



Integrated CAD tolerancing model based on difficulty coefficient evaluation and Lagrange multiplier

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Abstract

Tolerance allocation is a very important assignment. In fact, knowing how to allocate efficiently tolerances for assembly parts in a computer-aided design (CAD) and computer-aided manufacturing (CAM) system is a very key issue because the cost and quality depend directly on dimension tolerances. The traditional optimization approaches complain about a lack regarding the evaluation of manufacturing difficulty to solve tolerance allocation problem. In this paper, a new approach based on difficulty coefficient evaluation (DCE) and Lagrange multiplier (LM) method is presented to obtain an economical tolerance allocation. In fact, DCE technique, based on the failure mode, effects and criticality analysis (FMECA) tool, is used to quantify the manufacturing difficulty of mechanical parts, as well as the LM method is developed to optimize the proposed approach while respecting the functions and manufacturing requirements. A case study is established to verify the highlights of this work. Indeed, the computed result shows that the method can produce tolerance allocation economically and accurately according to difficulty requirements. Consequently, the proposed method is performed to exploit fully design for manufacturing and assembly (DFMA). For this, an integrated CAD/tolerancing model based on manufacturing difficulty (ICADTMD) is developed using graphical user interface (GUI) in MATLAB.

Keywords Tolerance allocation · Manufacturing difficulty · FMECA · LM · Minimal dimension chain

Abbreviations

CAD	Computer-aided design
CAM	Computer-aided manufacturing
DFMA	Design for manufacturing and assembly
DCE	Difficulty for manufacturing and assembly
DC	Difficulty coefficient: β
FMECA	Failure mode, effects and criticality analysis
GUI	Graphical user interface
LM	Lagrange multiplier
RPN	Risk priority number
MD	Manufacturing dimension
MO	Manufacturing operation
FR	Functional requirement
WC	Worst case
AUWC	Uniform allocation using WC approach

LMWC	Allocation using LM method according to WC approach
LMDC	Allocation using DCE and LM according to WC approach
C_m	Manufacturing cost
QL	Quality loss
C_T	Total cost

1 Introduction

Tolerancing is a key step in the product life cycle and process development. The optimization tolerance technique aims to improve the product quality and assemblability as well as reduce the overall costs and time to market. Especially, the tolerance allocation is an important engineering function having a direct impact on compliance with functional and manufacturing requirements. For decades, numerous researchers focused on this issue since a pertinent close relationship between manufacturing cost and tolerance exists. However, the traditional optimization approaches neglect the manufacturing difficulty quantification to solve tolerance

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allocation problem. In this respect, this work manifests an appropriate tolerance allocation CAD model based on DCE and LM using graphical user interface (GUI) in MATLAB. Hence, the CAD integration of the proposed approach offers to designer the possibility to easily simulate the proposed tolerance allocation model. In addition, the new method, named an integrated CAD/tolerancing model based on manufacturing difficulty (ICADTMD), demonstrates an aided tool for decision in the field of tolerancing.

This paper is organized as follows. In Sect. 2, a review of the literature is presented. Then, an overview of the proposed approach is described, followed by a description of the used cost model as well as the DCE and LM methods for tolerance allocation. In Sect. 4, a case study is presented and followed by the implementation of the proposed CAD model based on GUI execution. In addition, the comparative study according to the cost criterion is exposed in order to analyze the allocated tolerances and to establish a constructive discussion. The conclusions and perspectives for this work are presented in Sect. 6.

2 Literature review

Optimal tolerance allocation is a trade-off between functional requirements and manufacturing cost. During the early decades, various manufacturing cost-tolerance models have been proposed in [1–3]. These models are separated into four classes: (1) the reciprocal power functions, (2) the exponential functions, (3) the polynomial functions, and (4) the hybrid models, such as combined reciprocal power and exponential function, combined polynomial and exponential function, reciprocal power as well as exponential hybrid function. Various optimization methods have been developed to optimal allocation of tolerances as exposed in Sect. 2.1.

2.1 Tolerance allocation method considering cost and manufacturing process

To minimize manufacturing cost, the tolerance allocation is considered as an optimization problem constrained by functional and manufacturing requirements. Muthu et al. [4] used meta-heuristics techniques, as genetic algorithm (GA) and particle swarm optimization, to obtain optimal tolerances. Huang et al. [5] and Lu et al. [6] proposed nonlinear programming methods considering the practical manufacturing process and mechanism tolerance process. Cheng and Tsai [7, 8] explored a method to optimal statistical tolerance allocation using the exponential cost-tolerance function. Prabhakaran et al. [9] exploited a colony algorithm as an optimization method for both quality improvement and optimal tolerance allocation. Zong and Mao [10] proposed a methodology to obtain the least

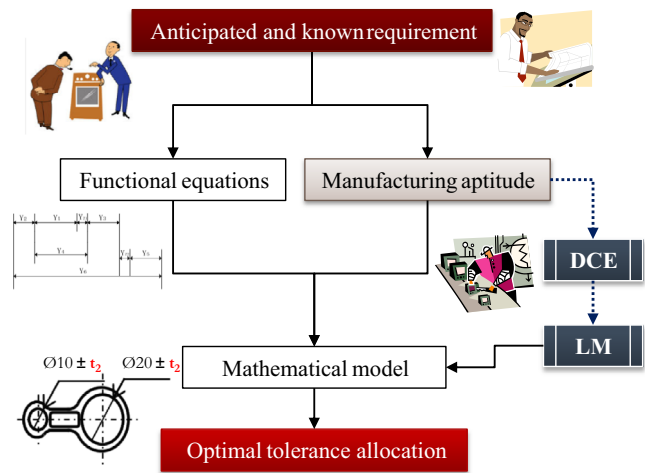


Fig. 1 Flowchart of the whole proposed tolerance allocation approach

manufacturing cost and to reduce quality loss (QL) for a mechanism with multiple correlation characteristics. Dinesh et al. [11] presented a model dedicated to the concurrent optimization of design and manufacturing tolerances in the case of a prismatic mechanism. Kumar et al. [12] proposed recent approach founded on LM method integrating Lambert W function. The optimization model considering manufacturing cost and QL was developed in [13–19] for optimum tolerances of mechanical assembly. An algorithm called teaching-learning-based optimization (TLBO) was used to achieve the global optimal tolerance allocation [20]. Wang et al. [21] introduced the variable coefficients reciprocal squared model (VCRSM) into tolerance allocation process aiming the resolving of multi-constraints into tolerance allocation problem in the case of aircraft mechanism. Kumar et al. [22] proposed a recent optimization algorithm, called bat algorithm (BA) for optimal tolerance allocation based on QL. Different methods,

