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ORIGINAL ARTICLE

Early failure modeling and analysis of CNC machine tools

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Abstract

Frequent early failures are the key factors restricting the reliability improvement of CNC (computer numerical control) machine tools. In order to eliminate the early failures of CNC machine tools as much as possible, this paper defined the early failures of CNC machine tools strictly and divided the early failures into sudden early failures and progressive early failures. According to the characteristics of sudden early failures, the corresponding reliability analysis model was established by fishbone diagram and 5M1E (man, machine, material, method, measurement, environment) method. Based on the failure time rather than failure time intervals, the reliability analysis model of progressive early failures, which can reflect the dynamic characteristics of CNC machine tool failures, was established by BBIP (bounded bathtub intensity process) method in NHPP (non-homogeneous Poisson process). The reliability evaluation indexes of progressive early failures were given, and the analysis method of the influence of the relationship between former failure and latter failure on the established reliability model was given. CNC machine tools made in China were taken as the example, and the reliability analysis models of sudden early failures and progressive early failures were established and analyzed, respectively. The conclusion that different product failures of the same model could not be analyzed by the same parameter model is obtained. The results also verify the applicability and correctness of the proposed method, which lay a foundation for the elimination of early failure and the improvement of reliability of machine tools.

Keywords CNC machine tools · Early failures · Reliability · Dynamic characteristics

Nomenclature

CNC	Computer numerical control
5M1E	Man, machine, material, method, measurement, environment
BBIP	Bounded bathtub intensity process
NHPP	Non-homogeneous Poisson process
HPP	Homogeneous Poisson process
RP	Renewal process
CMTEF	CNC machine tools early failures
SEF	Sudden early failures
PEF	Progressive early failures
TTT	Total test time
$MTBF_s$	Instantaneous mean time between failure
$MTBF_c$	Cumulative mean time between failure
S-PLP	Superposed power law process
S-LLP	Superposed log-linear process
t_0	The moment when machine tools are installed and debugged
t_1	Early failure time inflection point
t_2	Accidental failure time inflection point
t_3	The moment when the machine tools are scrapped
$F(t)$	Normal operation state fluctuation function of machine tools
$\delta(t)$	Pulse function
t_{t0}	Positive maximum fluctuation amplitude time
t_{t1}	Negative maximum fluctuation amplitude time
A	Maximum fluctuation amplitude that machine tools can bear
m	Total number of experimental machine tools
T_i	Timing censoring time of the i-th product
n_i	Total number of failures collected in $(0, T_i)$
S_K	The moment when the K-th failure occurs in the time series
N	Total number of m product failures in the statistical time

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u	Failure occurrence time
$p(u)$	Number of machine tools observed at time u
K	Number of failures observed at $u=S_K$
Δt	Very small time interval
$\lambda(t)$	Failure intensity function
$\lambda_c(t)$	Cumulative failure intensity function
$\omega(t)$	Cumulative failure intensity function
$R(t)$	Reliability function
$\lambda_1(t)$	Early failure process of products
$\lambda_2(t)$	Depletion failure process of products
λ_1	Model parameters of S-PLP
β_1	Model parameters of S-PLP
λ_2	Model parameters of S-PLP
β_2	Model parameters of S-PLP
α_1	Model parameters of S-LLP
α_2	Model parameters of S-LLP
γ_1	Model parameters of S-LLP
γ_2	model parameters of S-LLP
α	Model parameters of four-parameter method
β	Model parameters of four-parameter method
η	Model parameters of four-parameter method
θ	Model parameters of four-parameter method
a_1	Model parameters of five-parameter method
b_1	Model parameters of five-parameter method
c_1	Model parameters of five-parameter method
d_1	Model parameters of five-parameter method
k	Model parameters of five-parameter method
a	Model parameters of BBIP
b	Model parameters of BBIP
c	model parameters of BBIP
d	Model parameters of BBIP
τ_0	Standing point of failure intensity function
τ_1	Standing point of the first derivative of failure intensity function
$\hat{b}$	Estimated values of b
$\hat{c}$	Estimated values of c
$\hat{d}$	Estimated values of d
P	Goodness of fit evaluation index
N_{S_K}	Actual cumulative failure number of CNC machine tools observed at time S_K
N_{S_K}	Expected failure number at time S_K
ρ_{pxy}	Pearson correlation coefficient
ρ_{sxy}	Spearman's rank correlation coefficient
ρ_{kxy}	Kendall tau rank correlation coefficient
x	Failure time data
y	Failure time data
σ_x	Standard deviation failure time data of x
σ_y	Standard deviation failure time data of y
d_K	Difference between the ranks of two sets of failure time data of x and y
C	Number of elements with consistency in x and y failure time data
H_0	Zero hypothesis of BBIP model
H_1	Alternative hypothesis of BBIP model
Λ	likelihood ratio statistic
ρ_{xy}	Nonlinear relationship between the first failure and the second failure
a_0	Coefficients of ρ_{xy}
a_1	Coefficients of ρ_{xy}
a_2	Coefficients of ρ_{xy}
b_1	Coefficients of ρ_{xy}
b_2	Coefficients of ρ_{xy}
l_0	Maximum value of log-likelihood function of BBIP model with the same parameters
l_i	Maximum value of log-likelihood function of i-th BBIP model with different parameters

1 Introduction

Due to the design errors, material defects, process defects, machining errors, improper installation and debugging, improper operation, etc. [1], CNC machine tools fail frequently within half a year of use [2, 3], which greatly reduces their reliability. At the same time, the initial stage of product use is a critical time for users to judge the quality of the product psychologically. In this stage, the frequent failure will seriously damage the image of the enterprise [4, 5]. Therefore, it is of great significance to study the failures in this period and try to eliminate them in the product manufacturing enterprises to improve the reliability and market competitiveness of products [6].

The definition of CNC machine tool early failures in the current research is not accurate enough, and the characteristics of different types of failures are not considered when establishing the failure model. The accurate establishment of the early failure model of CNC machine tools is of great significance for the subsequent early failure analysis and elimination measures, so it is necessary to select the appropriate method to analyze the failure data of CNC machine tools. The current failure modeling methods of machine tools are mostly based on the failure time intervals. They assume that the failure data of products are independent identical distribution and consider that the repair of the products is a process of "repair as new" and the repaired products can reach the level of just leaving the factory, such as literature [7, 8]. However, the occurrence of CNC machine tool failure is a dynamic process. It is obviously inappropriate to use the time interval between failures as the analysis data. In addition, the theory of "repair as new" is generally for non-repairable products, while machine tools are repairable systems [9, 10]. Moreover, the theory also ignores the degradation of non-faulty parts of machine tools [11], and the quality of repaired products cannot be restored to its state of delivery. In fact, when CNC machine tools fail, it is generally only necessary to repair or replace the failed parts [12], which can make the reliability of the repaired products basically restore to the level before it was repaired, so it is more reasonable to regard the maintenance process as "repair as old" [13, 14]. In conclusion, it is more accurate to use the time of failure as the analysis data and use the random point process to describe the characteristics of CNC machine tool failure. The occurrence of machine tool failure is a random process. At present, there are three main methods currently used for random point process modeling, namely HPP (homogeneous Poisson process), NHPP, and RP (renewal process). HPP assumes that the failure intensity of the product is constant [15, 16], and it is proven that the failure intensity of machine tools is not constant [3], so the modeling method is not suitable for machine tools. RP is mainly used to describe the situation that product repair is "repair as new" [17, 18], and it is not suitable for failure modeling of machine tools. NHPP meets the minimum maintenance assumption of "repair as old" [10, 19], and it is considered that the repaired product can reach the level before its failure. NHPP can reflect the randomness and dynamics of machine tool failure, which is widely applied, as shown in literature [20, 21]. So, it is reasonable to select this method to establish the failure model of CNC machine tools. In addition, the former failure of products also has a certain impact on the latter failure, and this relationship has an important impact on the failure modeling of machine tools, which should also be valued and analyzed [22].

The rationality and integrity of failure data collection are the basis for the accurate establishment of the failure model and early failure cause tracing. The failure data should mainly come from the experimental data of manufacturing enterprises, the maintenance data of after-sales departments, the failure data of user self-repair, and the failure data of similar products. The collected failure data should include product model, production date, failure time, failure location, failure mode, failure cause, failure handling, and maintenance start and end time, etc. After the early failure model of CNC machine tools is established by the collected failure data and the selected method, the fitting effect of the established model to the failure data should be verified to determine the accuracy of the selected model. Then, the early failure analysis and prediction can be carried out according to the established model.

In this paper, the early failure of CNC machine tools is clearly defined, and the classification and collection methods of early failure are also introduced. The respective analysis models are established for different types of early failure, and the model is solved and the goodness of fit test is carried out. The appropriate indicators are selected to evaluate the reliability of CNC machine tools. The correlation between the former early failure and the latter early failure of CNC machine tools is also analyzed. The accuracy and rationality of the proposed method are verified by an example.

2 Early failure definition and classification

2.1 Early failure definition

A large number of researches and practices have shown that under the specified operating environment, operation, and maintenance, the relationship between the failure characteristic indexes of machine tools and time in its life cycle is generally shown as the shape of "bathtub", which is commonly called as the bathtub curve, as shown in Fig. 1 [23, 24].

In Fig. 1, t_0 is the moment when machine tools are installed and debugged; t_1 is the transition point between early failure period and accidental failure period of machine tools, which is called the early failure time inflection point; t_2 is the transition point between accidental failure period and depletion failure period of machine tools, which is called the accidental failure time inflection point; t_3 is the moment when the machine tools are scrapped. The early failure of CNC machine tools is defined as follows:

CNC machine tool early failures (CMTEF) refer to the failures that occur during the period from the completion of machine tool installation and commissioning to the rapid decline of failure rate in the early stage of use, specifically the failures occurring during the period $t_0 \sim t_1$ in Fig. 1.

In early failure period, the failure rate of machine tools will decrease rapidly with time. In accidental failure period, the failure rate of machine tools will basically stabilize. When products are used for a certain period of time, the failure rate will increase rapidly with time until they are scrapped. The failures in early failure period of CNC machine tools often have the following characteristics: failure occurrence rate is high but its decline speed is fast, large proportion of early failures in the total number of failures in the whole life cycle of the products, failure mode and failure causes are concentrated, etc. [25].

2.2 Early failure classification

In this paper, the early failures of CNC machine tools are divided into two types: sudden early failures and progressive early failures. The relevant definitions and analysis are as follows:

Sudden early failures (SEF) refer to those failures caused by "accidental causes" in the early use stage of machine tools. These failures occur irregularly, and they cannot be established with probability models and calculated.

The "accidental causes" here refer to those factors that cause sudden failures of machine tools, such as foreign matter intrusion, processing fluid pollution, excessive temperature, excessive humidity, noise pollution, large vibration, unstable voltage, improper operation, interference of purchased parts, and self-made parts, etc. These factors will cause sudden failures of the parts, which can reduce the stability of the performance of CNC machine tools. In severe cases, the function of

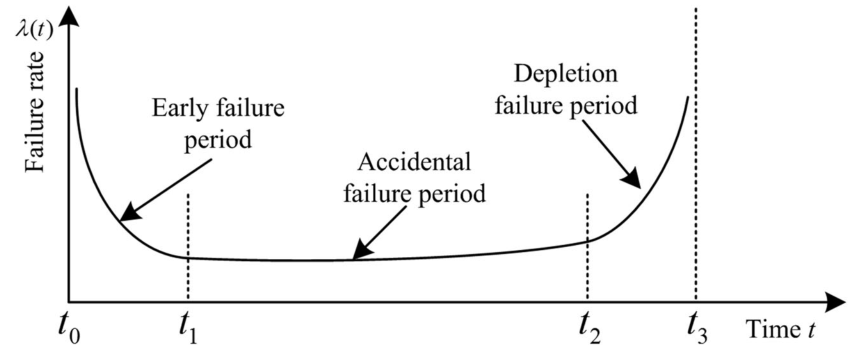


Fig. 1 Failure rate bath curve

machine tools will be lost. The occurrence of sudden early failures is irregular, and it is impossible to use quantitative probability model to model them [26], which is the important reason to restrict the improvement of machine tool reliability.

Sudden early failures can be regarded as a random impulse failure function. Due to the early failure causes of this type, the operating state of machine tools will suddenly fluctuate greatly. When the fluctuation reaches or exceeds the acceptable critical value, failures will occur. Assuming that the normal operation state fluctuation function of machine tools is $F(t)$, which will be impacted by pulse function $\delta(t)$ at t_{t0} and t_{t1}, and the maximum fluctuation amplitude that machine tools can bear is A, then the schematic diagram of sudden early failures of machine tools is shown in Fig. 2, and the mathematical model is shown in formula (1).

$$F(t_{t0}) + \delta(t_{t0}) \geq A \quad \text{or} \quad F(t_{t1}) + \delta(t_{t1}) \leq -A \tag{1}$$

Progressive early failures (PEF) refer to the failures caused by "essential cause" in the early failure period of machine tools. This kind of failures is subject to a certain probability distribution, and it can be predicted by building a mathematical model.

