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ORIGINAL ARTICLE

An advanced fuzzy approach for modeling the yield improvement of making aircraft parts using 3D printing

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Abstract

Forecasting the yield is critical for making aircraft parts using three-dimensional (3D) printing. However, the existing methods for the yield forecasting exhibit a common problem: they employ the logarithmic or log-sigmoid value, rather than the original value, of the yield to simplify the computation. To address this problem, an advanced fuzzy approach was proposed in this study. The focus of this study is to investigate the effectiveness of the advanced fuzzy approach for forecasting the yield of a 3D-printed aircraft part. The advanced fuzzy approach derived the direct-solution (DS) versions of the existing fuzzy yield learning models. These DS versions use the original yield value directly, thereby optimizing the forecasting performance. The proposed methodology was applied to the process of making an aircraft part using 3D printing to evaluate its effectiveness. Experimental results revealed significant improvements in the forecasting accuracy of the proposed methodology compared with the aforementioned methods. Furthermore, when the proposed methodology was applied to various fuzzy yield learning models, different improvements were obtained.

Keywords Yield · Forecasting · Direct-solution · 3D printing · Aircraft

1 Introduction

Three-dimensional (3D) printing has been extensively applied to the aircraft industry due to its potential for load carrying applications [15, 20, 25]. Now, it is possible to print multi-component aerospace (aircraft) structures and components with large multiple 3D printing systems [19]. However, researchers and practitioners still encounter several challenges while applying 3D printing in the aircraft industry. For example, the majority of the required alloys cannot be 3D-printed because the melting and solidification dynamics during the 3D printing process lead to intolerable microstructures with large columnar grains and periodic cracks [17]. Other challenges include the following:

(1) how to make an entire airplane,
(2) how to protect intellectual property rights,
(3) how to measure the performance of maintenance and repair tasks,
(4) how to enhance the precision of printing a 3D object, and
(5) quality control (QC) issues [26]

The last challenge is focused on this study. In this regard, Yang et al. [28] evaluated a 3D-printed aircraft part in terms of part density, tensile strength, dimensional accuracy, and surface roughness, while Li et al. [16] considered the good fusion quality between polylactic acid (PLA) resins and the infiltration quality of carbon fiber and melting PLA matrix during the 3D printing process. However, a universally accepted definition of the "quality" of a 3D-printed aircraft part is still lacking [7]. In addition, many manufacturers do not have strong confidence in the consistent quality and reliability of 3D-printed aircraft parts [7].

Among QC issues, how to measure and enhance yield is obviously the most critical one. Yield refers to the percentage of jobs that are not scrapped following quality problems in manufacturing. It is a critical performance measure in numerous manufacturing processes including 3D printing processes. Therefore, many researchers and practitioners have been endeavoring to improve yield. Related studies have usually been focused on precise and accurate yield analyses. Statistics, simulation, mathematical programming (MP), fuzzy logic, and

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artificial neural networks (ANNs) are the yield analysis techniques most frequently applied. For example, Yang and Tjia [27] attempted to improve the yield in the purification of an active pharmaceutical ingredient (API) by using batch distillation modeling and engineering principles, which are considered as applied statistics. Chen [2] proposed a fuzzy collaborative intelligence (FCI) approach to forecast the yield of a semiconductor product. In this method, experts configured fuzzy feed-forward neural networks to forecast the yield. The maximal consensus and radial basis function network approach were proposed to aggregate expert forecasts. Eberle et al. [10] established a four-step procedure to improve the yield of a pharmaceutical batch production process. In this procedure, statistical analyses were performed to approximate the relationship between loss factors (which reduce the yield) and the overall yield. John et al. [14] simulated the operations of fluid catalytic cracking risers in a modern refinery with various diameters to determine the diameter that optimized the yield. Parra et al. [21] studied the reverse water–gas shift reaction and observed that the moving-bed configuration had higher space–time yield (STY) than did the fixed-bed operation. Furthermore, they formulated a bi-objective MP model to determine a method for simultaneously optimizing the STY and adsorbent loading.

Operators become more experienced over time. Similarly, quality control engineers become more familiar with quality problems and their possible solutions. Equipment engineers eventually determine the optimal settings of machines. All these phenomena contribute to improvements in yield, which can be generally described using a yield learning model. However, considerable uncertainties are associated with yield improvement processes because the aforementioned activities all involve some extent of human intervention. Therefore, a probabilistic, stochastic, or fuzzy yield learning model that can manage such uncertainty must be used. Among the aforementioned models, the fuzzy yield learning model has been widely adopted because of its efficient computation and communication as well as its ability to incorporate subjective judgments [2, 4, 11].

In the fuzzy yield learning model, the relationship between time (or other factors) and yield is typically approximated using a fuzzy exponential function, in which parameters are generally converted into the logarithmic values to facilitate problem-solving [24]. Chen [3] fitted the relationship using an ANN that involved the log-sigmoid values of parameters. Nevertheless, both approaches present the same problem: the forecasting precision or accuracy is not optimized. To illustrate this problem, a hypothetical example is presented in Table 1, where $\tilde{Y}_t$ is a fuzzy yield forecast, such that $\tilde{Y}_t = (Y_{t1}, Y_{t2}, Y_{t3})$. Three fuzzy yield learning models exist for the same yield forecasting problem. Existing methods that use the logarithmic values employ the first model. Chen's ANN method, which is based on log-sigmoid yield values, employs the second model. However, in terms of the fuzzy yield forecast range $Y_{t3} - Y_{t1}$, neither method optimizes the forecasting precision. This paper proposes an advanced fuzzy approach that employs the third model, in which $Y_{t3} - Y_{t1}$ is minimized. The advanced fuzzy approach applies the polynomial fitting technique to several existing fuzzy yield learning models to directly use the original values of parameters, thus providing direct-solution (DS) versions. In the original fuzzy yield learning models, intractable nonlinear programming (NLP) problems are solved, whereas in the DS models, tractable polynomial programming (PP) problems are solved. The Karush–Kuhn–Tucker (KKT) conditions [1] for the PP problems are derived, and they can be solved using mathematical or optimization software. The benefits of applying the proposed methodology to various fuzzy yield learning models are compared, because these benefits may differ.