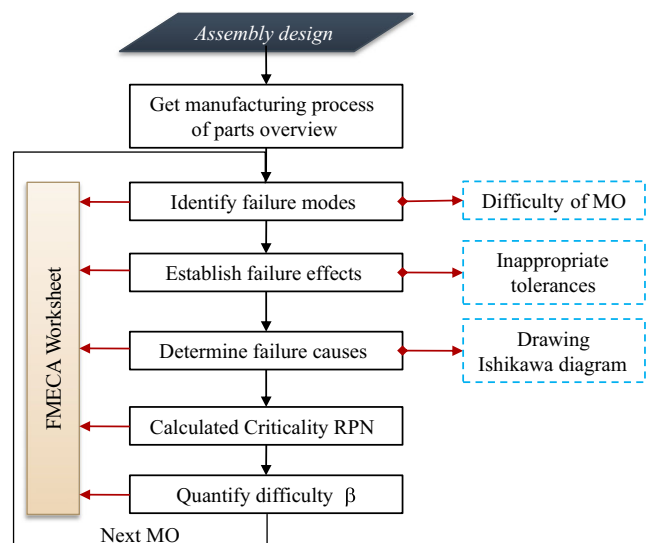


Fig. 2 Flowchart of DCE based on FMECA tool

Table 1 Score range criterions

Score (S)	Severity	Score (O)	Occurrence	Score (D)	Detection
1	Minor	1	Very unlikely	1	Obvious
2	Average	2	Remote	2	Possible
3	Major	3	Frequent	3	Unlikely
4	Catastrophic	4	Certain	4	Impossible

using the analysis of fuzzy factors in the optimal tolerance allocation, were established in many researches [23–25]. To reduce the uncertainties in the assembly process and increase customer satisfaction, Mingzhou et al. [26] developed a method of tolerance grading allocation based on the remanufacturing assembly uncertainty analysis.

2.2 CAD model for tolerance analysis and synthesis integration

In spite of the fact that there are enough data extracted from CAD models [27], the imperfection manifests in the simple verification. Hence, the digital mock up (DMU) are not sufficiently exploited in the field of tolerancing. In this respect, Hassani et al. [28] proposed a CAD model to evaluate the defect assembly parts for linear analytical expressions of functional characteristics using Monte Carlo (MC) simulation. Tlija et al. [29] and Louhichi et al. [30] proposed an innovative approach to integrate the tolerances in CAD models. In fact, the face displacements (translations and/or rotation) lead to obtain realistic configurations of parts. The small displacement torsor deduced from the geometrical tolerance determines the displacement parameters. The model is considered as a tolerance analysis tool while taking into account the assembly process planning and the contact types between assembly components. The proposed approach considers the dimensional, orientation, and positional tolerances. Nevertheless, the neglect of form defects generates,

especially in the case of a small clearance [31–33], non-compliant assemblies of compliant parts. Thus, Jbira et al. [34] improved the above model, cited in [29, 30], by considering the form defects in CAD model. This is performed to simulate and visualize the behavior of mechanical assembly in real configurations.

2.3 Synthesis and research objectives

The advantage of traditional LM method consists on the obtaining of an ideal solution with easy treatment within short time compared to non-traditional optimization technique [35]. Besides, LM method gives closed-form solution with precision for all types of real-time tolerancing problems [1, 2, 36–38]. However, the major drawbacks of the general allocation methods as in [4–25] are the neglect of the difficult cost reflecting the impediment of manufacturing operation to obtain machined dimensions and the integration lack of the manufacturing difficulty in tolerance allocation. Considering the improvements and inconveniences of the above methods, this paper proposes a new method of tolerance allocation based on DCE and LM optimization technique. The DCE is carried out by quantifying weight factors called difficulty coefficient β . This coefficient is computed using techniques for the study and analysis of reliability of the design or the processes FMECA tool and Ishikawa diagram. Therefore, the originality and novelty of the proposed approach is the fact to incorporate the β values in tolerance allocation

Fig. 3 Proposed FMECA worksheet and Ishikawa diagram

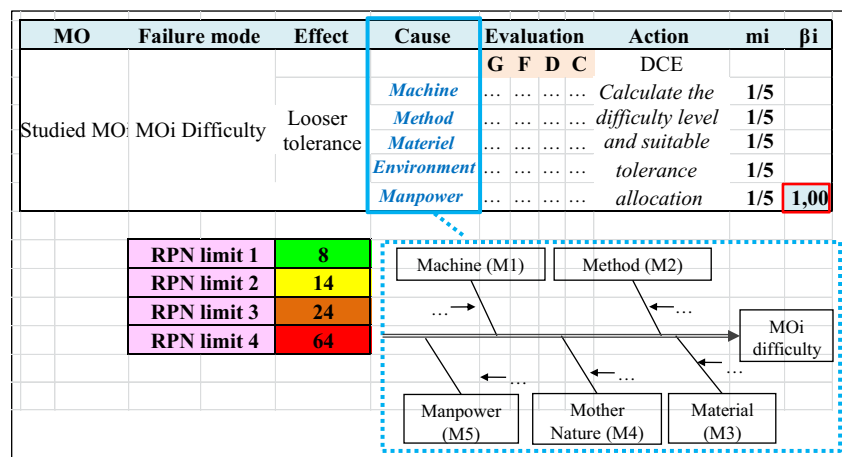


Table 2 Parameters of cost-tolerance model

Cost-tolerance model	C ₀	C ₁	C ₂
t _{a23}	296.29	19.47	23.82
t _{b12}	296.29	19.47	23.82
t _{e23}	82.42	16.66	19.98
t _{a14}	296.29	19.47	23.82
t _{c14}	160.45	86.67	29.19

$$C_m = \sum_{i=1}^n \beta_i C(t_i) \tag{2}$$

The total product cost is the summation of the manufacturing cost C_m and QL (Eq. 3):

$$C_T = C_m + QL \tag{3}$$

The QL is calculated according to Noorul et al. [40] as in Eq. 4.

$$QL = \frac{A}{9t_y^2} \sum_{i=1}^n t_i^2 \tag{4}$$

3.2 Tolerance allocation based on DCE and LM

The LM method contributes to the closed-form solution. Thus, LM is considered as a classical approach for constrained optimum problem while respecting the functional requirement

Table 3 Influencing MO and related β

MD	MO	β notations	β values
a23	Face milling	β_{a23}	1.48
b12	Face milling	β_{b12}	1.48
e23	Drilling	β_{e23}	1.10
a14	Face milling	β_{a14}	1.48
c14	Drilling	β_{c14}	1.10

(FR) of mechanism. In this paper, the LM concept is used in order to obtain efficiently and certainly optimal tolerances. In addition, the DCE allows quantifying and considering the manufacturing operations (MO) difficulty of parts in the tolerance allocation step. The flowchart of the proposed algorithm is shown in Fig. 1.

