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# Interval type-2 fuzzy logic systems optimized by central composite design to create a simplified fuzzy rule base in image processing for quality control application

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#### Abstract

A novel method that uses a Mandami interval singleton type-2 fuzzy logic system (IT2 SFLS) with the support of the central composite design (CCD) technique and the classic approach of composed base inference (CBI) is made to enhance the modeling and the construction of the fuzzy rule base. The IT2 SFLS has the potential to outperform the singleton type-1 fuzzy logic systems (T1 SFLS). The IT2 SFLS systems accounts for the uncertainties that can be added during the system modeling and construction: the uncertain rules created using noisy data. There is no way to take into account this uncertainty in the antecedent and consequent membership functions of a singleton type-1 fuzzy logic systems. Due to this uncertainty, an additional process is required to filter the measured data, but the uncertainty is still present in the structure of the T1 system. The main goal of the proposed model is to enhance the performance obtained in the dimensional features evaluation in a quality assurance process of the manufacturing of product parts. The experiments developed in a real facility show that the application of the proposed method produced better results than that obtained by the T1 SFLS benchmarking system.

Keywords IT2 FLS . Interval type-2 fuzzy logic . Image processing . Artificial vision . CCD

# 1 Introduction

The image processing for quality control is one of the most complicated tasks because it needs some devices and some phases such as filtering, segmentation, dimensioning, and

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pattern recognition, among others. All these phases provide uncertainty, which is presented in the form of variations caused by the devices that are needed to assemble an artificial vision system. Devices that conform an artificial vision system are camera, light source, and software to process the images [\[1](#page-9-0)]. In order to identify the variations presented in images, Jean-Yves Bouguet presented a calibration tool that provides some necessary parameters to know the variations produced by the camera. Figure [1](#page-1-0) shows the shape of the lens of the camera which produces a sketch that shows the defects in the lens. This figure shows the variations in the form of the lens that provide variations in the image [\(http://www.vision.](http://www.vision.caltech.edu/bouguetj/calib_doc/index.html#ref) [caltech.edu/bouguetj/calib\\_doc/index.html#ref](http://www.vision.caltech.edu/bouguetj/calib_doc/index.html#ref)). An example of variations in the image acquisition is the spatial position of the capture device, this position causes distortions on the size of the sample that appear in the picture [\[2](#page-9-0)], e.g., a dark image can produce the joint of the object and background, another effect is called parallax, that cause changes in the dimension of the sample (Fig. [2](#page-1-0)).

The parallax effect can be interpreted as a spatial positioning in a 3D environment that changes the geometry of the object [[3\]](#page-9-0) by perspective; this can transform a square into a rectangle or a rhombus or diamond (Fig. [3](#page-2-0)). At the

<span id="page-1-0"></span>

Fig. 1 Distortion of a lens, adapted from ([http://www.vision.caltech.edu/bouguetj/calib\\_doc/index.html#ref](http://www.vision.caltech.edu/bouguetj/calib_doc/index.html#ref))

same point, the distortion causes misclassification because the object cannot be positioned on the border of a pixel and these conditions cause loss of information or addition of it, e.g., Demant et al. in [\[4](#page-9-0)] mentions that at least a width of 570 pixels is needed to identify a barcode to obtain at least 5 pixels for thin lines due to degradation in the filtering and segmentation phases, only with this condition, the identification is accomplished, see Fig. [4.](#page-2-0) Other problems are: (A) the conformation of the sensor, Table [1,](#page-2-0) (B) the exposition time regulated by the F-number [\[5](#page-9-0)] see Fig. [5](#page-3-0), and (C) some perturbations caused by the environmental conditions produce a variation in the measurement data in an amount near to a one standard deviation as is mentioned by the National Institute of Standards and Technology (NIST) [[6](#page-9-0)]. This condition produces corrupted data and their values are increased or decreased due to the displacement of the membership function to the left or to the right due to the uncertainty [[7](#page-9-0)], (Fig. [6\)](#page-3-0). This uncertainty is called footprint of uncertainty.

