<span id="page-0-0"></span>**ORIGINAL ARTICLE** ORIGINAL ARTICLE



# Optimal maintenance control of machine tools for energy efficient manufacturing

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#### Abstract

Performance of machine tool tends to deteriorate in the production process. This deterioration increases the processing energy consumption and leads to more defectives and corresponding energy waste. Maintenance can be taken to restore the performance of machine tool and improve the energy efficiency, which has a significant impact on the total energy consumption and productivity. This paper proposes an approach to improve the energy efficiency of the production process through scheduling the maintenance actions of the machine tool, taking into account productivity, product quality, and energy consumption. The deteriorating machine tool is modeled as a discrete-time, discrete-state Markov process. Partially observable Markov decision process (POMDP) framework is applied to develop the maintenance decision-making model, where the joint observation of processing energy consumption and quality of manufactured workpiece is used to infer the status of the machine tool. An optimal maintenance policy maximizing the total expected reward about energy efficiency over a finite horizon is obtained, which consists of a sequence of decision rules corresponding to the optimal action for each belief vector. The characteristics of the optimal policy are illustrated through a numerical example and the effects of parameters on the policy are analyzed.

Keywords Energy efficiency . Machine tool . Maintenance control . Partially observable Markov decision process

# 1 Introduction

Industrial sector consumes a large amount of energy and results in serious greenhouse gas emissions, where the manufacturing plays a crucial role. According to Abdelaziz et al. [\[1\]](#page-7-0), the energy consumption of industrial sector in China accounts for 70% of the national total energy usage, of which the manufacturing consumes 85.2% of the final industrial energy usage. A comparison of the gas emissions between machine tool and sports utility vehicle (SUV) indicates that improving the energy efficiency of machine tool is of great importance [\[2](#page-7-0)]. Due to increasing energy demand and environmental concerns, energy efficient manufacturing is receiving more and more attention from both academic and industrial community.

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Performance of machine tool tends to deteriorate as the manufacturing process proceeds. This deterioration may lead to increased processing energy consumption [[3](#page-7-0)–[6](#page-7-0)] and poor quality of manufactured workpiece. Note that defective workpiece wastes energy significantly, especially if there are many machining operations before scrapping. Maintenance actions such as inspection, repair, or replacement are taken to restore the performance of machine tool and improve the energy efficiency [\[4,](#page-7-0) [7\]](#page-7-0). However, frequent maintenance may increase the downtime and decrease the productivity. Additionally, there will be a large amount of energy consumption to restart and maintain the machine tool in a ready position after each maintenance [[8,](#page-7-0) [9\]](#page-7-0).

Taking into consideration productivity, product quality, and energy consumption, this paper aims to propose an approach to improve the energy efficiency of the production process through scheduling the maintenance actions of the machine tool. Herein, the deteriorating process of the machine tool is modeled as a multi-state discrete-time Markov process, where the state of the machine tool is not directly observable and could be inferred from observations about processing energy consumption and quality of manufactured workpiece. Hence, this situation is partially observable. Generally, there are two sources resulting in the partial observability: (i) multiple states may give the same observation and (ii) observations are noisy: observing the same state can obtain different observations [[10](#page-7-0)]. In addition, there are four maintenance actions available  $A = \{a_1, a_2, a_3, a_4\}$  during each horizon. Action  $a_1$  is to do nothing, i.e., let the system continue operating. Action  $a_2$  is to observe the manufactured workpiece after manufacturing. Action  $a_3$  stops the machine tool for inspection, and maintenance is performed if it has failed. Action  $a_4$  is to perform maintenance directly without inspecting the system first. At each horizon, the decision-maker has to take an action selected from the available actions and then receives a reward about energy efficiency. To maximize the cumulative reward over a finite horizon, partially observable Markov decision process (POMDP) framework is applied in this paper.

The remainder of this paper is organized as follows. Related literature is reviewed in Section [3.](#page-2-0) Section [5](#page-3-0) models a deteriorating machine tool as a discrete-time, discrete-state time-homogeneous Markov process. Section [6](#page-7-0) investigates the observations of the manufactured workpiece for the Markov model. Section [7](#page-7-0) develops a maintenance decisionmaking model based on POMDP. Section [6](#page-7-0) analyzes the effects of some parameters on the optimal policy. Section [7](#page-7-0) concludes the paper by a brief summary.

