The International Journal of Advanced Manufacturing Technology (2019) 100:2007–2020
https://doi.org/10.1007/s00170-018-2738-7

ORIGINAL ARTICLE

Optimization method of tool axis vector based on kinematical characteristics of rotary feed axis for curved surface machining

Jian-wei Ma [1] · Guo-qing Hu [1] · Feng-ze Qin [1] · Wei-wei Su [1] · Zhen-yuan Jia [1]

Received: 3 April 2018 / Accepted: 17 September 2018 / Published online: 12 October 2018

Abstract

Complex surface parts are widely used in the industrial applications, and 5-axis NC machining with ball-end cutter is the commonly adopted method for curved surface parts. Due to the additional rotary feed axis comparing with 3-axis NC machining, the tool orientation control is complex for curved surface machining. With the more complexity of curved surface parts, it is a known problem that the large incoherent movement of the rotary feed axis will easily appear in curved surface machining, which may even be beyond the kinematical performances of the rotary feed axis in machine tool, so as to affect the machining quality of curved surface parts. In order to overcome this issue, an optimization method of tool axis vector based on the kinematical characteristics of the rotary feed axis for curved surface machining is proposed. Firstly, the optimizing interval of the toolpath for tool axis vector is selected based on the relationship between the rotation angle of rotary feed axis and the accumulation arc length of toolpath. Then, the equalization method of tool axis vector based on the quaternion method is used to optimize the tool axis vector with the kinematical characteristics of rotary feed axis. Finally, the angular change curve of rotary feed axis with the optimized angular value and cumulative arc length is adjusted by the principle of least-squares fitting after the local optimization of tool axis vector. Simulation and experiment on test parts are carried out to verify the validity of the proposed method, and the achievements are significant to improve the processing quality of complex curved surface parts.

Keywords Curved surface · Tool axis vector · Kinematical characteristics · Rotary feed axis · Angular velocity · Angular acceleration

1 Introduction

Complex surface parts are widely used in aerospace, shipbuilding, mold, and other fields. With the increasing requirements on the performance of high-end equipment, the local structural features of key parts and components become more and more complicated. For example, the shape of blades, impellers, and propellers is complex with the curvature changing greatly. And the traditional 3-axis NC machining cannot meet the processing requirements, in which case 5-axis NC machine tool has been widely developed. Compared with the 3-axis NC machine tool, 5-axis NC machine tool has two additional rotary freedoms. By programming tilt angles and yaw angles of the cutter relative to the workpiece during machining process, the cutting relationship between the cutter and the workpiece can be ensured and the processing efficiency can be improved. However, due to the two additional rotation axes, the control complexity of the tool axis vector is increased. Especially in the area for machining of the complex curved surface with sharp change curvature, the control of tool axis vector is more complex, which will easily lead to the tool axis vector change drastically in 5-axis NC machining. When the feed rate is too large, the kinematical characteristics for the rotary feed axis of machine tool may even be beyond the allowable range, which will directly affect the machining accuracy of the curved surface parts and the service life of the machine tool.

Usually, the tool axis vector is generated based on the surface geometry. For a complex curved surface with sharp change curvature, the adjacent tool axis vectors may cause a sudden change in kinematical characteristics. In order to make

✉ Jian-wei Ma
mjw2011@dlut.edu.cn

[1] Key Laboratory for Precision and Non-traditional Machining Technology of the Ministry of Education, School of Mechanical Engineering, Dalian University of Technology, Dalian 116024, China

the tool axis vector between adjacent tool contact positions change uniformly in 5-axis machining process and meet the kinematical characteristics of the rotary feed axis in the machine tool, the traditional method is just to reduce the feed rate to prevent the kinematical characteristics from the exaggerate, which to some extent reduces the processing efficiency. In this way, the planning of tool axis vector in the complex curve surface machining should not only meet the machining accuracy requirement but also avoid the exceeding of kinematical characteristics for the rotary feed axis. Meanwhile, the tool axis vector optimization method based on the kinematical characteristics of the rotary feed axis is also a hot and difficult point in the 5-axis NC machining.

There are lots of researches on the planning of tool axis vector in the 5-axis NC machining. To avoid the interference between tools and parts, Li et al. [1] proposed a novel generation method of interference-free inspection path for the impeller blades using an on-machine probe. By comparing the position relations between points in tool projection area and tool in tool coordinate system, Wang et al. [2] proposed a method to detect the tool interference based on the coordinate system transformation theory. Tang and Bohez [3] proposed a new collision avoidance strategy and combined it with the collision detection in 5-axis NC machining by using the bounding sphere algorithm. In toolpath planning, Yoon et al. [4] presented locally optimal cutting positions for cutting directions in the collision-free 5-axis milling of sculptured surfaces to optimize the toolpath based on the second-order approximations of machined strip width. Ho et al. [5] proposed a tool orientation smoothing couple with cutting error improvement method for controlling the nonlinear error to generate the toolpaths. Morishige et al. [6] applied a three-dimensional configuration space (C-space) and showed the relationship between all tool positions and postures and the existence of tool collision to generate the toolpath. Castagnetti et al. [7] proposed the domain of admissible orientation (DAO) method to optimize the rotation axis coordinate in the machine tool coordinate system, which could automatically avoid the singularity problem in post-processing and optimize the toolpath. Srijuntongsiri and Makhanov [8] proposed and analyzed a new numerical algorithm to reduce the kinematic errors of a 5-axis toolpath by using minimization of the variation of the rotation angles.

In order to optimize the tool axis vector, Choi et al. [9] established an analytic expression of tilt angle and yaw angle to optimize the tool axis vector. Grandguillaume et al. [10] presented a tool axis vector optimization method around the singular point based on B-spline curves. Li et al. [11] presented a new method for generating 5-axis toolpath with smooth tool motion based on the accessibility map (A-map) of the cutter at a point on the parts surface. Jun et al. [12] presented the methodology and developed the algorithm for optimizing and smoothing the tool orientation control in the C-space for 5-axis sculptured surface machining based on the machined surface error analysis. Lavernhe et al. [13] optimized the tool axis orientations to respect the high-speed machining. Geng et al. [14] analyzed the relationship between the nonlinear error and the rotation angle of the rotation axis, and proposed a rotation axis position optimization algorithm. For eliminating both local and rear gouging in the 5-axis machining, Li et al. [15] presented a cutter partition-based tool orientation optimization algorithm. Based on the improvement of the C-space method, Zhou et al. [16] proposed a safe space method that could generate continuous and uniform tool axis vectors before and after the interference region. In order to generate the interference-free 5-axis surface machining toolpaths, Wang and Tang [17] presented an approach to smooth the tool axis vector by directly involving the angular velocity limit in the search process. Luo et al. [18], considering the physical limit on velocity and acceleration of the rotary motions, presented a kinematics method to reduce the angular acceleration of the machine tool by optimizing the tool orientation between neighboring tool-workpiece contact points. In order to solve the problem of time-consuming computation in the direction of the tool axis, Wang et al. [19] proposed a new method for tool axis vector optimization based on the critical constraint, which could guarantee the smooth motion for the tool axis vector.

Based on the above, it can be seen that though the tool axis vectors are optimized, the previous researches are mainly focused on reducing the angle between adjacent tool axis vectors in the local coordinate system, which is rarely optimized under the machine coordinate system. Meanwhile, the kinematical characteristics of the machine tool, such as angular velocity and angular acceleration of the rotary feed axis, are less considered. Due to the extensive use of ball-end cutter in complex surface machining, any tool axis orientation does not affect the cutter-to-workpiece contact state and thus does not affect the machining quality of complex surface. In order to make the kinematical characteristics of rotary feed axis meet the constraints of machine tool without affecting the machining efficiency, a method is proposed to improve the motion characteristics of rotary feed axis by optimizing the tool axis vector in this study based on ball-end cutter, which can not only reduce angular velocity and angular acceleration of the rotary feed axis but also improve the processing quality of the curved surface with sharp change curvature. Firstly, the optimizing interval of the toolpath for tool axis vector is selected based on the relationship between the rotation angle of rotary feed axis and the accumulation arc length of toolpath. Then, the equalization method of tool axis vector based on the quaternion method is used to optimize the tool axis vector with the kinematical characteristics of rotary feed axis. Finally, the angular change curve of rotary feed axis with the optimized angular value and cumulative arc length is adjusted by the principle of least-squares fitting after the local optimization of tool axis vector. Simulation and experiment on test parts are carried out to verify the validity of the proposed method, and the achievements in this study are significant to improve the processing quality of complex curved surface parts.

The rest of this paper is organized as follows. In Section 2, the kinematical characteristics analysis of the rotary feed axis in machine tool is given. The optimization interval of tool axis vector is selected and the optimization method of tool axis vector is proposed in Section 3. Section 4 is the simulation and experiment results. Conclusions are summarized in Section 5.

2 Kinematical characteristics analysis of rotary feed axis in machine tool

In this section, the structure and the inverse kinematics of machine tool are introduced firstly, and then the calculation of angular velocity and angular acceleration for the rotary feed axis is given, which lay the basis for the optimization of tool axis vector in curved surface machining with the kinematical characteristics of rotary feed axis.

