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ORIGINAL ARTICLE

A chatter-free path optimization algorithm based on stiffness orientation method for robotic milling

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Abstract

Robots used in machining processes are more prone to mode coupling chatter due to low rigidity and asymmetrical structure. In this paper, a new stiffness orientation method is proposed to optimize milling path and improve stability of the robotic milling system. First, the principal stiffness directions of the robot are determined through experimental or calculated modal analysis. Then, based on kinematics, the principal stiffness directions are projected onto the processing plane. Second, a mode coupling chatter stability criterion for robotic milling is obtained based on machining force analysis and system dynamics stability. Finally, a robotic milling path optimization algorithm based on the stiffness orientation process is proposed. Experiments were performed to verify the benefits of pre-optimizing the machining path to avoid mode coupling chatter and improve machining accuracy.

Keywords Robotic milling · Mode coupling chatter · Stiffness orientation · Path optimization algorithm

1 Introduction

For aerospace, marine, automotive, and other manufacturing applications, numerous drilling, grinding, milling, and other cutting operations are required at assembly sites [1]. Due to limitations associated with on-site assembly, traditional multi-axis machining centers often cannot adapt to the manufacturing and machining requirements of complex structural parts [2]. The flexibility and low cost of industrial robots have led to their gradual use in the machining industry and robots are now widely used in mechanical engineering and manufacturing according to the American Robotics Association (RIA) [3]. However, cutting forces generated during the machining process can cause vibrations in robotic machining systems and easily lead to chatter, which seriously affects machining accuracy [4].

Chatter is the mutual vibration between a machine tool and workpiece and greatly affects mechanical manufacturing systems [5, 6]. In robotic machining systems, chatter is a complex elastic dynamics phenomenon mainly caused by self-excitation of the overall robot structure as it absorbs energy from the periodic cutting forces produced during the machining process [7]. When chatter occurs, precision is seriously affected, and thus surface quality and chatter may even damage the tool or robot body [8]. Moreover, chatter marks appear on the surface of the workpiece. Unlike regenerative chatter produced by traditional machine tools, robots are more prone to mode coupling chatter due to their weak rigidity and asymmetric structure.

At present, chatter in robotic machining processes is gradually attracting attention and a number of studies have been conducted in this area. Pan et al. [9] demonstrated serious chatter in robotic milling experiments and found the chatter frequency was close to the low-order natural frequency of the robot. In addition, a prediction criterion for mode coupling chatter was presented by establishing a simplified milling dynamics model. Following Pan's work, Cen et al. [10] optimized a dynamic model by considering the influence of cutting force in the chatter prediction criteria of robotic milling. Optimizing the feed rate of robotic milling was experimentally shown to suppress chatter. In addition, actively suppressing chatter may enable robotic milling to be used in various processing applications. To reduce chatter, Chen et al. [11] proposed a fuzzy-sliding mode force control that adjusts robot feed rate. Furthermore, Wang et al. [12] used an ARM-based

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microcontroller to control the cutting force, thereby improving the stability of a robotic cutting process.

However, in the previous study, there is no answer about how to get the principal directions of the robot, which affects the machining stability and is not easy to determine due to robotic asymmetry. Moreover, passive adjustment of cutting parameters has not been widely adopted since production efficiency is usually sacrificed. Active suppression requires additional auxiliary devices and is therefore not cost effective.

In this paper, for the first time, a stiffness orientation method combining modal shape and robot kinematics is proposed. Furthermore, a chatter-free milling path optimization algorithm is presented by choosing appropriate feed directions based on the stiffness orientation method. Henceforth, the paper is organized as follows: In Section 2, machining force in the robotic milling process is analyzed. In Section 3, the mode coupling chatter mechanism is shown. Since the robot itself has an asymmetrical structure, a stiffness orientation method using modal analysis and robot kinematics is presented in Section 4, and then the stiffness direction in the machining plane is obtained through a coordinate transformation process. In Section 5, the robotic milling chatter-free path optimization algorithm is presented. Finally, in Section 6, a case study is used to verify the benefits of pre-optimizing the machining path to avoid mode coupling chatter and improve machining accuracy.

2 Machining force analysis

During the milling process, machining force F is generally divided into tangential force F_t and radial force F_r, and fluctuates in different directions during machining, as shown in Fig. 1.

In general metal material milling process, the tangential cutting coefficient k_t is greater than the radial cutting force coefficient k_r. Therefore, the influence of radial force can be neglected in the model and the stability criterion of the robotic milling process can be obtained.

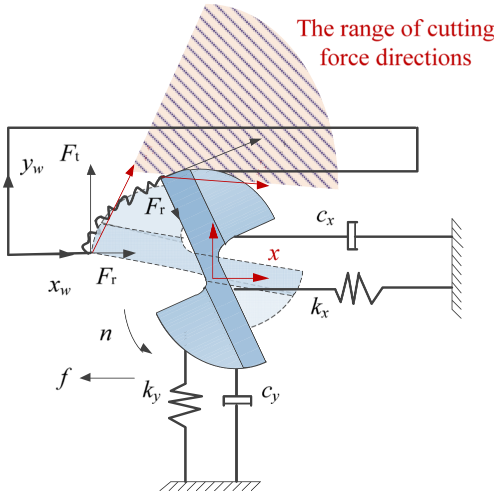


Fig. 1 Model for analyzing cutting force of two-flute end mill

3 Mode coupling chatter mechanism of robotic milling

Mode coupling chatter is a self-excited vibration phenomenon that occurs in multi-degree-of-freedom systems and is caused by simultaneous oscillations of different amplitude and phase. Machining force is the source of vibrational energy and chatter frequency is close to the low-order natural frequency of the robotic system. Under these conditions, elliptical chatter marks are left on the surface of the workpiece.

3.1 Dynamic modeling of robotic milling

To analyze mode coupling chatter during the milling process of a six-degree-of-freedom (6-DoF) robot, the coordinate system is first established, as shown in Fig. 2. Moreover, we made the following assumptions to simplify the analysis [13]: (1) Damping effects increase the stability of the system; therefore, to reduce complexity, an undamped system is assumed; (2) Machining force F is proportional to depth of cut b. (3) In general, the radial cutting force is smaller than the tangential cutting force. In order to simplify the model, the radial cutting force can be ignored. The main consideration is force F which acts in the opposite direction of the feed during the machining process.

Based on the above assumptions, an undamped two-degree-of-freedom (2-DoF) dynamic model was established, as shown in Fig. 2. The end mill rotates counterclockwise to feed the cutting tool along f.

To establish the dynamic model, $F = K_p y$, where F is machining force, y represents the vibration displacement in the Y direction, and K_p is process stiffness, which depends on the material of the workpiece, current WOC, feed speed, number of teeth, etc. Based on this, an undamped kinetic equation with two degrees of freedom can be defined as.

