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ORIGINAL ARTICLE

Alternative methods for the simultaneous monitoring of simple linear profile parameters

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Abstract

In many industrial processes, the quality characteristic of interest has a relation (linear or non-linear) with other supporting variable(s). Simple linear profile is a well-known term used for the quality characteristic, which is linearly associated with another descriptive variable, and the monitoring of simple linear profile parameters (i.e., slope, intercept, and error variance) is known as linear profiling. In the literature, a well-known approach named as EWMA_3 chart is used for the simultaneous monitoring of intercept, slope, and error variance. This approach is very efficient as compared to EWMA/R, Hotelling T^2, and Shewhart_3 charts but it is a tedious method, since distinct pair of control limits require individual charting constant for each parameter. In this study, new methods are designed for the simultaneous monitoring of simple linear profile parameters, which requires single charting constant and have several advantages such as simplicity, efficiency, and ease of applicability. The findings of this study reveal that newly designed control charts such as the Max − EWMA − 3 and SS − EWMA − 3 have almost similar performance with EWMA_3 chart. Specifically, Max − EWMA − 3 − C chart shows superiority among all other control charts. Further, importance of the stated proposals is highlighted by the real example from the field of chemical engineering.

Keywords Control chart · Linear profiling · Max statistic · MOX sensor · Simultaneous monitoring

1 Introduction

Usually, control charts are designed to monitor single quality characteristic (qualitative or quantitative) of a process, but in many manufacturing processes, quality characteristic have a relationship with other auxiliary variable(s). For example, in semiconductor manufacturing, flow of gasses is dependent on pressure of mass flow controller (cf. Kang and Albin [1]) and in electrical engineering, capacitor charge is dependent on capacitance level (cf. Riaz et al. [2]). When such quality characteristic is linearly associated with another explanatory variable, then it is termed as simple linear profiles and the monitoring of simple linear profile parameters (i.e., slope, intercept, and error variance) is known as linear profiling.

In the literature, many researchers discuss linear profiling. Two multivariate control charts for the stability of linear calibration curves are discussed by Mestek et al. [3]. Stover and Brill [4] proposed two phase I control charts for multilevel ion chromatography linear calibrations and Croarkin and Varner [5] designed a phase I study for integrated-circuit photomasks. Two well-known charts EWMA/R and Hotelling T^2 are proposed by Kang and Albin [1] and the simultaneous scheme named as EWMA_3 for the monitoring of intercept, slope, and error variance was developed by Kim et al. [6]. Mahmoud and Woodall [7] proposed phase I study based on F test in the simple linear profiles while Noorossana et al. [8] examined the quality characteristics of linear profile through multivariate cumulative sum control chart.

In simple linear profiles (SLP), effects of non-normal environments are studied by Noorossana et al. [9]. Phase II and phase I studies based on change point model was discussed by Zou et al. [10] and Mahmoud et al. [11], respectively. However, a comparative study between Shewhart method by Kim et al. [6] and Croarkin and Varner [5] was discussed by Gupta et al. [12] and a comprehensive overview on linear profiles was discussed by Woodall [13]. Noorossana and Amiri [14] proposed χ^2 and integrated MCUSUM control

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charts for the monitoring of linear profile parameters and a control chart for the monitoring of recursive residuals was proposed by Zou et al. [15]. Mixed models in linear profiles were discussed by Jensen et al. [16] and a control chart based on likelihood ratio in linear profiles was discussed by Zhang et al. [17]. A CUSUM-based approach was proposed by Saghaei et al. [18] and a study related to small sample size (one or two) in linear profiles was discussed by Mahmoud et al. [19]. In multivariate setup, monitoring of non-linear auto-correlated processes through support vector regression (SVR) control charts is studied by Khediri et al. [20]. Shang et al. [21] employed EWMA scheme for the categorical responses using likelihood ratio test based on logistic regression. Laursen et al. [22] proposed a control chart named comprehensive control (COCO) chart which is beneficial for the combine monitoring of univariate and multivariate quality characteristics of the process. They also implement the proposed methodology on chromatographic peak areas to monitor the purity level of biopharmaceutical medicine material. Retrospective studies based on change point model was discussed by Yeh and Zerehsaz [23] whereas Riaz and Touqeer [24] proposed run rule schemes to enhance the performance of both linear and multiple linear profile methodologies. Noorossana et al. [25] proposed a phase II study about linear profile parameters under random effect models and the effect of phase I estimation under EWMA_3 structure was discussed by Noorossana et al. [26]. However, in recent studies, efficient sampling strategies such as ranked set sampling (RSS) and its modified structure are encouraged by the researchers. Specifically, for the monitoring of linear profile parameters, Riaz et al. [2] examined phase II study under different ranked set strategies. Sogandi and Amiri [27] designed a phase II study for the monitoring of maximum likelihood estimator of generalized linear profile (GLP) model under monotonic change while Chiang et al. [28] proposed a multivariate setup (MEWMA chart) to monitor the SLP in the presence of within profile autocorrelation and a univariate setup (EWMA chart) for the monitoring of SLP in the presence of first-order autocorrelation between disturbance terms was discussed by Wang and Huang [29]. The zone control charting structure for the monitoring of linear profile parameters was proposed by Zhang et al. [30] while change point method for GLP model was discussed by Shadman et al. [31].

The recent studies such as the EWMA_3 chart for simple linear profiles are based on simultaneous structure, which is a tedious method such as each distinct pair of control limits require individual charting constant. For example, in case of linear profile parameters such as slope, intercept, and error variance, on fixed overall average run length 200, one my need 584.7 average run length for each individual chart under simultaneous structure, which is a tedious method for the practitioners. In the current study, new control charts are designed for the monitoring of simple linear profile parameters, which requires single charting constant and have several characteristics such as simple and efficient procedure and ease of applicability.

The organization of the rest of article is as follows. In section 2, the brief description of simple linear profile model, proposed schemes, and other competing schemes are discussed. In section 3, performance evaluations and discussion of results are reported. A case study on chemical engineering phenomena is given in section 4. Finally, in sections 5 and 6, summary, conclusions, scope, and recommendations are reported.

2 Linear profiles and charting structures

In this section, the theoretical structure of existing charts for simple linear profile model (i.e., EWMA/R chart, Hotelling T^2 chart, Shewhart_3 chart, and EWMA_3 chart) are presented and the discussion is made on the proposed joint methodologies.

2.1 Linear profile models

In many manufacturing processes, the quality characteristic is examined with an auxiliary information. For the monitoring of the process, j paired observations of the quality characteristic along an auxiliary information (Y_{ij}, X_j) are collected over time i. On the base of j paired measurements which are obtained over time i, i profiles are obtained by using the simple linear profile model which is defined as the following:

$$Y_{ij} = \beta_0 + \beta_1 X_j + \varepsilon_{ij}; j = 1, 2, \ldots, n; i = 1, 2, \ldots \quad (1)$$

where β_0 is the intercept (origin of the regression line or the expected value of quality characteristic, when there is no relation between quality characteristic and auxiliary information), β_1 is the slope (rate of change in quality characteristic with unit change in an auxiliary information), and ε is the error term which follows normal distribution with mean zero and constant variance σ^2 (i.e., $\varepsilon_{ij} \sim N(\mu, \sigma^2)$) (cf. by Kim et al. [6]). By following Kang and Albin [1], the ordinary least square estimates of the parameters (β_0 and β_1) are given by the following expressions:

$$\hat{\beta}_{1i} = \frac{\sum_{j=1}^{n} \left(X_j - \mu_X\right) Y_{ij}}{\sum\limits_{j=1}^{n} \left(X_j - \mu_X\right)^2} = \frac{S_{xy_i}}{S_{xx}}$$

$$\hat{\beta}_{0i} = \overline{Y}_i - \hat{\beta}_{1i} \mu_X$$

where $\overline{Y}_i = \sum_{j=1}^{n} Y_{ij}/n$, $\mu_X = \sum_{j=1}^{n} X_j/n$ and the conditional mean, variance, and covariance of $\hat{\beta}_{0i}$, $\hat{\beta}_{1i}$ are defined as follows:

$$E\left[\hat{\beta}_{0i}|X\right] = \beta_0,\ \ E\left[\hat{\beta}_{1i}|X\right] = \beta_1,$$

$\mathrm{Var}\left[\hat{\beta}_{0i}|X\right] = \sigma^2\left[\frac{1}{n}+\frac{1}{n}\right]$; $\mathrm{Var}\left[\hat{\beta}_{1i}|X\right] = \frac{\sigma^2}{S_{xx}}$; and $\mathrm{Cov}\left[\hat{\beta}_{0i}, \hat{\beta}_{1i}|X\right] = -\frac{\sigma^2\ \mu_X}{S_{xx}}$.

The mean square error is an unbiased estimator of the variance of error term (σ^2), which is defined as the following:

$$\mathrm{MSE}_i = \frac{\sum_{j=1}^{n} e_{ij}^2}{n-2}$$

where $e_{ij} = y_{ij} - \hat{y}_{ij}$ is the ith residual of the jth observation and $\hat{y}_{ij}$ is the ith fitted regression estimate for the jth observation. Kim et al. [6] reported that the $\hat{\beta}_{0i}$ and $\hat{\beta}_{1i}$ are negatively related (i.e., $\mathrm{Cov}\left[\hat{\beta}_{0i}, \hat{\beta}_{1i}|X\right] = -\sigma^2\ \mu_X/S_{xx}$) and to develop a simultaneous charting setup, independence between the parameters is required which is obtained by transforming the explanatory variable (i.e., $X_j^* = X_j - \mu_X$). After the transformation on X_j, a modified version of model (1) is obtained and named as transformed model which is defined as follows:

$$Y_{ij} = (B_0) + (B_1)X_j^{\ *} + \varepsilon_{ij} \tag{2}$$

where $B_0 = \beta_0 + \beta_1\mu_X + (\beta\sigma)\mu_X$, $B_1 = (\beta_1 + \beta\sigma)$ and β is the amount of shift in the slope of model (1). In rest of the article, the estimated version of B_0 and B_1 are expressed as $\left(\hat{b}_{0i}\right)$ and $\left(\hat{b}_{1i}\right)$, while estimated version of mean square error for model (2) is expressed by $\mathrm{M\hat{S}E}_i$. Moreover, it is noted that after the transformation on X_j,the covariance of $\hat{b}_{0i}$ and $\hat{b}_{1i}$ will be zero as the average of X_j^* is zero.

2.2 Proposed control charts

The simple linear model given in Eq. (1) is a basic model used in simple linear profiling but due to the limitation (e.g., independence of parameters), model (2) (transformed model) was preferred in many studies such as in Riaz et al. [2], Kim et al. [6], Mahmoud and Woodall [7], and many others. Recall that the ordinary least square estimates for the parameters of transformed model are represented by $\hat{b}_{0i}$, $\hat{b}_{1i}$ and $\mathrm{M\hat{S}E}_i$. Further, for the joint monitoring, $\hat{b}_{0i}$ and $\hat{b}_{1i}$ are normalized by using the standard score method and $\mathrm{M\hat{S}E}_i$ is normalized by the chi-square transformation which are defined as the following:

$$Z_{\hat{b}_{0i}} = \frac{\hat{b}_{0i} - B_0}{\sqrt{\sigma^2 / n}} \tag{3}$$

$$Z_{\hat{b}_{1i}} = \frac{\hat{b}_{1i} - B_1}{\sqrt{\sigma^2 / S_{xx}}} \tag{4}$$

$$A_{\widehat{\mathrm{MSE}}_i} = \Phi^{-1}\left[H\left\{\frac{(n-2)\widehat{\mathrm{MSE}}_i}{\sigma^2}; n-2\right\}\right] \tag{5}$$

where $\Phi^{-1}[.]$ is the inverse standard normal distribution function and $H\{.;(n-2)\}$ is termed as chi-square distribution function having $(n-2)$ degrees of freedom. In the recent literature, two more transformations are used for the dispersion parameter to gain approximate normal results such as three-parameter logarithmic transformation (cf. Castagliola [32]) and Johnson SB transformation (cf. Castagliola [33]). The description of these two transformations for mean square error is as follows:

$$B_{\widehat{\mathrm{MSE}}_i} = a_T + b_T \ln\left(\widehat{\mathrm{MSE}}_i + c_T\right) \tag{6}$$

$$C_{\widehat{\mathrm{MSE}}_i} = a_U + b_U \ln\left(\frac{\widehat{\mathrm{MSE}}_i - c_U}{d_U + c_U - \mathrm{M\hat{S}E}_i}\right) \tag{7}$$

where $a_T = A_T(n) - 2B_T(n)\ln\sigma$, $b_T = B_T(n)$, $c_T = C_T(n)\sigma^2$, $a_U = A_U(n)$, $b_U = B_U(n)$, $c_U = C_U(n)\sigma^2$, and $d_U = D_U(n)\sigma^2$. The values of these constants are reported in Table 1 for $n = 3, 4, 5, \ldots, 15$. In the rest of article, symbol "Δ" is used to differentiate the charts based on aforementioned three

Table 1 Constant for transformations (three-parameter logarithmic transformation and Johnson S_B transformation)

n	a_T	b_T	c_T	a_U	b_U	c_U	d_U
3	−0.6627	1.8136	0.6777	3.1936	1.1952	−0.2588	15.0770
4	−0.7882	2.1089	0.6261	3.3657	1.3983	−0.2438	12.5910
5	−0.8969	2.3647	0.5979	3.5402	1.5727	−0.2352	11.3120
6	−0.9940	2.5941	0.5801	3.7111	1.7281	−0.2295	10.5300
7	−1.0827	2.8042	0.5678	3.8768	1.8698	−0.2254	10.0000
8	−1.1647	2.9992	0.5588	4.0369	2.0010	−0.2224	9.6180
9	−1.2413	3.1820	0.5519	4.1918	2.1238	−0.2200	9.3280
10	−1.3135	3.3548	0.5465	4.3417	2.2396	−0.2181	9.1000
11	−1.3820	3.5189	0.5421	4.4869	2.3495	−0.2166	8.9170
12	−1.4473	3.6757	0.5384	4.6279	2.4544	−0.2152	8.7660
13	−1.5097	3.8260	0.5354	4.7648	2.5549	−0.2141	8.6400
14	−1.5697	3.9705	0.5327	4.8981	2.6515	−0.2132	8.5320
15	−1.6275	4.1100	0.5305	5.0279	2.7446	−0.2123	8.4400

transformations, used to obtain the normality of error variance. For example, when $\Delta = \mathrm{A}$, then the mean square error transformed by chi-square transformation $\left(A_{\widehat{\mathrm{MSE}_i}}\right)$ is used in the control chart structure. Moreover, $\Delta = \mathrm{B}$ and C are used to represent the mean square error based on three-parameter logarithmic transformation (given in Eq. (6)) and Johnson SB transformation (reported in Eq. (7)). Since, this study is further depending on standard scores in Eqs. (3), (4), and (5), the natural boundaries for these statistics are $(-\infty, \infty)$ or specifically, if the process is in control, the plotting statistics is mostly expected to range in the interval $(-3, 3)$. Moreover, the boundaries for plotting statistics (6) and (7) are similar because they are also transformed to gain approximate normality.