## 2 Literature review

#### 2.1 Approaches for energy efficient manufacturing

Many approaches for energy efficient manufacturing have been proposed on different aspects. Some approaches try to improve the energy efficiency of the machine tool on the design level, such as kinetic energy recovery system (KERS) [[11\]](#page-7-0), lightweight design [\[12](#page-8-0)]. These approaches can help the machine manufacturers develop energy efficient machine tools with advanced functions, but may be impractical for the plant managers when using the existing and relatively old machine tools. Focusing on the machining process, many research efforts have been made towards the optimization of cutting conditions and process parameters, such as Rajemi et al. [[13\]](#page-8-0), Mori et al. [[14](#page-8-0)], Hu et al. [[15\]](#page-8-0), and Oda et al. [\[3\]](#page-7-0). Whereas, machining processes differ in a variety of ways due to the different machines, workpieces, and environments. Most of these methods are proposed in given situations and may be not suitable for others.

Apart from above two aspects, the approach about production management has been regarded as one of the most economical and efficient ways for energy efficient manufacturing. Mouzon et al. [\[16](#page-8-0)] proposed an approach to determine the optimal production sequence which

would minimize the total energy consumption while optimizing total completion time. Shi et al. [\[17\]](#page-8-0) developed a decision-making model for energy saving by shutting down the machine tool when it was idle between processing steps. Shrouf et al. [[18](#page-8-0)] minimized the energy consumption of single machine through determining appropriate launch time of job processing, idle time, and operating time of the machine tool during a production shift. Chen et al. [[19\]](#page-8-0) further investigated the energy efficient manufacturing of a serial production line through scheduling machine startup and shutdown. Cao et al. [\[20](#page-8-0)] developed an energy efficient scheduling model to optimize the matching relationship between machine tools and production tasks based on the fact that the energy consumptions for processing the same workpiece in different machine tools were usually different. Most of these studies focus on optimizing the scheduling of machine startup and shutdown, or the scheduling of matching relationship between machine tools and production tasks. Meanwhile, the relationship between maintenance management and energy efficient manufacturing also receives some attention. Shao et al. [\[4](#page-7-0)] pointed out that replacing a worn cutting tool could decrease the processing energy consumption of the machine tool. Dietmair and Verl [\[7\]](#page-7-0) stated that replacing old components with energy efficient ones could be an effective way to improve the energy efficiency. Recently, Xu and Cao [\[21](#page-8-0)] developed mathematical models to evaluate the energy efficiency of the machine tool under periodic maintenance. They stated that the energy efficiency of machine tool could be improved through the optimization of maintenance scheduling. However, there is still lack of studies for improving the energy efficiency of machine tool through scheduling maintenance activities.

#### 2.2 Maintenance control about POMDP

For a large range of system, the internal state is partially observable and the related decision-making problem is formulated as a POMDP. POMDP has been widely used in many areas, such as machine maintenance and replacement, human learning and instruction, medical diagnosis and decision-making, and infrastructure management [[22,](#page-8-0) [23](#page-8-0)]. Anily and Grosfeld-Nir [[24](#page-8-0)] designed a production and inspection policy for a lot-sizing problem to guarantee a zero defective delivery with minimum total expected cost. Papakonstantinou and Shinozuka [[23\]](#page-8-0) used the POMDP framework to find the optimal inspection and maintenance policy for corroding reinforced concrete structure.

For the application of POMDP in machine maintenance, Ivy and Pollock [\[25\]](#page-8-0) studied a maintenance control problem with imperfect monitoring, silent failures and state-dependent repairs to minimize total expected cost. Meola [\[26](#page-8-0)] proposed a <span id="page-2-0"></span>method to investigate a maintenance decision-making problem considering the production cost about product quality, system failure cost, and maintenance cost. The former two papers treat cost as the decision objective. Integrating availability, process rate, and quality rate together, AlDurgam and Duffuaa [[27](#page-8-0)] developed a maintenance control model for a three-state POMDP system to maximize overall system effectiveness (OSE). Therein, the process rate and quality rate are assumed to be fixed in each state. Then AlDurgam and Duffuaa [\[28\]](#page-8-0) extended that model by assuming variable quality rate, and adding speed control actions along with maintenance actions. A distinguishing factor of this paper is that we intend to find the optimal maintenance policy for energy efficient manufacturing.