The "essential causes" here refer to those failure factors that can be modeled or quantified. The changes of parts quality characteristics caused by them are traceable and can be quantified by a certain statistical model. Progressive early failures obey a probability distribution, which can be analyzed by building a probability statistical model [27].

The failure generation process of machine tools can generally be described by NHPP [28–30]. According to the failure model, the early failure period of machine tools can be calculated, and the early failures of products can be obtained, which lays a foundation for subsequent failure analysis.

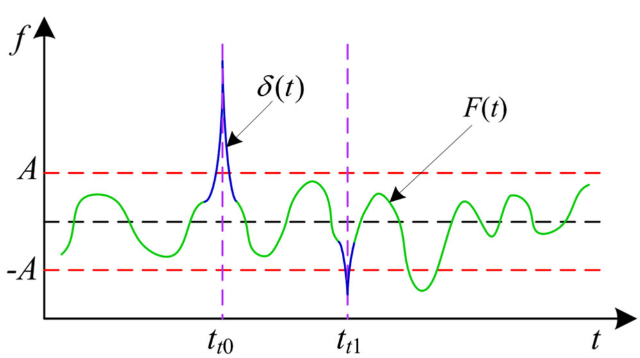


Fig. 2 Sudden early failure mathematical model

3 Early failure modeling

3.1 Sudden early failure modeling

The occurrence of sudden early failures is closely related to the capability and quality of the operators, the maintenance status of equipment, the coordination status among the parts with different materials, the process flow, the operating environment, and the parameters detection, etc., which is the 5M1E.

Fishbone diagram, also known as Ishikawa diagram and cause effect diagram, is an effective method to explore the "essential causes" of problems, which is simple, intuitive, convenient, and quick. In this paper, Fishbone diagram and 5M1E are used to discuss the sudden early failures, and the failure model is shown in Fig. 3.

Figure 3 is only a schematic diagram to illustrate how to establish the model from man, machine, material, method, measurement, and environment. In the specific case analysis, the required failure influencing factors can be added according to the process. When the collected failure data is less, the historical failure data of similar products can be used for analysis.

3.2 Progressive early failure modeling

The modeling process of progressive early failures of CNC machine tools in this paper is shown in Fig. 4.

The detailed analysis of each modeling step in Fig. 4 is as follows.

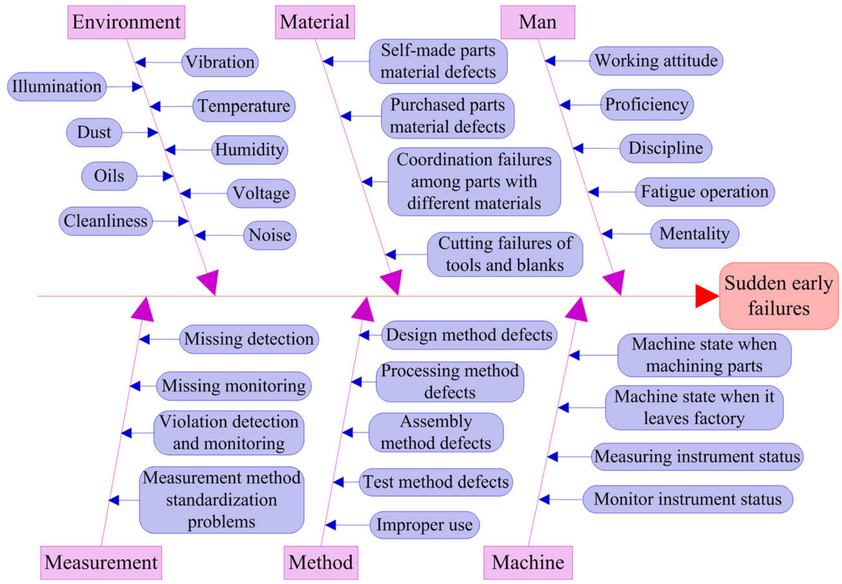


Fig. 3 Sudden early failure model

(1) Failure data collection

There are three main sources of failure data in this paper: the user's self-maintenance record, the after-sales maintenance record of products manufacturer, and the on-site failure detection record. The specific failure data collection process is as follows:

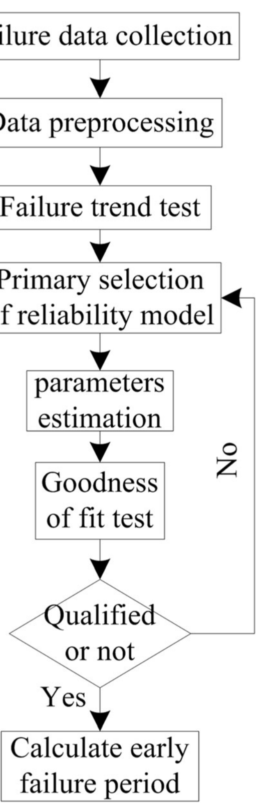


Fig. 4 Progressive early failure modeling process

(a) For the failures that can be repaired by users themselves, the maintenance personnel shall promptly repair them in time after the failures occur and record the product maintenance information;
(b) For the failures that users cannot repair by themselves, they should contact with the after-sales department of the product manufacturer in time. After receiving the users' maintenance requirements, the after-sales department should promptly repair the products and record the maintenance process;
(c) Regularly track the investigated products, so as to know the products status and deal with the problems encountered in time.

(2) Failure data preprocessing and trend test

(a) Failure data preprocessing

Generally, the failure data collected from the products use site cannot be used directly and needs to be transformed in advance. TTT (total test time) method is the most commonly used data transformation method [31]. This paper selects this method to preprocess the collected data, and the transformation process is as follows.

According to the principle of timing censoring, the failure data of m machine tools are statistically analyzed. Assuming that the timing censoring time of the i-th ($0 < i < m$) product is T_i, n_i failure data are collected in the time of $(0, T_i)$, and the moment when the j-th failure of i-th product occurs is t_{ij}. The failure occurrence times of the m machine tools are arranged in order of small to large, then $0 < S_1 \le S_2 \le \ldots \le S_K \le \ldots \le S_N$, where S_K represents the moment when the K-th failure occurs in the time series, and $N = n_1 + n_2 + \ldots + n_m$ is the total number of m product failures in the statistical time.

TTT method is used to process the collected failure data, and the transformed time series can be obtained as formula (2):

$$T(S_K) = \int_0^{S_K} p(u) d_u \quad (2)$$

where u represents the time of failure occurrence; $p(u)$ is the number of machine tools observed at time u, and when all failure processes are observed, $p(u) = m$ [30].

(b) Failure trend test

In order to improve the accuracy of reliability model selection, it is necessary to judge the trend of product failure data in advance. TTT graph method is particularly suitable for the trend judgment of failure data from multiple products [32]. The values of abscissa and ordinate of TTT graph can be calculated from formula (3):

$$\begin{cases} y_K = T(S_K)/T(S) = \int_0^{S_K} p(u) d_u / \int_0^T p(u) d_u \\ x_K = K/N \end{cases} \quad (3)$$

where $T = \max(T_1, T_2, \ldots, T_m)$, K is the number of failures observed at $u = S_K$.

Substituting the processed failure data into formula (3), a TTT graph of the product failures can be obtained, and then, the trend of product failure intensity can be judged. In general, there are four representations of TTT graphs, as shown in Fig. 5.

In Fig. 5, the scatter points in figure (a) are approximately a straight line, indicating that there is no tendency for product failures, and the product failure intensity is almost unchanged, which means that the system is relatively stable. The scattered points in figure (b) are concave, indicating that the product failure intensity is getting smaller and smaller. As time goes by, the number of product failures is less and less, and the product performance tends to be stable. Figure (c) has the opposite meaning to figure (b). The scatter points in figure (d) are *S*-shaped and show "concave before convex", which indicates that the product failure intensity first decreases with time and then increases with time, that is, the number of product failures decreases first and then increases with time, which is a typical "bathtub curve" trend.

(3) Model selection

(a) Modeling method primary selection

Traditional reliability modeling methods believe that the reliability model established must be the same as long as the product failure time intervals are the same [33]. However, in engineering practice, for different CNC machine tools of the same type with the same failure time intervals but different failure sequence, the reliability at the same time is not the same, as shown in Fig. 6.

In Fig. 6, product 1-1, product 1-2, and product 1-3, respectively, represent different CNC machine tools of the same model; 1, 2, 3, and 4, respectively, represent the serial number of failure; and $\triangle t$, $2\triangle t$, and $3\triangle t$, respectively, represent the failure interval time. It can be seen from Fig. 6 that products 1-1, 1-2, and 1-3 have the same failure time interval, then the reliability models established by traditional methods are the same, and the reliability of the three products at the same time is also the same. However, the failure rules of these three products are obviously different, and their reliability level at the same time is not the same. It can be seen that the reliability model based on the failure time interval is not consistent with the engineering practice. Many studies have proved that the maintenance of machine tools belong to the "minimum maintenance" of repairable products, and it can be considered that the reliability of repaired products can reach the level before failure, that is, the maintenance of CNC machine tools is a "repair as old" process [34].

Compared with the operation time of machine tools, the failure maintenance time can generally be ignored, so that the failure occurrence process of machine tools can be described by random point process. NHPP is the most commonly used method in random point process, and it is also the most suitable method for reliability modeling of CNC machine tools [35]. Therefore, this

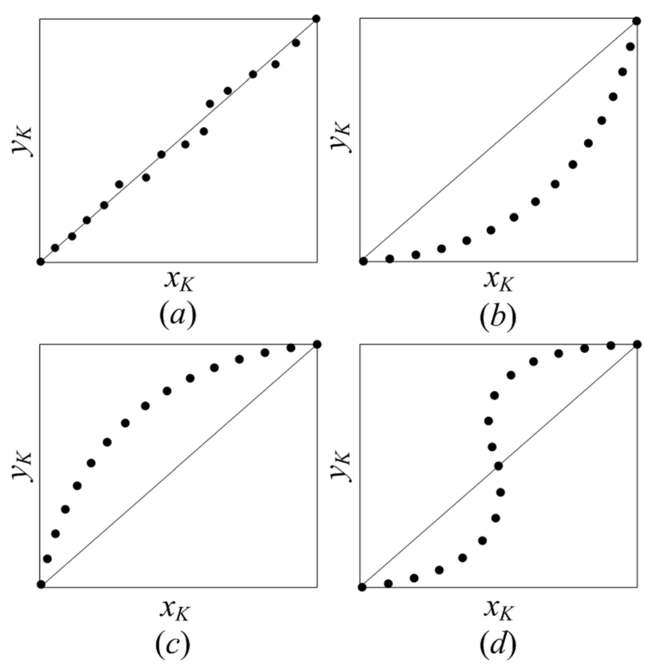


Fig. 5 Typical shape of TTT graphs. **a** System with stable failure rate. **b** System with reduced failure rate. **c** System with increased failure rate. **d** System with reduced first and increased then failure rate

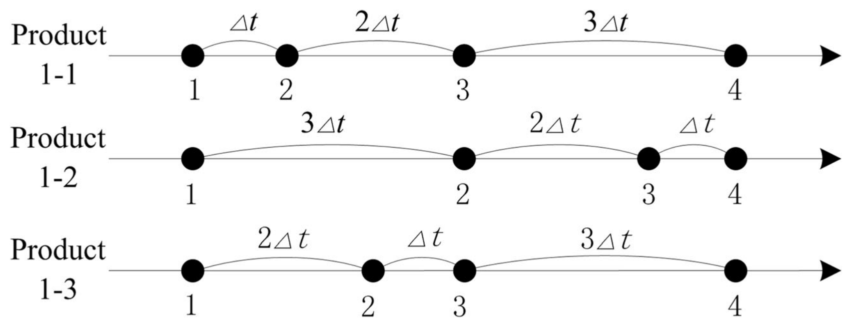


Fig. 6 CNC machine tool failure sequence

paper uses NHPP method to model the reliability of CNC machine tools.

(b) NHPP process basic principles

If a counting process $\{N(t), t \geqslant 0\}$ satisfies the following conditions, it is called an NHPP process with failure intensity function of $\lambda(t)$:

① $N(0) = 0$;
② $\{N(t), t \geqslant 0\}$ has independent increments;
③ $P\{N(t+\Delta t) - N(t) = 1\} = \lambda(t)\Delta t + o(\Delta t)$;
④ $P\{N(t+\Delta t) - N(t) \geq 2\} = o(\Delta t)$.

where $\lambda(t)\Delta t$ represents the failure probability of product within time Δt; cumulative failure intensity function $\omega(t)$ is the average failure number $E[N(t)]$ in time $(0,t]$, which is shown in formula (4):

$$E[N(t)] = \omega(t) = \int_0^t \lambda(t)dt \tag{4}$$

Then

$$P\{N(t+\Delta t)-N(t) = k\} = \frac{[\omega(t+\Delta t)-\omega(t)]^{\mathrm{k}}}{k!} e^{-[\omega(t+\Delta t)-\omega(t)]} \tag{5}$$

or

$$P\{N(t) = k\} = \frac{[\omega(t)]^{\mathrm{k}}}{k!} e^{-\omega(t)} \tag{6}$$

It can be seen from formulas (5) and (6) that $N(t+\Delta t) - N(t)$ obeys the Poisson distribution with a mean of $\omega(t+\Delta t) - \omega(t)$, or $N(t)$ obeys the Poisson distribution with a mean of $\omega(t)$.