In 2013, a compilation of applications of type-2 fuzzy logic systems was presented by Melin and Castillo in [[8\]](#page-9-0). In this work, some artificial vision and pattern recognition systems are described. The systems  $[9-14]$  $[9-14]$  $[9-14]$  are used to classify, evaluate, and to accomplish several quality tasks by image processing. The limited production of real applications of fuzzy systems appears in the type-1 and type-2 fuzzy logic applications because some [[15](#page-9-0)–[24](#page-9-0)] mention that there are multiple complications to generate soft computing models. The most mentioned complications are the following:

The hardness to establish the form of membership functions (MF's), [\[15](#page-9-0)–[17\]](#page-9-0).



Fig. 2 Effects of parallax in a measure, adapted from [\[2](#page-9-0)]

<span id="page-2-0"></span>

Fig. 3 3D positioning of a piece, adapted from [[3\]](#page-9-0). a Overhead, b oblique, c perspective.

- The development of the inference module by applying different techniques such as:
- Artificial neural networks (ANN) [\[16,](#page-9-0) [18](#page-9-0)–[20](#page-9-0)].
- Genetic algorithms (GA) [\[21](#page-9-0)–[23\]](#page-9-0).
- ANN and GA [\[24\]](#page-9-0).
- The setting of forms and limits for the MF's and the inference module for their FLS. The solution to this problem is partially proposed by Sahab and Hagras in [\[17\]](#page-9-0), they proposed that the MF can be generated with a free-form distribution assembly with a histogram of data. However, it is not necessarily a solution, the model does not need a particular form of MF.

On the other hand, there has not appeared in literature a specific method to model an intelligent system. The universal approximation theorem [[25](#page-9-0)] does not explain the number of inputs, rules, or fuzzy sets that are needed to generate a reliable system. Some efforts were made by several authors to generate specific algorithms to model this class of systems [\[26](#page-9-0)–[38\]](#page-9-0). To provide a solution for these complications, they can be used in the limits of specification to get the limits of the universe of disclosure so that it can be used in some methodologies presented in literature, but in this case, the literature survey provides a few solutions.

The aim of this work is to present a simple method to model the IT2 SFLS using CCD that allows modeling a multiple-input-single-output system in a fast and simple way. In Section 2, the theoretical foundations are presented. Section [3](#page-4-0) presents the construction process of the proposed IT2 SFLS/CCD model. Section [4](#page-7-0) presents the



The sensor of the field of view is too close to the 570 pixels

The loss of information in data transmission is 10%, approximately

The fact of the non-square pixels

The thermal excitation of the neighbor pixels in the sensor due to the charge of the photon

experimental results. The conclusion section is presented in Section [5](#page-8-0).

# 2 Theoretical foundations

The CCD is a technique used in design of experiments (DOE) to study the interaction between variables called factors. These factors have specified limits, and these limits are called levels. Levels, work in an interval. The edges of this interval are defined as high and low levels. A graphical model has three factors, by the Euclidean space; usually, these three factors change every state; the additional factors remain fixed in order to represent the model in three dimensions [\[39\]](#page-9-0). Jang's [\[40](#page-9-0)] showed a graphic representation of a model matrix used to create the rules, this model is equivalent to (1), where the dimension of space (quantity of variables)  $N$  represents the levels and k represents the factors that need to be evaluated, respectively. In this case, a  $2<sup>k</sup>$  model was selected. The models have maximum and minimum levels that are known as treatments or axial points. These points conform the rules of fuzzy sets. The CCD model is very useful at the beginning of experiments because it only needs a few tests to evaluate the behavior of the model in a confidence interval.

