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Optimal tolerance allocation for five-axis machine tools in consideration of deformation caused by gravity

Jinwei Fan[1] · Haohao Tao[1] · Ri Pan[1] · Dongju Chen[1]

Abstract
Due to the effect of gravity on machine tools, the small deformations inevitably exist. The existing tolerance allocation methods are based on the rigid body assumption, which ignore the small deformations. It will make optimization results inaccurate and increase manufacturing cost. Therefore, a new optimal tolerance allocation method, which integrates the small deformations, is presented in this paper. The establishment of a geometric error model based on tolerance is involved at first. Based on this model and multi-body system theory, the mapping relationship between tolerance and volumetric error of the five-axis machine tool (FAMT) is formulated. Secondly, the small deformations of the FAMT are obtained based on finite element analysis. Then, the optimal tolerance allocation model is established by integrating the small deformations into the constraint conditions. Thirdly, simulation analysis is carried out with this model by using a genetic algorithm. Then, the optimal tolerance allocation scheme is obtained, and the total manufacturing cost after optimization is reduced by approximately 11.5%. Finally, the volumetric errors of the FAMT are calculated based on the two tolerance allocation schemes. The results show that the volumetric errors are within the permitted ranges. Therefore, the proposed method in consideration of the small deformation is feasible and effective.

Keywords Five-axis machine tool · Rigid body assumption · The small deformation · Optimal tolerance allocation model · Total manufacturing cost

1 Introduction

Five-axis machine tools (FAMTs) with clear advantages of higher material removal rate, fewer setups [1, 2], and higher machining efficiency [3] are extensively adaptable for the machining of sculptured surfaces [4]. The surface quality of the workpiece is significantly influenced by the machining performance of the machine tools [5, 6]. Since the machine tools' machining accuracy is determined by the mutual coordination with the accuracy of machine tool components, improving individual components' accuracy may increase the manufacturing cost instead of enhancing the machining accuracy. Therefore, it is of great theoretical and practical significance to obtain an appropriate precision design scheme for machine tools cost-effectively while satisfying the desired machining accuracy requirement during the initial design stage of machine tools.

As indicated by many engineering practices, a bad precision design scheme requires the component with extremely tight or loose tolerance, which results in excessive manufacturing cost, unacceptable quality loss, and poor performance, on the contrary, an excellent precision design scheme with advantages of better performance, lower loss of quality, and reasonable tolerance allocation scheme for the component [7, 8]. In the design stage of machine tools, since the most guiding significance for precision design is tolerance parameters of machine tools' component, the tolerance is regarded as a bridge between accuracy requirement and manufacturing cost of the component [9]. The precision design of machine tools is therefore a problem of optimal tolerance allocation for components. Hence, a reasonable tolerance allocation scheme plays an important role in balancing machining performance requirement and manufacturing cost of machine tools.

✉ Ri Pan
raympan@126.com

Haohao Tao
Taohao_hao@163.com

[1] Beijing Key Laboratory of Advanced Manufacturing Technology, Beijing University of Technology, Beijing 100124, People's Republic of China

In the past, to reduce cost and meet the performance requirement of mechanisms simultaneously, considerable study has been carried out to obtain the optimal tolerance allocation scheme. Wu et al. [10] developed a method that simultaneously considers manufacturing cost and the risk of non-quality to allocate the part tolerance based on an improved genetic algorithm. Sivakumar et al. [11] introduced a multi-objective optimization method for concurrent design of tolerance by using Elitist Non-dominated Sorting Genetic Algorithm (NSGA-II) and Multi-Objective Particle Swarm Optimization (MOPSO). Sivakumar et al. [7] carried out an experiment on overrunning clutch assembly and knuckle joint assembly to contrast the computational effort of NSGA-II and MOPSO algorithms. Geetha et al. [12] took a wheel mounting assembly as an example to research the tolerance allocation by considering the cost (manufacturing cost and machine idle time cost) and machining time. The cost has been significantly reduced. Zhao et al. [13] proposed a tolerance optimization model by taking the minimum manufacturing cost and service quality cost as criteria and taking the component tolerance as a constraint to improve product performance and reduce the total cost. Ghali et al. [14] presented a CAD approach that simultaneously considers functional requirement and manufacturing process to obtain optimal tolerance allocation scheme. The results broaden the tolerance range of key machined dimensions while meeting the design requirement.

Obviously, the abovementioned researches focus on the tolerance allocation of the small-scale assembly, such as overrunning clutch assembly, the rotor key base assembly, knuckle joint assembly, and aim at the effect of tolerance on the dimension chain errors in the interior of an assembly, which have not involved the tolerance allocation of machine tools. In an early design stage of machine tools, the tolerance allocation schemes mainly depend on the engineer's experience. In addition, enterprises are difficult to assess whether tolerance allocation schemes are appropriate for newly designed machine tools. Therefore, it will lead to excessive high precision and manufacturing cost of machine tools. To solve the abovementioned problems, a majority of studies about the optimal tolerance allocation of machine tools have been carried out. Guo et al. [15] developed a state-space model with tolerance and assembly process. The optimal tolerance allocation scheme and the assembly process are achieved by the optimal control theory. Cai et al. [8] presented a method by using the possibility of failure and cost as criteria and using reliability and robustness as constraint conditions to allocate tolerance of parts for enhancing the accuracy of machine tools. Zhang et al. [16] took a special machine tool as an example to research the optimal tolerance allocation by taking the manufacture easiness index as criteria and taking cam surface quality as constraints. According to reference [17], the optimization model is established to maximize reliability and minimize total cost subject to the machining accuracy of the five-axis machining

center. The optimal tolerance allocation scheme was obtained based on the NSGA-II algorithm. Guo et al. [18] carried out research on the multi-objective optimization of machine tools by simultaneously considering accuracy robustness and quality loss to obtain the optimum tolerance scheme. Zhang et al. [19] developed a reliability model based on Rackwite–Fiessler and advanced first order and second moment. The optimal results are obtained by taking manufacturing cost as an optimization objective and taking the reliability of the machining precision as a constraint condition.

The aforementioned researches played a vital role in the tolerance allocation of machine tools and have obtained some positive results. However, there are some tricky problems that remain to be further addressed.

(1) The existing optimal tolerance allocation methods for machine tools are performed under the assumption that all components of the machine tool are rigid body, which ignores the small deformation of machine tools. In fact, it is inevitable that small deformations will exist due to the gravity effect. Furthermore, the machining accuracy requirement of machine tools is generally on the micron scale. Therefore, the small deformations have a major effect on the machining accuracy and manufacturing cost, which are indeed not negligible.

