The International Journal of Advanced Manufacturing Technology (2018) 97:1697–1710
https://doi.org/10.1007/s00170-018-2052-4

ORIGINAL ARTICLE

# The correlational design method of the dimension tolerance and geometric tolerance for applying material conditions

Yuguang Wu [1]

Received: 31 October 2017 / Accepted: 17 April 2018 / Published online: 1 May 2018


**Abstract**

This paper proposes a correlational design method for the dimension tolerance and geometric tolerance of the feature of size when applying a material condition. Based on the equivalent transformation method, the calculation equation of virtual size and extreme virtual size for a single feature during free assembly is expanded to the calculation of those of related features at the datum positioning and directional assembly, and then the application rules of the material conditions are established. Two kinds of design requirements are considered: guaranteeing the distance requirement between the surface of a single feature of size and its datum, and guaranteeing the minimum clearance and minimum interference between two features of size. For these design requirements, the correlational equations for the design of the dimension tolerance and geometric tolerance are developed. Two design cases are conducted to demonstrate the design method.

**Keywords** Virtual size · Extreme virtual size · Material condition · Dimensional tolerance · Geometric tolerance · Correlation design · Application criteria

## 1 Introduction

The virtual size is the size of the functional virtual boundary of a geometric feature. The virtual boundary of a feature of size is dependent on the actual size of the contour feature and geometric errors of the center feature. The virtual size is the actual mating size of the assembling feature at free assembly, or the size of a related actual mating envelope at the datum location assembly. The virtual size determines the assembly clearance or interference between two engaged features. In functional dimensioning and tolerancing of the assembly features, the extreme virtual size is the objective that must be guaranteed, and the material condition must be considered. The application of the material condition will improve the manufacturing benefits, and when both the feature of size and datum of size are applicable to the material condition, the greater manufacturing benefits will be achieved. In this situation, the tolerances on both the feature of size and datum of size should be specified cohesively and cannot be chosen arbitrarily, and there must be a relationship between the geometric tolerance specification of feature of size and datum of size. Currently, a design method for correlating the dimensional tolerance and geometric tolerance of the feature of size and the datum of size for applying the material condition is not available, and the application principle of the material condition is incomplete. Therefore, it is important to investigate the correlation and establish a design model and a method for the dimension and geometric tolerance of both the feature of size and datum of size.

The principles of the maximum and least material condition specifications by the virtual boundary are defined in ISO standards 2692 (1988) [1] and in the ASME standard (1994) [2]. In the last 20 years, a significant amount of research has been devoted to the development of specification models with a virtual boundary. These virtual boundaries have been used as virtual gauges since the early 1990s. Jayaraman and Srinivasan [3] proposed conditional tolerancing and virtual gauge methodologies to define the virtual boundary requirement (VBR). The major focus of their works is to consider the assembly of perfect form parts at the maximum material to determine if the assembly is possible or at the least material to determine the maximum displacement of the end surface. The fundamental hypothesis supposes that the displacement will be greater when the links are in the least material condition.

✉ Yuguang Wu
ygwu@hdu.edu.cn

[1] College of Mechanical Engineering, Hangzhou Dianzi University, Hangzhou 310018, People's Republic of China

Robinson [4] uses the maximum material parts among assembly specifications, tolerance specifications, and assembly tolerance analysis, which is an expansion of the maximum and least material condition modifiers. Etesami [5] presented a formulation to interpret the 2D position tolerance specifications. Simulated gauges are constructed from datum features as a set of constraint relationships. The measure of perfect-form position-imperfection is determined as the distance between the measured and the nominal feature positions subject to the datum constraint requirements. The derived formulation is applied to an example part with a hole-slot datum-priority frame. This formulation results in a three-variable optimization problem that is solved by the augmented Lagrange multipliers technique. The extension of the formulation to 3D is also discussed but without reference to a specific representation. Lehtihet and Gunasena [6] suggested models to predict the probability of an acceptable hole by combining the probability of the acceptable position and size of the hole. To obtain a statistical model for the position, they fit a two-dimensional Gaussian probability-distribution function to the tolerance zone for the position of the axis of the hole (as a circle).

The designer needs a method and principle to determine the tolerance specification for the geometric feature. Ballu et al. [7] proposed a series of preliminary rules regarding the application of material conditions, such as (1) for the assembly requirement and the minimum clearance requirement, the functional virtual boundary of a feature of size is the maximum material condition and (2) for the maximum clearance requirement and maximum deviation requirement, the functional virtual boundary of a feature of size is the least material condition. Dantan et al. [8] presented the notion of a quantifier. The quantifier can stand for the concepts of the functional requirements of an assembly, such as a functional requirement must be respected in at least one acceptable configuration of gaps, or a functional requirement must be respected in all acceptable configurations of gaps. According to the function and the processing requirements of an assembly, the process of tolerance synthesis is transferred into a formalized mathematical formula of the quantifier. With this approach, some rules are formalized to determine the modifier (MMC or LMC) for all functional requirements and for all assembly process requirements. These formalized formulas are if-then rules, such as the following: If {considered feature = feature of size} and {contact = floating} and {requirement must be respected in at least one acceptable configuration of gaps}, then the functional virtual boundary of this considered feature is the maximum material condition.

Chavanne and Anselmetti [9] outlined that the independence principle does not permit limiting the orientation inside the location zone in the particular case of a floating datum reference frame. Then, two main contributions are developed, the extension of material conditions on complex surfaces and the definition of a new association criterion to specify hybrid prismatic surfaces with a surface contact zone and a zone with clearance. With this goal, the paper suggests six propositions for possible extensions of the standards of tolerancing.

Tolerance synthesis is determined from a specification model. Some tolerance syntheses were developed from the tolerance zone approach by Fleming [10] and Robinson [11]. Others are based on variational geometry [12–14]. Lastly, some are based on vectorial tolerancing [15–18]. For example, in tolerance synthesis based on tolerance zones, inequalities expressing the fact that the related features must be in tolerance zones are introduced. The conditions on the features are known, while the unknowns are the values of the tolerances of these conditions.

Pairel et al. [19] presented a model of virtual fitting gauges. This model developed the algorithm considering the maximum material and least material conditions. In their following article [20, 21], they introduced the use of software by taking a pattern of holes as an example. These gauges concern the geometrical entities of the part, which are represented on the three-dimensional geometrical model of the part (CAD model). The topology of a gauge is related to that of the part. Recording these attributes is sufficient. The advantages of this representation are its simplicity, the semantic coherence (which can be guaranteed), the independence from the standards, their limits and their evolutions, and the extension of the tolerancing possibilities for the designer.

Singh et al. [22] used the T-Map model to analyze the issue of assembly tolerance allocation, which takes a self-aligning coupling of a vehicle to demonstrate the analysis and synthesis ability of the T-Map model. The T-Map can distinguish between related and unrelated actual mating envelopes, as described in the ASME/ISO standards. The model pursues consistency and compatibility with the standard tolerances, but its mathematical tool is complicated. Shen et al. proposed an improved simulation-based approach to tolerance and ease of assembly analyses for assemblies with pin/hole floating mating conditions [23] and for 3D slot features and tab features [24]. They explained the "bonus tolerance" and "shift tolerance." The bonus tolerance is the extra variation added to the geometric tolerance when the feature deviates from its maximum or minimum material condition size. The shift tolerance is the deviation from its datum's size deviation. The algorithms developed account not only for bonus/shift tolerances but also for feasibility of assembling. Later, they extended the T-Map math model for geometric and size tolerances to include the probabilistic representation of the one-dimensional clearance between an engaged tab and slot in an assembly of two parts [25]. It includes a method for assigning tolerances statistically when both the size and position tolerances are specified on an engaged tab and slot and gives the comparison between the frequency distributions for clearance both with and without the MMC being specified.

To improve the readability of the tolerance specification involved in the material condition and to facilitate the decoding of the CAT software, Anselmetti [26] proposed two complementary writings and several explanations for the application of the maximum and least material requirements. The new description is simply to place the diameter of the virtual state of the specified surface and of the datum in the specification. The meaning is exactly the same as the classical description. This description is consistent with the principle of independence of the tolerance, and the readability is improved, which can be used in a situation that does not require verification of the local size of the feature.

ISO and ASME GD&T standards described the function and significance of the material condition very clearly, but the design method for correlating the dimensional tolerance and geometric tolerance of the feature of size for applying the material condition is not available, and the principle of application of the material condition is incomplete. In this paper, the definition and calculation of the virtual size and the extreme virtual size, based on the guarantee of the functional requirement, is first discussed in Section 2. The application criteria of the material conditions for the related feature and datum feature are established in Section 3. The design algorithms for correlating dimensional tolerance and geometric tolerance of the feature of size are described in Section 4. Two design cases are demonstrated at the end of this paper.

## 2 The calculation of assembly virtual size

Although the virtual size is a notion of the single features at free assembly, it can also be used to represent the mating size of the related feature. This section will expand the virtual size of a single feature at free assembly into the calculation of the related feature at the datum positioning assembly and datum directional assembly.