The  $2<sup>k</sup>$  factorial design belongs to the CCD. The DOE is obtained by [\(1\)](#page-3-0) which is a combination between the factor levels. The principal effects of different points in Fig. [7](#page-4-0) are obtained by the rows of the matrix represented in Table [2.](#page-4-0) For technical purposes in the original model [\[39\]](#page-9-0) when both variables are at low level, they do not have symbolic representation as they are represented by the number one "1," in the case of variable  $a$  in low level and  $b$  in high level, it is represented by  $b$ ; in the case of  $a$  in high level and  $b$  in lower level, it is



Fig. 4 Positioning of thin lines on image

<span id="page-3-0"></span>Fig. 5 Exposition of the image, adapted from [\[5\]](#page-9-0)



represented by  $a$ ; if both variables are in high level, they are represented by ab. Finally, the relation matrix is given by (2), this matrix is obtained from Table [2.](#page-4-0)

$$
N^k \tag{1}
$$

Where  $N$  represents the variables and  $k$  represents the possible states of every variable in the control limits. The mathematical values for the lower and upper limits are obtained by (2).

$$
2^2 \tag{2}
$$

The adaptation of the method proposed by [[27](#page-9-0)–[29](#page-9-0)] produces the matrix  $(3)$ . Every point marked in Fig. [7](#page-4-0)  $(ab, b, a,$ 1) represents a row in the matrix  $(3)$ . Later to obtain the process limits the points  $(ab, b, a, l)$  are replaced with the specification limits for every variable, those limits represent the quality control limits and are used as labels for every factor and finally as fuzzy labels,

$$
2^{k} = \begin{bmatrix} a & b \\ a & B \\ A & b \\ A & B \end{bmatrix} = \begin{bmatrix} y_{1} \\ y_{b} \\ y_{a} \\ y_{ab} \end{bmatrix}
$$
 (3)

Fig. 6 Effects of type-2 uncertainty. Adapted from [\[6](#page-9-0)]

Where the lower control limit (LCL) of the first variable is equivalent or equal to "a," the upper control limit (UCL) level of "a," is equal to "A." The same case for variable "B," with these changes, the points of Fig. [7](#page-4-0) are replaced by column 3 of Table [3](#page-4-0) and given by (4),

$$
2^{k} = \begin{bmatrix} LCLa & LCLb \\ LCLa & UCLB \\ UCLA & LCLb \\ UCLA & UCLB \end{bmatrix}
$$
 (4)

Where  $2^k$  is the matrix that conforms the CCD model and every pair in the matrix is formed by the possible combinations of the lower and upper limits of control of the input variables.

In the case of missing information about the limits, they should be generated in an interpolation to obtain the crisp value of every limit that can be obtained by a regression model (5) obtained in [\[40](#page-9-0)]. The correlation coefficients yield  $(6-9)$  $(6-9)$  $(6-9)$ ,

$$
y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \varepsilon \tag{5}
$$

Where  $y$  is the output approximation of regression model,  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$ ,  $\beta_{12}$  are the regression coeficients, and  $\varepsilon$  is the error of aproximation,



<span id="page-4-0"></span>

Fig. 7 CCD model graphical adaptation. Adapted from [\[40\]](#page-9-0)

$$
\beta_0 = \overline{X} = \sum_{i=1}^n \frac{x_i}{n} = \frac{LCL_a + LCL_b + UCL_a + UCL_b}{n}
$$
 (6)

Where  $\overline{X}$  is the mean of  $x_i$  or the control limits (UCL, LCL) of the variables.

$$
\beta_1 = \frac{a-(1)}{2} - \frac{b+ab}{2} = \frac{UCL_a - UCL_b}{2} - \frac{LCL_a + LCL_b}{2} \tag{7}
$$
\n
$$
\beta_2 = \frac{ab + (1)}{2} - \frac{b+a}{2} = \frac{LCL_b + UCL_b}{2} - \frac{LCL_a + UCL_a}{2} \tag{8}
$$

$$
\beta_{12} = \frac{((1) - ab)}{2} - \frac{a - b}{2} = \frac{UCL_b - LCL_b}{2} - \frac{UCL_a - LCL_a}{2} \tag{9}
$$

Where *a* is the UCL<sub>a</sub>, *ab* is the LCL<sub>b</sub>, (1) is the UCL<sub>b</sub>, and b is the  $LCL$ <sub>a.</sub>

