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ORIGINAL ARTICLE

A general method for thermal error measurement and modeling in CNC machine tools' spindle

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Abstract
To improve the machining accuracy of the CNC machine tools, it is essential to establish a high-precision and strong-robustness thermal error model for further thermal error compensation work. In this paper, it presents a new method of thermal error measurement and modeling in CNC machine tools' spindle. In order to measure the thermal deformation of the machine tools' spindle more conveniently and efficiently, a five-point measurement method is proposed in this paper. The sensitive temperature points are identified by using the theory of partial correlation analysis and it has successfully reduced 12 raw temperature variables to 2~3 sensitive temperature variables. The thermal error model of the machine tools' spindle is exactly figured out based on the regression theory of weighted least squares support vector machine (WLS-SVM). This paper proposes a method of gene expression programming (GEP) algorithm to optimize the penalty parameter c and kernel function parameter σ involved in WLS-SVM. The parameter υ of the weighting value contained in WLS-SVM is to be optimized with the method of an improved normal distribution weighting rule (INDWR). The measurement and modeling experiments are carried out on i5M1 CNC machining center and its modeling accuracy reaches 0.7664 μm in the axial direction of the spindle. That the prediction accuracy under the variable working condition reaches 0.8168 μm proves that the model is still of high precision and robustness. Compared with other modeling methods, the experimental results have shown that this GEP-WLSSVM modeling method is superior to PSO-LSSVM and GA-LSSVM method.

Keywords Thermal error · Error molding · CNC machine tools · WLS-SVM · GEP · INDWR

1 Introduction

Among the factors affecting the machining accuracy of CNC machine tools, the thermal error has been regarded as the main error source [1], which accounts for 50–70% of the total errors during the machining process [2]. The key of improving the machining accuracy of CNC machine tools is to control the thermal error. Many solutions can be conducted to reduce the thermal error. One of the most economical and reliable solutions is thermal error compensation. The prerequisite of thermal error compensation is to establish an effective thermal error model of high precision and strong robustness.

Many scholars have made their significant devotions to the research of thermal error of CNC machine tool. For example, Liu et al. [3] first revealed the law of thermal tilt deformation of spindle, defined the concept of "Liu-Loop," first modeled and compensated the thermal drift error of CNC lathe's spindle in the world. The research has laid an important theoretical foundation for the optimization design of the spindle, and has great reference value. The commonly used thermal error modeling methods include least square support vector machine (LS-SVM) method [4], genetic algorithm (GA) and BP algorithm [5], Grey or BP neural network [6–9], fuzzy clustering analysis [10], time series analysis [11], reconstructed variable regression method [12], and many other modeling methods [13–15]. These thermal error models have achieved certain accuracy improvement. However, they still cannot meet the requirements of high robustness and generalization ability because of these shortcomings such as fitting accuracy inadequately, initial-conditions dependently, data-smoothness

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dependently, and over-learning easily. As for the grey system theory modeling method, it does not own the abilities of feedback and self-learning and will have poor prediction accuracy when changing the machine tools' working condition. As for the artificial neural network modeling method, it needs a huge number of training samples and costs a long time before the model is done. As for the genetic algorithm modeling method, its encoding method is not flexible enough to initialize the population. As for the least square support vector machine, it cannot overcome the influence caused by random errors from the samples when operating the training process and it is not easy to get the optimum parameters involved in its model.

In view of the shortcomings of the existing thermal error modeling method, this paper proposes a new method based on the weighted least square support vector machine (WLS-SVM) regression theory which essentially springs from LS-SVM theory. Support vector machine (SVM), put forward by Vapnik [16] based on statistical learning theory [17], can improve the generalization ability for using the principle of structural minimization, and smoothly solve the problems of lacking enough samples, nonlinearity, multiple dimensions, and local minimum. The least squares support vector machine (LS-SVM) is the extension of the standard SVM. The difference between the LS-SVM and the standard SVM is that the LS-SVM defines the 2-norm of error (relaxation variables) as the loss function and transforms the inequality constraints into equality constraints. The quadratic programming problem in SVM has been transformed into linear equations in LS-SVM, which reduce the computational complexity and enhance the learning speed. However, the objective function of LS-SVM is the sum of mean squares error and its corresponding Lagrange multipliers are proportional to the errors, which results in the losses of sparsity and reduction of robustness of the model. To solve this problem, Suykens proposed the weighted least squares support vector machine (WLS-SVM) method [18, 19] by giving certain weights to the fitting errors in the objective function. As for how to get the optimum weighting values, this paper proposes a new weighting method based on an improved normal distribution weighting rule (INDWR), which adaptively assigns different weights to these modeling fitting errors and reduces the influence of random errors from the samples as a result. On the other hand, as how to find the optimum parameters involved in LS-SVM model, this paper puts forward a new parameter optimization algorithm, gene expression programming (GEP) [20–22], which regards the two key parameters (penalty parameter c and kernel function parameter σ) as the GEP genes. It is executed by the dynamic change mechanism of the number of genes in chromosomes and evolutionary generations according to its mutation operator. The speed and accuracy of its convergence are greatly improved. Therefore, the thermal error model is to be established with the method of GEP-WLSSVM in this paper.

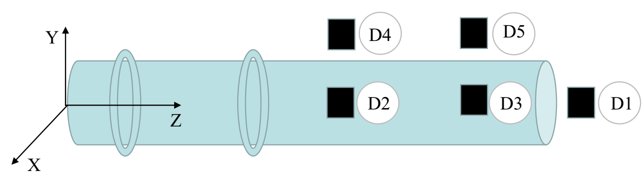
The framework of this paper is about to be conducted as follows: The first section makes a brief introduction to this paper. In Section 2, the five-point measurement method is proposed to efficiently measure the thermal deformation of the machine tools' spindle. In Section 3, the temperature-sensitive points are identified based on the partial correlation analysis theory. In Section 4, the modeling theory based on GEP-WLSSVM is expounded at length and then used to establish the thermal error mathematical model. Meanwhile, its modeling accuracy has been compared with other traditional modeling method. In Section 5, under variable working conditions, the reliability and robustness of this established thermal error model are verified to be still of high prediction accuracy and strong robustness. Finally, the conclusions are drawn in Section 6.