This remainder of the paper is organized as follows: Section 2 provides preliminary introductions to arithmetic operations on triangular fuzzy numbers (TFNs) and the polynomial fitting technique. In Section 3, the advanced fuzzy approach is introduced. The advanced fuzzy approach converts several fuzzy yield learning models into PP problems to be solved for the application. Then, the effectiveness of the advanced fuzzy approach is validated using the case of making an aircraft part using 3D printing, and the results are discussed in Section 4. The conclusions are provided in Section 5.

2 Preliminary

2.1 Arithmetic operations on TFNs

In this study, the parameters used in the fuzzy yield learning models were assumed to be TFNs. Therefore, some arithmetic operations on TFNs are introduced.

Definition 1 *TFN*

A TFN $\tilde{A} = (A_1, A_2, A_3)$ exhibits the following membership function:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - A_1}{A_2 - A_1} & if \quad A_1 \le x < A_2 \\ \frac{A_3 - x}{A_3 - A_2} & if \quad A_2 \le x < A_3 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Table 1 Hypothetical example ($t = 1$)

Model no.	Equation	$\ln Y_{t3} - \ln Y_{t1}$	$\log - \text{sig}(Y_{t3}) - \log - \text{sig}(Y_{t1})$	$Y_{t3} - Y_{t1}$
1	$\tilde{Y}_t = (0.895, 0.908, 0.922)e^{-\frac{(0.073, 0.235, 0.398)}{t}}$	0.1375	0.0243	0.1156
2	$\tilde{Y}_t = (0.867, 0.888, 0.909)e^{-\frac{(0.182, 0.321, 0.460)}{t}}$	0.1408	0.0240	0.1124
3	$\tilde{Y}_t = (0.937, 0.948, 0.958)e^{-\frac{(0.604, 0.802, 1.000)}{t}}$	0.1542	0.0246	0.1119

Theorem 1 [29] *Arithmetic operations on TFNs*

Let $\tilde{A}$ and $\tilde{B}$ be TFNs; then, the following arithmetic operations hold:

(1) Fuzzy addition:

$$\tilde{A}(+)\tilde{B} = (A_1 + B_1, A_2 + B_2, A_3 + B_3). \tag{2}$$

(2) Fuzzy subtraction:

$$\tilde{A}(-)\tilde{B} = (A_1 - B_3, A_2 - B_2, A_3 - B_1). \tag{3}$$

(3) Fuzzy multiplication:

$$\tilde{A}(\times)\tilde{B} \cong (A_1B_1, A_2B_2, A_3B_3) \text{ if } A_1, B_1 \geq 0 \tag{4}$$

(4) Fuzzy division:

$$\tilde{A}(/)\tilde{B} \cong (A_1/B_3, A_2/B_2, A_3/B_1) \quad \text{if} \quad A_1 \geq 0, B_1 > 0 \tag{5}$$

(5) Exponential operation:

$$e^{\tilde{A}} \cong \left(e^{A_1}, e^{A_2}, e^{A_3}\right) \quad \text{if} \quad A_1 \geq 0 \tag{6}$$

(6) Logarithmic operation:

$$\ln\tilde{A} \cong (\ln A_1, \ln A_2, \ln A_3) \quad \text{if} \quad A_1 > 0 \tag{7}$$

2.2 The polynomial fitting technique

The polynomial fitting technique is used to approximate a nonlinear function with a polynomial function. The approximation of an exponential function with a polynomial function was of concern because most yield learning models use exponential functions. The polynomial function $f(x)$ must satisfy the following requirements:

$$f(x) = \sum_{l=0}^{L} a_l x^l, \tag{8}$$

$$f(0) = 1, \tag{9}$$

$$|f(x) - e^x| \leq \varepsilon \quad \forall x \in [0, 2], \tag{10}$$

where a_l is a real number; $l = 1 \sim L$. ε is a small positive real number. The range [0, 2] was sufficient for yield forecasting. A numerical simulation was performed using MATLAB 2017. After the simulation, the polynomial functions $f(x)$, which minimize ε for various values of L, were obtained and are summarized in Table 2. The coefficients of determination (R^2) of the approximations are also provided in this table:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} \left(e^{x_i} - f(x_i)\right)^2}{\sum_{i=1}^{n} \left(e^{x_i} - \frac{\sum_{i=1}^{n} e^{x_i}}{n}\right)^2} \tag{11}$$

R^2 provides a measure of how well-observed outcomes (e^{x_i}) are replicated by the model ($f(x_i)$).

2.3 Solving the PP problem

A PP model can be formulated as follows,
(Model PP)

$$\text{Min}\, Z = f(\mathbf{x}), \tag{12}$$

subject to

$$\mathbf{g}(\mathbf{x}) \leq 0, \tag{13}$$

$$\mathbf{h}(\mathbf{x}) = 0, \tag{14}$$

$$\mathbf{x} \in R^{n_x}, \tag{15}$$

where $f(\mathbf{x})$, $\mathbf{g}(\mathbf{x})$, and $\mathbf{h}(\mathbf{x})$ are polynomial functions of $\mathbf{x}$. The Lagrangian function for this model is as follows:

$$L(\mathbf{x}, \boldsymbol{\mu}, \boldsymbol{\lambda}) = f(\mathbf{x}) + \sum_{i=1}^{n_h} \mu_i h_i(\mathbf{x}) + \sum_{j=1}^{n_g} \lambda_j g_j(\mathbf{x}). \tag{16}$$

Based on Eq. (15), the KKT conditions for the optimal solutions to this model are derived as follows [9]:

(a) Equality constraints:

$$\nabla_{\mathbf{x}} L(\mathbf{x}, \boldsymbol{\mu}, \boldsymbol{\lambda}) = 0, \tag{17}$$

Table 2 Polynomial functions $f(x)$ with various values of L

L	$f(x)$	ε	R^2
1	$f(x) = 1.0000 + 2.4019x$	1.5853	0.8883
2	$f(x) = 1.0000 + 0.4123x + 1.3231x^2$	0.2721	0.9973
3	$f(x) = 1.0000 + 1.1412x + 0.1112x^2 + 0.4533x^3$	0.0355	0.9999
4	$f(x) = 1.0000 + 0.9765x + 0.6043x^2 + 0.0229x^3 + 0.1145x^4$	0.0037	0.9999

$$\lambda_j g_j(\mathbf{x}) = 0; j = 1\sim n_g, \tag{18}$$

$$\mathbf{h}(\mathbf{x}) = 0. \tag{19}$$

(b) Inequality constraints:

$$\mathbf{g}(\mathbf{x}) \le 0, \tag{20}$$

$$\lambda_j \ge 0, j = 1\sim n_g, \tag{21}$$

These constraints can easily be obtained analytically by using mathematical or optimization software.