2.2.1 The Max − EWMA − 3 charting structures

The exponentially weighted moving average (EWMA) chart was firstly introduced by Roberts [34], as an effective technique to monitor small or moderate shifts in the process. Further, Chen et al. [35] proposed a modified EWMA chart termed as Max − EWMA for the joint monitoring of two parameters (location and scale). The derivation of probability density function (pdf), mean, and variance of Max − EWMA statistic is derived in Appendix A.1–2. Here, a similar Max − EWMA approach is used to monitor the linear profile parameters (i.e., intercept, slope, and error variance), which is further referred as Max − EWMA − 3 chart. The structure of Max − EWMA − 3 depends on EWMA statistics which are based on $Z_{\hat{b}_{0i}}$, $Z_{\hat{b}_{1i}}$ and transformed mean square errors $\left(\Delta_{\widehat{\mathrm{MSE}_i}} = A_{\widehat{\mathrm{MSE}_i}}, B_{\widehat{\mathrm{MSE}_i}} \text{ and } C_{\widehat{\mathrm{MSE}_i}}\right)$,

$$M_i = \lambda Z_{\hat{b}_{0i}} + (1-\lambda)M_{i-1}, \tag{8}$$

$$N_i = \lambda Z_{\hat{b}_{1i}} + (1-\lambda)N_{i-1}, \tag{9}$$

$$O_i = \lambda \Delta_{\widehat{\mathrm{MSE}_i}} + (1-\lambda)O_{i-1}, \tag{10}$$

where the initial values M_0, N_0, and O_0 equals to zero and λ is a smoothing (weight) parameter having values in the range of $0 < \lambda \leq 1$. The statistic and limit (derived in Appendix A.3) for three separate Max − EWMA − 3 charts (by equating $\Delta = \mathrm{A}$, B,and C) are reported as follows:

$$\mathrm{Max{-}EWMA{-}3{-}}\Delta_i : \begin{cases} \text{Statistic} & \mathrm{Max}(|M_i|, |N_i|, |O_i|), \\ \mathrm{UCL}_{\mathrm{ME}\Delta} & \sqrt{\dfrac{\lambda}{2-\lambda}}(1.32639 + 0.5859607 L_{\mathrm{Max}}) \end{cases}, \tag{11}$$

where L_{Max} is the control limit coefficient that is used to control the in-control (IC) run length behavior of the chart. It is noted that the Max − EWMA − 3 − A represent the charting structure based on first transformation of error variance reported in Eq. (5), while the Max − EWMA − 3 − B and Max − EWMA − 3 − C represents the charts based on transformations reported in Eqs. (6) and (7), respectively.

2.2.2 The EWMA − Max − 3 charting structures

Xie [36] proposed a reverse scheme by taking the max statistic initially and used the EWMA structure thereafter. In the structure of EWMA − Max − 3, max statistics are formulated based on $Z_{\hat{b}_{0i}}$, $Z_{\hat{b}_{1i}}$ and transformed mean square errors $\left(\Delta_{\widehat{\mathrm{MSE}_i}} = A_{\widehat{\mathrm{MSE}_i}}, B_{\widehat{\mathrm{MSE}_i}} \text{ and } C_{\widehat{\mathrm{MSE}_i}}\right)$, which are obtained as follows

$$\mathrm{Max}_\Delta = \mathrm{Max}\left(\left|Z_{\hat{b}_{0i}}\right|, \left|Z_{\hat{b}_{1i}}\right|, \left|\Delta_{\widehat{\mathrm{MSE}_i}}\right|\right), \tag{12}$$

The Max_A statistic is obtained by equating the $\Delta = A$ in Eq. (12), while Max_B and Max_C statistics are also obtained by equating the $\Delta = B$ and C in Eq. (12). Based on the Max statistics, three EWMA − Max − 3 statistics and their corresponding limits (derived in Appendix A.4) are defined as the following:

$$EWMA{-}Max{-}3{-}\Delta_i : \begin{cases} Statistic & \lambda Max_\Delta + (1-\lambda) EWMA{-}Max{-}3{-}\Delta_{i-1}, \\ UCL_{EM\Delta} & 1.32639 + 0.5859607 L_{EMax}\sqrt{\dfrac{\lambda}{2-\lambda}} \end{cases}, \tag{13}$$

where the width of the control limit is dependent on L_{EMax}, which is used to control the IC run length behavior of the chart. Moreover, the EWMA − Max − 3 − A represent the charting structure for Max_A statistic, while EWMA − Max − 3 − B and EWMA − Max − 3 − C charts are based on the Max_B and Max_C statistics, respectively.

2.2.3 The SS − EWMA − 3 charting structures

Another approach for the joint monitoring of process parameters based on sum of squares of EWMA statistics was proposed by Xie [36]. In stated study, this concept is used for the monitoring of linear profile parameters and referred as SS − EWMA − 3 chart. The structure of SS − EWMA − 3 charts depends on aforementioned EWMA statistics M_i, N_i, and O_i (given in Eqs. 8–10), are obtained by the following:

$$\mathrm{SS{-}EWMA{-}3{-}}\Delta_i : \begin{cases} \text{Statistic} & M_i^2 + N_i^2 + O_i^2 \\ \mathrm{UCL}_{\mathrm{SS}\Delta} & \dfrac{\lambda\left(3 + L_{\mathrm{SS}}\sqrt{6}\right)}{2-\lambda} \end{cases}, \tag{14}$$

Further, when $\Delta = \mathrm{A}$, the SS − EWMA − 3 − A represents a charting structure based on first transformation of error variance

$\left(A_{\widehat{\mathrm{MSE}_i}}\right)$, while for $\Delta = \mathrm{B}$ and C, the SS − EWMA − 3 − B and SS − EWMA − 3 − C represent the charts based on $B_{\widehat{\mathrm{MSE}_i}}$ and $C_{\widehat{\mathrm{MSE}_i}}$, respectively. However, the L_{SS} is a control limit coefficient that describes the width of control limit and the control limit is derived in Appendix A.5.

2.2.4 The EWMA − SS − 3 charting structures

In this scheme EWMA control charts are designed for the sum of square statistics, which are based on $Z_{\hat{b}_{0i}}$, $Z_{\hat{b}_{1i}}$ (cf. Eqs. 3 and 4) and transformed mean square errors $\left(\Delta_{\widehat{\mathrm{MSE}_i}} = A_{\widehat{\mathrm{MSE}_i}}, B_{\widehat{\mathrm{MSE}_i}}, \text{and } C_{\widehat{\mathrm{MSE}_i}}\right)$ (cf. Eqs. 5, 6, and 7). The sum of square statistics can be obtained by the following:

$$\mathrm{SS}_\Delta = Z_{\hat{b}_{0i}}{}^2 + Z_{\hat{b}_{1i}}{}^2 + \Delta_{\widehat{\mathrm{MSE}_i}}{}^2, \tag{15}$$

The SS_A statistic is obtained by equating the $\Delta = A$ in Eq. (15), while SS_B and SS_C statistics are also obtained by equating the $\Delta = B$ and C in Eq. (15). However, the EWMA − SS − 3 statistics (based on Eq. 15) with their corresponding limit (derived in Appendix A.6) are defined as the following:

$$EWMA\text{−}SS\text{−}3\text{−}\Delta_i : \begin{cases} Statistic & \lambda SS_\Delta + (1-\lambda) EWMA\text{−}\mathrm{SS}\text{−}3\text{−}\Delta_{i-1}, \\ UCL_{ESS\Delta} & 3 + L_{ESS}\sqrt{\dfrac{6\lambda}{2-\lambda}} \end{cases}, \tag{16}$$

where L_{ESS} is the control charting constant and used to control the IC run length behavior of a chart. Further, the EWMA − SS − 3 − A represent the charting structure based on first transformation of error variance reported in Eq. (5), while the EWMA − SS − 3 − B and EWMA − SS − 3 − C represents the charts based on the transformation given in Eqs. (6) and (7), respectively.

2.3 Existing control charts

For the comparison, several existing simple linear profile methods are considered such as the EWMA/R chart, Hotelling T^2 chart, Shewhart_3 chart, and EWMA_3 chart. Further, the structures of these charts are given below:

2.3.1 The EWMA/R chart

Kang and Albin [1] proposed a combined structure based on EWMA and R chart for the monitoring of simple linear profile parameters. Basically,EWMA chart has some limitations which are covered by incorporating the R chart. The ith statistic for EWMA chart is estimated by

$$Z_i = \lambda \bar{e}_i + (1-\lambda) Z_{i-1}, \tag{17}$$

where λ is the smoothing parameter which ranges from 0 to 1, $\bar{e}_i = \sum_{j=1}^{n} e_{ij}/n$ and the initial value of EWMA statistic is considered as zero. (i.e., $Z_0 = 0$). The process is said to be out-of-control (OOC) when Z_i is less than $\mathrm{LCL_E}$ or greater than $\mathrm{UCL_E}$. The control limits ($\mathrm{LCL_E}$ and $\mathrm{UCL_E}$) of EWMA chart based on charting constant (L_{ER}) are given as follows:

$$\mathrm{LCL_E} = -L_{\mathrm{ER}}\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[\frac{1}{n}\right]}; \tag{18}$$

$$\mathrm{UCL_E} = L_{\mathrm{ER}}\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[\frac{1}{n}\right]},$$

There exist two causes to combine R chart with EWMA chart, (i) to detect shifts in error variance under model (1) and (ii) to tackle the unusual error variance. Further, the ith statistic and control limits of R chart are defined as the following:

$$R_i = max_i(e_{ij}) - min_i(e_{ij}), \tag{19}$$

$$\mathrm{LCL_R} = \sigma(d_2 - L_{\mathrm{ER}} d_3); \quad \mathrm{UCL_R} = \sigma(d_2 + L_{ER} d_3), \tag{20}$$

where d_2 and d_3 are unbiased constants reported in Montgomery [37].

2.3.2 The Hotelling T^2chart

Kang and Albin [1] also proposed a multivariate control chart for the monitoring of slope and intercept known by Hotelling T^2 control chart. The ith statistic of Hotelling T^2 control chart is estimated by the following:

$$T_i^2 = (ZA_i - UA)^T \Sigma^{-1} (ZA_i - UA), \tag{21}$$

where

$$ZA_i = \left(\hat{\beta}_{0i}, \hat{\beta}_{1i}\right)^{\boldsymbol{T}}; UA = (\beta_0, \beta_1)^{\boldsymbol{T}}, \tag{22}$$

$$\Sigma = \begin{bmatrix} \sigma^2\left[\dfrac{1}{n} + \dfrac{\mu_X{}^2}{S_{XX}}\right] & -\sigma^2\dfrac{\mu_X}{S_{XX}} \\ -\sigma^2\dfrac{\mu_X}{S_{XX}} & \dfrac{\sigma^2}{S_{XX}} \end{bmatrix}, \tag{23}$$

The Hotelling T^2 statistic follows χ^2 distribution with p degrees of freedom. The upper control limit ($\mathrm{UCL_H} = \chi^2_{p,\,\alpha}$) is the αth quantile of χ^2 distribution and lower control limit is zero (i.e., $\mathrm{LCL_H} = 0$). p is the number of parameters to be studied, which are 2 in this case. If the process is unstable, then the Hotelling T^2 statistic follows non-central χ^2

distribution with non-centrality parameter (τ), which is obtain as follows:

$$\tau = n\left(\theta\sigma + \beta\sigma\overline{X}\right)^2 + (\beta\sigma)^2 S_{XX}, \tag{24}$$

where θ is the amount of shift in intercept for model (1) and β is the measure of shift in the slope of model (1).

2.3.3 The Shewhart_3 chart

In the literature, many researchers addressed linear profiling under EWMA structure but a popular memoryless proposal named as Shewhart_3 chart was introduced by Gupta [38]. In Shewhart_3 chart, individual chart for each parameter (i.e., slope, intercept, and error variance) are combined to evaluate the joint/simultaneous monitoring of the process. The structures of the individual chart for each parameter are expressed below:

$$\begin{array}{ll} \text{for intercept :} & \text{UCL or LCL} = B_0 \pm L_{\alpha/2}\sqrt{\sigma^2\left[\frac{1}{n} + \frac{\mu_X{}^2}{S_{xx}}\right]} \\ \text{for slope :} & \text{UCL or LCL} = B_1 \pm L_{\alpha/2}\sqrt{\frac{\sigma^2}{S_{xx}}} \\ \text{for error variance :} & \begin{array}{l} \text{UCL} = \frac{\sigma^2}{n-2} + \chi^2_{\frac{\alpha}{2},n-2} \\ \text{LCL} = \frac{\sigma^2}{n-2} - \chi^2_{1-\frac{\alpha}{2},n-2} \end{array} \end{array}, \tag{25}$$

where $L_{\alpha/2}$is the $\alpha/2^{th}$ quantile of student's t distribution, $\chi^2_{\alpha/2}$ and $\chi^2_{1-(\alpha/2)}$ are the upper and lower $\alpha/2^{th}$ quantiles of chi-square distribution having $n-2$ degree of freedom. The level of significance (α) is obtained by the definition of overall level of significance (i.e., $\alpha_{\text{overall}} = 1-(1-\alpha)^3$).