### 2.1 The virtual size and extreme virtual size of a single feature at free assembly

The virtual size of a single feature depends on the actual size of its contour feature and the geometric error of its center feature. The extreme virtual size of a single feature includes the maximum material virtual size (MMVS) and the least material virtual size (LMVS), and it can be calculated by the extreme size of the contour feature and the geometric tolerance of the center feature. In the design model shown in Fig. 1, the actual hole or shaft is substituted by the ideal geometry with the shape and dimension error, the ideal geometry has an identical local size, and the MMVS and LMVS can be represented by the extreme local size of the contour feature and the geometric error of the center feature.

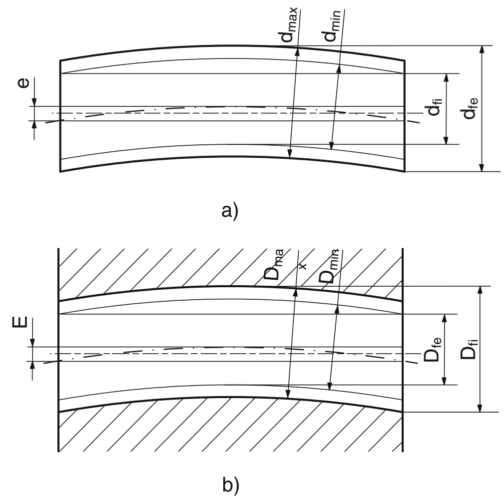


**Fig. 1** The feature of size and its representation model. **a** an external feature. **b** an internal feature

Suppose that the shape error of the center line of the hole and shaft be denoted by symbols $E$ and $e$, respectively, and the diameter and its tolerance be represented by the symmetrical tolerances $D \pm \Delta D/2$ and $d \pm \Delta d/2$, respectively, then the MMVS ($D_{MV}$, $d_{MV}$) and LMVS ($D_{LV}$, $d_{LV}$) of a hole and shaft can be represented as Eq. (1).

$$\begin{aligned} D_{MV} &= D - \Delta D/2 - E \\ d_{MV} &= d + \Delta d/2 + e \\ D_{LV} &= D + \Delta D/2 + E \\ d_{LV} &= d - \Delta d/2 - e \end{aligned} \tag{1}$$

The minimum clearance of a shaft/hole assembly is determined by the MMVS of both the shaft and hole, and the minimum interference of a shaft/hole assembly is determined by the LMVS of both shaft and hole. The calculation equations of the minimum clearance $C_{\min}$ and minimum interference $Y_{\min}$ are as follows.

$$\begin{aligned} C_{\min} &= D_{MV} - d_{MV} = D - d - \Delta D/2 - \Delta d/2 - E - e \\ Y_{\min} &= d_{LV} - D_{LV} = d - D - \Delta D/2 - \Delta d/2 - E - e \end{aligned} \tag{2}$$

To calculate the dimension tolerance and geometric tolerance conveniently, the interference value of a shaft/hole assembly is defined as a positive value in Eq. (2), i.e., the difference of the diameter of a shaft minus the diameter of a hole. According to Eq. (2), the minimum clearance and minimum interference can be declared as follows. $C_{\min}$ is the nominal dimension of a hole minus the nominal dimension of the shaft and minus both the radius tolerance and form tolerance. $Y_{\min}$ is the nominal dimension of a shaft minus the nominal dimension of the hole, and minus both the radius tolerance and form tolerance.

The design objective of an assembly feature includes the dimension tolerance and geometric tolerance, and when designing to guarantee the minimum clearance or the minimum interference between two assembly features, Eq. (2) creates a relationship between the design requirement and the design objective.

According to Eqs. (1) and (2), the form tolerance of an assembly feature will contribute to the virtual size and the assembly clearance and interference in a similar manner as with the radius tolerance, and therefore, the form tolerance of an assembly feature cannot be ignored when its value is close to the radius tolerance. It is the function mechanism of the form error and the geometric error to the extreme virtual size of a single feature, and the combined effect of the form error and the geometric error will improve the acceptability of a part when the material condition is considered.

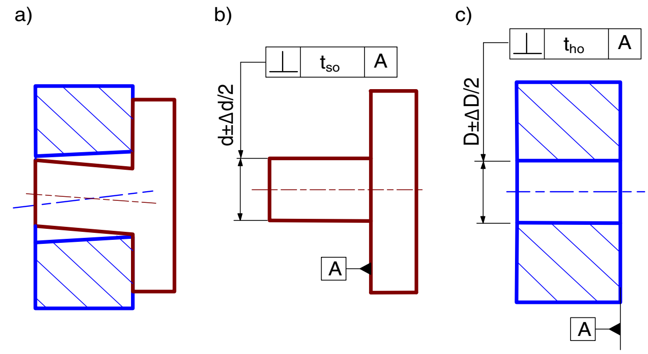


**Fig. 2** A directional assembly of two features of size with planar datums. a) The assembly model. b) the tolerance specification of the external assembly feature. c) the tolerance specification of the internal assembly feature.

## 2.2 The virtual size and extreme virtual size of related features at the datum directional assembly

An assembly of two related features is the assembly where their datum features remain in contact with one another. The geometric tolerance specification of a related feature with respect to its datum feature has two types of tolerance, i.e., the location tolerance and orientation tolerance. Therefore, the assembly of the related features includes the datum positioning assembly and datum directional assembly.

In datum directional assembly, the relative rotation of two related features is constrained by their datum feature, but the relative translational motion between two related features remains free. Figure 2 shows the datum directional assembly; the virtual size of two related features is calculated in the direction parallel to their datums. According to the GD&T standard, the orientation tolerance contains the form tolerance, and therefore, the form tolerance is neglected in the calculation of the virtual size of the related feature in the datum directional assembly. The MMVS and LMVS of the related feature in the datum directional assembly of the planar datum feature can be expressed as follows:

$$\begin{aligned} D_{MV} &= D - \Delta D/2 - t_{ho} \\ d_{MV} &= d + \Delta d/2 + t_{so} \\ D_{LV} &= D + \Delta D/2 + t_{ho} \\ d_{LV} &= d - \Delta d/2 - t_{so} \end{aligned} \tag{3}$$

where $t_{so}$ and $t_{ho}$ denote the orientation tolerance of the related feature, and the symbols $D$, $\Delta D$, $d$, and $\Delta d$ have the same meaning as those in Eq. (1). The minimum clearance $C_{\min}$ and minimum interference $Y_{\min}$ are as follows:

$$\begin{aligned} C_{\min} &= D_{MV} - d_{MV} \\ &= D - d - \Delta D/2 - \Delta d/2 - t_{ho} - t_{so} \\ Y_{\min} &= d_{LV} - D_{LV} \\ &= d - D - \Delta D/2 - \Delta d/2 - t_{ho} - t_{so} \end{aligned} \tag{4}$$

Comparing Eq. (4) with Eq. (2), there is no difference between the two except that the form tolerance in Eq. (2) is replaced by the orientation tolerance, and the minimum clearance and minimum interference have no relevance to the form tolerance of their planar datum features in the datum directional assembly.

However, this is not true when the two datum features are the features of size. In this situation, the contact of two datum features becomes another shaft/hole assembly, such as shown in Fig. 3. It is obvious that the clearance between two datum features will affect the assembly result of the two related features. Therefore, to guarantee the minimum clearance or the minimum interference of the assembly in the design, the clearance between the two datum features must be considered. Clearly, the minimum clearance between two related features will occur when both the related feature and datum feature are in the maximum material virtual condition (MMVC), and the minimum interference will occur when both the related feature and datum feature are in the least material virtual condition (LMVC). Based on the discussion above, $C_{\min}$ and $Y_{\min}$ in the datum directional assembly when the datum features are the features of size will be modified as follows:

$$\begin{aligned} C_{\min} &= D_{MV} - d_{MV} + (Q_{MV} - q_{MV}) l_o / l_D \\ Y_{\min} &= d_{LV} - D_{LV} - (Q_{LV} - q_{LV}) l_o / l_D \end{aligned} \tag{5}$$

In Eq. (5), $Q_{MV}$, $Q_{LV}$, $q_{MV}$, and $q_{LV}$ denote the MMVS and LMVS of both datum features, and the symbols $l_o$ and $l_D$ denote the contact length of the two related features and the contact length of the two datum features, respectively. For the assembly in Fig. 3a, $l_o = \min(l_d, L_D)$, $l_D = \min(l_q, L_Q)$. Because the evaluated direction of the MMVS and LMVS of the related feature are perpendicular to the evaluated direction of the dimension and error of the datum feature, the dimension and error of the datum feature have no contribution to the MMVS and LMVS of the related feature; therefore, $D_{MV}$, $D_{LV}$, $d_{MV}$, and $d_{LV}$ in Eq. (5) are the same as those in Eq. (3). Substituting the equations of MMVS and LMVS of the related feature and datum feature into Eq. (5), we obtain the relation equations of

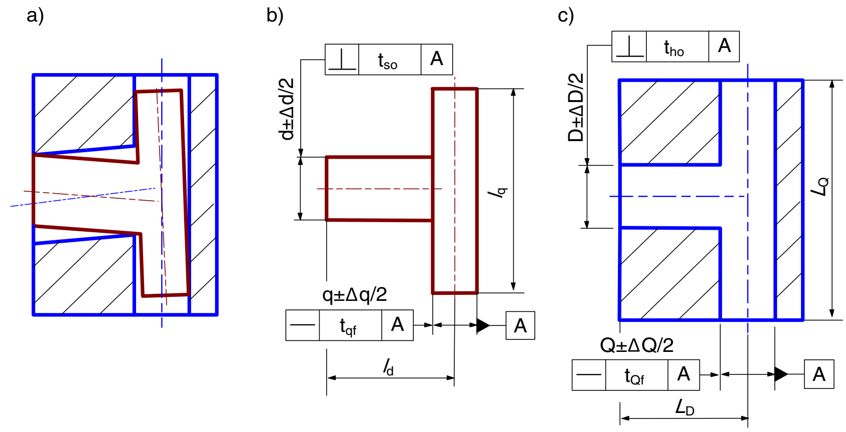


**Fig. 3** A directional assembly of two features of size with datum of size. a) The assembly model. b) the tolerance specification of the external assembly feature. c) the tolerance specification of the internal assembly feature.

