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ORIGINAL ARTICLE

Kinematic errors prediction for multi-axis machine tools' guideways based on tolerance

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Abstract

In this paper, a systematic approach on how to predict kinematic errors based on tolerance of machine tools' guideways is introduced. Firstly, the truncated Fourier series function is applied to fit curve of guideways surface. Since geometric profile errors are regarded as a bridge between tolerance and kinematic errors of machine tools' guideways, the mapping relationship between tolerance and geometric profile errors of machine tools' guideways is formulated, and the mapping relationship between geometric profile errors and kinematic errors of guideways is established. Then, kinematic errors prediction model based on tolerance of guideways is subsequently proposed. Finally, simulation verification is conducted with this method. Simulation results show the range of the predicted kinematic errors (positioning error, y direction and z direction straightness error, roll error, pitch error, and yaw error) is 17.12 μm, 56.57 μm, 70.71 μm, 28.28 μrad, 141.42 μrad, and 113.14 μrad, respectively. In order to verify the feasibility and effectiveness of the presented method, a measuring experiment is carried out on guideways of a gantry-type five-axis milling machine tools by using a dual-frequency laser interferometer. The measured and identified discrete data can be fitted precisely by Fourier curve fitting method. The fitting results show the range of the measured kinematic errors is 16.96 μm, 59.43 μm, 68.63 μm, 28.65 μrad, 135.40 μrad, and 111.58 μrad, respectively. The maximum residual errors between the predicted and measured values of kinematic errors are 1.67 μm, 5.19 μm, 5.50 μm, 1.87 μrad, 9.81 μrad, and 7.07μrad, respectively. Comparing with the measured results of kinematic errors, residual errors are considerably small and can be neglected. Therefore, there is no doubt that this method is effective enough for predicting kinematic errors and can be used to replace the measurement of kinematic errors. In the design stage of machine tools, this approach is convenient for engineers to derive the distribution of kinematic errors. And its basic idea can be applied to other type of machine tools' guideways.

Keywords Machine tools' guideways · Truncated Fourier series function · Kinematic errors · Tolerance · Geometric profile errors

1 Introduction

With a rapid development of precision machining for the complex parts, multi-axis machine tools are widely used in various manufacturing, the significance of improving machine tools' machining accuracy is well approved [1]. The structure of multi-axis machine tools consists of several components, such as machine bed, guideways, rotary axes, spindle, and worktable. Among these components, guideways are of paramount importance. Lead screw and pair of rails are the two main components of guideway. And geometric errors of machine tools' guideways, which play a crucially important role in the accuracy design of machine tools and weaken the whole system accuracy, are affected by profiles of the pair of rails and cumulative-lead error of screw. However, in the initial design stage of machine tools, only the information of tolerance of machine tools' key components is known. Since geometric errors of machine tools can play the role of guidance for the accuracy design of machine tools, designers and engineers obtain geometric errors only by making use of design experiences. Hence, it is important to develop a method for predicting geometric errors based on tolerance in the design process of new machine tools.

At present, considerable research works [2–9] for geometric errors modeling of multi-axis machine tools are devoted. Li

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et al. [10] proposed a novel 13-line identification method to identify geometric errors of the linear axes in five-axis machine tools by using a laser interferometer and adopted the MBS theory to establish geometric errors model simultaneously. Yang et al. [11] presented an identification and correction of position-independent geometric errors (PIGEs) method to improve the accuracy of machined parts based on screw theory. Through ball-bar tests, the performance of the proposed PIGEs identification model has been validated. Zhu et al. [12] proposed an integrated geometric errors modeling, identification, and compensation method for machine tools. Gao et al. [13] proposed arithmetic in reversely calculating machine errors to evaluate the actual accuracy of the ultra-precision machine based on coupling geometric errors and combining the MBS theory to simplify geometric errors modeling. According to the Ref. [14], a universal kinematic errors modeling method was proposed based on MBS theory. To date, many researchers used this method to establish geometric errors model of machine tools [15–20].

In addition, the geometric errors of guideways lay a foundation on the machining accuracy of multi-axis machine tools. Over the past few decades, geometric errors modeling of the machine tools' guideways was usually studied by many researchers. In order to measure the parallelism and straightness of a pair of rails for ultra-precision guideways, Hwang et al. [21] proposed a three-probe system for measuring the parallelism and straightness of a pair of rails for ultra-precision guideways. Ekinci et al. [22] considered the internal mechanisms causing motion errors and established the relationship between the motion errors and guideways' geometric errors to gain a deeper understanding of aerostatic guideways. Zha et al. [23] presented an approach to model and compensate the vertical straightness error of gantry type open hydrostatic guideways. Through carrying out an experiment to measure the straightness error at different points on the beam by using a laser interferometer, a static analysis model is established. Ekinci et al. [24] proposed a machine error modeling approach which considers the geometric errors of guideways, the relationship between the joint kinematic straightness and angular error was analyzed based on this approach, and this paper applied trigonometric function to fit the surface curve of guide rail. He et al. [25] presented a method for motion errors estimation of a linear motion bearing table based on hierarchical idea and formulated a map from the rail form errors to the motion errors of Slider Tier. According to Ref. [26], a method was proposed for estimating two-dimensional position errors and flatness based on measured guideway profiles. Profile measurements, estimates of motion errors, and geometric errors models are also considered to estimate the planar XY stage errors. Majda [27] established geometric errors model of linear guideway based on finite element method (FEM) and carried out the analytical and experimental examinations to analyze the influence of geometric errors of linear guideway on joint kinematic errors. Qi et al. [28] presented a method to predict linear motion errors caused by components profile errors and took a hydrostatic guideway as an example to study the influence of three-dimensional profile errors on straightness and error averaging effects. Tang et al. [29] gave a systematic approach on the relationship between straightness and angular errors and guideways surface in precise linear stage. By analyzing the characteristics of machining process for guide rail, a combination of trigonometric functions and quadratic function is selected in curve fitting based on the measurement results, which was not suitable for representing the surface of large-scale guideways in real situation.

As can be observed in the abovementioned studies, the majority of them focused on geometric errors modeling, identification and compensation of machine tools, and the establishment of relationship between geometric profile errors and geometric errors of small-scale guideways. However, in the design stage of machine tools, only the information of tolerance of machine tools' key components is known, geometric errors is unknown, since geometric error is a parameter generated after assembly of machine tools. In addition, geometric errors of machine tools provide the guidance for the accuracy design of machine tools during the design process of new machine tools. It is obvious that there is little research work reported in the literature concerning the establishment of geometric errors prediction model based on tolerance for guideways, i.e., there is lack of a complete theoretical analysis method in quantitative analysis of the relationship between tolerance and geometric errors of machine tools' guideways at present. Therefore, an effective approach to the prediction of geometric errors of guideways based on tolerance during the design stage of machine tools is a significant subject in practice.