# 3 Markov model of the deteriorating machine tool

#### 3.1 Modeling

Manufacturing system tends to wear, deteriorate, or break as the production process proceeds. It is very common to model the deteriorating system as a multi-state system and each state represents a degradation level. To describe the dynamic behavior of the deteriorating process, the concept of "state transition" is introduced. Particularly, a state represents a condition of the system, and a state transition represents a change of the system condition. However, there is some uncertainty in the state transition. If in some situations where each state transition depends only on the current state and not on the historical states, this deteriorating process has the Markov property and the process is termed as a Markov process. Markov process has been widely used to model the deteriorating process of manufacturing system in many studies, such as Ivy and Pollock [[25](#page-8-0)], Chiang and Yuan [[29](#page-8-0)], Ben-Zvi and Grosfeld-Nir [[30\]](#page-8-0), and Le and Tan [[31\]](#page-8-0). This paper assumes that the deteriorating process of machine tool can be modeled by a discrete-time, discrete-state time-homogeneous Markov process.

As shown in Fig. 1, the Markov model incorporates  $n$ states with progressively increasing levels of deterioration and the transition probability between any two adjacent states is  $p$ . Machine tool starts working in state 1 and eventually goes to state  $n$ . State 1 is the perfect state with the



Fig. 1 Markov model of the deteriorating machine tool

best performance, while state  $n$  is the failure state with the worst performance. This paper assumes that once reaching state  $n$ , the machine tool continues operating until it is stopped by some exogenous actions. Referring to Fig. 1, the state transition process can be represented by a state transition matrix  $P$ , given by Eq.  $(1)$ 

$$
P = \begin{bmatrix} 1-p & p & 0 & \cdots & 0 & 0 \\ 0 & 1-p & p & \cdots & 0 & 0 \\ 0 & 0 & 1-p & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1-p & p \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}
$$
 (1)

These states 1, 2,  $\ldots$ , *n* do not necessarily correspond to explicit physical events occurring in the machine tool, and they may represent the deteriorating process of machine tool at a high level of abstraction. For example, the state of a machine tool describes the integrated condition of various unobservable components. Thus, the state of the machine tool is assumed to be partially observable.

#### 3.2 Estimating the parameters

If the machine tool works in any state, it manufactures a workpiece per time step. During any step, a state transition occurs. Even though the state does not change, it is usually viewed as a special state transition. If the system is currently in state i,  $i = 1, 2, ..., n-1$ , the result of a transition is that the system either remains in state  $i$  with probability  $1-p$  or moves to the next state j,  $j = 2, 3, ..., n$ , with probability p. Thus, a state transition can be treated as a Bernoulli trial. Referring to probability theory, a series of state transitions form a Bernoulli process and the amount of time that the machine tool spends in state  $i$  until moving to state  $j$  is a geometric random variable with memoryless property [\[32\]](#page-8-0). Furthermore, we assume that the time that the machine tool spends in state  $n$  also obeys a geometric distribution with parameter  $p$ . Then, the total working time of the machine tool, T, is a random variable given by the sum of  $n$  geometric distributions with the common parameter  $p$ . Thus, the variable  $T$  obeys a Pascal distribution, and the probability mass function is given by

$$
\Pr(T = t) = {t-1 \choose n-1} p^n (1-p)^{t-n}, \ t \ge n \tag{2}
$$

For a given machine tool, the values of  $n$  and  $p$  can be estimated using this method: (i) build up the empirical distribution of the system working time using experimental data, and (ii) fit this empirical distribution with Pascal distribution and select the most appropriate values of  $n$  and  $p$ .

# <span id="page-3-0"></span>4 Observations for the partially observable Markov process

#### 4.1 Observations of manufactured workpiece

As explained already, the state of the machine tool is not directly observable and should be inferred from observations. Since there is a close relationship between machine tool's status and the observation of processing energy consumption and quality of manufactured workpiece, this paper uses this observation to infer the system state.

Processing energy consumption for the same workpiece increases as the machine tool deteriorates [[3](#page-7-0)–[6](#page-7-0)]. In order to use the POMDP framework to deal with the maintenance control of machine tools, it is reasonable to use a finite set of discretized energy states to describe the variation of the processing energy consumption for manufacturing workpiece. Note that the size of observation space and the computational burden increase severely as the number of energy states increases in the POMDP. For simplicity, two states separated by an energy threshold are used in this paper. If the processing energy consumption for a workpiece is more than the given energy threshold, the energy observation is recorded as  $H$ ; otherwise, it is recorded as L. Considering the uncertainties in the dynamics of deteriorating process and in the observation process, this paper assumes that the probability of observing an H increases as the machine tool deteriorates. In terms of quality performance, the observed quality of any workpiece may be good or defective, and the probability of observing a defective workpiece increases as the machine tool deteriorates [\[30,](#page-8-0) [33](#page-8-0)].

