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ORIGINAL ARTICLE

Machining condition-based stochastic modeling of cutting tool's life

Arash Zaretalab [1] · Mani Sharifi [2,3] · Sharareh Taghipour [4]

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Abstract

One of the major problems in the application of machining processes is the cutting tool life estimation. In this regard, different studies with various assumptions have been conducted to analyze tool wear characteristics under various cutting conditions to achieve different objectives. Traditional models for the analysis of tool life are mostly based on deterministic approaches, and the variations in cutting conditions are overlooked, and the tool life is not precisely matched with predicted values by these methods. In recent years, researchers have considered using the stochastic approach in forecasting tool life. Among them, Weibull distribution has special significance. One problem in using these approaches is the accurate estimation of tool's life distribution functions based on the empirical information. In other words, although many researchers have considered Weibull an appropriate distribution for the cutting tool life modeling, however, the estimation of its parameters has certain inherent complexities. In this research, a hybrid methodology is presented to determine the parameters of the tool life distribution, by using the design of experiment (DOE) based on Box-Behnken design (BBD), total time on the test (TTT) transform, and golden section search (GSS). The estimation method of Weibull distribution parameters in this paper is compared with well-known techniques such as the least square method and maximum likelihood estimation. Finally, the proposed methodology was implemented in a case study, and the results were reported. The values of R^2 for shape and scale parameters are 92.52% and 96.80%, respectively, which confirm the adequacy of the proposed methodology in the practical applications.

Keywords Stochastic tool's life modeling · Weibull distribution · TTT transform · Golden section search · Machining conditions

1 Introduction

Machining is an essential industry in developed countries [1]. Despite the developments of machines' tools and coating technologies, tool wear poses a significant challenge in cutting processes, reliability determination, and product quality [2]. Tool wear can negatively affect the quality of producing workpiece and surface roughness. It can also increase the production time and machining process cost [3].

Taylor's equation is adopted to predict the amount of flank wear, and hence the tool life [4], in which the relationship between cutting speed (V_c) and tool life (T) is defined as (V_c. $T^n = C$), where C and n depend on a specific composition of a workpiece and cutting tool materials [2].

Given the increasing industrial application of new machining technologies (e.g., high cutting speed or dry cutting), the existing equations for tool life should be updated based on constants. Moreover, although these equations can be used to predict the tool life, it is difficult for tools designers to receive more information about the extent of wear, wear specifications, or wear mechanisms [5]. Therefore, the tool life equations cannot be used in all cutting conditions [6, 7].

In the previous studies, tool life is assumed to be deterministic, which allows the calculation of the tool life accurately when the process parameters are provided. In this regard, Noel et al. [8] reported that ignoring the variation of tool life results in unsuccessful planning and low accessibility of the machining process. Therefore, many researchers have investigated stochastic approaches to model tool life. Hundreds of machining experiments in the study of Wager and Barash [9] confirmed that the real life of HSS tools is entirely different from

✉ Mani Sharifi
manisharifi@ryerson.ca

1 Department of Industrial Engineering, Amirkabir University of Technology, Tehran, Iran

2 Mechanical and Industrial Engineering Department, Ryerson University, Toronto, ON, Canada

3 Reliability, Risk and Maintenance Research Laboratory (RRMR Lab), Ryerson University, Toronto, ON, Canada

4 Mechanical and Industrial Engineering Department, Ryerson University, Toronto, ON, Canada

deterministic predictions. Based on these results, the authors reported that normal distribution is suitable for tool life as long as the complete breaking of the tool is concerned. In another study, using a normal distribution and Bayesian inference, a tool life model was developed for the turning and milling process [10]. Dasic et al. [11] emphasized on the easiness and symmetrical features of normal distribution. They argued that despite the observed data skewness, the normal distribution is suitable for reliability modeling [11]. However, the normal distribution allowed the inclusion of negative values in tool life. Moreover, Wager and Barash [9] found skewed data in their study on the life of the cutting tool and concluded that the normal distribution cannot represent the tool life's behavior, when it is not symmetric. Therefore, they recommended using the log-normal distribution or Weibull distribution for tool life modeling. Log-normal distribution has been used for tool wear modeling by some other researchers as well [12, 13].

Regarding observations on the metal cutting process, Ramalingam [13] concluded that given the changing rate of detachment over time, log-normal distribution is more suitable for tool life modeling. However, log-normal distribution does not realistically represent the tool life's distribution. This conclusion is made based on the failure rate function. According to this function, the failure rate increases over a certain period, after which it reduces to zero. Apart from a few cases, this type of failure rate function cannot be used in usual machining processes [14].

According to Rausand and Høyland [15], an inverse Gaussian distribution represents the distribution of tool life more realistically than a log-normal distribution. The shape of the failure rate function of this distribution resembles the shape of a log-normal distribution. Yet, over time, the failure rate function gets closer to a non-zero value. The studies conducted by Galante et al. [16] indicated the applicability of the inverse Gaussian distribution for tool life modeling.

The analyses of cutting tools wear using Bernstein distribution show that Bernstein distribution tends to inverse Gaussian distribution when the primary tool wear is zero [17, 18]. However, the main problem of Bernstein and inverse Gaussian distributions is the difficulty of estimating their parameters, which makes the practical use of these two distributions challenging.

Meanwhile, various studies represent Weibull distribution as a suitable distribution for cutting tool life modeling [19, 20]. Wager and Barash [9] proposed a Weibull distribution as a flexible distribution. The main advantage of this distribution is that it allows the modeling of several shapes of failure rate functions, including increasing, decreasing, and constant rate [14]. The study of Elwardany and Elbestwai [21] on the flexibility and shape of reliability function and failure rate function showed that a Weibull distribution is suitable when a tool breaks under the influence of a shock.

Pandit [22] showed that an exponential distribution as a particular subclass of Weibull distribution is an appropriate approximation of tool life, used in high-speed machining. Likewise, Elwardany and Elbestawi [21] suggested the possibility of using a Weibull distribution when the distinction among various failure modes is impossible.

Based on the surveyed literature, it can be concluded that no study has investigated the impact of machining conditions, such as spindle speed, feed rate, and depth of cut on the parameters of Weibull distribution. Using an empirical approach, this paper aims to propose a tool life model based on Weibull distribution. In this methodology, the Weibull parameters under various machining conditions are obtained using the Box-Behnken design, TTT transform, and golden section search.

In this paper, a hybrid methodology is developed to achieve two objectives. The first objective is to model the Weibull distribution's parameters precisely for a specific machining condition that, through it, the cutting tool life distribution is determined. The second objective is the identification of variations in tools' life distribution due to the changes in the machining variables. To achieve these objectives, in the proposed methodology, the design of the experiment is adopted using Box-Behnken design. The Total time on test (TTT) transform is conducted on the tool life data. Moreover, the golden section search (GSS) algorithm is applied to estimate the tool life's distribution. For this purpose, the relation between the Weibull distribution's parameters and the machining conditions is determined as a full quadratic model. Finally, the proposed method is implemented in a milling operation, and the results are reported.

The rest of the paper is organized as follows: Section 2 presents the problem definition in detail. In Section 3, the Weibull distribution estimators are compared. Section 4 describes the material and experimental planning for the case study. Section 5 is focused on the result and discussion, and finally, Section 6 presents the conclusions.

2 Problem definition

Accurate estimation of tool life and monitoring the cutting tool's performance during machining operations is critical since the quality of the end-product and productivity rate depend on the functional state of the tool [23]. Replacing tools in short periods of time leads to increased tool costs. On the other hand, replacing tools over long periods of time may lead to tool breakdowns during the operation and cause damage to the workpiece and even the machine tool. Therefore, accurate estimation of tool life distribution's parameters can lead to the optimization of tool replacement time and thus reduce the production costs [24, 25].

In this section, assuming the tool life follows a Weibull distribution, a mathematical approach to model the parameters of the tool life distribution is presented. This study aims to determine the relationship between the machining conditions and the parameters of the Weibull distribution and investigate how the tool life's parameters vary as the machining conditions change. As it is explained in Section 2.1, the design of experiment (DOE) is used based on BBD. The nomenclatures used in this paper are shown in Table 1.