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 0 & K_p \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} \tag{1}$$

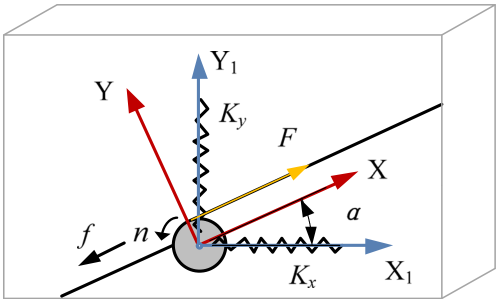
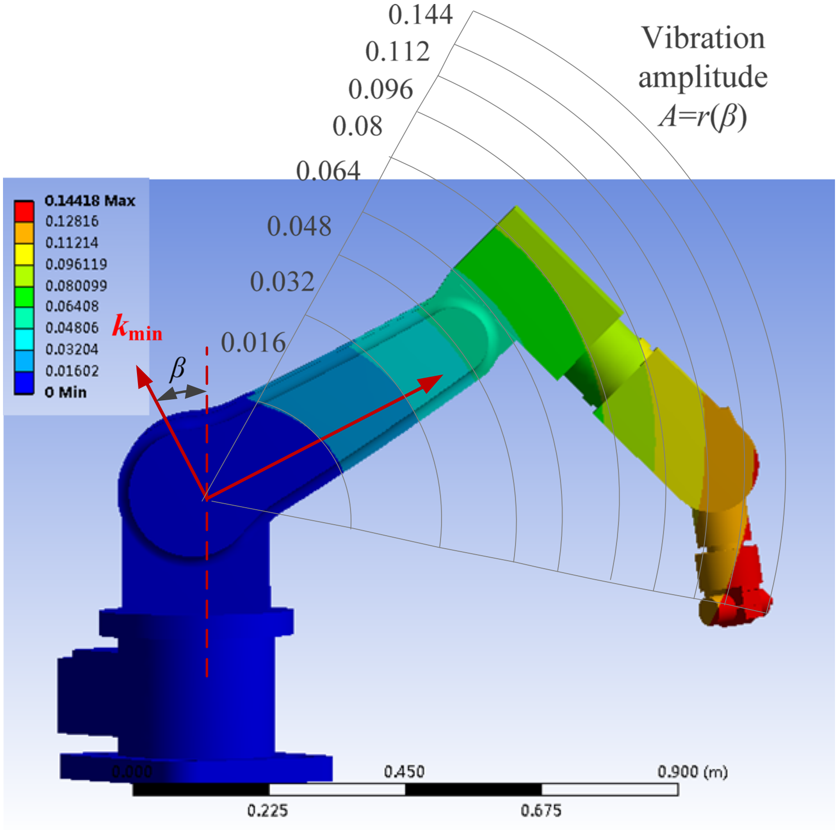


Fig. 2 Two-degree-of-freedom dynamic model of the robot

Fig. 3 Modal shape of the industrial robot



3.2 Stability analysis review

According to reference [11], the solution of Eq. (1) must be decoupled. To simplify Eq. (1), we can transform the differential equation in the X-Y coordinate system into an equation in the X_1-Y_1 coordinate system through a rotational transformation. The new equation can be derived as follows:

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \end{bmatrix} + \begin{bmatrix} K_x & 0 \\ 0 & K_y \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} -K_{\mathrm{P}}\cos\alpha\sin\alpha & K_{\mathrm{P}}\cos^2\alpha \\ -K_{\mathrm{P}}\sin^2\alpha & K_{\mathrm{P}}\sin\alpha\cos\alpha \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \tag{2}$$

Then, the characteristic equation is

$$\lambda^4 + \frac{K_x + K_y}{m}\lambda^2 + \frac{K_xK_y + \left(K_y - K_x\right)K_{\mathrm{P}}\sin\alpha\cos\alpha}{m^2} = 0 \tag{3}$$

For which the solution is

$$\lambda^2 = \frac{-\left(K_x + K_y\right) \pm \sqrt{\left(\Delta K\right)^2 + 2\Delta K K_{\mathrm{P}}\sin\left(2\alpha\right)}}{2m} \tag{4}$$

where $\Delta K = K_x - K_y$, assuming $\Delta K > 0$ (the y direction is the degree of freedom with the smallest stiffness) and the eigenvalues of Eq. (3) can be analyzed. If $\Delta K < 0$, the conclusion is the opposite of the following analysis.

If $\sin(2\alpha) > -(\Delta K/2K_{\mathrm{p}})$, the λ^2 values of Eq. (4) are real negative numbers and the four eigenvalues of the solution are two sets of imaginary numbers symmetrical about the real axis. According to the bounded-input bounded-output

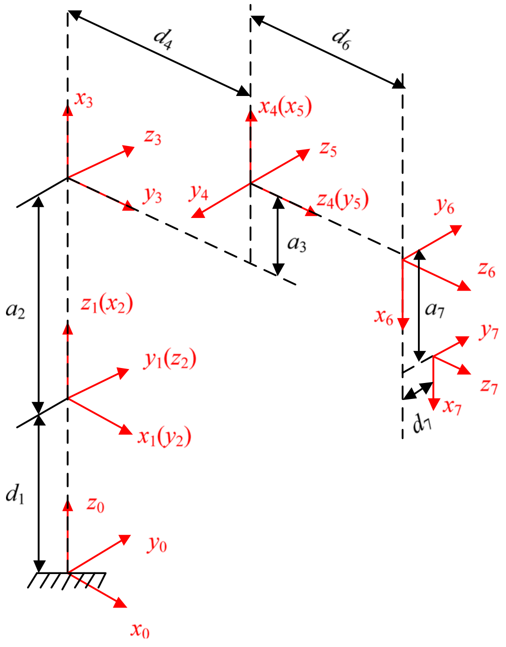


Fig. 4 Coordinate system of the industrial robot

Table 1 D-H parameters of ABB IRB 1200 industrial robot

i	$\alpha_{i\text{-}1}$	$a_{i\text{-}1}$	θ_i	d_i	Joint variable	Link parameter (mm)
1	0	0	0	d_1	θ_1	$a_2 = 350$
2	− 90°	0	90°	0	θ_2	$a_3 = 42$
3	0	a_2	0	0	θ_3	$d_1 = 399.1$
4	− 90°	a_3	0	d_4	θ_4	$d_4 = 351$
5	90°	0	0	0	θ_5	$d_6 = 82$
6	− 90°	0	180°	d_6	θ_6	

(BIBO) system stability criterion, the system is stable. Conversely, if $\sin(2\alpha) < -(\Delta K/2K_p)$, the system is unstable.