(c) Model comparison and selection

Figure 1 is obtained from the failure analysis of electronic products. Generally speaking, the failure intensity of machine tools does not strictly follow the curve trend shown in Fig. 1. Many test data show that the failure of CNC machine tools generally follows the law shown in Fig. 7 [3].

It can be seen from Fig. 7 that the failure intensity curve of CNC machine tools can be roughly divided into two parts: early failure period and depletion failure period. The failure process with the shape shown in Fig. 7 can be described by superposition of two independent NHPPs [35], and the failure intensity function is shown in formula (7).

$$\lambda(t) = \lambda_1(t) + \lambda_2(t) \tag{7}$$

where $\lambda_1(t)$ and $\lambda_2(t)$, respectively, represent the early failure process and the depletion failure process of products. According to the properties of NHPP, the superposed function $\lambda(t)$ is still a NHPP.

For products whose failure intensity curve is "bathtub" shape, the commonly used NHPP modeling methods are mainly as follows: S-PLP (superposed power law process) method proposed by Pulcini [36], S-LLP (superposed log-linear process) method proposed by Byeong [37], four-parameter method proposed by Ren (referred to as four-parameter method) [29], five-parameter method proposed by Wang (referred to as five-parameter method) [38], and the BBIP method proposed by Pulcini, which are, respectively, introduced as follows.

① S-PLP method model

S-PLP model is superimposed by two PLP models, and its failure intensity function is shown in formula (8):

$$\lambda(t) = \lambda_1\beta_1 t^{\beta_1-1} + \lambda_2\beta_2 t^{\beta_2-1} \\ \lambda_1, \lambda_2 > 0, 0 < \beta_1 < 1, \beta_1 > 1, t \geq 0 \tag{8}$$

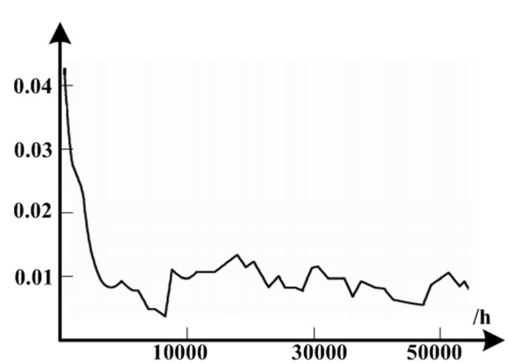


Fig. 7 CNC machine tool failure intensity curve

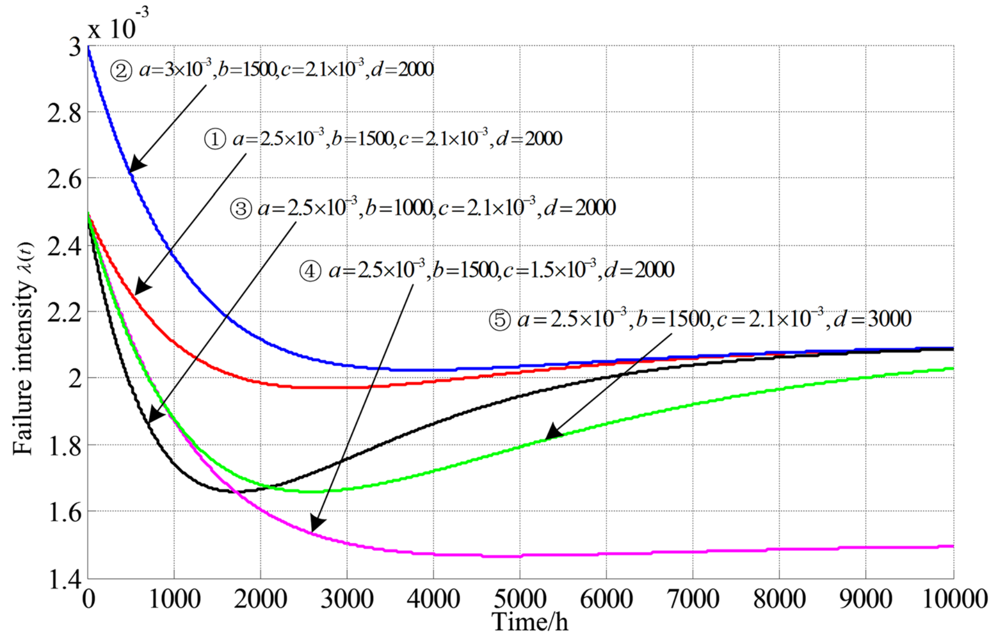


Fig. 8 Influence of model parameters on failure intensity

where λ_1, β_1, λ_2, and β_2 are the model parameters of S-PLP model, respectively.

From formula (8), we can see that the first half of S-PLP model is a monotonically decreasing function, and the second half is a monotonically increasing function. However, the failure intensity function will have a large mutation when $t \to 0^+$, which is quite different from the actual [39].

② S-LLP method model

S-LLP model is superimposed by two LLP models, and its failure intensity function is shown in formula (9):

$$\lambda(t) = \alpha_1 e^{-\gamma_1 t} + \alpha_2 e^{\gamma_2 t} \quad \alpha_1, \alpha_2, \gamma_1, \gamma_2 > 0, t \geq 0 \tag{9}$$

where α_1, γ_1, α_2, and γ_2 are the model parameters of S-LLP model, respectively.

It can be seen from formula (9) that S-LLP model is also composed of a monotone decreasing function and a monotone increasing function. $\lambda(t) = \alpha_1 + \alpha_2$ at $t \to 0^+$, and $\lambda(t)|_{t \to +\infty} = +\infty$ at $t \to +\infty$, which indicate that failure intensity of products will increase infinitely with time, which is not in line with the actual use process of the product. This phenomenon is not consistent with the actual.

③ Four-parameter method model

The four-parameter method is superimposed by two independent NHPP processes, and its failure intensity function is shown in formula (10):

Table 1 Simple correlation judgment criteria

Correlation coefficient	Criterion		Correlation degree
$r = \rho_{pxy}/\rho_{sxy}/\rho_{kxy}$	Negative correlation	$-0.3 < r < 0$	Almost irrelevant
		$-0.3 \leq r < -0.5$	Low correlation
		$-0.5 \leq r < -0.8$	Moderate correlation
		$-0.8 \leq r < -0.95$	Highly correlation
		$-0.95 \leq r \leq -1$	Significant correlation
		$r = -1$	Complete correlation
	Irrelevant	$r = 0$	Irrelevant
	Positive correlation	$0 < r < 0.3$	Almost irrelevant
		$0.3 \leq r < 0.5$	Low correlation
		$0.5 \leq r < 0.8$	Moderate correlation
		$0.8 \leq r < 0.95$	Highly correlation
		$0.95 \leq r < 1$	Significant correlation
		$r = 1$	Complete correlation

Table 2 CNC machine tool failure time

Number	Failure time/h						
1	165.45	361.09	849.17	2084.72	3523.88	4674.65	6084.25
	6723.53	7092.74	7714.06	8442.65	8854.21	9067.13	9298.65
2	58.17	383.42	961.21	1324.63	2143.55	2985.37	4150.11
	5355.79	5726.33	6161.28	6865.24	7348.07	7709.34	8352.20
	8640.85	8807.51	9196.45	9341.92			
3	476.96	801.77	1534.27	3050.08	4981.44	6266.49	6802.34
	7940.56	8609.25	8995.37	9249.48			
4	270.45	623.16	1426.7	2575.49	4555.63	5530.51	6272.36
	6705.57	7517.22	8362.06	8770.28	9047.13	9142.74	
5	312.65	571.25	1382.8	2327.47	4453.43	5263.38	6127.83
	6977.13	7396.41	7851.49	8224.3	8565.26	9095.33	
6	419.3	771.83	1727.76	2645.78	5073.93	5711.49	6633.67
	7178.07	8165.78	8409.92	8893.26	9439.01		
7	245.16	895.49	2228.81	4075.42	5442.35	6517.64	6876.78
	7657.45	8491.83	8905.77	9108.14	7657.45		
8	82.53	443.05	1263.18	2126.09	3278.32	4519.2	5825.36
	6328.7	6928.13	7214.55	8129.85	8712.21	8941.37	9224.64

$$\lambda(t) = \alpha\beta/(t+\alpha) + \eta[1-\theta/(t+\theta)] \quad \alpha,\beta,\eta,\theta > 0, t\geq 0 \tag{10}$$

where α, β, η, and θ are the model parameters of four-parameter method model, respectively.

In this modeling method, $\lambda(t) = \beta$ at $t \to 0^+$, and $\lambda(t) = \eta$ at $t \to +\infty$, which can better simulate the occurrence process of product failures.

④ Five-parameter method model

Five-parameter modeling method is a combination of bounded Burr XII failure intensity function and bounded intensity process. It is mainly used to describe the failure intensity change of the minimum maintenance system with the shape of "bathtub", and its failure intensity function is shown in formula (11):

$$\lambda(t) = a_1 + kb_1 t^{b_1-1}/(1+t^{c_1}) + c_1[1-\exp(-t/d_1)] \quad a_1, k, c_1, d_1 > 0, b_1 > 1, t\geq 0 \tag{11}$$

where a_1, b_1, c_1, d_1, and k are the model parameters of five-parameter method model, respectively.

Table 3 Failure modes and frequencies of sudden early failure

Failure mode	Number	Frequency
Software lost	1	0.03125
Guard plate, shield damage	1	0.03125
Machine does not operate according to the program instructions	1	0.03125
Liquid, gas, oil blockage	2	0.0625
Abnormal sound	2	0.0625
Unable to rotate and move	2	0.0625
Components damage	2	0.0625
No return to zero	2	0.0625
Moving parts are too jittery	2	0.0625
No action	2	0.0625
Liquid, gas, oil leakage	3	0.09375
Parts damage	3	0.09375
Reduced machining accuracy	4	0.125
Components function loss	5	0.15625

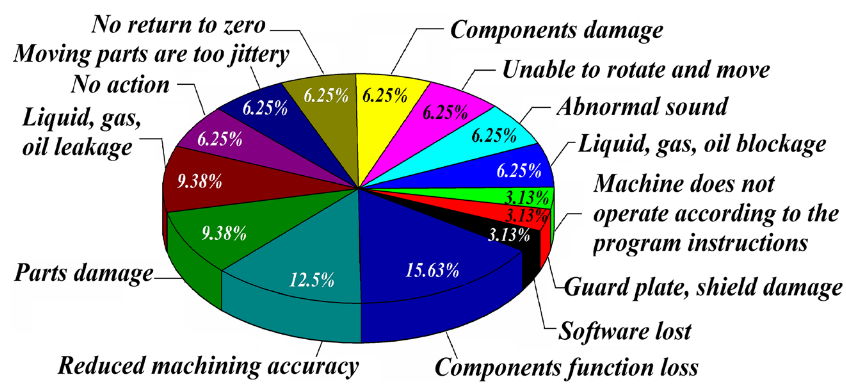


Fig. 9 Sudden early failure proportion

Although formula (11) can better simulate the process of product failures, the increase of model unknown parameters will inevitably lead to the increase in computational difficulty compared with the four-parameter modeling method. Generally speaking, on the premise that solution accuracy can be achieved, the fewer the parameters, the better.

⑤ BBIP method model

BBIP model is based on the Drenick theory, which consists of an LLP model and a bounded intensity process model, and its failure intensity function can be expressed as formula (12):

$$\lambda(t) = a \times \exp(-t/b) + c[1-\exp(-t/d)]$$
$$a, b, c, d > 0, t \geq 0 \tag{12}$$

where *a*, *b*, *c*, and *d* are the model parameters of BBIP model, respectively.