As is known, the fuzzy set of type-1 needs only a single parameter on every variable to obtain a membership function (10). Moreover, for IT2 fuzzification (Fig. [8](#page-5-0)), an interval type-2 fuzzy set  $[25]$  is represented by  $\ddot{A}$ , characterized by an interval type-2 membership function  $\mu_{\tilde{A}}(x, u)$ , where  $x \in X$  and  $u \in$  $J_x \subseteq [0, 1]$ :

$$
A = (x, \mu_A(x))|\forall x \in X \subseteq [0, 1]
$$
\n<sup>(10)</sup>

Table 2 Values for inputs of CCD states of every variable and combinatorial treatment

Variable		Condition		Symbolic representation Treatment	
$X_1$	$X_{2}$				
$\mathsf{A}$	В	Low	Low	$ab = 1$	A <sub>low</sub> , B <sub>low</sub>
$\mathbf{A}$	В	High	Low	$Ab = a$	$A_{high}$ , $B_{low}$
$\mathbf{A}$	B	Low	High	$aB = b$	A <sub>low</sub> , B <sub>High</sub>
$\mathbf{A}$	B	High	High	$AB = ab$	$A_{high}$ , $B_{High}$

Table 3 Equivalences of CCD states and quality control limits

CCD symbolic representation	Treatment	Quality control limits
$ab = 1$	A low, $B_{low}$	LCLa LCLb
$Ab = a$	$A_{high}$ , $B_{low}$	LCLa UCLB
$aB = b$	A <sub>low</sub> , B <sub>High</sub>	LCLA LCLb
$AB = ab$	$A_{high}$ , $B_{High}$	UCLA UCLB

$$
\tilde{\mathbf{A}} = \left\{ \left( \left( x, u \right), \mu_{\tilde{\mathbf{A}}}(x, u) \right) | \forall x \in X, \forall u \in j_x \subseteq [0, 1] \right\}
$$
(11)

Where A is a set that contains the values of the x variable and their membership values  $\mu_A(x)$ .

## 3 Constructing the IT2 SFLS/CCD model

As it is known, a matrix is composed onto a family of equations that represents several states of a non-linear function. The case of the IT2 SFLS/CCD requires an interval that contains the uncertainty generated at the inputs in a place of crisp number. This uncertainty needs to be treated, yet its task requires the calculation of an additional parameter to determine the spread of the data and the interval limits which are given by sigma (12),

$$
\sigma_{x_i} = \sum_{i=1}^n \left( \frac{x_i - \overline{x}_i}{n} \right) \tag{12}
$$

Where  $\sigma$  is the standard deviation of the data, *n* is the number of parameters,  $\overline{x}_i$  is the mean of the *i* variable, and  $x_i$ is the  $n$  state of the variable.

Once obtained, the correlation matrix ([2\)](#page-3-0) of the input variable  $(x_i)$ , the additional parameter  $\sigma_{x_i}$  is calculated the IT2 SFLS rule base. Calculating the lower interval  $\overline{N}$  given by (13) and  $\bar{N}$  upper interval limits given by (14), and their respective solution  $\bar{y}$  upper and y lower interval limits by interpolation, which are given by  $(15 \text{ and } 16)$ . Then Eq.  $(3)$  $(3)$  is converted into ([17](#page-5-0)) to get the rule base matrix of the IT2 fuzzy system, and schematic representation of the inputs for the model is presented in Fig. [8,](#page-5-0)

$$
\underline{N} = x_i - \sigma_{x_i} \tag{13}
$$

$$
\bar{N} = x_i + \sigma_{x_i} \tag{14}
$$

$$
\underline{y} = y_i - \sigma_{y_i} \tag{15}
$$

$$
\bar{y} = y_i + \sigma_{y_i} \tag{16}
$$

#### <span id="page-5-0"></span>Fig. 8 IT2 SFLS antecedents. Adapted from [[25](#page-9-0)]



$$
Rb = \begin{bmatrix} \frac{a}{a} & \overline{a} & \underline{b} & \overline{b} & \underline{y_1} & \overline{y_1} \\ \frac{a}{a} & \overline{a} & \underline{B} & \overline{B} & \underline{y_b} & \overline{y_b} \\ \frac{A}{a} & \overline{A} & \underline{b} & \overline{b} & \underline{y_a} & \overline{y_a} \\ \frac{A}{a} & \overline{B} & \overline{B} & \underline{y_{ab}} & \overline{y_{ab}} \end{bmatrix}
$$
(17)