2.3.4 The EWMA_3 chart

As discussed above that memoryless structure, Shewhart charts (cf. Shewhart [39]) are only useful for the detection of large shifts in the process parameters. However, memory type structure (EWMA) is suitable to detect small or moderate shifts in process parameters. In linear profiling, Kim et al. [6] proposed a memory type structure for the joint monitoring of linear profile parameters known as EWMA_3 control chart. The structure of the EWMA_3 chart is defined as the following:

$$\text{EWMA}_{Ii} = \lambda\left(\hat{b}_{0i}\right) + (1-\lambda)\text{EWMA}_{I[i-1]}, \tag{26}$$

$$\text{EWMA}_{Si} = \lambda\left(\hat{b}_{1i}\right) + (1-\lambda)\text{EWMA}_{S[i-1]}, \tag{27}$$

$$\text{EWMA}_{Ei} = \max\left\{\lambda\ln\left(\widehat{\text{MSE}_i}\right) + (1-\lambda)\text{EWMA}_{\text{E}[i-1]}, ln\left(\sigma^2\right)\right\}, \tag{28}$$

where EWMA_{Ii} is the ith EWMA statistic for intercept;EWMA_{Si} and EWMA_{Ei} are the ith EWMA statistics for slope and error variance, respectively; λ is the smoothing parameter that ranges between zero and one (i.e., $0<\lambda\le 1$). The means and variances of three EWMA statistics (reported in Eqs. 26, 27, and 28) are given as the following:

$$\text{E}(\text{EWMA}_{\text{Ii}}) = B_0, \text{E}(\text{EWMA}_{\text{Si}}) = B_1, \text{E}(\text{EWMA}_{\text{Ei}}) = \ln\left(\sigma^2\right), \tag{29}$$

$$\text{Var}(\text{EWMA}_{\text{Ii}}) = \frac{\lambda}{2-\lambda}\sigma^2\left[\frac{1}{n} + \frac{\mu_X{}^2}{S_{xx}}\right], \ \text{Var}(\text{EWMA}_{\text{Si}}) = \frac{\lambda}{2-\lambda}\frac{\sigma^2}{S_{xx}}, \tag{30}$$

$$\text{Var}(\text{EWMA}_{\text{Ei}}) = \text{Var}(\ln(\text{MSE}_{\text{i}})) \cong \frac{2}{n-2} + \frac{2}{(n-2)^2} + \frac{4}{3(n-2)^3} - \frac{16}{15(n-2)^5}.$$

(cf. Hamilton and Crowder [40]). Based on the abovementioned properties of the EWMA statistics, the asymptotic limits for each EWMA plotting statistic are given as follows:

$$\begin{array}{ll} \text{for EWMA}_{\text{Ii}} : & \text{UCL or LCL} = B_0 \pm L_I\sqrt{\frac{\lambda}{2-\lambda}\sigma^2\left[\frac{1}{n} + \frac{\mu_X{}^2}{S_{xx}}\right]} \\ \text{for EWMA}_{\text{Si}} : & \text{UCL or LCL} = B_1 \pm L_S\sqrt{\frac{\lambda}{2-\lambda}\frac{\sigma^2}{S_{xx}}} \\ \text{for EWMA}_{\text{Ei}} : & \text{UCL}_{\text{E}} = \ln\left(\sigma^2\right) + L_E\sqrt{\frac{\lambda}{2-\lambda}\text{Var}\left(\ln\left(\hat{\text{MSE}_i}\right)\right)} \end{array}, \tag{31}$$

where L_{I}, L_{S} ,and L_{E} are the control limit coefficients for intercept, slope, and standard deviation of error term, which are carefully selected against the prespecified IC average run length.

3 Performance evaluations

The performance of the proposed and existing control charts is evaluated by several measures, which are reported in this subsection. Moreover, the IC parameters of proposed/existing charts and the performance evaluation of the stated study are briefly discussed in the following section.

3.1 Performance measures

To compare the performance of proposed/existing charts, several performance measures are used such as average run length (ARL), extra quadratic loss (EQL), sequential extra quadratic loss (SEQL), relative average run length (RARL), sequential relative average run length (SRARL), and performance comparison index (PCI). The ARL is a well-known measure which is defined as average number of sample points before an out-of-control signal occur. Usually, the *ARL* is categorized as (i) in-control average run length (ARL_0) and (ii) out-of-control average run length (ARL_1). Further, other useful measures such as EQL, SEQL (refers to the EQL), RARL, and SRARL (refers to the RARL) are defined as follows:

$$\mathrm{EQL}=\frac{1}{\nabla_{\max}-\nabla_{\min}}\int_{\nabla_{\min}}^{\nabla_{\max}}\nabla^2\mathrm{ARL}(\nabla)d\nabla, \tag{32}$$

$$\mathrm{SEQL}_i=\frac{1}{\nabla_{\max}-\nabla_{\min}}\int_{\nabla_{\min}}^{\nabla_{\max}}\nabla^2\mathrm{ARL}(\nabla)d\nabla,\quad i=2,3,\ldots\ldots,\nabla_{\max}, \tag{33}$$

$$\mathrm{RARL}=\frac{1}{\nabla_{\max}-\nabla_{\min}}\int_{\nabla_{\min}}^{\nabla_{\max}}\frac{\mathrm{ARL}(\nabla)}{\mathrm{ARL}_{\mathrm{bmk}}(\nabla)}d\nabla, \tag{34}$$

$$\mathrm{SRARL}_i=\frac{1}{\nabla_{\max}-\nabla_{\min}}\int_{\nabla_{\min}}^{\nabla_{\max}}\frac{\mathrm{ARL}(\nabla)}{\mathrm{ARL}_{\mathrm{bmk}}(\nabla)}d\nabla,\quad i=2,3,\ldots\ldots,\nabla_{\max}, \tag{35}$$

$$\mathrm{PCI}=\frac{\mathrm{EQL}}{\mathrm{EQL}_{\text{best chart}}}, \tag{36}$$

where $\nabla_{\max}$ and $\nabla_{\min}$ are the maximum and minimum shift in the linear profile parameters. The average run length of particular chart at specific shift (∇) is termed as ARL(∇) and $\mathrm{ARL}_{\mathrm{bmk}}(\nabla)$ is the average run length of benchmark chart at shift (∇). A chart is said to be a best chart which have minimum EQL value, and in this study, the EWMA_3 chart is considered as benchmark chart. It is to be noted that SEQL, SRARL values of any chart related to $\nabla_{\max}$ are known as EQL and RARL, respectively. For the more details on performance measures, see Wu et al. [41] and Ou et al. [42].

3.2 Designing of in-control parameters and control limits

For the original IC simple linear model given in Eq. (1), it is assumed that the $\beta_0=3$ and $\beta_1=2$ by following Kim et al. [6] (i. e., $Y_{ij}=3+2X_j+\varepsilon_{ij}$), where the fixed values of explanatory variable are $X=2$, 4, 6,and 8, sample size ($n=4$) and the error term is $\varepsilon_{ij}\sim N(s;\mu_s=0,\sigma_s=1)$. The transformed model reported in Eq. (2) is obtained by substituting the $B_0=3+2\overline{X}+(\beta\sigma)\overline{X}$, $B_1=(2+\beta\sigma)X_j^*$, and the fixed transformed values of explanatory variable, $X^*=-3,-1$, 1,and 3 with average equals to zero. For the fixed overall $\mathrm{ARL}_0=200$, control limit coefficients used for the charts under discussion are reported in Table 2. For the computations, Monte Carlo simulation study is designed with 10^5 iterations.

3.3 Shifts for performance evaluation

In order to evaluate the performance of control charts under consideration, several amounts of shift in linear profile parameters are considered. The description of shifts in linear profile parameters are given as follows:

i. Shifts in intercept parameter (B_0 *to* $B_0+\theta(\sigma/\sqrt{n})$),
ii. Shifts in slope parameter $\left(\beta_1 \textit{ to } \beta_1+\beta\left(\sigma/\sqrt{S_{xx}}\right)\right)$,
iii. Shifts in slope parameter $\left(B_1 \textit{ to } B_1+\delta\left(\sigma/\sqrt{S_{xx}}\right)\right)$,
iv. Shifts in error variance (σ^2 *to* $\gamma\sigma^2$),
v. Joint shifts in intercept (B_0 *to* $B_0+\theta(\sigma/\sqrt{n})$) and slope parameter $\left(B_1 \textit{ to } B_1+\delta\left(\sigma/\sqrt{S_{xx}}\right)\right)$

It is noted that process is said to be IC when θ, β, and δ are equal to zero and $\gamma=1$ otherwise, process is said to be OOC. Moreover, shifts in β_1 can cause the change in both B_0 and B_1, while shifts in B_1 are the original change in the slope of transformed model.

3.4 Comparative analysis

The comparative results of proposed and existing charts are reported in terms of ARL, EQL, SEQL, RARL, SRARL, and PCI. Further, the performance of the charts under consideration is discussed in terms of percentage change in the ARL_1 which is obtained as the following:

Percentage change $=\frac{\mathrm{ARL}_0-\mathrm{ARL}_1}{\mathrm{ARL}_0}$,

(i) *Shifts in intercept parameter*: the intercept is the mean value of dependent variable when independent variable has zero values. The shift in intercept plays a key role in linear profiling which means the change in the origin of regression line. The results for charts under consideration at shifted intercept parameter are reported in Table 3. Which shows that (20%) upward shift in intercept parameter ($\theta=0.20$), may decrease 92.0% in the ARL_1 of EWMA/R chart, 68.3% in the $\mathrm{ARL}_1$of Hotelling T^2 chart, 61.1% in the $\mathrm{ARL}_1$of Shewhart_3 chart, approximately 92.0% in the ARL_1 of *E*WMA_3 chart, Max − EWMA − 3 charts, and SS − EWMA − 3 charts while all other charts have around 76.0% decrease in the ARL_1 values. The Max − EWMA − 3 − C chart have minimum EQL (labeled by “*”) and SEQL values at every shift as compared to all other charts. As consider EWMA_3 chart as a benchmark chart, the RARL (tagged by “+”) and SRARL values, which are less than one for Max − EWMA − 3 − C chart reveals the superiority of this chart against all

Table 2 In-control design parameters for each chart at fixed $ARL_0 = 200$

Parameters	EWMA/R	Shewhat_3	EWMA_3	Max-EWMA-3	SS-EWMA-3	EWMA-Max-3	EWMA-SS-3
Intercept	$L_{ER} = 3.1151$	$Z_{\alpha/2} = 3.14$	$L_I = 3.0156$	$L_{Max} = 2.91$	$L_{SS} = 3.63$	$L_{EMax} = 2.56$	$L_{ESS} = 2.9$
Slope	$L_{ER} = 3.1151$	$Z_{\alpha/2} = 3.14$	$L_S = 3.0109$	$L_{Max} = 2.91$	$L_{SS} = 3.63$	$L_{EMax} = 2.56$	$L_{ESS} = 2.9$
Error variance	$L_{ER} = 3.1151$	LCL = 0.001 UCL = 14.17	$L_E = 1.3723$	$L_{Max} = 2.91$	$L_{SS} = 3.63$	$L_{EMax} = 2.56$	$L_{ESS} = 2.9$
Smoothing parameter	$\lambda = 0.2$	–	$\lambda = 0.2$	$\lambda = 0.2$	$\lambda = 0.2$	$\lambda = 0.2$	$\lambda = 0.2$

others under study. The $Max-EWMA-3-C$ chart have minimum EQL, so it has PCI = 1, which is the minimum PCI among all other charts and also the evidence about the dominance of this chart.

Moreover, the ARL curves for shifted intercept parameter are plotted in Fig. 1a, which reveals that joint ($Max-EWMA-3$ and $SS-EWMA-3$) and simultaneous (EWMA_3) charts have similar performance but they have better performance as compared to EWMA/R, Hotelling T^2, Shewhart_3, $EWMA-Max-3$, and $EWMA-SS-3$ charts. Specifically, $Max-EWMA-3-C$ chart outperforms all others charts under consideration.

(ii) *Shifts in slope parameter of original model*: the slope is an important parameter in regression, which is the rate of change in dependent variable as change occurred in independent variable. When the slope is affected by the shift, then the original rate of change is disturbed between the dependent variable and independent variable, which may lead to the false conclusions.

Table 4 is about the results of shifted slope parameter for original model given in Eq. (1). The findings reveal that upward shift in slope parameter of original model ($\beta = 0.075$), may decrease 90.1, 69.7, and 60.4% in the ARL_1 of EWMA/R, Hotelling T^2, and Shewhart_3 charts, respectively. On the same shifted parameter, almost 92.0% reduction is reported in ARL_1 of EWMA_3, $Max-EWMA-3$, and $SS-EWMA-3$ charts and approximately 76.0% reduction reported for all other charts under consideration. In terms of EQL (tagged by "*") and SEQL, $Max-EWMA-3$, and $SS-EWMA-3$ charts have minimum values but $SS-EWMA-3-A$ chart has relatively better performance as compared to all other charts. Further, both $Max-EWMA-3$ and $SS-EWMA-3$ charts have RARL (labeled by "+") and SRARL values fewer than one but $SS-EWMA-3-A$ chart have minimum values, which reveals the superiority of this chart against all others under study. There is no PCI < 1, which is the evidence about the dominance of $SS-EWMA-3-A$ chart.

Moreover, the *ARL* curves for shifted slope parameter of original model are plotted in Fig. 1b, which shows that $Max-EWMA-3$ and $SS-EWMA-3$ charts have similar performance. Specifically, $SS-EWMA-3-A$ chart outperforms all others charts under consideration.