Let  $x$  denote the observation of manufactured workpiece taking into account both processing energy consumption and quality. Specifically, x takes values in the set  $O = \{1, 2, 3, 4\}$ , where  $x = 1$  indicates the observed quality is good and the energy observation is  $L$ ,  $x = 2$  indicates the observed quality is good and the energy observation is  $H$ ,  $x = 3$  indicates the observed quality is defective and the energy observation is L, and  $x = 4$  indicates the observed quality is defective and the energy observation is H.

## 4.2 Relationship between observations and system states

Observations are probabilistically related to system states and the relationship can be represented by a stateobservation matrix Q, whose element  $q_{kx}$  denotes the conditional probability that the observation is  $x$  given that the system is currently in state k. The value of  $q_{kx}$  can be estimated using this method: (i) Repeat the deteriorating process of machine tool for m times and in each process the total working time is  $t_1, t_2, \ldots, t_h, \ldots, t_m$ . This means in any process  $h$ ,  $t_h$  observations are gathered and they are recorded orderly as  $x(1)$ ,  $x(2)$ ,...,  $x(t_h)$ . (ii) Divide the observations  $x(1)$ ,  $x(2)$ ,...,  $x(t<sub>h</sub>)$  in order into *n* sub-groups containing an equal number of observations, i.e.,  $n$  subgroups of  $t<sub>h</sub>/n$ . Herein, *n* is the number of system states. (iii) Collect the observations of sub-groups  $k$  of all the m processes as a new group k, and  $q_{kx}$  is calculated by

$$
q_{kx} = \frac{\text{number of observation } x \text{ in group } k}{\text{total number of observations in group } k} \tag{3}
$$

## 5 Decision-making based on POMDP

In this POMDP, the process is discretized by the time points at which workpieces are finished. The POMDP is such a process: At the start of any horizon, the system is in state  $i$ , the decision-maker takes an action  $a$  and gets a reward  $ra$  i, while the system changes to state  $j$ . Then, a workpiece is processed and an observation  $x$  is received. At last, the information about system state is updated and the next horizon starts. Herein, a, without an index, denotes an arbitrary action in set A = {a1, a2,  $a_3, a_4$ , and ra I is the expected immediate reward for state i if the action  $a$  is taken. In terms of the four actions explained in Section [1,](#page-0-0) observing a workpiece consumes a fixed observation time  $t<sub>O</sub>$ , inspecting the system consumes a fixed inspection time  $t_I$ , and performing maintenance consumes a fixed maintenance time  $t_M$ . We further assume that only the failure state *n* could be identified through inspection, and the system will return to state 1 after maintenance.

## 5.1 State transition and observation

During each horizon, system state transition occurs twice. The first transition occurs immediately after taking action  $a$  and the state transition matrix is denoted as  $M^a$ . Its element ma ij is the conditional probability that the system will move to state  $j$  after taking action  $a$  if it is currently in state  $i$ . Corresponding to the four actions, the transition matrixes are expressed as

$$
M^{a_1} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & \cdots & 1 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix} \quad M^{a_2} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \\ 1 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \end{bmatrix}
$$

$$
M^{a_3} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 1 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \end{bmatrix}
$$

$$
(4)
$$

The second transition occurs during the manufacturing process, which reflects the deteriorating process of the system and is represented by a transition matrix P. Its element  $p_{jk}$  is the probability that the machine tool will move from state *i* to k. Assuming  $M^a$  and P are independent of each other, it can be considered the state changes according to the transition matrix  $P^a = M^a \times P$  with element pa ik representing the probability that the system moves from state *i* to *k* during a horizon.