2.1 Design of experiment based on Box-Behnken design

The RSM is a non-linear multivariate model that involves the DOE to provide reliable responses and fits the best surface to the data [26]. One of the RSM methods is BBD, which is a second-order design based on a 3-level partial factorial design [27, 28]. BBD was used in this research because it is more efficient than the central composite design (CCD) [29]. This method can estimate the values of the parameters in a quadratic model, making it possible for the responses to be modeled by a second-order polynomial as in Eq. 1:

$$Y = \beta_0 + \beta_1.x_1 + \beta_2.x_2 + \beta_3.x_3 + \beta_4.x_1.x_2 + \beta_5.x_1.x_3 + \beta_6.x_2.x_3 + \beta_7.x_1^2 + \beta_8.x_2^2 + \beta_9.x_3^2 \quad (1)$$

where x_1, x_2, and x_3 represent independent variables and γ represents the response variable. β_0 represents intercept, while β_1 to β_9 denote the model's coefficients. The number of experiments can be obtained through Eq. 2:

$$N = 2.k.(k-1) + C_0 \quad (2)$$

where N and k represent the number of experiments and the number of independent variables, respectively. C_0 indicates the number of center points in the experiment.

In this study, the spindle speed, feed rate, and depth of cut are the independent variables. It is worth noting that in all studies conducted using DOE in machining processes, the response variables are quantitative and measurable. Surface roughness, flank wear, machining time, and the end of the tool's life are some of the response variables commonly considered. However, in the current study, a Weibull distribution's parameters are considered response variables, which cannot be directly calculated based on a single machining experiment. Given different machining conditions, these experiments aim to realize how the tool life's parameters vary. To this end, Section 2.2 describes a mathematical transformation, called the total time on test (TTT) transform, and explains its application in the DOE.

Table 1 Nomenclatures

N	Number of experiments
n	Number of inserts (repetition)
k	Number of independent variables
C_0	Number of center points in the experiment
t_i	Failure time for the ith insert (repetition)
$T(t_i)$	Total time on test for the ith failure
$E(t)$	Life mathematical expectation
$F(t)$	Cumulative distribution function
$R(t)$	Reliability function
$h_f^{-1}(v)$	Total time on test transform
$G(v)$	Scaled total time on test transform
α	The shape parameter of the Weibull distribution
λ	Scale parameter of the Weibull distribution
n	Spindle Speed
f	Feed rate
ap	Depth of cut
SSE	Summation squared error
d	Width of the workpiece
L	Length of the workpiece

2.2 TTT transform

Consider an experiment in which a cutting tool machines a workpiece under specific machining conditions. The experiment with fixed machining conditions was repeated n times, and each time, the tool life is recorded. Then, the recordings are arranged in ascending order ($t_1, \ldots, t_n$). The tool life is assumed to follow a continuous distribution that has a strictly increasing cumulative distribution function with a finite mean. Given these conditions, the total time on test for ith failure $\{T(t_i)\}$ is defined as Eq. 3:

$$T(t_i) = \sum_{j=1}^{i} t_j + (n-i)t_i \quad (3)$$

In order to scale the TTT values for ith failure, each value should be divided by $\{T(t_n)\}$. Now, a plot is drawn with x axis and y axis representing (i/n; $i = 1, \ldots, n$) and $T(t_i)/T(t_n)$; $i = 1, \ldots, n$, respectively. The plot is called the TTT plot.

Rausand and Høyland [15] indicated that $T(t_i)$ can be calculated as Eq. 4, where $F_n(u)$ represents the function of experimental life distribution. Accordingly, they demonstrated that when n tends to infinity, $E(t)$ can be determined as Eq. 5, where $E(t)$ is the mathematical life expectation.

$$T(t_i) = n\int_0^{t_i}(1-F_n(u))du \quad (4)$$

$$E(t) = \frac{1}{n}\sum_{i=1}^{n} t_i = \int_0^{F^{-1}(1)}(1-F(u))du \quad (5)$$

Now, $h_f^{-1}(v)$ denotes the TTT transform on $F(t)$, which is expressed in Eq. 6:

$$h_F^{-1}(v) = \int_0^{F^{-1}(v)} (1-F(u))du; 0 \le v \le 1 \tag{6}$$

$$G(v) = \frac{h_f^{-1}(v)}{h_f^{-1}(1)}; for\, 0 \le v \le 1 \tag{7}$$

In Eq. (7), $G(v)$ denotes the scaled TTT transform on $F(t)$. Given that the focus of this study is on Weibull distribution if $F(u)$ is replaced by $\left\{1-e^{-(\lambda.u)\ \alpha}\right\}$, where $\lambda > 0$, $\alpha > 0$, $u \ge 0$, and $F^{-1}(v)$ is replaced by $(1/\lambda)$. $\{-Ln(1-v)\}^{1/\alpha}$, where $0 \le v \le 1$, the TTT transform, is achieved in Eq. 6. Consequently, Eq. 8 can be written for the Weibull distribution:

$$h_F^{-1}(v) = \int_0^{\frac{1}{\lambda}(-\ln(1-v))^{1/\alpha}} e^{-(\lambda u)^{\alpha}} du; 0 \le v \le 1 \tag{8}$$

Equation 8 can be rewritten as Eq. 9 with the change of variable $t = (\lambda.\ u)^{\alpha}$:

$$h_F^{-1}(v) = \frac{1}{\alpha\lambda} \int_0^{-\ln(1-v)} t^{\left(\frac{1}{\alpha}-1\right)} e^{-t} dt; 0 \le v \le 1 \tag{9}$$

Now, $G(v)$ is obtained for the Weibull distribution as follows:

$$G(v) = \frac{\frac{1}{\alpha\lambda}\int_0^{-\ln(1-v)} t^{\left(\frac{1}{\alpha}-1\right)} e^{-t} dt}{\frac{1}{\alpha\lambda}\int_0^{\infty} t^{\left(\frac{1}{\alpha}-1\right)} e^{-t} dt} = \frac{\frac{1}{\alpha\lambda}\int_0^{-\ln(1-v)} t^{\left(\frac{1}{\alpha}-1\right)} e^{-t} dt}{\frac{1}{\alpha\lambda}\Gamma\left(\frac{1}{\alpha}\right)} = \frac{\frac{1}{\alpha\lambda}\int_0^{-\ln(1-v)} t^{\left(\frac{1}{\alpha}-1\right)} e^{-t} dt}{\frac{1}{\lambda}\Gamma\left(\frac{1}{\alpha}+1\right)} = \frac{\frac{1}{\alpha}\int_0^{-\ln(1-v)} t^{\left(\frac{1}{\alpha}-1\right)} e^{-t} dt}{\Gamma\left(\frac{1}{\alpha}+1\right)} \tag{10}$$

The following points are essential to make about (v):

1. $G(v)$ is a transform on the Weibull distribution, which is independent of parameter λ.
2. The numerator of this fraction is an incomplete gamma function.
3. It cannot be stated as an explicit mathematical function and can be calculated only numerically.
4. Its exact value cannot be calculated for each v. However, there are several methods to approximate the value.

Rausand and Høyland reported a specific relationship between $h_F^{-1}(v)$ behavior and failure rate [15]. They demonstrated that if $h_F^{-1}(v)$ is a convex function, the F distribution would have an increasing failure rate, and if $h_F^{-1}(v)$ is a concave function, the F distribution has a decreasing failure rate. These conditions are two-way, assuming that F distribution is a continuous function and has a strictly increasing cumulative distribution.

The application of $G(v)$ is the estimation of TTT based on different values of v when $0 \le v \le 1$. Equation 11 defines the relationship between $G(v)$ and the scaled total time on the test:

$$\frac{h_f^{-1}(v)}{h_f^{-1}(1)} = \frac{T(t_i)}{T(t_n)} \tag{11}$$

Equation 11 holds for $i = 1, \ldots, n$, where $v = (i/n)$. Equation 11 is used in this research to analyze the parameters of the cutting tool life's distribution, which is modeled according to Weibull distribution. Section 2-3 will discuss how using convex optimization at each level of the BBD experiment based on TTT transform, the shape and scale parameters can be estimated. Finally, the relationship between the machining conditions and these parameters will be modeled.

