The International Journal of Advanced Manufacturing Technology (2019) 102:2891–2905
https://doi.org/10.1007/s00170-019-03334-9

ORIGINAL ARTICLE

Impact of surface machining complexity on energy consumption and efficiency in CNC milling

Junhua Zhao [1] · Li Li [1] · Yue Wang [2] · John W. Sutherland [2]

Received: 24 September 2018 / Accepted: 16 January 2019 / Published online: 7 February 2019

Abstract
Energy consumption of machining systems has been a great concern of many manufacturing enterprises. It is pointed out that complex properties of sculptured surface have important influence on CNC machining process where energy consumption and machining efficiency are treated as two evaluation indicators of machining system performance. This paper studies the impact of Surface Machining Complexity (SMC) on energy consumption and efficiency in CNC machining. By analyzing critical factors that influence machining power and efficiency, a pentagon model that refers to the workpiece, equipment, cutter, goal, and process is provided. Based on the pentagon model, a model for calculating SMC, which reflects the difficulty level of CNC machining, is developed. Furthermore, a detailed process of the solution using Fuzzy c-means clustering algorithm is introduced with a case study. Finally, the impact of SMC on energy consumption of machining system is discussed via a group of experiments. The experiments verified the effectiveness of the proposed method and present the increased trend between surface machining complexity and energy consumption, in particular considering the effect of surface curvature on machining energy consumption.

Keywords Energy consumption · Surface machining complexity (SMC) · Machining efficiency · Surface partitioning · Fuzzy c-means clustering algorithm

1 Introduction

In recent years, the energy consumption has aroused extensive concern, which makes people aware of the importance of energy saving and emission reduction. It is predicted that the world's energy consumption will increase by 56%, and the carbon dioxide emissions will increase by 46% from 2010 to 2040 [1], most focusing on industry field. The manufacturing industry, as a vital part of industry field, plays an important role in national economic development. Meanwhile, it has consumed enormous resources and energy, and has harmed the ecological environment. In 2012, the US energy information administration published the energy yearbook. It reported that industrial electricity accounted for 31% of the total electricity consumption, and manufacturing electricity consumption accounted for 90% of the industrial electricity consumption. The electricity consumed by machine tools account for 75% of the electricity in manufacturing [2, 3]. Therefore, sustainable development requires the reduction of resources and energy consumption, especially in mechanical manufacturing sector. However, the energy efficiency in processing is usually very low, for example, the average energy efficiency of machine tools is less than 30%. Gutowski concluded that the energy utilization rate of an automatic machining line is only 14.8% [4], and the energy consumed by milling machine is mainly in non-cutting period [5]. Gutowski reported that only about 15–25% of the total energy was used in processing [6].

With the development of industrial technology in modern society, a great variety of products with sculptured surface have been widely used in the automotive, aeronautical, ship, and die industries. Ideally, to machine workpiece with sculptured surface, the operators of the machine tools should have a good knowledge of surface geometric modeling and machinability. However, in reality, most of the machine tool operators have few knowledge of the machining complexity of the

✉ Li Li
cqulily@163.com

[1] College of Engineering and Technology, Southwest University, Chongqing 400715, China

[2] Environmental & Ecological Engineering, Purdue University, West Lafayette, IN 47906, USA

workpiece, and only uses experience to select processing parameters. This might cause unnecessary energy consumption of machine tools. As the complexity of the surface shape makes the energy consumption vary in different machining processes, the relationship between surface machining complexity (SMC) [7] and energy consumption is of great significance for energy management in the manufacturing industry [8] and the design of sculptured surface product.

2 Related work

At present, the relevant research on sculptured surface machining has mainly been done on the geometry of the sculptured surfaces [9], tool path generation [10] and optimization [11, 12], interference inspection and tool shaft orientation optimization [13], post-treatment of multi-axis machining cutter location files [14], and cutting parameter optimization [15].

Many of the studies on sculptured surface machining are based on geometric characteristics. Chen et al. [16] studied geometric and processing properties of sculptured surfaces in 3½½ axis CNC machining. This study provides a new tool path generation method that improves the machining efficiency. The sculptured surface is divided piecewise by fuzzy clustering, and the tool path is generated according to the surface curvatures. N Van Tuong and P Pokorný [17] described a method to partition sculptured surfaces based on the surface curvatures and the chain code technique, and then machine the surfaces separately by regions. Li et al. [7] divided the sculptured surface into patches according to the curvature characteristics. The surface machining complexity is proposed and calculated by considering the machining attributes, but the effect of the material is not considered in the processing attributes. The effects of the curvature of each sculptured surface patches are not considered. In these studies, the factors of machining complexity were considered incompletely. The researchers considered surface machining problems based on geometrical features of sculptured surfaces, especially curvature information, but did not analyze surface machining complexity.

Sonthi et al. [18] presented a feature recognition method based on curvature region representation. Curvature analysis is done for each part to the surface curvature properties. Based on these properties, the surface is divided into pure convex region, pure concave region, transition region, and plane region. Then, tool paths were planned on each of these regions. Lee, Ma, and Jegadesh [19] proposed the rolling ball algorithm to ensure a key region of the occurrence as well as the boundary of surface matching algorithm. From these literatures, it is clear that much work has been done in surface partitioning, which are aimed at the surface feature recognition, interference inspection, and tool path planning. Nonetheless, these methods do not discuss the complexity of machining feature based on surface patching, only considering the problem of tool path generation in certain regions.

Sridharan et al. [20] described the machining geometric features from the perspective of geometry and topology, and made an effective classification based on considering the machining method and tool path generation for machining geometric features. Chen et al. [21] proposed a CNC machining method for composite surface based on region division. The composite surface was divided into three types of regions, namely elliptical region, hyperbolic region, and parabolic region. These regional characteristics and the relevant mathematical models were developed to determine the tool parameters, step length, and path interval. Giri et al. [22] presented a freeform surface machining method, in which the selected surfaces were composed of convex curvature and maximum smoothness surface patches. Antonelli et al. [23] presented the method of subdivision surfaces integrated in a CAD system with an extensible geometric kernel, which is used to locally modify the limit surface of the subdivision scheme so as to tune the analytic properties without affecting its geometric shape and confirm the effectiveness of the proposed local correction patching method by some tests. Roman et al. [24] proposed sculptured surface partitioning scheme with this technique, 3½½-axis and the method of selecting an optimum number of sub-divisions along with actual machining experiments and then machines each patch using a fixed tool orientation, resulting in shorter machining time compared to traditional three-axis machining and comparable to simultaneous five-axis machining. These studies have made a simple division and classification in terms of machining geometrical feature of sculptured surface, and some put forward the machining methods. However, the influence of machining tools and other factors on processing characteristics are not taken into consideration.

In summary, deficiencies of previous studies focus mainly on the two aspects. First of all, the existing studies are mainly qualitative analysis simply for sculptured surface geometry characteristics and machining methods. Because of the complexity and variety of influencing machinability factors in sculptured surface machining, the related research of surface machining complexity is few. Realistically, few effective models are available for developing to evaluate machining complexity of sculptured surfaces. In addition, traditional machining experience of machine tool operators has been always used in practical machining process, resulting in great energy consumption. Therefore, it is of great importance to find a quantitative surface analysis and calculation method of surface machining complexity with the combination of geometric and processing properties. Furthermore, the complexity of measuring surface machining has guiding significance for selecting process parameters, process planning, and generating tool path, which is conductive to achieve the goal of reducing energy consumption. Inquiring the relationship between SMC and energy consumption is of necessity to predict energy consumption [25] and machining time in sculptured surface CNC machining. So research on energy consumption of surface machining is a vital part of energy-saving and emission reduction work, which makes great difference in the manufacturing sector.

In order to reduce the energy consumption in sculptured surface machining, the analysis of machining complexity is the prerequisite task. It is reported that surface machining complexity is determined by geometric properties and machining properties of sculptured surfaces. In this paper, a pentagon model for surface machining complexity is analyzed and developed from geometric properties and machining properties of sculptured surfaces. The machining complexity of sculptured surfaces is studied from the aspects of workpiece, equipment, cutter, goal and process to help the operator make reasonable process decisions before machining the surface and contribute to reduce energy consumption of machining sculptured surface. The focus of this paper is to explore the impacts of surface machining complexity on energy consumption and efficiency during sculptured surface machining process. In the first place, description and factor analysis of surface machining complexity are arranged in Section 3. Next, the mathematic model of surface machining complexity is developed in terms of crucial indicators influencing energy consumption and efficiency in sculptured surface machining based on the pentagonal model in Section 4. Moreover, as the geometry features of sculptured surface are more complex, how power consumed by machine tool in machining process and efficiency vary is verified intensively by the experiments in the paper in Section 5. Finally, conclusions are concluded in Section 6.