In action set  $A = \{a_1, a_2, a_3, a_4\}$ , only action  $a_2$  is to observe the manufactured workpiece and the observation could be used to update the information of system state. But there is no observation after using the actions  $a_1$ ,  $a_3$ , and  $a_4$ . Thus, the three actions cannot provide any useful information about system state. Referring to Smallwood and Sondik [\[22](#page-8-0)] and Papakonstantinou and Shinozuka [\[34](#page-8-0)], the observation matrixes are given by

$$
Q^{a_2} = Q, \quad Q^{a_1} = Q^{a_3} = Q^{a_4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 \end{bmatrix}
$$
 (5)

## 5.2 Belief vector

Since the state of machine tool is partially observable, at each time, the decision-maker only obtain a belief vector  $\pi = [\pi_1,$  $\pi_2, \ldots, \pi_n$  about the system state, where  $\pi_i$  represents the probability that the system is currently in state  $i$ . Let  $S$  be a set of all possible system states  $\{1, 2, \ldots, n\}$ . Belief  $\pi$  is a probability distribution over S and is a sufficient statistic of the history of actions and observations. This means knowing  $\pi$ , but not the full history, could obtain the same amount of information [[23\]](#page-8-0). For  $\pi_1 + \pi_2 + \cdots + \pi_n = 1$ , all vectors are contained in an  $n - 1$  dimensional simplex.

The belief vector  $\pi$  should be updated once receiving the observation x. Let  $\pi'$  denote the updated belief vector given that the prior belief vector is  $\pi$ , the selected action is a, and the observation is x. Its element  $\pi_k'$  is calculated by

$$
\pi_k' = \frac{\sum\limits_{i=1}^n \pi_i p_{ik}^a q_{kx}^a}{\beta(x|\pi, a)}
$$
\n
$$
(6)
$$

where  $\beta(x|\pi, a)$  is the conditional probability of observing x given belief  $\pi$  and action a, which is expressed as

$$
\beta(x|\pi, \, a) = \sum_{i=1}^{n} \sum_{k=1}^{n} \pi_i p_{ik}^a q_{kx}^a \tag{7}
$$

#### 5.3 Reward

The energy efficiency during any horizon is defined as the ratio of productivity to the energy consumption [\[35](#page-8-0)]. Define the time of manufacturing a workpiece as a unit time. The processing time for each workpiece is equal. Since only one workpiece is processed in each horizon, the productivity equals the reciprocal of time length of the horizon. The time length depends on the performed action, which may consist of a unit time for manufacturing, and possible observation time  $t<sub>O</sub>$ , or possible inspection time  $t_I$ , or possible maintenance time  $t_M$ . On the other hand, energy consumption of a horizon includes the processing energy consumption, energy loss for defective quality, and possible energy consumption for maintenance. Let  $E_L$  and  $E_H$  represent respectively the average processing energy consumption for energy observations L and H. Let  $E_D$  denote the average extra energy loss for a defective workpiece. Let  $E_M$  denote the average maintenance energy consumption, which may include the energy required to restart the machine tool and perform other maintenance operations. Let  $EE^{a}_{x}$  denote the immediate energy efficiency of any horizon given the action  $a$  and observation  $x$ . The expressions of  $EE^a$ <sub>x</sub> with various combinations of a and x are given in Table 1. Note that  $1/E_L$  is the maximum immediate energy efficiency.

Let  $\lambda^a_{x}$  denote the immediate reward during a horizon if action *a* is taken and the observation is *x*. For  $1/E_L$  is the maximum of  $EE^{a}_{x}$  in Table 1, we use  $1/E_L$  as a benchmark and define  $\lambda^a_{x}$  as the ratio of  $EE^a_{x}$  to  $1/E_L$ . Thus,  $\lambda a \; x$  is expressed as

$$
\lambda_x^a = E_L \cdot EE_x^a \tag{8}
$$

**Table 1** Expressions of *EEa x* with various combinations of a and x

Action	Observation $x$			
$\boldsymbol{a}$				
a <sub>1</sub> a <sub>2</sub> $a_3$ if not failed $a_3$ if failed	$E_L$ $(1+t_O)E_L$ $\overline{(1+t_I)E_L}$ $(1+t_I+t_M)(E_L+E_M)$	$E_H$ $(1+t_O)E_H$ $\overline{(1+t_I)E_H}$ $(1+t_I+t_M)(E_H+E_M)$	$E_L + E_D$ $(1+t_O)(E_L+E_D)$ $\overline{(1+t_I)(E_L+E_D)}$ $\overline{(1+t_I+t_M)(E_L+E_D+E_M)}$	$E_H + E_D$ $(1+t_O)(E_H+E_D)$ $\overline{(1+t_I)(E_H+E_D)}$ $\overline{(1+t_I+t_M)(E_H+E_D+E_M)}$
$a_4$	$(1+t_M)(E_L+E_M)$	$(1+t_M)(E_H+E_M)$	$(1+t_M)(E_L+E_D+E_M)$	$(1+t_M)(E_H+E_D+E_M)$