2 Five-point measurement method

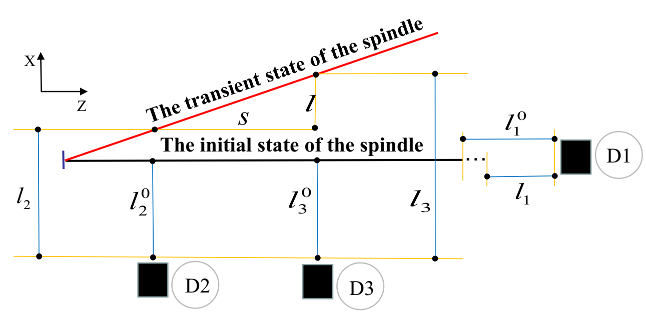
2.1 Measurement principle

To measure the thermally induced errors of CNC machine tools' spindle, a five-point measurement method is proposed as shown in Fig. 1. Five displacement sensors (D1, D2, D3, D4, D5) are used to measure the spindle's thermal errors along the *X*-, *Y*-, and *Z*-direction.

As shown in Fig. 1a, the one displacement sensor D1 is used to measure the thermal deformation error along the *Z*-direction, the two displacement sensors D2 and D3 are used to measure the thermal deflection error along the *X*-direction,



(a) Sensor arrangement



(b) Measuring principle

Fig. 1 The diagram of five-point measurement method. **a** Sensor arrangement. **b** Measuring principle and the other two displacement sensors D4 and D5 are used to measure the thermal deflection error along the *Y*-direction.

As shown in Fig. 1b, the thermal deformation *E* along the *Z*-direction can be calculated as follows.

$$E = l_1^0 - l_1 \tag{1}$$

where l_1^0 is the initial distance between the displacement sensor D1 and the spindle along the *Z*-directio and l_1 is transient distance between the displacement sensor D1 and the spindle along the *Z*-direction.

As shown in Fig. 1b, the thermal deflection error θ_x along the *X*-direction can be expressed as follows.

$$\theta_x \approx \tan\theta_x = \frac{l}{s} = \frac{\Delta l_3 - \Delta l_2}{s} = \frac{\left(l_3 - l_3^0\right) - \left(l_2 - l_2^0\right)}{s} \tag{2}$$

where *S* is the distance between displacement sensors D2 and D3 along the *Z*-direction, l_2^0 and l_3^0 are the initial distance between the displacement sensors (D2, D3) and the spindle along the *X*-direction, l_2 and l_3 are the transient distance between the displacement sensor (D2, D3) and the spindle along the *X*-direction.

Similarly, the thermal deflection error θ_y along the *Y*-direction can be measured.

2.2 Measurement experiment

Using the five-point measurement method, an experiment was carried out on an i5M1 machining center produced by Shenyang machine tool factory. As shown in Fig. 2, the five displacement sensors were used to measure the thermal errors of the spindle in three directions. The thermal deformation *E* was measured by the sensor D1, the thermal deflection error θ_x along the *X*-direction was measured by the sensors D2 and D3, the thermal deflection error θ_y along the *Y*-direction was measured by the sensors D4 and D5. The spindle of the machine tool ran at 8000 rpm for 5 h without loading, and then stopped for 1 h. These displacement sensors collected data every 3 min, so each sensor can get 120 thermal error data. The measuring results are shown in Fig. 3.

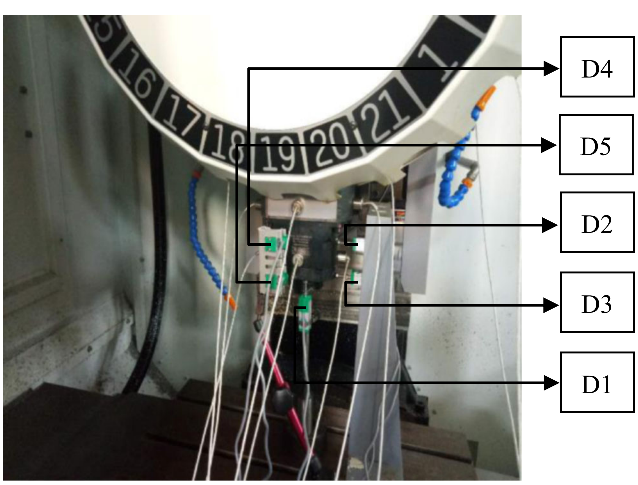


Fig. 2 Experimental arrangement

The measurement experiment shows that the thermal errors of the spindle increase with the running time increase. The maximum thermal deflection error along the *X*-direction is 0.0031°; the maximum thermal deflection error along the *Y*-direction is 0.0045°; the maximum thermal deformation along the *Z*-direction is 42.5 μm.

3 Temperature-sensitive points' identification

3.1 Identification principle

In order to improve the robustness of the thermal error model, the temperature variables used to establish the thermal error model should be as few as possible. Considering the complexity and characteristics of the thermal error model, a partial correlation analysis method is proposed to identify the temperature-sensitive points. The correlation coefficient is used to describe the degree of correlation among these factors, namely temperature variables and thermal error variables.

$$r_{xy} = \frac{\mathrm{cov}(x,y)}{\sqrt{D(x)D(y)}} = \frac{\sum\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sqrt{\sum\left(x_i - \bar{x}\right)^2 \sum\left(y_i - \bar{y}\right)^2}} \tag{3}$$

where $x,\ y \in t \cup \delta$, $t = [t_1, t_2, t_3, \cdots, t_{12}]$, $\delta = [E, \theta_x, \theta_y]$, *t* represents the temperature variables of the CNC machine tool, and δ represents the thermal error variables of the CNC machine tool.

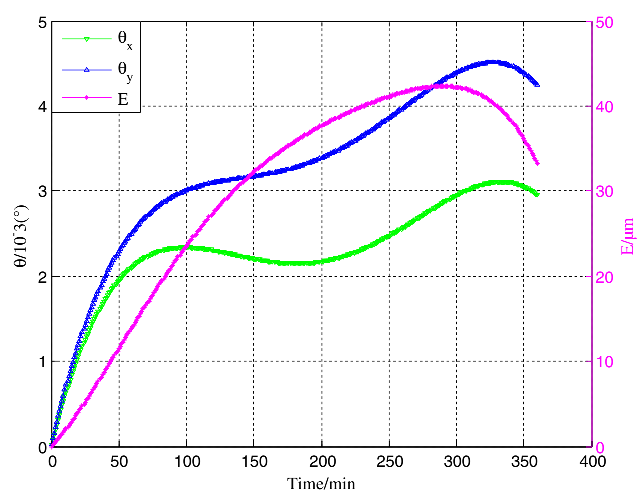


Fig. 3 The thermal errors of the spindle. $\theta_x = [\theta_{x1}, \theta_{x2}, \theta_{x3}, \cdots, \theta_{x120}]$ represents the *X*-direction thermal error; $\theta_y = [\theta_{y1}, \theta_{y2}, \theta_{y3}, \cdots, \theta_{y120}]$ represents the *Y*-direction thermal error; and $E = [E_1, E_2, E_3, \cdots, E_{120}]$ represents the *Z*-direction thermal error.