2.3 Estimating the parameters of the cutting tool life distribution based on the Weibull distribution

This section introduces a mathematical approach for estimating the Weibull distribution's parameters. This approach is used to obtain the relationship between these parameters and the machining conditions. In this study, a hybrid methodology is developed, which consists of a DOE based on BBD, TTT transform, and convex optimization using the golden section. In this methodology, based on BBD for each experiment, the spindle speed, feed rate, and the depth of cut are initially determined through the DOE. Then, a predefined number of inserts (n) are performed on the workpiece, and the tool life (t_i) is recorded for each insert. Then, using Eq. 3, $T(t_i)/T(t_n)$ ratio is calculated under the scaled total TTT. In the next stage, $G(v)$, which is obtained from Eq. 12, is fitted on the TTT plot data, such that the summation squared error (*SSE*) is minimum. Therefore,

$$Min\, SSE = \sum_{i=1}^{n} \left(G\left(\frac{i}{n}\right) - \frac{T(t_i)}{T(t_n)} \right)^2 \tag{12}$$

The optimization of the function above involves only one variable, namely, α, which is the shape parameter of the Weibull distribution for the tool life because $G(v)$ is independent of λ. Now, if *SSE* is calculated for various values of α, and all the obtained points are connected on a plot, a unimodal function emerges.

This study uses the golden section search (GSS) method to optimize Eq. 12. The GSS was extended to optimize the unimodal functions by Kiffer [30] and other researchers [31–34]. In Appendix A, a geometrical interpretation and the pseudocode of this method to optimize Eq. 12 and calculate the shape parameter are presented.

After estimating the shape parameter of the Weibull distribution (α), the relationship between the scale parameter and the distribution's mean is used to calculate the scale parameter

(λ), which is equal to $E(T) = \Gamma(1 + 1/\alpha)/\lambda$. On the other hand, based on the observations of the tool life for each level of BBD, $E(t)$ is calculated using $E(T) = \sum_{i=1}^{n} t_i/n$, where n is the number of the inserts. Then, the scale parameter for each level of the BBD experiment is obtained using α in Eq. 13.

$$\lambda = \frac{\Gamma\left(\frac{1}{\alpha} + 1\right)}{E(t)} \tag{13}$$

Figure 1 displays the proposed methodology, which combines a BBD-based DOE, TTT transform, and optimization through GSS to determine the distribution parameters of a tool life according to the machining conditions. This approach not only supports the stochastic modeling of the tool life, but it also represents the impact of the changing machining conditions on the tool life's distribution.

As Fig. 1 indicates, the BBD-based design of the experiment initially introduces the values for each machining parameter. A predefined number of repetitions (inserts) are needed to increase the accuracy of each experiment introduced by BBD. Then, using TTT transform on the obtained data, a convex optimization problem is applied. This problem contains a uni-modal and a single variable function, which is optimized by the GSS method. The output of the GSS is the shape parameter of the Weibull distribution (α, β) under a set of specified and constant values of the machining conditions. The scale parameter of the Weibull distribution (α, β) is calculated using the shape parameter and the mean of the Weibull distribution. This process is performed for all of the designed experiments in BBD. Finally, the relationship between each parameter of the tool life's distribution and the machining conditions is modeled using a full quadratic function.

3 Comparison of the Weibull distribution's estimators with the methods of LSM and MLE

In the proposed methodology, the Weibull distribution parameters are determined by the combination of TTT transform and GSS for each experiment. There are various methods to estimate the parameters of Weibull distribution, such as the least square method (LSM), maximum likelihood estimation (MLE), method of the logarithmic moment (MLM), percentile method, and the method of moments (MM) [35]. The estimation errors of these methods depend on the sample size (n). Thus, the estimation error for these methods is considerable for low values of n.

For this purpose, the combination of TTT transform and GSS performance and the two methods, namely, LSM and MLE, are compared with the Weibull distribution's parameters estimated in this study. LSM and MLE are implemented by MINITAB software, in which the density function of the Weibull distribution is considered $f(x) = \left(\frac{\alpha}{\beta}\right).\left(\frac{x}{\beta}\right)^{\alpha-1}.e^{-(x/\beta)^{\alpha}}$. In the proposed methodology, we considered that $\beta = (1/\lambda)$. To compare the performance of the methods in estimating the Weibull distribution's parameters, three sets of the parameters, including $(\alpha, \beta) = (2, 1000), (3, 1500), (4, 2000)$, are investigated. For the three Weibull distributions, 1000 samples of size five were simulated using the Monte Carlo method. For sample i, the Weibull distribution's parameters are estimated as $(\widehat{\alpha}_i, \widehat{\beta}_i)$. Then, the result of each parameter is calculated using Eqs. 14 and 15:

$$\overline{\alpha} = \frac{\sum_{i=1}^{1000} \widehat{\alpha}_i}{1000} \tag{14}$$

$$\overline{\beta} = \frac{\sum_{i=1}^{1000} \widehat{\beta}_i}{1000} \tag{15}$$

In this paper, the estimation error is determined through the normalized root to mean square error (NRMSE), provided in Eq. 16:

$$NRMSE = \sqrt{\frac{1}{1000}\sum_{i=1}^{1000}\left(\frac{\widehat{\alpha}_i - \alpha}{\alpha}\right)^2 + \left(\frac{\widehat{\beta}_i - \beta}{\beta}\right)^2} \tag{16}$$

The values of $\overline{\alpha}$ and $\overline{\beta}$ obtained from the estimators and the NRMSE are reported in Table 2 for the three methods. The estimated errors of the TTT transform and GSS of the proposed method are significantly lower than those determined by LSM and MLE. Figure 2 shows the reliability function of the Weibull distribution schematically under $\overline{\alpha}$ and $\overline{\beta}$ from each method. The figure shows that the accuracy of the TTT transform and GSS is better than those of LSM and MLE.

4 Material and experimental planning

To implement the proposed methodology, a three-axis CNC milling machine under various machining conditions shown in Table 3 is considered. This machining process was carried out on the steel 304 workpiece with 260-mm length and 40-mm width. Moreover, three-edged cemented carbide inserts were used as the cutting tool. The upper and lower bounds of the machining conditions were defined based on the milling machine's operational specifications. Table 3 shows the machining process's characteristics.

The cutting speed (V) is obtained using Eq. 17, where d is the cutting width and n_S is spindle speed:

$$V = \frac{\pi.d.n_S}{1000} \tag{17}$$

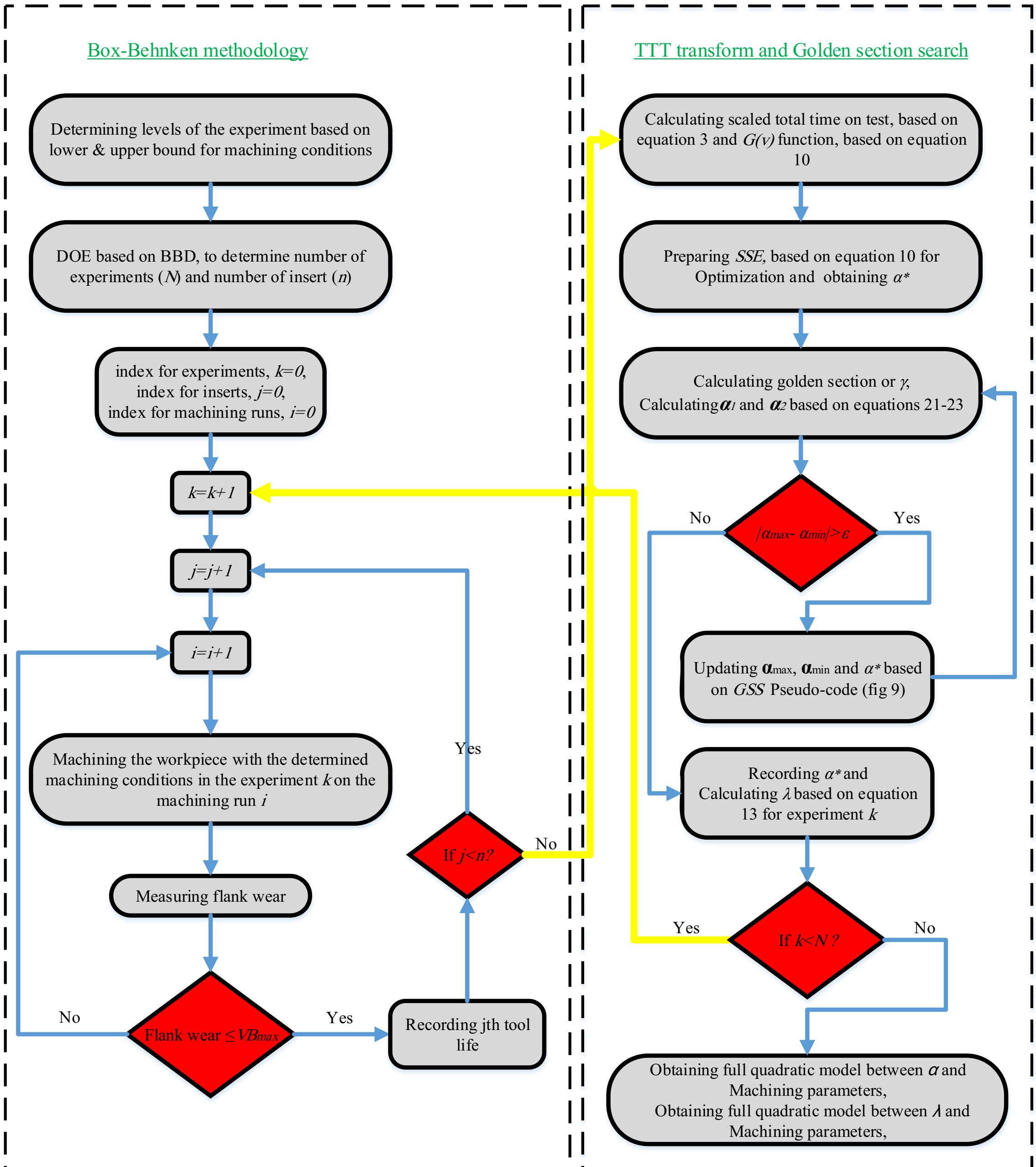

Fig. 1 The flowchart of the proposed methodology

Furthermore, the machining time on each workpiece is obtained by the "volume of removed material/material removal rate" [36] and is calculated using Eq. 18:

$$t = \frac{Vol}{n_S . f . ap . d} \tag{18}$$

In this formulation, the denominator is the material removal rate (n_S is the spindle speed, f is the feed rate, ap is the depth of cut, and d is the width of cut), and Vol, which is equal to $Vol = ap \times L \times d$, is the volume of the removed material from the workpiece surface, where L is the length of cut.

5 Results and discussion

Based on ISO 3685 (1993) [37], the tool life is defined as the period of time, which the tool can efficiently produce the workpiece with the required dimensions and surface roughness. Typically, flank wear has a critical impact on the quality of the workpiece. Therefore, V_B is usually taken as a criterion for the tool life:

1) The average width of flank wear land (VB_B) equals to 0.3 mm, if the wear patterns formed on the relief face of the cutting tool are regular.

Table 2 Results of the estimation of the parameters by LSM, MLE, and the TTT transform and GSS

α	β	Method	$\overline{\alpha}$	$\overline{\beta}$	NRMSE
2	1000	LSM	2.3349	992.2624	0.6917
		MLE	2.8933	978.1068	0.9087
		TTT transform and GSS	2.0242	967.8314	0.4971
3	1500	LSM	3.4776	1484.7196	0.6570
		MLE	4.3035	1470.6493	0.8748
		TTT transform and GSS	2.8381	1485.0398	0.3915
4	2000	LSM	4.6370	1980.1734	0.6492
		MLE	5.7382	1966.0513	0.8690
		TTT transform and GSS	3.6635	1999.8304	0.3614

2) The maximum width of flank wear land (VB_{max}) equals to 0.6 mm if the wear patterns formed on the relief face of the cutting tool are irregular.

In this study, the flank wear was measured from the captured images of the cutting tool using a machine vision. Vision systems are suitable for assessing the tool wear and surface quality of the workpiece. Some studies about the use of vision systems for tool wear are reported by Kurada and Bradley [38]. In this research, the workpiece width is equal to the cut width (or the milling tool diameter). After the first machining run, the insert was removed from the tool holder, and the image of the side profile of the tooltip was captured. The width of the flank wear land (VB_{Bmax}) was measured from the obtained profile. The above procedure was then repeated until VB_{Bmax} reaches 0.3 mm as the tool life criterion. At this stage, the insert is considered to be worn out or "failed." Recently, Broto et al. [39] have considered the same criterion for the end of tool life in a study on a milling process.

By considering $k = 3$ (namely, the spindle speed, feed rate, and the depth of cut), and $C_0 = 1$, the number of experiments equals to 13. In this cube design, the machining parameters are determined and based on the specified machining conditions, and each experiment is conducted with a predetermined number of repetitions. In the current research, five repetitions are considered. It is worth mentioning that the cube design is defined using a set of hypothetical points in the middle of each side and the center of a multidimensional cube. Table 4 presents the machining parameters for each experiment

Figure 3 a–d show the flank wear progress for the first insert of experiment number 4. The flank wear reaches the maximum permissible value due to the high amount of spindle speed and feed rate in this experiment.

In order to show the impact of the variation in the machining conditions on the flank wear, the flank wear progress for four randomly selected experiments are shown in Fig. 4 a–d. Based on Section 2.2 and the presented flowchart in Fig. 1, the values of $T(t_i)$ are calculated for each experiment. To do this,

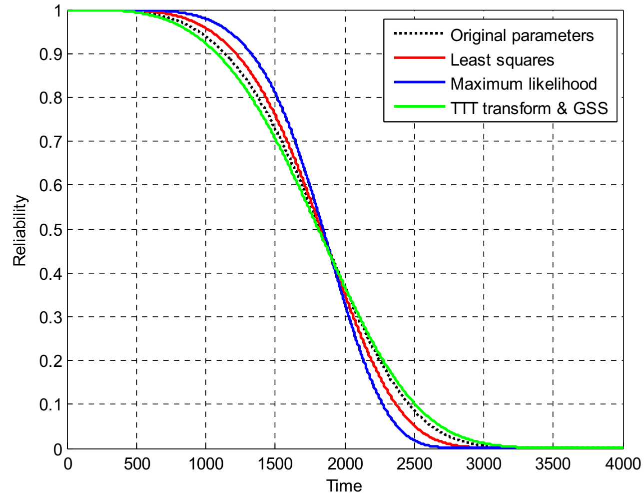

Fig. 2 Comparison of all estimators for $\alpha = 4$ and $\beta = 2000$

the first obtained data of the tool lifetime (t_i) are sorted in ascending order; then, based on Eq. 3, the TTT is calculated. Finally, these values are divided by $T(t_n)$. Table 5 presents the process of the required calculations for four levels of the design of experiments.

Then, the Weibull parameters in each level of the designing experiments should be determined. For this purpose, according to Section 3.2, $G(v)$ and SSE functions are obtained for each experiment based on Eqs. 10 and 12. As it is mentioned in Section 2.3 with SSE minimization, using the GSS, the shape parameter of the Weibull distribution is determined for each designed experiment based on BBD. Figure 5 a–d demonstrate the trend of implementing the iterations of the GSS for four experiments. In this figure, the blue line is the SSE function, the red points are the solutions in each iteration of the GSS, and the green points are the global optimum.

Figure 6 a–d show the TTT plot based on the description provided in Section 2.2 for the four experiments. The blue points are the values of the scaled total time on test transform

Table 3 Machining process characteristics

Workpiece	Shape: rectangular Material: Steel 304 Length (L): 260 mm Width (d): 40 mm
Machine tool	Three-axis CNC - Emco-PC MILL 100
Tool	Holder: T114-D042-16 Z03 TP16 Insert: Sandvik T-PU-N-16-03-04-H13A
Spindle speed (n)	Lower bound: 1000 rpm Upper bound: 2000 rpm
Feed rate (f)	Lower bound: 0.1 mm/rev Upper bound: 0.3 mm/rev
Depth of cut (ap)	Lower bound: 0.1 mm Upper bound: 0.2 mm
Cooling	None

Table 4 Experimental plan and the results for the tool life

Experiment no.	Machining conditions			Lifetime for the failed inserts				
	Spindle speed (n)	Feed rate (f)	Depth of cut (ap)	t_1	t_2	t_3	t_4	t_5
1	1000	0.1	0.15	1243.00	1865.54	1965.60	2363.06	690.22
2	2000	0.1	0.15	79.13	223.26	238.08	187.58	199.33
3	1000	0.3	0.15	1352.00	135.25	1071.02	178.48	387.48
4	2000	0.3	0.15	86.86	116.44	107.54	62.70	75.61
5	1000	0.2	0.10	1025.42	408.36	913.71	1198.66	507.00
6	2000	0.2	0.10	119.70	100.13	91.00	126.57	108.98
7	1000	0.2	0.20	392.95	178.27	686.51	499.95	990.49
8	2000	0.2	0.20	65.18	61.29	83.48	78.59	118.74
9	1500	0.1	0.10	1872.00	905.02	1132.54	1703.78	1693.75
10	1500	0.3	0.10	508.09	468.00	322.28	494.73	575.47
11	1500	0.1	0.20	813.45	1371.85	1490.39	1831.59	665.08
12	1500	0.3	0.20	48.41	143.94	100.08	64.87	121.33
13	1500	0.2	0.15	402.36	276.61	461.53	189.94	151.86

$T(t_i)/T(t_n)$, and $G(v)$ function is shown by the red line in Fig. 6. Finally, after calculating the shape parameter, to find the scale parameter, Eq. 13 is used.