$C_{\min}$ and $Y_{\min}$ with the dimension tolerance and geometric tolerance of all related features and datum features as follows.

$$\begin{aligned} C_{\min} &= D-d-\Delta D/2-\Delta d/2-t_{ho}-t_{so} \\ &\quad +\left(Q-q-\Delta Q/2-\Delta q/2-t_{Qf}-t_{qf}\right)l_o/l_D \\ Y_{\min} &= d-D-\Delta D/2-\Delta d/2-t_{ho}-t_{so} \\ &\quad -\left(Q-q+\Delta Q/2+\Delta q/2+t_{Qf}+t_{qf}\right)l_o/l_D \end{aligned} \quad (6)$$

Equation (6) shows that the dimension tolerance and form tolerance of the datum feature will contribute to the assembly of the related feature in the datum directional assembly when the datum features are the feature of size.

## 2.3 The virtual size and extreme virtual size of the related feature at the datum positioning assembly

When a related feature has a position tolerance, the position of the related feature is controlled by the position tolerance with respect to its datum, and therefore, the relative position between the two related features at the datum positioning assembly is depended on many factors, such as the positions of the related feature with respect to their datum feature, the extreme virtual size of both the two related features, the contact of the two datum features, and the dimension and geometric errors of the two datum features. When the two related features are the feature of size, the minimum clearance or interference will occur at two points on the surface of two the related features, and thus, it needs two distances to define the relative position between the two related features. Take a cylindrical shaft/hole assembly as an example, one is the distance between two upper straight generatrices of a cylindrical shaft and a cylindrical hole, while the other is the distance between two lower straight generatrices of a cylindrical shaft and a cylindrical hole. In contrast to datum positioning assembly, the free assembly of two features of size can be taken as a special case of the datum positioning assembly. In free assembly, the “assembly datum” is a pair of straight generatrices of both the cylindrical shaft and cylindrical hole, the “assembly objective” is another pair of straight generatrices, and the distance is the clearance or interference of the free assembly.

According to the analysis, a datum positioning assembly can be transformed into two equivalent shaft/hole free assemblies, where one is the assembly of the datum feature and the upper edges of two related features, such as two upper straight generatrices of both a cylindrical shaft and a cylindrical hole, while the other is the assembly of the datum feature and the lower edges of the related feature, such as two straight generatrices of both a cylindrical shaft and a cylindrical hole.

The calculation equation of minimum clearance and minimum interference in a datum positioning assembly can be derived in a similar way to those in the shaft/hole free assembly. Figure 4 shows a planar datum positioning assembly of a cylindrical shaft and a cylindrical hole, and the solid line outlines in Fig. 4a denote the planar datum and the target hole of one part, while the outline in hidden line outlines denote the planar datum and the target shaft of another part. When two planar datums are in contact, the datum positioning assembly is broken into two equivalent shaft/hole free assemblies, where one is the small diameter shaft/hole assembly constructed by the planar datum and lower straight generatrix of both parts, while the other is the large diameter shaft/hole assembly constructed by the planar datum and upper straight generatrix of both parts, as shown in Fig. 4b and c, respectively.

Figure 5 is the diagrammatic sketch for calculating the MMVS and LMVS in an equivalent shaft/hole assembly. Figure 5a and b shows the dimension chain of the MMVS and LMVS of the small diameter shaft and hole, respectively. The MMVS ($d_{\mathrm{MV}}$, $D_{\mathrm{MV}}$) and LMVS ($d_{\mathrm{LV}}$, $D_{\mathrm{LV}}$) of the small diameter shaft and hole are represented as follows:

$$\begin{aligned} D_{MV} &= L-d/2-\Delta d/4-t_{sp}/2 \\ D_{LV} &= L-d/2+\Delta d/4+t_{sp}/2 \\ d_{MV} &= L-D/2+\Delta D/4+t_{hp}/2\,, \\ d_{LV} &= L-D/2-\Delta D/4-t_{hp}/2 \end{aligned} \quad (7)$$

where variable $L$ is the nominal distance between the related feature and its datum feature, $t_{\mathrm{sp}}$ and $t_{\mathrm{hp}}$ are the position

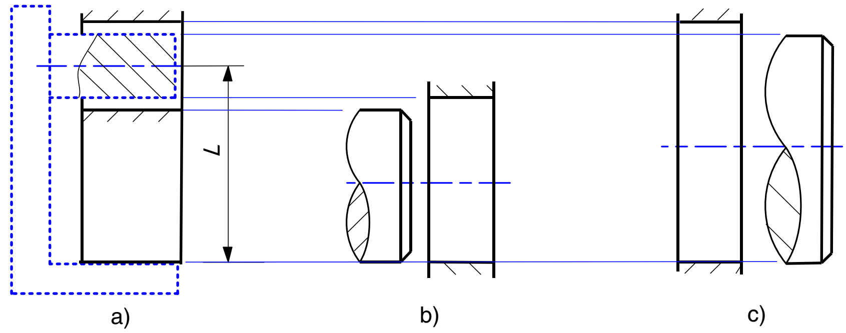


**Fig. 4** The equivalent relations between free assembly and planar datum positioning assembly. **a** The positioning assembly by planar datum. **b** The equivalent free small diameter shaft/hole assembly. **c** The equivalent free big diameter shaft/hole assembly

tolerance of the related feature with respect to the datum feature, the symbols $D$, $\Delta D$, $d$, and $\Delta d$ have the same meanings as in the previous equations. The calculation equations of minimum clearance $C_{\min}$ and minimum interference $Y_{\min}$ of the small diameter shaft/hole assembly are as follows:

$$\begin{aligned} C_{\min} &= D_{MV}-d_{MV} \\ &= D/2-d/2-\Delta D/4-\Delta d/4-t_{hp}/2-t_{sp}/2 \\ Y_{\min} &= d_{LV}-D_{LV} \\ &= d/2-D/2-\Delta D/4-\Delta d/4-t_{hp}/2-t_{sp}/2 \end{aligned}. \tag{8}$$

Figure 5c and d shows the dimension chains of the MMVS and LMVS of the large diameter shaft and hole, respectively. The calculation equations of the MMVS and LMVS of the large diameter shaft and hole are seen in Eq. (9), as follows:

$$\begin{aligned} D_{MV} &= L+D/2-\Delta D/4-t_{hp}/2 \\ D_{LV} &= L+D/2+\Delta D/4+t_{hp}/2 \\ d_{MV} &= L+d/2+\Delta d/4+t_{sp}/2 \\ d_{LV} &= L+d/2-\Delta d/4-t_{sp}/2 \end{aligned}, \tag{9}$$

and the calculation equation of the minimum clearance $C_{\min}$ and minimum interference $Y_{\min}$ of the large diameter shaft/hole assembly are seen in Eq. (10), as follows:

$$\begin{gathered} C_{\min} = D_{MV}-d_{MV} \\ = D/2-d/2-\Delta D/4-\Delta d/4-t_{hp}/2-t_{sp}/2 \\ Y_{\min} = d_{LV}-D_{LV} \\ = d/2-D/2-\Delta D/4-\Delta d/4-t_{hp}/2-t_{sp}/2 \end{gathered}. \tag{10}$$

The meanings of the symbols in Eq. (9) are the same as those in Eq. (7). Equations (10) and (8) are identical, and thus, the minimum clearance and minimum interference at the datum positioning assembly can be calculated by either of the two equivalent shaft/hole assemblies.

According to Eqs. (7) and (9), the related feature will add to the MMVS of the equivalent shaft and the LMVS of the equivalent hole by 1/4 of the dimension tolerance and 1/2 of the location tolerance, respectively, and will subtract from the MMVS of the equivalent hole and the LMVS of the equivalent shaft by 1/4 of the dimension tolerance and 1/2 of the location tolerance, respectively. There is no dimension tolerance and form tolerance of the planar datum feature in Eqs. (7) and (9), and the planar datum feature has no contribution to the MMVS and LMVS of the equivalent shaft and the equivalent hole.