 $For i = 1, a, b, ab$ 

Rule  $i : \text{IF } x1$  is a and  $x2$  is b then y is 1 (18)

Rule 
$$
i
$$
: IF  $x1$  is  $\tilde{F}_1^i$  and  $x2$  is  $\tilde{F}_1^i$  then  $yi = \tilde{G}^l$  (19)

Where *a* is the value of the *x* variable,  $\tilde{F}_1^i = \{m_1, m_2\}$  and  $\tilde{G}^l = y_i = \left\{ \bar{y}_i, \underline{y}_i \right\}.$ 

Using  $(13-16)$  $(13-16)$  $(13-16)$  $(13-16)$ ,  $(20-23)$  are obtained,

$$
m_{1} = \underline{a} = a - \sigma_a \tag{20}
$$

$$
m_{2} = \bar{a} = a + \sigma_a \tag{21}
$$

$$
\underline{y_i} = y - \sigma_y \tag{22}
$$

$$
\bar{y}_{i} = y + \sigma_y \tag{23}
$$

Where  $m_1$  represents the lower value function,  $m_2$  represents the upper value function,  $\alpha$  represents the lower limit of the variable a, the upper limit of a is obtained by  $\bar{a}$ , and  $\underline{b}$ represents the lower limit of the variable b, the upper limit of b is obtained by  $\bar{b}$ ; the interval for the output response is

Table 4 Parameters for the T1 FLS (mean and standard deviation of the samples)

Feature	$X_1$ (height)	$X_2$ (width)	
Sigma	2.15	1.31	
Lower control limit	263.396	181.090	
Mean	274.85	188.15	
Upper control limit	286.3	195.200	

Where  $Rb$  represents the fuzzy rule base matrix,  $a$  represents the lower limit of the IT2 for the variable  $a$ , the upper limit of the IT2 for the variable  $a$  is obtained by  $\bar{a}$ in the LCL<sub>a</sub>,  $\underline{A}$  represents the lower limit of the IT2 for the variable  $a$ , the upper limit for the IT2 for the variable  $a$  is obtained by  $\overline{A}$  in the UCL<sub>a,</sub>  $\underline{b}$  represents the lower limit of the IT2 for the variable  $b$ , the upper limit of the IT2 for the variable *b* is obtained by  $\bar{b}$  in the LCL<sub>b</sub>, and <u>*B*</u> represents the lower limit of the IT2 for the variable  $b$ , the upper limit of the IT2 for the variable *b* is obtained by  $\bar{B}$  in the UCL<sub>b</sub>. The interval for the output response is obtained by  $Y_i$ ,  $\overline{Y}_i$ . Those values represent the lower and upper output or response for a specific pair of states  $(a, b, ab, 1)$  or treatments of the inputs.

As it is known, the rule base for a T1 SFLS having  $m$ inputs  $x_1, x_2, ..., x_n$  and one output y, can be described by a fuzzy rule IF-THEN that shows the mathematical relationship between the input and output of the system which yields (18). In the particular case of IT2 SFLS, two parameters are needed for every input variable and two for the output that are expressed as fuzzy rules given by (19), it represents a relation between the input and output spaces of type-2 FLS [[25](#page-9-0)],



Fig. 9 Conversion of the system from type-1 to type-2. Adapted from [[25](#page-9-0)]

obtained by  $\underline{Y}_i$ ,  $\overline{Y}_1$ . Those values represent the lower and upper output or response for an specific pair of states or treatments of the inputs.