(2) Some of these tolerance optimization methods are carried out under the certain constraint condition (i.e., $0 < t \leq$ initial value). The given initial value will limit the search range of the optimal scheme. Thus, the obtained optimization results are not necessarily the optimal results.

In view of the limitations stated, this paper has proposed a new optimal tolerance allocation method to make a balance between machining accuracy and total manufacturing cost by integrating the small deformation into the constraint conditions, as outlined in the flowchart given in Fig. 1.

To conduct the optimal tolerance allocation method, the geometric error-tolerance model is initially established. Meanwhile, the volumetric error-tolerance model is formulated by the multi-body system (MBS) theory and homogeneous transform matrix (HTM). Then, the small deformation of machine tools due to the gravity effect is acquired by the finite element analysis (FEA). Finally, the optimal tolerance allocation model for five-axis machine tools (FAMTs) in consideration of the small deformation is established. The correctness and effectiveness of the proposed method are proved through simulation validation. The remaining part of this paper is arranged as follows. In Section 2, the geometric error model and the volumetric error model are established. Section 3 obtains the small deformation of FAMT. Section 4 establishes the optimal tolerance allocation model. In Section 5, the simulation validation is conducted. Finally, the conclusions are given in Section 6.

Fig. 1 Flowchart of the optimal tolerance allocation method
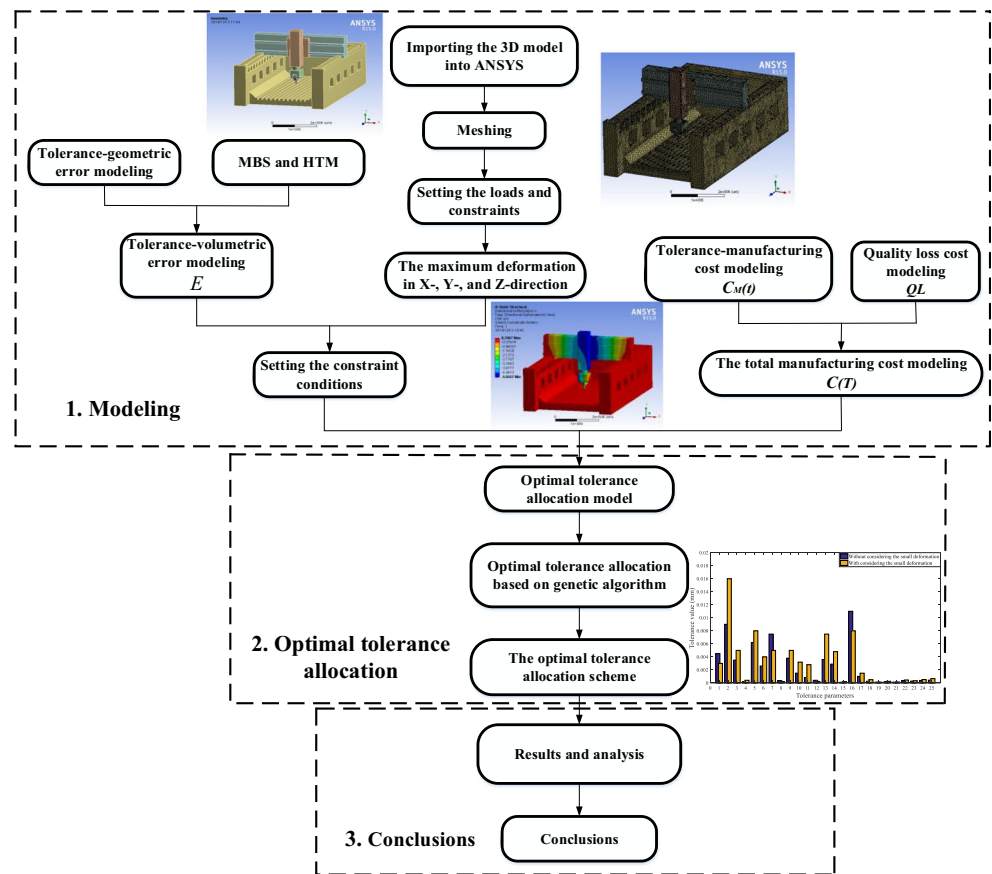
2 Volumetric error modeling based on the tolerance of five-axis machine tools' components

The definition of tolerance is the allowable variation range of actual parameter values which guarantee the quality and performance of components in the design and manufacture stage. Since only the information of tolerance of machine tools' component is known during the design stage of machine tools, the precision design of machine tools is a problem of optimal tolerance allocation for machine tools' components. Therefore, establishing the mapping relationship between machining accuracy and tolerance is the prerequisite and foundation for tolerance allocation. The volumetric error modeling process based on tolerance consists of two steps.

Step 1: Geometric error modeling based on tolerance In this paper, a five-axis machine tool (FAMT) is selected as an example. It contains X-, Y-, Z-, B-, and C-axis, as shown in Fig. 2. According to reference [20], a FAMT with two rotary axes has 37 geometric errors. Geometric errors of machine tools are usually divided into position-dependent geometric errors (PDGEs) and position-independent geometric errors [33]. Therefore, all 37 geometric errors of the researched FAMT and the tolerance that correspond to geometric errors are all

listed, as shown in Table 1. The definitions of all the tolerance parameters are listed in Table 2.

Since the values of geometric errors are unknown in the initial design stage of machine tools, designers obtain geometric errors only by utilizing design experiences. Therefore, it is of paramount importance to establish the mapping relationship between geometric errors and tolerance of machine tools. According to reference [9] cited in our manuscript, the surface profile error of large structural parts fulfill Dirichlet boundary conditions; hence, the surface profile error can be represented by a series of Fourier. Firstly, the mapping relationship

Table 1 The tolerance that corresponds to geometric errors

Geometric error	$\delta_x(x)$	$\delta_y(x)$	$\delta_z(x)$	$\varepsilon_x(x)$	$\varepsilon_y(x)$	$\varepsilon_z(x)$	$\delta_x(y)$	$\delta_y(y)$
Tolerance	t_1	t_2	t_3	t_4	t_3	t_2	t_5	t_6
Geometric error	$\delta_z(y)$	$\varepsilon_x(y)$	$\varepsilon_y(y)$	$\varepsilon_z(y)$	$\delta_x(z)$	$\delta_y(z)$	$\delta_z(z)$	$\varepsilon_x(z)$
Tolerance	t_7	t_7	t_8	t_5	t_9	t_{10}	t_{11}	t_{10}
Geometric error	$\varepsilon_y(z)$	$\varepsilon_z(z)$	$\delta_x(c)$	$\delta_y(c)$	$\delta_z(c)$	$\varepsilon_x(c)$	$\varepsilon_y(c)$	$\varepsilon_z(c)$
Tolerance	t_9	t_{12}	t_{13}	t_{13}	t_{14}	t_{13}	t_{13}	t_{15}
Geometric error	$\delta_x(b)$	$\delta_y(b)$	$\delta_z(b)$	$\varepsilon_x(b)$	$\varepsilon_y(b)$	$\varepsilon_z(b)$	ε_{xy}	ε_{xz}
Tolerance	t_{16}	t_{17}	t_{16}	t_{16}	t_{18}	t_{16}	t_{19}	t_{20}
Geometric error	ε_{yz}	ε_{xc}	ε_{yc}	ε_{xb}	ε_{bz}			
Tolerance	t_{21}	t_{22}	t_{23}	t_{24}	t_{25}			