After obtaining the shape and the scale parameters of the Weibull distribution in each level of the designed experiments, the relationship between these parameters with the machining conditions is modeled using a full quadratic function. Table 6 shows the shape and the scale parameters of the Weibull distribution for all experiment levels. Moreover, in this table, the values of the SSE function obtained in the optimization process using the GSS algorithm are presented. In Fig. 7, the tool reliability functions with 4 different categories of machining conditions are compared, which indicate that each reliability function has different values of Weibull distribution's parameters.

Tables 7 and 8 present the effects of machining conditions on the shape and scale parameters of the Weibull distribution. These tables show the analysis of variances (ANOVA) for the

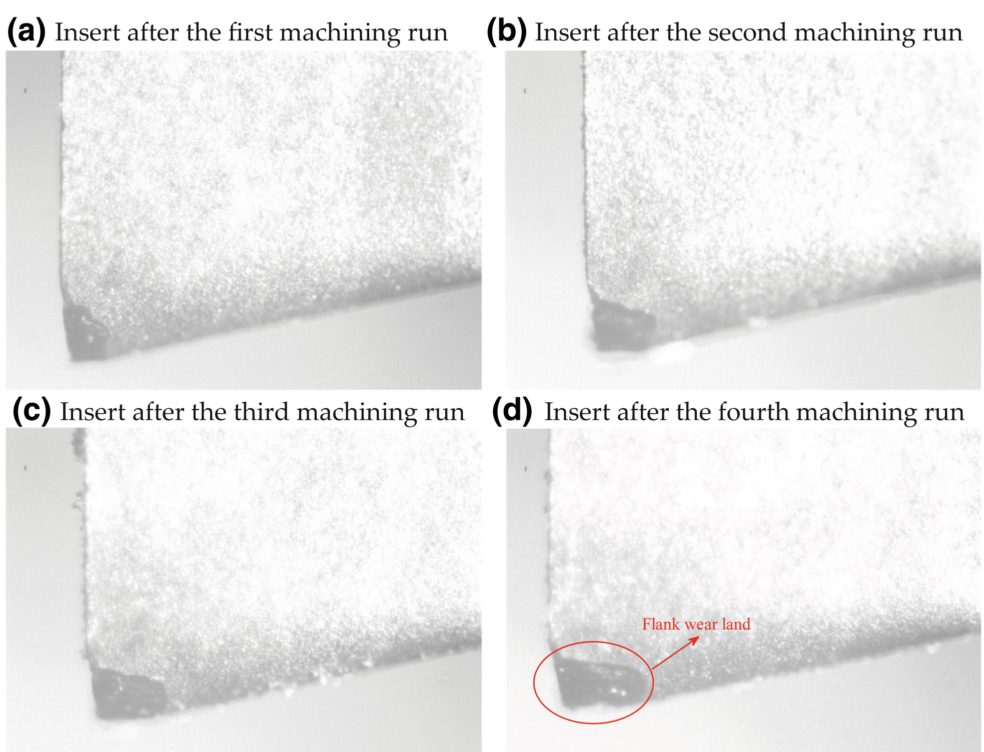

Fig. 3 Real images of the flank wear progress for the first failed to insert in experiment number 4 ($n = 2000$ rpm, $f = 0.3$ mm/rev, $ap = 0.15$ mm)

Table 5 Tool life data and TTT estimates

Experiment number 4					Experiment number 5				
i	t_i	$T(t_i)$	v	$T(t_i)/T(t_n)$	i	t_i	$T(t_i)$	v	$T(t_i)/T(t_n)$
1	62.70	313.51	0.2000	0.6980	1	408.36	2041.80	0.2000	0.5038
2	75.61	365.12	0.4000	0.8129	2	507	2436.36	0.4000	0.6011
3	86.86	398.89	0.6000	0.8881	3	913.71	3656.50	0.6000	0.9021
4	107.54	440.24	0.8000	0.9802	4	1025.42	3879.92	0.8000	0.9573
5	116.44	449.14	1.0000	1.0000	5	1198.66	4053.16	1.0000	1.0000
Experiment number 9					Experiment number 13				
i	t_i	$T(t_i)$	v	$T(t_i)/T(t_n)$	i	t_i	$T(t_i)$	v	$T(t_i)/T(t_n)$
1	905.02	4525.11	0.2000	0.6193	1	151.86	759.32	0.2000	0.5123
2	1132.54	5435.16	0.4000	0.7438	2	189.94	911.64	0.4000	0.6150
3	1693.75	7118.81	0.6000	0.9742	3	276.61	1171.63	0.6000	0.7904
4	1703.78	7138.87	0.8000	0.9770	4	402.36	1423.13	0.8000	0.9601
5	1872	7307.09	1.0000	1.0000	5	461.53	1482.30	1.0000	1.0000

Weibull parameters separately. In these tables, the degree of freedom (*DF*) is the number of changeable values in statistics and is obtained by subtracting the number of the evaluated parameters from the number of independent observations. The sequential sums of squares (SS_{Seq}) depend on the number of entire factors in the model. The adjusted sums of squares