3.2.1 Optimal tolerance using LM method

LM is a mathematical method for solving the optimization problems (Eq. 5), where ψ is the Lagrange’s multiplier.

$$\frac{\partial}{\partial T_i} (\text{cost function}) + \psi \frac{\partial}{\partial T_i} (\text{constraints}) = 0 \tag{5}$$

Each individual tolerance, which provides a minimum exponential cost-tolerance, can be determined in terms of a pivot parameter t_i according to the tolerancing approach: Eq. 6 in the case of statistical model and Eq. 8 for WC model, such as

Fig. 7 FMECA worksheet and Ishikawa diagram of drilling

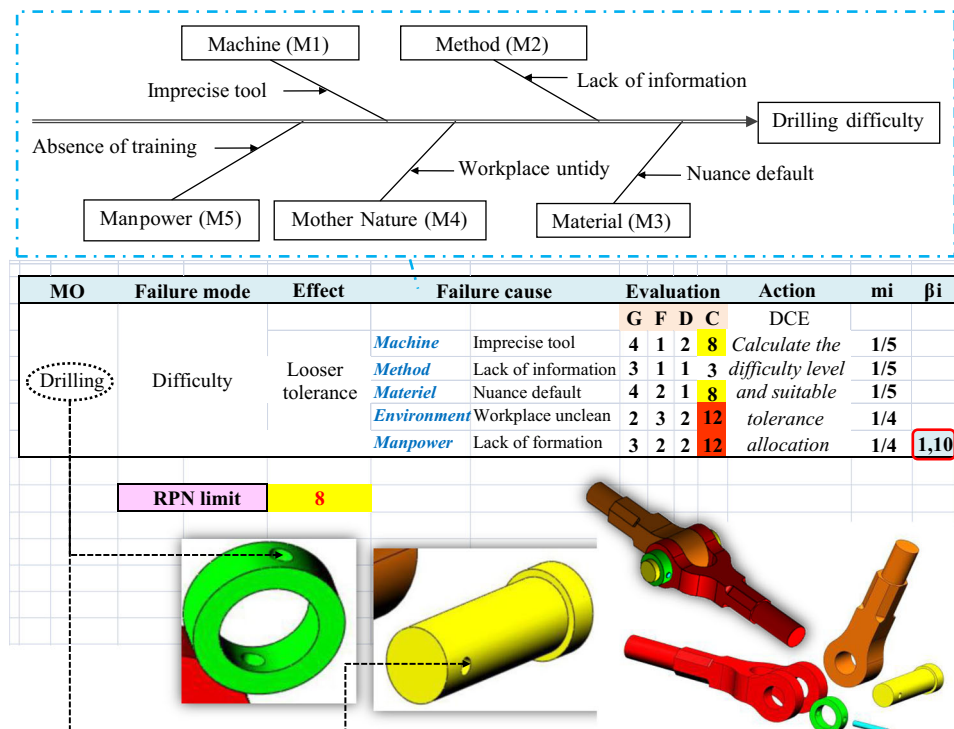


Fig. 8 Welcome user interface of ICADTMD model



lambert w(x) is the Lambert’s W function at the value *x* [41]. Consequently, the accumulated tolerance *t_y* of the FR can be calculated by Eqs. 7 and 9 using statistical and WC models respectively.

$$t_y = t_1 + \sum_{i=1}^n \frac{1}{C_{li}} \left[C_{11} \times t_1 + \ln \left(\frac{C_{li} \times C_{0i}}{C_{11} \times C_{01}} \right) \right]; i = 1 \text{ to } n \tag{9}$$

Statistical model

$$t_i = \frac{1}{C_{li}} \text{lambertw} \left[\left(\frac{C_{li}^2 \times C_{0i}}{C_{11} \times C_{01}} \right) \times t_1 \times \exp(C_{11} \times t_1) \right] \tag{6}$$

$$t_y^2 = t_1^2 + \sum_{i=1}^n \frac{1}{C_{li}} \text{lambertw} \left[\left(\frac{C_{li}^2 \times C_{0i}}{C_{11} \times C_{01}} \right) \times t_1 \times \exp(C_{11} \times t_1) \right]; i = 1 \text{ to } n \tag{7}$$

WC model

$$t_i = \frac{1}{C_{li}} \times \left[C_{11} \times t_1 + \ln \left(\frac{C_{li} \times C_{0i}}{C_{11} \times C_{01}} \right) \right] \tag{8}$$

3.2.2 Tolerance allocation based on DCE

The quantitative evaluation of the manufacturing difficulty of each part is performed using the DCE approach. The developed DCE uses the β coefficient. A β value is assigned to each MD driven by MO. The β value is proportional to the obtaining difficulty of MD. This coefficient is computed basing on FMECA method and Ishikawa diagram as developed in [31] and illustrated in Fig. 2. The piloting of the proposed method is carried out by the elected FMECA group.