(iii) *Shifts in slope parameter of transformed model*: the slope is the parameter, which describe the predicted values of dependent variable given independent variable. The shift in slope may cause the misleading results in the predicted values of dependent variable. The findings for the charts under consideration at shifted slope parameter of transformed model are reported in Table 5. Which shows that downward shift in slope parameter of transformed model ($\delta = -0.4$) may decrease 92.4% in the ARL_1 of EWMA/R chart, 95.2% in the $ARL_1$of Hotelling T^2 chart, 94.5% in the ARL_1 of $Shewhart-3$ chart, approximately 97.8% in the ARL_1 of EWMA_3 chart, $Max-EWMA-3$ charts, and $SS-EWMA-3$ charts, while all other charts have around 96.7% decrease in the ARL_1 values. In terms of minimum EQL, EWMA_3 and $Max-EWMA-3-C$ charts have similar behavior but in terms of minimum SEQL, for large shifts (−0.7 to −1.0), Hotelling T^2 chart have quite better performance, while for small shifts (−0.2 to −0.6) EWMA_3 and $Max-EWMA-3-C$ charts have relatively good performance. The same performance pattern is also observed with the values of SRARL that for large shifts (−0.7 to −1.0), Hotelling T^2 chart and for small shifts (−0.2 to −0.6) EWMA_3 and $Max-EWMA-3-C$ charts have relatively good performance. In terms of RARL (tagged by "+") and PCI, both EWMA_3 and $Max-EWMA-3-C$ charts have similar performance while both are superior then all other charts under discussion. Moreover, the ARL curves for shifted slope parameter of transformed model are plotted in Fig. 1c, which also reveals that both EWMA_3 and $Max-EWMA-3-C$ charts outperforms all others charts under the small shifts in slope parameter and when shifts are large Hotelling T^2 chart have better performance. In literature, the memoryless charting structures are used to detect the large shifts in process parameters, while memory type charts are used for the monitoring of small or

Table 3 Comparative analysis based on several performance measures for control charts in the presence of shifts in intercept parameter

Chart	Performance measure	θ										PCI
		0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	
EWMA/R	ARL	66.500	17.700	8.400	5.400	3.900	3.200	2.700	2.300	2.100	1.900	1.043
	SEQL	1.330	2.038	2.335	2.561	2.784	3.029	3.304	3.590	3.896	4.226*	
	SRARL	1.063	1.086	1.083	1.078	1.071	1.064	1.060	1.055	1.049	1.044+	
Hotelling T^2	ARL	137.700	63.500	28.000	13.200	6.900	4.000	2.600	1.800	1.500	1.200	1.575
	SEQL	2.754	5.294	6.903	7.493	7.529	7.329	7.058	6.782	6.554	6.382*	
	SRARL	1.665	2.395	2.841	2.897	2.758	2.557	2.355	2.172	2.014	1.880+	
Shewhart_3	ARL	151.400	77.900	33.800	15.500	7.700	4.300	2.700	1.900	1.500	1.200	1.775
	SEQL	3.028	6.144	8.201	8.912	8.892	8.567	8.164	7.778	7.454	7.192*	
	SRARL	1.781	2.733	3.337	3.417	3.240	2.985	2.732	2.507	2.314	2.150+	
EWMA_3	ARL	59.100	16.200	7.900	5.100	3.800	3.100	2.600	2.300	2.100	1.900	1.010
	SEQL	1.182	1.830	2.126	2.358	2.593	2.849	3.125	3.421	3.746	4.092*	
	SRARL	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000+	
Max − EWMA − 3 − A	ARL	61.288	16.686	7.987	5.143	3.841	3.102	2.601	2.276	2.071	1.900	1.013
	SEQL	1.226	1.893	2.187	2.411	2.642	2.893	3.163	3.451	3.764	4.103*	
	SRARL	1.019	1.026	1.024	1.021	1.018	1.016	1.014	1.012	1.009	1.007+	
Max − EWMA − 3 − B	ARL	60.886	16.329	7.985	5.155	3.827	3.085	2.626	2.286	2.077	1.893	1.013
	SEQL	1.218	1.871	2.162	2.393	2.628	2.880	3.154	3.449	3.766	4.104*	
	SRARL	1.015	1.017	1.014	1.014	1.013	1.011	1.010	1.009	1.007	1.006+	
Max − EWMA − 3 − C	ARL	58.870	16.167	7.892	5.133	3.801	3.056	2.587	2.257	2.038	1.891	1.000
	SEQL	1.177	1.824	2.121	2.356	2.593	2.845	3.116	3.405	3.715	4.052*	
	SRARL	0.998	0.998	0.998	0.999	1.000	0.999	0.998	0.997	0.994	0.993+	
SS − EWMA − 3 − A	ARL	59.991	17.073	8.491	5.509	4.131	3.330	2.806	2.443	2.196	2.032	1.075
	SEQL	1.200	1.883	2.220	2.488	2.756	3.040	3.342	3.659	3.995	4.358*	
	SRARL	1.008	1.021	1.035	1.046	1.053	1.058	1.061	1.062	1.061	1.061+	
SS − EWMA − 3 − B	ARL	59.166	17.509	8.452	5.559	4.136	3.318	2.818	2.454	2.190	2.026	1.077
	SEQL	1.183	1.884	2.230	2.497	2.768	3.050	3.350	3.669	4.004	4.364*	
	SRARL	1.001	1.021	1.039	1.049	1.057	1.061	1.064	1.065	1.064	1.063+	
SS − EWMA − 3 − C	ARL	61.343	17.384	8.482	5.519	4.130	3.311	2.810	2.437	2.193	2.031	1.077
	SEQL	1.227	1.922	2.254	2.513	2.777	3.056	3.353	3.668	4.002	4.363*	
	SRARL	1.019	1.037	1.049	1.056	1.062	1.065	1.066	1.067	1.065	1.064+	
EWMA − MAX − 3 − A	ARL	132.445	49.079	17.491	8.384	4.877	3.355	2.521	2.019	1.694	1.462	1.308
	SEQL	2.649	4.612	5.433	5.532	5.450	5.350	5.283	5.255	5.262	5.302*	
	SRARL	1.620	2.128	2.293	2.202	2.054	1.909	1.782	1.675	1.582	1.503+	
EWMA − MAX − 3 − B	ARL	131.559	48.352	17.583	8.400	4.864	3.305	2.501	2.035	1.693	1.465	1.304
	SEQL	2.631	4.565	5.388	5.504	5.427	5.323	5.252	5.228	5.242	5.285*	
	SRARL	1.613	2.109	2.274	2.190	2.044	1.899	1.772	1.666	1.575	1.497+	
EWMA − MAX − 3 − C	ARL	125.394	45.543	17.099	7.995	4.754	3.229	2.436	1.957	1.650	1.430	1.258
	SEQL	2.508	4.329	5.127	5.254	5.189	5.108	5.052	5.033	5.050	5.098*	
	SRARL	1.561	2.014	2.172	2.095	1.958	1.822	1.704	1.603	1.515	1.441+	
EWMA − SS − 3 − A	ARL	126.786	48.180	18.595	8.657	5.055	3.378	2.470	1.950	1.616	1.382	1.302
	SEQL	2.536	4.463	5.376	5.561	5.508	5.416	5.336	5.284	5.266	5.277*	
	SRARL	1.573	2.066	2.265	2.205	2.067	1.924	1.795	1.683	1.586	1.502+	
EWMA − SS − 3 − B	ARL	128.965	49.624	18.516	8.772	5.157	3.400	2.489	1.966	1.620	1.377	1.317
	SEQL	2.579	4.564	5.477	5.643	5.592	5.498	5.411	5.355	5.331	5.337*	
	SRARL	1.591	2.107	2.306	2.237	2.098	1.953	1.820	1.706	1.607	1.521+	
EWMA − SS − 3 − C	ARL	132.998	48.646	18.124	8.626	5.003	3.363	2.472	1.946	1.602	1.375	1.302
	SEQL	2.660	4.606	5.455	5.597	5.530	5.428	5.344	5.291	5.268	5.277*	
	SRARL	1.625	2.126	2.300	2.223	2.079	1.933	1.802	1.689	1.591	1.506+	

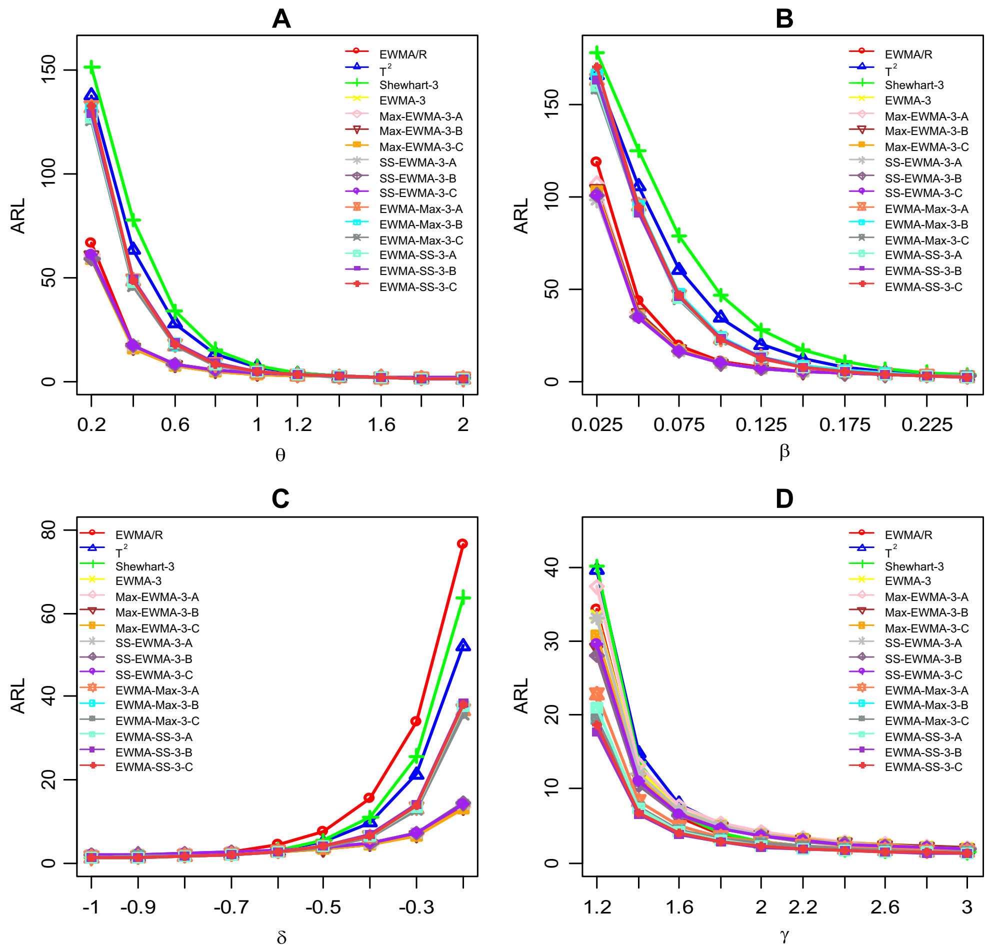

Fig. 1 **a**–**d** ARL curves of control charts with respect to different shifts in parameters

moderate shifts in process parameters. From the stated result, it is the evidence that under large shift memoryless (Hotelling T^2) chart have better performance while under small or moderate shifts memory type structures (both EWMA_3 and $Max-EWMA-3-C$) are performing well.

(iv) *Shifts in error variance of disturbance term*: in classical linear regression model, it is assumed that the error or disturbance term follows normal distribution with zero mean and fixed variance σ^2. When the variance of the IC model is affected by the shift, then the properties of the error term, which are also the properties of dependent variable, are disturbed badly. In such case, all the regression parameters are changed with the change in error variance. Table 6 is about the results of shifted error variance parameter for charts under consideration. Which reveals that upward shift in error variance parameter ($\gamma = 1.6$) may decrease 97.0, 96.1, 96.8, 96.4, 96.2, 96.6, 96.6, 96.5, 96.8, 96.7, 97.6, 97.8, 97.8, 97.9, 98.1, and 98.0% in the ARL_1 of EWMA/R, Hotelling T^2, Shewhart_3, EWMA_3, $Max-EWMA-3-A$, $Max-EWMA-3-B$, $Max-EWMA-3-C$, $SS-EWMA-3-A$, $SS-EWMA-3-B$ and $SS-EWMA-3-C$, $EWMA-Max-3-A$, $EWMA-Max-3-B$, $EWMA-Max-3-C$, $EWMA-SS-3-A$, $EWMA-SS-3-B$, and $EWMA-SS-3-C$ charts, respectively. In terms of EQL (tagged by "*") and SEQL, $EWMA-SS-3-B$ chart has relatively better performance as compared to all other charts. Further, all proposed structures have RARL (labeled by "+") and SRARL values fewer than one but $EWMA-SS-3-B$ chart have minimum values which reveals the superiority of this chart against all others under study. There is no PCI < 1, which is the evidence about the dominance of $EWMA-SS-3-B$ chart. Furthermore, the ARL curves for shifted error variance parameter are plotted in Fig. 1d, which also showed that $EWMA-SS-3-B$ chart outperforms all others charts under consideration.

(v) *Joint shifts in intercept and slope of transformed model*: in regression model, when both parameters (i.e., intercept and slope) are affected, then a change appears in the origin of the regression line as well as misleading results are produced for the predicted values. Table 7 is

Table 4 Comparative analysis based on several performance measures for control charts in the presence of shifts in slope of original model

Chart	Performance measure	β										PCI
		0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25	
EWMA/R	ARL	119.000	43.900	19.800	11.300	7.700	5.800	4.700	3.900	3.400	3.000	1.110
	SEQL	0.037	0.065	0.080	0.088	0.094	0.099	0.104	0.110	0.116	0.123*	
	SRARL	1.086	1.136	1.152	1.147	1.134	1.122	1.112	1.102	1.094	1.088+	
Hotelling T^2	ARL	166.000	105.600	60.700	34.500	20.100	12.200	7.800	5.200	3.700	2.700	2.140
	SEQL	0.052	0.118	0.179	0.220	0.242	0.251	0.252	0.248	0.243	0.236*	
	SRARL	1.317	1.790	2.271	2.568	2.669	2.641	2.546	2.422	2.291	2.165+	
Shewhart_3	ARL	178.300	125.000	79.200	46.700	27.900	17.100	10.900	7.100	5.000	3.600	2.846
	SEQL	0.056	0.134	0.216	0.276	0.311	0.327	0.332	0.329	0.322	0.314*	
	SRARL	1.377	1.984	2.670	3.151	3.362	3.384	3.295	3.152	2.989	2.828+	
EWMA_3	ARL	101.600	36.500	17.000	10.300	7.200	5.500	4.500	3.800	3.300	2.900	1.031
	SEQL	0.032	0.055	0.068	0.075	0.082	0.088	0.094	0.100	0.107	0.114*	
	SRARL	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000+	
Max-EWMA-3-A	ARL	107.274	37.499	17.392	10.295	7.226	5.580	4.494	3.774	3.264	2.911	1.037
	SEQL	0.034	0.057	0.070	0.078	0.084	0.090	0.096	0.102	0.108	0.114*	
	SRARL	1.028	1.035	1.032	1.027	1.022	1.020	1.018	1.015	1.012	1.010+	
Max-EWMA-3-B	ARL	104.169	37.380	17.471	10.269	7.194	5.510	4.490	3.786	3.288	2.926	1.036
	SEQL	0.033	0.056	0.069	0.077	0.083	0.089	0.095	0.101	0.108	0.114*	
	SRARL	1.013	1.019	1.021	1.019	1.015	1.012	1.010	1.009	1.008	1.007+	
Max-EWMA-3-C	ARL	102.865	35.895	17.022	10.158	7.065	5.418	4.432	3.727	3.249	2.884	1.018
	SEQL	0.032	0.055	0.067	0.075	0.081	0.087	0.093	0.099	0.106	0.112*	
	SRARL	1.006	1.002	0.999	0.998	0.995	0.993	0.992	0.991	0.990	0.990+	
SS-EWMA-3-A	ARL	98.167	35.011	16.431	9.871	6.976	5.356	4.345	3.695	3.207	2.877	1.000
	SEQL	0.031	0.053	0.065	0.073	0.079	0.085	0.091	0.097	0.104	0.110*	
	SRARL	0.983	0.973	0.970	0.968	0.967	0.968	0.968	0.968	0.969	0.970+	
SS-EWMA-3-B	ARL	101.115	35.389	16.432	9.887	6.902	5.332	4.364	3.716	3.235	2.874	1.003
	SEQL	0.032	0.054	0.066	0.073	0.079	0.085	0.091	0.097	0.104	0.111*	
	SRARL	0.998	0.990	0.983	0.978	0.974	0.972	0.972	0.972	0.973	0.974+	
SS-EWMA-3-C	ARL	101.032	34.777	16.661	9.885	6.943	5.350	4.357	3.686	3.224	2.865	1.002
	SEQL	0.032	0.053	0.066	0.073	0.079	0.085	0.091	0.097	0.104	0.111*	
	SRARL	0.997	0.985	0.979	0.977	0.974	0.973	0.973	0.972	0.972	0.973+	
EWMA-MAX-3-A	ARL	163.639	95.546	46.415	23.477	13.245	8.265	5.904	4.399	3.488	2.900	1.680
	SEQL	0.051	0.111	0.157	0.180	0.188	0.189	0.189	0.187	0.186	0.185*	
	SRARL	1.305	1.710	2.031	2.150	2.132	2.055	1.962	1.871	1.786	1.711+	
EWMA-MAX-3-B	ARL	166.288	95.279	47.384	24.306	13.434	8.457	5.988	4.407	3.472	2.900	1.702
	SEQL	0.052	0.112	0.158	0.183	0.191	0.193	0.192	0.190	0.189	0.188*	
	SRARL	1.318	1.721	2.047	2.179	2.165	2.088	1.995	1.901	1.813	1.734+	
EWMA-MAX-3-C	ARL	158.212	91.909	44.610	22.324	12.851	8.125	5.666	4.274	3.364	2.802	1.621
	SEQL	0.049	0.107	0.151	0.173	0.181	0.182	0.182	0.181	0.180	0.179*	
	SRARL	1.279	1.658	1.962	2.071	2.052	1.982	1.894	1.806	1.724	1.651+	
EWMA-SS-3-A	ARL	160.153	92.313	45.198	23.327	13.026	8.036	5.490	4.167	3.209	2.595	1.614
	SEQL	0.050	0.108	0.153	0.175	0.184	0.185	0.184	0.182	0.180	0.178*	
	SRARL	1.288	1.670	1.978	2.099	2.087	2.012	1.916	1.821	1.734	1.654+	
EWMA-SS-3-B	ARL	163.165	91.691	46.264	23.775	13.408	8.210	5.650	4.176	3.220	2.630	1.639
	SEQL	0.051	0.108	0.154	0.178	0.187	0.189	0.187	0.185	0.183	0.181*	
	SRARL	1.303	1.681	1.993	2.123	2.116	2.043	1.947	1.851	1.761	1.679+	
EWMA-SS-3-C	ARL	170.425	94.245	46.541	23.025	12.807	8.084	5.504	4.102	3.204	2.590	1.624
	SEQL	0.053	0.112	0.158	0.180	0.187	0.188	0.186	0.183	0.181	0.179*	
	SRARL	1.339	1.734	2.043	2.154	2.124	2.041	1.942	1.843	1.752	1.670+	

