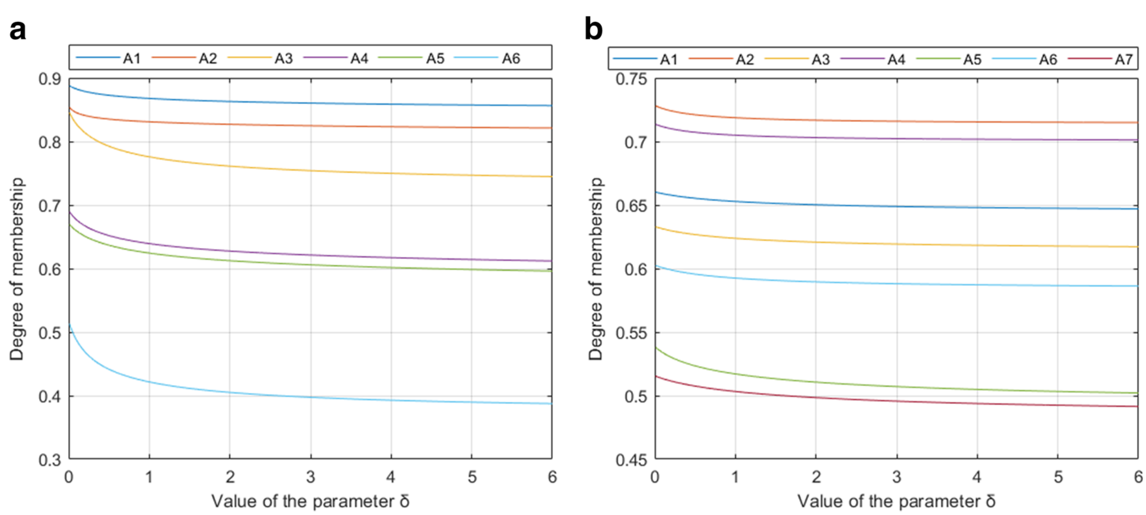

Fig. 6 Results of the third capability demonstration experiment. **a** AM machine and material selection example. **b** Optimal build direction selection example

alternatives is produced on the basis of the aggregation results of the operator in the generation of alternative sequence. The paper has also introduced an example of AM machine and material selection and an example of optimal build direction selection to illustrate the application of the presented method and demonstrated the effectiveness and capabilities of the method via a set of numerical experiments. The demonstration results show that the method is feasible and effective which can capture the interactions of criteria and the risk attitude of decision-makers and reduce the influence of extreme criterion values on the decision-making results. Future work will aim especially at studying the application of the presented method in more MCDM problems in additive manufacturing environment.

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Availability of data and materials The research data and implementation code for supporting the research findings of this paper have been deposited in the GitHub repository (https://github.com/YuchuChingQin/MethodforMCDMinDfAM).

Declarations

Ethical approval This paper does not contain any studies with human participants or animals performed by any of the authors.

Competing interests The authors declare that they have no competing interests.

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