Table 2 The definition of tolerance parameters

Tolerance parameter	Definition
t_1	The positioning tolerance of screw for X-axis guideway
t_2	Tolerance of straightness in Y–X plate
t_3	Tolerance of straightness in Z–X plate
t_4	Tolerance of parallelism for X-axis guideway
t_5	Tolerance of straightness in X–Y plate
t_6	The positioning tolerance of screw for Y-axis guideway
t_7	Tolerance of straightness in Z–Y plate
t_8	Tolerance of parallelism for Y-axis guideway
t_9	Tolerance of straightness in X–Z plate
t_{10}	tolerance of straightness in Y–Z plate
t_{11}	The positioning tolerance of screw for Z-axis guideway
t_{12}	Tolerance of parallelism for Z-axis guideway
t_{13}	The radial runout tolerance of the C-axis
t_{14}	The axial runout tolerance of the C-axis
t_{15}	The positioning tolerance of the C-axis
t_{16}	The radial runout tolerance of the B-axis
t_{17}	The axial runout tolerance of the B-axis
t_{18}	The positioning tolerance of the B-axis
t_{19}	The perpendicularity tolerance between X-axis and Y-axis
t_{20}	The perpendicularity tolerance between X-axis and Z-axis
t_{21}	The perpendicularity tolerance between Y-axis and Z-axis
t_{22}	The perpendicularity tolerance between X-axis and C-axis
t_{23}	The perpendicularity tolerance between Y-axis and C-axis
t_{24}	The perpendicularity tolerance between X-axis and B-axis
t_{25}	The perpendicularity tolerance between Z-axis and B-axis

Fig. 2 The structure diagram of the FAMT (1, machine bed; 2, workpiece; 3, X-axis slide carriage; 4, Y-axis slide carriage; 5, Z-axis slide carriage; 6, C-axis; 7, B-axis; 8, spindle; 9, cutting tool)

between tolerance and surface profile error of machine tools is formulated. Then, according to the assembly relationship of each part, the mapping relationship between surface profile error and geometric errors is established. Since geometric profile errors are regarded as a bridge between tolerance and geometric errors of machine tools, the mapping relationship between tolerance and geometric error is subsequently established. Due to limited space, only the results of the geometric error model for a linear axis and a rotary axis (i.e., X-axis and C-axis) are given, as shown in Appendix 1. More details of the modeling process can refer to the previous work [9]. In addition, perpendicularity errors are a direct reflection of perpendicularity tolerance.

Step 2: Volumetric error modeling In this study, the commonly utilized multi-body system (MBS) theory and the homogeneous transfer matrix (HTM) are adopted for establishing a volumetric error model [3, 20]. Based on MBS theory, a machine tool can be considered as a kind of MBS consisted of many rigid bodies. Choosing a FAMT with two rotational axes (as shown in Fig. 2) as a research objective, its topological structure is shown in Fig. 3, which describes the kinematic
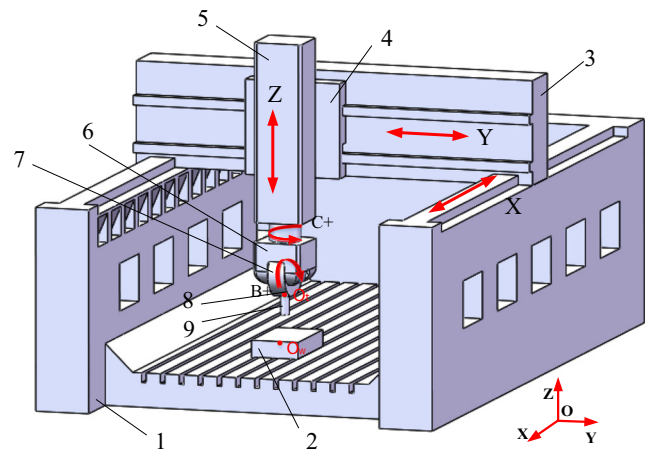
chain of the machine tool, i.e., the tool branch and workpiece branch. Taking a 4×4 HTM to denote the relative movement relationship between the two adjacent bodies.

In the workpiece branch, the ideal tool cutting point position can be described to the workpiece coordinate system (WCS), as shown in Eq. (1).

$$
r_w^i = \left([S1w]_p[S1w]_s\right)^{-1}[S13]_p[S13]_s[S34]_p[S34]_s[S45]_p[S45]_s \\
[S56]_p[S56]_s[S67]_p[S67]_s[S78]_p[S78]_s[S89]_p[S89]_s r_t
$$

(1)

where 1–9 represent the components of researched FAMT, as shown in Fig. 2; W represents the workpiece; $[S(j-1)j]_p$ and $[S(j-1)j]_s$ represents the relative position transformation matrix and relative motion transformation matrix between the rigid body $j-1$ and the adjacent lower body j respectively. $\mathbf{r}_t$ represents the tool cutting point position in the tool coordinate system (TCS). These are described in reference [3].

In the workpiece branch, the actual tool cutting point position can be described to the WCS, as shown in Eq. (2).

$$
r_w^a = \left([S12]_p[S12]_{pe}[S12]_s[S12]_{se}\right)^{-1}[S13]_p[S13]_{pe}[S13]_s[S13]_{se} \quad (2) \\
[S34]_p[S34]_{pe}[S34]_s[S34]_{se}[S45]_p[S45]_{pe}[S45]_s[S45]_{se}[S56]_p \\
[S56]_{pe}[S56]_s[S56]_{se}[S67]_p[S67]_{pe}[S67]_s[S67]_{se}[S78]_p[S78]_{pe} \\
[S78]_s[S78]_{se}[S89]_p[S89]_{pe}[S89]_s[S89]_{se} r_t
$$

where $[S(j-1)j]_{pe}$ and $[S(j-1)j]_{se}$ describe the relative position error transformation matrix and relative motion error transformation matrix between the rigid body $j-1$ and the adjacent lower body j respectively, which are described in reference [3].