2.1 Structure and inverse kinematics of machine tool

A 5-axis NC machine tool has three moving axes and two rotating axes. In this study, the structure of the machine tool with a double rotary table of *A*-axis and the *C*-axis is shown in Fig. 1. The *A*-axis has a typical working range of + 30° to − 120°, and the *C*-axis can generally rotate to 360°. The advantages of NC machine tool with a double rotary workbench of *AC* axis are simple structure, good rigidity, and low manufacturing cost.

For the machine tool with a specific structure, the control method of the tool axis vector based on the kinematical characteristics of the rotary feed axis in the machining process is given in this section. In 5-axis NC machining, the toolpath is described in the CAM software by tool position files. Usually, the tool position files are expressed in workpiece coordinate system with six coordinate parameters, such as $\{\mathbf{P}, \mathbf{V}\} = \{x, y, z, i^*, j^*, k^*\}$. The first three represent the coordinates of the tool tip point $\mathbf{P}[x, y, z]^{\mathrm{T}}$ and the last three represent the direction of the tool axis vector $\mathbf{V}[i^*, j^*, k^*]^{\mathrm{T}}$. However, the tool position files generated by the CAM software cannot be directly recognized by the machine tool. The tool position files in workpiece coordinate system need to be converted into the machine coordinate system, so as to be recognized by the machine tool.

In this study, the ball-end cutter is used in machining. No matter how the tool axis vector is varied, the tool center is fixed that means that the cutting area does not change with the tool axis vector variation. In the machining process of curved surface, the cutter contact point $\mathbf{P_C}$ between the cutter and the curved surface is as shown in Fig. 2. $\mathbf{S}(u, v)$ is the machining curved surface and $\mathbf{r}(\xi)$ is a machining toolpath. At the cutter contact point $\mathbf{P_C}$ along the toolpath, $\mathbf{N}$ is the unit normal vector of the curved surface and $\mathbf{V}$ is the unit tool axis vector. According to the cutter tip point $\mathbf{P}$, the coordinates of cutter contact point $\mathbf{P_C}$ can be obtained as

$$\mathbf{P}_C = \mathbf{P} + R{\cdot}\mathbf{V} - R{\cdot}\mathbf{N} \tag{1}$$

In which, R is the radius of the ball-end cutter.

For the given surface $\mathbf{S}(u, v)$, the geometric information of unit normal vector $\mathbf{N}$ and unit tangent vector $\mathbf{T}$ can be calculated as

$$\begin{cases} \mathbf{N} = \dfrac{\mathbf{S}_u(u,v) \times \mathbf{S}_v(u,v)}{|\mathbf{S}_u(u,v) \times \mathbf{S}_v(u,v)|} \\ \mathbf{T} = \dfrac{\mathbf{r}'(\xi)}{|\mathbf{r}'(\xi)|} \end{cases} \tag{2}$$

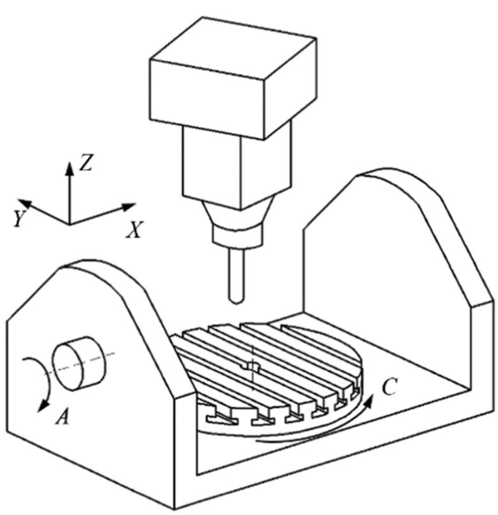


Fig. 1 Structure of 5-axis NC machine tool

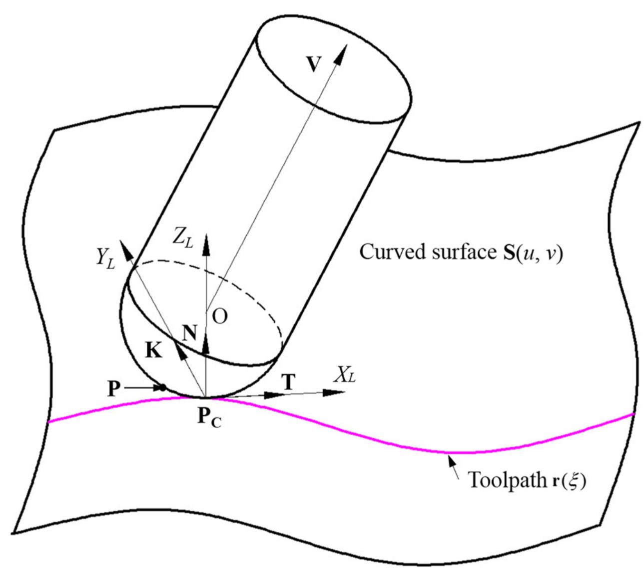


Fig. 2 Machining surface and local coordinate system

in which $\mathbf{S}_u(u, v)$ and $\mathbf{S}_v(u, v)$ are the first derivatives of the surface. $\mathbf{r}'(\xi)$ is the first derivative of a specific toolpath on the curved surface. $\mathbf{K} = \mathbf{N} \times \mathbf{T}$ is the cross product of unit normal vector and unit tangent vector at the cutter contact point.

In Fig. 2, the local coordinate system $P_C X_L Y_L Z_L$ is established by setting $\mathbf{T}$, $\mathbf{K}$, and $\mathbf{N}$ as the directions of X_L, Y_L, and Z_L at the cutter contact point $\mathbf{P_C}$. For the tool axis vector $\mathbf{V}$ in the machining process, the rotation angle α along the $\mathbf{K}$ direction is defined as tilt angle and the rotation angle β along the $\mathbf{N}$ direction is defined as yaw angle in the local coordinate system. Then the tool axis vector can be completely determined by the two angles in the local coordinate system as shown in Fig. 3. In the local coordinate system, the tool axis vector can be obtained as

$$\mathbf{V}_{axis}^{LCS} = \mathbf{Rot}(\mathbf{N}, \beta) \cdot \mathbf{Rot}(\mathbf{K}, \alpha) \cdot [0, 0, 1, 0]^T \tag{3}$$

in which $\mathbf{V}_{axis}^{LCS}$ represents the tool axis vector in the local coordinate system. The specific form of the rotation matrix is as

$$\begin{cases} \mathbf{Rot}(\mathbf{N}, \beta) = \begin{bmatrix} \cos\beta & -\sin\beta & 0 & 0 \\ \sin\beta & \cos\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \mathbf{Rot}(\mathbf{K}, \alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\alpha & 0 & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{cases} \tag{4}$$

The transformation from the machine coordinate system to the workpiece coordinate system belongs to the positive kinematic transformation. However, the tool position files in the workpiece coordinate system need to be transformed into the machine coordinate system. Therefore, the relationship between the workpiece coordinate system and the machine coordinate system needs to be established by the inverse kinematics transformation.

The tool axis vector $\mathbf{V}$ at the cutter contact point $\mathbf{P_C}$ in the workpiece coordinate system can be obtained by CAM software. As $\mathbf{V} = [i^* \, j^* \, k^* \, 0]^\mathrm{T}$, it can be converted into the machine coordinate system $OXYZ$ as $\mathbf{V_M}$ by

$$\begin{cases} \mathbf{V} = \mathbf{M}\mathbf{V}_M \\ \mathbf{V}_M = \mathbf{M}^{-1}\mathbf{V} \end{cases} \tag{5}$$

In which, $\mathbf{M}$ is the transformation matrix from the machine coordinate system to the workpiece coordinate system. $\mathbf{V_M} = [0\ 0\ 1\ 0]^\mathrm{T}$ is the initial tool axis vector in the machine coordinate system. Based on the structure of 5-axis NC machine tool as shown in Fig. 1, the rotary angles of axis A and axis C relative to the initial state are θ_A and θ_C respectively, then the transformation matrix is expressed as

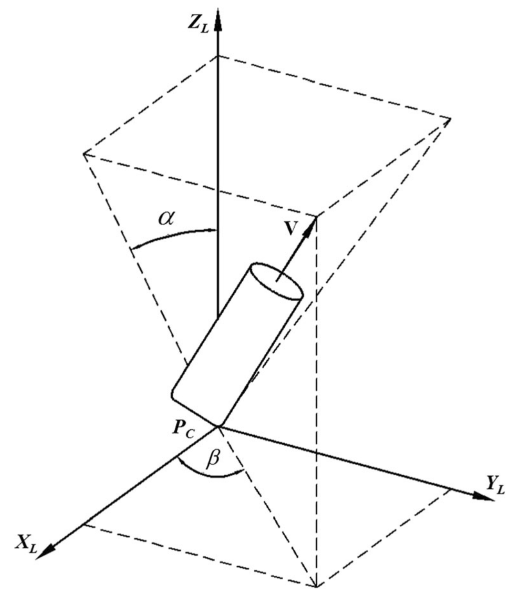


Fig. 3 Tool axis vector in local coordinate system

$$\begin{aligned} \mathbf{M} &= \mathbf{Rot}(\mathbf{Z}, \theta_C) \cdot \mathbf{Rot}(\mathbf{X}, \theta_A) \\ &= \begin{bmatrix} \cos\theta_C & -\sin\theta_C & 0 & 0 \\ \sin\theta_C & \cos\theta_C & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_A & -\sin\theta_A & 0 \\ 0 & \sin\theta_A & \cos\theta_A & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \cos\theta_C & -\cos\theta_A\sin\theta_C & \sin\theta_A\sin\theta_C & 0 \\ \sin\theta_C & \cos\theta_A\cos\theta_C & -\sin\theta_A\cos\theta_C & 0 \\ 0 & \sin\theta_A & \cos\theta_A & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{6}$$

Therefore, the tool axis vector in the workpiece coordinate system can be obtained as

$$\begin{cases} i^* = \sin\theta_C \cdot \sin\theta_A \\ j^* = -\cos\theta_C \cdot \sin\theta_A \\ k^* = \cos\theta_A \end{cases} \tag{7}$$

According to the above equation, the relationship between the angle of the rotary feed axis and the tool axis vector in the workpiece coordinate system can be obtained as

$$\begin{cases} \theta_A = \arctan2\left(\sqrt{i^{*2} + j^{*2}}, k^*\right) \\ \theta_C = \arctan2\left(i^*, j^*\right) \end{cases} \tag{8}$$

According to the above, the position of tool axis vector in workpiece coordinate system can convert to the G code that can be identified by machine tool in the machine coordinate system.