Considering the interaction between the variables, in order to obtain the essential connection between the variables more accurately, the partial correlation analysis theory was defined based on simple correlation in statistics. Assuming that there are a group of independent variables $x = [x_1, x_2, x_3, \cdots, x_n]$, the calculation method of the partial correlation coefficient between x_i and x_j includes three steps.

The first step is to build the correlation coefficient matrix **R** composed of the simple correlation coefficient calculated by Eq. (3).

$$\mathbf{R} = \left(r_{ij}\right)_{n\times n} = \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1n} \\ r_{21} & r_{22} & \dots & r_{2n} \\ \dots & \dots & \dots & \dots \\ r_{n1} & r_{n2} & \dots & r_{nn} \end{bmatrix} \tag{4}$$

The second step is to calculate the inverse matrix $\mathbf{R}^{-1}$ according to Eq. (4).

$$\mathbf{R}^{-1} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \dots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22} & \dots & \lambda_{2n} \\ \dots & \dots & \dots & \dots \\ \lambda_{n1} & \lambda_{n2} & \lambda_{n3} & \lambda_{nn} \end{bmatrix} \tag{5}$$

The third step is to calculate the coefficients of partial correlation:

$$|c_{ij}| = \left|\frac{-\lambda_{ij}}{\sqrt{\lambda_{ii}\lambda_{jj}}}\right| \tag{6}$$

The coefficients of partial correlation indicate the degree of correlation between two variables when other factors already exist in the model, the greater the coefficient value, the stronger the correlation.

3.2 Experimental analysis

To identify the temperature-sensitive points of the spindle of the i5M1 machining center, 12 temperature sensors were installed on the i5M1 machining center as shown in Fig. 2. The positions of these temperature sensors are shown in Table 1. The temperature measurement process was synchronized with the thermal error measurement process, and the corresponding data was also collected every 3 min, so each temperature sensor can get 120 temperature data. Figure 4 shows the measured results of 12 temperature sensors. It can be seen that the temperatures of the spindle fluctuate with the running time increase. The maximum temperature rise of the spindle was 18.7 °C that occurred in the spindle motor.

Table 1 The arrangement of temperature sensors

Temperature sensors	Positions
T1	Spindle box
T2, T3, T4	End cap of front bearing of the spindle
T5, T6, T7	End cap of rear bearing of the spindle
T8	Ball screw nut of *Z*-axis
T9, T10	Spindle motor
T11	Column
T12	Environment

According to the experimental results and the partial correlation analysis method, the partial correlation coefficients between 12 temperature variables and thermal errors E, θ_x, and θ_y can be obtained as shown in Table 2.

It can be concluded from Table 2 that t_3 and t_8 are the temperature-sensitive points of the thermal deformation E; t_2, t_7, and t_{11} are the temperature-sensitive points of the thermal deflection error θ_x; t_2, t_5, and t_8 are the temperature-sensitive points of the thermal deflection error θ_y. Therefore, the input samples to each thermal error model can be gathered in Tables 3, 4, and 5.

Using these temperature-sensitive points and their corresponding thermal error, the thermal error model is to be established in Section 4.

4 Thermal error modeling

4.1 WLS-SVM-based thermal error modeling

For a given training set $\{(T_i, \delta_i)|\, i = 1, 2, \cdots, n\}$ ($T_i \in R^d$ and $\delta_i \in R$ represent the temperature-sensitive value and the thermal errors of CNC machine tools in Section 3, n is the number of samples), according to the regression theory, the thermal error model can be expressed by the following equation.

$$\delta(T) = W^{\mathrm{T}}\phi(T) + b \tag{7}$$

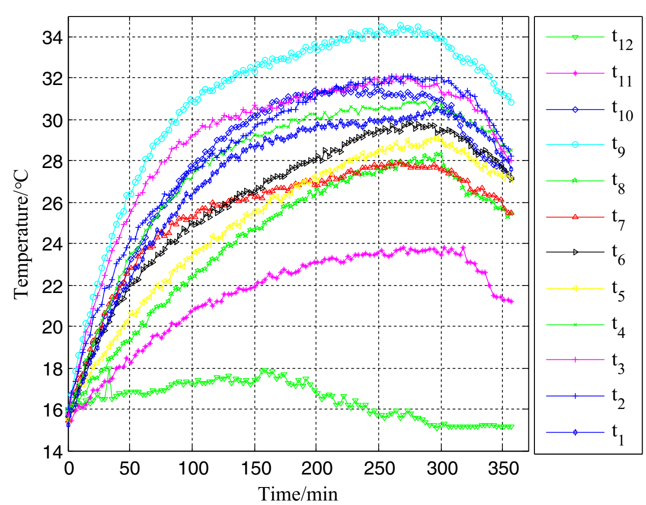


Fig. 4 The temperature curves. $t_i = [t_{i1}, t_{i2}, t_{i3}, \cdots, t_{i120}]$ represents the temperature value which was measured by temperature sensor Ti ($i = 1, 2, 3, \cdots, 12$).

Table 2 Partial correlation coefficients and significance

Temperature variables	Partial correlation coefficient ($\|c_{ij}\|$)			Significance (κ)		
	E	θ_x	θ_y	E	θ_x	θ_y
t_1	0.298	0.223	0.271	0.33	0.39	0.28
t_2	0.023	0.781	0.533	0.78	0.07	0.11
t_3	0.646	0.178	0.189	0.10	0.41	0.35
t_4	0.252	0.038	0.095	0.39	0.58	0.49
t_5	0.214	0.029	0.646	0.47	0.62	0.09
t_6	0.293	0.106	0.083	0.21	0.49	0.54
t_7	0.097	0.643	0.206	0.68	0.09	0.31
t_8	0.845	0.245	0.779	0.06	0.36	0.07
t_9	0.279	0.091	0.112	0.35	0.52	0.44
t_{10}	0.106	0.016	0.051	0.61	0.77	0.57
t_{11}	0.027	0.541	0.098	0.72	0.11	0.48
t_{12}	0.006	0.009	0.011	0.93	0.86	0.79

κ is the significant degree. The lower the κ value is, the higher the reliability of the correlation is. The bilateral t test calculating mothed is adopted to calculate the significance value κ

where b is the threshold value, W is the weight-coefficient vector, $W=[W_1, W_2, \cdots, W_n]^{\mathrm{T}}$, $\phi(\cdot)$ is the mapping function, $\phi(\cdot)=[\phi_1(\cdot), \phi_2(\cdot), \cdots, \phi_n(\cdot)]^{\mathrm{T}}$.