<span id="page-5-0"></span>

Referring to Smallwood and Sondik [[22\]](#page-8-0), ra i, the expected immediate reward for state  $i$  given the action  $a$ , is given by

$$
r_i^a = \sum_{k=1}^n \sum_{x=1}^4 p_{ik}^a q_{kx} \lambda_x^a \tag{9}
$$

where  $q_{kx}$  is the element of state-observation matrix Q not matrix  $Q^a$ . Actually, the observation result of processing energy consumption and quality of manufactured workpiece is always following the rule of matrix  $Q$  regardless of the existence of observation operation in the action. And the reward, under the given state  $i$  and action  $a$ , depends only on the real outcome of processing energy consumption and quality of manufactured workpiece. Observation operation could receive the information of actual outcome but cannot determine the outcome.



Let  $V_N(\pi)$  be the maximum expected reward that the system can accrue if there are N horizons remaining before the process terminates and the current belief is  $\pi$ .  $V_N(\pi)$  and the optimal action corresponding to belief  $\pi$  can be obtained through computing

$$
V_N(\pi) = \max_{a \in A} \left\{ \sum_{i=1}^n \pi_i r_i^a + \sum_{x=1}^4 \beta(x|\pi, a) V_{N-1}[U(\pi, a, x)] \right\}, N \ge 1
$$
\n(10)

with the initial condition  $V_0(\pi) = 0$ , where

$$
U(\pi, a, x) = \pi'
$$
\n<sup>(11)</sup>

To solve Eq. (10), we begin at the end of the process, i.e., there are  $N = 0$  horizons remaining to reach the end. Approximate value iteration algorithm is an efficient kind of method for solving POMDP. Herein, the grid-based approximation, a simple algorithm of approximate value iteration, is suggested. Using this method, the value function over a continuous belief space can be approximated by a finite set of grid





(c) Rule with 20 horizons remaining. (d) Rule with 50 horizons remaining.

Fig. 2 Four decision rules with different horizons remaining



(a) Rule with 5 horizons remaining. (b) Rule with 10 horizons remaining.



<span id="page-6-0"></span>points, and the value of an arbitrary belief point can be estimated based on the values of grid points through an interpolation-extrapolation rule. Other methods reviewed in Papakonstantinou and Shinozuka [\[36\]](#page-8-0), such as most likely state (MLS) method and point-based value iteration, can also be used to solve the problems with large state spaces.

### 5.5 Optimal maintenance policy

Through computing Eq.  $(10)$  $(10)$ , we can obtain the optimal action for any belief  $\pi$  if there are N horizons remaining. Let  $\Phi_N$ denote the optimal decision rule with  $N$  horizons remaining which specifies for each belief  $\pi$  the optimal action to take if there are N horizons remaining. Consequently, the optimal control policy over N horizons for the overall process is a sequence of decision rules  $\Phi = {\Phi_1, \Phi_2, ..., \Phi_N}$ . Next, the optimal policy is illustrated through a numerical example about a three-state machine tool. Parameters are shown in Table [2,](#page-5-0) and the state-observation matrix  $Q$  is assumed to be







(c) Optimal decision rule with  $t_0 = 0.1$ . (d) Optimal decision rule with  $t_1 = 4$ .