In view of the limitations stated, first, this paper presents a systematic approach to predict kinematic errors of guideways in multi-axis machine tools. In order to predict kinematic

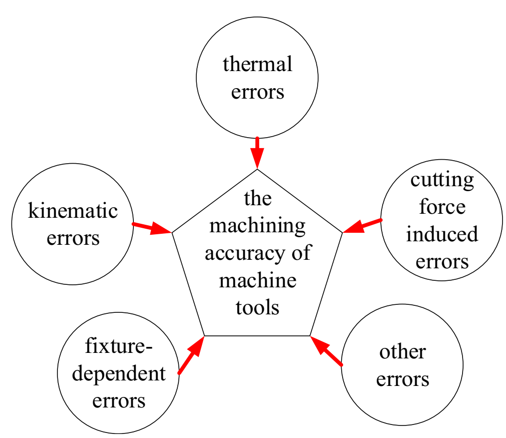

Fig. 1 The main error resources of machine tools

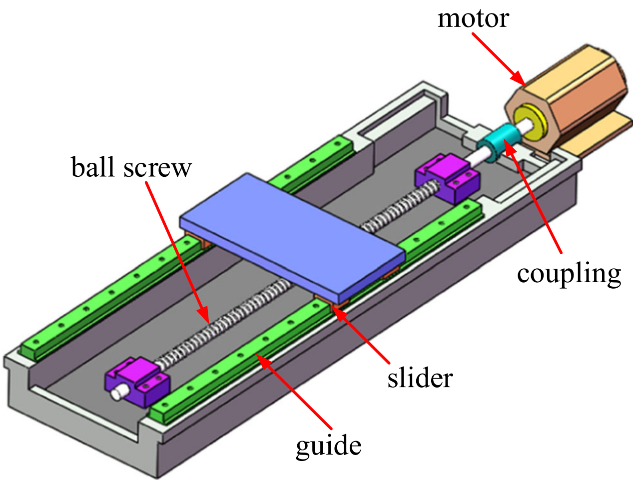

Fig. 2 The schematic diagram of the linear guideway

errors effectively, geometric profile errors are regarded as a bridge between tolerances and kinematic errors of machine tools' guideways. Therefore, the kinematic errors prediction model based on tolerance is established. Finally, a measuring experiment is conducted on guideways of multi-axis machine tools. Capability of the presented method is verified via comparison of predicted values and measured values. The rest of this paper is organized as follows: Section 2 gives a detailed account of kinematic errors of guideways. Section 3 presents the kinematic errors prediction model of guideways. In this model, the mapping relationship between tolerance and geometric profile errors of machine tools' guideways is formulated for the first time. Subsequently, the mapping relationship between geometric profile errors and kinematic errors of guideways is established. Therefore, kinematic errors prediction model based on tolerance of guideways is formulated naturally. The simulation verification and a measuring experiment are presented, then the predicted results are contrasted with measured results to prove the validity and feasibility of the method is discussed in Section 4. Finally, the conclusions are drawn in Section 5.

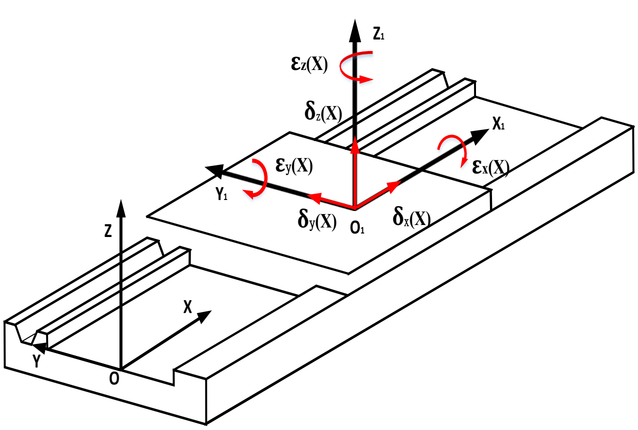

Fig. 3 The schematic diagram of kinematic errors of guideway

Table 1 Kinematic errors of *X*-axis

		X-axis
Positioning error		$\delta_x(x)$
Straightness error	y direction	$\delta_y(x)$
	z direction	$\delta_z(x)$
Roll error		$\varepsilon_x(x)$
Pitch error		$\varepsilon_y(x)$
Yaw error		$\varepsilon_z(x)$

2 Kinematic errors analysis of guideways

In the machining process, machining accuracy of machine tools was affected by many factors, such as geometric errors, thermal errors, cutting force induced errors, tool wear, fixture-dependent errors, and servo errors. Among them, geometric errors are one of the major contributors to machining accuracy of machine tools [30]. According to Ref. [29], geometric errors consist of kinematic errors and location/assembly errors; however, most of location/assembly errors can neglect. The main error resources of machine tools are given in Fig. 1. The linear guideway driven by ball screw was adopted for our studies, as shown in Fig. 2. It is comprised of guide, ball screw, slider, coupling, and motor.

According to BS ISO 230-1:2012 [31], each moving part of a machine tools has six degrees of freedom (DOFs) in the Cartesian coordinate system based on rigid body motion theory. Taking *X*-axis of machine tools as an example, when machine tools moves along the *X*-axis, six kinematic errors were generated, including positioning error, straightness error, pitch error, yaw error, and roll error. As shown in Fig. 3, six kinematic errors of guideway were analyzed, where $\delta_i(x)$ $(i = x, y, z)$ represent kinematic errors of *X*-axis along the *i* direction, and $\varepsilon_i(x)$ $(i = x, y, z)$ represent kinematic errors of *X*-axis around the *i* direction. Cumulative-lead error of screw affects the positioning error $\delta_x(x)$, geometric profile error of guideway in vertical plane affects the straightness error $\delta_y(x)$, geometric profile error of guideway in horizontal plane affects the straightness error $\delta_z(x)$, the parallelism error of two guideways affects the roll error $\varepsilon_x(x)$, the straightness error of guideway in vertical plane and length of moving parts affects the

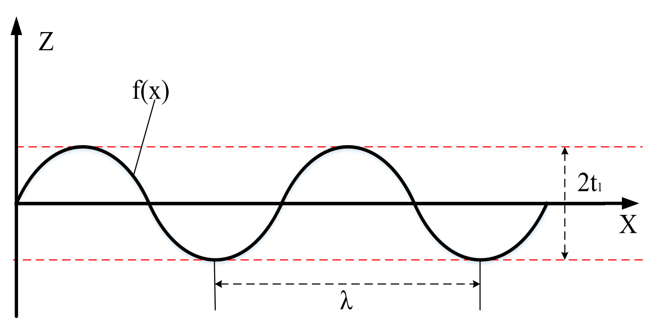

Fig. 4 The schematic diagram of the curve of guideways surface

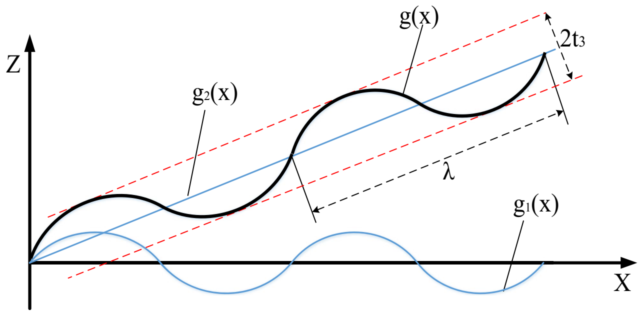

Fig. 5 The schematic diagram of cumulative-lead error

pitch error $\varepsilon_y(x)$, and the straightness error of guideway in horizontal plane and length of moving parts affect the yaw error $\varepsilon_z(x)$ [32]. Therefore, all kinematic errors of X-axis are listed, as shown in the Table 1.