#### a. Forecasting

Due to the aspects mentioned in the introduction, several factors provide uncertainty into the system and make the process complex in terms of producing an evaluation. One of the most critical operations on the image processing is the filtering, because of their loss of some information caused by the transition between the sample and the background. An image can be used to calculate the dimension of some pieces based on a pattern that allows to use to generate several states used for modeling the system using the interpolation produced by cross multiplication (24),

$$
Y = \frac{B \cdot X}{A} \tag{24}
$$

Where  $A$  is the mean of  $X_n$ ,  $B$  is  $Xi$ , and  $Y$  is the new value for some state of the variable with new state of  $X_n$ .

Table 5 Parameters for the IT2 FLS (mean and standard deviation of the samples)

Feature	$M X_1$ (height)		$M X_2$ (width)	
Mean interval	272.6	277	186.84	189.46
Upper control limit interval $283.225$ $288.454$			194.238	196.858
Lower control limit interval $260.395$ $264.69$			178.944 182.722	

#### b. Input-output data

The input consists of two parameters (height and width) that are used to obtain the quality features needed for the sample. The output was designed in order to be adjusted to a quality control chart with six intervals delimited by the standard deviations between the specification limits, Table [4](#page-5-0).

The variables are modeled as T1 fuzzy sets. Figure 9a shows the treatments of Table [2](#page-4-0) for generating a reference system that is used as a benchmark between T1 SFLS. In Fig. 9b, the noise generated by the processing of the images that are added to the model in order to obtain the footprint of uncertainty is shown in Fig. 9b; then the fuzzy sets of type-1 are converted to a IT2 SFLS, Fig. 9c, and expressed in Table 5. Figure 9d shows an example of the type-2 output and the values that could be obtained after processing the information by the expert system, obtaining several values for every sample.

## c. Rule base

The rule base for the hybrid IT2 SFLS/CCD model consists of the treatments generated by the CCD model. For example, rule 1 is given by (25) for a T1 SFLS, on the other hand, for an IT2 SFLS the rule yields ([26\)](#page-7-0) that includes the lower and upper limits of the variable to obtain the lower and upper membership grades required by the IT2 model,

if 
$$
X_1
$$
 is a and  $X_2$  is b THEN  $Y_1$  (25)

<span id="page-7-0"></span>



if  $X_1$  is  $(\underline{a}, \overline{a})$  and  $X_2$  is  $(\underline{b}, \overline{b})$  THEN  $(Y_1, \overline{Y}_1)$  (26)

Where *a* represents the variable  $X_i$ , *a* represents the lower limit of the variable a, the upper limit of a is obtained by  $\bar{a}$ , and  $b$  represents the lower limit of the variable  $b$ , the upper limit of b is obtained by  $\bar{b}$ ; the interval for the output response is obtained by  $Y_1$  on type-1 fuzzy model T1 SFLS and  $\overline{Y}_i$ ,  $\overline{Y}_1$ on type-2 fuzzy model. Those values represent the lower and upper output or response for a specific pair of states or treatments of the inputs.

## b. Test

Once the expert system is created, it is tested without training to generate an approximation and evaluate its performance. The tests were conducted with a database of 70 samples, where 40 samples were used to train the system and 30 to test the model.

## 4 Results

time spent in approximation

The approximations generated by the expert systems after 50 epochs of training show an error rate of 1.5 pixels in the

Table 6 Computational Epochs Time (sec) T1 SFLS IT2 SFLS 1 0.050 0.548 2 0.082 0.595 5 0.136 0.779 10 0.200 0.842 20 0.316 1.441 50 0.695 2.590 100 1.256 3.860 200 2.126 7.827 500 4.952 18.008

dimension for the predictions produced by T1 SFLS. On the other hand, the IT2 SFLS system shows error rates below 0.5 pixels in the prediction system (Fig. 10). The solid line shows the goal, the dotted line shows the predictions generated by the T1 SFLS, and the dashed line presents the evaluation given by the IT2 SFLS expert system. In contrast, the approximation generated by the T1 SFLS with 100 epochs of training shows better results and only a half of the error rate, Table 6, of the IT2 SFLS model with less computational time, Fig. 11, half of the computational time used for predictioning in IT2 SFLS is used in T1 SFLS requiring double training (50 epochs of training in IT2 SFLS vs. 100 epochs of training in T1 SFLS epochs) in type-1, Table [7.](#page-8-0)