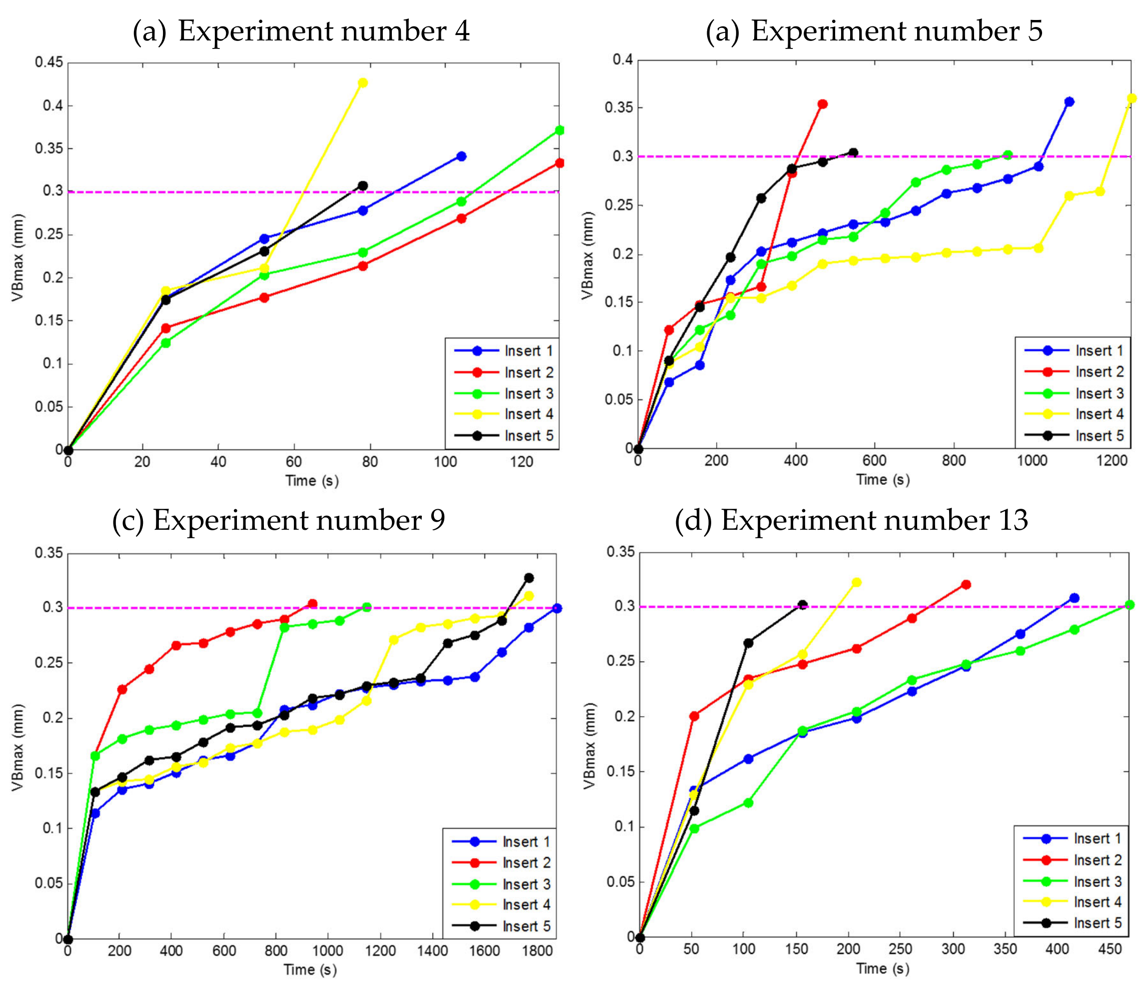

Fig. 4 Flank wear progress for 5 failed inserts in experiments numbers 4, 5, 9, and 13.

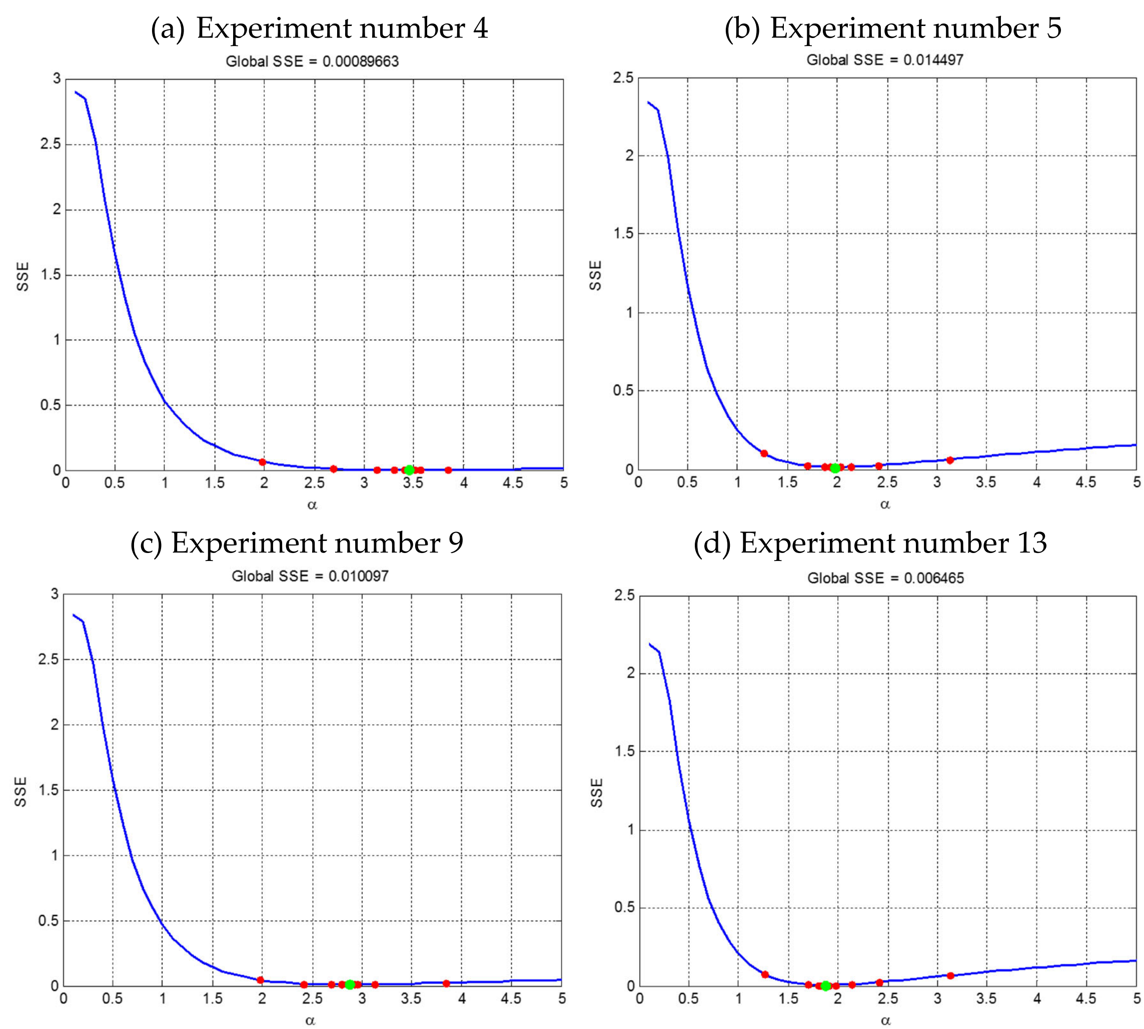

Fig. 5 SSE function and the resultant solutions in each iteration of the golden section search

(SS_{Adj}) are independent of the number of entire factors in the model. The adjusted mean squares (MS_{Adj}) are calculated based on dividing the adjusted sums of squares by the degree of freedom. F is calculated by dividing MS_{Adj} by the error mean square (MSE) for each factor in each row of the table. P-value (P) is used to determine the significance level of the factors.

$$R^2 = \%92.52, R^2_{Adjusted} = \%70.08$$

$$R^2 = \%96.80, R^2_{Adjusted} = \%87.19$$

In Table 7, the values of R^2 and the adjusted R^2 for the shape, parameters are 92.52% and 70.08 %, respectively, which show the appropriate correlation between the quadratic model and the results of the experiments. The obtained model for the shape parameter of the Weibull distribution has been explained in Eq. 19. According to Table 7, the $P-value$ for n and ap in the last column has the least value compared to other factors. These results show the significant impact of the spindle speed and the depth of cut on the shape parameter of the Weibull distribution. The impact of the spindle speed is more prominent than the depth of cut. This is further confirmed by $P-values$ of 0.023 and 0.072 for the spindle speed and the depth of cut, respectively. Moreover, according to Eq. 19 in Table 9, n and ap coefficients are negative, indicating an inverse relationship between these machining parameters and the shape parameter of the Weibull distribution.

In Table 8, the values of R^2 and the adjusted R^2 for the scale, parameters are 96.80% and 87.19 %, respectively, which show the appropriate correlation between the quadratic model and the results of the experiments. The full quadratic model for the scale parameter of the Weibull distribution is shown in Eq. 20 of Table 9. According to Table 8, the spindle speed has the greatest impact on the scale parameter based on $P-value$ (0.005). It should be noted that the significant impact of the cutting speed on the tool life has been confirmed by other studies, which are considered a deterministic tool life.