3 Kinematic errors prediction model of guideways

In the design stage of machine tools, only the information of tolerance of machine tools' key components is known. As is known to all, the definition of tolerance is the allowable variation range of actual parameter values of parts in the design and manufacture process. Tolerance is the key index for machine parts. It includes tolerance of form and position, and tolerance of dimension. Since kinematic errors of machine tools' key components are unknown during the design stage, designers and engineers obtain kinematic errors only by making use of design experiences. Therefore, it is important to predict kinematic errors of guideways in the initial design stage of machine tools based on tolerance.

3.1 Mapping relationship between tolerance and geometric profile errors

According to Refs. [24, 28, 29], the guideway surface generally presents a random trend. There are several different surface error forms of guideways due to different machining processes. These guideways geometric profile errors fulfill

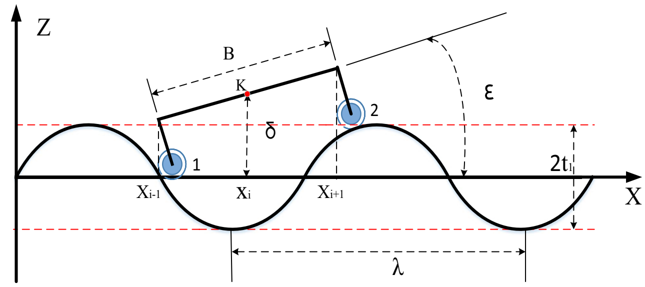

Fig. 7 Schematic for a slider moving along guideways

Dirichlet boundary conditions, hence the geometric profile error can be represented by a series of Fourier, and the change of the surface is within tolerance limits, as Eq. (1) presents.

$$f(x) = t_1 \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{\mathrm{n}} \sin\left(\frac{2\pi n x}{\lambda}\right) \tag{1}$$

According to the Ref. [24], with the increase of the term of Fourier series, the amplitude of each harmonic constituent gradually decreases. Hence, the influence of geometric profile errors on kinematic errors of guideway will decrease as well. The research subject of this paper focus on large-scale guideways, which were different from small-scale guideways, small changes of guideways surface have a little influence on kinematic errors. In addition, the first term of Fourier series with the largest wavelength has a marked impact on the kinematic errors of guideway. Therefore, according to the Ref. [24], in order to calculate the value of kinematic errors easily and effectively, we selected the first term of Fourier series to represent the curve of guideway surface, which is known as the truncated Fourier series function. Hence, it was applied to fit curve of guideways surface, as shown in Eq. (2).

$$f(x) = t_1 \sin\left(\frac{2\pi x}{\lambda}\right) \tag{2}$$

where $f(x)$ denotes curve of guideways surface in $Z-X$ plate, t_1 denotes tolerance of straightness in $Z-X$ plate and also denotes the amplitude of the curve $f(x)$, λ denotes wavelength of the curve $f(x)$, as is seen in Fig. 4.

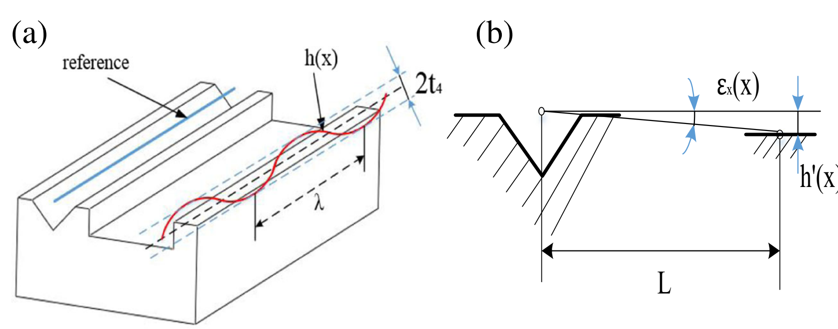

Fig. 6 The schematic diagram of parallelism error of the two guideways. a Parallelism error. b Roll error

Similarly, the curve of guideways surface in $Y-X$ plate can also be fitted by this approach, as shown in Eq. (3).

$$f_1(x) = t_2 \sin\left(\frac{2\pi x}{\lambda}\right) \quad (3)$$

where $f_1(x)$ denotes curve of guideways surface in $Y-X$ plate, t_2 denotes tolerance of straightness in $Y-X$ plate.

According to aforementioned analysis, the straightness error, the pitch error and the yaw error can be predicted by the curve of guideways surface ($f(x)$ and $f_1(x)$). However, the position error is mainly affected by the manufacturing accuracy of screw. Hence, the position error can be represented by cumulative-lead error of screw. According to the Ref. [33], cumulative-lead error is composed of two parts which are the lead deviation and the cumulative representative lead error. Hence a combination of monotone function and truncated Fourier series function can be applied to fit the change of cumulative-lead error, as shown in Fig. 5 and Eq. (4),

$$g(x) = g_1(x) + g_2(x) = t_3 \sin\left(\frac{2\pi x}{\lambda}\right) + ax \quad (4)$$

where $g_1(x)$ represents the lead deviation of screw, $g_2(x)$ represents the cumulative representative lead error of screw, $g(x)$ represents cumulative-lead error of screw, t_3 represents the positioning tolerance of screw, λ represents wavelength of the curve $g(x)$, and a represents the scale coefficient of cumulative-lead error.

The roll error is significantly affected by parallelism error of two guideways and length of carriage. Therefore, take one of guideways as the reference, another change can be fitted by the truncated Fourier series function, as is seen in Fig. 6 and Eq. (5).

$$h(x) = t_4 \sin\left(\frac{2\pi x}{\lambda}\right) \quad (5)$$

where $h(x)$ represents parallelism error of the two guideways and t_4 denotes tolerance of parallelism.

3.2 Mapping relationship between geometric profile errors and kinematic errors

Suppose that there is a carriage moves along curve of guideways surface ($f(x)$), B denotes the width of carriage, K denotes the midpoint of carriage, ε denotes angular error, and δ denotes linear error, as shown in Fig. 7. Before analyzing the relationship between geometric profile errors and kinematic errors, some assumptions are as follows:

(1) Geometric profile errors exist in guideways only.
(2) Only quasistatic condition is considered.

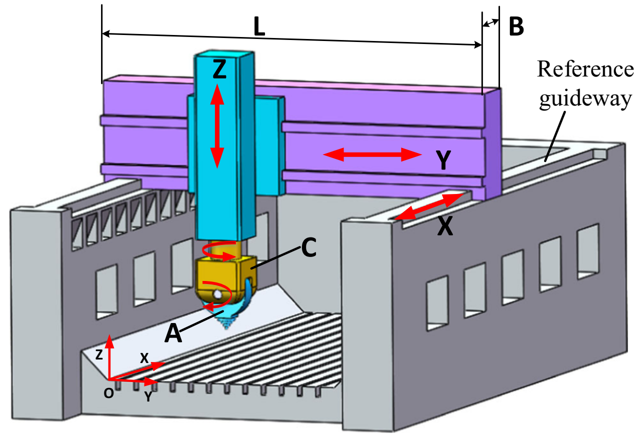

Fig. 8 The structure of the gantry-type five-axis milling machine tools

(3) The sizes of two points 1 and 2 are neglected; both of them are considered as rigid points.
(4) The deformation of guideways caused by carriage weigh and load is not taken into account.
(5) In order to avoid Bryan error, the straightness error is selected at the midpoint (point K) of carriage.