2.2 Calculation of angular velocity and angular acceleration for rotary feed axis

For the given toolpath $\mathbf{r}(\xi)$ on the machining curved surface $\mathbf{S}(u, v)$, the movement of each feed axis should satisfy the kinematical characteristics of the machine tool such as maximum speed, angular velocity, and angular acceleration. If the toolpath is complex, especially with sharp change curvature, it may cause the change of angular velocity and angular acceleration for the rotary feed axis of machine tool too large.

In order to satisfy the kinematical characteristics, the kinematics characteristics of the rotary feed axis need to be recognized firstly. Assuming the position of the rotary feed axis is θ and for the given toolpath $\mathbf{r}(\xi)$, the angular velocity and the angular acceleration of each rotary feed axis can be expressed as

$$\begin{cases} \boldsymbol{\omega} = \frac{d\theta}{dt} = \frac{d\theta}{d\xi} \cdot \frac{d\xi}{dt} = \theta_\xi \frac{d\xi}{dt} \\ \mathbf{a} = \frac{d^2\theta}{dt^2} = \theta_{\xi\xi} \left(\frac{d^2\xi}{dt^2} \right)^2 + \theta_\xi \frac{d^2\xi}{dt^2} \end{cases} \tag{9}$$

in which θ_ξ and $\theta_{\xi\xi}$ represent the first and second derivatives of the rotary feed axis angle for toolpath parameter ξ respectively. $\frac{d\xi}{dt}$ and $\frac{d^2\xi}{dt^2}$ represent the first and second derivatives of the toolpath parameter ξ for the processing time t.

In practice, it is difficult to express the toolpath with a clear equation. In order to recognize the motion state of the rotary feed axis in the machine tool, the discrete method is adapted to recognize the angular velocity and the angular acceleration for the rotary feed axis. For the toolpath $\mathbf{r}(\xi)$, there are n cutter contact points and $\mathbf{P}_{Ci}$ is the ith cutter contact point. $\{\mathbf{P}, \mathbf{V}\} = \{x, y, z, i^*, j^*, k^*\}$ is the tool position files corresponding to the cutter contact point. When the cutter moves from the cutter contact point $\mathbf{P}_{Ci}$ to $\mathbf{P}_{Ci+1}$, the tool axis vector changes from $\mathbf{V}_i$ to $\mathbf{V}_{i+1}$. The A-axis and C-axis angle values of the machine tool corresponding cutter contact positions can be calculated by Eq. (8), just changing from θ_A and θ_C to θ_{A+1} and θ_{C+1} respectively. L_i represents the approximate arc length between the cutter contact points $\mathbf{P}_{Ci}$ and $\mathbf{P}_{Ci+1}$, and the step length between adjacent cutter contact points is not fixed. When the feed rate is v, the processing time of adjacent cutter contact points is also different. For discrete points on the toolpath, the angular velocity and the angular acceleration of the rotary feed axis can be solved as

$$\begin{cases} \boldsymbol{\omega}_i = \frac{\theta_{i+1} - \theta_i}{L_i} \cdot v \\ \mathbf{a}_i = \frac{\boldsymbol{\omega}_{i+1} - \boldsymbol{\omega}_i}{L_i} \cdot v \end{cases} \tag{10}$$

Correspondingly, the specific calculation of the angular velocity for the A-axis and the C-axis of the machine tool can be obtained as

$$\begin{cases} \boldsymbol{\omega}_{Ai} = \frac{\theta_{Ai+1} - \theta_{Ai}}{L_i} \cdot v \\ \boldsymbol{\omega}_{Ci} = \frac{\theta_{Ci+1} - \theta_{Ci}}{L_i} \cdot v \end{cases} \tag{11}$$

in which θ_{Ai} and θ_{Ci} represent the rotation angles of A-axis and C-axis in the machine tool corresponding for the ith cutter contact point, respectively. ω_{Ai} and ω_{Ci} respectively represent the angular velocities of A-axis and C-axis in the machine tool corresponding for the ith cutter contact point.

Thus, the angular velocity for rotary feed axis of machine tool can be calculated as

$$\boldsymbol{\omega}_i = \begin{cases} \frac{\theta_{i+1} - \theta_i}{L_i} \cdot v = \sqrt{\boldsymbol{\omega}_{Ai}^2 + \boldsymbol{\omega}_{Ci}^2} & i = 1, \mathrm{K}, n-1 \\ 0 & i = n \end{cases} \tag{12}$$

In the same method, the angular acceleration for A-axis and C-axis of the machine tool can be calculated as

$$\begin{cases} \mathbf{a}_{Ai} = 2v \cdot \frac{\boldsymbol{\omega}_{Ai+1} - \boldsymbol{\omega}_{Ai}}{L_{i+1} + L_i} \\ \mathbf{a}_{Ci} = 2v \cdot \frac{\boldsymbol{\omega}_{Ci+1} - \boldsymbol{\omega}_{Ci}}{L_{i+1} + L_i} \end{cases} \tag{13}$$

Thus, the angular acceleration for rotary feed axis of machine tool can be calculated as

$$\mathbf{a}_i = \begin{cases} 2v \cdot \frac{\boldsymbol{\omega}_{i+1} - \boldsymbol{\omega}_i}{T_{i+1} + T_i} = \sqrt{\mathbf{a}_{Ai}^2 + \mathbf{a}_{Ci}^2} & i = 2, \mathrm{K}, n-1 \\ 0 & i = 1, n \end{cases} \tag{14}$$

Through the above analysis, the kinematical characteristics of the rotary feed axis of the machine tool is recognized, so the angular velocity and angular acceleration for rotary feed axis at any cutter contact point can be obtained.

3 Optimization of tool axis vector based on kinematical characteristics of rotary feed axis

In Section 2, the way how the position of tool axis vector converts from workpiece coordinate system to the machine coordinate system which can be directly recognized by the machine tool is obtained. Then, the method for recognizing the kinematical characteristics of the rotary feed axis of the machine tool is given. In this section, the above method is used to simulate the kinematical characteristics for the rotary feed axis of machine tool. According to the simulation results, the tool axis vector is reasonably adjusted to reduce the angular velocity of rotary feed axis during the curved surface machining. Firstly, the optimizing interval of the toolpath for tool axis vector is selected, and then, the equalization method of tool axis vector is given based on quaternion method. On this basis, the least-squares fitting method is used to optimize and smooth the relationship between the angle value of rotary feed axis and the cumulative arc length of toolpath, thus making the tool axis vector smooth better.

3.1 Optimizing interval of toolpath for tool axis vector

The kinematical calculation for the rotary feed axis of the machine tool at the cutter contact point on the toolpath is completed, and then the optimizing interval of toolpath for tool axis vector according to the calculation results can be selected. The interval with large change of tool axis vector is selected as the optimizing interval, and then optimize the tool axis vector. A complex surface with sharp change curvature is designed to verify the validity of the proposed method at last.

For a machining toolpath on a complex surface **S** with unknown equation in Fig. 4, there are n cutter contact points $\mathbf{P_C}$ on it. The arc length between current cutter contact point $\mathbf{P_{Ci}}$ and starting cutter contact point is defined as cumulative arc length L_C and $L_C = \sum L_i$. Angular value, angular velocity, and angular acceleration of rotary feed axis for each cutter contact point $\mathbf{P_{Ci}}$ can be obtained according to the formulas in Section 2. Through MATLAB simulink, the relationship between the angular value of rotary feed axis (A-axis and C-axis) and the cumulative arc length of toolpath is shown in Fig. 5.

The purpose of tool axis vector optimization is just to make the tool axis vector at the adjacent cutter contact points change smoothly. From Eqs. (7) and (8), the relationship can be established between the tool axis vector in workpiece coordinate system and the angle of rotary feed axis (A-axis and C-axis). In this way, each rotary feed angle corresponds to a specific tool axis vector. If the rotary feed angle change largely for the adjacent cutter contact points, the tool axis vector of the adjacent cutter contact points will change too much. Considering the large variation of tool axis vector will be affected by the large change of the rotary feed angle (A-axis and C-axis) in the machine tool, the angle value of rotary feed axis which corresponds to the adjacent cutter contact points should be ensured to change uniformly, so as to make the tool axis vector of adjacent cutter contact points smoothly transit. In order to reasonably choose the optimizing interval of toolpath for tool axis vector, the selection method for the optimizing interval is proposed based on the relationship between the rotation angle of rotary feed axis and the accumulation arc length of toolpath.