3 Description and factor analysis of surface machining complexity

Sculptured surface machining process is quite complex as a result of curvature frequently changing in the surface. Sculptured surface machining skill is limited by surface geometry characters and machining conditionings. Whether in terms of geometric property or machining property, some researchers made evaluation for sculptured surface machining, including the study of surface machining complexity [7].

Due to various factors in sculptured surface machining process, there exist some issues such as low machining efficiency, poor surface quality, and especially amount of energy consumption. In order to analyze the reasons of the above problems and settle, it is necessary to have a great knowledge of sculptured surface geometry and clearly evaluate the level of surface machining in multi-axis CNC process; the notation of surface machining complexity is needed to introduce to make an approximate judgment and estimate for surface part considering effects of the whole machining environment. It is closely related to parameters and cutter selection, and tool path generation as well as tool-axis orientation optimization.

Surface machining complexity (SMC) is a quantification that used to describe the complexity of surface topological geometry and the difficulty of machining a sculptured surface. It includes geometric complexity and machining complexity. Geometric complexity reflects the geometrical characteristics of the surface, which mainly refers to the curvature features, namely the radius and distribution of the curvature, as shown in the blue block in Fig. 1. The geometric complexity can be measured by the differences among the curvatures. Machining complexity refers to the difficulty of machining the surface. It is not only influenced by geometric properties, but also influenced by other effects such as equipment, cutters, processes, and goals, as shown in the green blocks in Fig. 1. In this study, we develop a pentagon model to describe various crucial factors of machining complexity, which are classified into five aspects: workpiece, machine tools, cutter, goal, and process, as shown in Fig. 1. That considers the five key indicators influencing energy consumption and machining time during machining process is the basis of measuring surface machining complexity in theory.

3.1 Workpiece

"Workpiece" is the direct factor that reflects the geometrical complexity of the machining object. It includes size, material, and curvature of the surface. The size of the workpiece directly determined machining time, energy consumption, carbon emission, the selection of machine tools, and cutters, and process plan. The material of the workpiece determines the type of cutters, process plan, and the final surface quality.

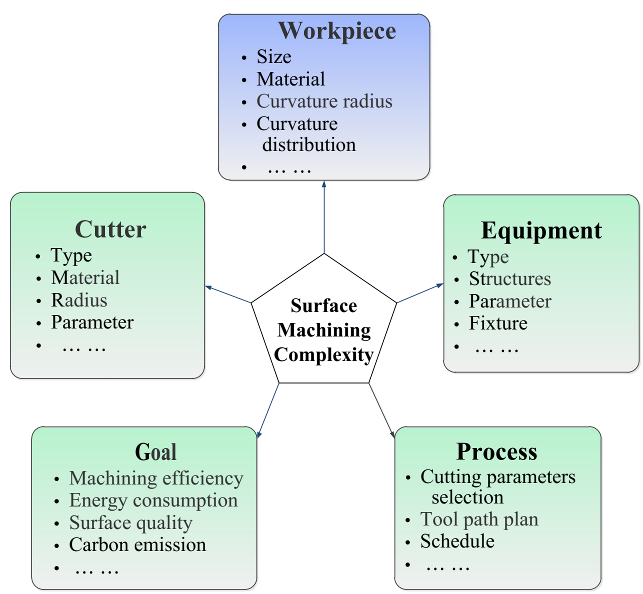


Fig. 1 The pentagon model for analyzing surface machining complexity

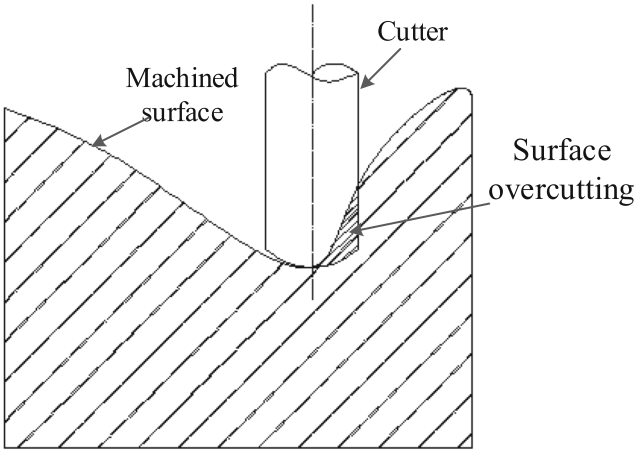


Fig. 2 The cutter partially overcuts the part surface while cutting

Curvature radius and its distribution reflect the geometrical features and its topological relations with other essential factors.

3.2 Equipment

"Equipment" refers to the machine tools, fixture etc. It includes the type, structure, parameter, and fixture. Types are such as three-axis machine tools and five-axis machine tools. For the same sculptured machine tools, the SMC will be different if different types of machine tools are used. Structure means choosing a different structure for the same type of machine tools, the SMC might be different. Parameter refers to the parameters of the machine tools, such as the range of spindle speed, feed, *X*-axis, *Y*-axis, and *Z*-axis. Fixture means different fixture will affect the value of SMC.

3.3 Cutter

"Cutter" includes the type, material, radius, and parameter. For machining the same sculptured surface, if operator chooses different cutter, the machining time and surface quality will be different. As shown in Fig. 2, if the cutter radius is larger than the curvature radius, interference will emerge. Therefore, to guarantee the surface quality and non-interference in the surface machining, an appropriate radius must be used. If a ball-end cutter is used in surface machining, the cutter curvature must be larger than the maximum normal curvature of the machined surface to avoid partial overcutting. Parameter refers the geometrical angle and the shape of the cutter, such as the rake, relief, and helix angle. Increasing the rake angle and relief angle can reduce the chip deformation, thereby reducing the cutting force, and improving the machining quality. However, if the rake and relief angle is too large, the strength and wear resistance of the tool will be reduced, increasing cutting force and the roughness of the machined surface, and bringing down the machinability. So changing the rake and relief angle to tend to be beneficial to processing properties has important significance for the surface machining.

3.4 Process

"Process" includes cutting parameters, tool path plan, and schedule. Choosing different cutting parameters, such as spindle speed, rate of feed, cutting depth and cutting width affects the SMC significantly. It is necessary to make process plan before machining. Process plan can apply different toolpath planning strategies to machining, which produces different machining efficiency [26], cutting forces [27], and even energy consumption. According to the distribution of curvature of the surface, selecting the appropriate tool path generation and schedule method is able to improve machining efficiency and reduce energy consumption, especially the non-cutting energy consumption in machining [28].

3.5 Goal

In machining system, "Goal" refers to surface quality, machining efficiency [8], energy consumption, or carbon emission. Choosing different machining goals, the SMC will be different. Surface quality is the most critical goal for assessing a part. The main factors influencing directly surface quality are

Table 1 Categories of sculptured surface

Mean curvaturek_G	Gaussian curvaturek_H	Types of point	Concavity	Categories
0	0	Plane point	–	Plane
> 0	> 0	Elliptic point	Concave	Concave ellipsoid
> 0	< 0	Elliptic point	Convex	Convex ellipsoid
< 0	> 0	Hyperbolic point	Concave	Concave saddle
< 0	< 0	Hyperbolic point	Convex	Convex saddle
0	> 0	Parabolic point	Concave	Concave cylinder
0	< 0	Parabolic point	Convex	Convex cylinder

When the Gaussian curvature is positive and the mean curvature is zero, the point does not exist, so it is not listed in the table.