about the results of proposed/existing charts, for the simultaneous shifts in intercept and slope of transformed model. At fixed shift in slope of transformed model ($\delta = 0.1$), shift in intercept parameter ($\theta = 0.05$) may result to 29.6, 30.1, 75.9, 76.0, 75.5, 44.7, and 75.5% decrease in the ARL_1 of EWMA/R, Shewhart_3, EWMA_3, Max − EWMA − 3 − C, SS − EWMA − 3 − C, EWMA − Max − 3 − C, and EWMA − SS − 3 − C charts, respectively. Further, fixed shift in slope parameter ($\delta = 0.1$), shift in intercept ($\theta = 0.25$) may cause 79.1, 48.2, 86.5, 86.7, 87.9, 66.9, and 87.9% reduction in the ARL_1 of EWMA/R, Shewhart_3, EWMA_3, Max − EWMA − 3 − C, SS − EWMA − 3 − C, EWMA − Max − 3 − C, and EWMA − SS − 3 − C charts. Moreover, at fixed shift in intercept ($\theta = 0.25$), shift in slope parameter ($\delta = 0.15$) may result to 80.8, 61.0, 91.1, 91.3, 92.1, 78.0, and 77.5% decrease in the ARL_1 of EWMA/R, Shewhart_3, EWMA_3, Max − EWMA − 3 − C, SS − EWMA − 3 − C, EWMA − Max − 3 − C, and EWMA − SS − 3 − C charts, respectively. In conclusion, the EWMA_3, Max − EWMA − 3 − C, and SS − EWMA − 3 − C charts have similar performance and better than all other charts under consideration. Specifically, Max − EWMA − 3 − C chart has relatively better performance among all others.

Overall, the Max − EWMA − 3 − C chart shows relatively better performance in all type of shifts except shift in the slope of original model and error variance parameter. For the shift in slope of original model, the SS − EWMA − 3 − A chart while for the shifts in error variance parameter, the EWMA − SS − 3 − B chart shows superiority among all other control charts. Usually, the EWMA chart is used for the detection of persistent shifts in the process parameters, while the Max statistic based on the EWMA plotting statistics (e.g., MAX − EWMA − 3), provides the intense performance in the detection ability of control chart. Abbas et al. [43] showed that the Johnson SB transformation is a most power full transformation as compare to other transformations. Therefore, MAX − EWMA − 3 chart based on Johnson SB transformation (i.e., Max − EWMA − 3 − C chart) has impressive property as compared to all other charts. Meanwhile, quadratic version of the EWMA statistics (i.e., SS − EWMA − 3 − A chart) is more effective for detecting the immense change in the slope parameter of original model, which disturbed the complete properties of the transformed model given in Eq. (2). Moreover, when shifts are introduced in error variance term, then all parameters and the properties of the dependent variable are affected, which are more precisely detected by the EWMA structure based on sum of square version of standardized parameters.

4 Illustrative example

In a chemical industry, several devices are designed with chemical gas sensors and machine learning algorithms to solve some complex tasks. Unfortunately, such devices may still be far from laboratory and industry requirements. In calibration, variability of chemical gas sensors affects the performance of the calibration model, when the device is moved to other sensing environments. Among several chemical gas sensors, metal oxide (MOX) gas sensors are very promising due to their sensitivity, operational ease, cost efficiency, rapid response, and their ability to detect high number of volatiles. Fonollosa et al. [44] provides an extensive study about calibration models and their transferability. In particular, frame work of their study is portrayed in Fig. 2.

In this experimental study, three material flow controllers (MFC's) are used to maintain stable total flow (400 mL/min) in the measurement chamber. Four chemicals (ethanol, methane, ethylene, and carbon monoxide) are considered and the measurements were performed using the sensor. For the measurements, 10 different concentrations of each compound were used. The concentrations used for ethanol and ethylene are 12.5, 25.0, 37.5, 50.0, 62.5, 75.0, 87.5, 100.0, 112.5, and 125.0 ppm while for methane and carbon monoxide 25, 50, 75, 100, 125, 150, 175, 200, 225, and 250 ppm are used. They built five similar sensor units, each unit consist of eight MOX sensors (named by TGS2602(5.65V), TGS2602(5V), TGS2610(5.65V), TGS2610(V), TGS2611(5.65V), TGS2611(5V), TGS2612(5.65V), and TGS2612(5V)) and was tested individually. Moreover, these five sensor units were tested several times over a period of 22 days in such a way that unit 1 was tested on 4th, 10th, 15th, and 21st day, unit 2 was tested on 1st, 7th, 11th, and 16th day, unit 3 was tested on 2nd, 8th, 14th, and 17th day, unit 4 was tested on 3rd and 9th day, while unit 5 was tested on 18th and 22nd day.

For an illustrative example, 500 sample values of resistance (R) for MOX sensor TGS2612(5V) against carbon monoxide (CO) concentration levels 25, 100, 125, and 150 ppm are considered. These in-control values are obtained from the first unit, first replication, and about the time interval 368.88–373.00 s. In this study, resistance (R) is considered as a dependent variable and concentrations of carbon monoxide (CCO) as an independent variable. Further, the implementation is developed with the following steps:

Step 1. For the IC regression model, following estimated model and accompanying summary statistics are obtained from the 500 sample values of R against fixed values of CCO

$$\begin{aligned}\hat{R} &= 71.71233 - 0.01371354\ \mathrm{CCO}\\ S.E. &= (0.0116901);\quad (0.0001058915)\\ R^2 &= 0.8934,\quad \text{Adjusted } R^2 = 0.8933\end{aligned}$$

Table 5 Comparative analysis based on several performance measures for control charts in the presence of shifts in slope of transformed model

Chart	Performance measure	δ									PCI
		−1	−0.9	−0.8	−0.7	−0.6	−0.5	−0.4	−0.3	−0.2	
EWMA/R	ARL	1.200	1.400	1.800	2.600	4.200	7.500	15.300	33.700	76.700	2.023
	SEQL	0.600	0.627	0.692	0.798	0.945	1.130	1.367	1.606	1.763*	
	SRARL	0.725	0.803	0.951	1.184	1.526	2.012	2.566	2.936	2.533+	
Hotelling T^2	ARL	1.100	1.200	1.500	1.900	2.900	4.900	9.600	21.200	52.200	1.407
	SEQL	0.550	0.541	0.584	0.621	0.707	0.813	0.952	1.103	1.226*	
	SRARL	0.642	0.681	0.752	0.869	1.066	1.347	1.684	1.963	1.747+	
Shewhart_3	ARL	1.104	1.230	1.493	2.025	3.073	5.396	10.931	25.654	63.867	1.588
	SEQL	0.550	0.553	0.584	0.648	0.736	0.865	1.032	1.232	1.384*	
	SRARL	0.651	0.687	0.773	0.914	1.139	1.481	1.921	2.290	2.007+	
EWMA_3	ARL	1.711	1.874	2.061	2.319	2.726	3.317	4.389	6.698	12.897	1.000
	SEQL	0.855	0.843	0.830	0.824	0.826	0.830	0.839	0.855	0.872*	
	SRARL	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000+	
Max-EWMA-3-A	ARL	1.729	1.888	2.075	2.344	2.738	3.364	4.484	6.712	13.523	1.013
	SEQL	0.865	0.852	0.839	0.831	0.833	0.838	0.849	0.862	0.883*	
	SRARL	1.011	1.010	1.009	1.007	1.008	1.013	1.010	1.015	1.014+	
Max-EWMA-3-B	ARL	1.730	1.894	2.076	2.333	2.734	3.343	4.457	6.761	13.356	1.011
	SEQL	0.865	0.852	0.839	0.829	0.831	0.836	0.847	0.862	0.881*	
	SRARL	1.011	1.010	1.008	1.004	1.005	1.009	1.009	1.012	1.011+	
Max-EWMA-3-C	ARL	1.714	1.882	2.060	2.306	2.715	3.312	4.373	6.600	13.177	1.000
	SEQL	0.855	0.847	0.831	0.822	0.824	0.829	0.838	0.852	0.872*	
	SRARL	1.003	1.003	0.999	0.996	0.997	0.997	0.995	0.999	1.000+	
SS-EWMA-3-A	ARL	1.885	2.028	2.206	2.500	2.927	3.607	4.754	7.131	14.043	1.082
	SEQL	0.945	0.917	0.895	0.890	0.892	0.899	0.909	0.923	0.943*	
	SRARL	1.095	1.081	1.078	1.078	1.081	1.083	1.078	1.079	1.073+	
SS-EWMA-3-B	ARL	1.890	2.020	2.203	2.494	2.949	3.609	4.775	7.208	14.275	1.083
	SEQL	0.945	0.913	0.892	0.887	0.893	0.898	0.908	0.923	0.944*	
	SRARL	1.093	1.076	1.073	1.078	1.082	1.083	1.081	1.084	1.076+	
SS-EWMA-3-C	ARL	1.884	2.023	2.214	2.502	2.932	3.606	4.737	7.183	13.934	1.080
	SEQL	0.940	0.912	0.894	0.889	0.891	0.898	0.907	0.922	0.941*	
	SRARL	1.090	1.078	1.077	1.077	1.080	1.081	1.078	1.078	1.071+	
EWMA-MAX-3-A	ARL	1.266	1.457	1.713	2.093	2.732	3.864	6.364	13.195	36.928	1.182
	SEQL	0.635	0.655	0.674	0.701	0.738	0.781	0.843	0.928	1.031*	
	SRARL	0.762	0.801	0.849	0.909	0.987	1.099	1.261	1.473	1.376+	
EWMA-MAX-3-B	ARL	1.273	1.445	1.707	2.092	2.703	3.876	6.377	13.154	37.691	1.185
	SEQL	0.635	0.647	0.673	0.699	0.732	0.780	0.841	0.925	1.033*	
	SRARL	0.756	0.796	0.848	0.904	0.984	1.100	1.259	1.479	1.383+	
EWMA-MAX-3-C	ARL	1.241	1.412	1.663	2.030	2.622	3.716	6.166	12.627	35.556	1.142
	SEQL	0.620	0.633	0.654	0.680	0.712	0.755	0.816	0.895	0.995*	
	SRARL	0.740	0.776	0.824	0.878	0.952	1.063	1.216	1.419	1.330+	
EWMA-SS-3-A	ARL	1.198	1.361	1.638	2.031	2.700	3.906	6.538	13.483	37.587	1.172
	SEQL	0.600	0.611	0.642	0.673	0.715	0.765	0.831	0.917	1.021*	
	SRARL	0.715	0.757	0.816	0.884	0.974	1.100	1.266	1.480	1.379+	
EWMA-SS-3-B	ARL	1.204	1.375	1.637	2.041	2.700	3.926	6.707	13.926	38.537	1.189
	SEQL	0.600	0.619	0.644	0.677	0.717	0.769	0.841	0.930	1.036*	
	SRARL	0.720	0.762	0.819	0.886	0.978	1.113	1.292	1.511	1.403+	
EWMA-SS-3-C	ARL	1.190	1.364	1.634	1.995	2.663	3.868	6.481	13.625	38.087	1.169
	SEQL	0.595	0.610	0.639	0.664	0.707	0.758	0.823	0.913	1.019*	
	SRARL	0.712	0.754	0.807	0.871	0.963	1.089	1.262	1.484	1.380+	

Table 6 Comparative analysis based on several performance measures for control charts in the presence of shifts in error variance