FMECA group The calculation procedure of the difficulty coefficient (DC), which is the coefficient β , is based on the elected FMECA group. The criteria to choose

Fig. 9 Main steps of ICADTMD model

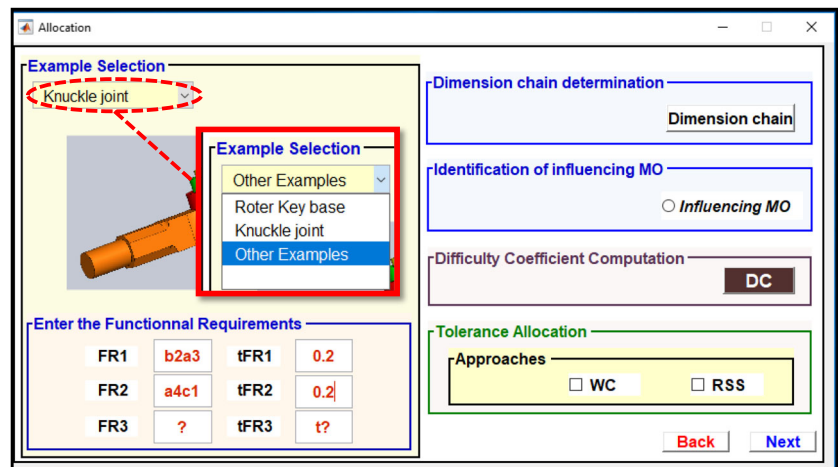
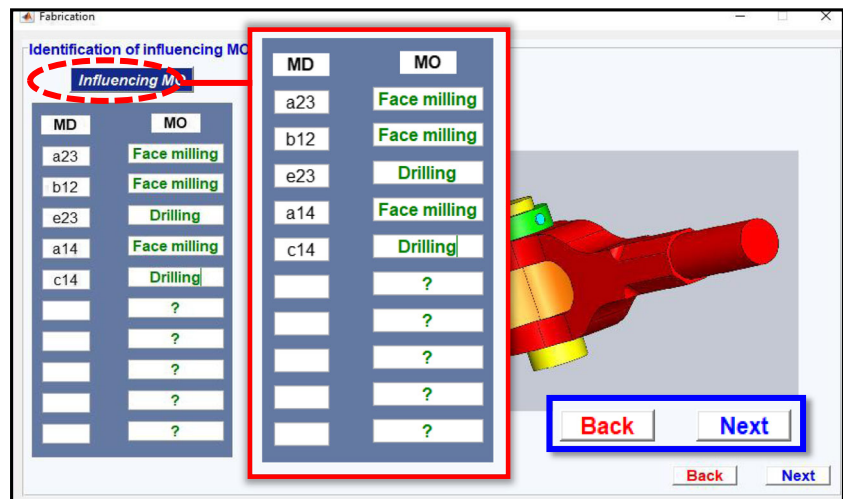


Fig. 10 User interface of influencing MO identification



FMECA group member are as follows: the great experience, the studied product knowledge, and the ability to provide the necessary information for the analysis. In fact, the FMECA group assesses the potential causes of failure and controls the data validity. An effective FMECA group pushes the right questions at the right time and encourages efficient communication between the various actors in the product development cycle. Therefore, seven qualified and specialized people are proposed to constitute FMECA group: 1—designer, 2—production expert, 3—maintenance expert, 4—technical pilot, 5—animator, 6—manufacture, 7—adjuster. The evaluation and quantification of FMECA parameters are absolutely performed by the FMECA group to properly calculate the DC according to the proposed approach (Fig. 2).

FMECA parameters The FMECA considers three parameters which are evaluated through interpreted linguistic expressions:

- (1) Severity (S) indicates the gravity of the failure mode effects
- (2) Occurrence (O) denotes the probability of a failure occurring
- (3) Detection (D) measures a failure’s visibility that is the attitude of a failure mode to be identified by controls or inspections

Each parameter is correlated with a score range [1; 4] (minimum of 1 to a maximum of 4 in this paper) according to Table 1 and to the FMECA group judgment. The

Fig. 11 User interface of DC computation

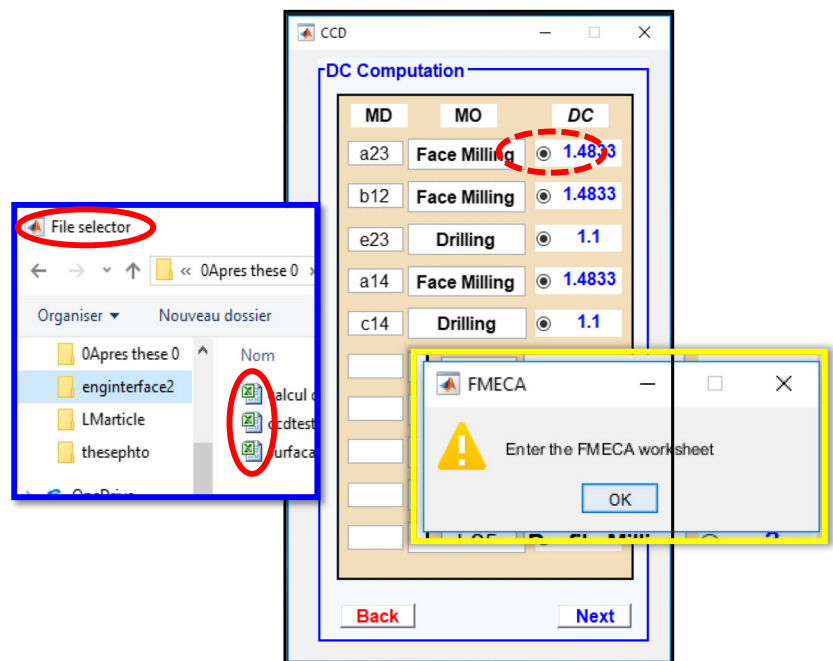
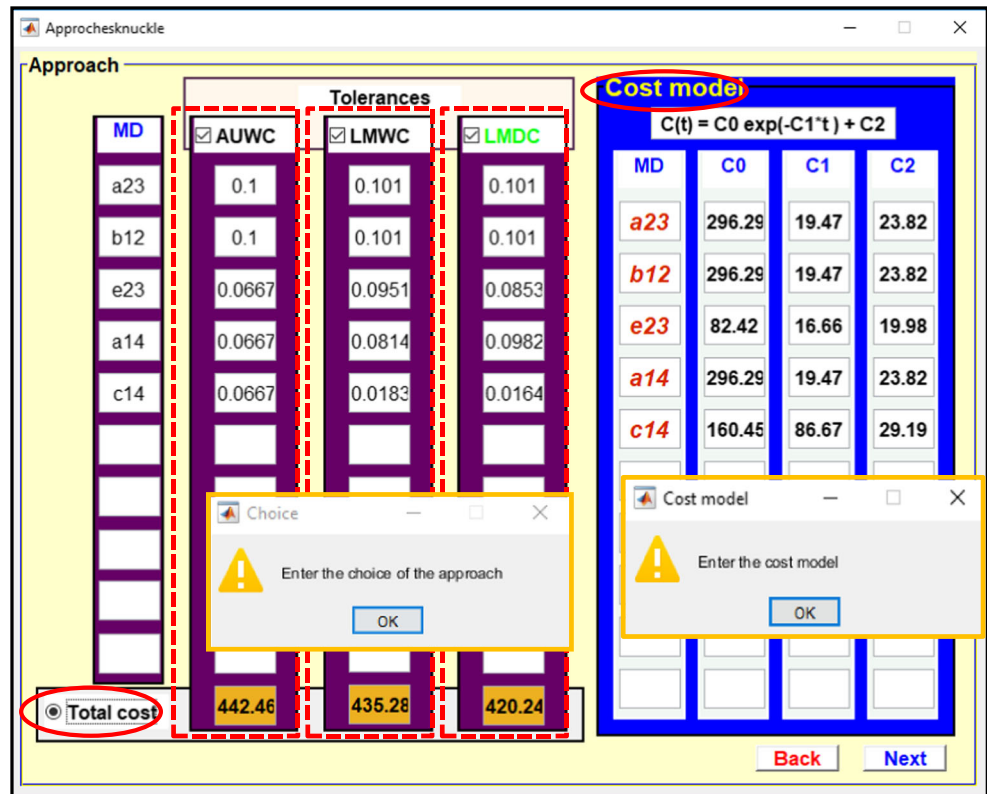