Based on the structural risk minimization principle, the optimization problem of Eq. (7) can be described as the following equation.

$$\min J(W,\xi_i)=\frac{1}{2}\|W\|^2+\frac{1}{2}c\sum_{i=1}^{n}\upsilon_i{\xi_i}^2 \tag{8}$$

$$\text{s.t.}:\quad \delta_i=W^T\cdot\phi(T_i)+b+\xi_i$$

$$\left(\xi_i\geq 0\quad i=1,2,\ldots,n\right)$$

where ξ_i is the fitting error, c is the penalty parameter, and υ_i is the weighting coefficient.

Table 3 The sample for modeling of thermal error θ_x

Input samples		Data$_1$ T_1	Data$_2$ T_2	Data$_n$ …	Data$_{119}$ T_{119}	Data$_{120}$ T_{120}
T	t_2	t_{21}	t_{22}	…	t_{2199}	t_{2120}
	t_7	t_{71}	t_{72}	…	t_{7119}	t_{7120}
	t_{11}	t_{111}	t_{112}	…	t_{11119}	t_{11120}
δ	θ_x	θ_{x1}	θ_{x2}	…	θ_{x119}	θ_{x120}

$T=t_2\cup t_7\cup t_{11}$, $t_i=[t_{i1}, t_{i2}, t_{23}, \cdots, t_{i118}, t_{i119}, t_{i120}]$, $i=2, 7, 11$; $T_j=[t_{2j}, t_{7j}, t_{11j}]$, $j=1, 2, 3, \cdots, 120$; $\delta=\theta_x\in\theta_{xk}$, $k=1, 2, 3, \cdots, 120$. Data$_n=[T, \delta]=[T_n, \theta_{xn}]=[t_{2n}, t_{7n}, t_{11n}, \theta_{xn}]$, $n=1, 2, 3, \cdots, 120$

The thermal error values of θ_x is shown in Fig. 3

The temperature values of t_2, t_7, and t_{11} is shown in Fig. 4

Table 4 The sample for modeling of thermal error θ_y

Input samples		Data$_1$ T_1	Data$_2$ T_2	Data$_n$ …	Data$_{119}$ T_{119}	Data$_{120}$ T_{120}
T	t_2	t_{21}	t_{22}	…	t_{2199}	t_{2120}
	t_5	t_{51}	t_{52}	…	t_{5119}	t_{5120}
	t_8	t_{81}	t_{82}	…	t_{8119}	t_{8120}
δ	θ_y	θ_{y1}	θ_{y2}	…	θ_{y119}	θ_{y120}

$T=t_2\cup t_5\cup t_8$, $t_i=[t_{i1}, t_{i2}, t_{23}, \cdots, t_{i118}, t_{i119}, t_{i120}]$, $i=2, 5, 8$; $T_j=[t_{2j}, t_{5j}, t_{8j}]$, $j=1, 2, 3, \cdots, 120$; $\delta=\theta_y\in\theta_{yk}$, $k=1, 2, 3, \cdots, 120$. Data$_n=[T, \delta]=[T_n, \theta_{yn}]=[t_{2n}, t_{5n}, t_{8n}, \theta_{yn}]$, $n=1, 2, 3, \cdots, 120$

The thermal error values of θ_y is shown in Fig. 3

The temperature values of t_2, t_5, and t_8 is shown in Fig. 4

To solve the optimization problem of Eq. (8), the Lagrange equation $L(W, \xi, \alpha, b)$ is established.

$$L(W,\xi,\alpha,b)=\frac{1}{2}\|W\|^2+\frac{1}{2}c\sum_{i=1}^{n}\upsilon_i\xi_i^2-\sum_{i=1}^{n}\alpha_i\left(W^T\cdot\phi(T_i)+b+\xi_i-\delta_i\right) \tag{9}$$

where α_i ($i=1, 2, \cdots, n$) is the Lagrange multipliers.

According to the KKT conditions, the following equations can be obtained by calculating the partial derivative of Eq. (9).

$$\begin{cases}\dfrac{\partial L}{\partial W}=0\rightarrow W=\sum\limits_{i=1}^{n}\alpha_i\phi(T_i)\\ \dfrac{\partial L}{\partial b}=0\rightarrow\sum\limits_{i=1}^{n}\alpha_i=0\\ \dfrac{\partial L}{\partial\xi_i}=0\rightarrow\alpha_i=c\upsilon_i\xi_i\\ \dfrac{\partial L}{\partial\alpha_i}=0\rightarrow\delta_i=W^T\times\phi(T_i)+b+\xi_i\end{cases}\quad i=1,2,\ldots n \tag{10}$$

The form of Eq. (10) can be further transformed into matrix so that the following equation can be obtained.

$$\begin{bmatrix}0 & \mathbf{1}_n^{\mathrm{T}}\\ \mathbf{1}_n & \mathbf{R}+\dfrac{1}{c}\mathbf{V}\end{bmatrix}\begin{bmatrix}b\\ \boldsymbol{\alpha}\end{bmatrix}=\begin{bmatrix}0\\ \boldsymbol{\delta}\end{bmatrix} \tag{11}$$

Table 5 The sample for modeling of thermal error E

Input samples		Data$_1$ T_1	Data$_2$ T_2	Data$_n$ …	Data$_{119}$ T_{119}	Data$_{120}$ T_{120}
T	t_3	t_{31}	t_{32}	…	t_{3199}	t_{3120}
	t_8	t_{81}	t_{82}	…	t_{8119}	t_{8120}
δ	E	E_1	E_2	…	E_{119}	E_{120}

$T=t_3\cup t_8$, $t_i=[t_{i1}, t_{i2}, t_{23}, \cdots, t_{i118}, t_{i119}, t_{i120}]$, $i=3, 8$; $T_j=[t_{3j}, t_{8j}]$, $j=1, 2, 3, \cdots, 120.\delta=E\in E_k$, $k=1, 2, 3, \cdots, 120$. Data$_n=[T, \delta]=[T_n, E_n]=[t_{3n}, t_{8n}, E_n]$, $n=1, 2, 3, \cdots, 120$