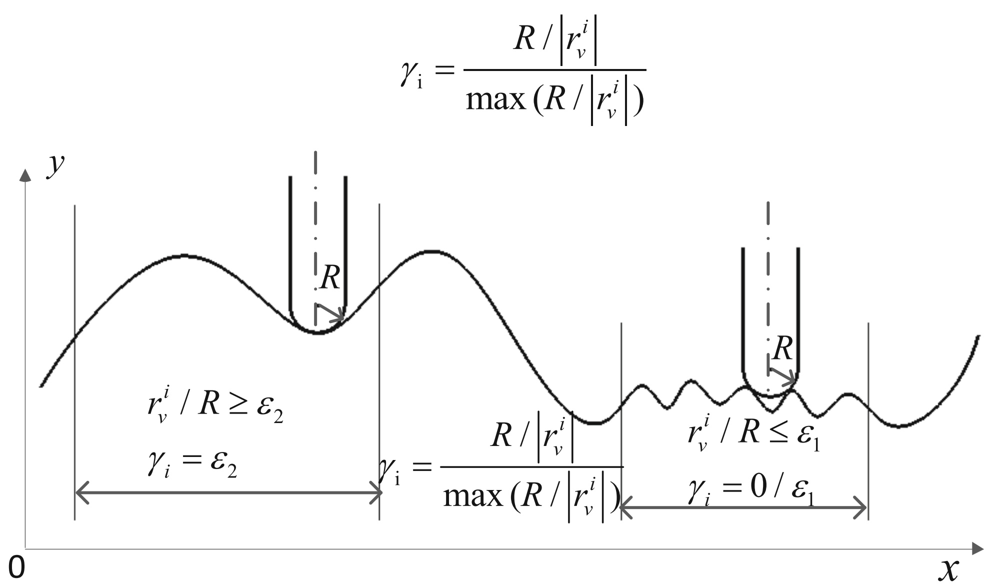


Fig. 3 Relation of cutter radius and curvature radius

machining step and spacing. If machining spacing is large, the residual height will rise, resulting in large surface roughness and inferior machining accuracy. Also, machining efficiency, energy consumption, and carbon emission have aroused more attentions in recent years due to the energy crisis and environment question.

4 Modeling and solution

4.1 Classification of sculptured surface

Before calculating SMC, a classification of sculptured surface is given. Curvature is one of the most important geometrical characteristics of surface. The curvature of the surface is partitioned into many types according to the mathematical meaning represented, containing principal curvature, normal curvature, mean curvature, and Gaussian curvature [7]. The average curvature and the Gaussian curvature have obvious geometric meanings. The positive and negative of the mean curvature represent the concavity and convexity of the surface, while the positive and negative of the Gaussian curvature represent the bending degree of curvature somewhere on the surface.

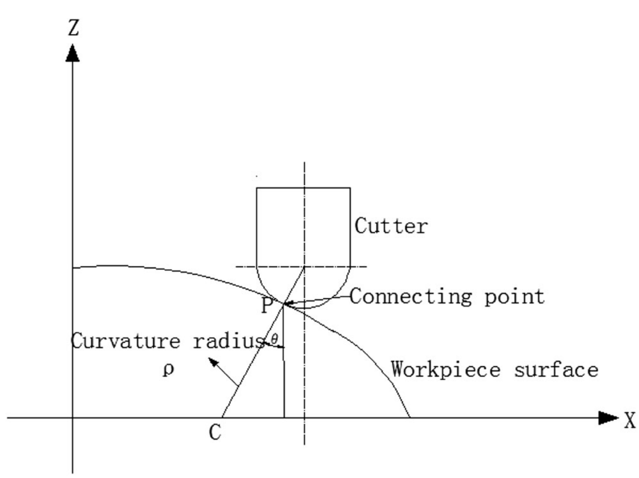


Fig. 4 The inclination θ formed between cutter axis and workpiece surface

By mapping a regular surface to a NURBS surface $S(u, v)$ in the u-v parametric space, the tangent space of each point on the surface can be decomposed to two tangent vectors, i.e.,$S_u'(u, v)$ and $S_v'(u, v)$ as follows [7].

$$\begin{cases} E(u, v) = S_u'(u, v) \cdot S_u'(u, v) \\ F(u, v) = S_u'(u, v) \cdot S_v'(u, v) \\ G(u, v) = S_v'(u, v) \cdot S_v'(u, v) \end{cases} \tag{1}$$

$$\begin{cases} L(u, v) = n(u, v) \cdot S_{uu}''(u, v) \\ M(u, v) = n(u, v) \cdot S_{uv}''(u, v) \\ N(u, v) = n(u, v) \cdot S_{vv}''(u, v) \end{cases} \tag{2}$$

Where E, F, and G are the fundamental quantities of the first class, and L, M, and N are the fundamental quantities of the second class.

The mean curvature k_H and the Gaussian curvature k_G of the surface are formulated as

$$k_H = \frac{EN - 2FM + GL}{2(EG - F^2)} \tag{3}$$

$$k_G = \frac{LN - M^2}{EG - F^2} \tag{4}$$

The type of points and the shape of the region can be categorized by the mean curvature and the Gaussian curvature, as shown in Table 1.

Table 2 Nomenclature of SMC

Symbol	Nomenclature	Unit
C_r^g	Surface geometrical complexity	mm^{-1}
C_r^m	Surface machining complexity	mm^{-1}
λ_i	The correction coefficient determined by the size of the workpiece's surface patch	–
η_i	The correction coefficient of the material of the workpiece and cutter	–
γ_i	The correction coefficient of cutter radius R and minimum curvature radius r_v^i of surface patch	–
β_i	The correction coefficient of equipment	–
φ_i	The correction coefficient of the process	–

4.2 Surface machining complexity (SMC) model

In this section, a mathematical model of the surface machining complexity is developed based on the pentagon model. According to the pentagon model, key indicators in the pentagon model that play an important part in energy consumption and efficiency of sculptured surface machining are quantized in the form of correction coefficients in the equation of SMC. The entire SMC is acquired by a linear superposition of machining complexity on each patch of the surface. As many conditions of the actual processing such as surface regions, curvature distribution, workpiece material, and cutter material are not consistent, the machining complexity of each patch of the surface cannot be described just by superposition. It is necessary to consider other factors. The mathematical model of the SMC can be represented by combining