4 Stiffness orientation process

Based on the chatter stability criterion, we know that the stability of the system depends on angle α, structural stiffness K_x and K_y, and process stiffness K_p. In this model, K_x and K_y are determined by the robot structure and posture and K_p is constant under a certain cutting setup. Angle α depends on the machining force direction and stiffness direction of the robot in the machining plane, which will form the basis of the proposed machining path optimization.

In milling applications, an electric spindle and milling cutter must be added at the end of the robot flange. To perform the modeling and analysis, stiffness directions of the robot in the machining plane must first be obtained. However, the principal stiffness directions are not easy to get due to robotic asymmetry. Then, we figure out that the results of the modal analysis can combine with robot kinematics to obtain the stiffness directions in the machining plane.

The stiffness orientation method for robotic milling is summarized below (see Fig. 7):

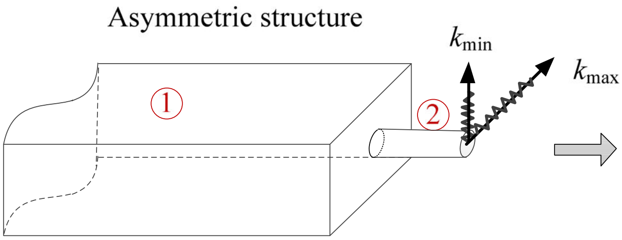


Fig. 5 Stiffness of asymmetrical structure

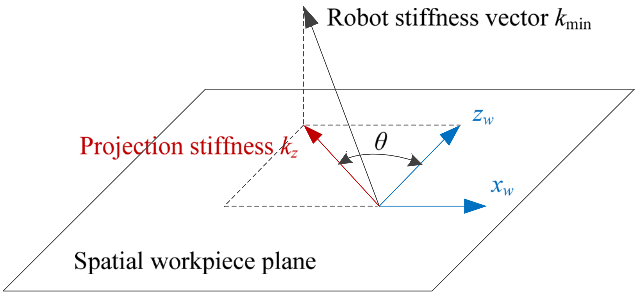


Fig. 6 Coordinate transformation process

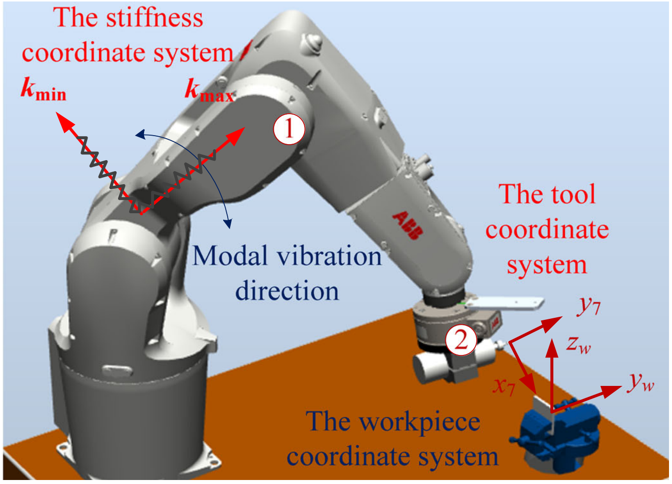
(1) Determine the robot's machining posture and workpiece coordinate system, establish a robot kinematics model, and obtain a homogeneous transformation matrix.

(2) A modal experiment is performed to obtain a low-order mode shape, which is the direction of energy concentration. This direction is defined as the principal stiffness direction of the robot and is expressed in kinematics, depending on the six joint angles. We can also obtain the mode shape by finite element analysis, which is consistent with the experimental results.

(3) Converting the principal stiffness direction of the robot into the machining plane to obtain the principal stiffness direction.