3 Deriving DS versions of fuzzy yield learning models

3.1 DS version of the Tanaka–Watada (TW) model

The TW fuzzy yield learning model has the following form [23]:

$$\tilde{Y}_t = \tilde{Y}_0 e^{-\frac{\tilde{b}}{t}}, \tag{22}$$

where $\tilde{Y}_t$ is the fuzzy forecast of the average yield within time period t, with $0 \le \tilde{Y}_t \le 1$ and $t = 1 \sim T$. $\tilde{Y}_0$ is the asymptotic (or final) yield that $\tilde{Y}_t$ should converge to, where $0 \le \tilde{Y}_0 \le 1$. $\tilde{Y}_0$ is generally a real-valued function of the point defect density per unit area, chip area, or a set of parameters distinctive to the yield model under consideration [11]. $\tilde{b}$ is the learning constant, where $\tilde{b} \ge 0$. A larger value of $\tilde{b}$ indicates an improvement in the yield at a higher learning speed. Without loss of generality, $\tilde{Y}_0$ and $\tilde{b}$ can be expressed using TFNs as

$$\tilde{Y}_0 = (Y_{01}, Y_{02}, Y_{03}), \tag{23}$$

$$\tilde{b} = (b_1, b_2, b_3). \tag{24}$$

According to the arithmetic operators on TFNs,

$$Y_{t1} \cong Y_{01} e^{-\frac{b_3}{t}}, \tag{25}$$

$$Y_{t2} \cong Y_{02} e^{-\frac{b_2}{t}}, \tag{26}$$

$$Y_{t3} \cong Y_{03} e^{-\frac{b_1}{t}}. \tag{27}$$

Equivalently,

$$Y_{01} \cong Y_{t1} e^{\frac{b_3}{t}}, \tag{28}$$

$$Y_{02} \cong Y_{t2} e^{\frac{b_2}{t}}, \tag{29}$$

$$Y_{03} \cong Y_{t3} e^{\frac{b_1}{t}}. \tag{30}$$

The left- and right-hand sides of Eq. (22) can be converted into logarithmic values as follows:

$$\ln \tilde{Y}_t = \ln \tilde{Y}_0 (-) \frac{\tilde{b}}{t}. \tag{31}$$

Applying the arithmetic on TFNs to Eq. (30) provides

$$\ln Y_{t1} \cong \ln Y_{01} - b_3/t, \tag{32}$$

$$\ln Y_{t2} \cong \ln Y_{02} - b_2/t, \tag{33}$$

$$\ln Y_{t3} \cong \ln Y_{03} - b_1/t. \tag{34}$$

Based on Eqs. (32), (33), and (34), several MP models have been formulated to derive the values of $\tilde{Y}_0$ and $\tilde{b}$. For example, Tanaka and Watada [23] solved the following linear programming problem:

(Model TW)

$$\text{Min}\, Z_{TW} = \sum_{t=1}^{T} (\ln Y_{t3} - \ln Y_{t1}) \tag{35}$$

subject to

$$\ln Y_t \ge \ln Y_{t1} + s(\ln Y_{t2} - \ln Y_{t1}), \tag{36}$$

$$\ln Y_t \le \ln Y_{t3} + s(\ln Y_{t2} - \ln Y_{t3}), \tag{37}$$

$$\ln Y_{t1} = \ln Y_{01} - b_3/t, \tag{38}$$

$$\ln Y_{t2} = \ln Y_{02} - b_2/t, \tag{39}$$

$$\ln Y_{t3} = \ln Y_{03} - b_1/t, \tag{40}$$

$$\ln Y_{01} \le \ln Y_{02} \le \ln Y_{03} \le 0, \tag{41}$$

$$0 \le b_1 \le b_2 \le b_3, \tag{42}$$

$$t = 1\sim T,$$

where Y_t is the average yield (i.e., the actual value) within time period t, and s is a predefined real constant representing the required satisfaction level, with $0 \le s \le 1$. The objective function minimizes the sum of the ranges of the logarithmic values of the fuzzy yield forecasts.

The DS version of the TW model is as follows:

(Model DS-TW)

$$\text{Min}\, Z_{DS-TW} = \sum_{t=1}^{T} (Y_{t3}-Y_{t1}) \tag{43}$$

subject to

$$Y_t \ge Y_{t1} + s(Y_{t2}-Y_{t1}), \tag{44}$$

$$Y_t \le Y_{t3} + s(Y_{t2}-Y_{t3}), \tag{45}$$

$$Y_{01} = Y_{t1} + \frac{0.9765}{t} b_3 Y_{t1} + \frac{0.6043}{t^2} b_3^2 Y_{t1} + \frac{0.0229}{t^3} b_3^3 Y_{t1} + \frac{0.1145}{t^4} b_3^4 Y_{t1}, \tag{46}$$

$$Y_{02} = Y_{t2} + \frac{0.9765}{t} b_2 Y_{t2} + \frac{0.6043}{t^2} b_2^2 Y_{t2} + \frac{0.0229}{t^3} b_2^3 Y_{t2} + \frac{0.1145}{t^4} b_2^4 Y_{t2}, \tag{47}$$

$$Y_{03} = Y_{t3} + \frac{0.9765}{t} b_1 Y_{t3} + \frac{0.6043}{t^2} b_1^2 Y_{t3} + \frac{0.0229}{t^3} b_1^3 Y_{t3} + \frac{0.1145}{t^4} b_1^4 Y_{t3}, \tag{48}$$

$$0 \le Y_{01} \le Y_{02} \le Y_{03} \le 1, \tag{49}$$

$$0 \le b_1 \le b_2 \le b_3, \tag{50}$$

$$t = 1 \sim T.$$

The objective function directly minimizes the sum of the ranges of the fuzzy yield forecasts. Equations (46), (47), and (48) are obtained by applying the polynomial approximations in Table 2 to Eqs. (28), (29), and (30).