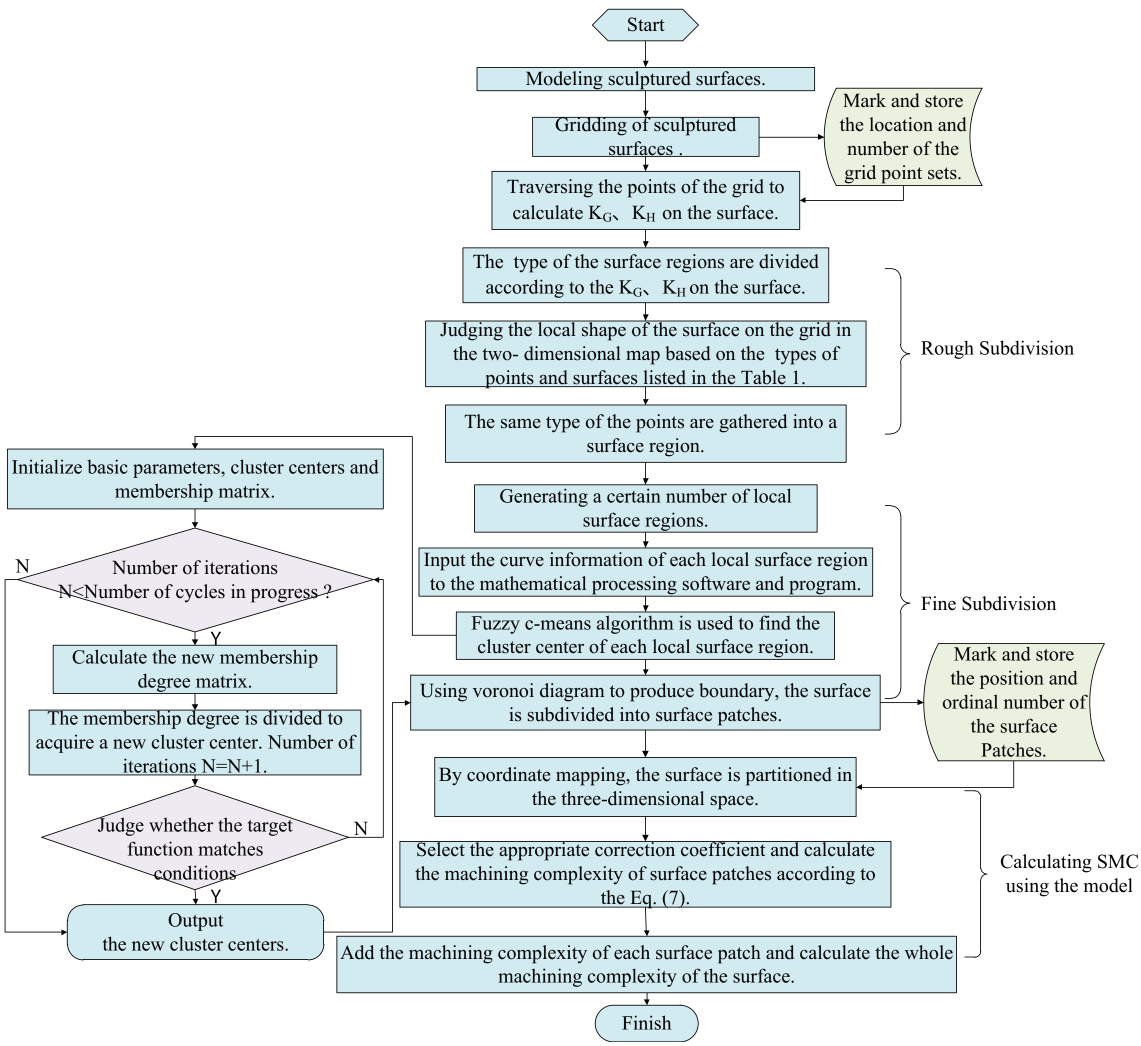
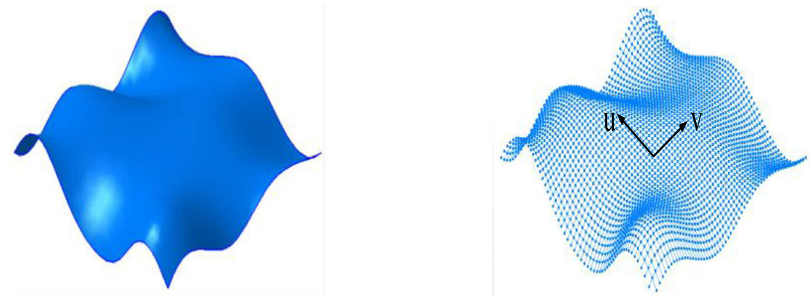


Fig. 5 The flowchart of surface machining complexity calculation procedure



(a) A sculptured surface model (b) Surface discrete point cloud

Fig. 6 **a** A sculptured surface model **b** Surface discrete point cloud

geometrical complexity model C_r^g (mm^{-1}) and machining complexity model C_r^m (mm^{-1}). The geometrical complexity model C_r^g is given as follows:

$$C_r^g = \lambda_i \left| \frac{1}{r_{max}^i} - \frac{1}{r_{min}^i} \right| \tag{5}$$

Where r_{max}^i and r_{min}^i are the maximum curvature radius and the minimum curvature radius of each region on the sculptured surface, respectively, and $\frac{1}{r_{max}^i} = (K_H)_{max}$, $\frac{1}{r_{min}^i} = (K_H)_{min}$.

λ_i is correction coefficient, which is determined by the size of the workpiece's surface patch *i*. It refers to the ratio of the size of surface projection area and surface area. As the sizes of the patches vary, it is not easy to calculate SMC. Therefore, compare the surface area with the projection surface area by using the projection method of comparison [7]. The size of the workpiece λ_i can be obtained as follows.

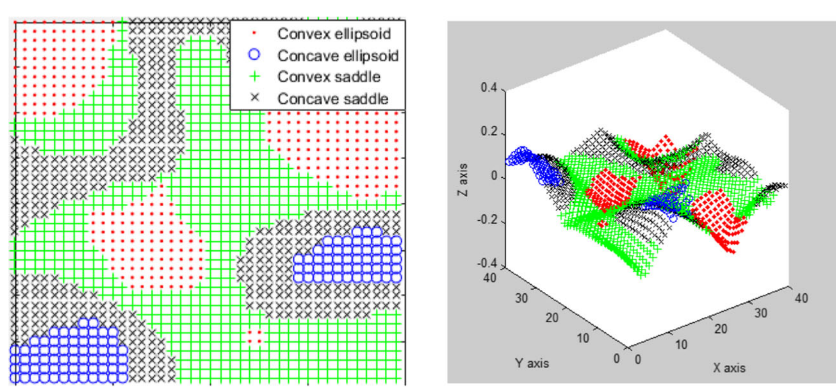
$$\lambda_i = \frac{D_i'}{D_i} \tag{6}$$

Surface machining complexity model C_r^m can be computed by the equation below,

$$C_r^m = \sum \eta_i \gamma_i \beta_i \varphi_i C_r^g. \tag{7}$$

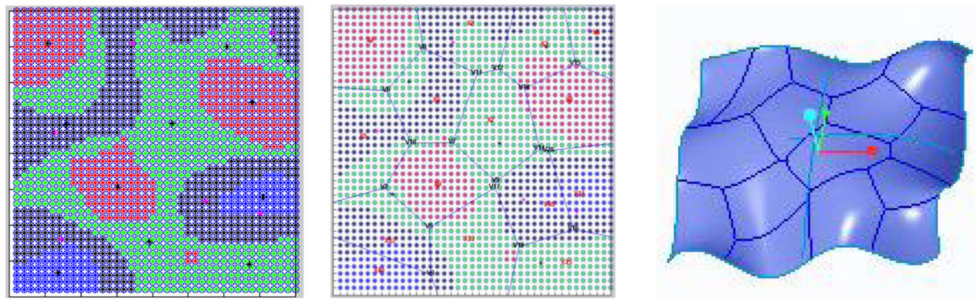
Since the machining process is a comprehensive function of machining tool, cutter and material, surface machining complexity is influenced by various factors. Some factors are positive for machining, and others are negative. Where all the coefficients in the left of C_r^g are to update the effects of various factors in machining process. Where $\eta_i$is the correction coefficient of the material of the workpiece and cutter. Hardness, including the workpiece material hardness and tool material hardness, is an important index to measure the mechanical performance. Thus, this study uses the ratio of workpiece material hardness to cutter material hardness to calculate the impact of material. For example, in this paper, the Brinell hardness value of the workpiece (6061 aluminum alloy) is 90~95, and the Brinell hardness value of the cutter (tungsten steel) is 170, if we take the maximum value of the workpiece 95, so $\eta_i = 95/170 = 0.559$.

γ_i is the correction coefficient of cutter radius *R* and minimum curvature radius r_v^i of patch *i*. Cutter diameter has a great influence on the performance of CNC machining [7, 29]. In the machining scenario, as shown in Fig. 3, two situations of the relationship between the cutter radius *R* and local curvature radius r_v^i are considered. The path spacing ε_1 and ε_1 are constants ($\varepsilon_2 > \varepsilon_1$), which are determined by the machining condition. If the curvature of the surface is quite small, the influence of γ_i can be neglected, since cutter radius has a little influence on the performance of CNC machining in such



(a) Two-dimensional map (b) Three-dimensional map

Fig. 7 Results of rough partitioning surface regions



(a) (b) (c)

Fig. 8 **a** Surface region clustering center. **b** Fine surface patch clustering boundary. **c** Segmentation of sculptured surface in 3D space

circumstance ($r_v^i/R\leq\varepsilon_1$). If r_v^i is larger than R ($r_v^i/R\geq\varepsilon_2$), γ_i will be larger. In the remaining cases, γ_i is an appropriate intermediate value. When the cutter radius is less than the minimum curvature radius of surface patch, the following expression can be used to calculate γ_i [7]:

$$\gamma_{\mathrm{i}}=\frac{R/|r_v^i|}{\max\left(R/|r_v^i|\right)}. \tag{8}$$

β_i is the correction coefficient of equipment. In practical machining, the inclination θ is formed between cutter axis and workpiece surface as shown in Fig. 4, which is an important parameter to measure CNC machine tool machining. The inclination θ is fixed in three-axis machining. However, the inclination θ varies with the cutter axis in multi-axis machining, and meet the following constraint [30].

$$\theta\in(0,2\pi), 0\leq\beta_i=|\cos\theta|\leq 1 \tag{9}$$

The cos θ is defined by the tool orientation projection in curvature radius of cutter contacting in workpiece surface during the surface machining process, measured the changes of tool orientation directly [20]. In three-axis machining, the tool axis corresponds with machine tool spindle (Z-axis), so the β_i value is considered as 1.000. The inclination θ reflects the changing tool posture in a five-axis machining process, and the β_i value is changing around the machining surface continually. The β_i value is taken as the mean value, and that is 0.500.