As shown in Fig. 6, through Eq. (8), the angular sequence of the rotary feed axis at each cutter contact point on the mth toolpath can be obtained. The vector $\mathbf{P}_{Ci}\mathbf{P}_{Ci-1} = (b_i, c_i)$ is defined, in which $b_i = L_i$ represents the arc length between adjacent cutter contact points and $c_i = \theta_i - \theta_{i-1}$ represents the angular difference for the rotary feed axis of adjacent cutter contact points. The angle between the vectors of the adjacent cutter contact points can be obtained as

$$_m\theta_i = \arccos\left(\frac{\mathbf{P}_{Ci}\mathbf{P}_{Ci-1}\cdot\mathbf{P}_{Ci}\mathbf{P}_{Ci+1}}{\|\mathbf{P}_{Ci}\mathbf{P}_{Ci-1}\|\|\mathbf{P}_{Ci}\mathbf{P}_{Ci+1}\|}\right) \quad i = 2,\cdots,n-1 \tag{15}$$

The angle between the vectors of the adjacent cutter contact points can reflect the variation of c value of the adjacent vectors. The smallest angle between vectors of the adjacent cutter contact points is, the greatest angular velocity of the adjacent cutter contact points changes, which will lead to a larger angular acceleration. So the angle between the vectors of the adjacent cutter contact points is small, from Eq. (13), and the larger angular acceleration occurs. In order to avoid excessive angular acceleration of the rotary feed axis, the interval with small angle should be selected by using the average angle value as a standard. Then, the average value of the included angle can be calculated as

$$^*_m\theta = \frac{1}{n-2}\sum_{i=2}^{n-1}|_m\theta_i| \tag{16}$$

And the optimizing interval of toolpath for the tool axis vector is selected by comparing the value of the angle at a cutter contact point and the average value of the included angle, just as

$$|_m\theta_i| < {^*_m\theta} \quad e \le i \le f \tag{17}$$

in which e and f are the first and last cutter contact points respectively that satisfy Eq. (17) in the optimizing interval. Through the above discussion, the optimizing interval $[e, f]$ can be selected, which provides the preparation for the following optimization of tool axis vector.

3.2 Optimization of tool axis vector for curved surface machining

The optimizing interval of toolpath for the tool axis vector can be chosen properly by the proposed method, which not only reflect the geometric feature information of the machined surface parts well but also can select the optimizing interval by G code when the surface equation is unknown. Next, the tool axis vector will be optimized in the optimizing interval of toolpath for curved surface machining.

For the optimizing interval of toolpath for tool axis vector, the angular velocity for the rotary feed axis of the machine tool is too large due to the sharp change of the tool axis vector, which may be beyond the allowable range of the maximum angular velocity of the machine tool, and even affects not only the machining quality but also the service life of machine tool. In order to prevent the angular velocity of the rotary feed axis from exceeding the kinematical characteristics of the machine tool, the equalization method of tool axis vector is proposed based on the quaternion method to optimize the tool axis vector, so as to limit the tool axis vector in the feasible boundary.

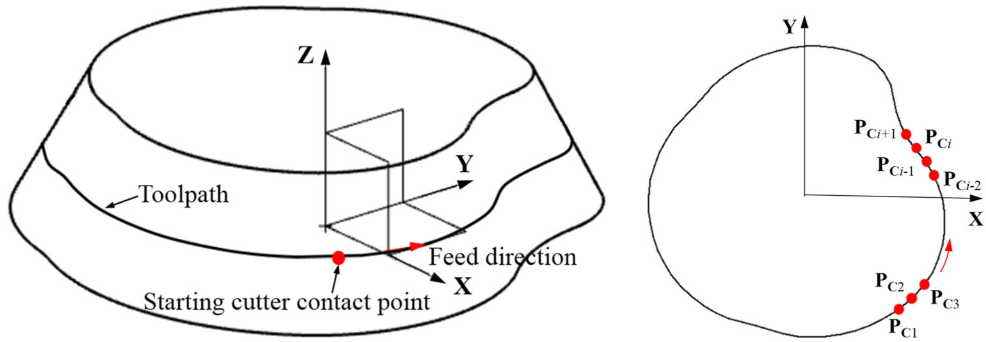


Fig. 4 Machining surface and toolpath

Through Section 3.1, the optimizing interval $[e, f]$ for tool axis vector has been selected. In optimizing process of tool axis vector for curved surface machining, the starting tool axis vector at the cutter contact point $\mathbf{P}_{\mathbf{C}e}$ is $\mathbf{V}_1$ and the ending tool axis vector at cutter contact point $\mathbf{P}_{\mathbf{C}f}$ is $\mathbf{V}_n$. The tool axis vector $\mathbf{V}_i$ is optimized by the quaternion method in the interval of $[e, f]$.

As shown in Fig. 7, θ_Q represents the angle formed by tool axis vectors $\mathbf{V}_1$ and $\mathbf{V}_n$, $\lambda\theta_Q$ represents the angle formed by tool axis vectors $\mathbf{V}_1$ and $\mathbf{V}_i$, and $(1\text{-}\lambda)\theta_Q$ represents the angle formed by tool axis vectors $\mathbf{V}_i$ and $\mathbf{V}_n$. In this way, the tool axis vector $\mathbf{V}_i$ between $\mathbf{V}_1$ and $\mathbf{V}_n$ can be obtained by the quaternion interpolation as

$$\mathbf{V}_i = \kappa(\lambda)\cdot\mathbf{V}_1 + \mu(\lambda)\cdot\mathbf{V}_n \qquad i\in[e, f] \tag{18}$$

in which $\kappa(\lambda)$ and $\mu(\lambda)$ are coefficients about parameter variable λ, $\lambda \in [0, 1]$. In order to determine the coefficients, Eq. (18) should do dot product with $\mathbf{V}_1$ and $\mathbf{V}_n$ respectively and can be expressed by the following formula, as

$$\begin{cases} \mathbf{V}_1\cdot\mathbf{V}_i = \kappa(\lambda)\mathbf{V}_1\cdot\mathbf{V}_1 + \mu(\lambda)\mathbf{V}_1\cdot\mathbf{V}_n \\ \mathbf{V}_n\cdot\mathbf{V}_i = \kappa(\lambda)\mathbf{V}_n\cdot\mathbf{V}_1 + \mu(\lambda)\mathbf{V}_n\cdot\mathbf{V}_n \end{cases} \tag{19}$$

For $\mathbf{V}_1$, $\mathbf{V}_n$, and $\mathbf{V}_i$ are all the unit tool axis vector, Eq. (19) can be translated as

$$\begin{cases} \cos(\lambda\theta_Q) = \kappa(\lambda) + \mu(\lambda)\cos\theta_Q \\ \cos[(1-\lambda)\theta_Q] = \kappa(\lambda)\cos\theta_Q + \mu(\lambda) \end{cases} \tag{20}$$

And then, the coefficients are calculated as

$$\begin{cases} \kappa(\lambda) = \dfrac{\sin(\theta_Q\cdot(1-\lambda))}{\sin\theta_Q} \\ \mu(\lambda) = \dfrac{\sin(\theta_Q\cdot\lambda)}{\sin\theta_Q} \end{cases} \tag{21}$$

in which the calculation for the parameter variable λ and the angle θ_Q is as

$$\begin{cases} \lambda = \dfrac{i-e}{f-e} \\ \theta_Q = \arccos\dfrac{\mathbf{V}_1 \cdot \mathbf{V}_n}{|\mathbf{V}_1\|\mathbf{V}_n|} \end{cases} \tag{22}$$

By Eqs. (18)–(22), the optimized tool axis vector $\mathbf{V}_i$ can be calculated as

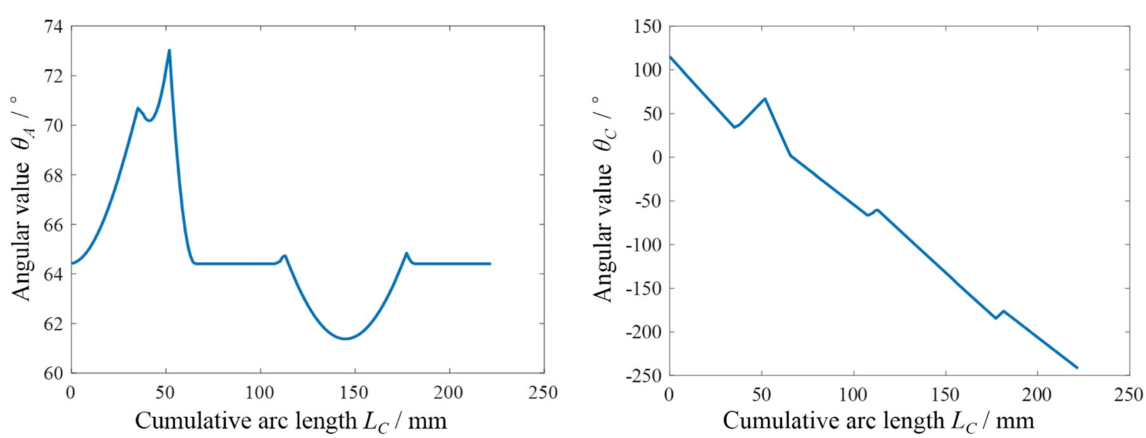


Fig. 5 Relationship between angular value of rotary feed axis and cumulative arc length of toolpath