Chart	Performance measure	γ										PCI
		1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3	
EWMA/R	ARL	34.300	12.000	6.100	3.900	2.900	2.300	1.900	1.700	1.500	1.400	1.259
	SEQL	124.696	80.576	60.240	48.712	41.393	36.388	32.767	30.073	28.024	26.439*	
	SRARL	1.012	0.998	0.964	0.925	0.890	0.864	0.840	0.820	0.803	0.789+	
Hotelling T^2	ARL	39.600	14.900	7.900	5.100	3.800	3.000	2.500	2.200	2.000	1.800	1.456
	SEQL	128.512	85.813	65.447	53.679	46.115	40.906	37.128	34.317	32.201	30.575*	
	SRARL	1.091	1.134	1.135	1.113	1.088	1.066	1.044	1.025	1.008	0.994+	
Shewhart_3	ARL	40.100	13.500	6.500	4.000	2.800	2.200	1.800	1.600	1.500	1.400	1.309
	SEQL	128.872	85.487	64.175	51.831	43.881	38.388	34.405	31.429	29.191	27.490*	
	SRARL	1.099	1.114	1.070	1.014	0.961	0.918	0.882	0.852	0.829	0.812+	
EWMA_3	ARL	33.500	12.700	7.200	5.100	3.900	3.200	2.800	2.500	2.300	2.100	1.428
	SEQL	124.120	80.343	60.783	49.957	43.178	38.572	35.320	32.969	31.247	29.969*	
	SRARL	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000+	
Max-EWMA-3-A	ARL	37.326	13.279	7.697	5.345	4.117	3.399	2.874	2.523	2.263	2.055	1.478
	SEQL	126.878	83.385	63.213	52.041	45.014	40.256	36.862	34.352	32.466	31.032*	
	SRARL	1.057	1.069	1.065	1.063	1.061	1.061	1.059	1.053	1.047	1.040+	
Max-EWMA-3-B	ARL	29.110	11.527	6.765	4.861	3.814	3.129	2.718	2.405	2.171	1.975	1.362
	SEQL	120.959	76.609	57.728	47.430	41.043	36.735	33.688	31.475	29.828	28.582*	
	SRARL	0.934	0.911	0.916	0.923	0.932	0.939	0.944	0.947	0.948	0.947+	
Max-EWMA-3-C	ARL	30.861	11.685	6.858	4.916	3.816	3.161	2.677	2.361	2.144	1.943	1.373
	SEQL	122.219	77.942	58.704	48.216	41.695	37.293	34.161	31.853	30.132	28.831*	
	SRARL	0.961	0.941	0.939	0.944	0.950	0.955	0.958	0.957	0.955	0.952+	
SS-EWMA-3-A	ARL	33.073	12.089	7.094	4.899	3.806	3.101	2.665	2.306	2.078	1.900	1.388
	SEQL	123.810	79.735	60.131	49.351	42.593	38.014	34.754	32.347	30.526	29.144*	
	SRARL	0.994	0.982	0.977	0.976	0.975	0.974	0.972	0.968	0.962	0.956+	
SS-EWMA-3-B	ARL	27.981	10.529	6.319	4.518	3.531	2.907	2.519	2.234	2.013	1.830	1.303
	SEQL	120.146	75.305	56.340	46.108	39.763	35.486	32.459	30.251	28.603	27.354*	
	SRARL	0.918	0.875	0.868	0.871	0.876	0.881	0.885	0.886	0.886	0.884+	
SS-EWMA-3-C	ARL	29.548	11.077	6.545	4.628	3.567	2.981	2.520	2.217	2.010	1.820	1.325
	SEQL	121.276	76.705	57.551	47.134	40.636	36.255	33.143	30.845	29.127	27.821*	
	SRARL	0.941	0.909	0.903	0.905	0.906	0.909	0.910	0.908	0.905	0.901+	
EWMA-MAX-3-A	ARL	22.911	8.352	4.899	3.481	2.775	2.304	1.989	1.786	1.638	1.524	1.137
	SEQL	116.495	70.587	51.876	41.885	35.743	31.637	28.731	26.613	25.042	23.865*	
	SRARL	0.842	0.756	0.727	0.716	0.712	0.712	0.713	0.713	0.713	0.713+	
EWMA-MAX-3-B	ARL	19.437	7.384	4.390	3.159	2.575	2.156	1.894	1.703	1.554	1.475	1.076
	SEQL	113.997	67.613	49.359	39.704	33.815	29.907	27.159	25.163	23.680	22.582*	
	SRARL	0.790	0.685	0.655	0.645	0.644	0.648	0.652	0.655	0.657	0.660+	
EWMA-MAX-3-C	ARL	19.508	7.533	4.426	3.200	2.564	2.166	1.892	1.694	1.563	1.458	1.079
	SEQL	114.047	67.737	49.508	39.845	33.936	30.009	27.250	25.238	23.748	22.642*	
	SRARL	0.791	0.689	0.661	0.651	0.649	0.652	0.656	0.658	0.660	0.663+	
EWMA-SS-3-A	ARL	20.954	7.407	4.203	2.952	2.311	1.943	1.685	1.534	1.400	1.317	1.051
	SEQL	115.084	68.715	50.023	40.056	33.924	29.823	26.928	24.817	23.244	22.062*	
	SRARL	0.813	0.709	0.667	0.645	0.633	0.628	0.624	0.622	0.621	0.621+	
EWMA-SS-3-B	ARL	17.628	6.564	3.858	2.745	2.149	1.814	1.613	1.461	1.360	1.283	1.000
	SEQL	112.694	65.908	47.729	38.141	32.261	28.331	25.572	23.572	22.093	20.993*	
	SRARL	0.763	0.642	0.604	0.587	0.578	0.575	0.574	0.575	0.576	0.579+	
EWMA-SS-3-C	ARL	18.681	6.786	3.946	2.817	2.218	1.853	1.631	1.475	1.385	1.298	1.016
	SEQL	113.450	66.777	48.421	38.722	32.779	28.802	25.998	23.956	22.452	21.336*	
	SRARL	0.779	0.662	0.622	0.604	0.596	0.592	0.590	0.590	0.590	0.593+	

Table 7 ARL results of control charts for simultaneous shifts in intercept and slope

θ	Chart	δ									
		0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
0.05	EWMA/R	179.10	169.90	156.60	140.80	123.20	105.10	88.80	73.40	60.20	49.60
	Shewhart_3	193.18	180.64	162.76	139.73	119.05	98.79	79.91	63.73	50.47	40.90
	EWMA_3	157.60	114.70	74.80	48.30	32.20	22.50	16.90	13.20	10.70	8.90
	Max-EWMA-3-C	156.63	113.18	73.85	48.06	31.53	22.40	16.70	13.13	10.55	8.77
	SS-EWMA-3-C	157.46	109.89	75.16	49.01	33.63	23.30	17.74	13.96	11.36	9.44
	EWMA-MAX-3-C	187.76	165.96	141.35	110.60	86.16	63.61	47.24	35.59	26.55	20.32
	EWMA-SS-3-C	193.96	176.45	148.11	117.82	88.93	67.79	50.26	37.58	28.57	21.82
0.1	EWMA/R	139.50	133.60	121.11	115.50	103.50	90.40	78.30	65.70	55.60	46.30
	Shewhart_3	184.63	172.96	154.66	137.51	115.32	95.28	77.85	62.38	50.83	40.31
	EWMA_3	122.10	94.60	66.40	44.90	30.70	21.90	16.60	13.10	10.60	8.90
	Max-EWMA-3-C	118.71	94.15	64.80	44.09	30.10	21.96	16.28	12.74	10.57	8.74
	SS-EWMA-3-C	118.33	90.65	62.10	43.18	30.03	22.50	17.05	13.39	10.98	9.32
	EWMA-MAX-3-C	166.16	151.35	129.84	103.92	80.08	60.81	44.76	34.05	25.69	20.05
	EWMA-SS-3-C	182.81	160.81	137.89	109.33	84.75	63.72	48.04	35.81	26.72	21.25
0.15	EWMA/R	96.80	94.20	90.30	85.10	78.50	70.90	63.00	55.30	47.70	40.90
	Shewhart_3	165.25	160.43	143.16	128.64	109.11	91.59	74.65	60.80	49.55	39.75
	EWMA_3	84.60	70.80	54.50	39.60	28.50	20.90	16.10	12.80	10.40	8.80
	Max-EWMA-3-C	83.85	69.37	54.43	38.85	28.13	20.98	15.90	12.71	10.24	8.75
	SS-EWMA-3-C	83.70	67.61	49.40	36.63	26.81	20.51	15.94	12.78	10.71	9.03
	EWMA-MAX-3-C	147.70	133.11	116.55	92.12	72.80	56.76	42.52	32.64	24.62	19.20
	EWMA-SS-3-C	156.09	139.77	119.23	96.98	75.99	57.61	43.93	33.59	26.06	20.10
0.2	EWMA/R	64.80	63.80	62.10	59.70	56.60	52.90	48.50	44.00	39.20	34.60
	Shewhart_3	148.30	143.85	132.16	116.66	101.51	87.21	70.79	57.63	47.31	38.28
	EWMA_3	57.10	51.10	42.40	33.30	25.40	19.50	15.40	12.40	10.20	8.70
	Max-EWMA-3-C	56.41	50.39	41.34	32.61	25.06	19.41	15.27	12.31	10.21	8.57
	SS-EWMA-3-C	56.89	48.89	38.62	30.20	22.91	18.34	14.39	12.00	10.09	8.60
	EWMA-MAX-3-C	122.92	111.80	96.54	80.60	64.98	49.94	38.87	29.91	23.26	18.00
	EWMA-SS-3-C	128.73	117.62	102.76	82.99	65.86	50.41	39.14	30.13	24.16	18.35
0.25	EWMA/R	44.30	43.80	42.90	41.80	40.30	38.40	36.10	33.60	30.80	28.10
	Shewhart_3	130.54	125.22	114.09	103.59	91.89	78.02	67.20	55.43	45.70	36.84
	EWMA_3	39.50	36.50	32.30	27.10	22.00	17.80	14.40	11.90	10.00	8.50
	Max-EWMA-3-C	39.34	35.99	31.89	26.62	21.71	17.44	14.30	11.68	9.96	8.43
	SS-EWMA-3-C	40.08	35.29	29.26	24.28	19.45	15.85	13.19	11.10	9.57	8.27
	EWMA-MAX-3-C	97.89	90.30	78.87	66.29	54.29	43.95	34.53	27.30	21.04	17.09
	EWMA-SS-3-C	104.17	94.25	82.15	68.69	55.86	45.06	35.22	27.02	21.72	17.35
0.3	EWMA/R	31.00	30.80	30.50	29.90	29.20	28.30	27.10	25.70	24.20	22.50
	Shewhart_3	112.33	107.85	100.24	91.56	81.92	71.51	61.72	50.87	42.88	35.20
	EWMA_3	28.20	26.90	24.70	22.00	18.80	15.70	13.20	11.20	9.60	8.30
	Max-EWMA-3-C	27.85	26.03	24.31	21.52	18.66	15.43	13.26	11.12	9.48	8.18
	SS-EWMA-3-C	28.78	26.37	22.81	19.61	16.56	13.92	11.97	10.18	8.86	7.78
	EWMA-MAX-3-C	77.49	70.35	64.96	55.96	46.20	37.33	29.97	24.39	19.58	15.87
	EWMA-SS-3-C	81.49	74.76	65.02	55.96	45.99	37.43	30.13	24.33	19.53	15.73

Step 2. For the analysis purpose, the standard deviation of resistance (R) is calculated ($\hat{\sigma}^2 = 0.09030$), smoothing parameter is fixed at 0.20 ($\lambda = 0.20$), sample size is considered as four ($n = 4$), and the CCO transformed values are taken as ($CCO^* = -75, 0, 25, 50$). To obtain the charting constant

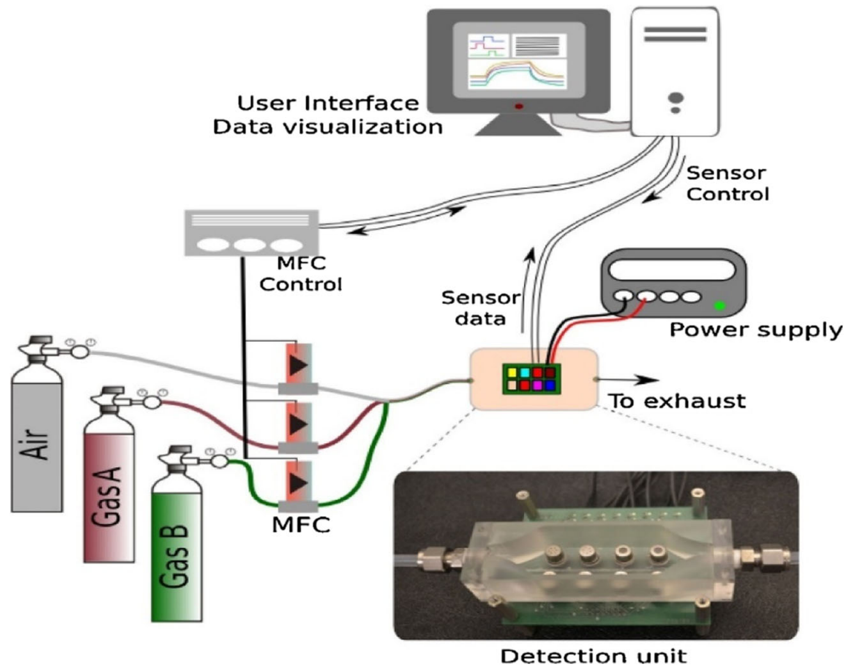

Fig. 2 Experimental setup of the study (cf. Fonollosa et al. [44])

of all charts at fixed $ARL_0 = 200$, bootstrap study is designed and obtained the following values; $L_{Max} = 3.0$, $L_{EMax} = 2.5$, $L_{SS} = 4.0$, $L_{ESS} = 3.57$, $L_I = 2.95$, $L_S = 3.3$, and $L_e = 3.1$.

Step 3. Once, the limits for EWMA_3, $Max - EWMA - 3 - C$, $EWMA - Max - 3 - C$, $SS - EWMA - 3 - C$, and $EWMA - SS - 3 - C$ charts are obtained, only first 200 profiles are used as IC profiles, which are pink shaded in Fig. 3a–g.

Step 4. To check the detection ability of these sensors, variations are introduced in the flow of gas by start/stop feature of selected gas release (cf. Fonollosa et al. [44]; Fig. 2). From this interval, hundred profiles belonging to time interval 60.94–61.94 s are considered as OOC profiles, shaded with white color in Fig. 3a–g.

In Fig. 3a, the EWMA_3 chart for the intercept parameter is plotted against the control limits ($LCL_I = 73.03661$, $UCL_I = 73.13076$), which reveals a single false alarm indexed with 169th point in the IC profiles. Further, under OOC profiles, this chart detects almost all points instead of 7 points having an index of 249–255. The EWMA_3 chart for the slope parameter is plotted in Fig. 3b along its control limits ($LCL_S = 0.01275253$, $UCL_S = 0.01467456$), which detected 83 OOC profiles out of 100 OOC profiles, while other profiles are IC with indexes of 201–212 and 214. Figure 3c is about the EWMA_3 chart for the error variance parameter with its control limits ($LCL_E = 0.04482076$, $UCL_E = 0.1357891$). It is noted that for ease, $EWMA_{Ei}$ is originally designed for the monitoring of ln(MSE) but in this real example, the $EWMA_{Ei}$ is designed for the monitoring of MSE. The result revealed a single false alarm (indexed with 79th point) in the IC profiles and under OOC profiles all points are detected OOC of control.