Based on aforementioned assumptions, the relationship between geometric profile errors and kinematic errors can be obtained, as shown in Eqs. ((6)–(11)).

$$\delta_z(x) = \frac{f(x_{i-1}) + f(x_{i+1})}{2} \quad (6)$$

$$\varepsilon_y(x) = \frac{f(x_{i+1}) - f(x_{i-1})}{B} \quad (7)$$

$$\delta_y(x) = \frac{f_1(x_{i-1}) + f_1(x_{i+1})}{2} \quad (8)$$

$$\varepsilon_z(x) = \frac{f_1(x_{i+1}) - f_1(x_{i-1})}{B} \quad (9)$$

$$\delta_x(x) = g(x) \quad (10)$$

$$\varepsilon_x(x) = \frac{h'(x)}{L} = \frac{h(x_{i-1}) + h(x_{i+1})}{2L} \quad (11)$$

where L denotes the length of carriage.

3.3 Mapping relationship between tolerance and kinematic errors

Substituting Eq. (2) into Eqs. ((6) and (7)), Z directional straightness error $\delta_z(x)$ and pitch error $\varepsilon_y(x)$ at position x_i can be expressed in Eqs.((12) and (13)).

$$\delta_z(x) = \frac{t_1\left(\sin\left(\frac{2\pi x_{i+1}}{\lambda}\right) + \sin\left(\frac{2\pi x_{i-1}}{\lambda}\right)\right)}{2} \quad (12)$$

Table 2 XL-80 Laser head performance indicators

Index items	Parameter
System accuracy (ppm)	± 0.5
Laser precision (ppm)	0.05
Resolution (μm)	0.001
Maximum measurement speed (m/s)	4
Maximum sampling frequency (KHz)	50
Measuring range (m)	0–80
Warm-up time (min)	< 6

$$\varepsilon_y(x) = \frac{t_1\left(\sin\left(\frac{2\pi x_{i+1}}{\lambda}\right) - \sin\left(\frac{2\pi x_{i-1}}{\lambda}\right)\right)}{B} \tag{13}$$

Similarly, Y directional straightness error $\delta_y(x)$ and yaw error $\varepsilon_z(x)$ at position x_i can be obtained, as shown in Eqs. ((14) and (15)).

$$\delta_y(x) = \frac{t_2\left(\sin\left(\frac{2\pi x_{i+1}}{\lambda}\right) + \sin\left(\frac{2\pi x_{i-1}}{\lambda}\right)\right)}{2} \tag{14}$$

$$\varepsilon_z(x) = \frac{t_2\left(\sin\left(\frac{2\pi x_{i+1}}{\lambda}\right) - \sin\left(\frac{2\pi x_{i-1}}{\lambda}\right)\right)}{B} \tag{15}$$

The position error is mainly affected by manufacturing accuracy of screw. Hence the position error at position x_i can be directly represented by cumulative-lead error of screw, as shown in Eq. (16).

$$\delta_x(x) = t_3 \sin\left(\frac{2\pi x}{\lambda}\right) + ax \tag{16}$$

Substituting Eq. (5) into Eq. (11), the roll error at position$x_i$can be expressed in Eq.(17).

$$\varepsilon_x(x) = \frac{t_4\left(\sin\left(\frac{2\pi x_{i+1}}{\lambda}\right) + \sin\left(\frac{2\pi x_{i-1}}{\lambda}\right)\right)}{2L} \tag{17}$$

Based on this approach, kinematic errors can be predicted in the initial design stage of machine tools. The new approach is beneficial for improving accuracy in error compensation and deriving precisely the distribution of kinematic errors