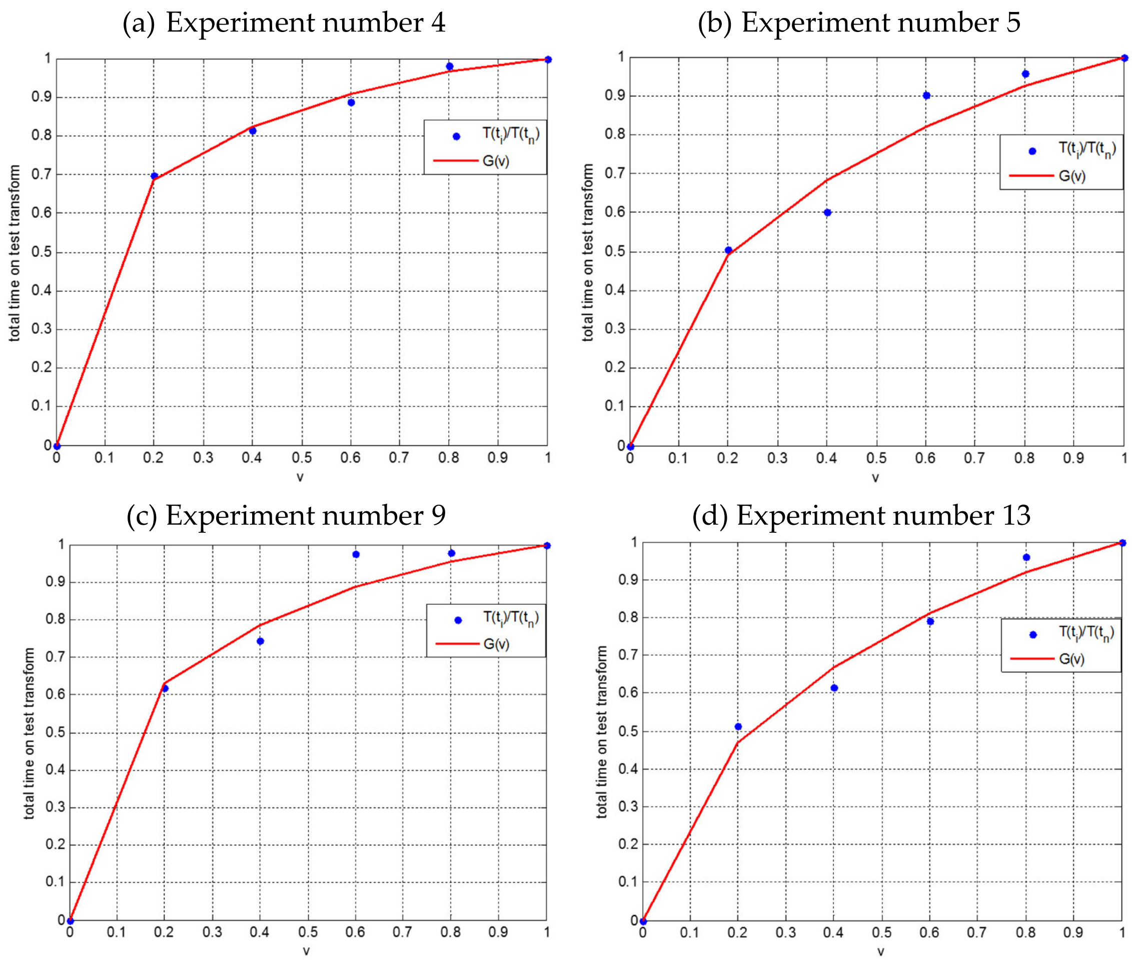

Fig. 6 TTT plot for 5 failed inserts of experiments numbers 4, 5, 9, and 13

Furthermore, in the obtained results, the effects of the feed rate with a *P-value* of 0.028 and the depth of cut with $P-value$ of 0.080 on the scale parameter are noticeable. Moreover, the interaction factor, $f\times ap$ with $P-value$ of 0.086, has a

Table 6 Shape and scale parameters and SSE value of the experimental plan

Number of experiment	Machining conditions			α	λ	*SSE*
	Spindle speed (n)	Feed rate (f)	Depth of cut (ap)			
1	1000	0.1	0.15	1.98924	0.0005453	0.0162
2	2000	0.1	0.15	2.44646	0.0047813	0.0499
3	1000	0.3	0.15	0.89286	0.0016913	0.0357
4	2000	0.3	0.15	3.44661	0.0100082	0.0009
5	1000	0.2	0.10	1.97024	0.0010936	0.0145
6	2000	0.2	0.10	6.41528	0.0085209	0.0002
7	1000	0.2	0.20	1.50150	0.0016422	0.0052
8	2000	0.2	0.20	3.69836	0.0110792	0.0080
9	1500	0.1	0.10	2.87611	0.0006099	0.0101
10	1500	0.3	0.10	4.15211	0.0019175	0.0093
11	1500	0.1	0.20	2.12786	0.0007174	0.0105
12	1500	0.3	0.20	2.00489	0.0092573	0.0043
13	1500	0.2	0.15	1.87706	0.0029944	0.0065

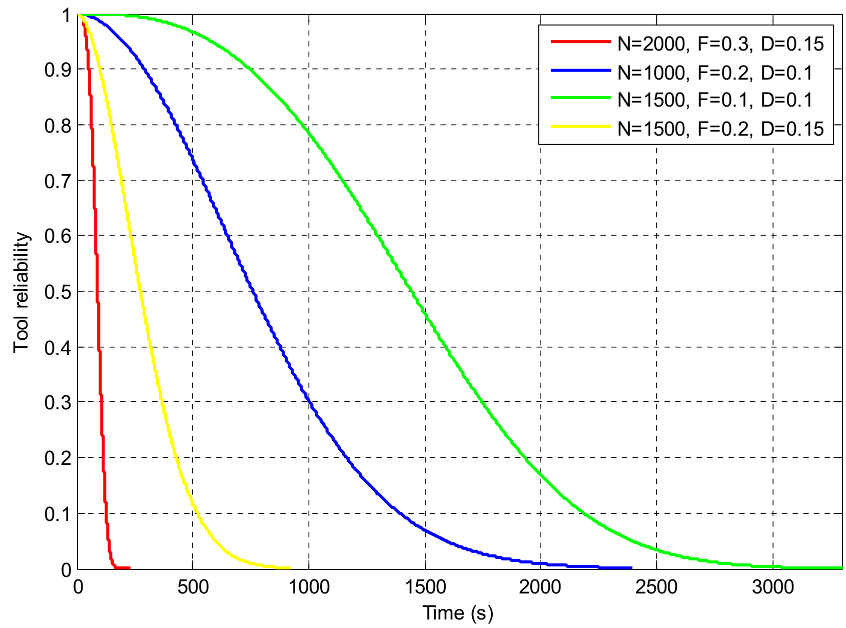

Fig. 7 Tool reliability function for experiments numbers 4, 5, 9, and 13

significant effect on the scale parameter. Finally, due to the negative coefficients of n, ap, and f in Eq. 20, all three parameters, the spindle speed, the feed rate, and the depth of cut, have an inverse relationship with the Weibull scale parameter.

6 Conclusion

In this paper, a hybrid methodology was developed to achieve two objectives. The first objective was to precisely model the Weibull distribution's parameters for a specific machining condition that, through which, the cutting tool life distribution was determined. The second objective was identifying the variations in the tool life's distribution according to the changes in the machining variables. To achieve these objectives, the design of the experiment was adopted using the Box-Behnken design for arranging the experiments. The total time on test (TTT) transform was conducted on the obtained data of the tool life. Moreover, the golden section search (GSS) algorithm was applied to estimate the tool life's distribution. To achieve this, the relation between the Weibull distribution parameters and the machining conditions was determined as a full quadratic model. Finally, the proposed method was implemented in a milling operation, and the obtained results were reported. The appropriate correlation between the Weibull distribution's parameters and the obtained data from the cutting tool

Table 7 Analysis of variance for α

Source	DF	SS_{Seq}	SS_{Adj}	MS_{Adj}	F	P
Regression	9	22.9286	22.9286	2.5476	4.12	0.135
Linear	3	16.4094	16.4094	5.4698	8.85	0.053
n	1	11.6472	11.6472	11.6472	18.85	0.023
f	1	0.1396	0.1396	0.1396	0.23	0.667
ap	1	4.6225	4.6225	4.6225	7.48	0.072
Square	3	3.6675	3.6675	1.2225	1.98	0.295
$n \times n$	1	0.1081	0.4866	0.4866	0.79	0.440
$f \times f$	1	1.0015	0.0478	0.0478	0.08	0.799
$ap \times ap$	1	2.5579	2.5579	2.5579	4.14	0.135
Interaction	3	2.8517	2.8517	0.9506	1.54	0.366
$n \times f$	1	1.0989	1.0989	1.0989	1.78	0.275
$n \times ap$	1	1.2636	1.2636	1.2636	2.05	0.248
$f \times ap$	1	0.4893	0.4893	0.4893	0.79	0.439
Residual error	3	1.8535	1.8535	0.6178		
Total	12	24.7821				

Table 8 Analysis of variance for λ

Source	DF	SS_{Seq}	SS_{Adj}	MS_{Adj}	F	P
Regression	9	0.000187	0.000187	0.000021	10.07	0.042
Linear	3	0.000155	0.000155	0.000052	25.05	0.013
n	1	0.000108	0.000108	0.000108	52.45	0.005
f	1	0.000033	0.000033	0.000033	15.95	0.028
ap	1	0.000014	0.000014	0.000014	6.75	0.080
Square	3	0.000014	0.000014	0.000005	2.22	0.265
$n \times n$	1	0.000010	0.000008	0.000008	3.84	0.145
$f \times f$	1	0.000002	0.000001	0.000001	0.40	0.574
$ap \times ap$	1	0.000001	0.000001	0.000001	0.59	0.499
Interaction	3	0.000018	0.000018	0.000006	2.95	0.199
$n \times f$	1	0.000004	0.000004	0.000004	2.02	0.250
$n \times ap$	1	0.000001	0.000001	0.000001	0.49	0.534
$f \times ap$	1	0.000013	0.000013	0.000013	6.34	0.086
Residual error	3	0.000006	0.000006	0.000002		
Total	12	0.000193				