The $Max - EWMA - 3 - C$ chart is plotted in Fig. 3d against upper control limit ($UCL_{MEC} = 1.028091$), which shows a single false alarm indexed with 169th point in the IC profiles and detect all OOC profiles. The $SS - EWMA - 3 - C$ chart is given in Fig. 3e along its control limit ($UCL_{SSC} = 1.421995$), which detected three false alarms (indexed with 79th, 166th, and 169th point) in the IC profiles and under OOC profiles, all points are detected OOC. Further, the $EWMA - Max - 3 - C$ and $EWMA - SS - 3 - C$ charts are plotted in Fig. 3f, g against their corresponding control limits ($UCL_{EMC} = 1.814691$, $UCL_{ESSC} = 6.265986$). The findings reveal that both charts also detect all OOC signals under the OOC profiles.

In conclusion, this illustrative example shows that the performance of MOX gas sensor's resistant for the carbon monoxide volatility is affected by the flow adjustment of gasses. The findings exhibit that all control charts detect abnormal behavior of the resistance in the presence of flow adjustment but $Max - EWMA - 3 - C$ chart and $SS - EWMA - 3 - C$ chart have relatively better performance as compare to the other charts under consideration.

5 Summary and conclusions

Control chart is a key device used for the monitoring of a production process. Usually, control charts are designed for

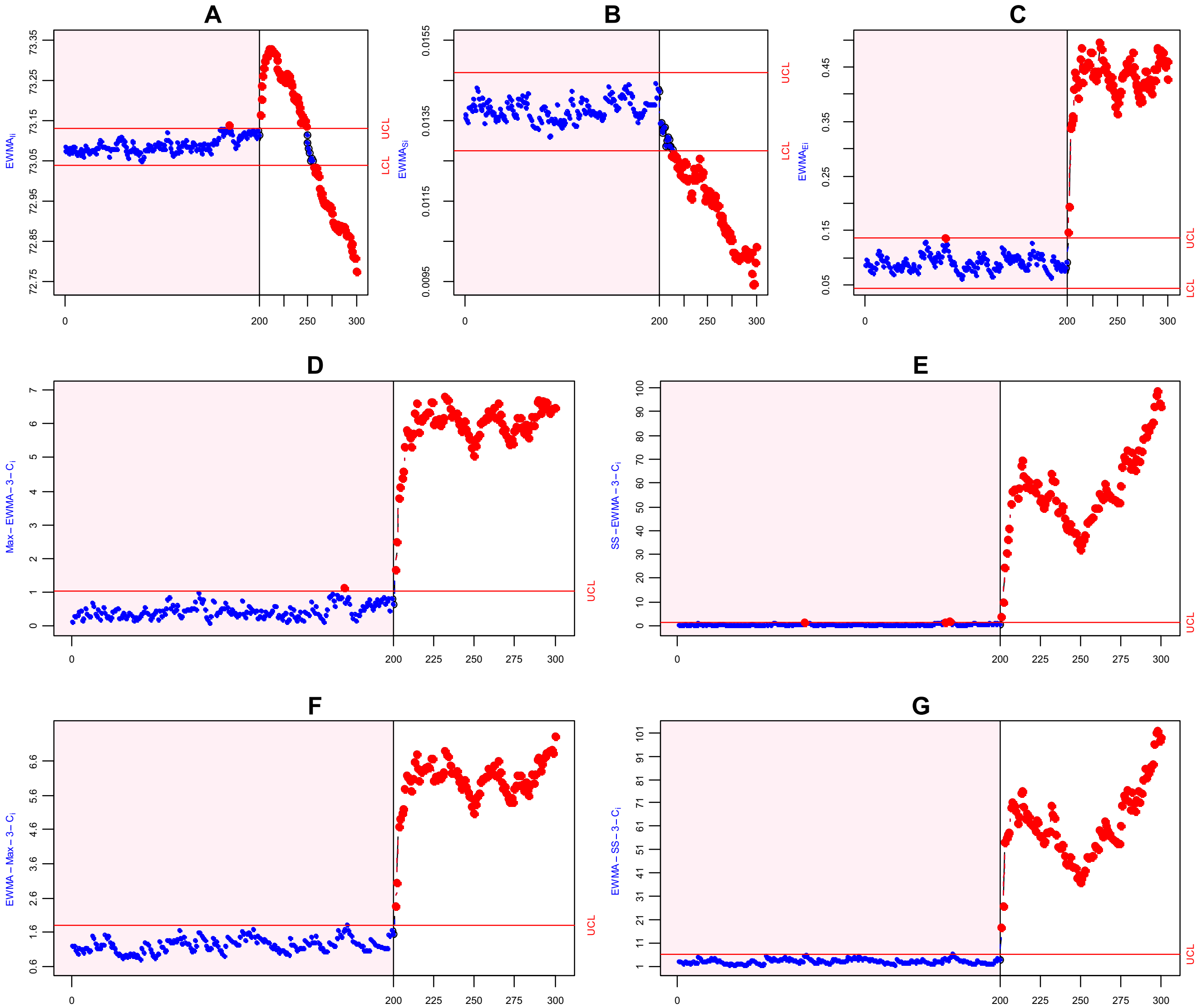

Fig. 3 **a–g** Control charts for illustrative example

the monitoring of single quality characteristic and/or for simultaneous monitoring of quality characteristics. Generally, in many manufacturing processes, quality characteristic of interest has a relation with other auxiliary variable(s). When such quality characteristic is linearly associated with another explanatory variable, the relationship is termed as simple linear profiles and the monitoring of simple linear profile parameters (i.e., slope, intercept, and error variance) is known as linear profiling. In recent literature, simple linear profiles are monitored through simultaneous structure, which is a tedious method; such as each distinct pair of control limits required individual charting constant. In the stated study, new control charting methodologies are designed for the monitoring of simple linear profile parameters, which are simple, relatively efficient, and easy to implement. The findings of this study reveal that newly designed control charts such as Max − EWMA − 3 and SS − EWMA − 3 have almost similar performance with EWMA_3 chart. Specifically, the Max − EWMA − 3 − C chart shows relatively better performance in the presence of shifts in intercept, slope of transformed model and the simultaneous shifts in intercept and slope. However, in the presence of shifts in the slope of original model SS − EWMA − 3 − A chart and for the shifts in error variance parameter, EWMA − SS − 3 − B chart shows superiority among all other control charts.

6 Limitations and recommendations

This study is purely designed for simple linear profiles under the ideal assumption of normality. The assumption of normality may not work in all real situations. Therefore, one may extend this study for non-linear profiles and multiple linear profiles under normal or non-normal environments.

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Appendix

Cumulative distribution function (CDF) and probability density function (pdf) of *Max − EWMA* statistic

Suppose, an jth observation for k^{th} statistic on i^{th} sampling point is represented by Y_{kij}. Whereas, sample size is indexed by j; $j = 1, 2, \ldots, n$, statistics are indexed by k; $k = 1, 2, \ldots, p$, and sampling points are indexed by i; $i = 1, 2, \ldots$,. Let, n simple random samples on i^{th} sampling point are drawn from normal distribution with mean (μ) and variance (σ^2). Then the p sample means are represented by $\overline{Y}_{1i}, \overline{Y}_{2i}, \ldots, \overline{Y}_{pi}$ and it is assumed that these p sample means are independent. The *EWMA* statistics for all these p independent sample means are defined as

$$
\begin{aligned}
W_{1i} &= \lambda\overline{Y}_{1i} + (1-\lambda)W_{1,i-1} \\
W_{2i} &= \lambda\overline{Y}_{2i} + (1-\lambda)W_{2,i-1} \\
W_{3i} &= \lambda\overline{Y}_{3i} + (1-\lambda)W_{3,i-1} \\
&\vdots \\
W_{pi} &= \lambda\overline{Y}_{pi} + (1-\lambda)W_{p,i-1}
\end{aligned}
$$

The $Max-EWMA$ statistic based on abovementioned p *EWMA* statistics is defined as follows

$$M_i = \max\left(|W_{1i}|.|W_{2i}|.|W_{3i}|.\ldots.|W_{pi}|\right)$$

$$M_i = \max\left(\prod_{k=1}^{p}|W_{ki}|\right)$$

$$F(h;\sigma_{W_{ki}}) = P(M_i \le h),$$

$$= P\left\{(|W_{1i}|\le h).(|W_{2i}|\le h)\ldots\left(|W_{pi}|\le h\right)\right\},$$

$$= P(|W_{1i}|\le h).P(|W_{2i}|\le h)\ldots P\left(|W_{pi}|\le h\right),$$

$$= \left\{2\Phi\left(\frac{h}{\sigma_{W_{ki}}}\right)-1\right\}^p; h\ge 0$$

Therefore, the pdf may be obtained as

$$f(h;\sigma_{W_{ki}}) = \frac{d}{dh}F(h;\sigma_{W_{ki}})$$

$$f(h;\sigma_{W_{ki}}) = \frac{d}{dh}\left\{2\Phi\left(\frac{h}{\sigma_{W_{ki}}}\right)-1\right\}^p$$

$$f(h;\sigma_{W_{ki}}) = p\left\{2\Phi\left(\frac{h}{\sigma_{W_{ki}}}\right)-1\right\}\left[\frac{d}{dh}\left\{2\Phi\left(\frac{h}{\sigma_{W_{ki}}}\right)-1\right\}-\frac{d}{dh}(1)\right]$$

$$f(h;\sigma_{W_{ki}}) = p\left\{2\Phi\left(\frac{h}{\sigma_{W_{ki}}}\right)-1\right\}\left[\left\{2\phi\left(\frac{h}{\sigma_{W_{ki}}}\right)\frac{d}{dh}\left(\frac{h}{\sigma_{W_{ki}}}\right)\right\}-0\right]$$

$$f(h;\sigma_{W_{ki}}) = p\left\{2\Phi\left(\frac{h}{\sigma_{W_{ki}}}\right)-1\right\}\left\{2\phi\left(\frac{h}{\sigma_{W_{ki}}}\right)\left(\frac{1}{\sigma_{W_{ki}}}\right)\right\}$$

$$f(h;\sigma_{W_{ki}}) = \frac{2p}{\sigma_{W_{ki}}}\phi\left(\frac{h}{\sigma_{W_{ki}}}\right)\left\{2\Phi\left(\frac{h}{\sigma_{W_{ki}}}\right)-1\right\}$$

where $\Phi(.)$ and $\phi(.)$ are known as standard normal CDF and standard normal pdf respectively.

Mean and variance of *Max − EWMA* statistic (M_i)

The mean of the Max statistic $E(M_i)$ can be obtained as follows:

$$E(M_i) = \int_0^\infty h\, f(h;\sigma_{W_{ki}})\, dh$$

$$E(M_i) = \int_0^\infty \frac{4ph}{\sigma_{W_{ki}}}\phi\left(\frac{h}{\sigma_{W_{ki}}}\right)\Phi\left(\frac{h}{\sigma_{W_{ki}}}\right)dh-\int_0^\infty \frac{2ph}{\sigma_{W_{ki}}}\phi\left(\frac{h}{\sigma_{W_{ki}}}\right)dh$$

By using the substitution $t = \frac{h}{\sigma_{W_{ki}}}$, where $h = t.\sigma_{W_{ki}}$ and $dt = \frac{1}{\sigma_{W_{ki}}}dh$

$$E(M_i) = \int_0^\infty 4pt\phi(t)\Phi(t)dt-\int_0^\infty 2pt\phi(t)dt$$

and by the numerical computation when $p = 3$, the mean is

$$E(M_i) = 1.32639(\sigma_{w_{ki}})$$

For the variance of Max statistic, the $E\left(M_i^2\right)$ is obtained as follows:

$$E\left(M_i^2\right) = \int_0^\infty h^2\, f(h;\sigma_{W_{ki}})\, dh$$

$$E\left(M_i^2\right) = \int_0^\infty \frac{4ph^2}{\sigma_{W_{ki}}}\phi\left(\frac{h}{\sigma_{W_{ki}}}\right)\Phi\left(\frac{h}{\sigma_{W_{ki}}}\right)dh-\int_0^\infty \frac{2ph^2}{\sigma_{W_{ki}}}\phi\left(\frac{h}{\sigma_{W_{ki}}}\right)dh$$

By using the substitution $t = \frac{h}{\sigma_{W_{ki}}}$, where $h = t.\sigma_{W_{ki}}$ and $dt = \frac{1}{\sigma_{W_{ki}}}dh$

$$E\left(M_i^2\right) = \int_0^\infty 4pt^2\phi(t)\Phi(t)dt-\int_0^\infty 2pt^2\phi(t)dt$$

and by the numerical computation when $P = 3$

$$E\left(M_i^2\right) = 2.10266(\sigma_{W_{ki}})$$

So, the variance of max statistic is obtained as follows

$$\sigma_{M_i}^2 = E\left(M_i^2\right)-\left(E(M_i)\right)^2$$

$$\sigma^2_{M_i} = 0.5859607\left(\sigma^2_{W_{ki}}\right)$$

Derivation of the upper control limit for *Max − EWMA − 3* chart

The structure of $Max-EWMA-3-A$ chart depends on three EWMA statistics M_i, N_i and O_i reported in equations 8–10 and the means and variances of M_i, N_i and O_i are obtained as $E(M_i) = E(N_i) = E(O_i) = 0$

$$\sigma^2_{M_i} = \sigma^2_{N_i} = \sigma^2_{O_i} = \frac{\lambda}{2-\lambda}\sigma^2_{Z_{\hat{b}_0}}$$

where the $\sigma^2_{Z_{\hat{b}_0}}, \sigma^2_{Z_{\hat{b}_1}}$ and $\sigma^2_{Z_{\hat{MSE}}}$ is equal to one. Therefore, by using the expression given in Appendix (A.2) the mean and variance of $Max-EWMA-3-A_i$ statistic are obtained as follows

$$E(Max\text{–}EWMA\text{–}3\text{–}A_i) = 1.32639$$

$$\sigma^2_{Max-EWMA-3-A_i} = 0.34335$$

Therefore, the upper control limit of $Max-EWMA-3-A$ chart (UCL_{MEO}) can be obtained as

$$UCL_{MEA} = \left[E(Max\text{–}EWMA\text{–}3\text{–}A_i) + L_{Max}\sqrt{\sigma^2_{Max-EWMA-3-A_i}}\right]\sigma^2_{M_i}$$

$$UCL_{MEA} = (1.32639 + L_{Max}0.5859607)\sqrt{\frac{\lambda}{2-\lambda}}$$

On the same lines, one may obtain the limits of $Max-EWMA-3-B$ chart (UCL_{MEB}) and $Max-EWMA-3-C$ chart (UCL_{MEC}).