Eventually, the volumetric error model can be obtained by substituting Eqs. (1) and (2) into Eq. (3) and ignoring the high-order terms.
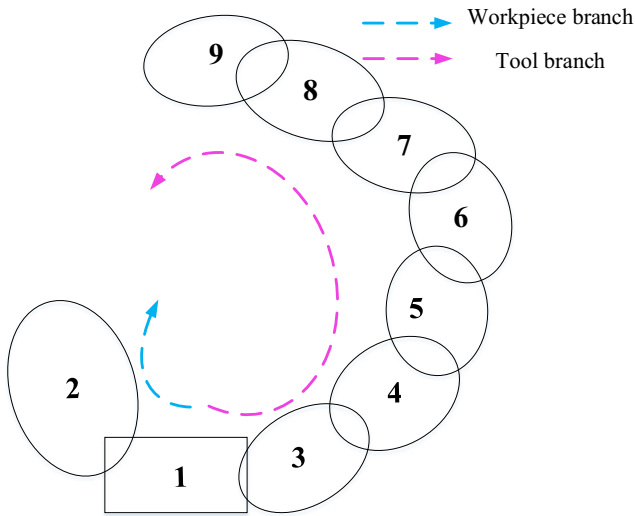
Fig. 3 The topological structure of the FAMT

Fig. 4 The deformation analysis of the researched machine tool

$$E = \begin{pmatrix} E_x \\ E_y \\ E_z \\ 1 \end{pmatrix} = r_w^a - r_w^i \tag{3}$$

where $E_v(v = x, y, z)$ denotes the volumetric error in the v direction. Therefore, the volumetric error model of the FAMTs is displayed in Eq. 12 in Appendix 2.

Finally, the volumetric error-tolerance model of the FAMTs can be obtained based on the geometric error-tolerance model and the volumetric error model.

$$E \xrightarrow{+G} = E_t = E(T, D) \tag{4}$$

where $E = [E_x, E_y, E_z, 0]^T$ denotes the volumetric error vector; $G = [g_1, g_2, g_3, \ldots, g_{37}]^T$ denotes the vector consisting of 37 geometric errors; $E_t = [E_{tx}, E_{ty}, E_{tz}, 0]^T$ denotes the

The deformation of tool tip for the researched machine tool in X-direction

The deformation of tool tip for the researched machine tool in Y-direction

The deformation of tool tip for the researched machine tool in Z-direction
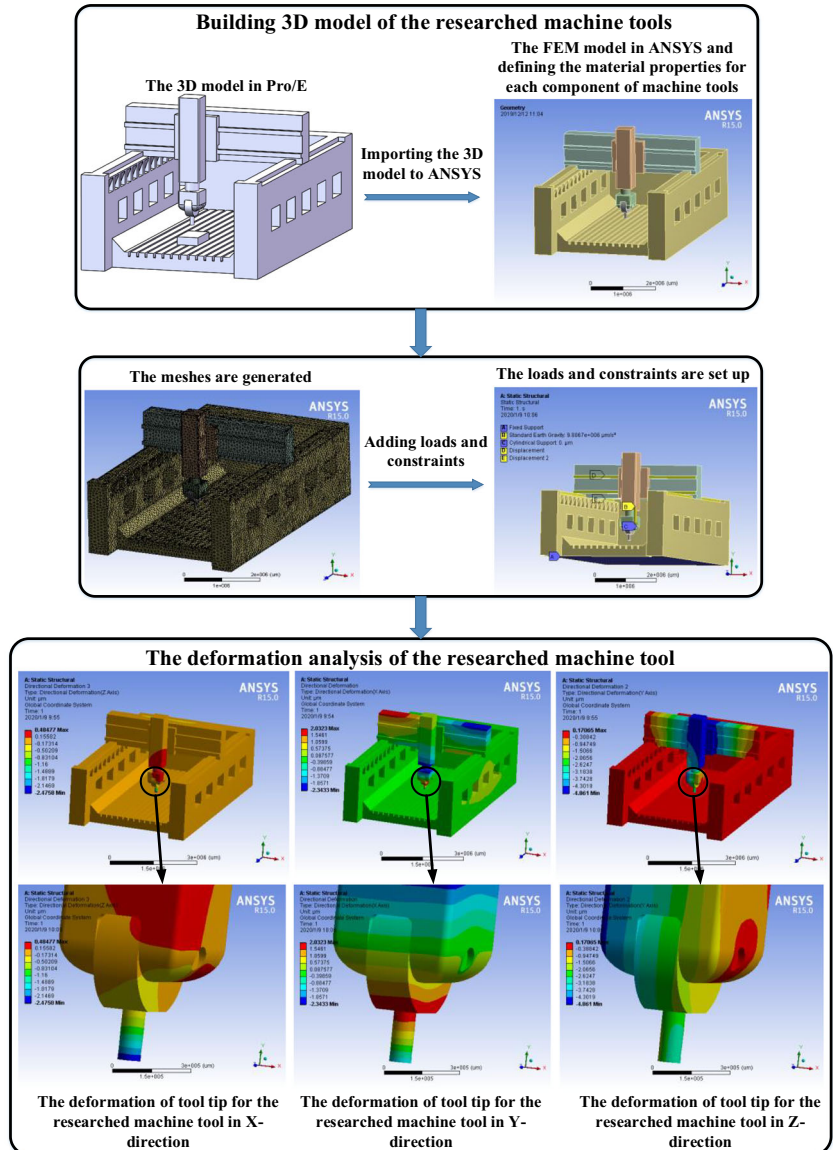
Table 3 The value of economic tolerance and economic MC

The economic tolerance	Value (mm)	The economic MC (CNY)	The economic tolerance	Value (mm)	The economic MC (CNY)
t_{h1}	0.003	1.8×10^4	t_{h14}	0.005	3×10^4
t_{h2}	0.04	1.35×10^4	t_{h15}	0.04/1000	1.8×10^4
t_{h3}	0.05	2.4×10^4	t_{h16}	0.007	2.1×10^4
t_{h4}	0.05	1.05×10^4	t_{h17}	0.006	2.4×10^4
t_{h5}	0.03	0.75×10^4	t_{h18}	0.04/1000	2.4×10^4
t_{h6}	0.002	1.05×10^4	t_{h19}	0.02/1000	1.5×10^4
t_{h7}	0.03	0.9×10^4	t_{h20}	0.02/500	1.5×10^4
t_{h8}	0.04	0.9×10^4	t_{h21}	0.02/500	1.5×10^4
t_{h9}	0.05	0.75×10^4	t_{h22}	0.025/300	1.2×10^4
t_{h10}	0.02	0.75×10^4	t_{h23}	0.025/300	1.2×10^4
t_{h11}	0.002	0.75×10^4	t_{h24}	0.02/300	1.2×10^4
t_{h12}	0.03	0.75×10^4	t_{h25}	0.02/300	1.2×10^4
t_{h13}	0.008	3×10^4			

volumetric error vector based on the tolerance of machine tools' component; $T = [t_1, t_2, t_3, \ldots, t_{25}]^T$ denotes the vector consisting of 25 tolerance; and $D = [x, y, z, b, c]^T$ denotes the position vector of motion axis for the FAMT.