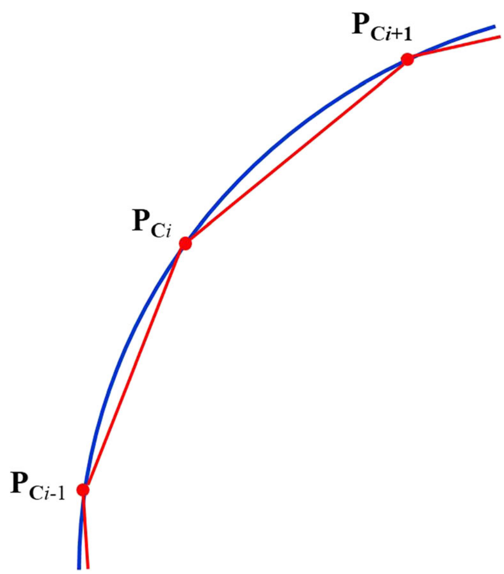


Fig. 6 Calculation of angle between vectors of adjacent cutter contact points

$$\mathbf{V}_i = \frac{\sin\left(\theta_Q - \frac{i-e}{f-e}\theta_Q\right)}{\sin\theta_Q}\mathbf{V}_1 + \frac{\sin\left(\frac{i-e}{f-e}\theta_Q\right)}{\sin\theta_Q}\mathbf{V}_n, \quad i \in [e+1, f-1] \tag{23}$$

From the above, the tool axis vector at each cutter contact point in the interval $[e, f]$ can be gotten and converted to the angle value for the rotary feed axis of the machine tool according to Eq. (8). In this way, the optimized angular sequence $\mathbf{S}\{\mathbf{S}_{Aopt}, \mathbf{S}_{Copt}\}$ for the rotary feed axis can be obtained as

$$\begin{cases} \mathbf{S}_{Aopt} = \left\{\theta_{Aopt\,1}, \cdots, \theta_{Aopt\,n}\right\} \\ \mathbf{S}_{Copt} = \left\{\theta_{Copt\,1}, \cdots, \theta_{Copt\,n}\right\} \end{cases} \tag{24}$$

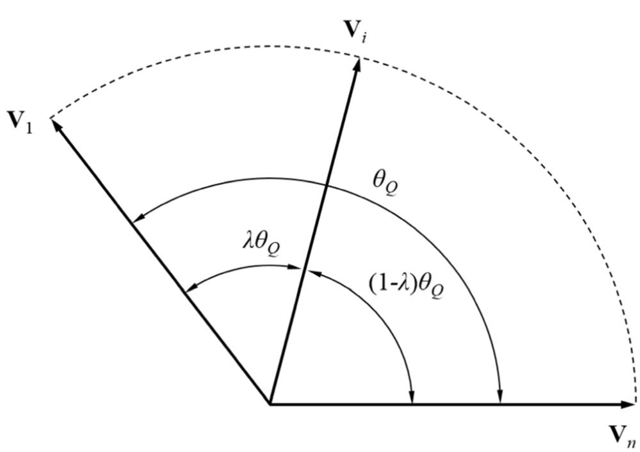


Fig. 7 Equalization method of tool axis vector

By MATLAB simulink, the relationship between the optimized angular value of rotary feed axis (*A*-axis and *C*-axis) and the cumulative arc length of toolpath is shown in Fig. 8.

In order to further reduce the angular velocity for the rotary feed axis of the machine tool, the relationship between the optimized angular value of rotary feed axis and the cumulative arc length of toolpath is analyzed and the cutter contact point with the largest curvature in the toolpath curve is selected. And then a method based on the principle of least-squares fitting is proposed to adjust the shape of the angular change curve for rotary feed axis with the optimized angular value and cumulative arc length, and makes the angular change curve *F* of rotary feed axis satisfy the following constraint as

$$F = \min \sum_{i=\varepsilon-3}^{\varepsilon+3} \omega_i \left(\theta'_{opt}(L_i) - \theta_{opt\,i}\right)^2 \tag{25}$$

in which ω_i represents the weight coefficient of each cutter contact point. To make the calculation simple, the weight coefficient is set 1. $\theta'_{opt}(L_i)$ represents the re-optimized angular value of the rotary feed axis at cutter contact point.

As shown in Fig. 9, the maximum curvature is at the cutter contact point $\mathbf{P}_{Cj}$, which is selected in the angular change curve of rotary feed axis with the optimized angular value and cumulative arc length. In order to adjust the shape of the angular change curve for rotary feed axis near this cutter contact point, the angular change curve of rotary feed axis respected the Eq. (25) is calculated in the cutter contact point interval of $[\mathbf{P}_{Cj\text{-}3}, \mathbf{P}_{Cj+3}]$. So the re-optimized angular sequence $\mathbf{S}\{\mathbf{S'}_{Aopt}, \mathbf{S'}_{Copt}\}$ of rotary feed axis is finally obtained as

$$\begin{cases} \mathbf{S}'_{Aopt} = \left\{\theta'_{Aopt\,1}, \cdots, \theta'_{Aopt\,n}\right\} \\ \mathbf{S}'_{Copt} = \left\{\theta'_{Copt\,1}, \cdots, \theta'_{Copt\,n}\right\} \end{cases} \tag{26}$$

Through MATLAB simulink, the relationship between the re-optimized angular value of rotary feed axis (*A*-axis and *C*-axis) and the cumulative arc length of toolpath is shown in Fig. 10.

From the above analysis and Figs. 5, 8, and 10, it can be concluded that the optimization method of tool axis vector based on the kinematical characteristics of rotary feed axis proposed in this study can make the angular change curve of rotary feed axis smoother without fluctuation, which is beneficial for the curved surface machining.

4 Experimental results and discussion

To further illustrate the effectiveness of the proposed optimization method of tool axis vector, a curved surface with

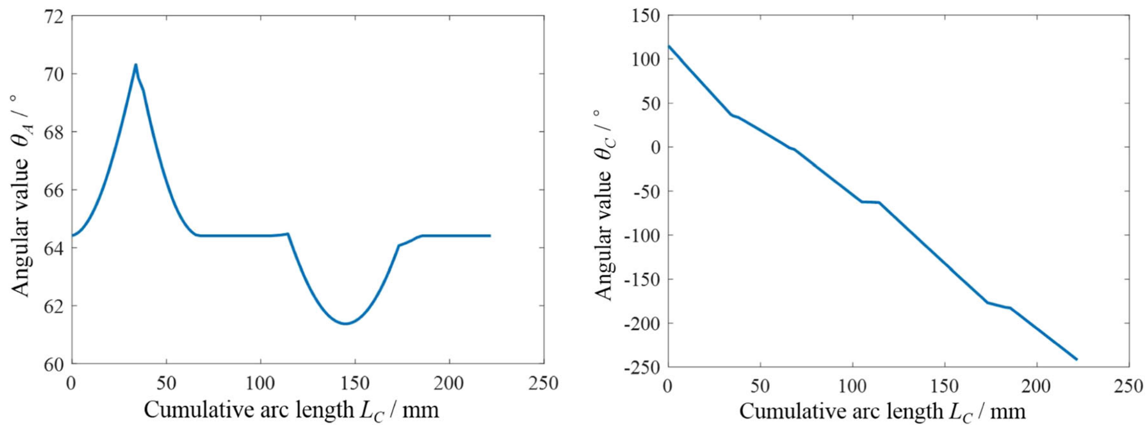


Fig. 8 Relationship between optimized angular value of rotary feed axis and cumulative arc length of toolpath

different curvatures is designed and to carry out the experiment, and the ring cut method is used in experimental processing. As shown in Fig. 4, the initial cutter position files of the curved surface can be obtained by the CAM software, and the kinematical characteristics for the rotary feed axis of the machine tool can be calculated. To achieve a better transition of the tool axis vector during the curved surface machining and reduce the calculation cost, the unreasonable interval of toolpath for the tool axis vector is firstly selected, and then the proposed method is used to optimize the tool axis vector.

4.1 Simulation analysis

Taking a toolpath at the middle round of the curved surface in Fig. 4 as an example, the optimizing interval of toolpath for the tool axis vector can be obtained based on the kinematical calculation according to Section 2 and Section 3.1. As shown in Fig. 11, the blue curve represents the optimizing interval and the corresponding cutter contact point sequences are {26–47, 69–72, 116–122}. For example, the initial tool axis vector for cutter contact points 69–72 are {(− 0.828, 0.358, 0.432), (− 0.810, 0.398, 0.430), (− 0.791, 0.438, 0.428), (− 0.786, 0.447, 0.427)} and the corresponding rotation angles of rotary feed axis are {(64.407, − 66.599), (64.504, − 63.854) (64.689, − 60.987), (64.735, − 60.385)}. Based on the optimization method of tool axis vector in Section 3.2, the optimized tool axis vectors are obtained as {(− 0.828, 0.358, 0.432), (− 0.810, 0.398, 0.430), (− 0.791, 0.438, 0.428), (− 0.786, 0.447, 0.427)}, and the corresponding rotation angles of rotary feed axis are {(64.420, − 62.413), (64.434, − 62.564), (64.447, − 62.714), (64.461, − 62.865)}.