The thermal error values of E is shown in Fig. 3

The temperature values of t_3 and t_8 is shown in Fig. 4

where $\mathbf{1}_n = [1, 1, \cdots, 1]^{\mathrm{T}}$, $\mathbf{R} = \{k(T_i, T_j)|\, i, j = 1, 2, \ \dots, n\}$, $k(T_i, T) = \exp[-\|T_i - T\|^2/(2\sigma^2)]$ is the Gauss kernel function, $\mathbf{V} = diag\left(\upsilon_1^{-1}, \upsilon_2^{-1}, \cdots, \upsilon_n^{-1}\right)$, $\boldsymbol{\delta} = [\delta_1, \delta_2, \cdots, \delta_n]^{\mathrm{T}}$, $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \cdots, \alpha_n]^{\mathrm{T}}$. Furthermore, $\boldsymbol{\alpha}$ and b can be figured out as the following equations by using Eq. (11).

$$b = \frac{\mathbf{1}_n^{\mathrm{T}}\left(\mathbf{R} + \frac{1}{c}\mathbf{V}\right)^{-1}\boldsymbol{\delta}}{\mathbf{1}_n^{\mathrm{T}}\left(\mathbf{R} + \frac{1}{c}\mathbf{V}\right)^{-1}\mathbf{1}_n} \tag{12}$$

$$\boldsymbol{\alpha} = \left(\mathbf{R} + \frac{1}{c}\mathbf{V}\right)^{-1}(\boldsymbol{\delta} - \mathbf{1}_n b) \tag{13}$$

Therefore, the final thermal error model can be obtained as the following mathematical expression.

$$\delta(T) = \sum_{i=1}^{n} \alpha_i k(T_i, T) + b \tag{14}$$

In Eq. (14), there are three variables to be determined. The first variable is the weighting coefficient υ_i, the second variable is the penalty parameter c, and the third variable is the width σ of Gauss kernel function. To improve the accuracy and robustness of the thermal error model, these three parameters will be optimized by using the method of GEP algorithm and the method of improved normal distribution weighting rule (INDWR) as proposed in this paper.

4.2 Parameters optimization

The parameters selection of the WLS-SVM directly affects the convergence, stability, and accuracy of the established thermal error model. In this paper, these three parameters are optimized in two steps. The first step is using the GEP algorithm to optimize the penalty coefficient c and the kernel function width parameter σ, and the second step is to calculate the weighting coefficient υ_i based on the improved normal distribution rule. Therefore, the GEP-WLSSVM method can be obtained and then applied to establish the thermal error model.

4.2.1 Parameters optimization for c and σ using GEP

GEP [20–22] is a new genetic algorithm (proposed by C. Ferreira, a Portuguese scientist) based on the genome and phenotype. It inherits and develops the genetic algorithm (GA) and the genetic programming (GP). Generally, each GEP gene consists of a head and a tail. The head could be contained functions and terminators. The tail could be only contained terminators. Functions usually include common operators, mathematical functions, and custom functions. Terminators usually include inputs to the GEP program, constants, or non-parametric functions. The length of tail and head meets the following GEP rule:

$$L(t) = L(h) \times (n-1) + 1 \tag{15}$$

where, $L(t)$ is the length of the tail, $L(h)$ is the length of the head, and n is the maximum number of operations of functions concerned. This formula guarantees the validity of each GEP gene.

Compared with GA and GP, GEP has a stronger global searching capability. Thus, it is proposed to optimize the two parameters in LS-SVM. The chromosome encoding method and the selection of fitness function are the two main parts of the optimization for the penalty coefficient c and the kernel function width parameter σ.

Considering the effectiveness and characteristic of c and σ on the performance of the thermal error model established by the LS-SVM method, two kinds of GEP encoding methods are used to optimize the parameters c and σ respectively. The Hzero algorithm is used to optimize the penalty parameter c, and the GEP-PO algorithm is used to optimize the parameter of σ. The two methods both have constant domain in each GEP gene; the length of the constant domain is equal to the length of tail. The Hzero encoding method is shown as below, for example:

01
?3 $C1 = \{15, 30, 125, 550, 965\}$

where $C1$ is random constant array. The head length of this gene is 0; the tail length and the constant domain length are 1. The sign "?" represents the terminator, and the number 3 means that the constant in the "?" position is replaced by the constant of the ordinal number of 3 (the ordinal number starts from 0) in the constant array. In the case, c takes 550. The GEP-PO encoding method is shown as below, for example:

01234567890123456789
q* + /?*q???????? 30413412
$C2 = \{0.7094, 2.1088, 4.5787, 3.9610, 4.7975\}$

where $C2$ is random constant array from the range of [0,5]. The gene has a head length of 7 and its constant domain length and tail length are 8, so the length of the gene is 23. The corresponding expression tree of the gene is shown as below:

So, σ can be obtained as:

$$\sigma = \sqrt{(3.9610 + 4.7975 \times 2.1088) \times \sqrt{3.9610}/0.7094} = 6.2846$$

In order to simultaneously encode the value of c and σ with the two encoding methods, the chromosome should be composed of two different encoding genes in the program, and the two parts are separated by a space, for example:

01 01234567890123456789
?3 q* + /?*q???????? 30413412
$C1 = \{15, 30, 125, 550, 965\}$
$C2 = \{0.7094, 2.1088, 4.5787, 3.9610, 4.7975\}$

All the evolutionary algorithms need to evaluate the adaptability of the newly generated chromosomes. The fitness is a measure of the ability of the species to adapt to the environment. Generally, the fitness function values are used to evaluate the chromosomes. In this study, the fitness function was chosen as the following equation.

$$f_i(c,\sigma) = \frac{1}{\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\delta_i - \delta_i^{'}\right)^2} + 1} \tag{16}$$

where δ_i is the actual thermal error, $\delta_i^{'}$ is the output value of LS-SVM, and n is the number of the samples.

In the GEP-LS-SVM method, the next generation of individuals is selected by the method of roulette wheel selection, so that the probability that each individual enters the next generation is proportional to the fitness of itself. The greater the fitness is, the more possibility the individual is to be selected. The genetic operators in this study include selection, replication, mutation, transposition, recombination, inversion, and insertion. In this paper, according to the characteristics of thermal error data and temperature data from the CNC machine tool, all the parameters of GEP are shown in Table 6.

To conclude, the optimization process includes the following eight steps and its flowchart is shown in Fig. 5.

1. Using the encoding method of Hzero and GEP-PO to encode each GEP gene to initialize the population according to the parameters c and σ.
2. Reading the samples and train the LS-SVM model based on the current parameters to obtain the outputs of the LS-SVM.
3. Calculating the adaptive values according to the Eq. (16), sorting the adaptive values in order to reserve the individual with the highest adaptability.