3 Finite element simulation

The major limitation of the MBS theory is that it is based on assumption that all components of the machine tool are a rigid body. It ignores the small deformation of machine tools, which was caused by the gravity of machine tools. This assumption will result in lessening the range of constraint conditions and tightening the tolerance requirement of critical components. Therefore, the total manufacturing

Table 4 GA parameter's value

Parameters	Value/method
Population size	100
Selection process method	Roulette wheel selection
Crossover probability	0.5
Crossover operator	Single point
Mutation probability	0.03
Mutation operator	Random gene mutation
Replacement strategy	Complete replacement
Termination criteria	1000 iterations
A (CNY)	2×10^4
H (mm)	0.07
Δ (mm)	0.05
C_f (CNY)	32.5×10^4

cost will of course increase. To obtain the small deformation is the prerequisite for determining the constraint range of optimal tolerance allocation. As is known to all, the FEA has been widely used as a means to obtain the small deformation in the initial design stage of machine tools. The FEA process can be summarized in five steps as shown in Fig. 4.

Step 1: The three-dimensional (3D) model of the researched machine tool is established by utilizing the SolidWorks software as illustrated in Fig. 2.
Step 2: Importing 3D model into the ANSYS workbench 15.0 software. The material properties for each component of machine tools are defined.
Step 3: The meshes of the researched machine tool are generated. There are 196,740 elements.
Step 4: The loads and constraints required for the FEA are set up. Setting fixed constraints on the base of machine tools, cylindrical support constraint on the B-axis, and displacement constraint on X-, Y-, and Z-axis. To account for load effect like gravity, standard earth gravity values are given.
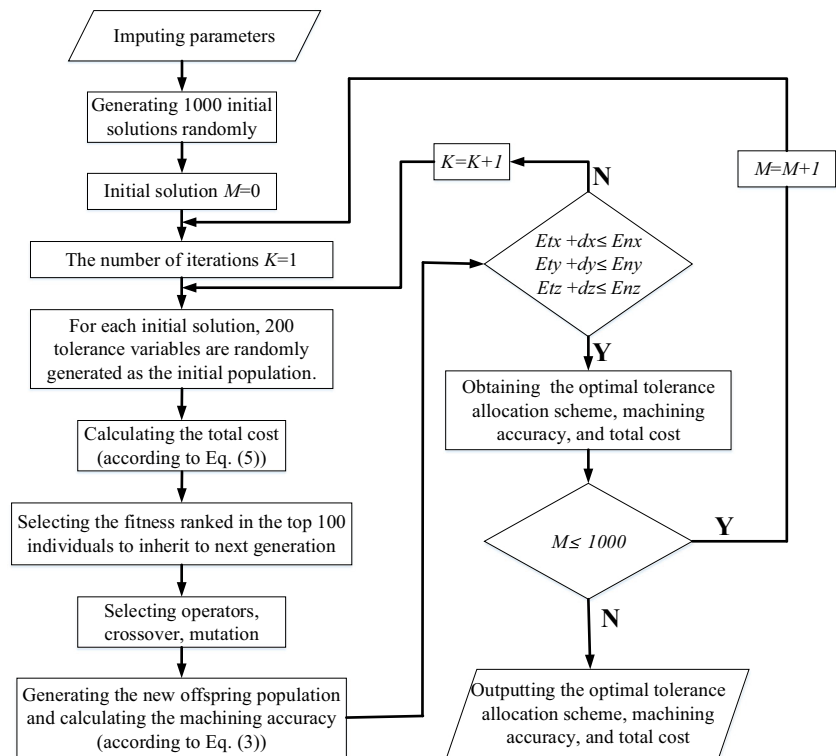Step 5: The deformation of the researched machine tool is obtained by finite element software

According to Fig. 4, the maximum deformation of tool tip (d_x, d_y, d_z) in X-, Y-, and Z-direction is − 2.48 μm, − 1.37 μm, and − 3.74 μm, respectively, which are relatively large. Moreover, the machining accuracy requirement of machine tools is generally on the micron scale. Hence, the small deformations of machine tools cannot be ignored while optimal tolerance allocation during the initial design stage of machine tools.

Fig. 5 The flowchart of GA

4 Optimal tolerance allocation for five-axis machine tools considering gravity effect

Optimal allocation is a way of getting the optimal allocation scheme for a problem that satisfies the presetting accuracy requirement and the least cost. In the initial design stage of machine tools, since the most guiding significance for precision design is the tolerance parameters of the machine tools' component, the precision design of machine tools is a problem of optimal tolerance allocation for machine tools' components. Hence, the optimal tolerance allocation scheme plays an important role in balancing the machining accuracy requirement and total manufacturing cost of machine tools. Optimal tolerance allocation consists of two steps. The first step is to establish a mapping relationship between tolerance and machining accuracy (as mentioned in Section 2). The second step, regarded as the most important, is optimal tolerance allocation modeling, which consists of two parts: the total manufacturing cost-tolerance modeling and setting constraint conditions.

4.1 The total manufacturing cost-tolerance modeling

Generally speaking, the total manufacturing cost (TMC) can be divided into two types: manufacturing cost (MC) and quality loss cost (QLC). MC is the cost occurred before the machine tool reach the customer. QLC is the cost occurred after the machine tool has been put into operation. Tight tolerance

will lead to high in MC and low in QLC. On the contrary, loose tolerance will lead to high in QLC and low in MC. Therefore, reaching an economic balance between MC and QLC is a prerequisite of obtaining an optimal tolerance allocation scheme.