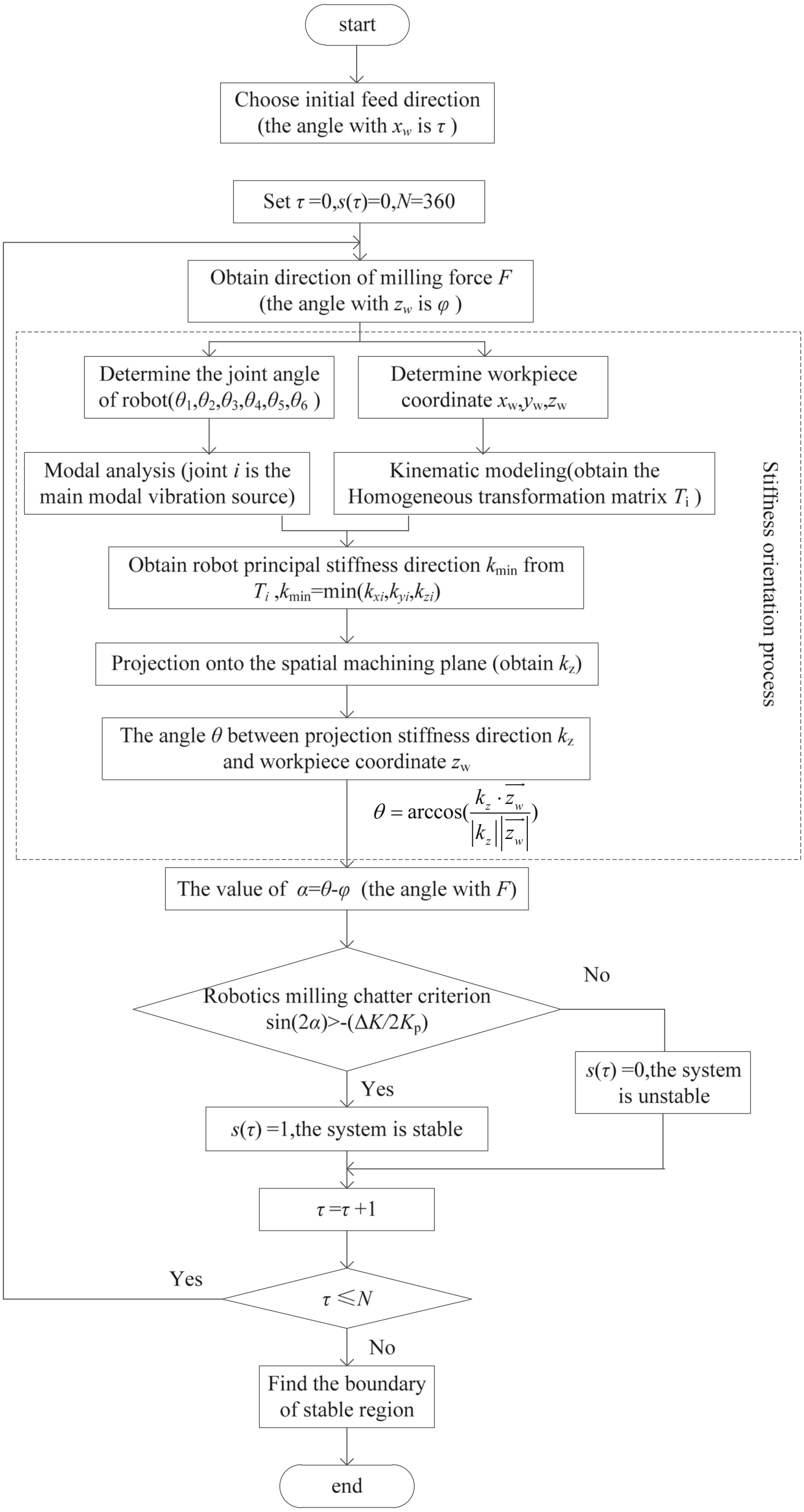
4.1 Modal analysis

Robots have a variety of options for installation, such as floor mounted and wall mounted. There are different postures in a different machining process, which usually lead to structural asymmetry. Due to these two reasons, the principal stiffness direction of the robot is undetermined. Therefore, modal analysis is needed to find the energy concentration direction, and then determine the principal stiffness direction.



Two types of methods are commonly used for modal analysis: calculated (finite element analysis) and experimental. Each order corresponds to one modality and has its own specific frequency, damping, and modal parameters. The low-order modal shape of the industrial robot (ABB IRB1200) used in this study was obtained through modal experiments, which is described by the result of using ANYSYS workbench 14.5. The mesh generation is performed by using the

Fig. 7 Flowchart of path generation algorithm



solid element. Because the robotic structure is complex, the tetrahedral mesh is adopted. The model is meshed into 9285 elements, 17,707 nodes. When simulating, the joints are set to rigid connection. The modal shape is joint 2 pitching motion under a certain robot posture, as shown in Fig. 3. The amplitude of vibrations increases with radius until the tool end reaches the maximum value. The principal stiffness direction is defined as $k_{\min}$. In addition, when the robot pose changes slightly, it will not affect the mode shape.

4.2 Kinematic modeling of robot movement

Robot kinematics usually refers to movements of the robot links, which consists of rotational and translational motion. The D-H method, proposed by Denavit and Hartenberg in 1956 [14], is frequently used to establish coordinate systems for robotic links. Here, a coordinate system for each link is established and the relationship between each link is defined. Thus, a homogeneous transformation matrix from the base to the end of the robot can be obtained.

Two versions of the D-H method exist depending on whether the modified or standard coordinate system is used. In this paper, a kinematic model of the ABB IRB1200 robot is established using the modified coordinate system, as shown in Fig. 4, where x_7-y_7-z_7 is the tool coordinate system and a_7 and d_7 describe the distance from the robot end flange.

The D-H parameters of the robot obtained in the established coordinate system are listed in Table 1.

4.3 Tool coordinate system

The homogeneous transformation matrix between the i-1 joint and i joint coordinate system of the robot can be represented as

$$ {}^{i-1}_{i}T = Rot(x_{i-1}, \alpha_{i-1})\cdot Trans(x_{i-1}, a_{i-1})\cdot Rot(z_i, \theta_i)\cdot Trans(z_i, d_i) \quad (5) $$

The tool coordinate system of the robot is (x_7, y_7, z_7), as shown in Fig. 4. Position of the milling cutter can be obtained in the base coordinate system via the robot kinematics homogeneous transformation matrix:

$$ T_7 = {}^0_1T\cdot{}^1_2T\cdot{}^2_3T\cdot{}^3_4T\cdot{}^4_5T\cdot{}^5_6T\cdot{}^6_7T = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6) $$

4.4 Stiffness coordinate system of the robot

Robots differ from traditional machine tools since their structural stiffness is usually less than 1 N/μm due to series assembly, speed reducers, etc. The ABB IRB1200 industrial robot has six single-axis rotational joints represented by six degrees of freedom, and vibrations of the various robot poses are different. Low-order vibration modes are the main cause of mode coupling chatter. In this paper, the direction of the modal vibration under a certain posture is defined as the principal stiffness directions of the robot vibration under this posture.

The industrial robot is a complex asymmetrical 6-DoF structure; therefore, changes in position and posture cause the direction of the principal stiffness to vary. In addition, when the robot is used in the milling process, it is necessary to install the spindle, fixtures, milling tool, force sensor, and other devices. Thus, an important task in robot mode coupling chatter analysis is determining the stiffness directions of the robot.

In Fig. 5, Structure 1 represents an asymmetrical structure and Structure 2 is a model representing the direction of stiffness to be determined. The overall structure still takes Structure 1 as the main body. With analog robotic milling devices, the robot body is the main structure of the robotic milling system and the spindle-cutter system is the additional robot structure. Moreover, the principal stiffness directions of the robot body are the principal stiffness directions of the robotic milling system.

To define the principal stiffness directions of the robot, the low-order natural frequency is obtained through modal analysis of the robot during the machining position in a particular posture. The corresponding modal vibration directions of the

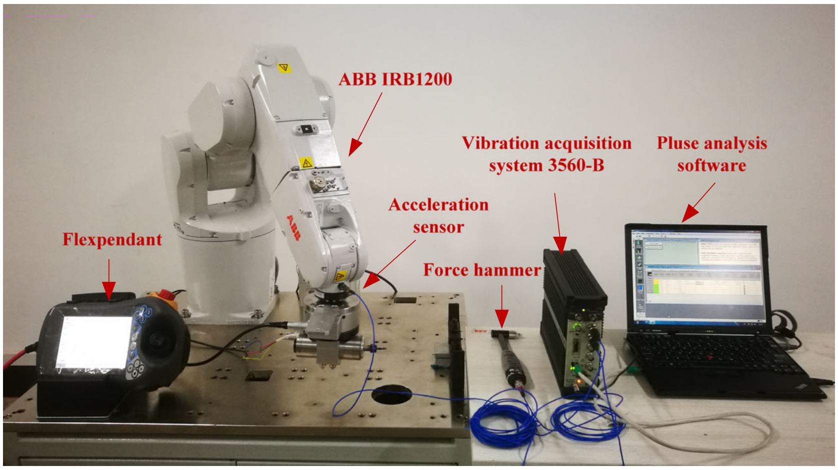


Fig. 8 Modal test equipment for ABB IRB1200 industrial robot

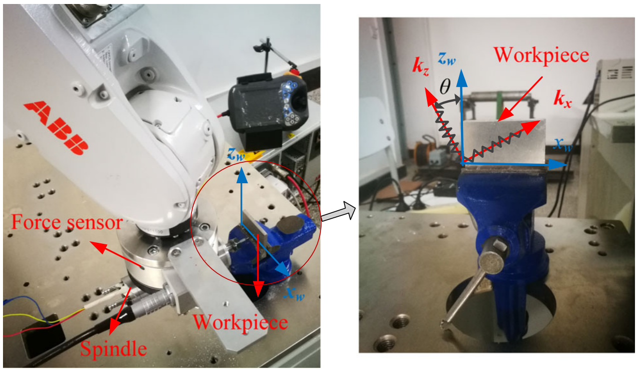


Fig. 9 Coordinate transformation process

robot are the principal stiffness directions of the robot. Using the modal shape obtained through modal analysis, the principal stiffness directions $k_{\max}$ and $k_{\min}$ of the robot (Structure 1) can be defined. The modal shape is assumed to be the pitching vibration of Joint 2, as shown in Fig. 3. Therefore, the principal stiffness directions of the robot in the machining posture can be derived from matrix T_2, which is given by