Fig. 12 User interface of tolerance allocation approaches



risk priority number (RPN) is defined as the product of these three parameters (Eq. 10):

$$RPN = S \times O \times D \tag{10}$$

The RPN assigns a weight to each MO difficulty. The higher value of RPN corresponds to the worst product reliability or MO obtaining.

FMECA worksheet To respect the above practical requirements, a suitable FMECA worksheet for the analysis must be decided and be updated for each MO as clarified in

Fig. 3. The different sources of the failure cause are the 5 Ms shown in the diagram of Ishikawa: machine (M1), method (M2), material (M3), mother nature (M4), and manpower (M5). Figure 3 exposed the proposed FMECA worksheet and Ishikawa diagram for an example of MO. Initially, the β value is equal 1.

β computation The β is the aggregation of five parameters m_i as shown in Fig. 3. Those parameters are assumed to be dependent to RPN value and limits. The FMECA workgroup fixes the RPN limits according to manufacturing requirements. In this work, the RPN limits are considered according to FMECA group equal to:

Fig. 13 Comparative approaches

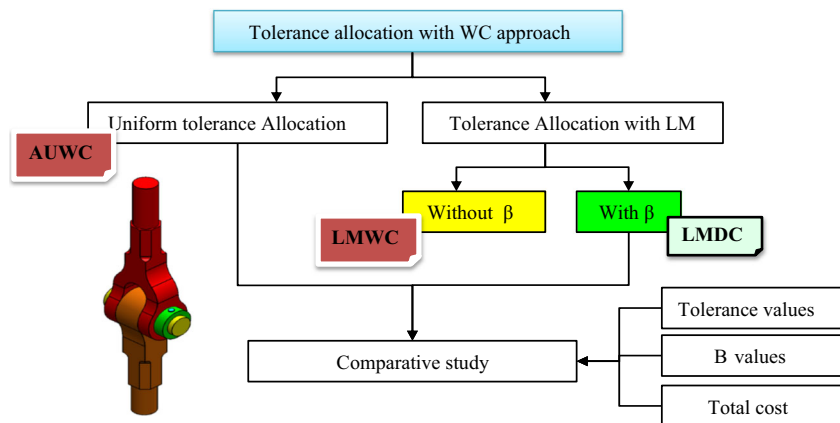


Table 4 Comparison of allocated tolerance of FR1

	t_{a23}	t_{b12}	Accumulated tolerance	Cost (€)
AUWC	0.1	0.1	0.2	200.104
LMWC	0.101	0.101	0.202	197.78
LMDC	0.101	0.101	0.202	197.78

- Limit 1 = Score 2 × Score 2 × Score 2 = 8
- Limit 2 = Score 2 × Score 3 + Score 2 × Score 4 = 14
- Limit 3 = Score 2 × Score 3 × Score 4 = 24
- Limit 4 = Score 4 × Score 4 × Score 4 = 64

The mathematical formula of β is expressed in Eq. 11, where m_i can have four possible attributes as follows:

- $m_i = 1/5$ if $1 \leq RPN \leq 8$, where RPN value is considered negligible,
- $m_i = 1/4$ if $8 < RPN \leq 14$, where RPN value is considered average,
- $m_i = 1/3$ if $14 < RPN \leq 24$, where RPN value is considered elevated,
- $m_i = 1/2$ if $24 < RPN \leq 64$, where RPN value is considered forbidden.

$$\beta = \sum_{i=1}^5 \overline{m}_i \tag{11}$$

DCE synthesis In order to clarify in an attractive way, the proposed FMECA worksheet, Fig. 4 presents an illustrative FMECA worksheet of face milling (MO). In this case, the β value is equal to 1.48 which is the summation of the m_i average values. In addition, “Mi”% is introduced to manifest the percentage of each contribution in β computation as included in FMECA worksheet of face milling in Fig. 7.

During the implementation of FMECA, the operators, leaders, and officials must be solicited as much as possible to collect the maximum information and circumvent any possible problems.

Table 5 Comparison of allocated tolerance of FR2

	t_{e23}	t_{a14}	t_{c14}	Accumulated tolerance	Cost (€)
AUWC	0.0667	0.0667	0.0667	0.2000	240.480
LMWC	0.0951	0.0814	0.0183	0.1948	237.501
Proposed LMDC	0.0853	0.0982	0.0164	0.1999	222.413

4 Case study

4.1 Description of the proposed example

In this paper, the knuckle joint assembly, used by several researches as in [10, 12, 20], is considered as the case study problem. This example is chosen to use the proposed cost model. The mechanical joint is used to connect two intersecting cylindrical rods which are under the tensile load. The system permits an angular movement between the cylindrical rods. A typical knuckle joint has the following parts: *a*—fork end, *b*—eye end, *c*—collar, *d*—tapper pin end, and *e*—pin as shown in Fig. 5. The mechanism contacts are between the following geometric entities (Fig. 6):

- Faces of parts *a* and *e* → $e2 = a1$
- Faces of parts *a* and *b* → $a2 = b1$
- Axes of the cylindrical faces of parts *c*, *d*, and *e* → $e3 = c2 = d2$

The above contacts are designed by thick line in Fig. 6. The functional requirements FRs are the gaps between the faces *a3* and *b2* of parts *a* and *b* respectively as well as between the faces *a4* and *c1* of parts *a* and *c*, respectively. These FRs are shown in Fig. 5 and in detail in Fig. 6. The variation range of acceptable gaps must be within or equal to 0.2 mm.