Theorem 2 *The DS-TW model is valid for $b_3 \le 2$.*

Proof

The third requirement of the polynomial fitting technique is $x \le 2$. In constraint Eq. (28),

$$x = \frac{b_3}{t} \le 2. \tag{51}$$

Because $t \ge 1$,

$$b_3 \le 2t \le 2 \cdot 1 = 2. \tag{52}$$

Hence, Theorem 1 is proved.

The Lagrangian function of the DS-TW model is as follows:

$$\begin{aligned} L_{Y_t,\lambda} &= \sum_{t=1}^{T} (Y_{t3}-Y_{t1}) + \sum_{t=1}^{T} (\lambda_{1t}(Y_{t1} + s(Y_{t2}-Y_{t1})-Y_t)) + \sum_{t=1}^{T} (\lambda_{2t}(Y_t-Y_{t3}-s(Y_{t2}-Y_{t3}))) \\ &+ \sum_{t=1}^{T} \left(\mu_{1t}\left(Y_{01}-Y_{t1}-\frac{0.9765}{t} b_3 Y_{t1}-\frac{0.6043}{t^2} b_3^2 Y_{t1}-\frac{0.0229}{t^3} b_3^3 Y_{t1}-\frac{0.1145}{t^4} b_3^4 Y_{t1}\right)\right) \\ &+ \sum_{t=1}^{T} \left(\mu_{2t}\left(Y_{02}-Y_{t2}-\frac{0.9765}{t} b_2 Y_{t2}-\frac{0.6043}{t^2} b_2^2 Y_{t2}-\frac{0.0229}{t^3} b_2^3 Y_{t2}-\frac{0.1145}{t^4} b_2^4 Y_{t2}\right)\right) \\ &+ \sum_{t=1}^{T} \left(\mu_{3t}\left(Y_{03}-Y_{t3}-\frac{0.9765}{t} b_1 Y_{t3}-\frac{0.6043}{t^2} b_1^2 Y_{t3}-\frac{0.0229}{t^3} b_1^3 Y_{t3}-\frac{0.1145}{t^4} b_1^4 Y_{t3}\right)\right) \\ &+ \lambda_3(Y_{02}-Y_{03}) + \lambda_4(Y_{01}-Y_{02}) + \lambda_5(b_2-b_3) + \lambda_6(b_1-b_2) \end{aligned} \tag{53}$$

The KKT conditions are as follows:

(a) Equality constraints:

$$\frac{\partial L}{\partial Y_{01}} = \sum_{t=1}^{T} \mu_{1t} + \lambda_4 = 0 \tag{54}$$

$$\frac{\partial L}{\partial Y_{02}} = \sum_{t=1}^{T} \mu_{2t} + \lambda_3 - \lambda_4 = 0 \tag{55}$$

$$\frac{\partial L}{\partial Y_{03}} = \sum_{t=1}^{T} \mu_{3t} - \lambda_3 = 0 \tag{56}$$

$$\frac{\partial L}{\partial b_1} = \lambda_6 - \sum_{t=1}^{T} \left(\mu_{3t}\left(\frac{0.9765}{t} Y_{t3} + \frac{1.2086}{t^2} b_1 Y_{t3} + \frac{0.0687}{t^3} b_1^2 Y_{t3} + \frac{0.4580}{t^4} b_1^3 Y_{t3}\right)\right) = 0 \tag{57}$$

$$\frac{\partial L}{\partial b_2} = \lambda_5 - \lambda_6 - \sum_{t=1}^{T} \left(\mu_{2t}\left(\frac{0.9765}{t} Y_{t2} + \frac{1.2086}{t^2} b_2 Y_{t2} + \frac{0.0687}{t^3} b_2^2 Y_{t2} + \frac{0.4580}{t^4} b_2^3 Y_{t2}\right)\right) = 0 \tag{58}$$

$$\frac{\partial L}{\partial b_3} = -\sum_{t=1}^{T} \left(\mu_{1t}\left(\frac{0.9765}{t} Y_{t1} + \frac{1.2086}{t^2} b_3 Y_{t1} + \frac{0.0687}{t^3} b_3^2 Y_{t1} + \frac{0.4580}{t^4} b_3^3 Y_{t1}\right)\right) - \lambda_5 = 0 \tag{59}$$

(b) Inequality constraints:

$$Y_{t1} + s(Y_{t2}-Y_{t1}) - Y_t \le 0, \tag{60}$$

$$Y_t - Y_{t3} - s(Y_{t2}-Y_{t3}) \le 0, \tag{61}$$

$$Y_{01} - Y_{02} \le 0, \tag{62}$$

$$Y_{02} - Y_{03} \le 0, \tag{63}$$

$$Y_{03} - 1 \le 0, \tag{64}$$

$$b_1 - b_2 \le 0, \tag{65}$$

$$b_2 - b_3 \le 0, \tag{66}$$

$$\lambda_{1t} \sim \lambda_{2t}, \lambda_3 \sim \lambda_6, \mu_{1t} \sim \mu_{3t} \ge 0. \tag{67}$$

The solutions to the KKT conditions, that is, the optimal solutions to the DS-TW problem, can be easily obtained analytically by using mathematical or optimization software. The DS model exhibits this advantage over the original model.

3.2 DS versions of other fuzzy yield learning models

The DS versions of other fuzzy yield learning models can be similarly derived. For example, Peters [22] solved the following quadratic programming (QP) problem to fit a fuzzy yield learning process:

(Model P)

$$\text{Max}\, Z_P = \bar{s} \quad (68)$$

subject to

$$\sum_{t=1}^{T} (\ln Y_{t3} - \ln Y_{t1}) \le Td, \quad (69)$$

$$\bar{s} = \frac{\sum_{t=1}^{T} s_t}{T}, \quad (70)$$

$$\ln Y_t \ge \ln Y_{t1} + s_t(\ln Y_{t2} - \ln Y_{t1}), \quad (71)$$

$$\ln Y_t \le \ln Y_{t3} + s_t(\ln Y_{t2} - \ln Y_{t3}), \quad (72)$$

$$\ln Y_{t1} = \ln Y_{01} - b_3/t, \quad (73)$$

$$\ln Y_{t2} = \ln Y_{02} - b_2/t, \quad (74)$$

$$\ln Y_{t3} = \ln Y_{03} - b_1/t, \quad (75)$$

$$\ln Y_{01} \le \ln Y_{02} \le \ln Y_{03} \le 0, \quad (76)$$

$$0 \le b_1 \le b_2 \le b_3, \quad (77)$$

$$0 \le s_t \le 1, \quad (78)$$

$$t = 1 \sim T,$$

where d is the prespecified required average range, with $d \ge 0$. The objective function maximizes the average satisfaction level. The DS version of this model is as follows:

(Model DS-P)

$$\text{Max}\, Z_{DS-P} = \bar{s} \quad (79)$$

subject to

$$\sum_{t=1}^{T} (Y_{t3} - Y_{t1}) \le Td, \quad (80)$$

$$\bar{s} = \frac{\sum_{t=1}^{T} s_t}{T}, \quad (81)$$

$$Y_t \ge Y_{t1} + s_t(Y_{t2} - Y_{t1}), \quad (82)$$

$$Y_t \le Y_{t3} + s_t(Y_{t2} - Y_{t3}), \quad (83)$$

$$Y_{01} = Y_{t1} + \frac{0.9765}{t} b_3 Y_{t1} + \frac{0.6043}{t^2} b_3^2 Y_{t1} + \frac{0.0229}{t^3} b_3^3 Y_{t1} + \frac{0.1145}{t^4} b_3^4 Y_{t1}, \quad (84)$$

$$Y_{02} = Y_{t2} + \frac{0.9765}{t} b_2 Y_{t2} + \frac{0.6043}{t^2} b_2^2 Y_{t2} + \frac{0.0229}{t^3} b_2^3 Y_{t2} + \frac{0.1145}{t^4} b_2^4 Y_{t2}, \quad (85)$$

$$Y_{03} = Y_{t3} + \frac{0.9765}{t} b_1 Y_{t3} + \frac{0.6043}{t^2} b_1^2 Y_{t3} + \frac{0.0229}{t^3} b_1^3 Y_{t3} + \frac{0.1145}{t^4} b_1^4 Y_{t3}, \quad (86)$$

$$0 \le Y_{01} \le Y_{02} \le Y_{03} \le 1, \quad (87)$$

$$0 \le b_1 \le b_2 \le b_3, \quad (88)$$

$$0 \le s_t \le 1, \quad (89)$$

$$t = 1 \sim T.$$

Moreover, the objective function can be replaced with another function that is more suitable for practical applications, such as the mean absolute percentage error (MAPE):

$$\text{Min}\, Z_{DS-P2} = \frac{\sum_{t=1}^{T} \left| Y_t - \frac{Y_{t1} + Y_{t2} + Y_{t3}}{3} \right|}{T}, \quad (90)$$

for which the fuzzy yield forecast is defuzzified using the center of gravity (COG) method.

Donoso et al. [8] solved another QP model to fit a fuzzy learning process:

(Model D)

$$\text{Min}\, Z_D = w_1 \sum_{t=1}^{T} (\ln Y_{t2} - \ln Y_t)^2 + w_2 \sum_{t=1}^{T} (\ln Y_{t3} - \ln Y_{t1})^2 \quad (91)$$

subject to

$$\ln Y_t \ge \ln Y_{t1} + s(\ln Y_{t2} - \ln Y_{t1}), \quad (92)$$

$$\ln Y_t \le \ln Y_{t3} + s(\ln Y_{t2} - \ln Y_{t3}), \quad (93)$$

$$\ln Y_{t1} = \ln Y_{01} - b_3/t, \quad (94)$$

$$\ln Y_{t2} = \ln Y_{02} - b_2/t, \quad (95)$$

$$\ln Y_{t3} = \ln Y_{03} - b_1/t, \quad (96)$$

$$\ln Y_{01} \le \ln Y_{02} \le \ln Y_{03} \le 0, \quad (97)$$

$$0 \le b_1 \le b_2 \le b_3, \quad (98)$$

$$t = 1 \sim T,$$

where w_1 and w_2 are the prespecified weights, with $w_1 + w_2 = 1$ and $w_1, w_2 \in [0, 1]$. The objective function minimizes the weighted sum of the squared deviations from the core and the squared ranges. The DS version of Model D is as follows:

(Model DS-D)

$$\text{Min}\, Z_{DS-D} = w_1 \sum_{t=1}^{T} (Y_{t2} - Y_t)^2 + w_2 \sum_{t=1}^{T} (Y_{t3} - Y_{t1})^2 \tag{99}$$

subject to

$$Y_t \geq Y_{t1} + s(Y_{t2} - Y_{t1}), \tag{100}$$

$$Y_t \leq Y_{t3} + s(Y_{t2} - Y_{t3}), \tag{101}$$

$$Y_{01} = Y_{t1} + \frac{0.9765}{t} b_3 Y_{t1} + \frac{0.6043}{t^2} b_3^2 Y_{t1} + \frac{0.0229}{t^3} b_3^3 Y_{t1} + \frac{0.1145}{t^4} b_3^4 Y_{t1}, \tag{102}$$

$$Y_{02} = Y_{t2} + \frac{0.9765}{t} b_2 Y_{t2} + \frac{0.6043}{t^2} b_2^2 Y_{t2} + \frac{0.0229}{t^3} b_2^3 Y_{t2} + \frac{0.1145}{t^4} b_2^4 Y_{t2}, \tag{103}$$

$$Y_{03} = Y_{t3} + \frac{0.9765}{t} b_1 Y_{t3} + \frac{0.6043}{t^2} b_1^2 Y_{t3} + \frac{0.0229}{t^3} b_1^3 Y_{t3} + \frac{0.1145}{t^4} b_1^4 Y_{t3}, \tag{104}$$

$$0 \leq Y_{01} \leq Y_{02} \leq Y_{03} \leq 1, \tag{105}$$

$$0 \leq b_1 \leq b_2 \leq b_3, \tag{106}$$

$$t = 1 \sim T.$$

Chen and Lin [4] formulated two NLP problems by using their FCI approach:

(Model CLI)

$$\text{Min}\, Z_{CLI} = \sum_{t=1}^{T} (\ln Y_{t2} - \ln Y_{t1})^{o(k)} \tag{107}$$

subject to

$$\ln Y_t \geq \ln Y_{t1} + s(k)(\ln Y_{t2} - \ln Y_{t1}), \tag{108}$$

$$\ln Y_t \leq \ln Y_{t3} + s(k)(\ln Y_{t2} - \ln Y_{t3}), \tag{109}$$

$$\ln Y_{t1} = \ln Y_{01} - b_3/t, \tag{110}$$

$$\ln Y_{t2} = \ln Y_{02} - b_2/t, \tag{111}$$

$$\ln Y_{t3} = \ln Y_{03} - b_1/t, \tag{112}$$

$$\ln Y_{01} \leq \ln Y_{02} \leq \ln Y_{03} \leq 0, \tag{113}$$

$$0 \leq b_1 \leq b_2 \leq b_3, \tag{114}$$

$$t = 1 \sim T.$$

(Model CLII)

$$\text{Max}\, Z_{CLII} = \sqrt[m(k)]{\sum_{t=1}^{T} {s_t}^{m(k)}} \tag{115}$$

subject to

$$\sum_{t=1}^{T} (\ln Y_{t2} - \ln Y_{t1})^{o(k)} \leq T d(k)^{o(k)}, \tag{116}$$

$$\ln Y_t \geq \ln Y_{t1} + s_t(\ln Y_{t2} - \ln Y_{t1}), \tag{117}$$

$$\ln Y_t \leq \ln Y_{t3} + s_t(\ln Y_{t2} - \ln Y_{t3}), \tag{118}$$

$$\ln Y_{t1} = \ln Y_{01} - b_3/t, \tag{119}$$

$$\ln Y_{t2} = \ln Y_{02} - b_2/t, \tag{120}$$

$$\ln Y_{t3} = \ln Y_{03} - b_1/t, \tag{121}$$

$$\ln Y_{01} \leq \ln Y_{02} \leq \ln Y_{03} \leq 0, \tag{122}$$

$$0 \leq b_1 \leq b_2 \leq b_3, \tag{123}$$

$$0 \leq s_t \leq 1, \tag{124}$$

$$t = 1 \sim T,$$

where $o(k)$, $s(k)$, $m(k)$, and $d(k)$ are optimization parameters specified by expert k, with $k = 1 \sim K$; $o(k), m(k) \in Z^+$, $s(k) \in [0, 1]$, and $d(k) \geq 0$. In Model CLI, the objective function minimizes the high-order sum of ranges, whereas in the Model CLII, the objective function maximizes the generalized mean of the satisfaction levels. The DS versions of these two models are as follows:

(Model DS-CLI)

$$\text{Min}\, Z_{DS-CLI} = \sum_{t=1}^{T} (Y_{t2} - Y_{t1})^{o(k)} \tag{125}$$

subject to

$$Y_t \geq Y_{t1} + s(k)(Y_{t2} - Y_{t1}), \tag{126}$$

$$Y_t \leq Y_{t3} + s(k)(Y_{t2} - Y_{t3}), \tag{127}$$

$$Y_{01} = Y_{t1} + \frac{0.9765}{t} b_3 Y_{t1} + \frac{0.6043}{t^2} b_3^2 Y_{t1} + \frac{0.0229}{t^3} b_3^3 Y_{t1} + \frac{0.1145}{t^4} b_3^4 Y_{t1}, \tag{128}$$

$$Y_{02} = Y_{t2} + \frac{0.9765}{t} b_2 Y_{t2} + \frac{0.6043}{t^2} b_2^2 Y_{t2} + \frac{0.0229}{t^3} b_2^3 Y_{t2} + \frac{0.1145}{t^4} b_2^4 Y_{t2}, \tag{129}$$

$$Y_{03} = Y_{t3} + \frac{0.9765}{t} b_1 Y_{t3} + \frac{0.6043}{t^2} b_1^2 Y_{t3} + \frac{0.0229}{t^3} b_1^3 Y_{t3} + \frac{0.1145}{t^4} b_1^4 Y_{t3}, \tag{130}$$

$$0 \leq Y_{01} \leq Y_{02} \leq Y_{03} \leq 1, \tag{131}$$

$$0 \leq b_1 \leq b_2 \leq b_3, \tag{132}$$

$$t = 1 \sim T,$$

(Model DS-CLII)

$$\text{Max } Z_{CLII} = \sqrt[m(k)]{\sum_{t=1}^{T} s_t{}^{m(k)}} \tag{133}$$

subject to

$$\sum_{t=1}^{T} (Y_{t2}-Y_{t1})^{o(k)} \leq Td(k)^{o(k)}, \tag{134}$$

$$Y_t \geq Y_{t1} + s_t(Y_{t2}-Y_{t1}), \tag{135}$$

$$Y_t \leq Y_{t3} + s_t(Y_{t2}-Y_{t3}), \tag{136}$$

$$Y_{01} = Y_{t1} + \frac{0.9765}{t} b_3 Y_{t1} + \frac{0.6043}{t^2} b_3^2 Y_{t1} + \frac{0.0229}{t^3} b_3^3 Y_{t1} + \frac{0.1145}{t^4} b_3^4 Y_{t1}, \tag{137}$$

$$Y_{02} = Y_{t2} + \frac{0.9765}{t} b_2 Y_{t2} + \frac{0.6043}{t^2} b_2^2 Y_{t2} + \frac{0.0229}{t^3} b_2^3 Y_{t2} + \frac{0.1145}{t^4} b_2^4 Y_{t2}, \tag{138}$$

$$Y_{03} = Y_{t3} + \frac{0.9765}{t} b_1 Y_{t3} + \frac{0.6043}{t^2} b_1^2 Y_{t3} + \frac{0.0229}{t^3} b_1^3 Y_{t3} + \frac{0.1145}{t^4} b_1^4 Y_{t3}, \tag{139}$$

$$0 \leq Y_{01} \leq Y_{02} \leq Y_{03} \leq 1, \tag{140}$$

$$0 \leq b_1 \leq b_2 \leq b_3, \tag{141}$$

$$0 \leq s_t \leq 1, \tag{142}$$

$$t = 1 \sim T.$$

Similarly, the KKT conditions for these DS versions can be analytically derived and solved.

4 Application of the proposed methodology to making an aircraft part using 3D printing

The case of making an aircraft part using 3D printing was adopted to validate the effectiveness of the proposed methodology. 3D printing has been widely applied to the design, manufacturing, and maintenance of aircraft [13, 18]. The quality issues of 3D-printed aircraft parts are always a critical concern to the researchers and practitioners in this field. In this case, each 3D-printed aircraft part was judged as "acceptable" or "unacceptable." The percentage of acceptable 3D-printed aircraft parts in a batch is defined as the yield of the batch. The yields of ten batches are illustrated in Fig. 1.