$$T_2 = {}^0_1T \cdot {}^1_2T = \begin{bmatrix} c_2 & -c_1s_2 & -s_1 & 0 \\ c_2s_1 & -s_1s_2 & c_1 & 0 \\ -s_2 & -c_2 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

where $c_1 = \cos(\theta_1)$, $c_2 = \cos(\theta_2)$, $s_1 = \sin(\theta_1)$, $s_2 = \sin(\theta_2)$, and d_1 is the length of robot link 1. Then the low stiffness direction $k_{\min}$ of the robot is

$$k_{\min} = \left\{ k \middle| min\left(|k_{x2}|, |k_{y2}|, |k_{z2}|\right) \right\} = \left[-c_1s_2, -s_1s_2, -c_2\right]^T \quad (8)$$

4.5 Coordinate transformation process

In Section 4.1, the stiffness directions of the robot can be obtained. However, to analyze the stability of mode coupling chatter, principal stiffness directions of the robot must be determined in the machining plane. The machining stiffness direction k_z (transformed stiffness) can then be defined, as shown in Fig. 6.

Milling cutter is known to perpendicular the spatial machining plane. Then, the unit normal vector of the plane is derived from T_7:

$$n = T_7(1:3,3) = \overrightarrow{z_7}(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6) \quad (9)$$

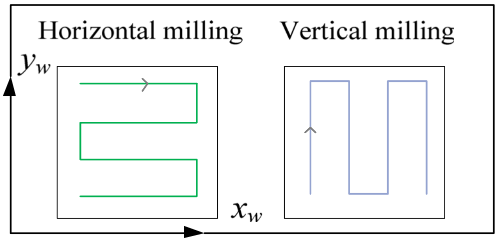


Fig. 10 Plane milling path

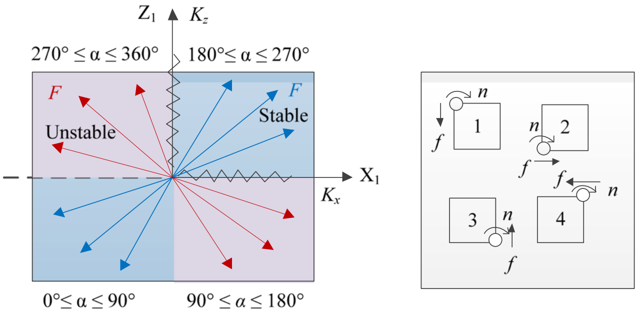


Fig. 11 Chatter analysis associated with α angle

The projection vector of the robot stiffness vector on the spatial machining plane is

$$k_z = k_{\min} - (k_{min} \cdot n)n \quad (10)$$

The angle between the mapped vector in the machining plane and workpiece coordinate is θ, which is given by

$$\theta = \arccos\left(\frac{k_z \cdot \overrightarrow{z_w}}{|k_z| |\overrightarrow{z_w}|}\right) \quad (11)$$

5 Path optimization algorithm

According to the established coordinate system and mode coupling chatter criterion, an algorithm for robotic milling path optimization can be derived, as shown in Fig. 7. First, the modal shape, which is the vibration of joint *i*, is obtained using a modal test or finite element analysis. Second, the principal stiffness direction of the robot is represented by a vector associated with the robot kinematics. Then, the vector is transformed into a new vector under the workpiece coordinate system, with angle θ between the mapped vector in the machining plane and workpiece coordinate. Therefore, the relationship between the analytical coordinate system and stiffness coordinate system can be described by α. Finally, the mode coupling

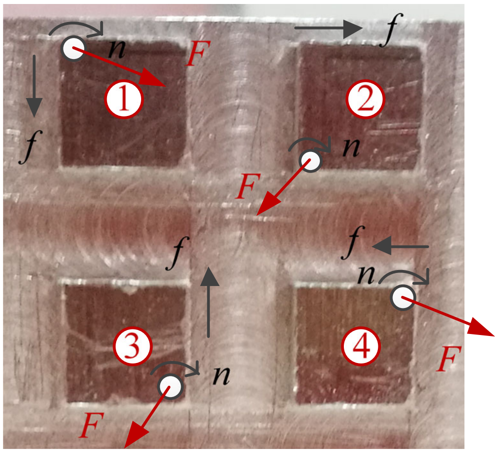


Fig. 12 Four machining directions used in milling experiments

chatter criterion is used as the basis of the algorithm, to avoid chatter and optimize the milling path of the robot.

A summary of the above procedure is as follows:

- Step 1: Determine the initial feed direction, expressed by the angle τ. Determine the angular increment, which is set to 1 in this paper.
- Step 2: Perform the stiffness orientation process proposed in Section 4 to obtain the principal stiffness direction in the machining plane.
- Step 3: Determine the stability of robotic milling based on the modal coupling chatter criterion in Section 3.
- Step 4: Repeat the steps until the stability of each feed direction in the machining plane is obtained, then the boundary is detected to form a stable machining area.

6 Case study

6.1 Modal testing of the industrial robot

The identification of natural vibration characteristics of the robot (also known as modal analysis) includes identifying natural frequencies, damping ratios, and modal shapes. The natural vibration characteristics of the structure reflect the actual vibration response generated by various sources of vibration within a certain frequency range. This method is often used to study structural dynamics and in this paper, the aim is to obtain the stiffness directions of the robot. The experimental setup included a three-axis acceleration sensor (356A24, PCB Piezotronics Sensor Technology Co., Ltd., China), force hammer (086C01, PCB), vibration acquisition system (3560-B), and pulse analysis software, as shown in Fig. 8. The low-order natural frequency was found to be 22 Hz and the modal shape was Joint 2 pitching motion.