φ_i is the correction coefficient of the process [31]. The value of φ_i will vary with different process plans. It is difficult for a variety of process plans to make quantization. It depends on the machine tool operator's decision.

Given the items referring to calculating SMC, the nomenclature table of SMC is presented in Table 2.

4.3 Solution

In this study, the process to solve the SMC is shown in Fig. 5. First, a sculptured surface model is built and roughly divided. Then the surface is finely divided using fuzzy c-means algorithm. The surface is divided into several surface patches by coordinate mapping in three-dimensional space. Finally, the SMC of each surface patch is calculated by Eq. (7), and the formation of the overall surface machining complexity is accumulated by each surface patch.

4.3.1 Rough subdivision

The rough subdivision process of the whole surface is to demine region types. A sculptured surface is divided into various shape types according to the difference of mathematical meanings of the curvature, especially mean curvature and Gaussian curvature. Some region types of sculptured surface are described by analyzing the signs of the average curvature and Gaussian curvature as shown in Table 1. Fig. 6(a) shows a sculptured surface model using NURBS format. Based on the curvature and the precision requirement of the machining, the surface is divided into appropriate parameter lines along the u and v direction. This study chooses the 40 by 40 parameter lines to partition the surface into a grid, and discrete the grid into point clouds, as shown in Fig.6(b). Then, the Gauss curvature and mean curvature of each discrete point on the grid of the surface are obtained transversely. The type of each discrete point is judged with curvature data according to Table 1 using software developed by us. The same type of point sets is gathered in a plane region, and when the type of two adjacent points changes, these points become the boundary point. So the boundary of the surface is formed by connecting all boundary points on the surface. As shown in Fig. 5, the surface is separated into a number of regions. The rough partitioning surface in form of two-dimensional map is displayed as shown in Fig. 7(a). The three-dimensional map of rough subdivision surface is shown in Fig. 7(b).

4.3.2 Fine subdivision

Fuzzy c-means clustering algorithm (FCM) is a widely used clustering algorithm because of the simple and fast

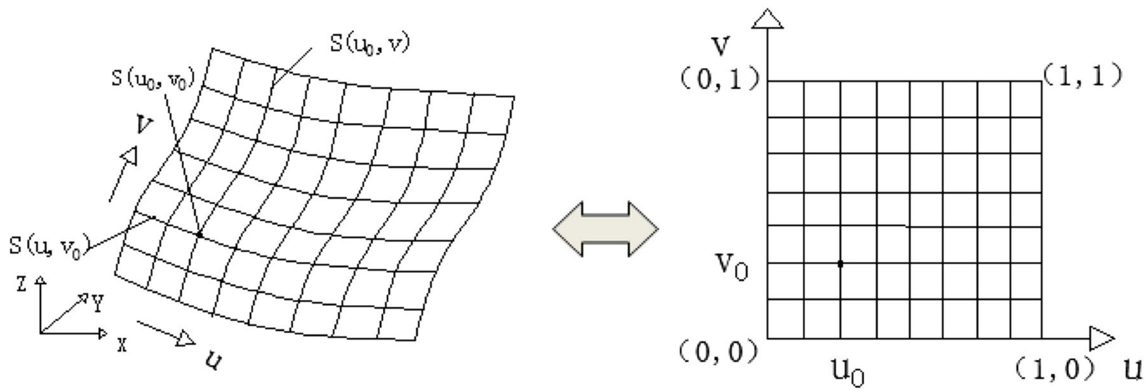


Fig. 9 Surface mapping between Cartesian space and parameter plane

feature. After the rough partition of the surface, we further use Fuzzy c-mean algorithm to find the clustering center of each surface region, then make fine subdivision of the sculptured surface, and classify the similar curvature points into a group, forming several surface patches. If there are too many surface patches to calculate SMC, the surface patches will be combined by similarity of surface shapes and process characteristics. By optimizing the objective function, we obtain the membership of each sample point and all cluster centers, to determine the sample point affinities and automatically classify the sample data.

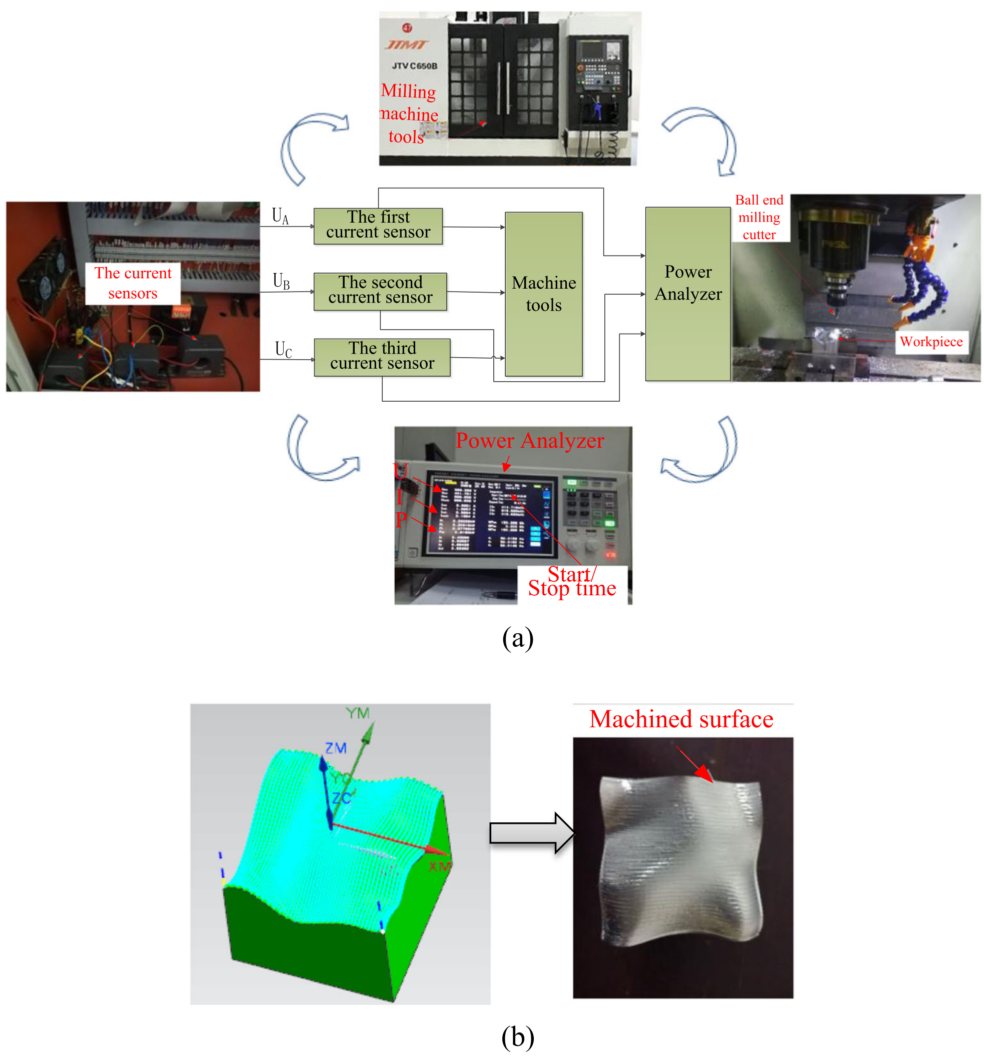


Fig. 10 a The machine tool and test platform used in the experiment. b Tool path and machined surface

Table 3 Parameters of the machine tool

CNC milling machine tool name	Workbench area (width *length) mm^2	Spindle maximum speed r/min	CNC machine system	*X*, *Y*, *Z* axis fast-moving feed speed m/min	Rated power of spindle motor kW
JTV C650B	1270 × 400	6000	FANUC-Oi-MateMc	18–18-15	5.5