However, this is not true when the datum feature is a feature of size. Figure 6 shows the datum positioning assembly of the related feature, where the datum feature is a feature of size. The four boundaries of both the related feature and the datum feature will combine to form two equivalent shaft/hole assemblies with different diameters, as shown in Fig. 6b and c. It is obvious that the clearance between the two datum features will affect the relative positions of the two related features. Therefore, the determination of MMVS and LMVS of the two equivalent shaft/hole assemblies must consider the MMVC and the LMVC of two datum features.

Figure 7 shows the dimension chains of the MMS ($d_{\max}$, $D_{\min}$) and LMS ($d_{\min}$, $D_{\max}$) of the equivalent shaft/

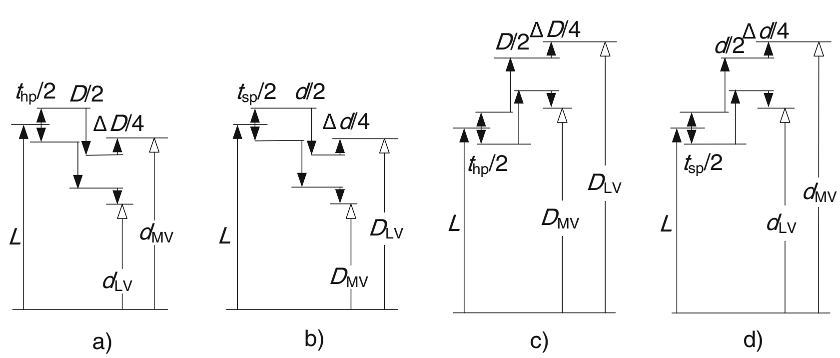


**Fig. 5** The dimension chain for calculating the extreme virtual size of equivalent shaft/hole assembly with planar datum. **a** The small diameter "shaft." **b** The small diameter "hole." **c** The large diameter "hole." **d** The large diameter "shaft"

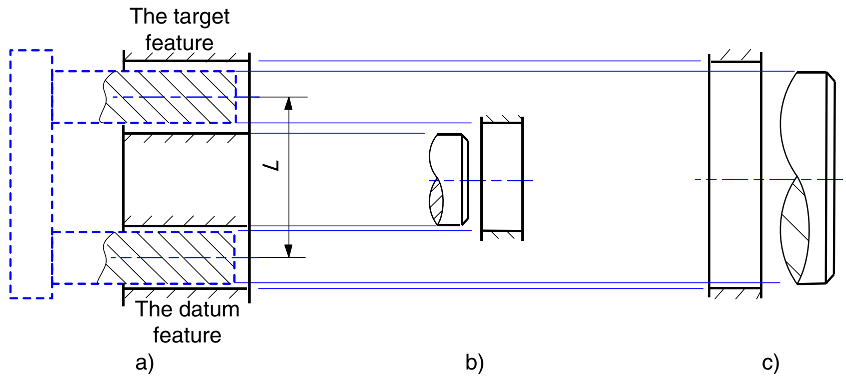


**Fig. 6** The equivalent relations of free assembly and datum positioning assembly with datum of size. **a** The datum positioning assembly of a feature of size. **b** The equivalent free small diameter shaft/hole assembly. **c** The equivalent free big diameter shaft/hole assembly

hole assembly. The dimension and the tolerance of both the datum features are represented by $Q \pm \Delta Q/2$ and $q \pm \Delta q/2$, the form tolerances of both datum features are represented by $t_{\mathrm{Qf}}$ and $t_{\mathrm{qf}}$, respectively. Figure 7a and b are the dimension chains for the equivalent shaft/hole assembly with a small diameter, and Fig. 7c and d are the dimension chains for the equivalent shaft/hole assembly with a large diameter. The MMVS ($d_{\mathrm{MV}}$, $D_{\mathrm{MV}}$) and LMVS ($d_{\mathrm{LV}}$, $D_{\mathrm{LV}}$) of the small diameter shaft/hole assembly are represented by Eq. (11), as follows:

$$\begin{array}{l} d_{MV} = L - Q/2 - D/2 + \Delta Q/4 + \Delta D/4 + t_{Qf}/2 + t_{hp}/2 \\ d_{LV} = L - Q/2 - D/2 - \Delta Q/4 - \Delta D/4 - t_{Qf}/2 - t_{hp}/2 \\ D_{MV} = L - q/2 - d/2 - \Delta q/4 - \Delta d/4 - t_{qf}/2 - t_{sp}/2 \\ D_{LV} = L - q/2 - d/2 + \Delta q/4 + \Delta d/4 + t_{qf}/2 + t_{sp}/2 \end{array}, \quad (11)$$

and the MMVS ($d_{\mathrm{MV}}$, $D_{\mathrm{MV}}$) and LMVS ($d_{\mathrm{LV}}$, $D_{\mathrm{LV}}$) of the large diameter shaft/hole assembly are represented by Eq. (12), as follows:

$$\begin{array}{l} d_{MV} = L + q/2 + d/2 + \Delta q/4 + \Delta d/4 + t_{qf}/2 + t_{sp}/2 \\ d_{LV} = L + q/2 + d/2 - \Delta q/4 - \Delta d/4 - t_{qf}/2 - t_{sp}/2 \\ D_{MV} = L + Q/2 + D/2 - \Delta Q/4 - \Delta D/4 - t_{Qf}/2 - t_{hp}/2 \\ D_{LV} = L + Q/2 + D/2 + \Delta Q/4 + \Delta D/4 + t_{Qf}/2 + t_{hp}/2 \end{array}. \quad (12)$$

According to Eqs. (11) and (12), the minimum clearance and minimum interference of the small diameter shaft/hole assembly and large diameter shaft/hole assembly are the same, which are represented by Eq. (13), as follows:

$$\begin{array}{l} C_{\min} = (D + Q - d - q)/2 - (\Delta D + \Delta d + \Delta Q + \Delta q)/4 \\ \qquad - \left(t_{Qf} + t_{qf} + t_{hp} + t_{sp}\right)/2 \\ Y_{\min} = (d + q - D - Q)/2 - (\Delta D + \Delta d + \Delta Q + \Delta q)/4 \\ \qquad - \left(t_{Qf} + t_{qf} + t_{hp} + t_{sp}\right)/2 \end{array}. \quad (13)$$

Because the calculation equations of $C_{\min}$ and $Y_{\min}$ for both the small and large diameter shaft/hole assemblies are identical, the minimum clearance and minimum interference at the datum positioning assembly can be calculated by either of the two equivalent shaft/hole assemblies. According to Eq. (13), when the datum feature is a feature of size, the datum features affect the minimum clearance and minimum interference at the datum positioning assembly in the same way as the related feature. This verifies the necessity of applying the material condition of both the related feature and datum feature at the same time when the datum feature is a feature of size.

## 3 The application criteria for the material condition

It is a complex issue to apply the material condition in the design of the dimension tolerance and geometric tolerance of the related feature, and thus the application of the material condition is difficult. Therefore, it is very important to

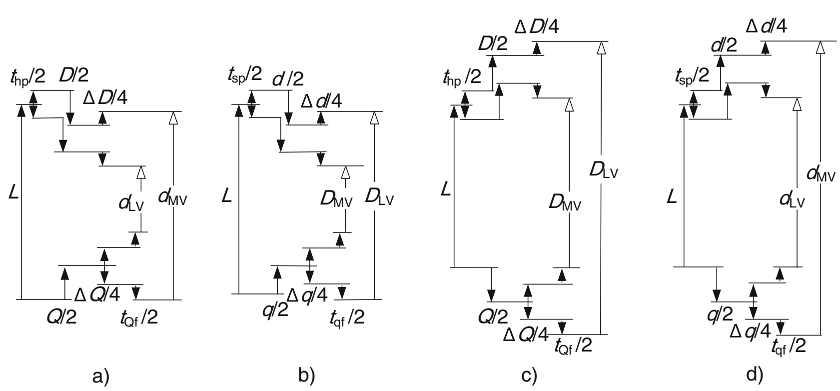


**Fig. 7** The dimension chain for calculating the extreme virtual size of equivalent shaft/hole assembly with datum of size. **a** The small diameter "shaft." **b** The small diameter "hole." **c** The big diameter "hole." **d** The big diameter "shaft"

establish the application criteria of the material condition. Based on the discussions and the related equations in the last section, this section will provide 8 applicable rules of the material condition as follows.

Rule 1 (The applicable geometry of the material condition): The geometric feature that can apply the material condition is the feature of size only, which includes the related feature and the datum feature. The reciprocity requirement can be applied to the related feature to compensate the dimension tolerance from the geometric tolerance and not the datum feature.

The size of the feature of size is related to the material condition of the part, while the non-feature of size has no size and is unrelated to the material condition of the part. The aim of the application of the material condition of the datum feature is to improve the ability of the related feature to guarantee the design requirement, and so it is not necessary to apply the reciprocity requirement for the datum feature.