As is known to all, a substantial amount of function has been proposed to establish the mapping relationship between tolerance and MC [21]. These functions are systematically analyzed and derived by regression analysis based on the actual MC data from the manufacturing community. In this study, the exponential model was chosen to represent the MC-tolerance model is formulated as follows.

$$C_M(t) = \sum_{i=1}^{25} C(t_i) + C_f = \sum_{i=1}^{25} C(t_{hi}) e^{-(t_i - t_{hi})} + C_f \qquad (5)$$

where t_i represents the ith ($i = 1, 2, 3, …, 25$) tolerance parameter; $C(t_i)$ represents the MC of tolerance parameter t_i; C_f represents the fixed cost; t_{hi} represents the economic tolerance of tolerance parameter t_i; and $C(t_{hi})$ represents the economic MC of t_{hi}.

QL means that the machining accuracy of machine tools cannot meet the user's requirement or is deviated from its target value during the actual machining process. Product quality is closely related to quality loss. The greater the quality loss, the worse the product quality is. In order to accurately estimate the QLC, Taguchi [22] developed the QLC model and is formulated as follows:

Table 5 Comparison of optimal tolerance allocation scheme with and without considering the small deformation

Tolerance parameter	Without considering the small deformation (mm)	With considering the small deformation (mm)
t_1	0.002	0.0025 (↑)
t_2	0.03	0.054 (↑)
t_3	0.025	0.041 (↑)
t_4	0.036	0.04 (↑)
t_5	0.022	0.015 (↓)
t_6	0.0015	0.0023 (↑)
t_7	0.026	0.035 (↑)
t_8	0.03	0.028 (↓)
t_9	0.056	0.045 (↓)
t_{10}	0.011	0.02 (↑)
t_{11}	0.002	0.003 (↑)
t_{12}	0.015	0.025 (↑)
t_{13}	0.005	0.01 (↑)
t_{14}	0.002	0.008 (↑)
t_{15}	0.06/1000	0.05/1000 (↓)
t_{16}	0.006	0.01 (↑)
t_{17}	0.005	0.0085 (↑)
t_{18}	0.05/1000	0.05/1000
t_{19}	0.015/1000	0.02/1000 (↑)
t_{20}	0.0175/500	0.025/500 (↑)
t_{21}	0.015/500	0.025/500 (↑)
t_{22}	0.0195/300	0.03/300 (↑)
t_{23}	0.021/300	0.024/300 (↑)
t_{24}	0.015/300	0.015/300
t_{25}	0.015/300	0.021/300 (↑)

$$C_{QL} = \frac{A}{\Delta^2} H^2 \qquad (6)$$

$$C(T) = C_M(t) + C_{QL} = \sum_{i=1}^{25} C(t_{hi}) e^{-(t_i - t_{hi})} + C_f + \frac{A}{\Delta^2} H^2 \quad (7)$$

where Δ represents the required specification of the product accuracy; A represents the QLC caused by unqualified product; H represents the actual machining accuracy of the product; $|H| \leq \Delta$ represents the qualified product; and $|H| > \Delta$ represents the unqualified product. The closer that actual machining accuracy of the product is to require specification value, the smaller the total quality loss cost is and the higher the machining performance of machine tools can be obtained.

Based on the above manufacturing cost and quality loss analysis, the total manufacturing cost objective function of the machine tool can be formulated as follows:

4.2 Setting constraint conditions

According to references [8, 16, 17, 19, 23], the constraint conditions were set under the assumption that all components of the machine tool are the rigid body. However, the small deformations will inevitably exist due to the effect of gravity on machine tools. Ignoring the small deformations will tighten the tolerance requirement of critical components and increase the total manufacturing cost. Therefore, it is of paramount importance to integrate the small deformations into the constraint conditions during the initial design stage of machine

Table 6 The comparisons of the total manufacturing cost with and without considering the small deformation

	Without considering the small deformation	With considering the small deformation	Saving in cost
TMC (CNY)	127.5×10^4	112.8×10^4	11.5%

tools. In addition, some of these tolerance optimization methods are carried out under a certain constraint condition (i.e., $0 < t \leq$ initial value). The given initial value will limit the search range of the optimal scheme. Thus, the obtained optimization results are not necessarily the optimal results.

Based on the above analysis, in this paper, the constraint conditions are set as follows:

$$\begin{cases} E_{tx} + d_x \leq E_{nx} \\ E_{ty} + d_y \leq E_{ny} \\ E_{tz} + d_z \leq E_{nz} \\ \quad t_i > 0 \end{cases} \tag{8}$$

where E_{tx}, E_{tx}, and E_{tx} represent the machining accuracy of the researched machine tool based on the tolerance of machine tools' component in the x, y, and z direction, respectively. E_{nx}, E_{ny}, and E_{nz} represent the nominal machining accuracy of the researched machine tool in the x, y, and z direction, respectively. In light of the design requirement of the researched machine tool, $E_{nx} = E_{ny} = E_{nz} = 20\mu m$.

4.3 Optimal tolerance allocation model

Based on the abovementioned analysis, the optimal tolerance allocation model is developed to minimize the MC and QLC subject to the machining accuracy and tolerance of machine tools constraints. Hence, the optimal tolerance allocation scheme is a trade-off between machining accuracy and the total manufacturing cost, which can be obtained by utilizing the following optimization model:

$$\begin{cases} Min \quad C(T) \\ \quad S.t. \\ E_{tx} \leq E_{nx} - d_x \\ E_{ty} \leq E_{ny} - d_y \\ E_{tz} \leq E_{nz} - d_z \\ \quad t_i > 0 \end{cases} \tag{9}$$

5 Simulation validation

5.1 The optimal tolerance allocation

Optimal tolerance allocation is a key procedure to reduce the total manufacturing cost of FAMTs reasonably while satisfying the desired machining accuracy. To obtain an optimal tolerance allocation scheme, many intelligent optimization algorithms are developed in a large number of literature, including the Newton iteration algorithm [24], genetic algorithm (GA) [25, 26], ants colony algorithm [27], particle swarm optimization algorithm [28, 29], bat algorithm [30], elitist non-dominated sorting genetic algorithm (NSGA-II), and multi-objective particle swarm optimization (MOPSO) [7].

In this study, GA with the advantage of convenient calculation, high precision and robustness, and the wide application range is selected to solve this objective optimization problem. The flowchart of GA is shown in Fig. 5. All the parameters can be obtained by referring to the accumulated design experience and test information of the enterprise for decades and references [31, 32]; those are listed in Tables 3 and 4.

5.2 Results and analysis

The optimal tolerance allocation scheme and the TMC of the researched FAMT are all obtained by the MATLAB software based on the GA, as shown in Tables 5 and 6, where "↓" denotes the value of tolerance calculated with considering the small deformation is smaller than that without considering the small deformation, "↑" denotes the value of tolerance calculated with considering the small deformation is bigger than that without considering the small deformation.