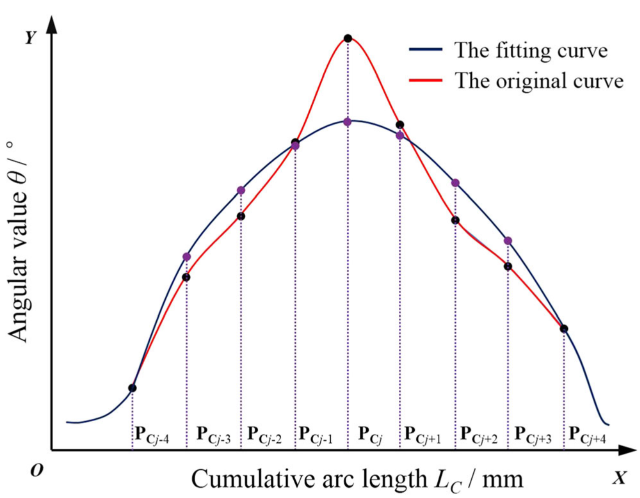


Fig. 9 Principle of least-squares fitting to adjust shape of angular change curve

Before optimizing of the tool axis vector, the relationship between the angular velocity and the angular acceleration of rotary feed axis and the cumulative arc length of toolpath can be obtained and is shown in Fig. 12. From Fig. 12, it can be found that the angular velocity for the rotary feed axis of the machine tool fluctuates frequently, thus causing the angular acceleration of the rotary feed axis large. In this way, the optimization method of tool axis vector proposed in this study is used to optimize the tool axis vector. After optimizing of tool axis vector, the kinematics characteristics for the rotary feed axis of the machine tool are shown in Fig. 13.

In order to obviously reflect the kinematical characteristics of the rotary feed axis before and after optimizing of the tool axis vector, the curve for the relationship between the kinematical characteristics of rotary feed axis and the cumulative arc length of toolpath before and after optimizing of tool axis vector is plotted in the same figure, as shown in Fig. 14. Analyzing the optimization result of tool axis vector, the maximum angular velocity is reduced from 24°/s to 12°/s and the maximum angular acceleration is reduced from 127°/s^2 to 89°/s^2. It should be noted that the processing efficiency is determined by the feed rate. In this study, the tool axis vector is optimized to reduce the angular velocity and acceleration of rotary feed axis of the machining tool, and the feed rate in the machining process is invariant, so the processing efficiency is

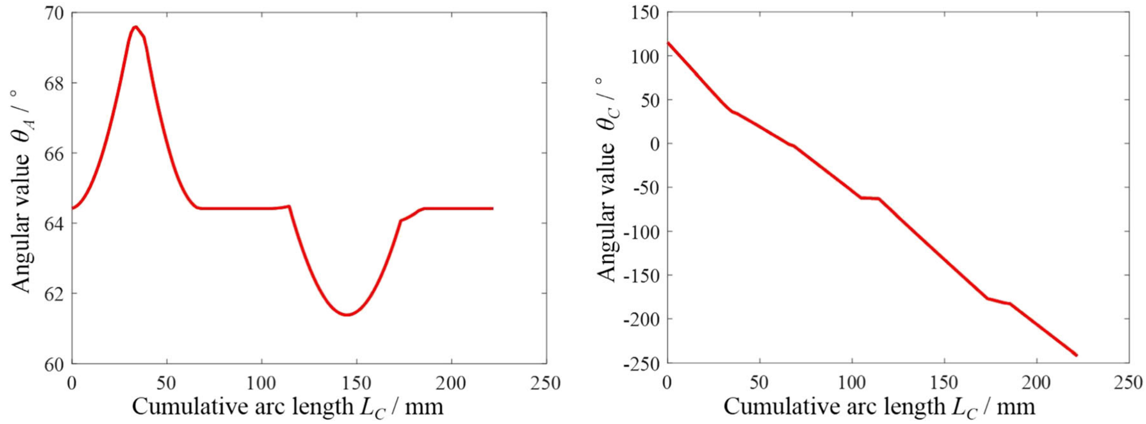


Fig. 10 Relationship between re-optimized angular value of rotary feed axis and cumulative arc length of toolpath

invariant. However, with the same feed rate, the optimization of the tool axis vector with the kinematical characteristics of rotary feed axis can make the quality of the machined curved surface better.

4.2 Experimental verification

In order to prove the effectiveness of the optimization method for tool axis vector proposed in this study and the correctness of the simulation analysis, experiment is carried out with the curved surface parts based on a 5-axis NC machine tool. The material of experimental workpiece is 7075 aviation aluminum alloy, and the machining curved surface is a workpiece with continuous change curvature as shown in Fig. 4. The ball-end cutter used is SANDVIK 1B230-0800-XA 1630 solid carbide coating ball-end cutter with two flutes. The diameter is 8 mm, the nominal helical angle is 30°, and the whole length is 50 mm. According to the engineering experience for aluminum alloy curved surface machining, the machining parameters are empirically selected as the spindle speed n = 3000 r/min, the feed rate f = 250 mm/min, and the cutting depth a_p = 1 mm. The machining process is shown in Fig. 15 with the scallop height between adjacent toolpaths as 0.02 mm.

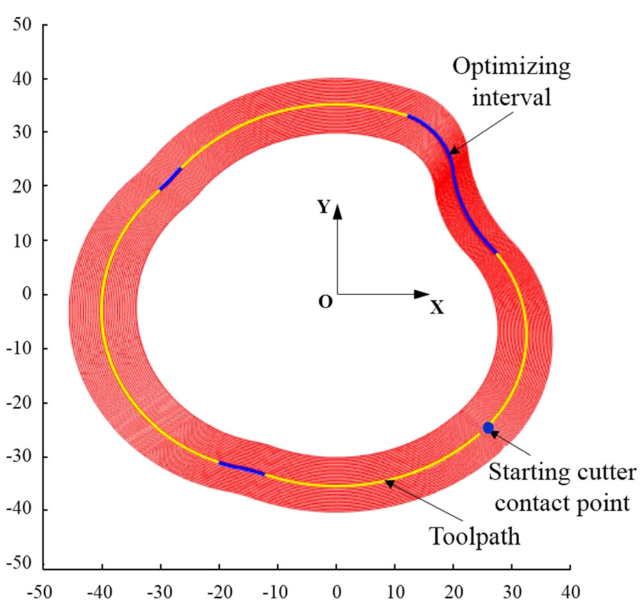


Fig. 11 Optimizing interval on toolpath for tool axis vector

As shown in Fig. 16, the machining result for optimization method of the tool axis vector proposed in this study is compared with that of conventional method for tool axis vector planning. Because the angular velocity and angular acceleration for the rotary feed axis of machine tool are large, that may make the kinematical characteristics of the machine tool change a lot and lead to the chatter mark on the machined curved surface, so as to affect the machining quality [10]. While the proposed optimization method for tool axis vector can notably decrease the change of kinematical characteristics for rotary feed axis, consequently the chatter mark can be avoided, so as to significantly improve the machining quality.

For the machined curved surface, the surface contour is firstly measured at the same position (as shown in Fig. 16) by the three coordinate measuring machine, and then the measured data points are fitted as shown in Fig. 17. It can be obviously found that the local machined surface with sharp change curvature is more smooth after optimization of tool axis vector based on the kinematical characteristics of rotary feed axis.

In order to further explain that the machining effect is improved after optimization, the surface roughness of the machined workpiece with sharp change curvature is measured, as shown in Fig. 18 for the circle position. The surface roughness is measured by 120-mm phase grating interference roughness profiler (PGI 840) in the direction of A and B respectively. That because the surface roughness

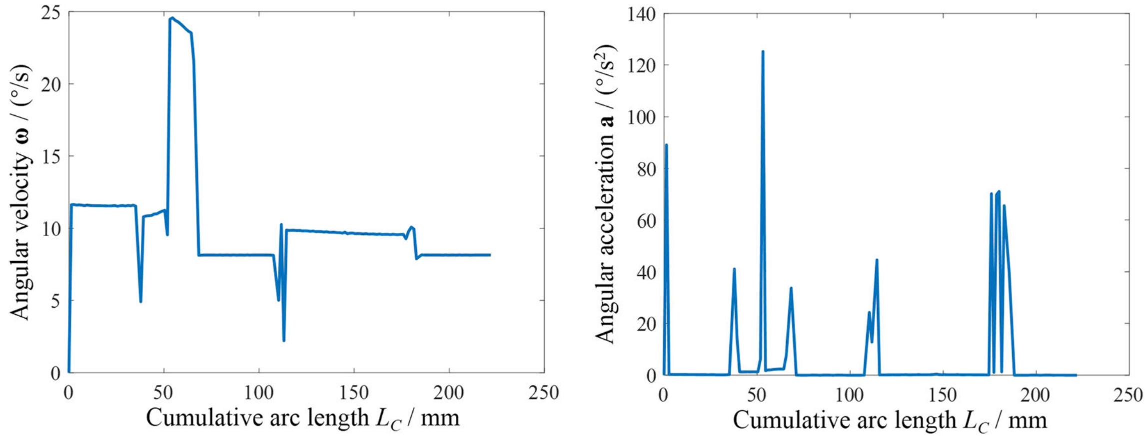


Fig. 12 Relationship between kinematical characteristics of rotary feed axis and cumulative arc length of toolpath before optimizing