Derivation of the upper control limit for *EWMA − Max − 3* chart

The structure of $EWMA-\ Max\ -3-A$ chart depends on a Max statistic (Max_A) (cf. equation 12), which consist three statistics $Z_{\hat{b}_0}, Z_{\hat{b}_1}$ and $Z_{\hat{MSE}}$ and the mean and variance of Max_A statistic are

$$E(Max_{Ai}) = 1.32639\left(\sigma_{Z_{\hat{b}_0}}\right)$$

$$\sigma^2_{Max_{Ai}} = 0.34335\left(\sigma^2_{Z_{\hat{b}_0}}\right)$$

Whereas, $\sigma^2_{Z_{\hat{b}_0}}, \sigma^2_{Z_{\hat{b}_0}}$ and $\sigma^2_{Z_{\hat{MSE}}}$ is equal to one, therefore the mean and variance of Max_A statistics is

$$E(Max_{Ai}) = 1.32639$$

$$\sigma^2_{Max_{Ai}} = 0.34335$$

The $EWMA-\ Max\ -3-A_i$ statistic and its mean and variance are defined as follows

$$EWMA\text{–}Max\text{–}3\text{–}A_i = \lambda Max_A + (1-\lambda)EWMA\text{–}Max\text{–}3\text{–}A_{i-1}$$

$$E(EWMA\text{–}Max\text{–}3\text{–}A_i) = 1.32639$$

$$\sigma^2_{EWMA-Max-3-A_i} = 0.34335\frac{\lambda}{2-\lambda}$$

Therefore, the upper control limit of $EWMA-\ Max\ -3-A$ chart is

$$UCL_{EMA} = E(EWMA\text{–}Max\text{–}3\text{–}A_i) + L_{EMax}\sqrt{\sigma^2_{EWMA-Max-3-A_i}}$$

$$UCL_{EMA} = 1.32639 + L_{EMax}\sqrt{0.34335\frac{\lambda}{2-\lambda}}$$

$$UCL_{EMA} = 1.32639 + 0.5859607L_{EMax}\sqrt{\frac{\lambda}{2-\lambda}}$$

One may obtain the upper control limit of $EWMA-\ Max\ -3-B$ chart (UCL_{EMB}) and $EWMA-\ Max\ -3-C$ chart (UCL_{EMC}) by using the abovementioned procedure.

Derivation of the upper control limit for *SS − EWMA − 3* chart

The structure of $SS-EWMA-3-A$ chart depends on three EWMA statistics (M_i, N_i and O_i) reported in equations 8–10 and the means and variances of M_i, N_i and O_i are obtained by

$$E(M_i) = E(N_i) = E(O_i) = 0$$

$$\sigma^2_{M_i} = \sigma^2_{N_i} = \sigma^2_{O_i} = \frac{\lambda}{2-\lambda}\sigma^2_{Z_{\hat{b}_0}}$$

As discussed earlier that $\sigma^2_{Z_{\hat{b}_0}}, \sigma^2_{Z_{\hat{b}_1}}$ and $\sigma^2_{Z_{\hat{MSE}}}$ is equal to one and $\frac{SS-EWMA-3-A_i}{\sigma^2_{M_i}} = \frac{SS-EWMA-3-A_i}{\sigma^2_{M_i}} + \frac{SS-EWMA-3-A_i}{\sigma^2_{M_i}} + \frac{SS-EWMA-3-A_i}{\sigma^2_{M_i}} \sim \chi^2_{(p)}$, therefore, the mean and variance of $SS-EWMA-3-A_i$ statistics are obtained follows

$$E(SS\text{–}EWMA\text{–}3\text{–}A_i) = p$$

$$\sigma^2_{SS-EWMA-3-A_i} = 2p$$

In this study, $p = 3$ is considered due to three monitoring statistics M_i, N_i and O_i. So, the above expressions are represented as follows

$$E(SS\text{–}EWMA\text{–}3\text{–}A_i) = 3$$

$$\sigma^2_{SS-EWMA-3-A_i} = 6$$

hence, the upper control limit of $SS-EWMA-3-A$ chart (UCL_{SSO}) can be obtained as

$$UCL_{SSA} = \left[E(SS\text{–}EWMA\text{–}3\text{–}A_i) + L_{SS}\sqrt{\sigma^2_{SS-EWMA-3-A_i}}\right]\sigma^2_{M_i}$$

$$UCL_{SSA} = \left[3 + L_{SS}\sqrt{6}\right]\frac{\lambda}{2\text{–}\lambda}$$

$$UCL_{SSA} = \frac{\lambda\left(3 + L_{SS}\sqrt{6}\right)}{2\text{–}\lambda}$$

On the same lines, one may obtain the limits of $SS-EWMA-3-B$ chart (UCL_{SSB}) and $SS-EWMA-3-C$ chart (UCL_{SSC}).

Derivation of the upper control limit for *EWMA − SS − 3* chart

The structure of $EWMA-SS-3-A$ chart depends on a SS statistic (SS_A) which contain three statistics $Z_{\hat{b}_0}, Z_{\hat{b}_1}$ and $Z_{\widehat{MSE}}$ having unit variance (i.e. $\sigma^2_{Z_{\hat{b}_0}} = \sigma^2_{Z_{\hat{b}_1}} = \sigma^2_{Z_{\hat{MSE}}} = 1$). As discussed above that

$$\frac{SS_{Ai}}{\sigma^2_{Z_{\hat{b}_0}}} = \frac{Z_{\hat{b}_0 i}^{\;2}}{\sigma^2_{Z_{\hat{b}_0}}} + \frac{Z_{\hat{b}_1 i}^{\;2}}{\sigma^2_{Z_{\hat{b}_0}}} + \frac{Z_{\widehat{MSE}i}^{\;2}}{\sigma^2_{Z_{\hat{b}_0}}} \sim \chi^2_{(p)}$$

Therefore, the mean and variance of SS_{Ai}statistic are obtained as follows

$$E(SS_{Ai}) = p$$

$$\sigma^2_{SS_{Ai}} = 2p$$

In stated study, three statistics $Z_{\hat{b}_0}, Z_{\hat{b}_1}$ and $Z_{\widehat{MSE}}$ are considered for monitoring purpose. Therefore $p = 3$ in this case and the above expressions are as follows

$$E(SS_{Ai}) = 3$$

$$\sigma^2_{SS_{Ai}} = 6$$

The $EWMA-SS-3-A_i$ statistic and its mean and variance are defined as

$$EWMA\text{–}SS\text{–}3\text{–}A_i = \lambda SS_A + (1\text{–}\lambda)EWMA\text{–}SS\text{–}3\text{–}A_{i\text{–}1}$$

$$E(EWMA\text{–}SS\text{–}3\text{–}A_i) = 3$$

$$\sigma^2_{EWMA-SS-3-A_i} = 6\frac{\lambda}{2\text{–}\lambda}$$

So, the upper control limit of $EWMA-SS-3-A$ chart is obtained as follows

$$UCL_{ESSA} = E(EWMA\text{–}SS\text{–}3\text{–}A_i) + L_{ESS}\sqrt{\sigma^2_{EWMA-SS-3-A_i}}$$

$$UCL_{ESSA} = 3 + L_{ESS}\sqrt{\frac{6\lambda}{2\text{–}\lambda}}$$

One may obtain the upper control limit of $EWMA-SS-3-B$ chart (UCL_{ESSB}) and $EWMA-SS-3-C$ chart (UCL_{ESSC}) by using the abovementioned procedure.

References

1. Kang L, Albin S (2000) On-line monitoring when the process yields a linear. J Qual Technol 32:418–426
2. Riaz M, Mahmood T, Abbasi SA, Abbas N, Ahmad S (2017) Linear profile monitoring using EWMA structure under ranked set schemes. Int J Adv Manuf Technol 91:2751–2775
3. Mestek O, Pavlík J, Suchánek M (1994) Multivariate control charts: control charts for calibration curves. Fresenius J Anal Chem 350: 344–351
4. Stover FS, Brill RV (1998) Statistical quality control applied to ion chromatography calibrations. J Chromatogr A 804:37–43
5. Croarkin MC, Varner RN (1982) Measurement assurance for dimensional measurements on integrated-circuit photomasks. NBS Technical Note 1164, U.S. Department of Commerce, Washington, D.C., USA. International Organization for Standardization, 1996, Linear calibration using reference materials. ISO 11095:1996, Geneva, Switzerland
6. Kim K, Mahmoud MA, Woodall WH (2003) On the monitoring of linear profiles. J Qual Technol 35:317–328
7. Mahmoud MA, Woodall WH (2004) Phase I analysis of linear profiles with calibration applications. Technometrics 46:380–391
8. Noorossana R, Amiri A, Vaghefi SA, Roghanian E (2004) Monitoring quality characteristics using linear profile, In Proceedings of the 3rd International Industrial Engineering Conference 246–255
9. Noorossana R, Vaghefi SA, Amiri A (2004) The effect of non-normality on monitoring linear profiles, in proceedings of the 2nd international industrial engineering conference. Riyadh, Saudi Arabia
10. Zou C, Zhang Y, Wang Z (2006) A control chart based on a change-point model for monitoring linear profiles. IIE Trans 38:1093–1103
11. Mahmoud MA, Parker PA, Woodall WH, Hawkins DM (2007) A change point method for linear profile data. Qual Reliab Eng Int 23: 247–268
12. Gupta S, Montgomery DC, Woodall WH (2006) Performance evaluation of two methods for online monitoring of linear calibration profiles. Int J Prod Res 44:1927–1942
13. Woodall WH (2007) Current research on profile monitoring. Production 17:420–425
14. Noorossana R, Amiri A (2007) Enhancement of linear profiles monitoring in phase II. AMIRKABIR J Sci Technol 18:19–27
15. Zou C, Zhou C, Wang Z, Tsung F (2007) A self-starting control chart for linear profiles. J Qual Technol 39:364–375
16. Jensen WA, Birch JB, Woodall WH (2008) Monitoring correlation within linear profiles using mixed models. J Qual Technol 40:167–183
17. Zhang J, Li Z, Wang Z (2009) Control chart based on likelihood ratio for monitoring linear profiles. Comput Stat Data Anal 53: 1440–1448
18. Saghaei A, Mehrjoo M, Amiri A (2009) A CUSUM-based method for monitoring simple linear profiles. Int J Adv Manuf Technol 45: 1252–1260
19. Mahmoud MA, Morgan JP, Woodall WH (2010) The monitoring of simple linear regression profiles with two observations per sample. J Appl Stat 37:1249–1263
20. Khediri IB, Weihs C, Limam M (2010) Support vector regression control charts for multivariate nonlinear autocorrelated processes. Chemom Intell Lab Syst 103:76–81

21. Shang Y, Tsung F, Zou C (2011) Phase II profile monitoring with binary data and random predictors. J Qual Technol 43:196–208
22. Laursen K, Rasmussen MA, Bro R (2011) Comprehensive control charting applied to chromatography. Chemom Intell Lab Syst 107: 215–225
23. Yeh A, Zerehsaz Y (2013) Phase I control of simple linear profiles with individual observations. Qual Reliab Eng Int 29:829–840
24. Riaz M, Touqeer F (2015) On the performance of linear profile methodologies under runs rules schemes. Qual Reliab Eng Int 8: 1473–1482
25. Noorossana R, Fatemi SA, Zerehsaz Y (2015) Phase II monitoring of simple linear profiles with random explanatory variables. Int J Adv Manuf Technol 76:779–787
26. Noorossana R, Aminmadani M, Saghaei A (2016) Effect of phase I estimation error on the monitoring of simple linear profiles in phase II. Int J Adv Manuf Technol 84:873–884
27. Sogandi F, Amiri A (2017) Monotonic change point estimation of generalized linear model-based regression profiles. Commun Stat Simul Comput 46:2207–2227
28. Chiang JY, Lio YL, Tsai TR (2017) MEWMA control chart and process capability indices for simple linear profiles with within-profile autocorrelation. Qual Reliab Eng Int 33:1083–1094
29. Wang YHT, Huang WH (2017) Phase II monitoring and diagnosis of autocorrelated simple linear profiles. Comput Ind Eng 112:57–70
30. Zhang Y, Shang Y, Zhang M, Wu X (2017) Zone control charts with estimated parameters for detecting prespecified changes in linear profiles. Commun Stat Simul Comput 1–26
31. Shadman A, Zou C, Mahlooji H, Yeh AB (2017) A change point method for phase II monitoring of generalized linear profiles. Commun Stat Simul Comput 46:559–578
32. Castagliola P (2005) A new S^2-EWMA control chart for monitoring the process variance. Qual Reliab Eng Int 21:781–794
33. Castagliola P, Celano G, Fichera S (2010) A Johnson's type transformation EWMA-S^2 control chart. International Journal of Quality Engineering and Technology 1:253–275
34. Roberts SW (1959) Control chart tests based on geometric moving averages. Technometrics 1:239–250
35. Chen G, Cheng SW, Xie H (2001) Monitoring process mean and variability with one EWMA chart. J Qual Technol 33:223–233
36. Xie H (1999) Contributions to qualimetry (Doctoral dissertation, The University of Manitoba, Canada
37. Montgomery DC (2009) Introduction to statistical quality control. Wiley, New York
38. Gupta S (2010) Profile monitoring-control chart schemes for monitoring linear and low order polynomial profiles (Doctoral dissertation, Arizona State University)
39. Shewhart WA (1980) Economic control of quality of manufactured product, New York: 1931. Reprinted by ASQC, Milwaukee
40. Hamilton MD, Crowder SV (1992) Average run lengths of EWMA control charts for monitoring a process standard deviation. J Qual Technol 24:44–50
41. Wu Z, Jiao J, Yang M, Liu Y, Wang Z (2009) An enhanced adaptive CUSUM control chart. IIE Trans 41:642–653
42. Ou Y, Wu Z, Tsung F (2012) A comparison study of effectiveness and robustness of control charts for monitoring process mean. Int J Prod Econ 135:479–490
43. Abbas N, Riaz M, Does RJMM (2014) Memory-type control charts for monitoring the process dispersion. Qual Reliab Eng Int 30:623–632
44. Fonollosa J, Fernández L, Gutiérrez-Gálvez A, Huerta R, Marco S (2016) Calibration transfer and drift counteraction in chemical sensor arrays using direct standardization. Sensors Actuators B Chem 236:1044–1053