As shown in Fig. 6, the dimensions constituting dimension chains of FR1 and FR2 are enumerated basing on minimal dimension chain approach in Eqs. 12 and 13.

$$FR1 = b2a3 = a23 - b12 \tag{12}$$

$$FR2 = a4c1 = e23 - a14 - c14 \tag{13}$$

The above dimensions are called blue print dimensions. The transfer of these dimensions into MDs is performed directly according to a suitable chosen manufacturing process of parts. Therefore, *a23*, *b12*, *e23*, *a14*, and *c14* are supposed in our case the MDs.

By applying tolerance chain relationship and respecting the acceptable gap of FRs, the above-mentioned equations are computed in WC approach as expressed in Eqs. 14

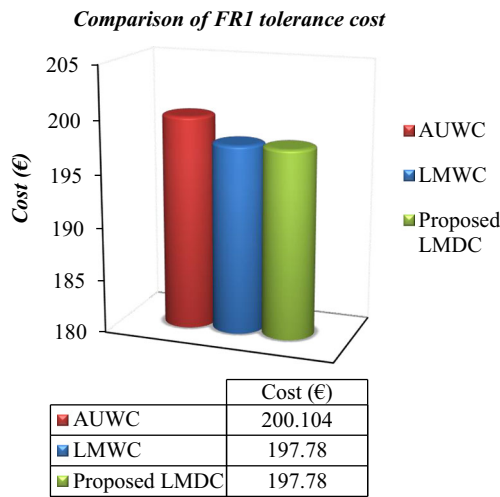


Fig. 14 Comparison of FR1 total cost

and 15, where 0.2 is the functional requirement value of FR1 and FR2:

$$t_{a23} + t_{b12} \leq 0.2 \tag{14}$$

$$t_{e23} + t_{a14} + t_{c14} \leq 0.2 \tag{15}$$

Thus, the Lagrangian function for the problem is implemented as Eq. 16.

$$\begin{aligned}
 f(t_{a23}, t_{b12}, t_{e23}, t_{a14}, t_{c14}) = & \sum_{i=1}^n C_{0i}e^{-C_{1i}t_i} + C_{2i} \\
 & + \psi_1(t_{a23} + t_{b12} - 0.2) \\
 & + \psi_2(t_{e23} + t_{a14} + t_{c14} - 0.2)
 \end{aligned} \tag{16}$$

To solve the problem using the LM approach in WC, the tolerances t_{a14} is taken in this studied case, as a pivot parameter and to be substitute into Eq. 9. These pivot tolerances can

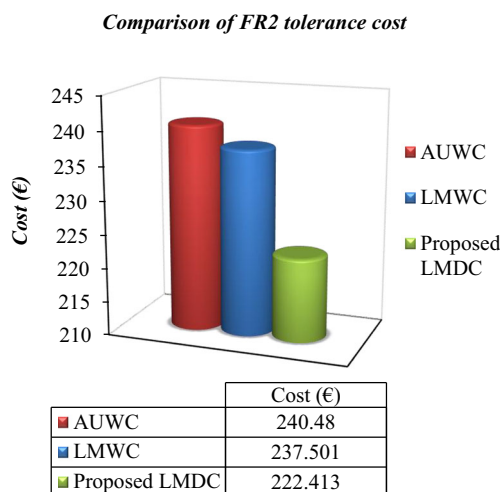


Fig. 15 Comparison of FR2 total cost

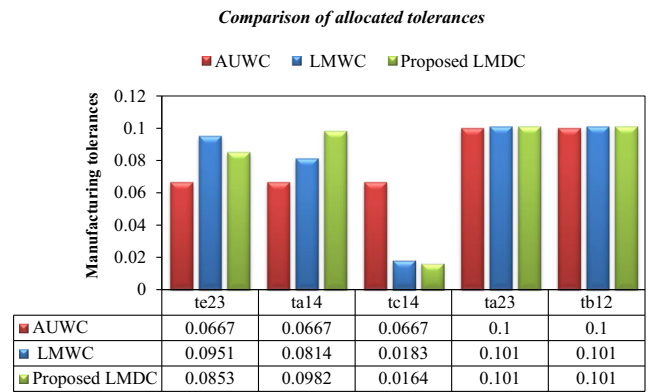


Fig. 16 Comparison of allocated manufacturing tolerance

be used to obtain the expression of other tolerances according to Eq. 8. The parameters of tolerance cost values are shown in Table 2.

Moreover, the tolerances deduced from LM method are multiplied by the ratio r_{LMDC} according to the proposed approach. The r_{LMDC} is calculated as indicated in Eq. 17, where n is the MDs number of each FR and $\sum \beta$ is the summation of β_{c14} , β_{a14} , β_{e23} , β_{a23} , and β_{b12} in this case study:

$$r_{LMDC_i} = \frac{n\beta_i}{\sum \beta_i} \tag{17}$$

The values of β of each MOs are computed after completing their worksheets as illustrated in Fig. 7 and shown in Table 3. Figure 7 shows FMECA worksheet and Ishikawa diagram of the drilling operation which affects e23 and c14.

At this stage, the implementation of the proposed method is introduced in order to accomplish the comparative study. In fact, the interfaces of ICADTMD offer an aided tool for decision in the tolerance analysis and allocation field.