To evaluate the predictions obtained, the root mean square error (RMSE) given by  $(27)$  is used and the results of the tests are described in Table [7](#page-8-0) and presented in Fig. [12](#page-8-0). The obtained predictions are below the typical variation of 2% as is mentioned in [\[4\]](#page-9-0) as a common variation in that kind of processes; also, a variation of 2% of the measured feature shows higher variations as is shown in Fig. [13.](#page-8-0) These variations are in orders of two standard deviations but still are on the specified interval. In the case of uncertain inputs, one of the samples can be qualified as a non-conformity item because of the variation



Fig. 11 Computational time used for prediction,  $($   $)$  T1 SFLS,  $($ ... $)$  IT2 SFLS

<span id="page-8-0"></span>Table 7 Expert systems, approximation RMSE

Epochs	$MSE(\sigma)$		
	T1 SFLS	<b>IT2 SFLS</b>	
1	0.84052361	0.82949805	
2	0.83456868	0.82075272	
$\overline{\mathcal{L}}$	0.81566363	0.79255869	
10	0.78213732	0.73798459	
20	0.70749078	0.60189317	
50	0.43913090	0.15540070	
100	0.08292205	0.03677361	
200	0.02654346	0.03656664	
500	0.02715470	0.03545607	



Fig. 12 RMSE of prediction ( $\Box$ ) T1 SFLS, (...) IT2 SFLS

produced by the measurement system, but it is not the case of this proposed application. If the uncertain inputs increase the measurements the conformity interval, it is displaced from  $[-6, 6]$  to  $[-4, 8]$ , then the lower limit with  $-2\%$  on the specification appears in the  $-2\sigma$  range and still accomplishing the specification. On the other hand, if the uncertainty decreases the value of the measurements, the interval is displaced from  $[-6, 6]$  to  $[4, -8]$  and the upper limit, in this case, with  $+2\%$  of the specification appears in the range of  $2\sigma$ .

Fig. 13 Interval generated by a 2% variation (uncertainty) on the image processes, (- - -) lower and upper limits,  $($   $)$  goal,  $($ . . .) prediction

$$
RMSE = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n}}
$$
 (27)

Where  $y_i$  represents the approximation,  $\hat{y}_i$  represents the goal, n represents the total number of samples, and i represents the sample processed.

## 5 Conclusions

The proposal exposed above shows a simplified method to assemble interval singleton type-2 fuzzy logic systems based in central composite design. The use of IT2 model addresses the problem of thin lines and provides a tool to contain the uncertainty from the positioning of the lines themselves. This approach solves both problems presented above.

The error converges to the same value after 200 epochs of training without worrying about the type of system used, see Table [6](#page-7-0) and Fig. 12.

The use of the interval type-2 produces better results due to the freedom provided by the form of the output in a set, instead of a crisp value. Also, the IT2 absorbs the uncertainty produced by the fuzzy rule base design and implementation.

The proposal can be used in an industrial application for online quality control with the chance to evaluate all samples with the same criteria over time.

Some parameters can be eliminated with the use of design experiments techniques, this would avoid all the mathematical modeling complexities. Because these factors do not have a relation with the final quality of the sample, e.g., the color of the sample is not important to determinate the dimension of the piece, among others. For they do not have the interaction with the principal variables, the elimination of these variables cannot produce variations on the final evaluation.

The IT2 system is useful in processes such as the visual inspection processes where the exact definition of a feature cannot be interpreted with precision in a mathematical form. The type-2 model provides the chance to generate a mathematical definition of these features in the form of intervals defined by crisp limits. Then the function that evaluates the



<span id="page-9-0"></span>object could be a sentence that describes all necessary features to evaluate the quality of the goods produced to satisfy the client expectations.

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