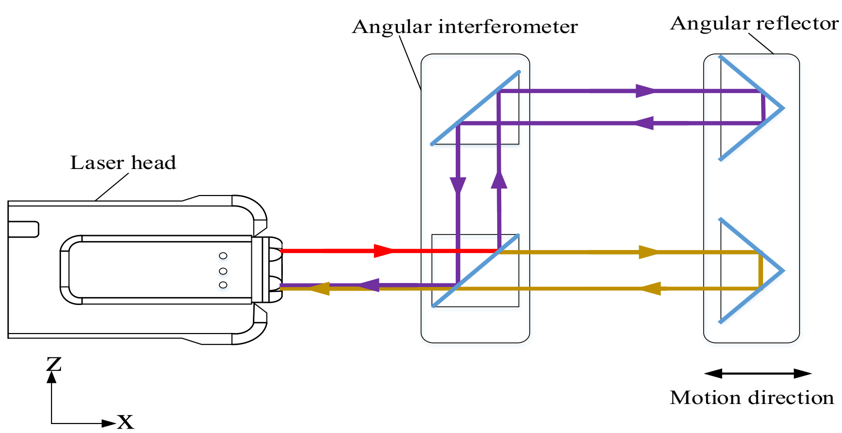

Fig. 9 The measuring principle diagram of laser interferometer

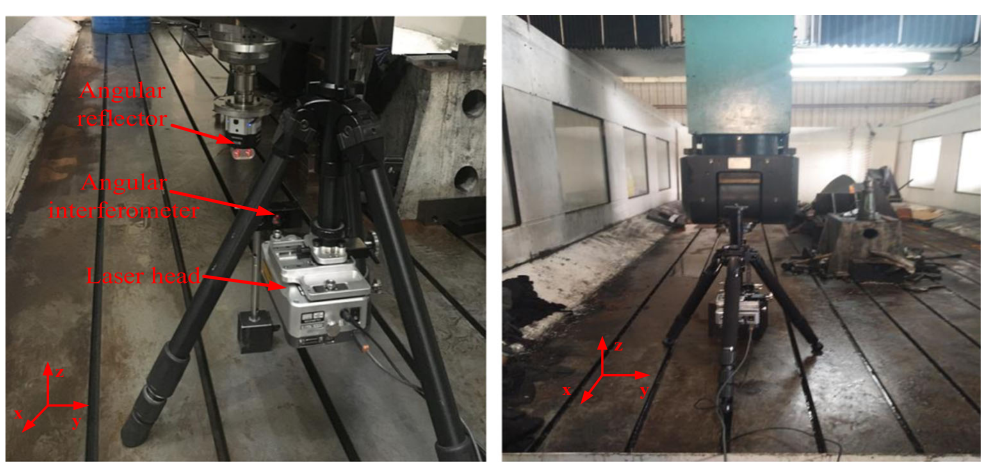

Fig. 10 The scenes of measurement with laser interferometer

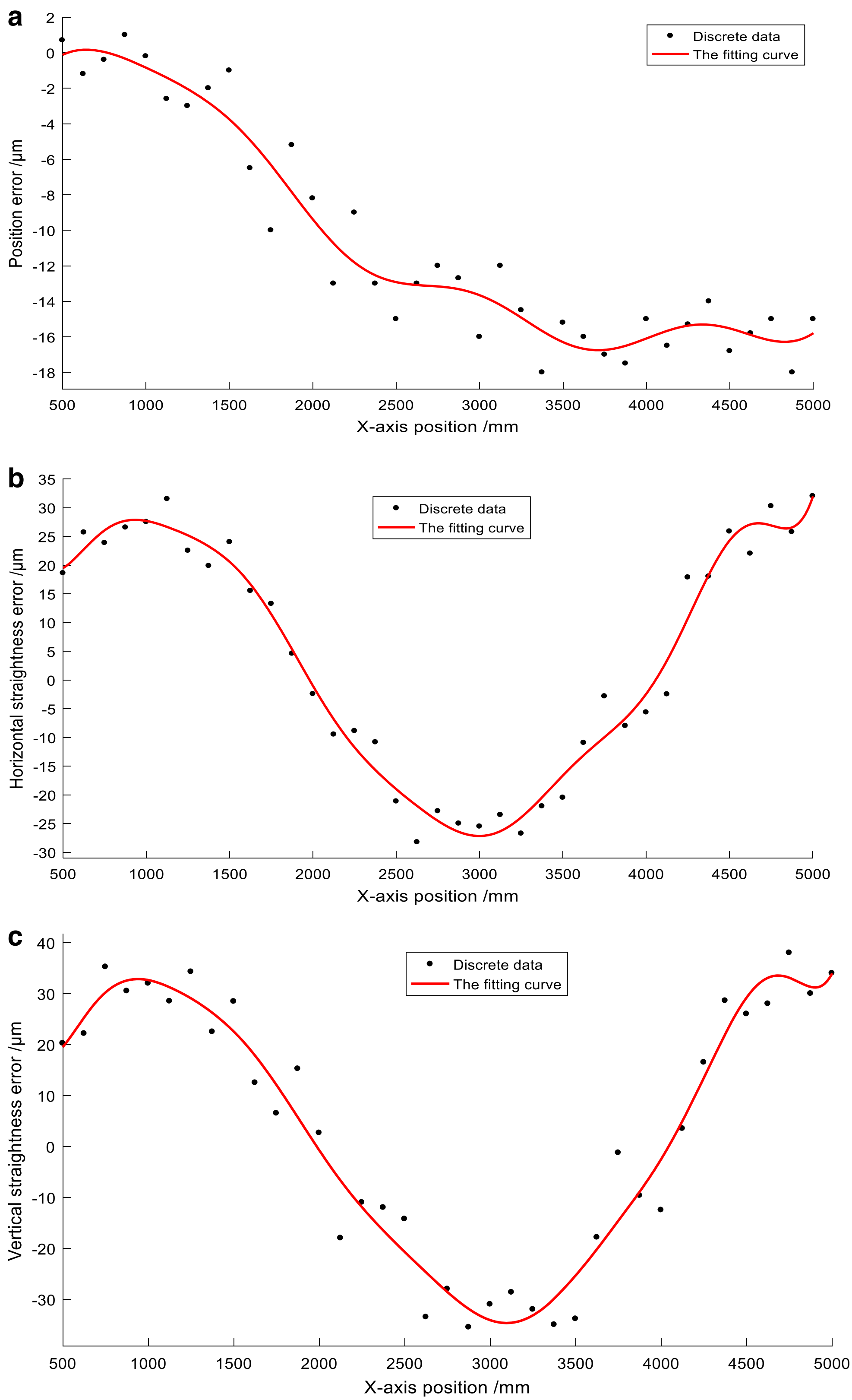

Fig. 11 The scatter diagram of discrete data and the Fourier curve diagram for kinematic errors of *X*-axis. **a** Position error. **b** Horizontal straightness error. **c** Vertical straightness error. **d** Roll error. **e** Yaw error. **f** Pitch error

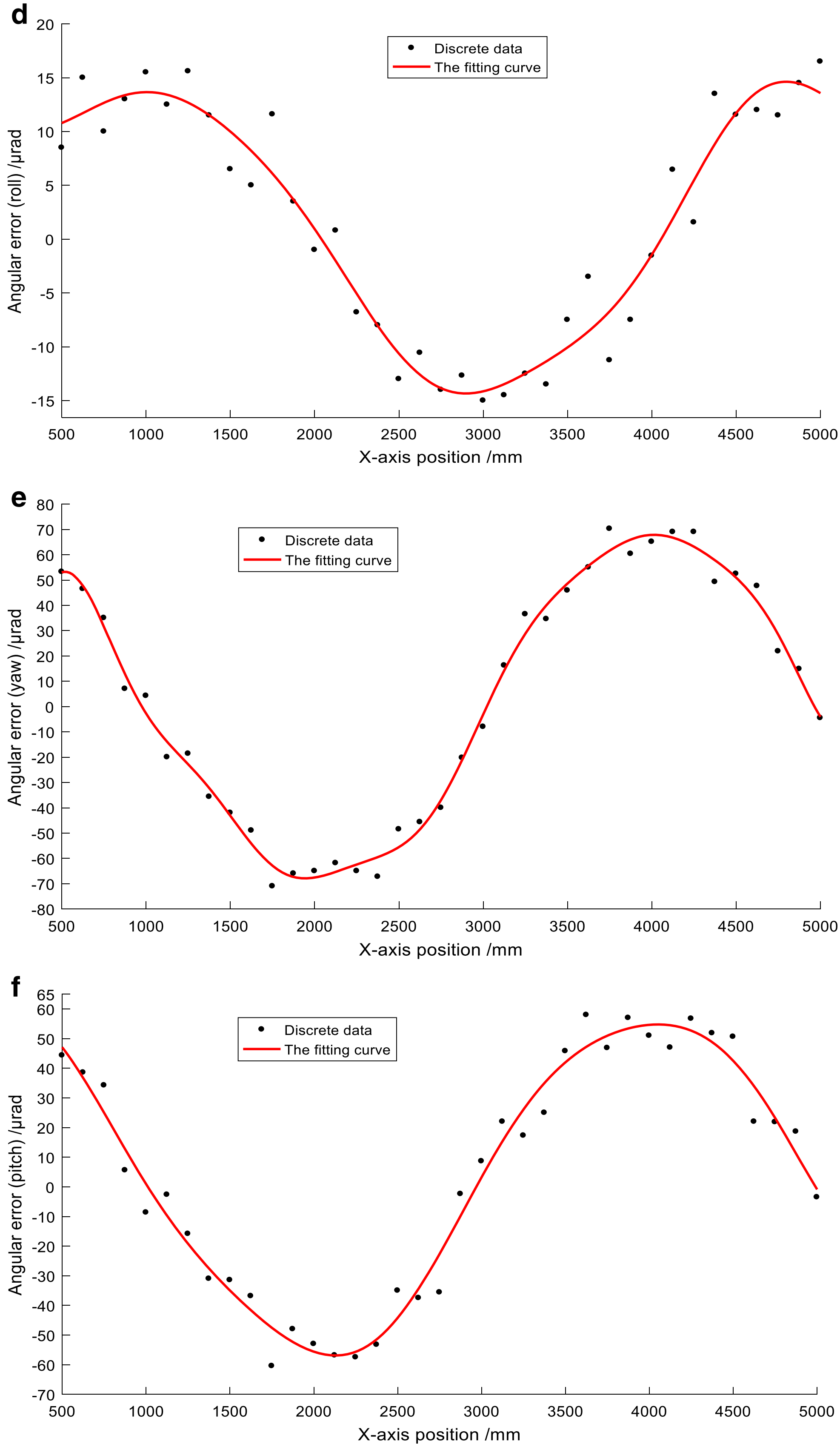

Fig. 11 (continued)

during the design stage. Thus, it is useful and practical for machine tools designer.