Rule 2 (The application of the material condition for single feature free assembly): The material condition can be applied for the combined effect of the dimension tolerance and the straightness tolerance or the flatness tolerance of the single feature, which includes the envelope requirement, maximum material requirement and its reciprocity requirement, and the least material requirement and its reciprocity requirement.

The envelope requirement is a special maximum material requirement, where the geometric tolerance is set at zero value, and so the envelope requirement has no reciprocity requirement because the geometric tolerance must be positive.

Rule 3 (The application of the material condition for the related feature and datum directional assembly): The material condition can be applied for the combined effect of the dimension tolerance and the orientation tolerance (parallelism tolerance, perpendicularity tolerance and angularity tolerance) of the related feature at the datum directional assembly, which includes the maximum material requirement, the least material requirement and their reciprocity requirement.

Rule 4 (The application of the material condition for the related feature and datum positioning assembly): The material condition can be applied for the combined effect of the dimension tolerance and the location tolerance (coaxially tolerance, symmetry tolerance and true position tolerance) of the related feature at the datum positioning assembly, which includes the maximum material requirement, the least material requirement and their reciprocity requirement.

Rule 5 (The application of the material condition for the datum feature): If the datum feature is a feature of size, it must apply the same material condition as the related feature at the datum positioning assembly. Meanwhile, its dimension tolerance and geometric tolerance must apply the same material condition.

When the datum feature is a feature of size, the difference between the MMVS (or LMVS) and the actual virtual size of the datum feature will contribute to the MMVS (or LMVS) of the related feature. The MMVS or LMVS of a datum feature is determined by the maximum material size or least material size of the contour feature and the form tolerance of the center feature, and the actual virtual size of the datum feature is determined by the actual size of the contour feature and the form error of the center feature.

Rule 6 (The effect of the datum feature applying the material requirement on the related feature): In the datum positioning assembly, the surplus amount of the actual virtual size of the datum feature to its extreme virtual size (MMVS or LMVS) will compensate for the extreme virtual size of the related feature. In the datum directional assembly, the surplus amount of the actual virtual size of the datum feature to its extreme virtual size (MMVS or LMVS) will compensate for the extreme virtual size of the related feature with the ratio of the assembly length of the related feature divided by the assembly length of the datum feature.

Rule 7 (The usage of the maximum material requirement and its reciprocity requirement): The maximum material requirement and its reciprocity requirement can be applied when the design requirement is to guarantee the minimum clearance or the minimum size of a hole and the maximum size of a shaft.

Rule 8 (The usage of the least material requirement and its reciprocity requirement): The least material requirement and its reciprocity requirement can be applied when the design requirement is to guarantee the minimum interference or the maximum size of a hole and the minimum size of a shaft.

Figure 8 is an example of the usage of the maximum material requirement. The pattern feature with four through holes of Ø6.1–6.2 mm has true position tolerance relative to the three datum features A, B, and C, as shown in Fig. 8a. Datum A is a planar feature, datum B is a hole with diameter of Ø17.6–17.7 mm, and datum C is a straight slot. Because plane A is a non-feature of size, plane A cannot apply the material condition, and both hole B and slot C are features of size, and therefore, they can apply the same material condition with the related feature, i.e., maximum material requirement. In Fig. 8b, datum A is reassigned as a feature of size, and therefore, datum feature A can apply the maximum material requirement. In the tolerance specification shown in Fig. 8b, three datum features apply the maximum material requirement, and it is legal according to the application rules described above.

## 4 The correlation design of dimension tolerance and geometric tolerance

This section will discuss two design objects, where one is a paired assembly feature, and the other is a single related feature. The design objective of the paired assembly feature is to guarantee the minimum clearance or the minimum

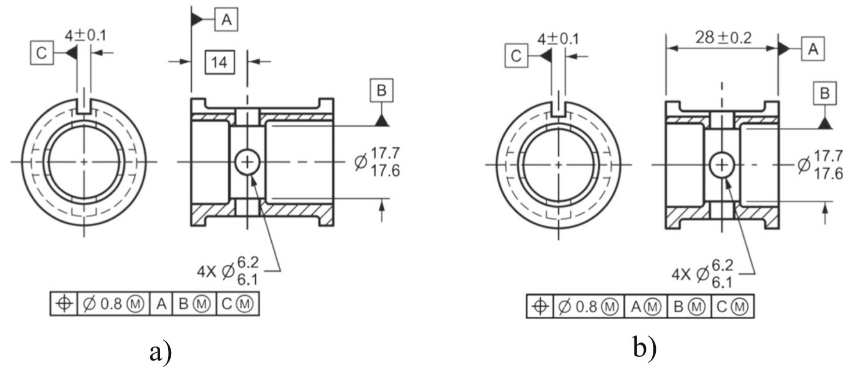


**Fig. 8** The application of the material condition of the datum feature. **a** Two datum features applying MMC. **b** Three datum features applying MMC

interference between two assembly related features, and the design objective of the single related feature is to guarantee the minimum or maximum distance from the surface of the related feature to its datum feature. This section will describe the design method and calculation equation for two design objects.

## 4.1 The tolerance correlation design for a single related feature

### 4.1.1 The design method for the planar datum feature

When a related feature is a feature of size, there are two measuring positions on the surface relative to its datum plane, i.e., the farthest point and the nearest point. For example, these two measuring points of a cylinder relative to its planar datum are on the farthest and the nearest straight generatrix of the cylinder relative to the datum. Figure 9 shows the true position tolerance of the two related features and their planar datum. Figure 9a shows an external feature and its planar datum, where the maximum and minimum distances of the surface of the external feature to the datum plane are $S_n$ and $S_f$, respectively. Figure 9b shows an internal feature and its planar datum, where the maximum and minimum distances of the surface of the internal feature to the datum plane are $H_n$ and $H_f$, respectively. These maximum and minimum distances are the design requirements of the correlation design. According to the geometric tolerance specifications in Fig. 9, the variation range of the maximum and minimum distances is determined by the dimension tolerance and position tolerance of the related feature, which are the design objectives. Therefore, the relationship between the design requirements and the dimension and geometric tolerance must be established first.

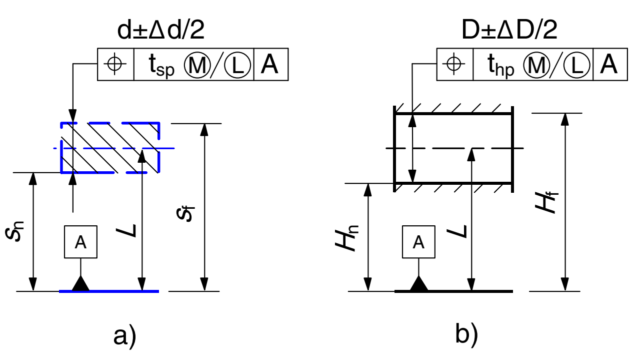


**Fig. 9** The tolerance correlation design of planar datum positioning assembly for guaranteeing the distance requirement. **a** The external feature. **b** The internal feature

Suppose that the related feature will be assembled with its counterpart, i.e., an external related feature is assembled with an internal feature, or an internal related feature is assembled with an external feature, then the relationship can be established by means of the shaft/hole datum positioning assembly. According to Eqs. (7) and (9), the hypothetical assembly will be transformed into two equivalent shaft/hole free assemblies, and as a result, the MMVS and LMVS of both the equivalent shaft and equivalent hole are the maximum and minimum value of these distances $S_n$, $S_f$, $H_n$ and $H_f$. In the engineering design, each distance of $S_n$, $S_f$, $H_n$ and $H_f$ has two design requirements, i.e., it must be greater than the design value or lesser than the design value, therefore, there are 8 design requirements for the maximum and minimum value of these four distances. These requirements can be repressed by 8 inequalities, such as $S_n \le S_{n,max}$, $S_n \ge S_{n,min}$, $S_f \le S_{f,max}$, $S_f \ge S_{f,min}$, $H_n \le H_{n,max}$, $H_n \ge H_{n,min}$, $H_f \le H_{f,max}$ and $H_f \ge H_{f,min}$. According to the transformation of the assembly in Fig. 4, each distance and its minimum and maximum values (MMVS and LMVS) of the equivalent shaft and equivalent hole, as well as the calculation equations of the MMVS and LMVS, the two design requirements and the material conditions applied, are listed in Table 1.