Table 6 The parameters of GEP

Parameter names	Values
Function set	{+ − × /}
Terminator set	{?}
Evolutionary generation	100
Population size	40
Constant set size	8
Constant range	[0,5]
Mutation rate	0.2
Inversion rate, insertion rate	0.1, 0.2
DC inversion rate, DC insertion rate	0.1, 0.2
Two-point recombination rate	0.3
Single-point recombination rate	0.2
Constant mutation rate	0.2
DC mutation rate	0.1

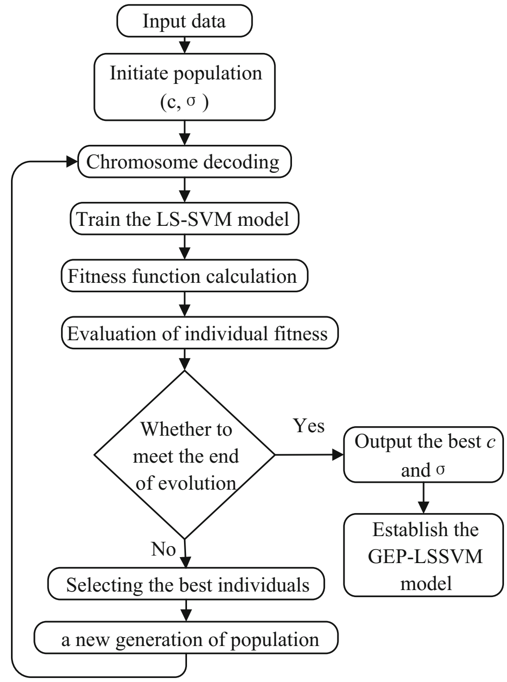


Fig. 5 The flowchart of GEP-LSSVM method

4. Executing the mutation. In this paper, each chromosome possesses two GEP genes, and each gene mutates in one gene locus once a time.
5. Executing the *IS* insert Sequence, the *RIS* insert Sequence, and the *Gene* insert Sequence, performing the single-point recombination, two-point recombination, and gene recombination with the parameters in Table 6.
6. If the execution result reaches the predetermined maximum algebra or the fitness function value converges to the presetting precision, continue to step 7, otherwise return to step 2.
7. Selecting and recording the optimum chromosomes and then decoding the chromosomes to obtain the optimum parameters c and σ.
8. Establishing the LS-SVM model with the optimum parameters c and σ.

4.2.2 Parameter optimization for υ_i using INDWR

The function of weights in WLS-SVM is mainly to eliminate the influence of random errors during the modeling process, and the appropriateness of the weights directly determines the performance of the model. It is necessary to optimize the weight-selection criteria. Therefore, a weighting rule based on improved normal distribution weighting rule (INDWR) is proposed and the weighting coefficient υ_i in Eq. (8) can be expressed by the following equations.

$$v_i = \begin{cases} \exp\left(\frac{\|\xi_i|-\mu|^2}{u_1 s^2}\right) & |\xi_i| < \mu \\ \exp\left(\frac{\|\xi_i|-\mu|^2}{u_2 s^2}\right) & |\xi_i| \geq \mu \end{cases} \tag{17}$$

$$\mu = \frac{1}{n}\sum_{i=1}^{n} |\xi_i| \tag{18}$$

$$S = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (|\xi_i|-\mu)^2} \tag{19}$$

where the fitting error ξ_i can be obtained by GEP-LSSVM method and u_1 and u_2 are the width adjustment parameters. Through lots of numerical experiments with thermal error value and temperature value, the range of the parameters u_1 and u_2 is eventually determined as $u_1 \in [9, 17]$, $u_2 \in [6, 13]$. In this paper, the parameters of u_1 and u_2 are determined as $u_1 = 11.34$, $u_2 = 8.52$.

Compared with the standard normal distribution weighting method whose curve distribution is symmetrical, this proposed weighting method based on the improved normal distribution weighting rule whose curve distribution expressed by the Eq. (17) can better reflect the influence of the error data points of the samples in the model. The calculating process includes the following five steps and its flowchart is shown in Fig. 6.

1. Input the sample Data$_i$($i = 1, 2, 3, \cdots, 120$) in Section 3 combined with the optimized parameters to establish the LS-SVM model.
2. Calculate the values of the model fitting errors ξ_i.
3. Calculate the values of weights ν_i by Eq. (17).
4. Establish the WLS-SVM model with the weights ν_i.

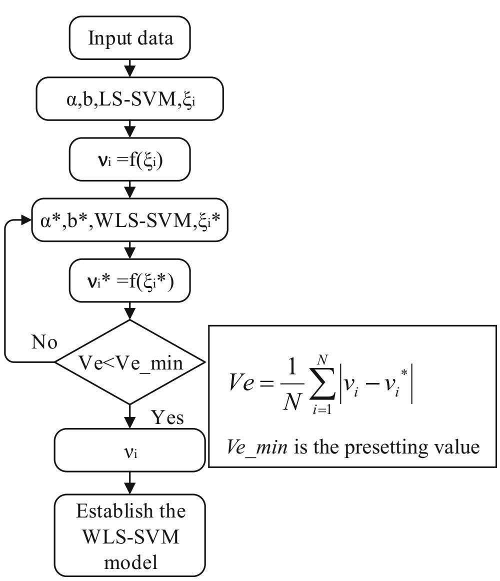


Fig. 6 The flowchart of adaptive WLS-SVM method

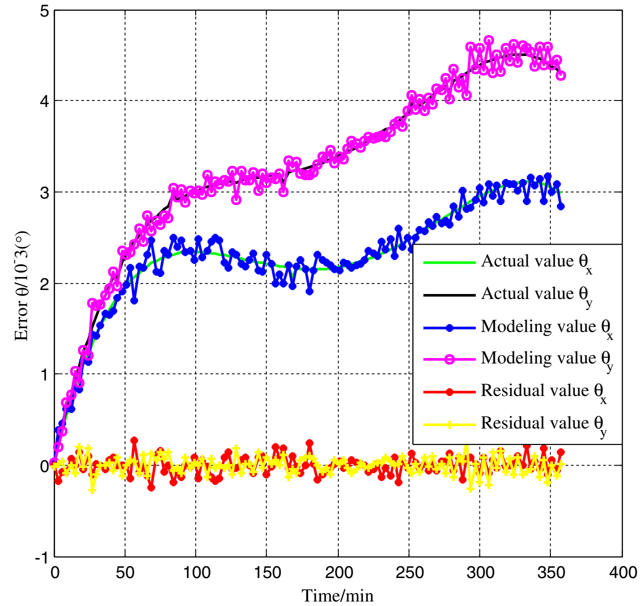


Fig. 7 The modeling results of thermal error (θ_x, θ_y)

5. If the weights meet the presetting value, choose the weights and end the calculation, otherwise return to step 2.

4.3 Modeling experiment

In this part, the mathematical model of the spindle thermal error of CNC machine tool is about to be established by the theory of GEP-WLS-SVM modeling method which has been elaborated above. The general modeling process can be carried out by the following four steps:

1. Calculate the optimum parameters (c, σ) based on the GEP algorithm to achieve the GEP-LSSVM method.

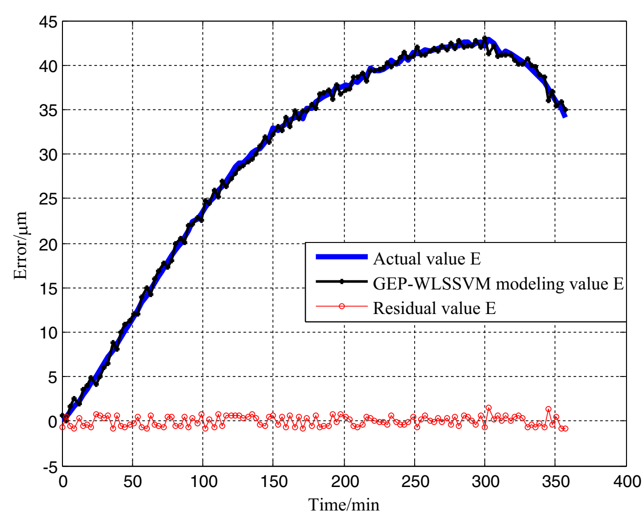


Fig. 8 The modeling results of thermal error E

Table 7 A portion of the parameters of mathematical expressions for θ_x, θ_y, and E based on GEP-WLSSVM modeling method

θ_x	c	457.1	σ	2.585	b	−4.592	MSE	0.0184	R^2	0.982	$n(SVs)$	45
	i	1	2	3	4	5	……	41	42	43	44	45
	α_i	−5.918	1.833	−2.594	−1.510	0.175	……	−87.00	204.7	−83.11	138.76	47.95
	υ_i	0.334	0.065	0.172	0.005	0.452	……	0.844	0.092	0.532	0.776	0.002
θ_y	c	819.2	σ	0.225	b	−3.729	MSE	0.0027	R^2	0.989	$n(SVs)$	81
	i	1	2	3	4	5	……	77	78	79	80	81
	α_i	33.2	21.7	−83.4	31.6	15.0	……	−21.2	48.9	34.2	−77.6	−31.2
	υ_i	0.406	0.232	0.235	0.036	0.140	……	0.693	0.224	0.008	0.251	0.052
E	c	22.672	σ	5.451	b	−0.447	MSE	0.0009	R^2	0.997	$n(SVs)$	76
	i	1	2	3	4	5	……	72	73	74	75	76
	α_i	−2.329	2.776	−5.433	7.245	21.345	……	12.652	22.627	−3.897	14.361	−20.392
	υ_i	0.350	0.076	0.745	0.998	0.167	……	0.432	0.623	0.574	0.734	0.443

c and σ is the penalty coefficient and the kernel function width; $n(SVs)$ is the number of support vector machine; α_i ($i = 1, 2, \cdots, n(SVs)$) are the Lagrange multipliers; b is the threshold value

2. Given each Data$_i$($i = 1, 2, 3, \cdots, 120$) of θ_x, θ_y and E in Section 3, train these samples by using GEP-LSSVM method.
3. Calculate their fitting error ξ_i.
4. Calculate the weighting value υ_i by Eq. (17) based on the improved normal distribution weighting rule.
5. Calculate the values of $\boldsymbol{\alpha}$ and b by Eqs. (12) and (13) to establish GEP-WLSSVM-based thermal error model, so that their mathematical expression can be obtained.

The input samples of the training data used to carry out modeling experiment are respectively shown in Tables 4, 5, and 6.

By using the GEP-WLSSVM modeling method, the thermal error regression results of θ_x, θ_y, and E are respectively shown in Figs. 7 and 8. Their mathematical expressions share the equation as Eq. (14). A portion of their coefficients and parameters are shown in Table 7.The values of MSE and R^2 are used to evaluate the performance of the regression model and they can be defined by the following equations:

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{\delta}_i - \delta\right)^2 \tag{20}$$

$$R^2 = \frac{\left(n\sum_{i=1}^{n}\hat{\delta}_i\delta_i - \sum_{i=1}^{n}\hat{\delta}_i\sum_{i=1}^{n}\delta_i\right)^2}{\left(n\sum_{i=1}^{n}\hat{\delta}_i^2 - \left(\sum_{i=1}^{n}\hat{\delta}_i\right)^2\right)\left(n\sum_{i=1}^{n}\delta_i^2 - \left(\sum_{i=1}^{n}\delta_i\right)^2\right)} \tag{21}$$

where n is the number of samples, δ_i is the ith actual value, and $\hat{\delta}_i$ is the ith output value.

From these diagrams above, it can be concluded that the maximum value of residual error of E is 0.7664 μm and 0.238e−3° to θ_x, 0.291e−3° to θ_y. It can be seen that the GEP-WLSSVM modeling method has high accuracy and can precisely estimate the change trend of thermal error in machine tool.

Table 8 The comparison of modeling parameters and effect

Algorithm	C	σ	MSE	R^2
GEP	22.672	5.451	0.0507	0.997
GA	45.023	6.876	0.0729	0.978
PSO	37.806	3.750	0.0843	0.943

4.4 Comparison of modeling performance

In order to further verify the modeling performance of GEP-WLSSVM method, it has been compared with the GA-LSSVM [23] prediction model and PSO-LSSVM [24] prediction model by using the training data of samples:

Data$_n = [T, \delta] = [T_n, E_n] = [t_{3n}, t_{8n}, E_n]$, $n = 1, 2, 3, \cdots, 120$.