6.2 Transformation process

Through the above analysis, the low-order natural frequency and modal shape were obtained; thus, the principal stiffness directions of the robot can be represented. Then, the stiffness in the machining plane can be defined using a transformation process. The robotic machining posture was found to be 35.87°, 66.26°, − 14.76°, 0.39°, 38.42°, and 37.6°. In the above analysis, the weak stiffness direction of the robot $k_{\min} = (0.45, 0.30, 0.84)$, stiffness k_z in the machining plane makes an angle θ with the workpiece in the z_w-direction, and $\theta = 28°$ (Fig. 9).

6.3 Plane milling

Plane milling is a common machining application. However, mode coupling chatter stability will vary depending on the machining path. To find the appropriate milling path for improving machining stability and accuracy, the feed direction of the milling

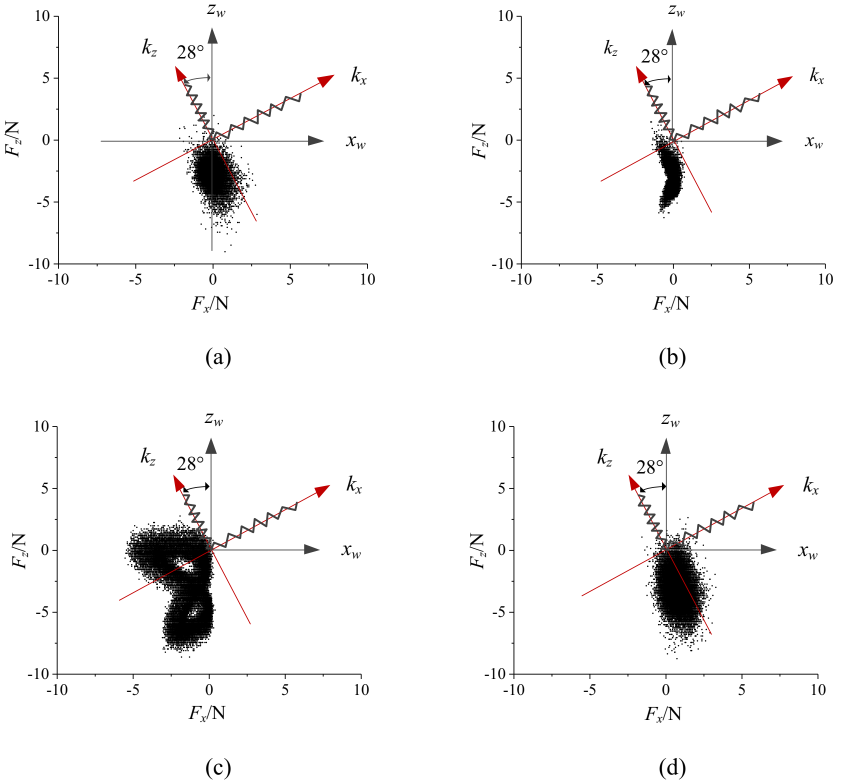
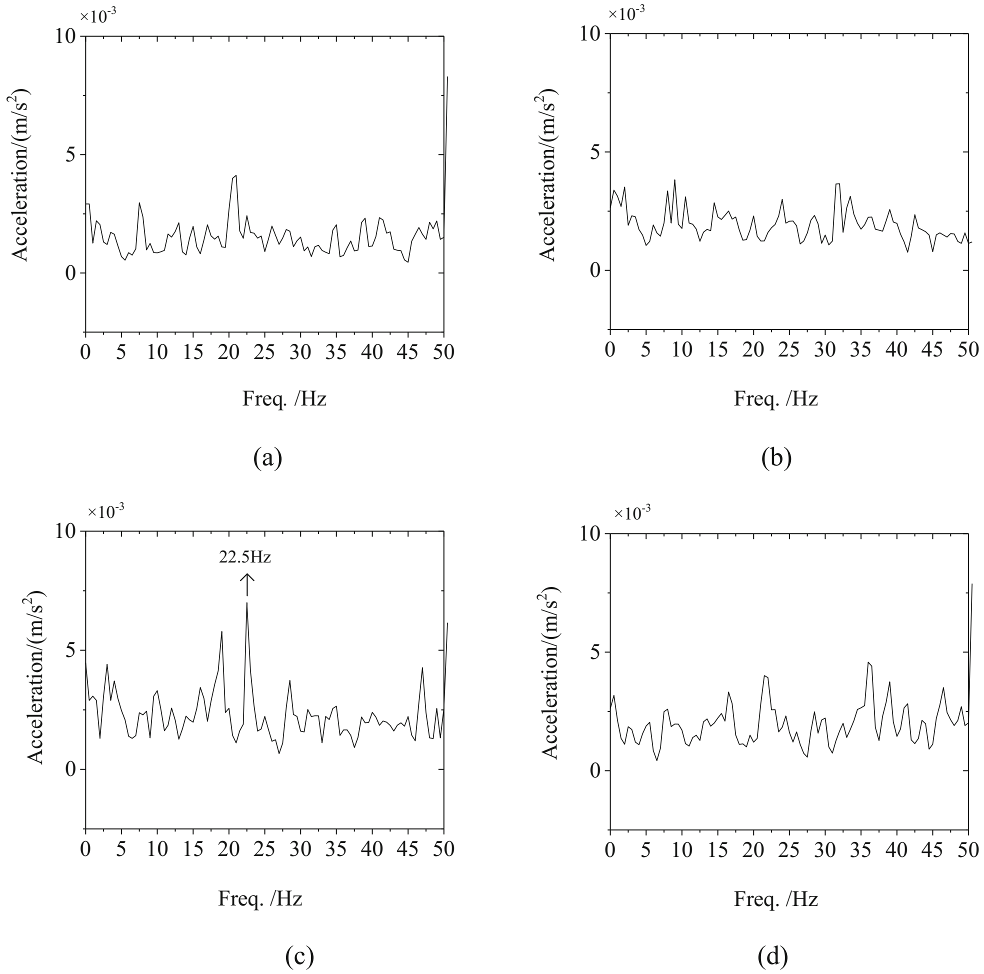


Fig. 13 Cutting forces of half-slot milling based on four different machining paths: **a** direction 1, **b** direction 2, **c** direction 3, and **d** direction 4

Fig. 14 Frequency spectrums of half-slot milling based on four different machining paths: **a** direction 1, **b** direction 2, **c** direction 3, and **d** direction 4



process is planned. Figure 10 shows two common milling plane machining paths for horizontal and vertical milling.