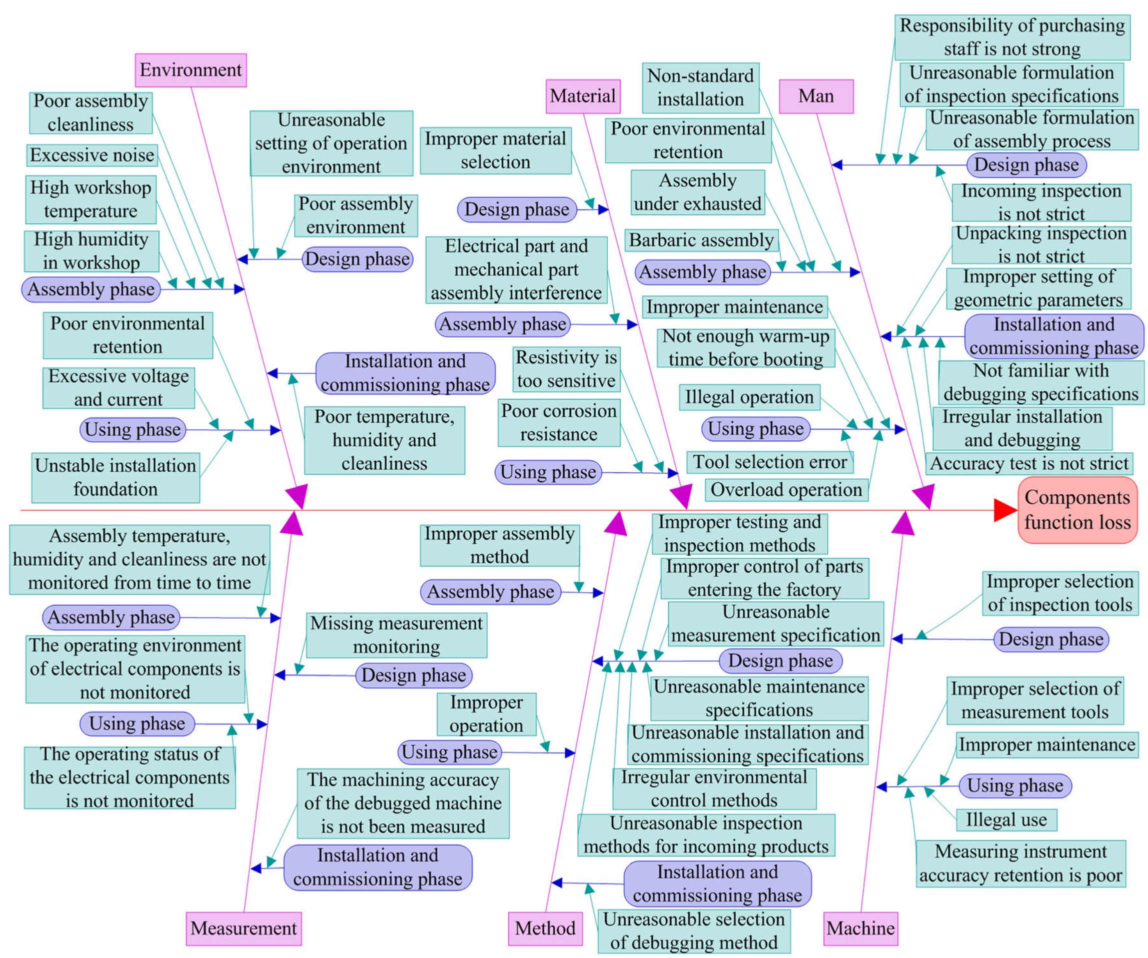


Fig. 10 Component function loss early failure model

When $t = 0$, $\lambda(t)|_{t=0} = a$, and a represents the initial value of the failure intensity function $\lambda(t)$. When $t \to +\infty$, $\lambda(t)|_{t\to\infty} = c$, and c represents the progressive value of failure intensity function $\lambda(t)$. This model not only has the form of "bathtub" in function shape, but also has the boundedness of the boundary intensity function, and its form is simple and easy to calculate.

When the characteristics of product failure rate and the properties of analyzed data are not known, the best and the most suitable model can be obtained directly by hypothesis testing. Instead, we can first compare the properties of the alternative models with the characteristics of product failure, so as to eliminate some inappropriate alternative models. If there are still some alternative models suitable for the analyzed data, the most suitable model can be selected from the remaining alternative models by hypothesis testing.

In summary, four-parameter method and BBIP method are the most suitable NHPP methods for failure intensity modeling. Among these two methods, BBIP is more advantageous in terms of model structure, calculation process, and applicability [40]. Therefore, the BBIP method is used to model and analyze the failures of CNC machine tools.

(d) Model property analysis

Formula (12) calculates the first partial derivative of time t, and formula (13) can be obtained as follows:

$$\lambda'(t) = -\frac{a}{b}e^{\left(-\frac{t}{b}\right)} + \frac{c}{d}e^{\left(-\frac{t}{d}\right)} \tag{13}$$

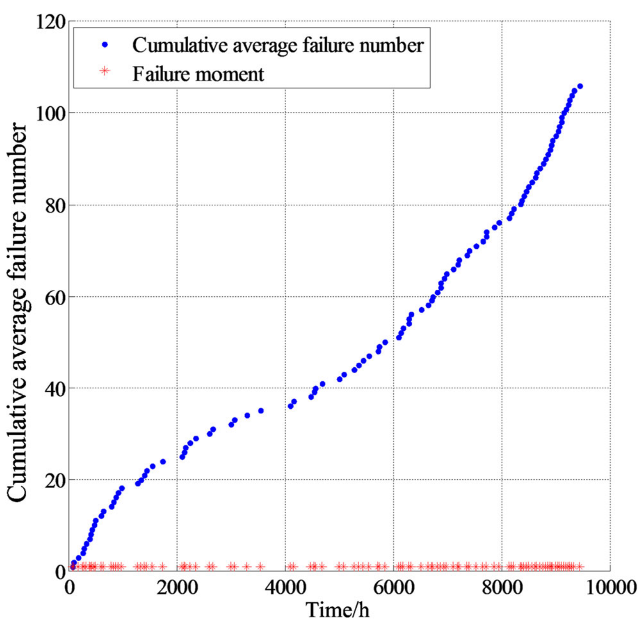


Fig. 11 CNC machine tool failure trend and cumulative failure change trend

When $t = 0$, $\lambda'(t)|_{t=0} = -a/b + c/d$. Because λ(t) is a function with the shape of "bathtub", so $\lambda'(t)|_{t=0} < 0$, that is, there must be formula (14) as follows:

$$bc < ad \tag{14}$$

It can be seen from the value of $\lambda'(t)$ at $t = 0$ that the smaller b is, the faster $\lambda(t)$ decreases, so b represents the decline rate of failure intensity. The larger d is, the slower $\lambda(t)$ approaches c.

Let $\lambda'(t) = 0$, the only standing point τ_0 of $\lambda(t)$ can be obtained as shown in formula (15):

$$\tau_0 = \frac{bd}{d-b}\ln\left(\frac{ad}{bc}\right) \qquad b \neq d \tag{15}$$

It is known from the meaning of τ_0 that it is a positive number, and $b < d$ should be satisfied.

Let formula (13) calculate the partial derivative of time t, and formula (16) can be obtained as follows:

$$\lambda''(t) = \frac{a}{b^2}e^{\left(-\frac{t}{b}\right)} - \frac{c}{d^2}e^{\left(-\frac{t}{d}\right)} \tag{16}$$

Substituting the value of τ_0 into formula (16), formula (17) can be obtained as follows:

$$\lambda''(t)\Big|_{t=\tau_0} = \left(\frac{ad}{bc}\right)^{\frac{d}{b-d}} \times \frac{a(d-b)}{b^2 d} \tag{17}$$

As can be seen from formula (17), $\lambda''(t)\Big|_{t=\tau_0} > 0$. Since there are $\lambda'(t)\Big|_{t=\tau_0} = 0$ and $\lambda''(t)\Big|_{t=\tau_0} \neq 0$ when $t = \tau_0$, τ_0 must be the only extreme value point of $\lambda(t)$, and τ_0 is the minimum value of $\lambda(t)$ when $bc < ad$ and $b < d$ are satisfied. The minimum value of $\lambda(t)$ can be obtained by substituting τ_0 into

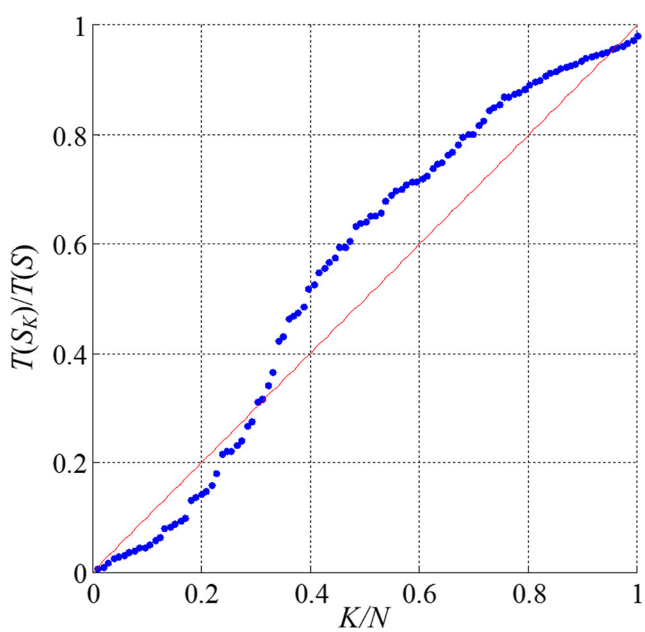


Fig. 12 Progressive early failure TTT diagram

Table 4 Estimation of BBIP model with same parameters

Parameters	a	b	c	d	l_0
Eight machine tools	4.26×10^{-3}	775.29	3.87×10^{-3}	15462.63	− 799.57

formula (12) as shown in formula (18):

$$\lambda(t)|_{t=\tau_0} = c + \frac{a(b-d)}{b} \times \left(\frac{ad}{bc}\right)^{\frac{d}{b-d}} \tag{18}$$

To ensure that $\lambda(t)$ is a single valley function with the "bathtub" shape, formula (19) must be satisfied.

$$\begin{cases} a, b, c, d > 0 \\ bc < ad \\ b < d \end{cases} \tag{19}$$

Let $\lambda''(t) = 0$, the only standing point of $\lambda'(t)$ can be obtained as shown in formula (20):

$$\tau_1 = \frac{bd}{d-b} \ln\left(\frac{ad^2}{b^2c}\right) \tag{20}$$

Comparing formulas (15) and (20), since $b < d$, there must be $\tau_1 > \tau_0 > 0$.

Let formula (16) calculate the partial derivative of time t, and formula (21) can be obtained as follows:

$$\lambda'''(t) = -\frac{a}{b^3} e^{\left(-\frac{t}{b}\right)} - \frac{c}{d^3} e^{\left(-\frac{t}{d}\right)} \tag{21}$$

Substituting the value of τ_1 into formula (21), and formula (22) can be obtained as follows:

$$\lambda'''(t)|_{t=\tau_1} = \left(\frac{ad^2}{b^2c}\right)^{\frac{d}{b-d}} \times \frac{a(b-d)}{b^3 d} < 0 \tag{22}$$

It can be seen from $\lambda''(t)|_{t=\tau_1} = 0$ and formula (22) that τ_1 must be an inflection point of $\lambda(t)$, and the slope of $\lambda(t)$ is the largest at $t = \tau_0$. From the above analysis, we can also know that $\lambda(t)$ decreases monotonically in the interval $[0, \tau_0]$, increases monotonically in the interval $[\tau_0, +\infty]$, and τ_0 is the early failure inflection point. $\lambda(t)$ is a concave function in the interval $[0, \tau_1]$, and a convex function in the interval $[\tau_1, +\infty]$.

To illustrate the influence of different parameters on $\lambda(t)$, Fig. 8 is obtained by control variable method.

In Fig. 8, comparing line ② and line ①, it can be seen that the value of a determines the starting point of $\lambda(t)$, and the larger the value of a, the higher the initial failure rate of products. Comparing line ③ and ①, we can know that the value of b determines the decline speed of failure intensity in the early failure period of products, and the smaller the b value, the faster the $\lambda(t)$ decreases, the shorter the early failure period is. As far as product manufacturing companies and users are concerned, the smaller the b value, the better. Comparing line ④ and line ①, the value of c determines the asymptotic value of failure intensity. It can be seen from the figure that the smaller the c value, the more gentle the $\lambda(t)$ is, and the better the performance of products. Comparing line ⑤ and line ①, the larger the value of d is, the slower the failure intensity curve approaches to its gradual value, and the longer the products is in the accidental failure period. From the above analysis, we can see that for the BBIP model, the smaller the values of a, b, and c, and the larger the value of d, the better the reliability of products.

Table 5 Estimation of BBIP model with different parameters

Machine tool number	Parameters				
	$\hat{a}_i$	$\hat{b}_i$	$\hat{c}_i$	$\hat{d}_i$	l_i
1	4.45×10^{-3}	731.43	3.76×10^{-3}	12732.81	− 103.65
2	4.73×10^{-3}	878.54	4.18×10^{-3}	10484.27	− 129.61
3	3.91×10^{-3}	772.06	3.58×10^{-3}	17327.11	− 84.85
4	4.17×10^{-3}	746.52	3.68×10^{-3}	13774.45	− 97.78
5	4.33×10^{-3}	753.19	4.02×10^{-3}	15664.66	− 97.95
6	3.87×10^{-3}	684.08	3.70×10^{-3}	14865.35	− 91.60
7	3.62×10^{-3}	701.13	3.41×10^{-3}	15208.56	− 84.60
8	4.07×10^{-3}	912.44	3.72×10^{-3}	13281.38	− 104.33

Table 6 Machine tool goodness of fit evaluation index value

Evaluation index	Machine tool number							
	1	2	3	4	5	6	7	8
P	0.9326	0.9454	0.9406	0.9371	0.9383	0.9527	0.9443	0.9477

(4) Model parameter estimation

Substituting formula (12) into formula (4), the average failure number of CNC machine tools in (0, t] time can be obtained as shown in formula (23):

$$\begin{aligned} E[N(t)] &= \omega(t) = \int_0^t \lambda(t)dt \\ &= ab - cd - ab \times \exp(-t/b) \\ &+ c[t + d \times \exp(-t/d)] \end{aligned} \tag{23}$$

It can be seen from formula (23) that $E[N(t)]|_{t=0} = 0$ at $t = 0$, which is consistent with the engineering practice.