4 Experimental validation

In order to verify the accuracy, feasibility and effectiveness of the proposed kinematic errors prediction model of machine tools' guideways, a measuring experiment is carried out on a linear axis of the gantry-type five-axis milling machine tools (XKAS2525). Taking X-axis as an example (Y-axis and Z-axis can be done in the same method), the work stroke of X-axis is 5000 mm, the distance of a pair of guideways (L) is 2500 mm, and the width of carriage (B) is 1000 mm, as seen in Fig. 8.

4.1 Measurement and prediction of kinematic errors

A dual-frequency laser interferometer has the characteristics of high precision, high resolution and quick response [7], it has been extensively used in high precision displacement measurements. In this study, six kinematic errors of guideways can be measured and equivalently identified with nine-line method by utilizing the dual-frequency laser interferometer (XL-80 by Renishaw in the UK). The main components of the dual-frequency laser interferometer measurement system are laser head, linear optics kit, angular optics kit, straightness measurement kit (vertical direction), and straightness measurement kit (horizontal direction). The key parameters of XL-80 are shown in Table 2.

Before measuring, the stability of laboratory environment and working condition of NC machine tools should be guaranteed. Firstly, it is needed to make sure that there is no vibration source in the surrounding environment. Secondly, in order to remove the effect of environment temperature on the measurement results, there is need for NC machine tools to warm-up for half an hour before measurement. In the measuring process, the ambient temperature is controlled at 20 °C within ± 2 °C to minimize thermal errors. Moreover, in order to improve the measurement stability, the measurement of kinematic errors was carried out three times, and then the ultimate error values are the average results of three times identified and measured values. Meanwhile, to eliminate the setup errors as far as possible, the experimental system is carefully installed.

The yaw error of X-axis guideway is considered as an example for the measurement. The measuring principle of laser interferometer is shown in Fig. 9. The angular interferometer is mounted on the worktable, while the angular reflector is mounted on the moving spindle. Figure 10 represents the scenes of measurement with laser interferometer. The work stroke of X-axis guideway is chosen 37 equidistant discrete points. Lingering for a few seconds before carriage moves to next measurement position, and then the yaw error can be measured at the same time. Similarly, other kinematic errors can also be measured by using this method.

Table 3 The R-square values of the fitting curve

$\delta_x(x)$	$\delta_y(x)$	$\delta_z(x)$	$\varepsilon_x(x)$	$\varepsilon_y(x)$	$\varepsilon_z(x)$
0.9241	0.9602	0.9245	0.9202	0.9824	0.9613

Fourier curve fitting is carried out by MATLAB 2016b on a computer having a 3.10 GHz frequency based on nonlinear least squares method; the scatter diagram of discrete data and the Fourier curve diagram for kinematic errors of X-axis guideway can be obtained, as seen in Fig. 11. In order to further verity the validity of the data fitting method, R-square is calculated as shown in Table 3. This shows that the fitting method used in this paper possesses higher fitting accuracy. Therefore, the measured value can be truly represented by the fitting curve. In order to predict kinematic errors, $t_i(i = 1, 2, 3, 4)$ can be obtained from BS ISO 8636-2:2007 [34], and $B/\lambda = 0.25$ can be seen in [24], main parameters are listed in Table 4. In addition, for predicting the roll error, take one of guideways as the reference, as shown in Fig. 8.

On the basis of Eqs. ((11)–(16)) and Table 4, kinematic errors of X-axis guideway can be predicted, as shown in Fig. 12.

4.2 Result comparison

By comparing with the measured results, the feasibility and effectiveness of the proposed kinematic errors prediction model of machine tools' guideways have been further validated. Results of the comparison between the predicted and measured results are presented in Fig. 12. The red lines represent the fitting curve of measured results, the blue lines represent the predicted results, and the green lines represent residual errors which are calculated by the subtraction between the measured error data and the predicted error data. From Fig. 12, it can be seen that the minimum and maximum predicted position error value of X-axis guideway are − 17 and 0.12 μm, respectively, which means the range of the predicted position error $\delta_x(x)_p$ is 17.12 μm, and the minimum and

Table 4 Main parameters

Main parameters for predicting	Value
Tolerance of straightness in $Z-X$ plate (t_1)	0.05 mm
Tolerance of straightness in $Y-X$ plate (t_2)	0.04 mm
The positioning tolerance of screw (t_3)	0.003 mm
Tolerance of parallelism (t_4)	0.05 mm
The scale coefficient of cumulative-lead error (a)	0.000004
Wavelength (λ)	4000 mm
The width of carriage (B)	1000 mm
The distance of a pair of guideways (L)	2500 mm