Finally, to validate the effectiveness of the presented method more intuitively, the comparisons of the tolerance of machine tools' components with and without considering the small deformation are displayed in Fig. 6. It can be clearly seen that the ranges of the vast majority of tolerance parameters have been properly enlarged. In addition, the volumetric errors of the researched FAMT are calculated based on two tolerance allocation schemes. The comparisons of the TMC and volumetric errors of the machine tool with and without considering the small deformation are displayed in Fig. 7 a and b, respectively, where "1" represents without considering the small deformation and "2" represents with considering the small deformation. It can be seen that the maximum value of volumetric error calculated with and without considering the small deformation is 0.034 mm and 0.025 mm, respectively. Although the volumetric errors with considering the small deformation are larger than that without considering the small deformation, both of them are within the range of the accuracy requirement for the FAMT. Furthermore, as shown in Fig. 7a, compared with the TMC without considering the small deformation, the TMC with considering the small deformation is
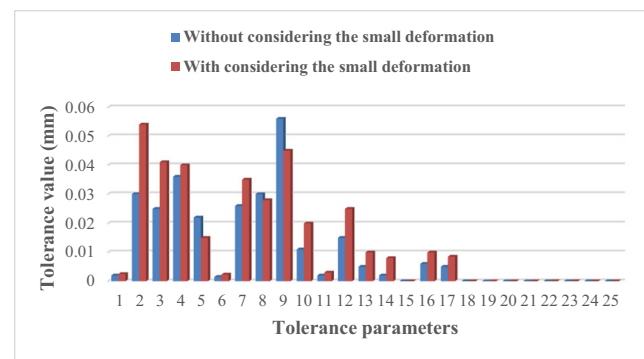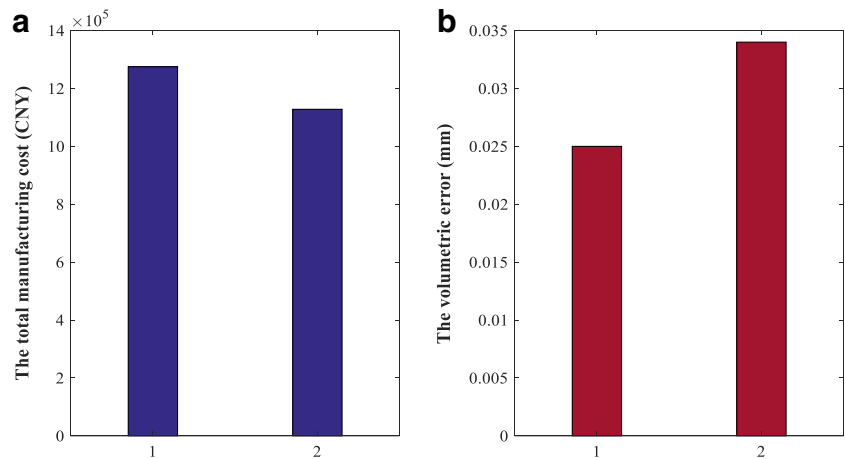

Fig. 6 The comparisons of the tolerance of machine tools' components with and without considering the small deformation

Fig. 7 The comparisons of the TMC and volumetric error of the machine tool with and without considering the small deformation

reduced by approximately 11.5%. Hence, integrating the small deformations into the constraint conditions can widen the range of tolerance cost-effectively and ensure that a FAMT satisfies the machining accuracy requirements simultaneously. Therefore, it can be concluded that the proposed optimal tolerance allocation model is correct and effective.

6 Conclusions

In this study, a novel optimal tolerance allocation method is developed to reduce the total manufacturing cost and meet the machining accuracy of a FAMT simultaneously. Compared with the traditional methods [8, 16, 17, 19, 24], the advantages of the new approach are listed as follows: (1) The small deformations of machine tools are integrated into the constraint conditions of the optimal tolerance allocation model. It can satisfy the machining accuracy requirement of machine tools and widen the tolerance range of the difficult manufacturing components and also reduce the total manufacturing cost. (2) In the design stage of machine tools, since only the information of tolerance of machine tools' component is known, the volumetric error-tolerance model and the MC-tolerance model are established. (3) The constraint condition of tolerance is set as $t_i > 0$, which widen the search range of the optimal tolerance allocation scheme.

To prove the practicability and effectiveness of the presented method, the simulation validation is carried out. The results show that the ranges of the vast majority of tolerance parameters are properly enlarged and the total manufacturing cost after optimization is reduced by approximately 11.5%. Additionally, the machining accuracy of the FAMT satisfies the desired requirement. Therefore, the optimal tolerance allocation method is accurate and effective. Furthermore, the presented method can provide a reference for obtaining the optimal tolerance allocation scheme of multi-axis NC machine tools.

Appendix 1

$$\begin{cases} \delta_x(x) = t_1 \sin\left(\frac{2\pi x}{\lambda}\right) + ax \\ \delta_y(x) = \frac{t_2\left(\sin\left(\frac{2\pi x_{i+1}}{\lambda}\right) + \sin\left(\frac{2\pi x_{i-1}}{\lambda}\right)\right)}{2} \\ \delta_z(x) = \frac{t_3\left(\sin\left(\frac{2\pi x_{i+1}}{\lambda}\right) + \sin\left(\frac{2\pi x_{i-1}}{\lambda}\right)\right)}{2} \\ \varepsilon_x(x) = \frac{t_4\left(\sin\left(\frac{2\pi x_{i+1}}{\lambda}\right) + \sin\left(\frac{2\pi x_{i-1}}{\lambda}\right)\right)}{2L} \\ \varepsilon_y(x) = \frac{t_3\left(\sin\left(\frac{2\pi x_{i+1}}{\lambda}\right) - \sin\left(\frac{2\pi x_{i-1}}{\lambda}\right)\right)}{B} \\ \varepsilon_z(x) = \frac{t_2\left(\sin\left(\frac{2\pi x_{i+1}}{\lambda}\right) - \sin\left(\frac{2\pi x_{i-1}}{\lambda}\right)\right)}{B} \end{cases} \quad (10)$$

$$\begin{cases} \delta_x(c) = t_{13}\sin^2(\theta_{ci}) \\ \delta_y(c) = t_{13}\sin(\theta_{ci})\cos(\theta_{ci}) \\ \delta_z(c) = t_{14}\sin(\theta_{ci}) \\ \varepsilon_x(c) = \frac{t_{13}\sin(\theta_{ci})\cos(\theta_{ci})}{L_c} \\ \varepsilon_y(c) = \frac{t_{13}\sin^2(\theta_{ci})}{L_c} \\ \varepsilon_z(c) = t_{14}\sin(\theta_{ci}) \end{cases} \quad (11)$$