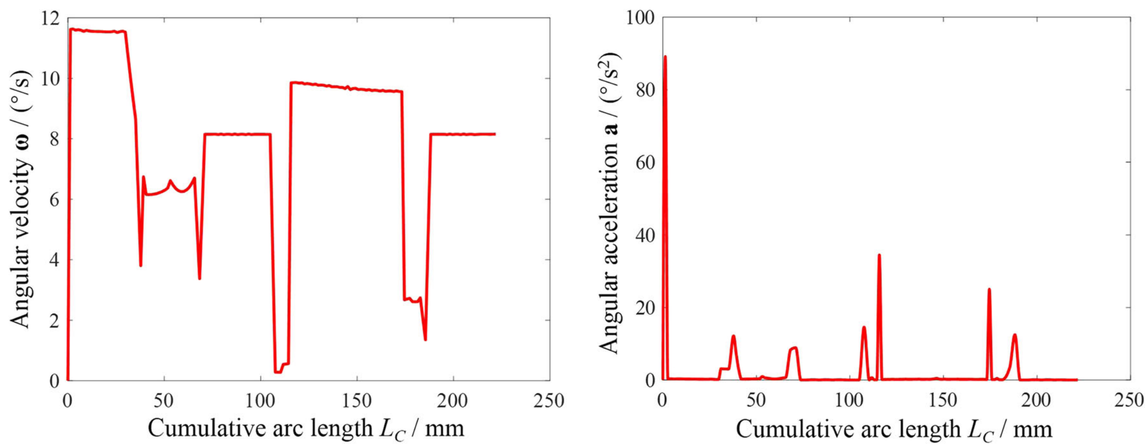


Fig. 13 Relationship between kinematical characteristics of rotary feed axis and cumulative arc length of toolpath after optimizing

measured in direction *A* mainly reflects the scallop height between adjacent toolpaths and the surface roughness measured in direction B can verify the optimization effect for tool axis vector. The measured results before and after optimizing of the tool axis vector are shown in Fig. 19 and Fig. 20.

From the experimental results as shown in Table 1, some results can be obtained as follows. The surface roughness in direction *A* is almost the same before and after optimizing for tool axis vector, which verifies that the optimization of tool axis vector does not affect the cutter-to-workpiece contact state and thus does not affect the machining quality of

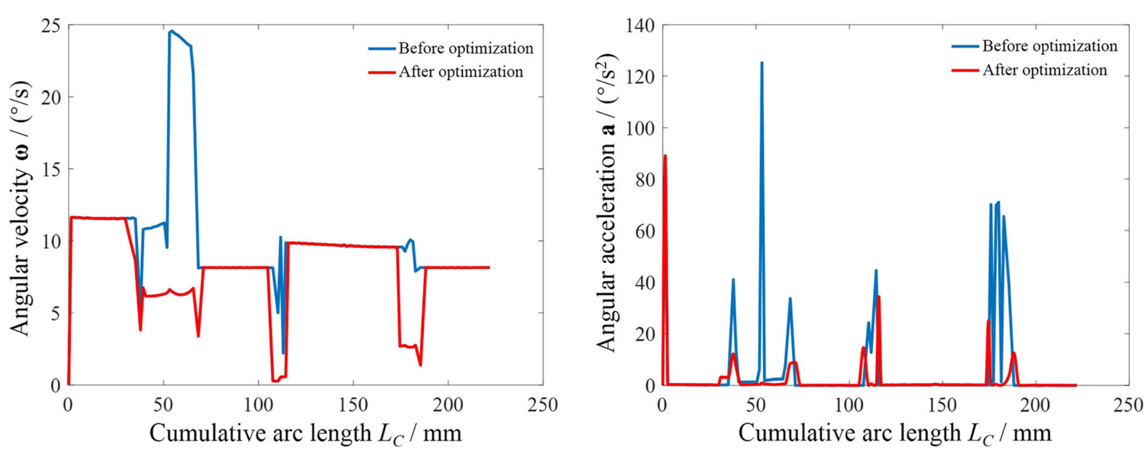


Fig. 14 Kinematical characteristics of rotary feed axis before and after optimizing

Fig. 15 Machining process of workpiece with continuous change curvature

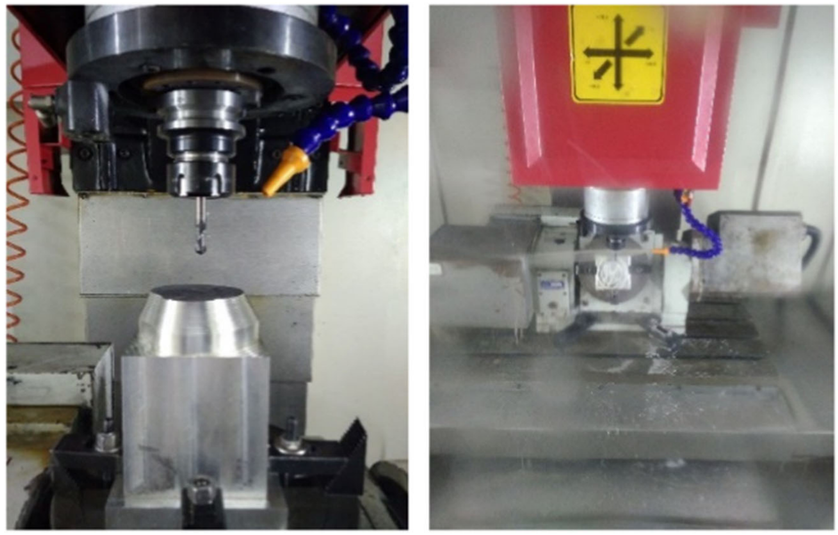
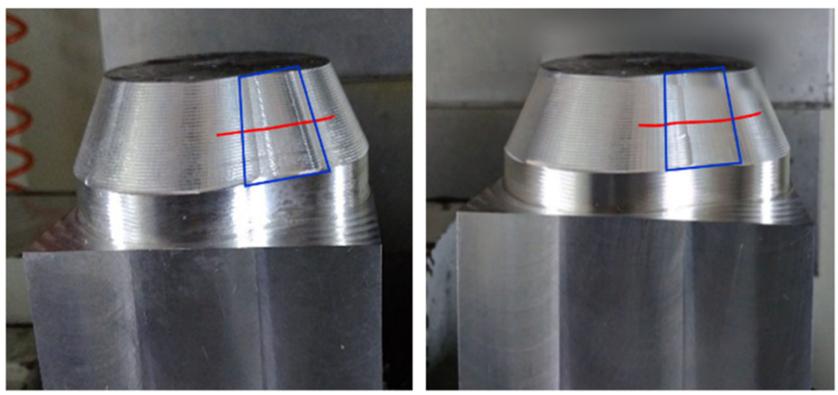


Fig. 16 Machining results of workpiece before and after optimizing for tool axis vector



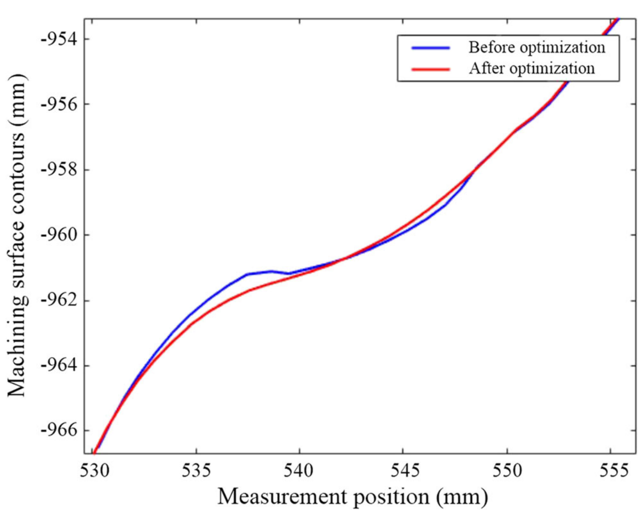


Fig. 17 Fitting curve of machining surface contour before and after optimizing for tool axis vector

complex curved surface due to the use of ball-end cutter, and the surface roughness in direction *A* can be improved by reducing the scallop height value between adjacent toolpaths. In direction *B*, it is clear that the surface roughness with the optimized tool axis vector is significantly better than that with tool axis vector before optimizing, shown as the surface roughness parameters in Table 1. The Ra value decreases from 1.6143 to 1.1868 μm with a decrease of 26.5%, and the Rz value decreases from 6.4960 to 5.1970 μm with a decrease of 20.0%. In this way, the proposed optimization method of tool axis vector for the curved surface machining can not only reduce angular velocity and angular acceleration of the rotary feed axis but also improve the processing quality of the curved surface parts with sharp change curvature.