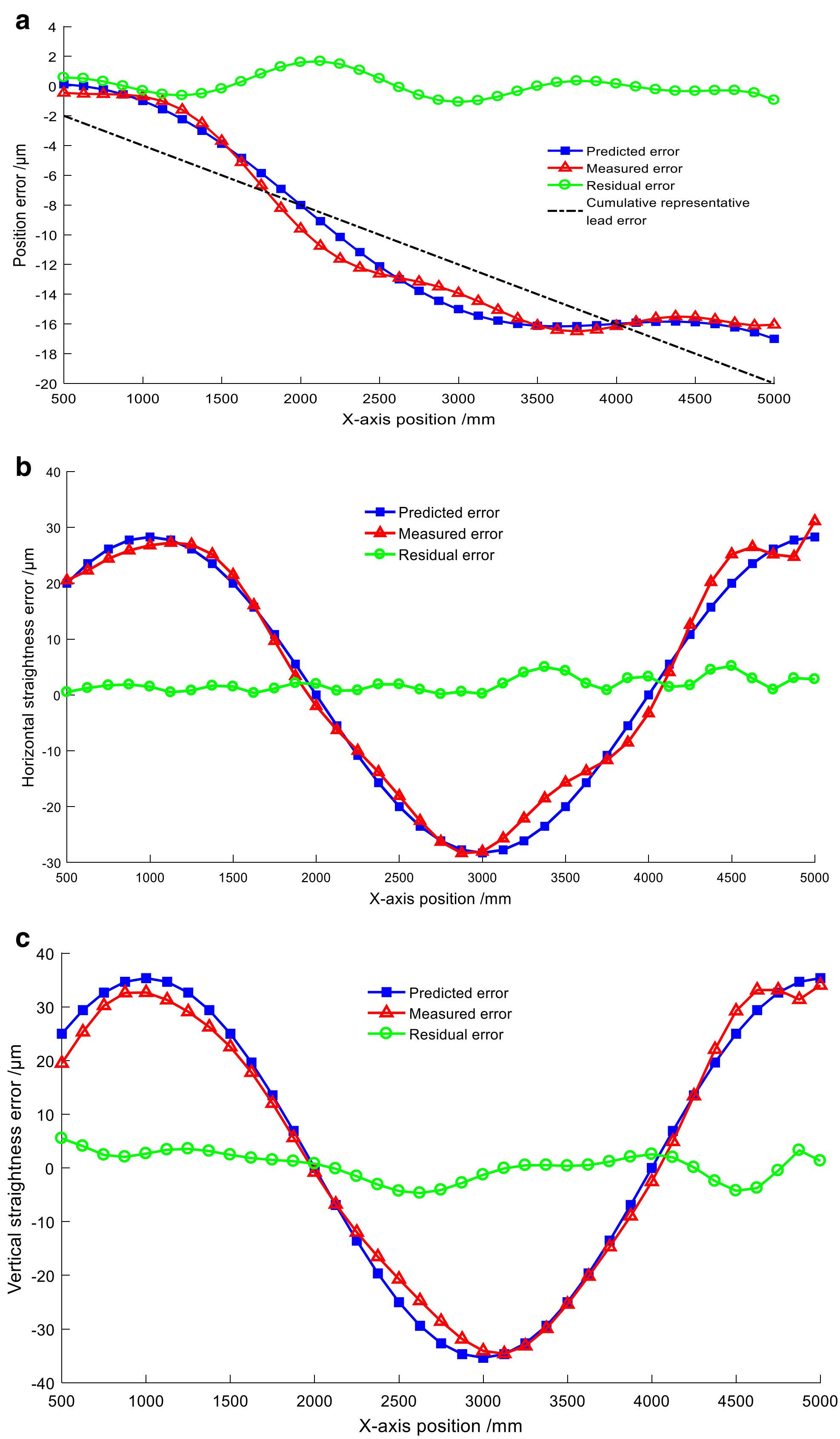

Fig. 12 Results of the comparison between the predicted and measured data of kinematic errors. **a** Position error. **b** Horizontal straightness error. **c** Vertical straightness error. **d** Roll error. **e** Yaw error. **f** Pitch error

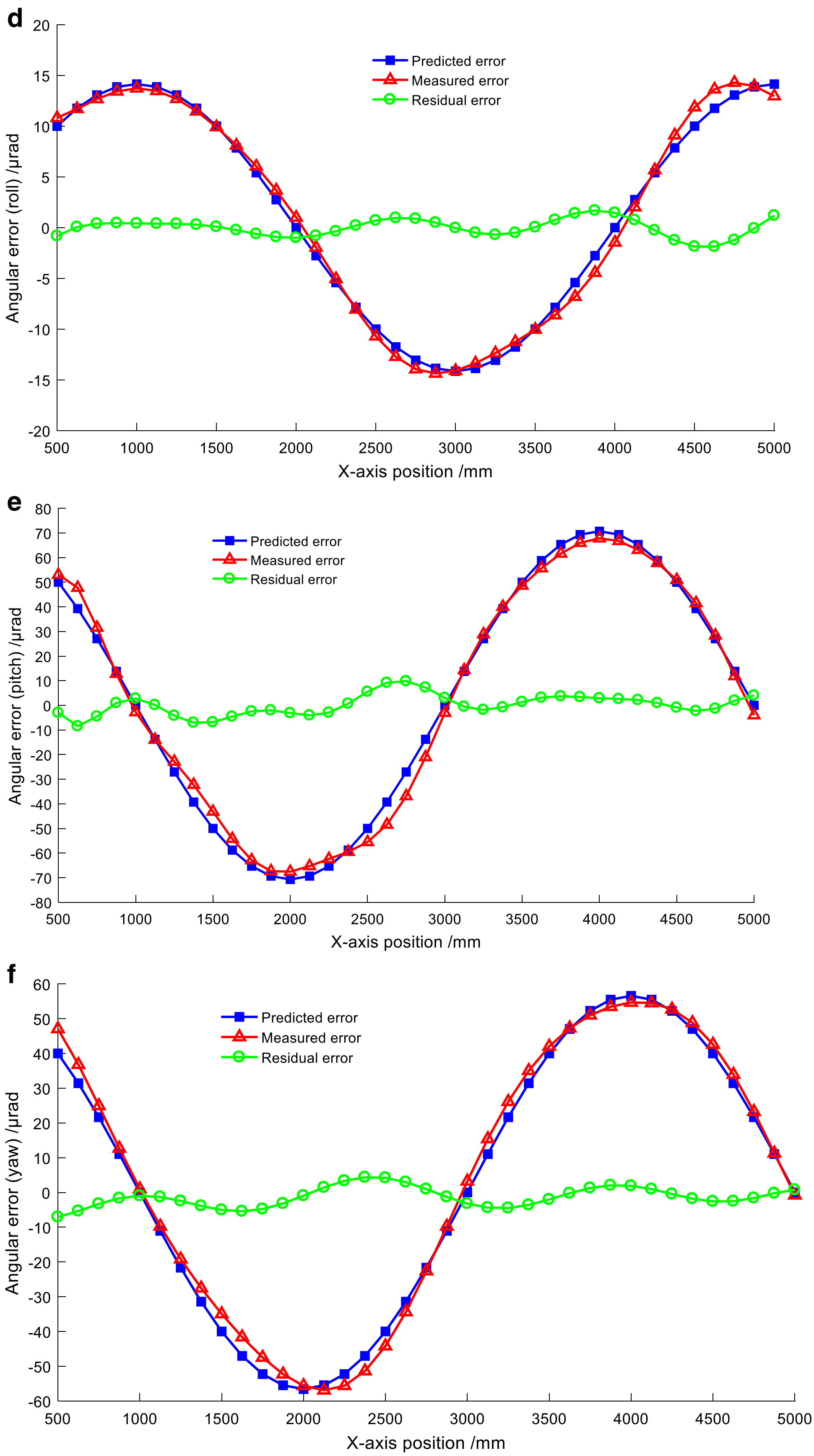

Fig. 12 (continued)

Table 5 The range of predicted and measured values of kinematic errors

	$\delta_x(x)$ (μm)	$\delta_y(x)$ (μm)	$\delta_z(x)$ (μm)	$\varepsilon_x(x)$ (μrad)	$\varepsilon_y(x)$ (μrad)	$\varepsilon_z(x)$ (μrad)
Predicted	17.12	56.57	70.71	28.28	141.42	113.14
Measured	16.96	59.43	68.63	28.65	135.40	111.58
D value	0.16	2.86	2.08	0.37	6.02	1.56

Table 6 The maximum and average residual errors between the predicted and measured values of kinematic errors

	$\delta_x(x)$ (μm)	$\delta_y(x)$ (μm)	$\delta_z(x)$ (μm)	$\varepsilon_x(x)$ (μrad)	$\varepsilon_y(x)$ (μrad)	$\varepsilon_z(x)$ (μrad)
Maximum	1.67	5.19	5.50	1.87	9.81	7.07
Average	0.57	1.93	2.22	0.70	3.42	2.65

maximum measured position error value of *X*-axis guideway are − 16.50 and − 0.46 μm, respectively, which means the range of the measured position error $\delta_x(x)_m$ is 16.96 μm. The difference value (*D* value) between the range of measured result and predicted result is 0.16 μm. Similarly, the range of the measured and predicted results, and *D* value of other kinematic errors can be obtained, as shown in Table 5.