The joint probability density likelihood function of m CNC machine tools' failures can be obtained by formula (12) as shown in formula (24):

$$\begin{aligned} L = \prod_{i=1}^{m} \Big\{ & \prod_{j=1}^{n_i} \left[a \times \exp(-t_{ij}/b) + c\left[1 - \exp(-t_{ij}/d)\right] \right\} \\ & \times \exp\Big\{ -ab + cd + ab \times \exp(-T_i/b) \\ & - c[T_i + d \times \exp(-T_i/d)] \Big\} \Big\} \end{aligned} \tag{24}$$

Formula (25) can be obtained by taking the logarithm of both sides of formula (24) as follows:

$$\begin{aligned} l = \ln L &= \sum_{i=1}^{m} \left\{ \sum_{j=1}^{n_i} \ln\left\{ a \times \exp(-t_{ij}/b) + c\left[1 - \exp(-t_{ij}/d)\right] \right\} \right. \\ & \left. - \{ab - cd - ab \times \exp(-T_i/b) + c[T_i + d \times \exp(-T_i/d)]\} \right\} \\ &= \sum_{i=1}^{m} \sum_{j=1}^{n_i} \ln\left\{ a \times \exp(-t_{ij}/b) + c\left[1 - \exp(-t_{ij}/d)\right] \right\} \\ & - \sum_{i=1}^{m} \{ab - cd - ab \times \exp(-T_i/b) + c[T_i + d \times \exp(-T_i/d)]\} \end{aligned} \tag{25}$$

It can be seen from formula (23) that the total failure number N of the m machine tools in (0, T] can be expressed as formula (26):

$$\begin{aligned} N &= \sum_{i=1}^{m} n_i \\ &= \sum_{i=1}^{m} \{ab - cd - ab \times \exp(-T_i/b) + c[T_i + d \times \exp(-T_i/d)]\} \end{aligned} \tag{26}$$

Formula (27) can be obtained by transforming formula (26) as follows:

$$a = \frac{\sum_{i=1}^{m} \left[n_i + c\left(d - T_i - de^{-\frac{T_i}{d}}\right) \right]}{b \sum_{i=1}^{m} \left(1 - e^{-\frac{T_i}{b}}\right)} \tag{27}$$

Substituting formulas (26) and (27) into formula (25), a function with three parameters can be obtained as shown in formula (28):

$$\begin{aligned} l &= \ln L \\ &= \sum_{i=1}^{m} \sum_{j=1}^{n_i} \ln \left\{ \frac{\sum_{i=1}^{m} \left[n_i + c\left(d - T_i - de^{-\frac{T_i}{d}}\right) \right]}{b \sum_{i=1}^{m} \left(1 - e^{-\frac{T_i}{b}}\right)} \times \exp(-t_{ij}/b) + c\left[1 - \exp(-t_{ij}/d)\right] \right\} \\ & -N \end{aligned} \tag{28}$$

In order to ensure the meaning of formula (28), there must be formula (29).

$$\frac{\sum_{i=1}^{m} \left[n_i + c\left(d - T_i - de^{-\frac{T_i}{d}}\right) \right]}{b \sum_{i=1}^{m} \left(1 - e^{-\frac{T_i}{b}}\right)} > 0 \tag{29}$$

As can be seen from formula (12), $b > 0$. It is known from the meaning of T_i that $T_i > 0$. At this time, $b \sum_{i=1}^{m} \left(1 - e^{-\frac{T_i}{b}}\right)$, then formula (29) can be equivalent to formula (30).

Table 7 Early failure period of machine tools

Early failure period	Different parameters								Same parameters
	Machine tool number								8 machine tools
	1	2	3	4	5	6	7	8	
t_0/h	2347.5	2496.0	2585.1	2399.6	2460.1	2239.9	2305.5	2711.9	2521.3
Error	7.40%	1.01%	− 2.47%	5.07%	2.49%	12.56%	9.36%	− 7.03%	/

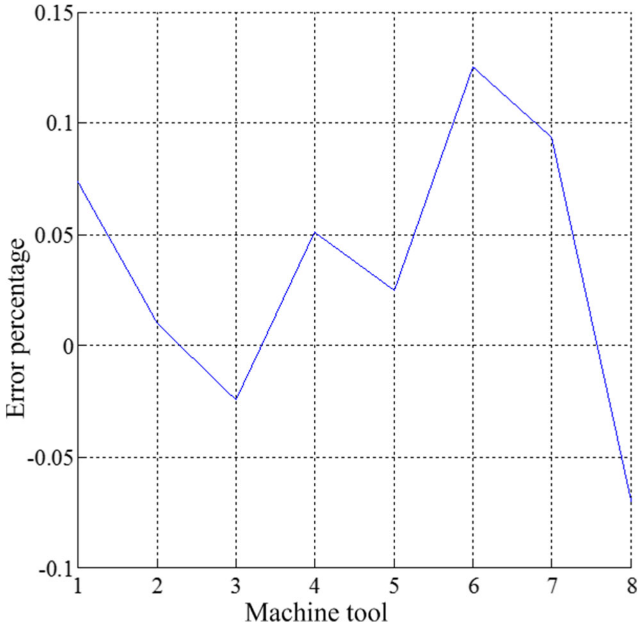


Fig. 13 Progressive early failure TTT diagram

$$\sum_{i=1}^{m}\left[n_{\mathrm{i}}+c\left(d-T_{\mathrm{i}}-de^{-\frac{T_{\mathrm{i}}}{d}}\right)\right]>0 \tag{30}$$

The parameter estimation problem of $\lambda(t)$ can be finally transformed into the minimization problem of formula (28) under the nonlinear constraint, and the solving model can be shown in formula (31).

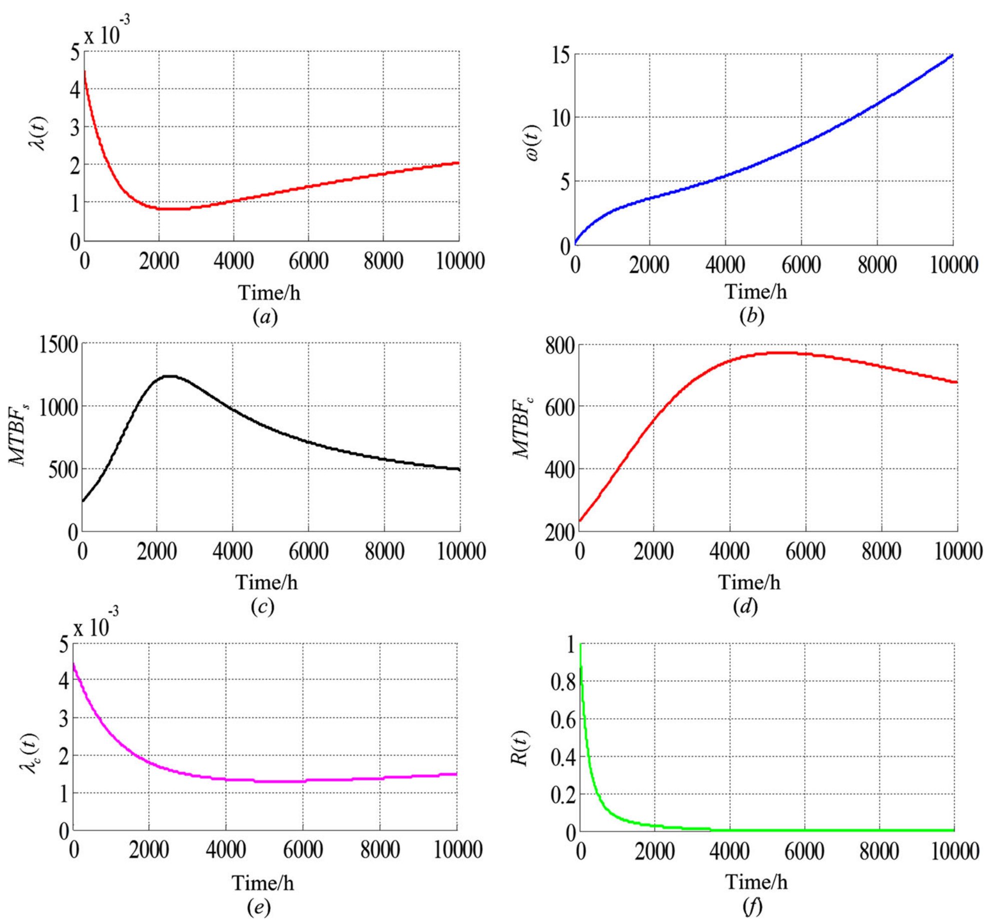
$$\min-\sum_{i=1}^{m}\sum_{j=1}^{n_{\mathrm{i}}}\ln\left\{\frac{\sum_{i=1}^{m}\left[n_{\mathrm{i}}+c\left(d-T_{\mathrm{i}}-de^{-\frac{T_{\mathrm{i}}}{d}}\right)\right]}{b\sum_{i=1}^{m}\left(1-e^{-\frac{T_{\mathrm{i}}}{b}}\right)}\times\exp\left(-t_{\mathrm{ij}}/b\right)+c\left[1-\exp\left(-t_{\mathrm{ij}}/d\right)\right]\right\}+N$$
$$s.t.\quad\begin{cases}-\sum_{i=1}^{m}\left[n_{\mathrm{i}}+c\left(d-T_{\mathrm{i}}-de^{-\frac{T_{\mathrm{i}}}{d}}\right)\right]<0\\ -b<0\\ -c<0\\ -d<0\end{cases} \tag{31}$$

Solving formula (31), we can get the estimated values of b, c, and d as $\widehat{b}$, $\widehat{c}$, and $\widehat{d}$, respectively. Substituting them into formula (27), we can get the estimated value of a as $\widehat{a}$.

(5) Goodness of fit test

The purpose of goodness of fit test is to find out the degree of conformity between the selected failure model and the failure data. According to the reference [41], the goodness of fit evaluation index P of the failure model of CNC machine tools is set as formula (32).

Fig. 14 Reliability index function of machine tool 1. **a** Failure intensity function of machine tool 1. **b** Average cumulative failure number function of machine tool 1. **c** Instantaneous mean time between failure function of machine tool 1. **d** Cumulative mean time between failure function of machine tool 1. **e** Cumulative failure intensity function of machine tool 1. **f** Reliability function of machine tool 1



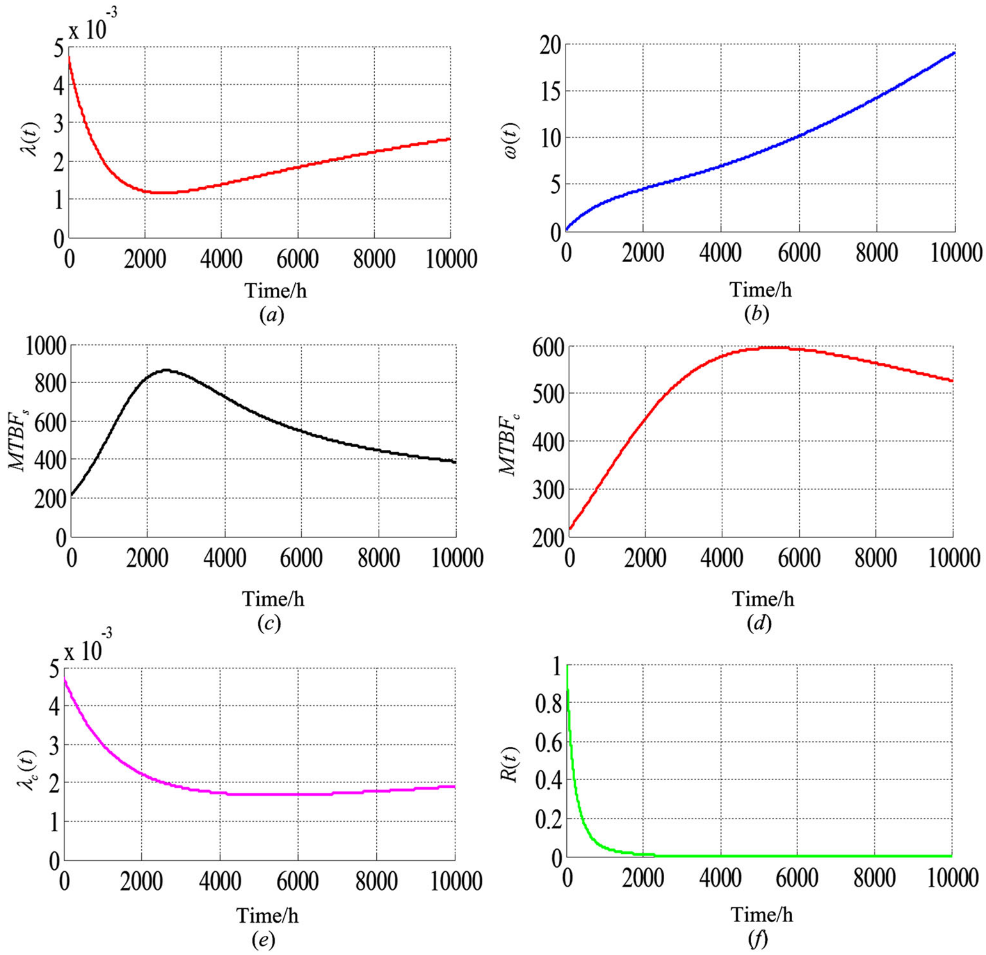


Fig. 15 Reliability index function of machine tool 2. **a** Failure intensity function of machine tool 2. **b** Average cumulative failure number function of machine tool 2. **c** Instantaneous mean time between failure function of machine tool 2. **d** Cumulative mean time between failure function of machine tool 2. **e** Cumulative failure intensity function of machine tool 2. **f** Reliability function of machine tool 2

$$P = 1-\sqrt{\frac{\sum_{S_K=0}^{T}\left(N_{S_K}-\overline{N}_{S_K}\right)^2}{\sum_{S_K=0}^{T} N_{S_K}^2}} \tag{32}$$

where N_{S_K} is the actual cumulative failure number of CNC machine tools observed at time S_K; $\overline{N}_{S_K}$ is the expected failure number at time S_K.