Fig. 3 Optimal policies with varied parameters and eight horizons remaining

As explained already, all beliefs are contained in an  $n-1$ dimensional simplex, for a three-state system, we can use the points  $(\pi_1, \pi_2)$  in the two-dimensional simplex  $\{0 \le \pi_1 \le 1,$  $0 \leq \pi_2 \leq 1$ ,  $\pi_1 + \pi_2 \leq 1$ } to represent the beliefs. The gridbased approximations and bilinear interpolation rule are used to solve Eq. [\(10\)](#page-5-0) with  $\pi_1$  and  $\pi_2$  discretized on a grid with increments of 0.01. Four decision rules with different horizons remaining are portrayed by Fig. [2a](#page-5-0)–d. Firstly, it is apparent each belief space is divided into a finite number of regions and all the beliefs of each region corresponds to an action which is the optimal action of the beliefs in this region. Herein, the unsmooth nature of the boundaries of these regions is due to the discretization of  $\pi_1$  and  $\pi_2$ . These figures provide the decision-maker with graphic decision rules showing the optimal action for any belief if the number of remaining horizons is given. For example, if there are five horizons remaining and the decision-maker believes the system is in state 1 with probability 0.4 and in state 2 with probability 0.3, Fig. [2a](#page-5-0) shows the optimal action is  $a_1$ . Secondly, this optimal policy is visualized by diagrams. Diagrams are convenient for problems containing only two or three states, but are impractical for those systems with more than three states. This is the reason for choosing a three-state system as an example. For systems with



(a) Optimal decision rule with  $p = 0.1$ . (b) Optimal decision rule with  $E_H = 1.8$ .



<span id="page-7-0"></span>more states, the belief space can also be divided into finite numbers of regions and all the beliefs in each region have the common optimal action. Thirdly, the rule with a given number of remaining horizons does not change with the total number of horizons of the overall process. As an application of general POMDP, more common characteristics of the value function  $V_N(\pi)$ , such as its convexity and piecewise linearity can be seen in Papakonstantinou and Shinozuka [\[23\]](#page-8-0).

## 6 Sensitivity analysis

This section shows the effects of some parameters on the optimal control policy of above example. Figure [3a](#page-6-0)–d portrays the optimal decision rules corresponding to the changes of parameters in Table [2,](#page-5-0) when there are 10 horizons remaining.

Parameter  $p$  is an indicator of system deteriorating rate. Figure [3a](#page-6-0) shows the optimal decision rule with  $p = 0.1$ . It can be noticed that action  $a_4$  plays an important role in this rule and the region for  $a_3$  decreases sharply compared with it in Fig. [2b](#page-5-0). This is because the machine tool deteriorates more quickly as  $p$  increases and frequent maintenance is necessary, and it may be better to execute maintenance directly than to inspect the system first.

Difference between  $E_H$  and  $E_L$  also affects the result of decision rule. Intuitively, if the ratio of  $E_H$  to  $E_L$  is large, the decision-maker will increase the frequency of performing  $a_2$ ,  $a_3$  $a_3$  or  $a_4$ . Figure 3b shows the decision rule with  $E_L = 1$  and  $E_H$ /  $E<sub>L</sub> = 1.8$ , where the region for action  $a<sub>2</sub>$  or  $a<sub>3</sub>$  is larger than it in Fig. [2b](#page-5-0). This is reasonable because more energy is consumed for manufacturing as the increase of  $E_H/E_L$ , and the decisionmaker would rather take other actions frequently than do nothing in order to maintain the system in an energy efficient state.

Figure [3](#page-6-0)c shows the optimal decision rule with  $t<sub>O</sub> = 0.1$ . Intuitively, the decision-maker would tend to observe the workpiece more frequently and know more information about system state due to the decreased observation time. This intu-ition is proved in Fig. [3c](#page-6-0) where the region for action  $a_2$  is increased compared with it in Fig. [2b](#page-5-0).

Figure [3](#page-6-0)d shows the optimal decision rule with  $t_1 = 4$ . The outline of these regions in Fig. [3d](#page-6-0) is similar to those in Fig. [2b](#page-5-0), but the region for action  $a_3$  is decreased. This is because the decision-maker would decrease the frequency of performing  $a_3$  as the inspection time increases so as to keep a high production efficiency.

## 7 Conclusions

This paper proposes an approach to improve the energy efficiency of the production process through scheduling the maintenance actions of the machine tool, taking into account productivity, product quality, and energy consumption. POMDP

framework is applied to develop the maintenance decisionmaking model, and the decision is made based on the observation information about processing energy consumption and quality of manufactured workpiece. Using this approach, the optimal maintenance policy maximizing the total expected reward about energy efficiency over a finite horizon is obtained, which is a sequence of decision rules corresponding to optimal actions. This approach, which does not need to redesign the machine tool or optimize the process, provides a cost-efficient way for energy efficient manufacturing.

In this study, two energy states separated by an energy threshold are used to represent the processing energy consumption of manufactured workpiece. In order to provide more precise information about system state for the decision-making, future works will try to use the more accurate number of energy states to describe the characteristics of processing energy consumption.

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