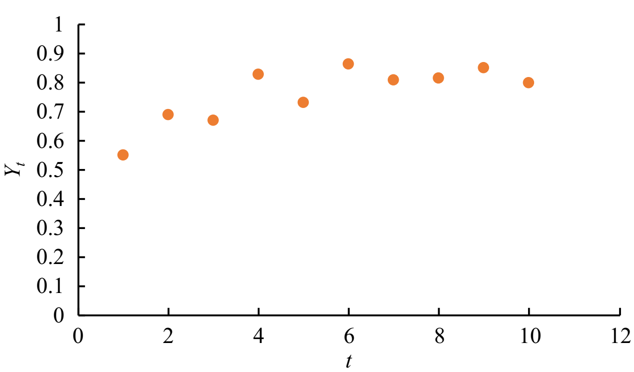

Fig. 1 The yields of ten batches

The data of the first five periods were used to generate each model. The remaining data were used for testing or evaluating the forecasting performance. The parameter settings in the models are summarized in Table 3. For a rational comparison, the parameter settings in the DS model were identical to those in the original model. Moreover, the values of the same parameter in various models were the same.

Table 3 Parameter settings in the models

Model	Parameter setting
TW	$s = 0.3$
DS-TW	$s = 0.3$
P	$d = 0.25$
DS-P	$d = 0.25$
D	$w_1 = 0.5$; $w_2 = 0.5$; $s = 0.3$
DS-D	$w_1 = 0.5$; $w_2 = 0.5$; $s = 0.3$
CLI	$o(k) = 2$; $s = 0.3$
DS-CLI	$o(k) = 2$; $s = 0.3$
CLII	$o(k) = 2$; $d(k) = 0.25$; $m(k) = 2$
DS-CLII	$o(k) = 2$; $d(k) = 0.25$; $m(k) = 2$

The fuzzy yield learning models fitted using these methods are summarized in Table 4.

The experimental results yielded the following findings:

(1) The values of b_3 in all models were lower than 2. Therefore, according to Theorem 2, the approximation of the exponential function with a polynomial function was valid.

Table 4 Fuzzy yield learning models fitted using the aforementioned methods

Method	Fuzzy yield learning model
TW	$\tilde{Y}_t = (0.7503, 0.9491, 0.9491)e^{-\frac{(0.5424,0.5424,0.5424)}{t}}$
DS-TW	$\tilde{Y}_t = (0.7419, 0.9472, 0.9472)e^{-\frac{(0.5398,0.5398,0.5398)}{t}}$
P	$\tilde{Y}_t = (0.7416, 0.7416, 1.0000)e^{-\frac{(0.0000,0.2957,0.2957)}{t}}$
DS-P	$\tilde{Y}_t = (0.7405, 0.7405, 1.0000)e^{-\frac{(0.0000,0.2950,0.2950)}{t}}$
D	$\tilde{Y}_t = (0.7740, 0.8828, 0.9791)e^{-\frac{(0.5424,0.5424,0.5424)}{t}}$
DS-D	$\tilde{Y}_t = (0.7732, 0.8742, 0.9785)e^{-\frac{(0.5398,0.5398,0.5398)}{t}}$
CLI	$\tilde{Y}_t = (0.7662, 0.9037, 0.9694)e^{-\frac{(0.5424,0.5424,0.5424)}{t}}$
DS-CLI	$\tilde{Y}_t = (0.7419, 0.9472, 0.9472)e^{-\frac{(0.5398,0.5398,0.5398)}{t}}$
CLII	$\tilde{Y}_t = (0.7416, 0.7416, 0.9523)e^{-\frac{(0.2957,0.2957,0.2957)}{t}}$
DS-CLII	$\tilde{Y}_t = (0.7405, 0.7405, 1.0000)e^{-\frac{(0.2378,0.2950,0.2950)}{t}}$

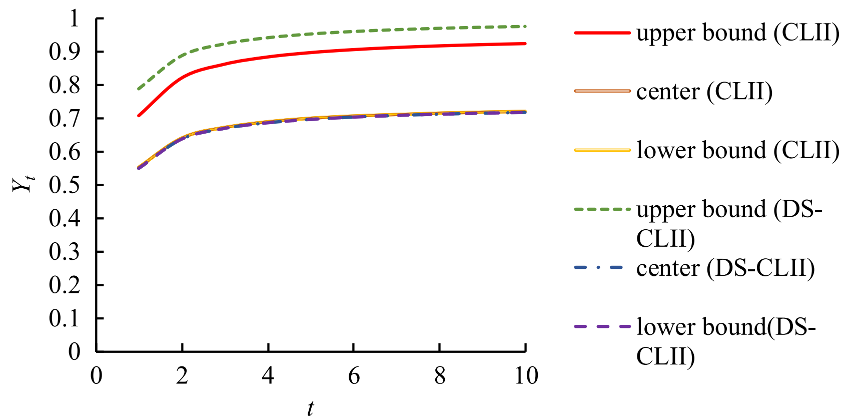

Fig. 2 The difference between Models CLII and DS-CLII

(2) Among the original models, Model CLII was most different from its DS version, as illustrated in Fig. 2, indicating that the fitted fuzzy yield learning model may be reasonably different after considering the original value, rather than the logarithmic or log-sigmoid value, of the yield. By contrast, Model CLI and its DS version were very similar, as illustrated in Fig. 3.

(3) The forecasting performances of all methods were compared using the mean absolute error (MAE), MAPE, root mean squared error (RMSE), and the average range, which were calculated as follows:

$$\mathrm{Min\,MAE} = \frac{\sum_{t=1}^{T} |Y_t - \hat{Y}_t|}{T}, \tag{143}$$

$$\mathrm{Min\,MAPE} = \frac{\sum_{t=1}^{T} \frac{|Y_t - \hat{Y}_t|}{Y_t}}{T} \cdot 100\%, \tag{144}$$

$$\mathrm{Min\,RMSE} = \sqrt{\frac{\sum_{t=1}^{T} \left(Y_t - \hat{Y}_t\right)^2}{T}}, \tag{145}$$

$$\text{Min The Average Range} = \frac{\sum_{t=1}^{T} (Y_{t3} - Y_{t1})}{T}, \tag{146}$$

The COG method was applied to defuzzify each fuzzy yield forecast as follows:

$$\hat{Y}_t = \frac{Y_{t1} + Y_{t2} + Y_{t3}}{3}, \tag{147}$$

The results are summarized in Fig. 4. The forecasting accuracy of the DS model was superior to that of the original model in terms of MAE, MAPE, and RMSE. The forecasting precision of the DS model was also comparable to that of the original model.