Moreover, the maximum and average residual errors between the predicted and measured values of kinematic errors are shown in Table 6. Figure 13 is the distribution of absolute residual errors value of kinematic errors. The vertical axis of the histogram is the value of absolute residual errors, and horizontal axis is the amount of measurement points. According to Fig. 13, comparing with the measured results of kinematic errors, the vast majority of residual errors are considerably small. Residual errors may be caused by two primary causes: (1) the existence of non-geometric error sources, such as control errors, dynamitic errors, assembly inaccuracy, and thermal errors for example, and (2) some assumptions were utilized for predicting kinematic errors. The results demonstrate that the core thought of new approach can be applied to predict kinematic errors of machine tools' guideways based on tolerance in the initial design stage of machine tools.

5 Conclusion

The main objective of this paper is to highlight the effectiveness of the proposed kinematic errors prediction method for a gantry-type five-axis milling machine tools' guideways based on tolerance. Comparing with previous methods [24, 25, 29], the advantages of this new method are as follows: (1) the curve of guideways surface can be fitted precisely by the truncated

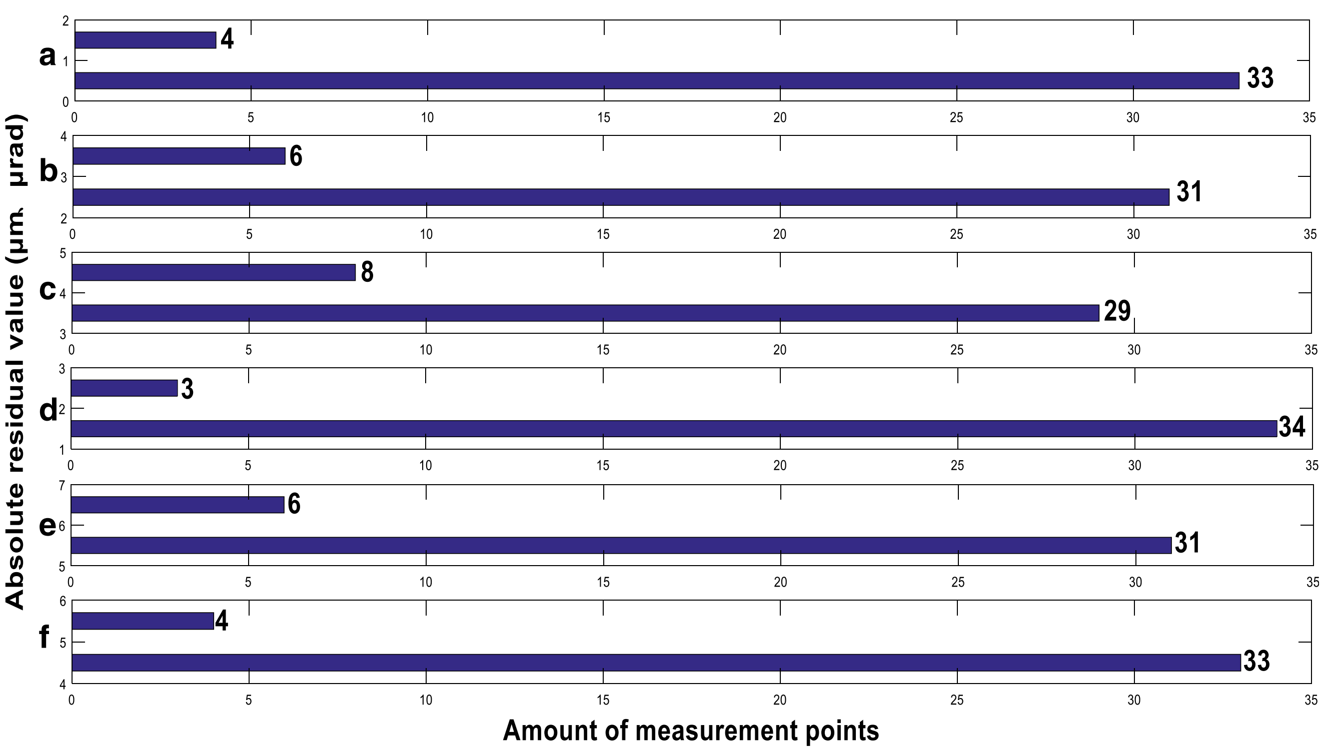

Fig. 13 The distribution of absolute residual between the predicted and measured results of kinematic errors. **a** Position error. **b** Horizontal straightness error. **c** Vertical straightness error. **d** Roll error. **e** Yaw error. **f** Pitch error

Fourier series function. Thus, it does not require to be measured by any special measuring devices. (2) The mapping relationship between tolerance and kinematic errors is firstly formulated by means of taking geometric profile errors as a bridge between tolerance and kinematic errors of machine tools' guideways. (3) It is beneficial for researchers and practicing engineers to predict kinematic errors of machine tools' guideways based on tolerance in the initial design stage of machine tools, and obtain the distribution of kinematic errors. (4) It can be straightforwardly applied in different guideways type.

To test the practicability and effectiveness of the proposed method, a measuring experiment is carried out. By utilizing the Fourier curve fitting approach, the measured and identified discrete data can be truly represented by the fitting curve. The range of the measured kinematic errors $\delta_x(x)$, $\delta_y(x)$, $\delta_z(x)$, $\varepsilon_x(x)$, $\varepsilon_y(x)$, $\varepsilon_z(x)$ are 16.96 μm, 59.43 μm, 68.63 μm, 28.65 μrad, 135.40 μrad, and 111.58μrad, respectively, and the range of the predicted kinematic errors are 17.12 μm, 56.57 μm, 70.71 μm, 28.28 μrad, 141.42 μrad, and 113.14 μrad, respectively. Hence the predicted results are in coincidence with the measured result. Furthermore, the maximum residual errors between the predicted and measured results of kinematic errors are 1.67 μm, 5.19 μm, 5.50 μm, 1.87 μrad, 9.81 μrad, and 7.07 μrad, respectively. Comparing with the measured results of kinematic errors, residual errors are considerably small and can be neglect. Therefore, there is no doubt that this method is accurate, feasible, and effective enough for predicting kinematic errors in the initial design stage of machine tools and can be substitute for measurement of kinematic errors. The new approach is beneficial for improving accuracy in error compensation and deriving precisely the distribution of kinematic errors during the design stage, which is useful and practical for machine tools designer. Its core thought can be applied to other type of machine tools' guideways.

Despite the progress made in this paper, the non-geometric error sources are not taken into consideration. However, some other errors, such as thermal errors, cutting force induced errors, tool wear, fixture-dependent errors, and servo errors, also contribute to the measured results of machine tools' guideways. Therefore, all of the errors, or at least most of them would be a focus of future research. In view of the limitation of space and experimental conditions, every guideways are impossible to be verified conclusively. However, we will take as many guideways as possible into account in future studies.

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