FCM initializes the cluster center before performing an iterative process. The performance of the algorithm depends on the initial cluster center because it cannot ensure that FCM converges to an optimal solution. In this study, the clustering centers are determined according to the number of rough subdivision surface regions. Specifically, the cluster centers of regions in the surface are found using Fuzzy c-means clustering algorithm in terms of the size of regions and surface, which decides the proper number of patches. The FCM iterative function will run several times to get the surface clustering centers as shown in Fig. 8(a). The operation process of Fuzzy C-means clustering algorithm is embedded into the flowchart of the SMC calculations of sculptured surface, as shown in Fig. 4. The main steps of the algorithm [32] are as follows:

Step 1: Setting cluster category number as c, $2 \le c \le n$, n is the number of data; setting iteration number as N and iteration stop threshold as ε, and setting $N = 0$ at the beginning of iteration; initializing clustering center, and membership matrix U are to satisfy the constraint conditions in Eq. (10);

$$\sum_{i=1}^{c} u_{ij} = 1, \forall j = 1, \dots, n \tag{10}$$

Step 2: Use Eq. (11) to calculate the new membership matrix U, and set $m = 2$;

$$u_{ij} = \frac{1}{\sum_{k=1}^{c} \left(\frac{d_{ij}}{d_{kj}}\right)^{2/(m-1)}} \tag{11}$$

Step 3: Use Eq. (12) to calculate clustering centers c_i, $i = 1, \dots, c$;

$$c_i = \frac{\sum_{j=1}^{n} u_{ij}^m x_j}{\sum_{j=1}^{n} u_{ij}^m} \tag{12}$$

Step 4: According to Eq. (12), determine and calculate the objective function. The algorithm stops if it is less than the set threshold, or it changes less than the threshold relative to the previous target function value; otherwise, returns to step 2, and set up $N = N + 1$.

$$J(U, c_1, \dots, c_c) = \sum_{i=1}^{c} J_i = \sum_{i=1}^{c} \sum_{j}^{n} u_{ij}^m d_{ij}^2 \tag{13}$$

As the fuzzy c-means clustering algorithm cannot get the surface boundary obviously, the Voronoi diagram is adopted. The Voronoi diagram is also called Tyson polygon or Dirichlet diagram, which is a series of polygons connected by vertical bisectors of two adjacent points. Voronoi diagram can define the boundary of each surface patch according to the location

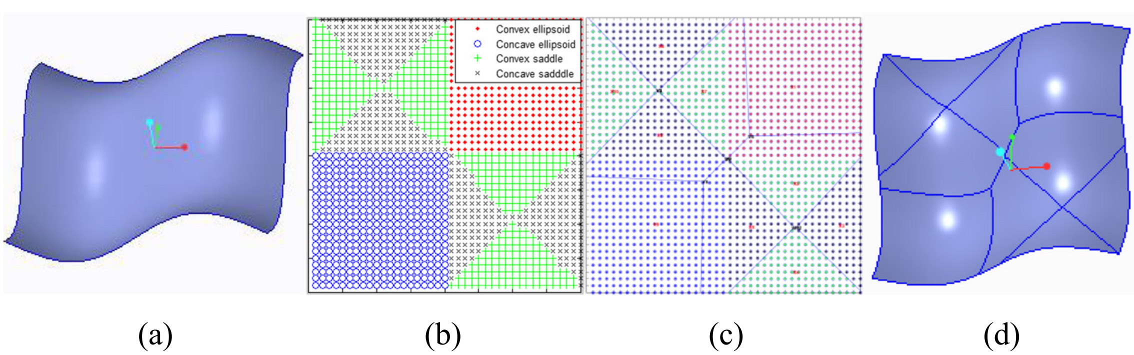
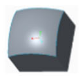
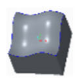
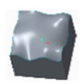
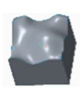
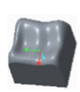


Fig. 11 The process of partitioning surface: **a** the model of a sculptured surface model, **b** surface rough partitioning, **c** surface fine partitioning in 2D map, **d** surface fine subdivision in 3D space

Table 4 Coefficients of machining complexity of sculptured surfaces

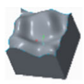
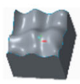
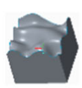
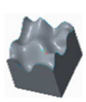
Models	Number of patches	Correction coefficient and curvature differences	The numerical values	SMC(C_r^m)
	1	λ_i	0.910	0.032
		γ_i	1.000	
		r_i	0.062	
	10	λ_i	0.869,0.840,0.845,0.848,0.832,0.850,0.830,0.828,0.867, 0.846	0.310
		γ_i	1.000,0.588,0.420,0.427,0.550,0.427,0.641,0.588,0.985, 0.427	
		r_i	0.065,0.131,0.126,0.130,0.130,0.130,0.126,0.130,0.065, 0.131	
	15	λ_i	0.781,0.747,0.893,0.671,0.888,0.975,0.965,0.857,0.982, 0.894,0.821,0.920, 0.960,0.866,0.894	0.502
		γ_i	1.000,0.171,0.342,0.312,0.223,0.192,0.298,0.575,0.219, 0.438,0.822,0.233, 0.209,0.586,0.370	
		r_i	0.313,0.064,0.129,0.154,0.123,0.077,0.091,0.151,0.060, 0.152,0.284,0.120, 0.098,0.141,0.166	
	15	λ_i	0.813,0.782,0.848,0.868,0.856,0.787,0.869,0.863,0.886, 0.860,0.847,0.893, 0.900,0.865,0.766	0.814
		γ_i	0.770,0.230,0.602,0.143,0.437,0.721,0.469,0.225,0.602, 0.173,0.190,0.264, 0.249,0.667,0.281	
		r_i	0.406,0.181,0.600,0.098,0.155,0.379,0.208,0.098,0.342, 0.100,0.119,0.122, 0.128,0.302,0.114	
	16	λ_i	0.657,0.704,0.945,0.646,0.804,0.855,0.647,0.763,0.581, 0.859,0.912,0.622, 0.818,0.848,0.816,0.610	1.386
		γ_i	0.793,0.228,0.253,0.408,0.793,0.279,0.572,0.224,0.478, 0.396,0.181,0.947, 0.304,0.259,0.272,0.548	
		r_i	0.629,0.173,0.178,0.254,0.801,0.322,1.033,0.234,0.369, 0.335,0.200,0.712, 0.375,0.197,0.309,0.391	

of cluster centers and the nearest neighbor principle of points on the surface. Fig. 8(b) shows the boundary of the surface patches using Voronoi diagram. To facilitate the calculation of SMC for the subsequent surfaces, the cluster center and boundary vertices of each surface patch are marked. Then the vertices of each surface patch boundary on the two-dimensional map are projected on the surface in 3D space according to the position distribution on the grid by coordinate mapping as shown in Fig. 9, so the surface is segmented as shown in Fig.8(c).

5 Case study

5.1 Experiment setup

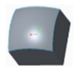
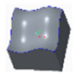
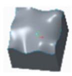
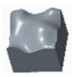
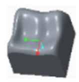
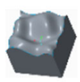
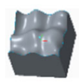
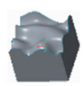
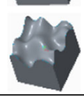
A vertical machining Center (JTV C650B) is used in the experiment, as shown in Fig. 10(a). The technical parameters are shown in Table 3. A Hioki 6001C power analyzer was used to monitor the real-time power of the machine tools during the cutting milling process. For simplicity, only the powers in semi-finishing process are analyzed.