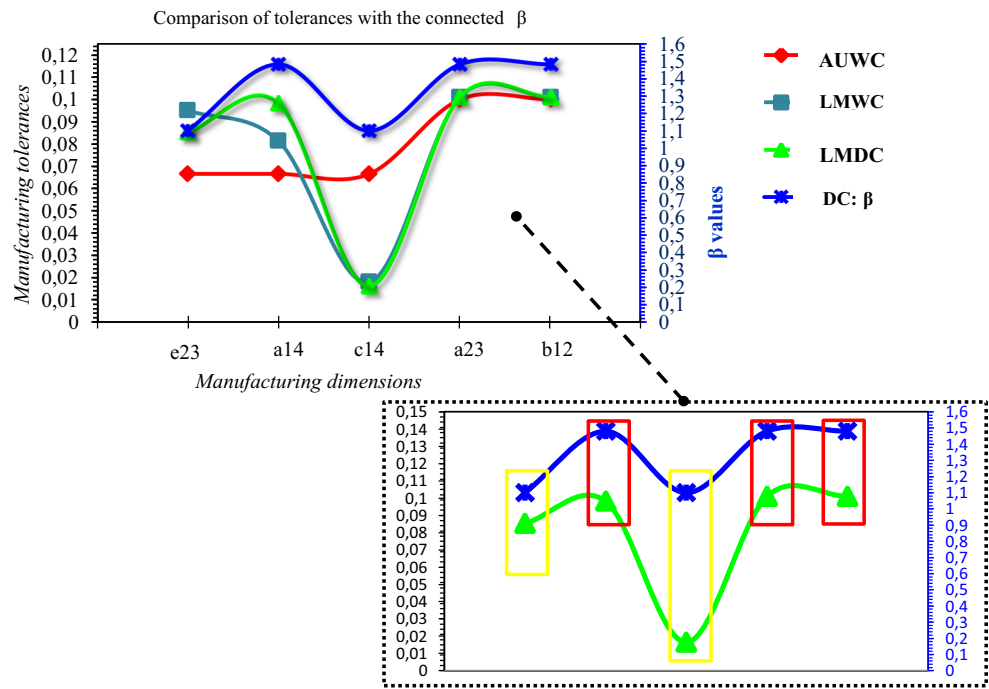
4.2 Implementation of the proposed approach: ICADTMD model

In fact, the proposed LMDC approach using simultaneous DCE and LM approach is a major step which manifests tolerance allocation approach in the ICADTMD model. Figure 8 presents the welcome user interface of ICADTMD model. Furthermore, the main steps of ICADTMD model are enclosed in the user interface of Fig. 9.

The principal proposed ICADTMD steps are the following:

- Step1: Dimension chain determination: This step is detailed in [39, 42]).
- Step 2: Identification of influencing MO: Fig. 10 shows the user interface of influencing MO identification. The technique of extraction the MOs that affect MDs from CAD model is developed in [39].

Fig. 17 Comparison of allocated tolerances and corresponding β values



- Step 3: Difficulty coefficients computation: Fig. 11 presents the user interface of DCs computation according to DCE procedure which is described in this paper.
- Step 4: Exposition of tolerance allocation approaches. Figure 12 elucidates the user interface of the developed tolerance allocation approaches.

In the following, the descriptions of the user interfaces and dialog box of ICADTMD model are established. Obviously, Fig. 9 encompasses the steps of ICADTMD model. In addition, the example choice is performed using the popup menu indicated in Fig. 9.

Moreover, a click on the button “next” of Fig. 9 leads to open the user interface of influencing MO identification (Fig. 10). This is achieved by simply clicking on the button “influencing MO” (Fig. 10). In addition, a click on button Next includes the display of the user interface of DC computation (Fig. 11). Besides, a click on radio-button of each influencing MO (Fig. 11) allows reading of the suitable

FMECA worksheet. Hence, the DC values are exposed. A warning box is programmed to enter the FMECA worksheets and except their XL pages appear in the file selector (Fig. 11).

A click on button next of Fig. 11 introduces the user interface of tolerance allocation approaches (Fig. 12). In fact, the user interface of Fig. 12 offers to the designer the possibility to visualize the allocated tolerances according to three tolerance allocation alternatives. In addition, a click on the radio-button total cost (Fig. 12) leads to calculate the total cost of each alternative. This is carried out according to the cost model which is modifiable by the designer according to the industry requirements. Warning box is performed to enter the choice of tolerance allocation approaches and to enter the cost model as illustrated in Fig. 12.

It is to highlight that the tree alternatives of tolerance allocation respect the functional requirements FRs. However, the proposed approach LMDC is the most economical according to the supposed difficulty coefficients. In order to recapitulate, the ICADTMD model establishes a CAT tool to facilitate decision-making for the designer in a context of collaborative

Table 6 Assembly total cost and related allocated tolerance

	t_{e23}	t_{a14}	t_{c14}	t_{a23}	t_{b12}	Cost (€)
AUWC	0.0667	0.0667	0.0667	0.100	0.100	440.584
LMWC	0.0951	0.0814	0.0183	0.101	0.101	435.281
Proposed LMDC	0.0853	0.0982	0.0164	0.101	0.101	420.193

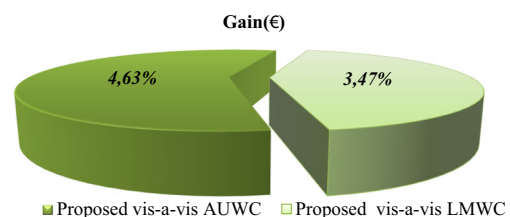


Fig. 18 Provided gain of knuckle joint assembly

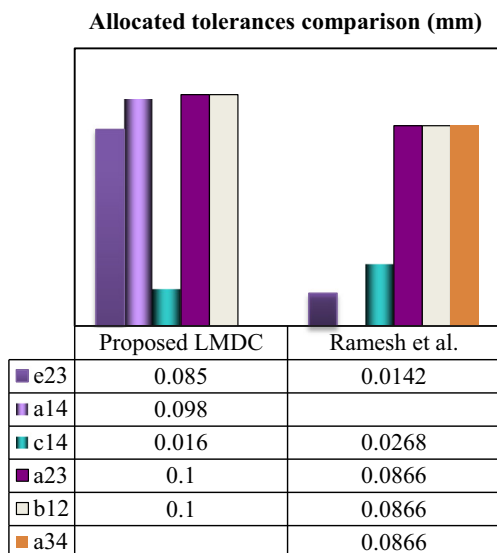


Fig. 19 Comparison of allocated manufacturing tolerance with author [12]

engineering as well as the field of interactive exchange. In addition, this work introduces an innovative tolerance allocation approach coupling between DCE procedure and LM optimization method. As a matter of fact, Sect. 5 exposes the results analysis of the three tolerance allocation alternatives, which are the comparative approaches in this work, as well as the comparative study.