Table 1 gives the correspondence between the design objective with the equivalent hole and shaft. For example, the design requirement that guarantees the minimum distance from the inner side of a solid cylinder to the datum plane is $S_n \ge S_{n,min}$, the design objective is the MMVS

**Table 1** The design objectives, design requirements, and the material condition applied when the datum feature is a plane feature

| The design object | | The extreme values and their equation | | | The minimum value ≥ the min. dist. $L_{\min}$ | | The maximum value ≤ the max. dist. $L_{\max}$ | |
|---|---|---|---|---|---|---|---|---|
| | | The min. value | The max. value | Eq. | The design requirement | M/L applied | The design requirement | M/L applied |
| External feature | $S_n$ | $D_{MV}$ | $D_{LV}$ | (7) | $D_{MV} \geq L_{\min}$ | Ⓜ | $D_{LV} \leq L_{\max}$ | Ⓛ |
| | $S_f$ | $d_{LV}$ | $d_{MV}$ | (9) | $d_{LV} \geq L_{\min}$ | Ⓛ | $d_{MV} \leq L_{\max}$ | Ⓜ |
| Internal feature | $H_n$ | $d_{LV}$ | $d_{MV}$ | (7) | $d_{LV} \geq L_{\min}$ | Ⓛ | $d_{MV} \leq L_{\max}$ | Ⓜ |
| | $H_f$ | $D_{MV}$ | $D_{LV}$ | (9) | $D_{MV} \geq L_{\min}$ | Ⓜ | $D_{LV} \leq L_{\max}$ | Ⓛ |

($D_{MV}$) of the hole in the equivalent shaft/hole assembly in Table 1, and the material modifier "M" must be used. If the design requirement is to guarantee the maximum distance from the inner side of a solid cylinder to the datum plane, the LMVS ($D_{LV}$) of the hole in the equivalent shaft/hole assembly in Table 1 is the design objective, i.e., $D_{LV} \leq S_{n,\max}$, and the material modifier "L" must be used. The symbols $D_{MV}$, $D_{LV}$ etc. in Table 1 are from Eqs. (7) and (9). Using Eqs. (7) and (9), these 8 design requirements guarantee the extreme values of the four distances and the dimension tolerance and the true position tolerance can be expressed as a set of the inequality (14). The dimension tolerance and geometric tolerance can be allocated and designed, according to the tolerance standard and the design practice about the tolerance value between the dimension tolerance and geometric tolerance.

$$\begin{aligned}
&L-d/2-\Delta d/4-t_{sp}/2 \geq S_{n,\min}\\
&L-d/2+\Delta d/4+t_{sp}/2 \leq S_{n,\max}\\
&L+d/2-\Delta d/4-t_{sp}/2 \geq S_{f,\min}\\
&L+d/2+\Delta d/4+t_{sp}/2 \leq S_{f,\max}\\
&L-D/2-\Delta D/4-t_{hp}/2 \geq H_{n,\min}\\
&L-D/2+\Delta D/4+t_{hp}/2 \leq H_{n,\max}\\
&L+D/2-\Delta D/4-t_{hp}/2 \geq H_{f,\min}\\
&L+D/2+\Delta D/4+t_{hp}/2 \leq H_{f,\max}
\end{aligned} \tag{14}$$

#### 4.1.2 The design method for the datum of size

When two datum features are features of size, the four distances between the contour boundaries of the related feature and their datum feature are shown in Fig. 10. Based on the same equivalent transformation method, each distance and its minimum and maximum values (MMVS and LMVS) of the equivalent shaft and equivalent hole, as well as the calculation equation of the MMVS and LMVS, two design requirements and the material conditions applied, are listed in Table 2. It should be noted that the datum feature must use the same material conditions as the related feature.

Using Eqs. (11) and (12), the 8 design requirements used to guarantee the extreme values of the four distances, the dimension tolerance and true position tolerance of the related feature, can be expressed as a set of inequalities (15), as follows:

$$\begin{aligned}
&L-d/2-\Delta d/4-u/2-\Delta u/4-t_{uf}/2-t_{sp}/2 \geq S_{n,\min}\\
&L-d/2-\Delta d/4-u/2+\Delta u/4+t_{uf}/2+t_{sp}/2 \leq S_{n,\max}\\
&L+d/2-\Delta d/4+u/2-\Delta u/4-t_{uf}/2-t_{sp}/2 \geq S_{f,\min}\\
&L+d/2+\Delta d/4+u/2+\Delta u/4+t_{uf}/2+t_{sp}/2 \leq S_{f,\max}\\
&L-D/2-\Delta D/4-u/2-\Delta u/4-t_{uf}/2-t_{hp}/2 \geq H_{n,\min}\\
&L-D/2+\Delta D/4-u/2+\Delta u/4+t_{uf}/2+t_{hp}/2 \leq H_{n,\max}\\
&L+D/2-\Delta D/4+u/2-\Delta u/4-t_{uf}/2-t_{hp}/2 \geq H_{f,\min}\\
&L+D/2+\Delta D/4+u/2+\Delta u/4+t_{uf}/2+t_{hp}/2 \leq H_{f,\max}
\end{aligned} \tag{15}$$

In these 8 inequalities, all parameters are known in addition to the dimension tolerance and position tolerance of the related feature. It should be noted that, although the datum feature used in Fig. 10 is an internal feature, it has the same calculation equation as when the datum feature is an external feature.

### 4.2 The tolerance correlation design for the paired assembly feature

In the correlation design of the dimension tolerance and geometric tolerance using a material condition for the paired assembly with related shaft/hole feature, the design requirements are guaranteeing the minimum clearance and the

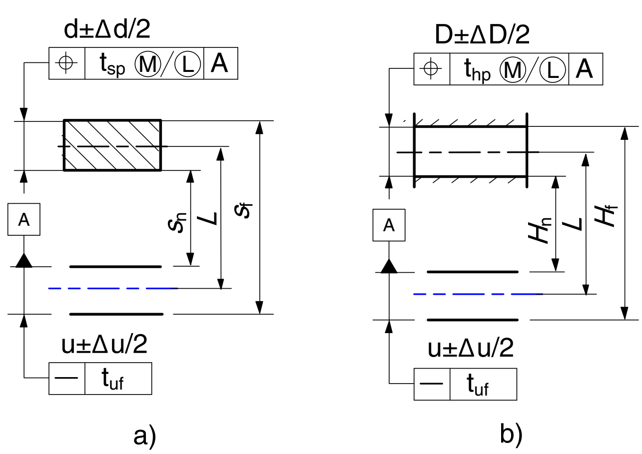


**Fig. 10** The tolerance correlation design of datum of size positioning assembly for guaranteeing the distance requirement. **a** The external feature. **b** The internal feature

**Table 2** The design objectives, design requirements, and the material condition applied when the datum feature is a feature of size

| The design object | | The extreme values and their equation | | | The minimum value ≥ the min. dist. $L_{\min}$ | | The maximum value ≤ the max. dist. $L_{\max}$ | |
|---|---|---|---|---|---|---|---|---|
| | | The min. value | The max. value | Eq. | The design requirement | M/L applied | The design requirement | M/L applied |
| External feature | $S_{\mathrm{n}}$ | $D_{\mathrm{MV}}$ | $D_{\mathrm{LV}}$ | (11) | $D_{\mathrm{MV}} \geq L_{\min}$ | Ⓜ | $D_{\mathrm{LV}} \leq L_{\max}$ | Ⓛ |
| | $S_{\mathrm{f}}$ | $d_{\mathrm{LV}}$ | $d_{\mathrm{MV}}$ | (12) | $d_{\mathrm{LV}} \geq L_{\min}$ | Ⓛ | $d_{\mathrm{MV}} \leq L_{\max}$ | Ⓜ |
| Internal feature | $H_{\mathrm{n}}$ | $d_{\mathrm{LV}}$ | $d_{\mathrm{MV}}$ | (11) | $d_{\mathrm{LV}} \geq L_{\min}$ | Ⓛ | $d_{\mathrm{MV}} \leq L_{\max}$ | Ⓜ |
| | $H_{\mathrm{f}}$ | $D_{\mathrm{MV}}$ | $D_{\mathrm{LV}}$ | (12) | $D_{\mathrm{MV}} \geq L_{\min}$ | Ⓜ | $D_{\mathrm{LV}} \leq L_{\max}$ | Ⓛ |

minimum interference between two surfaces of both the shaft and hole. The design equations of the dimension tolerance and geometric tolerance for the three assemblies, i.e., free assembly, datum directional assembly, and datum positioning assembly, are listed in Eqs. (16), (17), and (18), respectively.

$$\begin{array}{l} \Delta D/2+\Delta d/2+E+e \leq D-d-c \\ \Delta D/2+\Delta d/2+E+e \leq d-D-y \end{array}, \tag{16}$$

$$\begin{array}{l} \Delta D/2+\Delta d/2+t_{ho}+t_{so} \leq \\ D-d-c+\left(Q-q-\Delta Q/2-\Delta q/2-t_{Qf}-t_{qf}\right)\dfrac{l_2+0.5l_o}{l_1+0.5l_q} \\ \Delta D/2+\Delta d/2+t_{ho}+t_{so} \leq \\ d-D-y-\left(Q-q+\Delta Q/2+\Delta q/2+t_{Qf}+t_{qf}\right)\dfrac{l_2+0.5l_o}{l_1+0.5l_q} \end{array}, \tag{17}$$

$$\begin{array}{l} \Delta D/2+\Delta d/2+t_{hp}+t_{sp} \leq \\ D-d-2c+Q-q-\Delta Q/2-\Delta q/2-t_{Qf}-t_{qf} \\ \Delta D/2+\Delta d/2+t_{hp}+t_{sp} \leq \\ d-D-2y+q-Q-\Delta Q/2-\Delta q/2-t_{Qf}-t_{qf} \end{array}. \tag{18}$$

In the design equation above, the parameters on the left of the inequality are the design objective, and those on the right of the inequality are the known quantity and the design requirement. The parameter $c$ denotes the minimum clearance, the parameter $y$ denotes the minimum interference, and the other parameters are the nominal dimension and distance of the related feature and their datum feature. The dimension tolerance and geometric tolerance can be allocated and designed according to the fit and tolerance standard and the precision grade of the shaft and hole.