	16	λ_i	0.803,0.773,0.806,0.758,0.769,0.767,0.884,0.785,0.787, 0.775,0.739,0.754, 0.772,0.720,0.718,0.696	1.548
		γ_i	0.440,0.712,0.438,0.478,0.636,0.464,0.450,0.432,0.248, 0.522,0.510,0.468, 0.532,0.964,0.674,0.710	
		r_i	0.303,0.527,0.609,0.374,0.325,0.391,0.320,0.241,0.178, 0.273,0.294,0.318, 0.383,0.686,0.509,0.540	
	19	λ_i	0.740,0.763,0.791,0.817,0.742,0.877,0.754,0.779,0.792, 0.586,0.835,0.801, 0.874,0.709,0.812,0.646,0.867,0.790,0.754	1.634
		γ_i	0.740,0.287,0.457,0.580,1.000,0.488,0.785,0.398,0.471, 0.318,0.373,0.441, 0.365,0.578,0.666,0.352,0.578,0.719,0.702	
		r_i	0.422,0.277,0.229,0.375,0.506,0.310,0.420,0.367,0.283, 0.258,0.286,0.440, 0.396,0.236,0.292,0.485,0.272,0.438,0.425	
	21	λ_i	0.688,0.807,0.688,0.602,0.716,0.878,0.840,0.816,0.875, 0.812,0.805,0.713, 0.778,0.806,0.808,0.894,0.592,0.782,0.831,0.753,0.678	1.722
		γ_i	0.456,0.480,0.448,0.806,0.856,0.290,0.356,0.518,0.227, 0.600,0.775,0.788, 0.264,0.434,0.483,0.303,0.572,0.470,0.356,0.218,1.000	
		r_i	0.333,0.288,0.419,0.539,0.460,0.156,0.241,0.340,0.124, 0.411,0.420,0.436,0.246,0.290,0.462,0.232,0.419,0.274, 0.317,0.222,0.643	
	21	λ_i	0.659,0.811,0.710,0.792,0.718,0.730,0.748,0.829,0.774, 0.739,0.721,0.725,0.784,0.829,0.590,0.642,0.718,0.682, 0.688,0.647,0.713	2.216
		γ_i	0.426,0.659,0.381,0.628,0.603,0.632,0.825,0.475,0.388, 0.399,1.000,0.712,0.577,0.769,0.916, 0.916,0.894,0.239,0.781,0.539,0.758	
		r_i	0.365,0.532,0.327,0.442,0.299,0.288,0.470,0.209,0.233, 0.199,0.470,0.394,0.286,0.417,0.447,0.542,0.671,0.204, 0.374,0.402,0.563	

The machining process is under the condition of dry cutting. The tool path and machined surface are shown in Fig. 10(b).

The material of the workpiece is C6061 aluminum alloy. The size of the block is $60 \times 60\ 60 \times \mathrm{mm}^3$. The machining strategy is the iso-scallop height tool path generation method on the whole surface. In the finish milling process, the spindle speed and feed rate are 3000 rpm and 500 mm/min respectively. The residual height is set to be 0.05 mm. It is important to stress that all the surfaces in the research are machined by tool paths generated by the same strategies and parameters completely as shown in Fig.10(b), not using the strategy of each patch corresponding to a unique tool path to machine separately. Two ball end milling cutters made from tungsten steel are used. The one with the diameter of 8 mm is used in rough machining, and the one with the diameter of 4 mm is used in finishing machining.

Table 5 Results of SMC and the corresponding energy consumption and machining time

No.	Surface model	C_r^m /mm^{-1}	Energy Consumption E/kWh	Machining time T/h
1		0.032	0.138	0.158
2		0.310	0.145	0.167
3		0.502	0.150	0.171
4		0.814	0.154	0.176
5		1.386	0.166	0.186
6		1.548	0.171	0.195
7		1.634	0.178	0.203
8		1.722	0.184	0.213
9		2.216	0.200	0.226

5.2 Experiment data

This section uses experiment data to illustrate the process of partitioning surface and calculating machining complexity. First, commercial CAD software Creo3.0 is used to develop a NURBS surface model as shown in Fig. 11(a). Next, the rough partitioning of the surface is shown in Fig. 11(b) with the method mentioned above. According to the distribution of curvature, the surface is divided into one convex ellipsoid surface, one concave ellipsoid surface, four convex hyperbolic parabolic surfaces, and four concave hyperbolic parabolic surfaces, respectively. The surfaces are shown in different colors. What's more, with the FCM algorithm and Voronoi diagram partitioning the sculptured surface finely, certain surface patches can be acquired. The fine partitioning surface is obtained as shown in Fig. 11(c). Then the surface is segmented by the coordinate map, and the segmented surface is shown in Fig. 11(d). Finally, the SMC is calculated by Eq. (7), which equals 0.310.

To explore the relation of SMC and energy consumption, physical machining of nine models and energy consumption measurement are conducted in this section. All the models are milled by the same machine tools with the same material, process plan, goal, and cutters (the material of the model is 6061 aluminum alloy). Each model is dispersed into 1600 points (40 × 40). As mentioned in Section 4.2, we can get $\eta_i = 0.559$ ($\eta_i = 95/170 = 0.559$), $\beta_i = 1.000$, $\varphi_i = 1.000$. The other correction coefficients and calculation results are listed in Table 4. Table 5 shows the experimental data on energy consumption and machining time for each model.

5.3 Results and discussion

The data of measured energy consumption and machining time in semi-finishing machining process are fitted and the fitting equations are reported respectively, as shown in Fig. 10. Fig. 12(a) shows the relationship between SMC and energy consumption. Fig. 12(b) shows the relationship between SMC and machining time. R^2 represents the correlation size between SMC and energy consumption as well as machining time in Fig. 12(a) and (b). It can be seen from the two diagrams that energy consumption and machining time increase as SMC increases. The result is basically a quadratic growth. It can be seen from Fig. 12 that there is no significant difference between the fitted

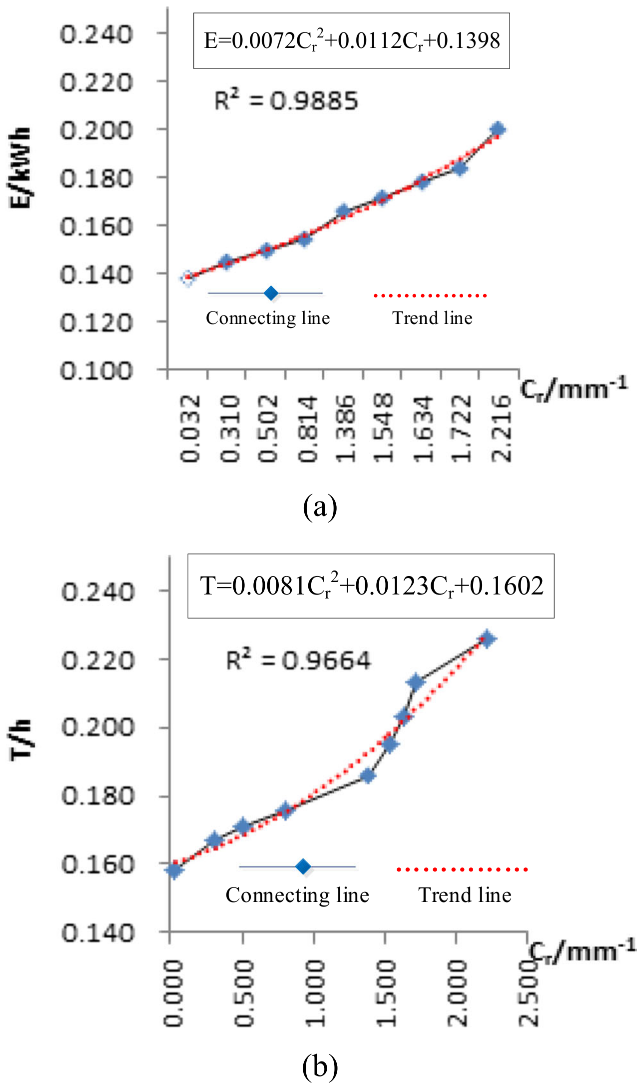


Fig. 12 **a** The relationship between SMC and energy consumption. **b** The relationship between SMC and machining efficiency of sculptured surfaces

data and the real-time measured data. The variance between the fitted data and the real-time measured data in Fig. 12(a), (b) is analyzed and calculated separately. The two average percentage deviations of the fitted data are less than 5%, which indicates that there is no significant difference between the fitted data and the real-time measured data. So, the two evaluation indicators for the machining system performance, energy consumption, and machining efficiency as well as SMC are reasonably relevant. The energy consumption increased by 45% from the minimum to the maximum with the SMC changing in the paper. It also suggests that the geometric modeling of sculptured surface should not be designed too complex as long as the function reaches the requirement, otherwise unnecessary energy consumption is produced and low efficiency in machining process.