Appendix 2

$$
\begin{cases}
\begin{aligned}
E_x =\ & \delta_x(x) + \delta_x(y) + \delta_x(z) - \varepsilon_z(x)y + \varepsilon_y(x)z + \varepsilon_y(y)z - \varepsilon_{xy}y + \varepsilon_{xz}z + \delta_x(c)\cos(c) - \varepsilon_y(b)\sin(c) - \delta_y(c)\sin(c) + \varepsilon_y(x)q_{3z} \\
& + q_{4z}(\varepsilon_y(x) + \varepsilon_y(y)) + q_{5z}(\varepsilon_y(x) + \varepsilon_y(y) + \varepsilon_y(z) + \varepsilon_{xz}) + q_{6z}(\varepsilon_y(x) + \varepsilon_y(y) + \varepsilon_y(z) + \varepsilon_{xc} + \varepsilon_{xz} + \varepsilon_y(c)\cos(c) + \varepsilon_x(c)sin(c)) \\
& - \varepsilon_z(x)q_{3y} - q_{4y}(\varepsilon_z(x) + \varepsilon_z(y) + \varepsilon_{xy}) - q_{5y}(\varepsilon_z(x) + \varepsilon_z(y) + \varepsilon_z(z) + \varepsilon_{xy}) + \delta_x(b)\cos(b)\cos(c) + (\delta_z(b) - l)\cos(c)\sin(b) \\
& - l\cos(b)(\varepsilon_y(x) + \varepsilon_y(y) + \varepsilon_y(z) + \varepsilon_{xc} + \varepsilon_{xz}) - (\varepsilon_z(c) + \varepsilon_z(x) + \varepsilon_z(y) + \varepsilon_z(z) + \varepsilon_{xy})(q_{6y}\cos(c) + q_{6x}\sin(c)) - \delta_x(b)\varepsilon_y(x)\sin(b) \\
& - \varepsilon_x(b)l\sin(c) - l\cos(b)\cos(c)(\varepsilon_y(b) + \varepsilon_y(c) + \varepsilon_x(c)) - \varepsilon_{bz}l\cos(b)\sin(c) + (\varepsilon_z(c) + \varepsilon_z(x) + \varepsilon_z(y) + \varepsilon_z(z) + \varepsilon_{xb} + \varepsilon_{xy})l\sin(b)\sin(c) \\[4pt]
E_y =\ & \delta_y(x) + \delta_y(y) + \delta_y(z) - \varepsilon_x(x)z - \varepsilon_x(y)z - \varepsilon_{yz}z + \delta_y(b)\cos(c) + \delta_y(c)\cos(c) + \delta_x(c)\sin(c) - \varepsilon_x(x)q_{3z} - q_{4z}(\varepsilon_x(x) + \varepsilon_x(y)) \\
& - q_{5z}(\varepsilon_x(x) + \varepsilon_x(y) + \varepsilon_x(z) + \varepsilon_{yz}) - q_{6z}(\varepsilon_x(x) + \varepsilon_x(y) + \varepsilon_x(z) + \varepsilon_{yc} + \varepsilon_{yz} + \varepsilon_x(c)\cos(c) - \varepsilon_y(c)\sin(c)) + \varepsilon_z(x)q_{3x} + l\varepsilon_x(b)\cos(c) \\
& + q_{5x}(\varepsilon_z(x) + \varepsilon_z(y) + \varepsilon_{xy} + \varepsilon_z(z)) + \delta_z(b)\sin(b)\sin(c) - l\cos(b)(\varepsilon_x(x) + \varepsilon_x(y) + \varepsilon_x(z) + \varepsilon_{yc} + \varepsilon_{yz}) + (\varepsilon_{bz} - \varepsilon_x(c))l\cos(b)\cos(c) \\
& + q_{6x}\cos(c)(\varepsilon_z(c) + \varepsilon_z(x) + \varepsilon_z(y) + \varepsilon_z(z) + \varepsilon_{xy}) - q_{6y}\sin(c)(\varepsilon_z(c) + \varepsilon_z(x) + \varepsilon_z(y) + \varepsilon_z(z) + \varepsilon_{xy}) + q_{4x}(\varepsilon_z(x) + \varepsilon_z(y) + \varepsilon_{xy}) \\
& + \cos(b)\sin(c)(\delta_x(b) + \varepsilon_y(c) - \varepsilon_y(b)) - l\cos(c)\sin(b)(\varepsilon_z(c) + \varepsilon_z(x) + \varepsilon_z(y) + \varepsilon_z(z) + \varepsilon_{xb} + \varepsilon_{xy}) \\[4pt]
E_z =\ & \delta_z(c) + \delta_z(x) + \delta_z(y) + \delta_z(z) - q_{4x}(\varepsilon_y(x) + \varepsilon_y(y)) + \sin(b)((\varepsilon_y(b)l + \varepsilon_y(c)l - \delta_x(b)) - l\sin(c)(\varepsilon_x(x) + \varepsilon_x(y) + \varepsilon_x(z) + \varepsilon_{yc} + \varepsilon_{yz})) \\
& + q_{6y}(\varepsilon_x(c) + \varepsilon_{yc}\cos(c) + \sin(c)(\varepsilon_y(x) + \varepsilon_y(y) + \varepsilon_y(z) + \varepsilon_{xc} + \varepsilon_{xz})) + \varepsilon_x(x)q_{3y} + q_{4y}(\varepsilon_x(x) + \varepsilon_x(y)) + \varepsilon_x(b)\varepsilon_y(z)\varepsilon_{xb}l\cos(c) \\
& - q_{5x}(\varepsilon_y(x) + \varepsilon_y(y) + \varepsilon_y(z) + \varepsilon_{xz}) + q_{6x}(\sin(c)(\varepsilon_x(x) + \varepsilon_x(y) + \varepsilon_x(z) + \varepsilon_{yc} + \varepsilon_{yz}) - \cos(c)(\varepsilon_y(y) + \varepsilon_y(z) + \varepsilon_{xc} + \varepsilon_{xz} + \varepsilon_y(x)) - \varepsilon_y(c)) \\
& + q_{5y}(\varepsilon_{yz} + \varepsilon_x(y) + \varepsilon_x(z) + \varepsilon_x(x)) + l\cos(c)\sin(b)(\varepsilon_y(x) + \varepsilon_y(y) + \varepsilon_y(z) + \varepsilon_{xc} + \varepsilon_{xz}) + \varepsilon_x(x)y + \delta_z(b)\cos(b) - \varepsilon_y(x)q_{3x}
\end{aligned}
\end{cases}
\tag{12}
$$

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