Generally speaking, when $P > 0.9$, it can be considered that the fitting degree between the assumed model and the failure data is acceptable [40], and the larger the value of P, the higher the fitting degree between the assumed model and the failure data, that is, the more appropriate the selected model is.

4 Product reliability evaluation and early failure correlation analysis

4.1 Reliability evaluation

Generally speaking, the commonly used reliability evaluation indexes can be divided into instantaneous reliability indexes and cumulative reliability indexes. Instantaneous reliability evaluation indexes mainly include instantaneous failure intensity function $\lambda(t)$ and $MTBF_s$ (instantaneous mean time between failure). Cumulative reliability indexes mainly include average cumulative failure number $\omega(t)$, cumulative failure intensity function $\lambda_c(t)$, $MTBF_c$ (cumulative mean time between failure), and reliability function $R(t)$. The meanings and calculation formulas of $\lambda(t)$ and $\omega(t)$ have been given above, and other reliability evaluation indexes are introduced as follows:

(1) Instantaneous mean time between failure

$MTBF_s$ refers to the time taken for a failure of products, which can be obtained by formula (33).

$$MTBF_s = 1/\lambda(t) = 1/\hat{a} \times \exp\left(-t/\hat{b}\right) + \hat{c}\left[1-\exp\left(-t/\hat{d}\right)\right] \tag{33}$$

(2) Cumulative mean time between failure

$MTBF_c$ represents the reliability level of CNC machine tools in time $[0, t]$, which can be obtained by formula (34).

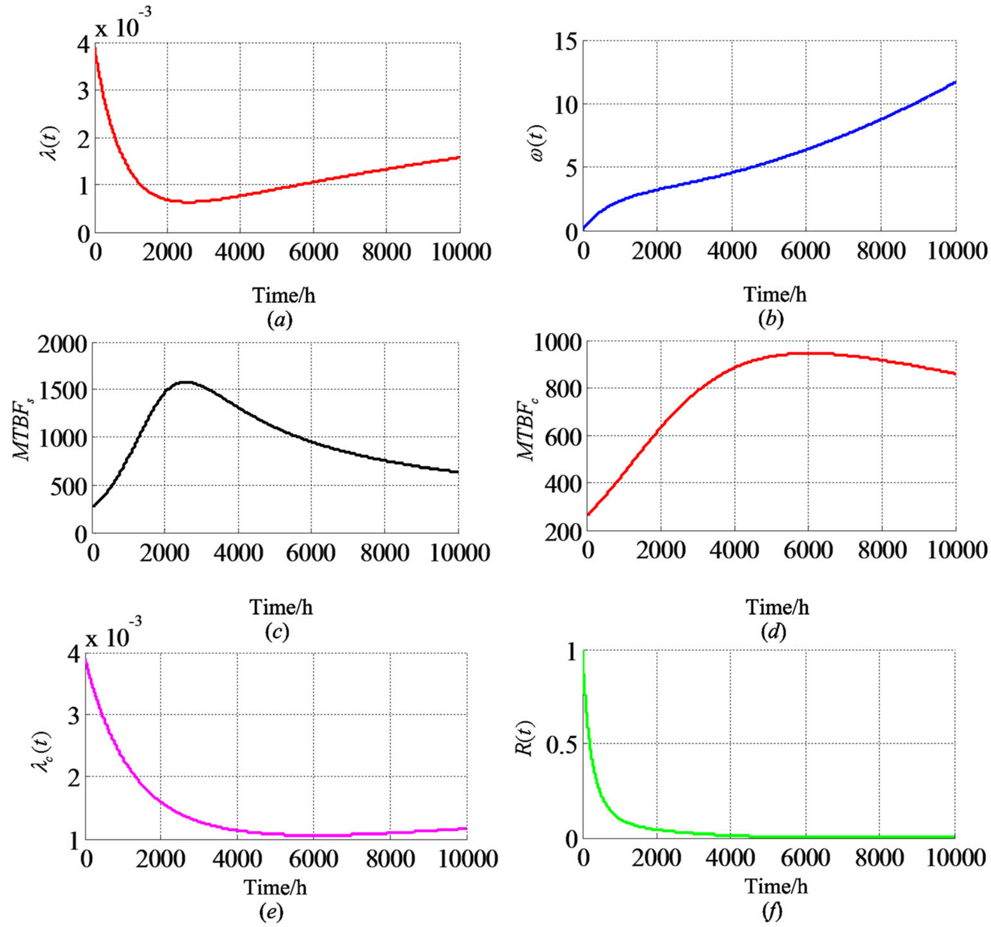


Fig. 16 Reliability index function of machine tool 3. **a** Failure intensity function of machine tool 3. **b** Average cumulative failure number function of machine tool 3. **c** Instantaneous mean time between failure function of machine tool 3. **d** Cumulative mean time between failure function of machine tool 3. **e** Cumulative failure intensity function of machine tool 3. (**f**) Reliability function of machine tool 3

$$MTBF_c = \frac{t_2 - t_1}{\int_{t_1}^{t_2} \lambda(t) d_t} = \frac{t_2 - t_1}{\omega(t)|_{t_1}^{t_2}} = \frac{t_2 - t_1}{\left\{ \widehat{a}\widehat{b} - \widehat{c}\widehat{d} - \widehat{a}\widehat{b} \times \exp\left(-t/\widehat{b}\right) + \widehat{c}\left[t + \widehat{d} \times \exp\left(-t/\widehat{d}\right)\right] \right\}\Big|_{t_1}^{t_2}} \tag{34}$$

(3) Cumulative failure intensity function

$\lambda_c(t)$ can be directly given by formula (35) as follows [42]:

$$\lambda_c(t) = \frac{1}{MTBF_c} = \frac{\left\{ \widehat{a}\widehat{b} - \widehat{c}\widehat{d} - \widehat{a}\widehat{b} \times \exp\left(-t/\widehat{b}\right) + \widehat{c}\left[t + \widehat{d} \times \exp\left(-t/\widehat{d}\right)\right] \right\}\Big|_{t_1}^{t_2}}{t_2 - t_1} \tag{35}$$

(4) Reliability function

$R(t)$ is mainly used to reflect the probability that products can operate normally at time t, which can be obtained by formula (36).

$$R(t) = \exp\left(-\int_0^t \lambda(t) dt\right) = \exp\left(-\widehat{a}\widehat{b} + \widehat{c}\widehat{d} + \widehat{a}\widehat{b} \times \exp\left(-t/\widehat{b}\right) - \widehat{c}\left[t + \widehat{d} \times \exp\left(-t/\widehat{d}\right)\right]\right) \tag{36}$$

4.2 Early failure correlation analysis

Generally speaking, there is a strong or weak connection between the former failure and the latter failure of products. Exploring this connection can lay a foundation for failure prediction and preventive maintenance. Considering that the functional relationship between the failures of products is weak, the correlation analysis method can be used to study the relationship. When the relationship between failure time data is obvious, the data can be analyzed by Pearson correlation coefficient method, Spearman's rank correlation coefficient method and Kendall tau rank correlation coefficient method. The calculation methods are as follows.

Pearson correlation coefficient ρ_{pxy} can be obtained by formula (37).

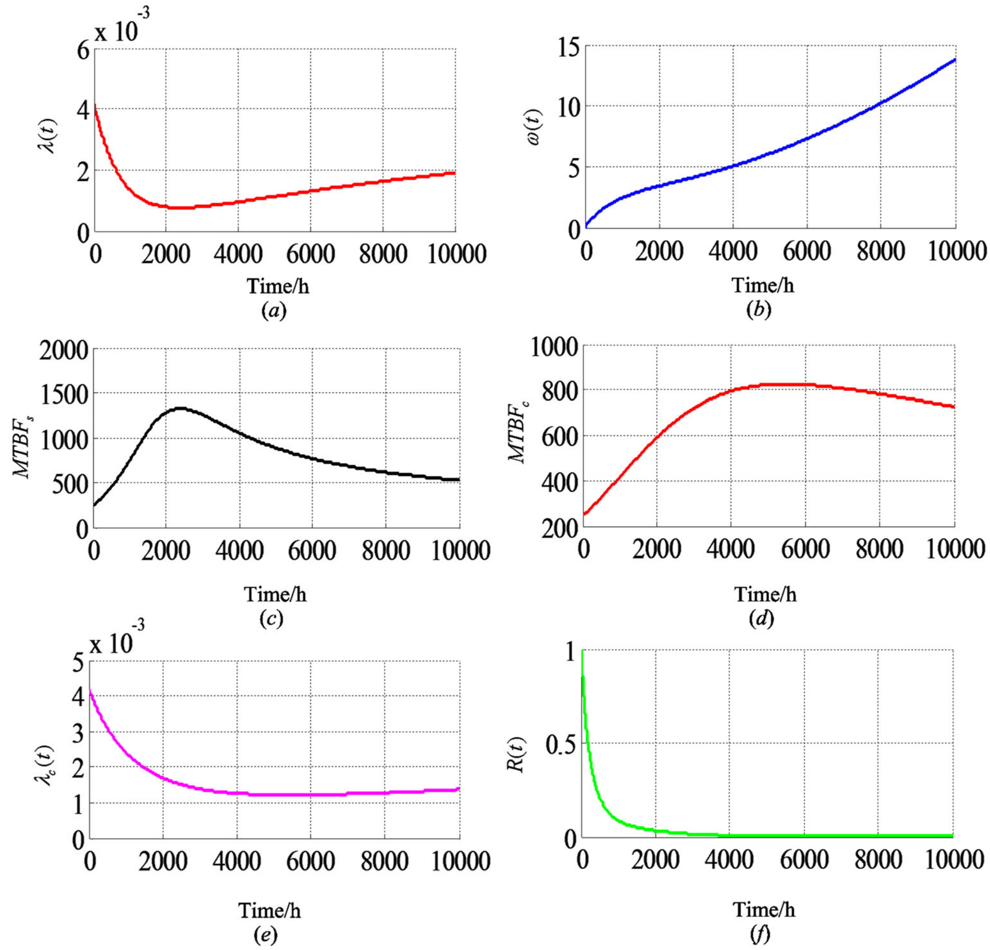


Fig. 17 Reliability index function of machine tool 4. **a** Failure intensity function of machine tool 4. **b** Average cumulative failure number function of machine tool 4. **c** Instantaneous mean time between failure function of machine tool 4. **d** Cumulative mean time between failure function of machine tool 4. **e** Cumulative failure intensity function of machine tool 4. **f** Reliability function of machine tool 4

$$\rho_{\mathrm{pxy}} = \frac{\mathrm{cov}(x,y)}{\sigma_{\mathrm{x}}\sigma_{\mathrm{y}}} = \frac{E(xy)-E(x)E(y)}{\sqrt{E(x^2)-E^2(x)}\sqrt{E(y^2)-E^2(y)}} \tag{37}$$

where cov(x, y) represents the covariance between two sets of failure time data of x and y; σ_{x} and σ_{y} are the standard deviation of two sets of failure time data of x and y, respectively.

Spearman's rank correlation coefficient ρ_{sxy} can be obtained by formula (38).

$$\rho_{\mathrm{sxy}} = 1-\frac{6\sum_{K=1}^{N} d_{\mathrm{K}}^{2}}{N(N^2-1)} \tag{38}$$

where d_{K} represents the difference between the ranks of two sets of failure time data of x and y; N is the total failure number; K is the number of failures observed at $u = S_{\mathrm{K}}$.