Inequality (16) is the design equation of the dimension tolerance and straightness or flatness tolerance of a related feature for the free assembly, the parameters $D$ and d denote the nominal dimension of the hole and shaft, the parameters $\Delta D$ and $\Delta d$ are the dimension tolerances of the hole and shaft, and the parameters $E$ and $e$ are the straightness or flatness tolerance of the related feature.

Inequality (17) is the design equation of the dimension tolerance and orientation tolerance of the related feature for the datum directional assembly, the parameters $\theta$, $l_1$, and $l_2$ denote the positional parameter of the related feature relative to their datum feature, the parameters $l_{\mathrm{o}}$ and $l_{\mathrm{q}}$ are the contact length of the two related features and the contact length of the two datum features, respectively, the parameters $Q$ and $q$ are the nominal dimensions of the datum shaft and the datum hole, the parameters $\Delta Q$ and $\Delta q$ are the dimension tolerances of the datum shaft and the datum hole, and the parameters $t_{\mathrm{Qf}}$ and $t_{\mathrm{qf}}$ are the form tolerances of the datum hole and datum shaft. The general datum directional assembly is sketched in Fig. 11, where the related feature is directed by the angle $\theta$, and it represents the datum directional assembly with three orientation tolerances, i.e., the parallelism tolerance, the perpendicularity tolerance and angular tolerance. It should be noted that the third item in the inequality does not exist when the datum feature is a plane.

Inequality (18) is the design equation of the dimension tolerance and position tolerance of the related feature for the datum positioning assembly, and all parameters have the same meaning as those in Eq. (17). It should be noted that all parameters related to the datum feature on the right of the inequality will not exist when the datum feature is a plane.

## 5 The design CASE

### 5.1 The correlation design case for guaranteeing the distance requirement of a single feature

Figure 12a is a design instance of a single feature, and Fig. 12b is a design model to verify the design in Fig. 12a. The

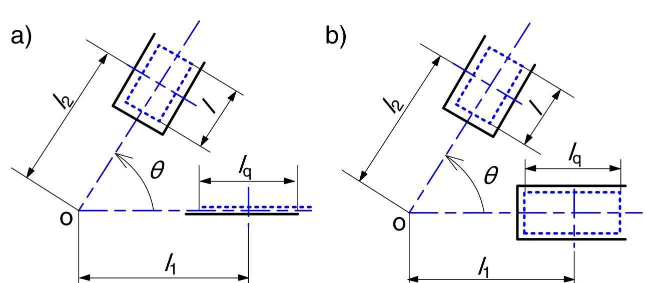


**Fig. 11** The general scheme of the datum directional assembly. a) the directional assembly with planar datum. b) the directional assembly with datum of size

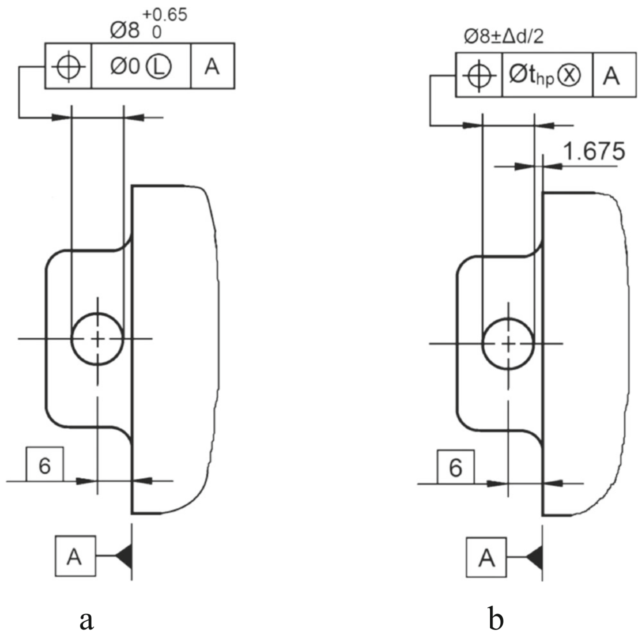


**Fig. 12** The design of dimension and true position tolerance for guaranteeing the minimum distance. **a** The design instance. **b** The design model

tolerance specification in Fig. 12a can guarantee the minimum distance of 1.65 mm from the nearest point on the inner surface of the hole with nominal diameter of Ø8 mm to the datum plane A, which is a design requirement of the correlation design of the dimension tolerance and position tolerance of the hole.

The minimum distance, 1.675 mm, is equivalent to the LMVS of the small equivalent shaft in an equivalent shaft/hole free assembly (Fig. 9a and Table 1), and it must use the least material condition, i.e., the dimension tolerance and position tolerance of an Ø8 mm hole must meet the least material requirement. By using the fifth inequality in Eq. (14) and the nominal dimension and position value and design requirement, we have the following:

$$6-4-\Delta D/4-t_{hp}/2\geq 1.675.$$

After organizing it, we then obtain the following:

$$\Delta D+2t_{hp}\leq 1.3,$$

The inequality gives the relation between the diameter tolerance and position tolerance of the Ø8 mm hole, and the diameter tolerance and position tolerance can be determined by the trial and error method and according to the number relation between the two. We have two selections: (1) the tolerance grade is selected as IT15, the diameter tolerance $\Delta D = 0.58$ mm, and the position tolerance will be 0.36 mm; and (2) the tolerance grade is selected as IT16, the diameter tolerance $\Delta D = 0.9$ mm, and the position tolerance will be 0.2 mm. Obviously, the first selection is more reasonable. If the diameter and its tolerance will be expressed by Ø8 + 0.65/0 mm, as shown in Fig. 12a, the nominal diameter will become Ø8.325 mm, the diameter tolerance will be ± 0.325 mm, and the position tolerance will be $t_{hp} \leq 0$, according to the fifth inequality in Eq. (14). Thus, the final position tolerance is zero, which is the same as the specification shown in Fig. 12a.

### 5.2 The correlation design case for the paired assembly feature with datum positional assembly

In a two-pin part and a two-hole part assembly with floating mating conditions, the design objectives are the dimension tolerance and position tolerance of the pin and hole of both parts under the condition that both pins will be completely inserted into two holes.

The tolerance designs for the assemblies shown in Fig. 13 must consider the size variation of a related feature, as well as their positional variation, including the bonus tolerance and shift tolerance, floating mating conditions, and feasibility of assembling. Therefore, the material condition for the dimension tolerance and position tolerance of both the related feature and the datum feature of the two parts must be considered. The small diameter pin and hole (Ø15 mm, Ø15.5 mm) is positioned relative to the large diameter pin and hole (Ø28 mm, Ø28.5 mm) with the position tolerance. Although these true position tolerance specifications have other datum features, such as datum features A, B and datum features J, K (the datum features B and K are the back of the two parts respectively and are not labeled in the figure), these datum features cannot constrain the relative distance between the small diameter pin (hole) and large diameter pin (hole) and consider that the assembly requirements guarantee no interference exists in the pins and holes assembly, thus it is reasonable that the

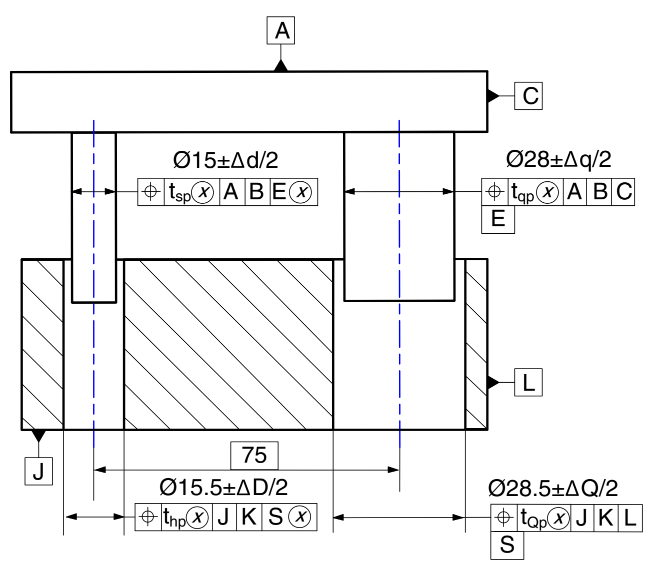


**Fig. 13** The correlation design case of dimension tolerance and geometric tolerance for datum positioning assembly

relative position between the small diameter pin (hole) and large diameter pin (hole) are controlled with the true position tolerance. According to the assembly requirement, this assembly can be taken as the datum positioning assembly, where the tolerance specification of both the datum feature and the related feature of the two parts are unknown. The assembly requirement in this example is to guarantee that the minimum clearance $c$ between the small diameter pin and small diameter hole must be greater than or equal to zero ($c \geq 0$) to maintain contact between the large diameter pin and hole.