(4) The forecasting precision, which was measured as the average range of fuzzy yield forecasts, did not always exhibit similar results. This result was not unexpected, because the existing fuzzy yield learning models maximize the sum of satisfaction levels, rather than minimizing the average range, to optimize the forecasting precision. The direct minimization of the average range is difficult in the existing fuzzy yield learning models.

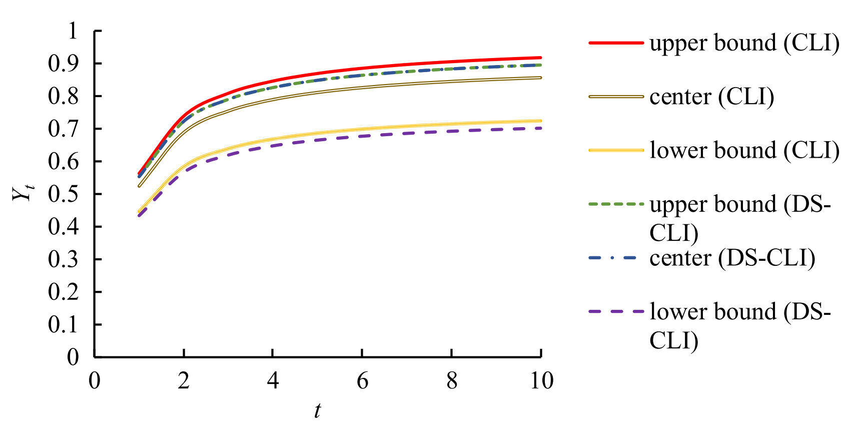

Fig. 3 Difference between Models CLI and DS-CLI

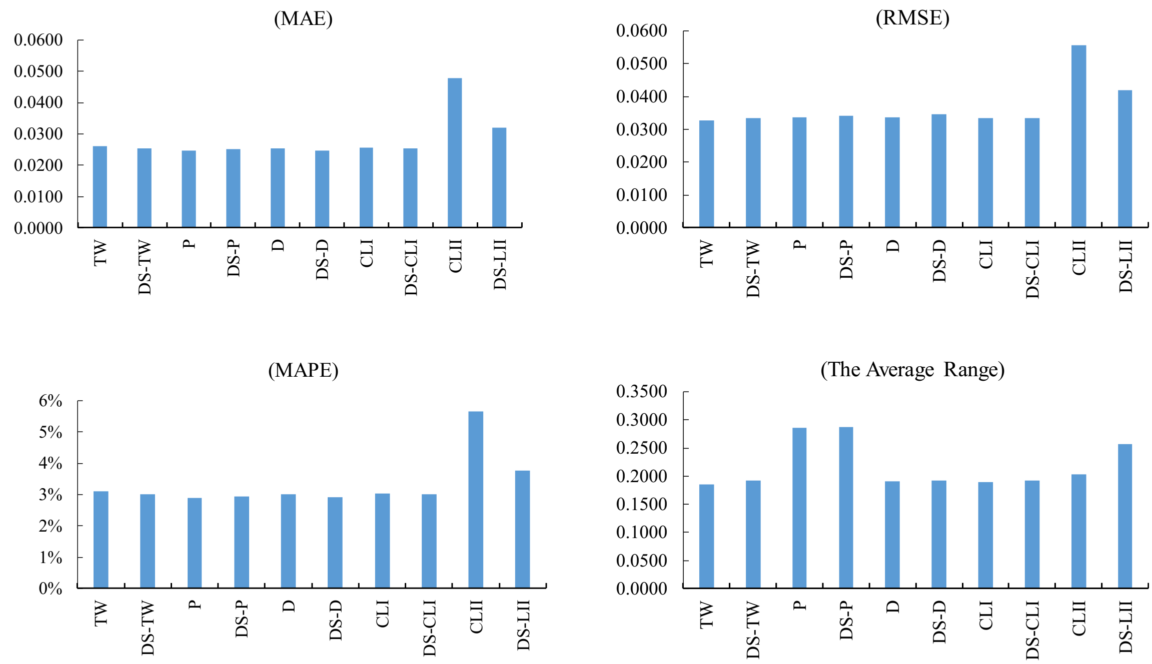

Fig. 4 Forecasting performance of all methods

(5) The most substantial advantage of the DS models over the original models was the reduction in the MAPE. Model DS-CLII exhibited an MAE reduction of 33.4% when compared with Model CLII. The MAPEs of the DS models were, on average, 7.7% lower than those of the original models.

5 Conclusions

Yield is a crucial performance measure for many manufacturing processes. The yield of products must be continually monitored, enhanced, and forecasted. This study aimed to resolve a crucial problem in yield forecasting; that is, the use of the logarithmic or log-sigmoid value, instead of the original value, of the yield to simplify the computation. To this end, an advanced fuzzy approach was proposed. The advanced fuzzy approach derived the DS versions of some existing fuzzy yield learning models. These DS versions directly employ the original value of the yield, thereby optimizing the forecasting performance.

To evaluate its effectiveness, the proposed methodology was applied to the case of making an aircraft part using 3D printing. On the basis of the experimental results, the following conclusions were obtained:

(1) The forecasting accuracy was substantially improved by directly considering the original value of the yield. Therefore, the MAE, MAPE, and RMSE results obtained by solving DS models were superior to those obtained by solving the original model.

(2) The treatments in this study were most effective when applied to the CLII model discussed by Chen and Lin [4] and least effective when applied to the CLI model of Chen and Lin [4].

Although the proposed methodology can theoretically be applied to other fields for modeling various learning processes, the polynomial fitting technique may not always achieve the required level of accuracy and modifications may be necessary. Several FCI methods for yield forecasting, including that of Chen and Wang [6], are based on the logarithmic or log-sigmoid value of the yield. The forecasting accuracy of such FCI methods can be enhanced by directly considering the original value of the yield. In addition, the joint applications of 3D printing and cloud computing to the aircraft industry are receiving attentions [5, 12], to which the proposed methodology may apply to enhance the effectiveness.

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