Kendall tau rank correlation coefficient ρ_{kxy} can be obtained by formula (39).

$$\rho_{\mathrm{kxy}} = \frac{2(C-D)}{N(N-1)} \tag{39}$$

where C represents the number of elements with consistency in x and y failure time data; D represents the number of elements with inconsistencies in x and y failure time data; N is the total failure number.

The correlation judgment criteria between the two sets of data represented by ρ_{pxy}, ρ_{sxy}, and ρ_{kxy} are shown in Table 1.

When the correlation of failure time data obtained from formulas (37) to (39) is moderate or weaker, the complex correlation of these data should be analyzed. At this time, we can use polynomial, one or multiple regression equations, information entropy and mutual information, Fourier series, and other methods to analyze and solve the data correlation.

5 Application

In this paper, a CNC machine tool made in China is taken as an example for early failure modeling and analysis. Based on the historical failure data of the products to be tested and similar products, the time threshold for failure data collection is set as 9500 h by the reference [39]. During this time, 138 failure data of 8 CNC

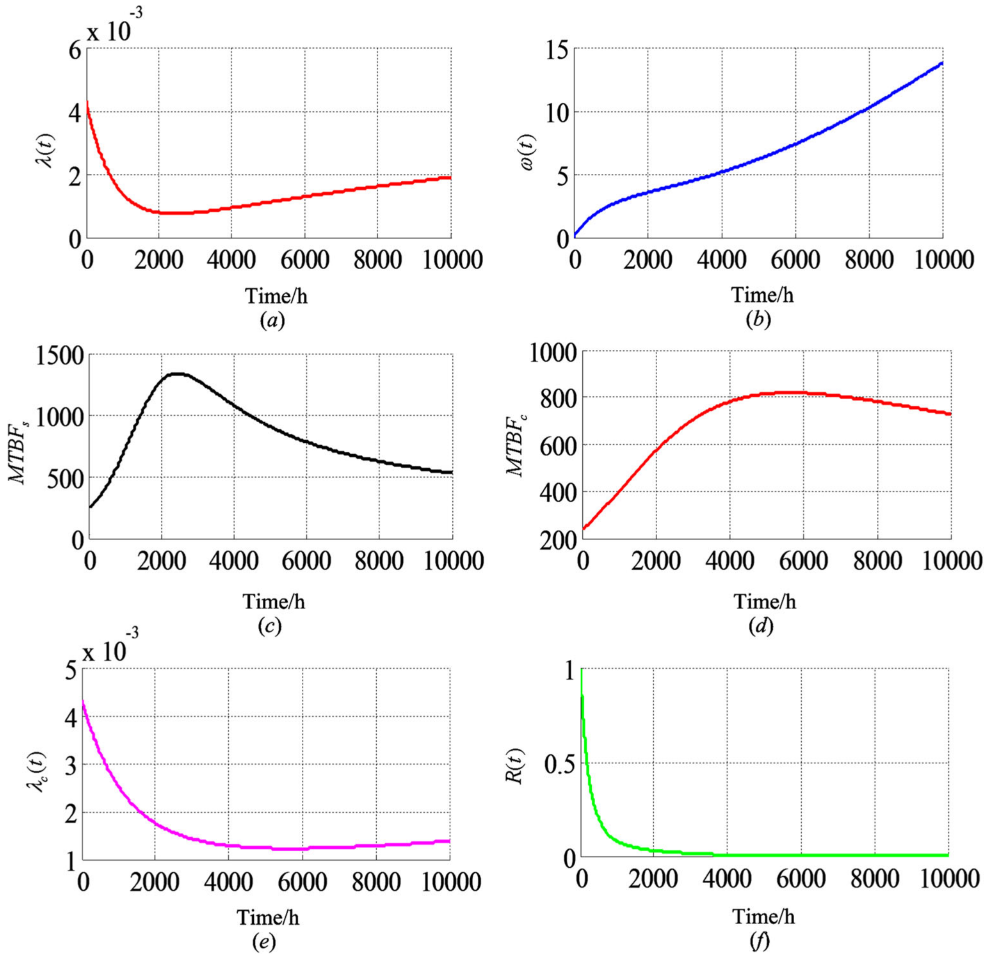


Fig. 18 Reliability index function of machine tool 5. **a** Failure intensity function of machine tool 5. **b** Average cumulative failure number function of machine tool 5. **c** Instantaneous mean time between failure function of machine tool 5. **d** Cumulative mean time between failure function of machine tool 5. **e** Cumulative failure intensity function of machine tool 5. **f** Reliability function of machine tool 5

machine tools were obtained from after-sales department, user maintenance department, and irregular investigation, and 106 failure data were left after eliminating the sudden early failures, as shown in Table 2.

5.1 Sudden early failure modeling

Among the failure data collected, there are 32 sudden failures, the failure frequency is shown in Table 3, and the failure proportion is shown in Fig. 9.

It can be seen from Table 3 and Fig. 9 that in the sudden early failures of the CNC machine tools, component function loss, reduced machining accuracy, parts damage and liquid, gas, oil leakage are the main failure modes, which account for 45.28% of the total failures.

In this paper, the failure number of component function loss is the most, so its sudden early failure model is built by the proposed method, as shown in Fig. 10.

5.2 Progressive early failure modeling

5.2.1 Failure trend test

Analyzing the failure data in Table 2, we can get the change trend of machine tool failure number and cumulative average failure number with time, as shown in Fig. 11.

As can be seen from Fig. 11, the cumulative average failure number of machine tools shows a trend of non-monotonic change, specifically, the failure rate first decreases and then increases, and the corresponding failure interval time increases first and then decreases. In the early failure period, the number of machine tool failures is obviously more, and the time between two adjacent failures is smaller, which indicates that the machine failures occur frequently during this period.

In order to further make a clear judgment on the change of machine tool failure with time, the failure data in Table 2 is preprocessed by formula (2), and a TTT diagram is obtained by formula (3) as shown in Fig. 12.

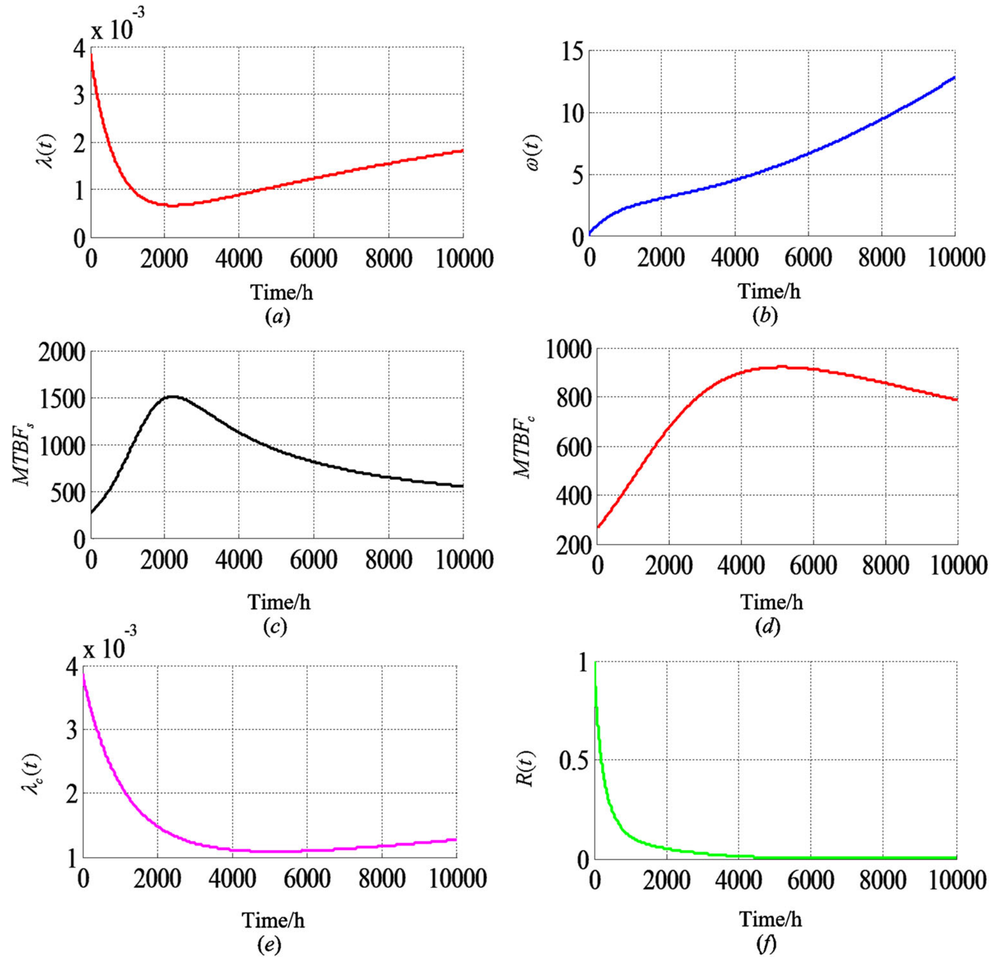


Fig. 19 Reliability index function of machine tool 6. **a** Failure intensity function of machine tool 6. **b** Average cumulative failure number function of machine tool 6. **c** Instantaneous mean time between failure function of machine tool 6. **d** Cumulative mean time between failure function of machine tool 6. **e** Cumulative failure intensity function of machine tool 6. **f** Reliability function of machine tool 6

It can be seen from the comparison between Figs. 5 and 12 that the failure intensity of the machine tools conform to the "bathtub curve", so BBIP model can be used to describe the failure occurrence process.

5.2.2 Parameter estimation and test

Assume that the failure process of these eight machine tools can be described by the BBIP model with the same parameters, and the maximum value of its log-likelihood function is l_0. Under this assumption, the corresponding model parameters can be obtained by Table 2, formula (27) and formula (31) as shown in Table 4.

In fact, the failure process of these eight machine tools may not be completely the same considering the different manufacturing process and using environment of machine tools, and the different technology and quality of operators.

Suppose that the eight machine tools have different failure model parameters, and the maximum value of their respective log-likelihood function is l_i ($i = 1, 2, \ldots, 8$). The parameters of each machine tool can be obtained by Table 2, formula (27), and formula (31), as shown in Table 5.

In order to verify which of the above two hypotheses is more suitable to describe the failure process of machine tools, this paper uses the method proposed in literature [43] to test the two hypotheses. The test process is as follows:

H_0 (zero hypothesis): The BBIP model parameters of each machine are the same;
H_1 (alternative hypothesis): The BBIP model parameters of each machine are different;

If H_0 is correct, the likelihood ratio statistic should obey the chi-square distribution with 4 freedom degrees approximately. The likelihood ratio statistic can be obtained from formula (40) as follows:

$$\Lambda = -2\left(l_0 - \sum_{i=1}^{8} l_i\right) \tag{40}$$

Substituting the data in Tables 4 and 5 into formula (40), $\Lambda = 10.40$. According to the quantile table of chi-square distribution, the critical value $p = 9.4877$ when the significant level is 0.05 and the freedom degree is 4. There is $\Lambda > p$, so H_0

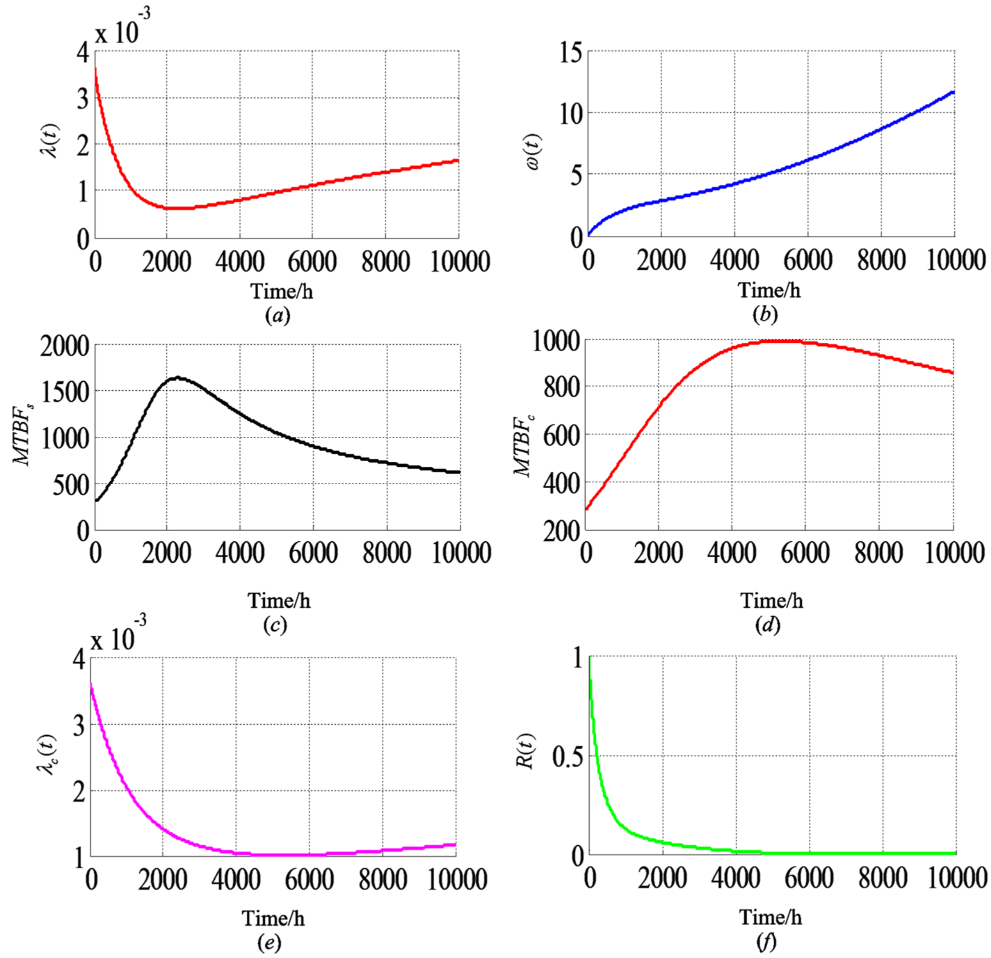


Fig. 20 Reliability index function of machine tool 7. (**a**) Failure intensity function of machine tool 7. (**b**) Average cumulative failure number function of machine tool 7 **c** Instantaneous mean time between failure function of machine tool 7. **d** Cumulative mean time between failure function of machine tool 7. **e** Cumulative failure intensity function of machine tool 7. **f** Reliability function of machine tool 7

should be rejected, which means that the BBIP model parameters of different machine tools are different.