5 Results and discussion

5.1 Comparative study between allocation approaches

In this work, the compared approaches, explained in Fig. 13, are the following: (1) uniform allocation using WC approach (AUWC), (2) allocation using LM method according to WC approach (LMWC), and (3) allocation using simultaneous DCE and LM according to WC approach (LMDC). The

Table 7 Tolerance analysis regarding to β values

CF	Proposed LMDC		Ramesh et al.
	t	β	t
e23	0.085	1.1	0.014
a14	0.098	1.48	
c14	0.016	1.1	0.027
a23	0.10	1.48	0.087
b12	0.10	1.48	0.087
a34			0.087

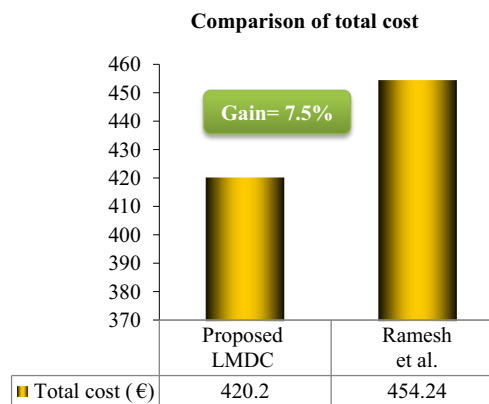


Fig. 20 Comparison of total cost with [12]

criteria of the comparative study are the tolerance values, the β values, and the total cost of assembly.

In the three approach cases, traditional LM method is employed to obtain optimal tolerance as given in Tables 4 and 5 for knuckle joint assembly while respecting FR1 and FR2 respectively. The obtained results are assessed with the LMDC approach. However, the DCE for FR1 leads to equals β s, in fact, $\beta_{b12} = \beta_{a23} = 1.48$. The optimal tolerances which satisfy the FR1 using traditional LM are illustrated in Table 4. The analysis of Table 4 shows, on the one hand, that the tolerance allocation in WC using LMWC and proposed LMDC approaches contributes to the same total cost and on the other hand guarantees a little gain compared to AUWC approach as illustrated in Fig. 14. Nevertheless, the proposed LMDC method is highlighted using FR2 as shown in Table 5 and Fig. 15.

The contributions of the proposed method are pointed out and highlighted more explicitly basing on the results analysis given in Table 5. Indeed, the LMDC approach leads to obtain optimal tolerances that satisfy the FRs and to widen the difficult MDs. For example, a14 with $\beta_{a14} = 1.48$, is more difficult than e23 which has $\beta_{e23} = 1.10$. Thus, the new obtained t_{a14} is upper than t_{e23} ($t_{a14} = 0.098 \text{ mm} > t_{e23} = 0.085 \text{ mm}$) as illustrated in Table 5. This fact guarantees absolutely optimal quality and cost. The achieved manufacturing cost using the proposed method is 222.413€ which is more economical than 237.501€ using traditional LMWC (Fig. 15). Hence, the proposed method leads to obtain a gain of 6.353% per assembly compared to LMWC method. Moreover, the percentage of the achieved gain is 7.513% per assembly when compared to AUWC approach which values 240.48€ as elucidated in Table 5 and Fig. 15. Moreover, the obtained accumulated tolerance is 0.199 mm, which is firmly appreciated compared to 0.2 mm using WC approach. The assembly total cost is the summation of FR1 and FR2 total cost.

Moreover, Fig. 16 embellishes the tolerance allocation values. The analysis of Fig. 16 according to the β values

shows that the proposed approach LMDC is perfectly proportionate to the β value fluctuations (Fig. 17). This verdict signifies that the proposed LMDC widens the tolerances of MDs which are supposed difficult to obtain. Nevertheless, the proposed LMDC accepts to reduce the tolerance value of MD which is supposed easy to obtain. Moreover, the examination of Fig. 17 leads to extrapolate that the proposed LMDC approach is the best method to ensure the assembly functionality, the quality, and the lowest assembly tolerance cost.

In order to clarify the assembly total cost computation, Table 6 summarizes the allocated tolerances as well as the knuckle joint assembly total cost for the AUWC, LMWC, and proposed LMDC approaches.

Indeed, the proposed approach based on DCE and LM which is LMDC promotes an economic gain (EG) as elucidated in Fig. 18:

- $EG = (440.584 - 420.2) \times 100 / 440.584 = 4.63\%$ per assembly compared to AUWC
- $EG = (435.29 - 420.2) \times 100 / 435.29 = 3.47\%$ per assembly compared to LMWC

Therefore, from the obtained results analysis, the proposed method generates an economical cost achievement and grants privileges to concurrent engineering environment by coupling of the DCE and LM approaches.

5.2 Comparison with R. Kumar's approach

In this work, the allocated tolerances using the LMDC approach are compared with tolerance values according to those in R. Kumar et al. [12]. Figure 19 shows the allocated manufacturing tolerances using the two above approaches: The two approaches do not use the same MDs; thus, the comparison should be based on the total cost. The exploration of Fig. 15 with the associated β values according to the proposed DCE procedure is summarized in Table 7. Hence, the allocated tolerance analysis regarding β values and tolerance values which are developed in [12] shows that

- The proposed LMDC enlarges the tolerances of MDs which are supposed difficult to obtain as b12 which has $\beta = 1.48$ (Table 7).
- Nevertheless, the proposed LMDC accepts to diminish the tolerance value of MD which is supposed feasible to obtain as c14 which has $\beta = 1.01$ (Table 7).

Consequently, the total cost computation (Fig. 20) proves that the proposed LMDC approach is more economical than the tolerance allocation approach in [12]. Besides, a gain of 7.5% is procured per assembly using the proposed LMDC.

6 Conclusions

The optimal tolerance is vital to minimize the manufacturing cost. This paper presents a new approach for tolerance allocation-based simultaneous on DCE and LM procedures. The proposed LM method solves the tolerance allocation problems by quantifying manufacturing dimension difficulty and minimizing the tolerance cost. In addition, a new CAD model named ICADTMD integrated CAD/tolerancing model based on manufacturing difficulty is performed to achieve a support tool for decision in the field of analysis and synthesis tolerance. Moreover, design for manufacturing and assembly (DFMA) is involved while respecting functional requirements using the ICADTMD model. A case study shows that the proposed methodology widens the manufacturing dimension tolerances representing the difficulty source. As a result, the optimal tolerance using the proposed LMDC method reduces the total assembly cost considering manufacturing cost and quality loss. Thus, the proposed method is both economical and successful. Future works will focus on the consideration for genetic algorithm and geometrical tolerances. In addition, the implementation of the proposed approach in different industrial manufactories is also among the desired outlooks.

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