6.4 Stability criterion analysis

According to the analysis, the choice of feed direction depends on α, which affects the stability criterion of the mode coupling chatter. The range of α is 0 to 360°. According to modal analysis, stiffness in the X-direction was found to be greater than stiffness in the Z-direction. As shown in Fig. 11, the principal stiffness directions of the robot in the machining plane are K_x and K_z. When $\alpha \in [0^\circ\ 90^\circ] \cup [180^\circ\ 270^\circ]$, then $\sin(2\alpha) + (\Delta K/2K_p) > 0$. Therefore, chatter does not occur within this range. When $\alpha \in [90^\circ\ 180^\circ] \cup [270^\circ\ 360^\circ]$, chatter may occur depending on other parameters such as K_p. The system was found to be stable in areas where cutting forces represented by blue arrow (shown in Fig. 11) are located, but chatter may also occur in the area where the red cutting forces are located. Therefore, mode coupling chatter may occur when the cutting forces are located in the second and fourth quadrants of the stiffness coordinate system in the machining plane.

Based on modal analysis of the robot, the principal stiffness directions of the robot are $k_{\max}$ and $k_{\min}$. To meet modeling requirements, the principal stiffness directions of the robot were mapped onto the machining plane. Full-slot and half-slot milling were performed according to four different machining paths, as shown in Fig. 11. The direction of the cutting force is F. In plane milling, directions 1 and 3 correspond to the vertical milling case, whereas directions 2 and 4 represent the horizontal milling case. The effect of pose variation on

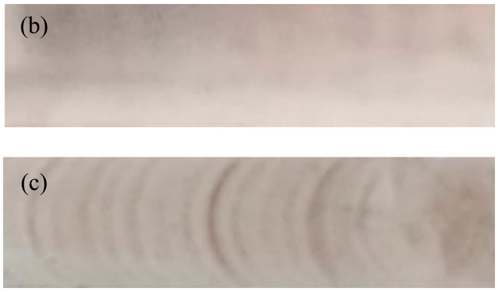


Fig. 15 Machined surface of the workpiece during **b** stable milling and **c** chatter

natural frequency and modal shape were ignored in a short milling path. Under the same milling machining parameters, stiffness directions of the robot during milling were found to seriously influence milling chatter stability. Therefore, stiffness orientation in robotic machining plane is a prerequisite for ensuring the stability of robotic milling.

6.5 Half-slot milling experiment

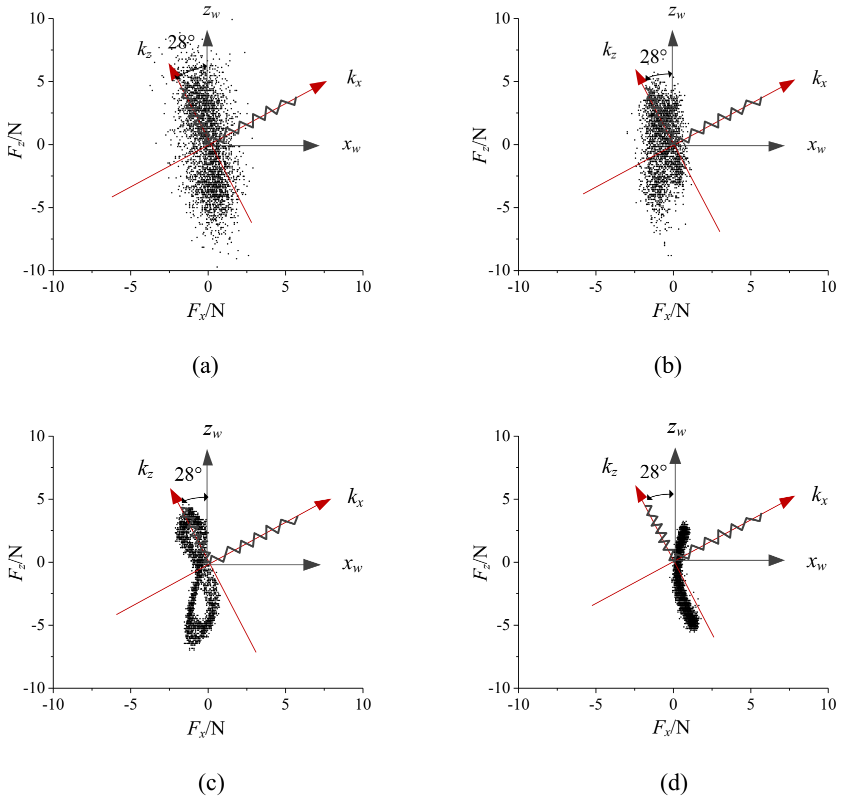
The ABB IRB1200 industrial robot was used to carry out milling experiments under different paths, as shown in Fig. 11. The spindle rotation speed was set to 16,000 rpm, depth of cut was 0.1 mm, and feed speed was 30 mm/min. A triaxial force sensor (ABB, China) was used to measure cutting force and a triaxial acceleration sensor (356A24, PCB) was used to acquire the vibration acceleration signals of the robot. Based on the analysis, mode coupling chatter may occur when cutting force directions are located in the second and fourth quadrants of the stiffness coordinate system in the machining plane.

Half-slot milling was performed in four directions on an aluminum alloy (Al1060) workpiece, as shown in Fig. 12. The diameter of the double edge milling cutter was 2 mm and milling forces were measured in three different directions. The machining plane is the *x*-*z* plane and cutting forces in *x*- and *z*-directions were plotted to observe the relationship between cutting force distribution and weak stiffness direction of the robot, as shown in Fig. 13.

As seen in Fig. 13, machining in direction 3 results in cutting forces distributed in the second and third quadrants of the stiffness coordinate system and mode coupling chatter may occur. The distribution of the cutting forces in direction 3 is larger, suggesting that direction 3 produces larger cutting forces than the other three machining directions under the same cutting conditions. In addition, the cutting forces of direction 3 follow an elliptical trajectory.

In order to identify the mode coupling chatter, a low-frequency analysis is made of the frequency measured in the range of 0–50 Hz. The sample rate is 200 samples/s. Meanwhile, three liner averages method is used to acquire signals and the sample time is 2 s. Acceleration signals acquired during the cutting process are shown in Fig. 14. The peak of each frequency in direction 3 is larger. The main peak appears at 22.5 Hz, which is close to the natural frequency of the robot. This suggests that mode coupling chatter occurs in the vertical robotic milling system. The horizontal milling method is considered stable.

Figure 15 shows the machined surface of the workpiece between the stable and the chatter. It is clear for the stable milling that the machined surface is smooth with high quality but shows apparent vibration marks when chatter occurs.



(a) (b)

(c) (d)

Fig. 16 Cutting force of full-slot milling based on four different machining paths. **a** direction 1, **b** direction 2, **c** direction 3, and **d** direction 4.