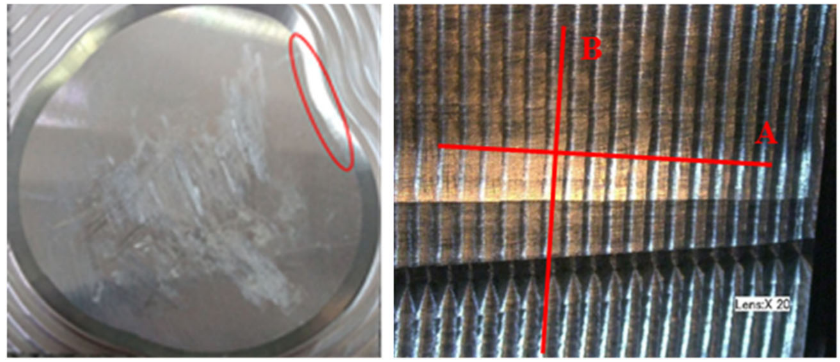


Fig. 18 Measure position of surface roughness

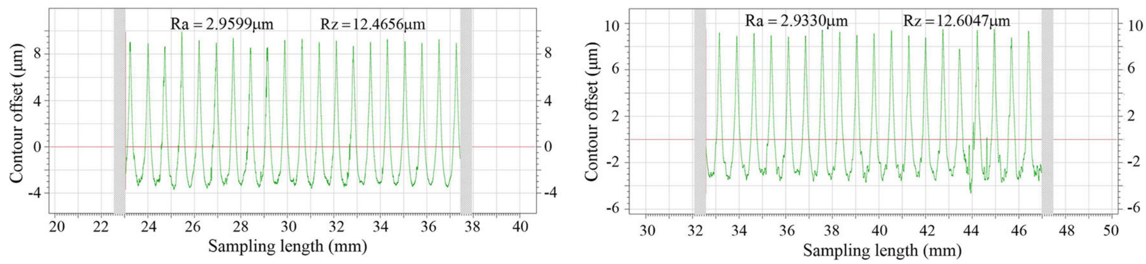


Fig. 19 Surface roughness in direction *A* before and after optimizing for tool axis vector

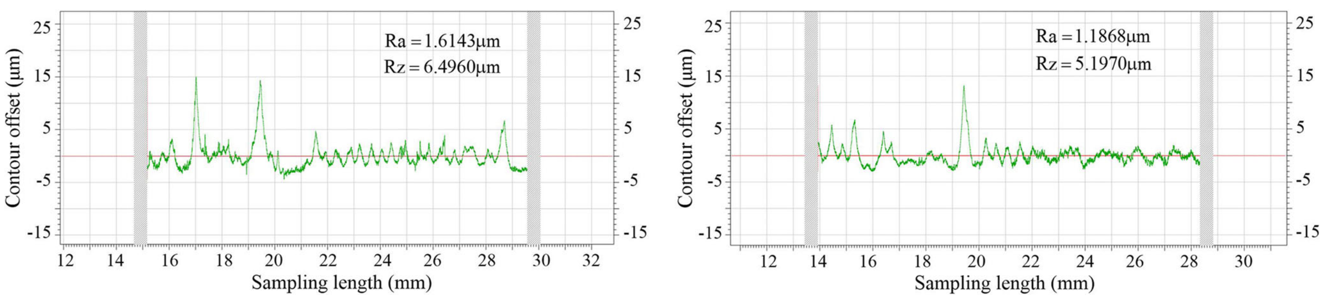


Fig. 20 Surface roughness in direction *B* before and after optimizing for tool axis vector

Table 1 Comparison results of surface roughness before and after optimizing for tool axis vector

Method of measurement	Direction *A*		Direction *B*	
	Ra (μm)	Rz (μm)	Ra (μm)	Rz (μm)
Before optimizing	2.9599	12.4656	1.6143	6.4960
After optimizing	2.9330	12.6047	1.1868	5.1970
Reduce percentage	0.90%	−1.10%	26.5%	20.0%

5 Conclusions

In 5-axis NC machining process of the complex surface, the sharp change curvature of the curved surface will cause the tool axis vector to change greatly, which may make angular velocity and angular acceleration of the rotary feed axis change a lot, so as to affect the machine tool life and the machining quality. To overcome this problem, this study presents an optimization method of tool axis vector based on the kinematical characteristics of the rotary feed axis for curved surface machining. Based on the calculation of angular velocity and angular acceleration for the rotary feed axis, the optimizing interval of the toolpath for tool axis vector is firstly selected based on the relationship between the rotation angle of rotary feed axis and the accumulation arc length of toolpath, so as to reduce the computation amount. Then taking the tool axis vector at both ends of the optimizing interval as constraints, the equalization method of tool axis vector based on the quaternion method is used to optimize the tool axis vector with the kinematical characteristics of rotary feed axis, and the angular change curve of rotary feed axis with the optimized angular value and cumulative arc length is adjusted by the principle of least-squares fitting after the local optimization of tool axis vector. Through the comparison of simulation results before and after optimizing of tool axis vector, it can be concluded that the optimized tool axis vector can significantly reduce angular velocity and angular acceleration of the rotary feed axis in the machine tool. Finally, the experimental investigations on a test workpiece in 5-axis ball-end milling are carried out to verify the validity of the proposed optimization method, and the machining results show that the local machined surface with sharp change curvature is more smooth and the surface roughness with the optimized tool axis vector is significantly better than that with tool axis vector before optimizing. To sum up, the achievements in this study are significant to reduce angular velocity and angular acceleration of rotary feed axis and make the machine tool run more smoothly, which further can improve the processing quality of complex curved surface parts.

Acknowledgements The authors wish to thank the anonymous reviewers for their comments which led to improvements of this paper.

Funding information The project is supported by Science Challenge Project of China (No. TZ2018006-0101-02), National Natural Science Foundation of China (No. 51575087 and No. 51675081), National Science and Technology Major Project of China (No. 2016ZX04001-002), Innovation Project for Supporting High-level Talent in Dalian (No. 2016RQ012), Science Fund for Creative Research Groups (No. 51621064), and the Fundamental Research Funds for the Central Universities.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

References

1. Li WL, Wang G, Zhang G, Li QD, Yin ZP (2017) Interference-free inspection path generation for impeller blades using an on-machine probe. IEEE/ASME Trans Mechatronics 22(3):1218–1226
2. Wang GX, Shu QL, Wang J, Wang WS (2014) Tool interference checking for five axis NC machining of sculptured surfaces. China Mechanical Engineering 25(3):299–303
3. Tang TD, Bohez ELJ (2015) A new collision avoidance strategy and its integration with collision detection for five-axis NC machining. Int J Adv Manuf Technol 81(5–8):1247–1258
4. Yoon JH, Pottmann H, Lee YS (2003) Locally optimal cutting positions for 5-axis sculptured surface machining. Computer Aided Design 35(1):69–81
5. Ho MC, Hwang YR, Hu CH (2003) Five-axis tool orientation smoothing using quaternion interpolation algorithm. Int J Mach Tools Manuf 43(12):1259–1267
6. Morisige K, Kase K, Takeuchi Y (1996) Tool path generation using C-space for 5-axis control machining. J Japan Society of Precision Engineering 62(1):1783–1787
7. Castagnetti C, Duc E, Ray P (2008) The domain of admissible orientation concept: a new method for five-axis tool path optimisation. Computer Aided Design 40(9):938–950
8. Srijuntongsiri G, Makhanov SS (2015) Optimisation of five-axis machining G-codes in the angular space. Int J Prod Res 53(11): 3207–3227
9. Choi BK, Park JW, Jun CS (1993) Cutter-location data optimization in 5-axis surface machining. Comput Aided Des 25(6):377–386
10. Grandguillaume L, Lavernhe S, Tournier C (2016) A tool path patching strategy around singular point in 5-axis ball-end milling. Int J Prod Res 54(24):7480–7490
11. Li LL, Zhang YF, Li HY, Geng L (2011) Generating tool-path with smooth posture change for five-axis sculptured surface machining based on cutter's accessibility map. Int J Adv Manuf Technol 53(5–8):699–709
12. Jun CS, Cha K, Lee YS (2003) Optimizing tool orientations for 5-axis machining by configuration-space search method. Computer Aided Design 35(6):549–566
13. Lavernhe S, Tournier C, Lartigue C (2008) Optimization of 5-axis high-speed machining using a surface based approach. Comput Aided Des 40(10):1015–1023
14. Geng C, Yu D, Zheng LM, Zhang H (2012) Algorithm of optimization of rotation axes in free-form surface machining. J Mechanical Engineering 48(23):127–134
15. Li XY, Lee CH, Hu PC, Zhang Y, Yang FZ (2017) Cutter partition-based tool orientation optimization for gouge avoidance in five-axis machining. Int J Adv Manuf Technol 95(5–8):2041–2057
16. Zhou B, Zhao JB, Liu WJ (2013) Method study on optimization of tool axis vector for complex surface of 5-axis CNC. J Mechanical Engineering 49(7):184–192
17. Wang N, Tang K (2007) Automatic generation of gouge-free and angular-velocity-compliant five-axis toolpath. Computer Aided Design 39(10):841–852
18. Luo M, Wu B, Li S, Zhang D (2009) Five-axis tool orientation optimization based on kinematical method. J Mechanical Engineering 45(9):158–163
19. Wang J, Zhang DH, Luo M, Wu BH (2013) A global tool orientation optimization method for five-axis CNC machining of sculptured surfaces. Acta Aeronautica et Astronautica Sinica 34(6): 1452–1462