5.2.3 Goodness of fit test

Substituting the data in Tables 2 and 5 into formula (32), the evaluation values of the goodness of fit for each machine tool can be obtained as shown in Table 6.

It can be seen from Table 6 that $P_i > 0.9$ ($i = 1, 2, ..., 8$), which indicates that the parameter models obtained are better for fitting the failure data. Therefore, the model can be used to describe and analyze the failure process of the machine tools.

5.2.4 Early failure period

It can be seen from the above analysis that the failure process of 8 machine tools cannot be described by the BBIP model with the same parameters. In order to reflect the error between the early failure period obtained by the same parameter model and the early failure period obtained by different parameter models, the data in Tables 4 and 5 are substituted into formula (15), then the early failure period of each machine tool can be obtained as shown in Table 7 and the visual representation of the error can be obtained shown in Fig. 13.

The "-" in the error column in Table 7 indicates that the early failure period of machine tool obtained by the same parameter model is smaller than the early failure period obtained by the different parameter model.

It can be seen from Table 7 and Fig. 13 that the early failure periods of different machine tools are not the same. If the same parameter model is used to calculate the early failure period of each machine tool, it will inevitably bring errors in the calculation results, which will adversely affect the subsequent early failure analysis. It can also be seen from Table 7 and Fig. 13 that the early failure period calculated by the same parameter model and different parameter models are quite different, which also proves that the H_0 hypothesis should be rejected.

5.3 Reliability evaluation and correlation analysis

5.3.1 Reliability evaluation

The data in Table 5 is sequentially substituted into formula (12), formula (22), and formula (33) to formula (36), and the reliability

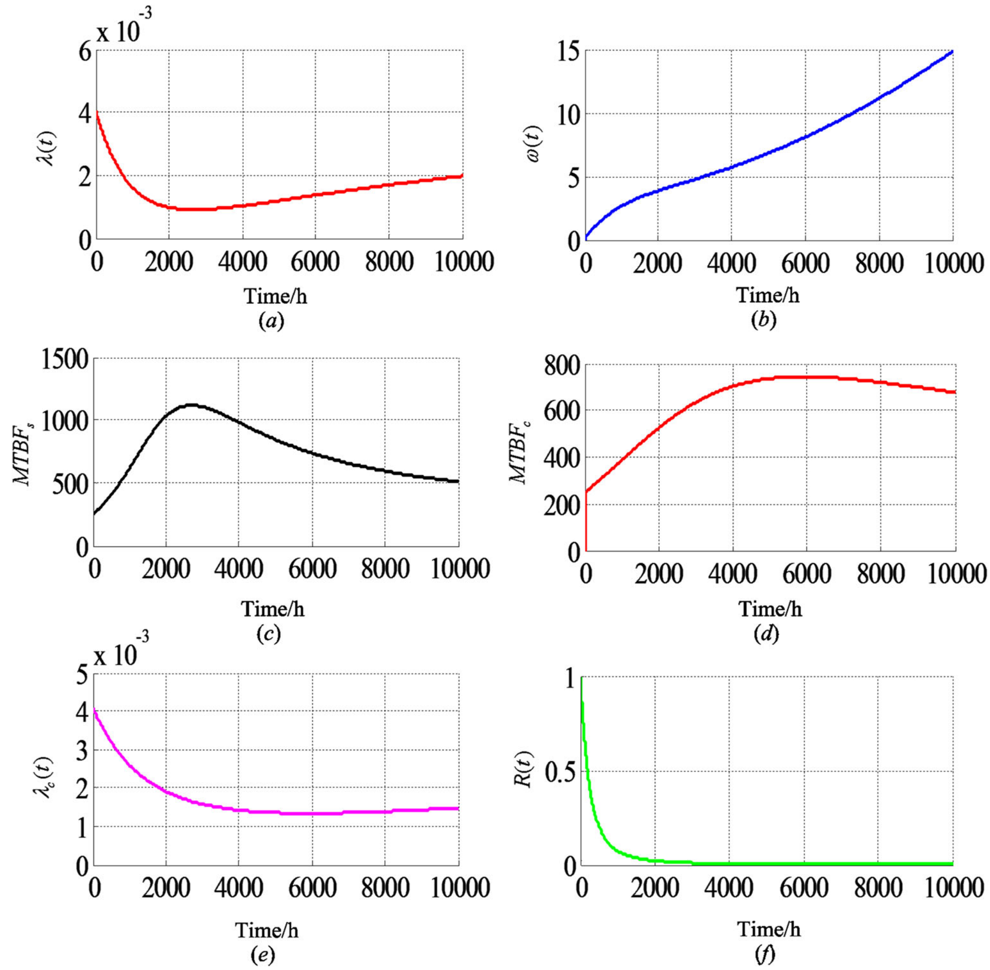


Fig. 21 Reliability index function of machine tool 8. **a** Failure intensity function of machine tool 8. **b** Average cumulative failure number function of machine tool 8. **c** Instantaneous mean time between failure function of machine tool 8. **d** Cumulative mean time between failure function of machine tool 8. **e** Cumulative failure intensity function of machine tool 8. **f** Reliability function of machine tool 8

index function of each machine tool can be obtained as shown in Figs. 14, 15, 16, 17, 18, 19, 20, and 21, respectively.

From Figs. 14, 15, 16, 17, 18, 19, 20, and 21, it can be seen that the reliability indexes of each machine tool change with time. We can know that in the early failure period, the overall trend of the failure intensity is decreased, and then increases with time from Figure (a), Figure (c), Figure (d), and Figure (e). It can also be seen from the comparison between Figure (c) and Figure (d) that the value of $MTBF_c$ is lower than the value of $MTBF_s$ at the same time due to the early failure.

5.3.2 Failure correlation analysis

For CNC machine tools, when a failure occurs, its reliability will inevitably change, and the performance of products cannot be “restored as new”, which will affect the occurrence of its latter failure.

In order to find the relationship between the former failure and the latter failure, the correlation between them should be analyzed. In this paper, only the first three failures of the tested machine tools are analyzed as an example. The data in Table 2 are substituted into formula (37) to formula (39) in turn, and then ρ_{pxy}, ρ_{sxy}, and ρ_{kxy} are obtained as shown in Table 8.

In Table 8, $\rho(i, j)$ represents the correlation coefficient between the i-th failure and the j-th failure of the machine tools.

It can be seen from Tables 1 and 8 that the relationship between the first failure and the second failure of the machine tools is moderate correlation, and the relationship between the second failure and the third failure is significant correlation, so it can be considered that the second failure of the machine tools has a significant impact on its third failure, while the relationship between the first failure and the second failure may be a more complex nonlinear relationship that needs to be further analyzed.

Table 8 Correlation coefficient between failures

Correlation coefficient	$\rho(1, 2)$	$\rho(2, 3)$
ρ_{pxy}	0.7547	0.9347
ρ_{sxy}	0.6667	0.9762
ρ_{kxy}	0.5000	0.9286

Table 9 Failure data fitting parameters and effect

Coefficient	a_0	a_1	b_1	a_2	b_2	ω	Fitting effect
Value	636.60	220.20	39.35	218.50	21.03	0.02815	0.9502

The relationship between the first failure and the second failure ρ_{xy} can be obtained as follows:

$$\rho_{xy} = a_0 + \sum_{k=1}^{2} [a_k \cos(k\omega t) + b_k \sin(k\omega t)] \tag{41}$$

The coefficients in formula (41) and the fitting effect can be obtained by MATLAB, as shown in Table 9.

It can be seen from Tables 1 and 9 that there is a strong correlation between the first failure and the second failure.

The relationship between the fitting function and the failure data can be obtained by formula (41) and Table 9 as shown in Fig. 22.

It can be seen from Table 9 and Fig. 22 that there is a simple or complex correlation between the former failure and the latter failure of machine tools, and the occurrence of the former failure will have an important impact on the latter failure. When establishing a failure model for CNC machine tools, the correlation between failures should be taken into account so as not to affect the accuracy of the analysis results.

6 Conclusion

Early failure seriously restricts the improvement of the quality of CNC machine tools. The existing research on early failure of CNC machine tools lacks a clear definition of early failure and does not classify early failures according to their different characteristics and assumes that all early failures obey the unified distribution. At the same time, the characteristics of early failure of CNC machine tools are not considered when establishing the early failure model. When analyzing multiple CNC machine tools, it is assumed that they have the same model parameters. In addition, the relationship between the former failure and latter failure is not considered when establishing the early failure model of CNC machine tools. The above reasons make the final early failure model of CNC machine tools not appropriate and accurate.

To address those problems, the early failures of CNC machine tools are divided into sudden early failures and progressive early failures herein, and they are strictly defined. Considering the unpredictability of sudden early failures, the analysis model of this kind of early failure is established by 5M1E method.

According to the randomness of progressive early failures, the failure occurrence time rather than failure interval time is taken as the analysis data, and the mathematical model of progressive early failures is established by BBIP method.

Machine tools made in China are analyzed as an example, and the conclusion that different product failures of the same model could not be analyzed by the same parameter model is obtained. The reliability of the product is also evaluated by its instantaneous reliability indexes and cumulative reliability indexes. The relationship between the former failure and the latter failure of

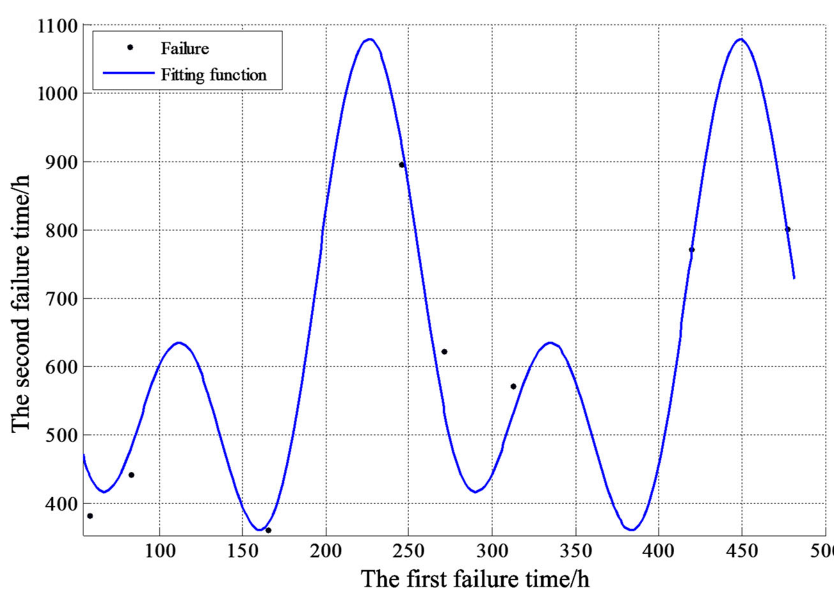


Fig. 22 Relevance between the first failure and second failure of machine tools

CNC machine tools is discussed by simple correlation and complex correlation. It is concluded that the occurrence of the former failure will have an important impact on the latter failure.

The research results lay a foundation for the accurate establishment of CNC machine tool early failure model. The accuracy of the early failure model is the premise of the early failure mechanism analysis and early failure elimination measures establishment.

Therefore, the method proposed in this paper not only enriches the existing early failure analysis methods, but also provides a basis for shortening the early failure period of CNC machine tools and improving their reliability.

Authors' contributions Yulong Li analyzed the data and wrote the manuscript. Xiaogang Zhang and Yan Ran were the major contributors in collecting the data. Genbao Zhang polished the manuscript. All authors read and approved the final manuscript.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval Not applicable.

Consent to participate Not applicable.

Consent to publish Not applicable.

Code availability Not applicable.

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