Table 9 Regression models for the parameters of Weibull distribution

$\alpha = 9.35623 - 0.00184806 \times n + 1.87674 \times f - 94.4347 \times ap + 1.84567e{-}006 \times n^2 - 14.4686 \times f^2 + 423.147 \times ap^2 - 0.0224818 \times n \times ap + 0.0104826 \times n \times f - 69.9483 \times ap \times f$	(19)
$\lambda = 0.0263548 - 2.20641e{-}005 \times n - 0.0406462 \times f - 0.163598 \times ap + 7.44103e{-}009 \times n^2 - 0.0598173 \times f^2 + 0.291717 \times ap^2 + 2.00967e{-}005 \times n \times ap + 2.0404e{-}005 \times n \times f + 0.361617 \times ap \times f$	(20)

life in the full quadratic models indicates the adequacy of the proposed methodology in practical applications. Based on the results, the values of R^2 for the shape and scale parameters are 92.52% and 96.80 % , respectively.

For the future study, a mathematical optimization model can be developed considering the relationship between the machining conditions and the tool life distribution. In this case, the tools' replacement policies and the machining conditions can be simultaneously need optimized.

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Appendix A

To explain the golden section (Heath et al. [40]), a geometrical interpretation is provided. It is assumed that a straight line of length R is divided into two parts so that the ratio of the longer part to the whole line is equal to the ratio of the shorter part to the longer part. In other words, if it is assumed that line AB is divided by point C, such that the length of AC and CB denoted by r_1 and r_2, respectively, and $r_2 < r_1$, then

$$s = \frac{r_1}{r_2} = \frac{r_2}{R} \tag{21}$$

Given that $R = r_1 + r_2$, Eq. 18 is rewritten as follows:

$$s = \frac{r_2}{R} = \frac{r_2}{r_1 + r_2} \tag{22}$$

Dividing the numerator and the denominator of Eq. 22 by r_2 and using Eq. 21, we obtain the following:

$$s = \frac{{}^{r_2}/_{r_2}}{{}^{r_1}/_{r_2} + {}^{r_2}/_{r_2}} = \frac{1}{s+1} \tag{23}$$

Equation $s^2 + s - 1 = 0$ is used to calculate s so that the roots of this equation are obtained through $s = \left(-1 + \sqrt{5}\right)/2$ and $s = \left(-1 - \sqrt{5}\right)/2$. The positive root of this equation is called the golden section, which is denoted by γ in this paper. This ratio is essentially used by the GSS to optimize the uni-modal functions. This paper seeks to determine the specific value of α denoted by α^*, which minimizes the SSE. To this end, interval $[\alpha_{min}, \alpha_{max}]$ is initially defined to find the optimized point. Then, α_1 and α_2 are calculated, using Eqs. 24, 25, and 26:

$$\gamma = \frac{-1 + \sqrt{5}}{2} \tag{24}$$

$$\alpha_1 = \gamma.\alpha_{\min} + (1 - \gamma).\alpha_{\max} \tag{25}$$

$$\alpha_2 = \gamma.\alpha_{\max} + (1 - \gamma).\alpha_{\min} \tag{26}$$

Following the calculation of α_1 and $\alpha_2 SSE$ for each point is calculated and denoted by $SSE(\alpha_1)$ and $SSE(\alpha_2)$, respectively. If $SSE(\alpha_1) < SSE(\alpha_2)$, α^* belongs to interval $[\alpha_{min}, \alpha_2]$; otherwise, α^* belongs to $[\alpha_1, \alpha_{max}]$. Given this clarification, each iteration of the GSS involves two search intervals, which only one of them will be selected for the following searches. The lengths of these intervals need to be equal. The explanation provided above expresses the first GSS iteration. In the next iterations, the search interval will be updated, and Eqs. 25 and 26 will be used to obtain α_1 and α_2.The algorithm will continue likewise until the stop condition is met. The stop condition is defined by the GSS when the length of the search interval is less than ε. Figure 8 shows the first iteration of the GSS algorithm in the interval $[\alpha_{min}, \alpha_{max}]$, where, e.g., $SSE(\alpha_1) < SSE(\alpha_2)$.

Furthermore, two characteristics of the golden sections are initially described to define the convergence rate of the GSS. The first characteristic is $1 - \gamma = \gamma^2$, and the second is $\varphi = 1 + \gamma = (1/\gamma)$, leading to the definition of the golden ratio. In the latter, φ is named the golden ratio, which equals to $\varphi = 0.5 \times \left(1 + \sqrt{5}\right)$. If the number of GSS iterations needed to reach the stop condition equals NOI (number of iterations), the convergence rate of the GSS is φ^{NOI} [41]. Likewise, Shao et al. [42] showed that the length of the search interval by $NOI = 15$ could decrease to less than 1% of the length of the primary interval in the GSS algorithm. Figure 9 shows the pseudo-code of the GSS algorithm.

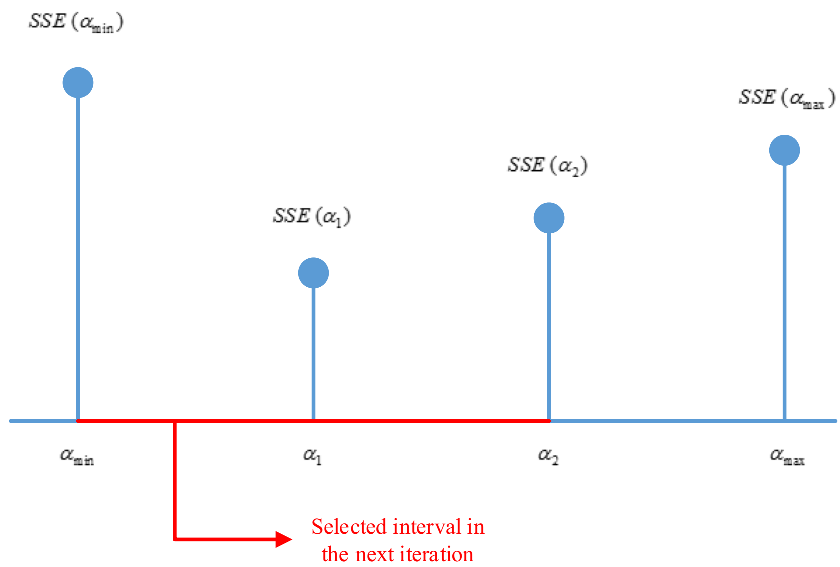

Fig. 8 The first iteration in the golden section search algorithm

Fig. 9 Pseudo-code of the golden section search algorithm

Parameter Setting $(\varepsilon, \alpha_{\min}, \alpha_{\max})$

$\gamma \leftarrow \frac{-1+\sqrt{5}}{2}$ *% calculating golden section*

$\alpha_1 \leftarrow \gamma.\alpha_{\min} + (1-\gamma).\alpha_{\max}$

$\alpha_2 \leftarrow \gamma.\alpha_{\max} + (1-\gamma).\alpha_{\min}$

While $|\alpha_{\max} - \alpha_{\min}| > \varepsilon$ *% checking the stop condition*

  if $SSE(\alpha_1) < SSE(\alpha_2)$

    $\alpha_{\max} \leftarrow \alpha_2$

    $\alpha_2 \leftarrow \alpha_1$

    $\alpha_1 \leftarrow \gamma.\alpha_{\min} + (1-\gamma).\alpha_{\max}$

    $\alpha^* \leftarrow \alpha_1$ *% updating* α^* *by* α_1

  Else

    $\alpha_{\min} \leftarrow \alpha_1$

    $\alpha_1 \leftarrow \alpha_2$

    $\alpha_2 \leftarrow \gamma.\alpha_{\max} + (1-\gamma).\alpha_{\min}$

    $\alpha^* \leftarrow \alpha_2$ *% updating* α^* *by* α_2

  End if

End While

Reporting output (α^*)

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