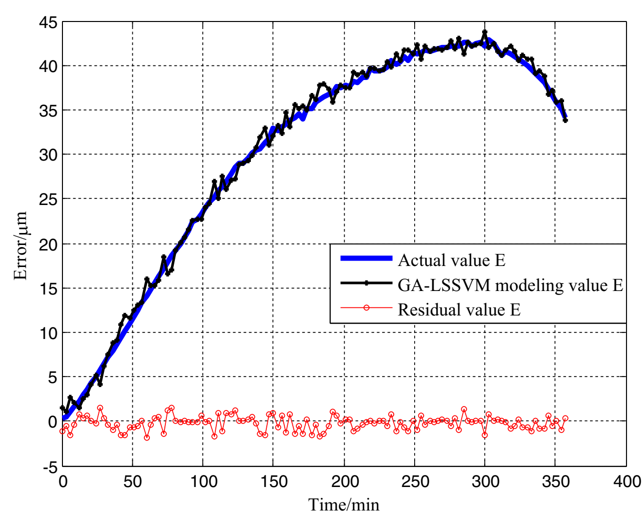


Fig. 9 The modeling results based on GA-LSSVM

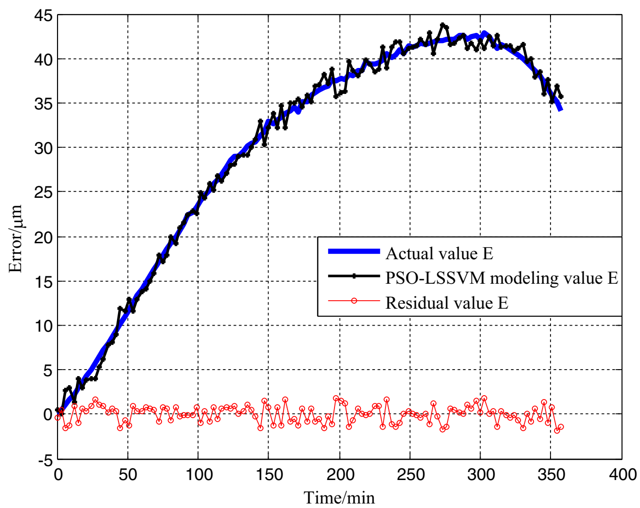


Fig. 10 The modeling results based on PSO-LSSVM

The parameters for each model are shown in Table 8. The modeling results are shown in Figs. 9 and 10. Their residual values are compared in Fig. 11.

It can be clearly seen from the comparison results. For GEP-WLSSVM model, the training samples with a relative fitting error of less than 8% are up to 93% and the maximum fitting error is 0.7664 μm. The mean square error (MSE) of thermal error E is 0.0507 μm. The MSEs of GA-LSSVM and PSO-LSSVM model are 0.0729 μm and 0.0843 μm respectively. It can be assuredly concluded that the thermal error modeling method of CNC machine tool based on GEP-WLSSVM has high prediction accuracy, robust performance, and generalization ability, which is superior to the method of GA-LSSVM and PSO-LSSVM.

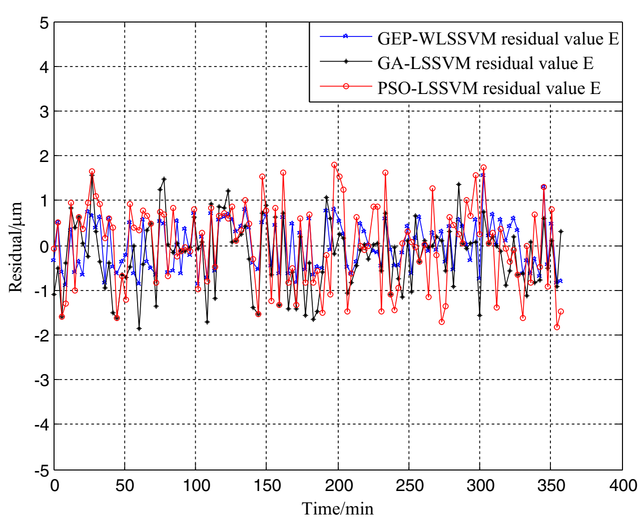


Fig. 11 The comparison of residual values

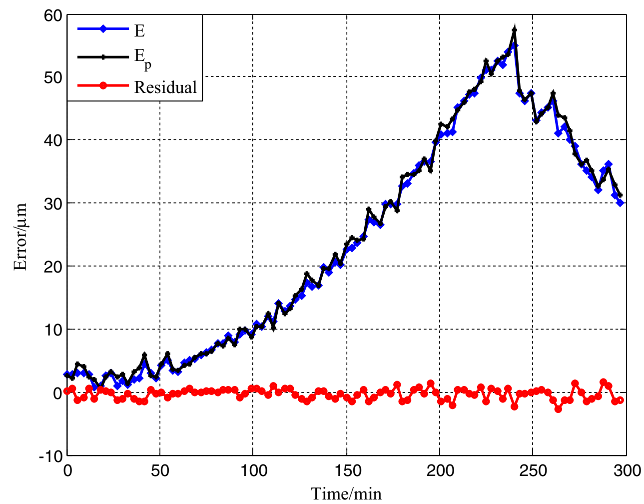


Fig. 12 The modeling results under variable conditions

5 Verification of prediction accuracy

In order to verify the feasibility, robustness, and generalization ability of the GEP-WLSSVM method for the modeling of spindle thermal error, this model is applied to predict and analyze the axial thermal error E under variable working conditions. The machine tool spindle started from cold state and ran 2 h at 5000 rpm speed then 2 h at 9000 rpm speed, and finally stopped for 1 h to cool down naturally. During the whole test, the two temperature sensors (T3 and T8) and the displacement sensor D1 took the data of samples every 3 min. Then, 100 groups of temperature data were input into the GEP-WLSSVM model, and the prediction value E_p of the axial thermal error E of the spindle can be output. The prediction effect of the model can be proved by comparing the prediction value E_p with the thermal error data E measured by the displacement sensor D1.The comparison between the prediction and the measured results is shown in Fig. 12. The parameters of the mathematical model are shown in Table 9.

It can be concluded that the residual value range is [− 0.6648,0.8168] and the prediction model still has high prediction ability with high robustness under variable working conditions. It proves the feasibility and effectiveness of the GEP-WLSSVM modeling method.

Table 9 The parameters of the mathematical model

Parameter	C	σ	MSE	R^2
Value	11.31370	4.34603	0.063307	0.96684

6 Conclusions

Establishing a highly accurate mathematical model of thermal error is of great significance to the further thermal error compensation. In this study, GEP-WLSSVM algorithm is proposed to have created a high-precision and robustness thermal error model of CNC machine tools. This modeling accuracy reaches 0.7664 μm in the axial direction. It can be drawn with the following conclusions.

1. The five-point measurement method is a simple but flexible way for measuring the thermal deformation of the CNC machine tools' spindle.
2. Partial correlation analysis method can effectively identify the sensitive temperature points.
3. The gene expression programming (GEP) algorithm can be used to optimize the parameters involved in LS-SVM.
4. The improved normal distribution weighting rule (INDWR) can be used to obtain the weighting values of WLS-SVM to boost the regression precision.
5. The GEP-WLSSVM modeling method can be used to model and predict the thermal error of the spindle of CNC machine tools, and lay a foundation for the next compensation work.
6. Compared with GA-LSSVM and PSO-LSSVM, the model built with the GEP-WLSSVM method is of higher prediction accuracy, stronger robustness, and generalization ability.

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