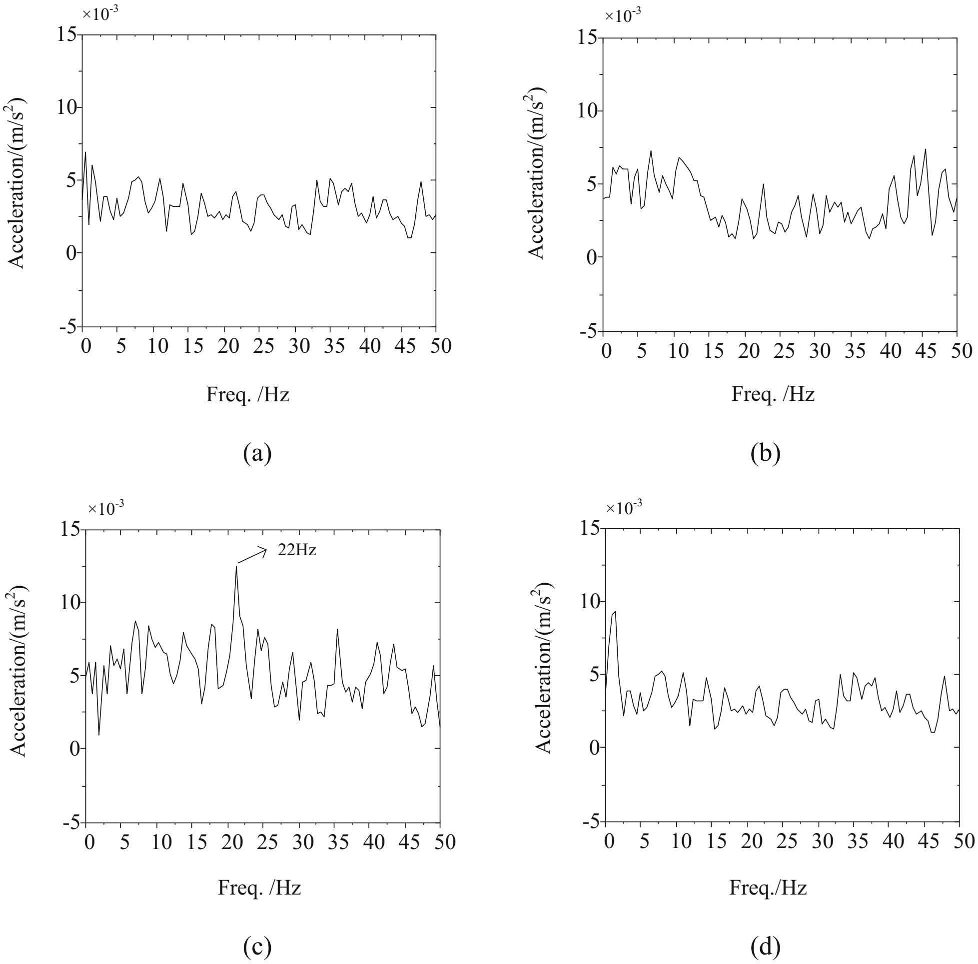


Fig. 17 Frequency spectrums of full-slot milling based on four different machining paths. **a** direction 1, **b** direction 2, **c** direction 3, and **d** direction 4.

6.6 Full-slot milling experiment

Similar to the half-slot milling experiment, double edge milling cutters with a diameter of 2 mm were used to perform milling experiments in four directions. The distribution of cutting force is presented in Fig. 16 and frequency domains of the acceleration signals are shown in Fig. 17.

The same trends were observed in full-slot milling and half-slot milling. Cutting forces in direction 3 are more distributed in the second quadrant and values are larger, moreover, the cutting force distribution again becomes elliptical. In the frequency domain, the peak frequency of the acceleration signals is at the natural frequency of the robot; therefore, stability of the robotic milling system is poor. Horizontal milling was shown to be better than vertical milling when performing slot milling for the same robot position and posture.

The results suggest that avoiding cutting force directions in the second and fourth quadrants can improve stability based on the orientation of stiffness in the machining plane. Thus, path planning using the stiffness orientation method can reduce cutting forces and avoid chatter during machining.

7 Conclusions

In this paper, robotic milling process was modeled and analyzed to simplify the 2-DoF dynamic model. Mode coupling chatter often occurs during the milling process due to the effects of machining forces caused by the poor rigidity and asymmetric structure. Introducing modal analysis and robot kinematics, a new stiffness orientation method was proposed. Further, a path optimization algorithm was presented to select appropriate feed directions. Then an optimization rule for robotic milling was proposed that is to avoid machining force directions locating in the second and fourth quadrants of the stiffness coordinate system. Finally, experiments showed smaller machining forces and improved system stability obtained using the stiffness orientation optimization criterion for selecting milling paths.

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References

1. Odf R, BURLEY G, NAING S (2001) Error budgeting for assembly-centric design of aerostructures. J Am Inst Aeronaut Astronaut 9:33–43
2. Zhanxi W, Xiaoyu Z, Feifei LI et al (2017) Review on the research developments of robot machining system and cutting chatter behaviors (in Chinese). Journal of Vibration and Shock 36:147–155
3. DePree J. and C. Gesswein (2008) Robotic machining white paper project. Halcyon Development Robotic Industries Association (RIA)
4. Liu Y, Wang Z, Liu K, Zhang YM (2017) Chatter stability prediction in milling using time-varying uncertainties. Int J Adv Manuf Technol 89(9–12):2627–2636
5. Tlusty J, Polacek M(1963)The stability of machine tools against self-excited vibrations in machining. International research in production engineering, ASME
6. Liu Y, Meng L, Liu K, Zhang YM (2016) Chatter reliability of milling system based on first-order second-moment method. Int J Adv Manuf Technol 87(1–4):801–809
7. Shi H(2018) A summary of the theory on machine tool chatter. Metal cutting theory. Springer, Cham:231–267
8. Merritt HE (1965) Theory of self-excited machine-tool chatter: contribution to machine-tool chatter research. ASME J Eng Ind 87: 447–454
9. Pan Z, Hui Z et al (2006) Chatter analysis of robotic machining process. J Mater Process Technol 173(3):301–309
10. Cen L, Melkote SN (2017) CCT-based mode coupling chatter avoidance in robotic milling. J Manuf Process 29:50–61
11. Chen S, Zhang T, Zou Y (2017) Fuzzy-sliding mode force control research on robotic machining. J Robot 2017:8 https://doi.org/10.1155/2017/8128479
12. Wang Z, Keogh P (2017)Active vibration control for robotic machining. ASME 2017 International Mechanical Engineering Congress and Exposition. American Society of Mechanical Engineers:V002T02A079-V002T02A079
13. Gasparetto A (1998) A system theory approach to mode coupling chatter in machining. ASME J Dyn Syst Meas Control 120:545–547
14. Denhavit J, Hartenberg R S(1955) A kinematic notation for lower-pair mechanisms based on matrices. ASME J Appl Mech