6 Conclusions

The shape characteristics of sculptured surfaces play a vital role in machining performance, especially energy consumption and machining efficiency during machining process, deciding machining complexity. In this paper, a connation model and mathematic model are developed to analyze and calculate machining complexity of sculptured surfaces respectively, and the relationship between SMC and energy consumption, SMC and machining time are researched and verified by the experiments. Compared to other related work, our approach has the following advantages:

- Through analyzing critical factors that influence energy consumption, a pentagon model that considers the workpiece, equipment, cutter, goal, and process is provided.
- The mathematic model is proposed to calculate the value of SMC for different sculptured surfaces, and then a solution based on fuzzy c-means clustering algorithm is given in detail.
- From the experiments, we can draw a conclusion that surface machining complexity has a great relation to energy consumption and machining time.

The results of the experiments demonstrate the feasibility of the proposed methods, and this study can give a new design idea and research method to save energy and improve machining efficiency before rough machining and semi-finishing machining. Moreover, the results of SMC are beneficial to help designers to get a better knowledge about the machining complexity of their design, so that they can evaluate the difficulty level of sculptured surface machining, further make predictions for energy consumption and machining time during the process to improve the design according to energy and time. The experiments in the study are done only focusing on the semi-finishing process. The number of the experiment is not enough, which will be our further work to get more data to demonstrate the methods. In addition, some factors may not be considered in the SMC model and need to be found further. The research and findings in the study are able to be applied to the machining process, which is conductive to sculptured surface machining. As a foundation research, our methods can be extended to study the relationship between SMC and cutting parameters, SMC and tool path optimization, etc.

Acknowledgements We would like to thank Miss Magdalene Jackson for her helpful language revision. We are grateful to other people in the research group of "Environmental and Ecological Engineering of Purdue University" for collaboration and discussion.

Funding information This research was supported by National Natural Science Foundation of China (No.51875480), National Natural Science Foundation of China (No.51405396), Fundamental Science and Frontier Technology Foundation of Chongqing (No. cstc2016jcyjA0422), and

Fundamental Research Funds for the Central Universities National Natural Science Foundation of China (XDJK2017B051).

Publisher's note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

References

1. Zhong Q, Tang R, Peng T (2016) Decision rules for energy consumption minimization during material removal process in turning. J Clean Prod 140:1819–1827
2. EIA. Annual energy review (2011) Last visited: March 2015 http://www.eia.gov/totalenergy/data/annual/index.cfm
3. Zhou L, Li J, Li F, Meng Q, Li J, Xu X (2016) Energy consumption model and energy efficiency of machine tools: a comprehensive literature review. J Clean Prod 112:3721–3734
4. Gutowski TG, Branham MS, Dahmus JB, Jones AJ, Thiriez A (2009) Thermodynamic analysis of resources used in manufacturing processes. Environ Sci Technol 43(5):1584–1590
5. Gutowski T, Dahmus J, Thiriez A (2006) Electrical energy requirements for manufacturing processes. Energy 2
6. Gutowski T, Murphy C, Allen D, Bauer D, Bras B, Piwonka T (2005) Environmentally benign manufacturing: observations from Japan, Europe and the United States. J Clean Prod 13(1):1–17
7. Li L, Chen B, Liu F, Li C (2014) Complexity analysis and calculation for sculptured surface in multi-axis CNC machining based on surface subdivision. Int J Adv Manuf Technol 71(5–8):1433–1444
8. Cai W, Liu F, Xie J, Liu P, Tuo J (2017) A tool for assessing the energy demand and efficiency of machining systems: energy benchmarking. Energy 138:332–347
9. Nategh MJ, Parvaz H (2017) Development of computer aided clamping system design for workpieces with freeform surfaces. Comput Aided Des 95:52–61. https://doi.org/10.1016/j.cad.2017.10.003
10. Chen ZC, Khan MA (2014) A new approach to generating arc length parameterized nurbs tool paths for efficient three-axis machining of smooth, accurate sculptured surfaces. Int J Adv Manuf Technol 70(5–8):1355–1368
11. Lazoglu I, Manav C, Murtezaoglu Y (2009) Tool path optimization for free form surface machining. CIRP Ann Manuf Technol 58(1):101–104
12. Li L, Deng X, Zhao J, Zhao F, Sutherland JW (2017) Multi-objective optimization of tool path considering efficiency, energy-saving and carbon-emission for free-form surface milling. J Clean Prod 172:3311–3322. https://doi.org/10.1016/j.jclepro.2017.07.219
13. Li LL, Zhang YF, Li HY, Geng L (2011) Generating tool-path with smooth posture change for five-axis sculptured surface machining based on cutter's accessibility map. Int J Adv Manuf Technol 53(5–8):699–709
14. Zhou X, Liu X, Li M, Wang Z, Meng X (2017) Post-processor development of a five-axis machine tool with optimization tool radius compensation. Int J Adv Manuf Technol 88(5–8):1505–1522
15. Song Q, Ju G, Liu Z, Ai X (2014) Subdivision of chatter-free regions and optimal cutting parameters based on vibration frequencies for peripheral milling process. Int J Mech Sci 83(3):172–183
16. Chen ZC, Dong Z, Vickers GW (2001) Optimal 3½½-axis CNC machining of sculptured parts using fuzzy pattern clustering//Proceedings of the International ICSC Congress on Soft Computing and Intelligent Systems for Industry Symposium on Intelligent Systems for Industry 26-29
17. Van Tuong N, Pokorný P (2010) A practical approach for partitioning free-form surfaces. Int J Comput Integr Manuf 23(11):992–1001
18. Sonthi R, Kunjur G, Gadh R (1997) Shape feature determination using the curvature region representation//Symposium on solid modeling & applications. SMA '97 Proceedings of the fourth ACM symposium on Solid modeling and applications 285–296
19. Lee YS, Jegadesh G, Ma Y (2000) Rolling-ball method and contour marching approach to identifying critical regions for complex surface machining. Comput Ind 41(2):163–180
20. Sridharan N, Shah JJ (2004) Recognition of multi axis milling features: part i-topological and geometric characteristics. J Comput Inf Sci Eng 4(3):242–250
21. Chen CK, Wu CT (2002) The region division and NC machining of compound surfaces. J Mater Process Technol 121(1):5–14
22. Giri V, Bezbaruah D, Bubna P, Choudhury AR (2005) Selection of master cutter paths in sculptured surface machining by employing curvature principle. Int J Mach Tools Manuf 45(10):1202–1209
23. Antonelli M, Beccari CV, Casciola G, Ciarloni R, Morigi S (2013) Subdivision surfaces integrated in a cad system. Comput Aided Des 45(11):1294–1305
24. Roman A, Bedi S, Ismail F (2006) Three-half and half-axis patch-by-patch nc machining of sculptured surfaces. Int J Adv Manuf Technol 29(5–6):524–531
25. Li L, Li C, Tang Y, Li L (2017) An integrated approach of process planning and cutting parameter optimization for energy-aware CNC machining. J Clean Prod 16. https://doi.org/10.1016/j.jclepro.2017.06.034
26. Ozturk E, Tunc LT, & Budak E (2007) Machining parameter and strategy selection in multi-axis milling of sculptured surfaces
27. Souza AFD, Berkenbrock E, Diniz AE, Rodrigues AR (2015) Influences of the tool path strategy on the machining force when milling free form geometries with a ball-end cutting tool. J Braz Soc Mech Sci Eng 37(2):675–687
28. Hu L, Liu Y, Lohse N, Tang R, Lv J, Peng C (2017) Sequencing the features to minimise the non-cutting energy consumption in machining considering the change of spindle rotation speed. Energy 139:935–946
29. Cao QY, Zhao J, Zhu L (2016) The effect of curvature radius of sculptured surface on finish milling tool path selection. Int J Adv Manuf Technol 89(9–12):1–9
30. Cao Q, Zhao J, Han S, Chen X (2012) Force coefficients identification considering inclination angle for ball-end finish milling. Precis Eng 36(2):252–260
31. Romero PE, Dorado R, Díaz FA, Rubio EM (2013) Influence of pocket geometry and tool path strategy in pocket milling of uns a96063 alloy ☆. Procedia Eng 63(63):523–531
32. Uğurhan Kutbay.(2018). Partitional clustering.IntechOpen