According to the analysis above, the maximum material condition must be used and the first inequality in Eq. (18) is applicable. Because the tolerance specifications of both the large diameter pin and hole are the design objectives, the design equation will be rewritten as follows:

$$\Delta D/2 + \Delta Q/2 + \Delta d/2 + \Delta q/2 + t_{hp} + t_{sp} + t_{Qf} + t_{qf} \leq D - d - Q - q$$

There are 8 unknowns in the inequality, and thus, the trial-and-error method is necessary to obtain the design result. According to the design principle of dimension and geometric tolerances, in general, and the processing capability of the shaft and hole, the same tolerance grade is assigned to both pins and both holes. Suppose the dimension tolerance grade is selected as IT13, and the form tolerance grade is selected as IT12, then the dimension tolerance can be found in the tolerance standard. They are $\Delta D = 0.33$ mm, $\Delta Q = 0.33$ mm, $\Delta d = 0.27$ mm, and $\Delta q = 0.27$ mm, and the straightness tolerances are $t_{\mathrm{Qf}} = 0.12$ mm and $t_{\mathrm{qf}} = 0.12$ mm. When these values are used in the inequality, the sum of the true position tolerance of both the small diameter pin and hole is $t_{\mathrm{hp}} + t_{\mathrm{sp}} \leq 0.16$ mm. The outcome of the true position tolerance of the small diameter pin and hole is 0.08 mm after the sum of the tolerance is allocated on average, and it is smaller than the form tolerance, so the initial result is not reasonable. Under the second trial and error, the dimension tolerance grade of both the pin and hole are improved to IT12, the dimension tolerances are $\Delta D = 0.21$ mm, $\Delta Q = 0.21$ mm, $\Delta d = 0.18$ mm, and $\Delta q = 0.18$ mm, and the form tolerance is unchanged. After calculating the inequality by the above values, the sum of the true position tolerance of both the small diameter pin and hole become $t_{\mathrm{hp}} + t_{\mathrm{sp}} \leq 0.37$ mm. The true position tolerance of both the small diameter shaft and hole are 0.18 and 0.19 mm, respectively. By using the average allocation method, the result is more reasonable than the first result. Although the true position tolerance of both the large diameter shaft and hole cannot be designed by this method, they can be selected by using the same or lower tolerance grade, and because they are the datum feature of the small diameter shaft and hole, the straightness tolerance is required.

## 6 Conclusions

To guarantee the correctness of the application of the material condition and the specification of the dimension tolerance and geometric tolerance of both the related feature and datum feature, it is important to establish the application criteria of the material condition and the design method of the dimension tolerance and the geometric tolerance. Unfortunately, they are not available and are incomplete at present design society. This paper establishes these application criteria and design method. First, it expands the notion of the virtual size and extreme virtual size of a single feature at free assembly to the situation of the related feature at the datum directional assembly and the datum positioning assembly. The virtual size and the extreme virtual size of the related feature at the different assembly schemes are studied and the relevant calculation equations of the MMVS and LMVS are constructed. Then, based on these equations, the correlation design method of the related feature and datum feature under the different assembly schemes is developed, which includes the correlation design of the dimension tolerance and geometric tolerance of the paired feature and single feature. For the paired assembly feature, the design objectives are the minimum clearance and minimum interference between the surfaces of the engaged feature, and for the single feature, the design objective is the minimum distance and the maximum distance between the surfaces of the related feature to its datum feature. The proposed method improves the design of the tolerance specification at the application of the material condition and can provide to the engineer to specify the dimension tolerance and geometric tolerance of both the feature of size and datum of size. Third, the function mechanisms of the material condition are investigated, and the application rules of the material requirement are established. These rules consider the geometry and relative position of both the related feature and datum feature, as well as the tolerance specifications, which can provide the basis for engineers to specify the material conditions.

**Funding information** This study was funded by the National Natural Science Foundation of China under grants No. 51675147.



## References

1. International standard (2014) Geometrical product specifications (GPS)—geometrical tolerancing—maximum material requirement (MMR), least material requirement (LMR) and reciprocity requirement (RPR). Int Stand ISO 2692:2014
2. ASME Standard (2009) Dimensioning and tolerancing—engineering drawing and related documentation practices, ASME Y14.5M-2009, ASME, USA

3. Jayaraman R, Srinivasan V (1989) Geometric tolerancing: I. Virtual boundary requirements. IBM J Res Dev 33(2):90–104
4. Robinson DM (1998) Geometric Tolerancing for assembly with maximum material parts, In: Elmaraghy HA (ed) geometric design tolerancing: theories, standards and applications. Springer, USA, pp 242–253
5. Etesami F (1991) Position tolerance verification using simulated gaging. Int J Robot Res 10(4):358–370
6. Lehtihet EA, Gunasena UN (1988) Models for the position and size tolerance of a single hole, manufacturing metrology, ASME PED-29, Presented at the Winter Annual Meeting of the ASME, Chicago, Illinois, November, 49–63
7. Ballu A, Mathieu L (1999) Choice of functional specifications using graphs within the framework of education. In: Houten FV, Kals H (ed) Global consistency of tolerances. Springer, Netherlands, pp 197–206
8. Dantan JY, Mathieu L, Ballu A, Martin P (2005) Tolerance synthesis: quantifier notion and virtual boundary. Comput Aided Des 37(2):231–240
9. Chavanne R, Anselmetti B (2011) Functional tolerancing: virtual material condition on complex junctions. Comput Ind 63(3):210–221
10. Fleming A (1987) Analysis of uncertainties and geometric tolerances in assemblies of parts. PhD Thesis, University of Edimburgh
11. Robinson DM (1998) Geometric tolerancing for assembly. In: PhD thesis. Cornell University, May
12. Gerth RJ, Hancock WM (2000) Computer aided tolerance analysis for improved process control. Comput Ind Eng 38(1):1–19
13. Gupta S, Turner JU (1993) Advanced tolerance analysis and synthesis for geometric tolerances, international forum on dimensional tolerancing and metrology. CTRD 27:187–198
14. Gupta S, Turner JU (1993) Variational solid modeling for tolerance analysis. IEEE Comput Graph Appl 13(3):64–74
15. Giordano M, Duret D (1993) Clearance space and deviation space, application to three-dimensional chain of dimensions and positions, CIRP seminar on computer aided tolerancing. ENS Cachan, Cachan
16. Sacks E, Joskowicz L (1997) Parametric kinematic tolerance analysis of planar mechanisms. Comput Aided Des 29(5):333–342
17. Sacks E, Joskowicz L (1998) Parametric kinematic tolerance analysis of general planar systems. Comput Aided Des 30(9):707–714
18. Teissandier D, Couétard Y, Gérard A (1999) A computer aided tolerancing model: proportioned assembly clearance volume. Comput Aided Des 31(13):805–817
19. Pairel E, Hernandez P, Giordano M (2005) Virtual gauges representation for geometrical tolerances in CAD-CAM Systems, Proceedings of the 9th International CIRP Seminar on Computer-Aided Tolerancing, Arizona:Tempe, AZ, USA
20. Pairel E (2006) Three-dimensional verification of geometric tolerances with the 'fitting gauge' model. J Comput Inf Sci Eng 7(1):26–30
21. Pairel E, Hernandez P, Giordano M (2007) Virtual gauge representation for geometric tolerances in CAD-CAM systems. In: Davidson JK (ed) Models for Computer Aided Tolerancing in Design and Manufacturing. Springer, Netherlands, pp 3–12
22. Singh G, Ameta G, Davidson JK, Shah JJ (2013) Tolerance analysis and allocation for design of a self-aligning coupling assembly using tolerance-maps. J Mech Des 135(3):031005
23. Shen Z, Shah JJ, Davidson JK (2005) Simulation-based tolerance and assemblability analysis of assemblies with multiple pin/hole floating mating conditions, Proceedings of IDETC/CIE 2005 ASME 2005 international design engineering Conferences & computers and information in Engineering conference, September 24–28, 2005, Long Beach, California, USA
24. Shen Z, Shah JJ, Davidson JK (2005) A complete variation algorithm for slot and tab features for 3D simulation-based tolerance analysis, Proceedings of IDETC/CIE 2005 ASME 2005 international design engineering Conferences & computers and information in Engineering conference, September 24–28, 2005, Long Beach, California, USA
25. Ameta G, Davidson JK, Shah JJ (2010) Statistical tolerance allocation for tab-slot assemblies utilizing tolerance-maps. J Comput Inf Sci Eng 10(1):011005
26. Anselmetti B, Laurent P (2016) Complementary writing of maximum and least material requirements with an extension to complex surfaces, 14th CIRP Conference on Computer Aided Tolerancing (CAT). Procedia CIRP 43:220–225