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ORIGINAL ARTICLE

An optimized feedrate scheduling method for CNC machining with round-off error compensation

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Abstract
Feedrate scheduling is one of the most important factors of computer numerical control (CNC) machining and has become a crucial research problem. In order to improve the machining accuracy and motion smoothness, an optimized feedrate scheduling method considering round-off error compensation based on S-shaped acceleration/deceleration (ACC/DEC) algorithm is proposed in this paper. There are two main stages, namely initial feedrate scheduling and parameters calculation of round-off error compensation. In the stage of initial feedrate scheduling, a novel time rounding principle is introduced to reduce the round-off error. Meanwhile, the motion parameters of each section can be calculated based on the proposed feedrate scheduling method. The constant feedrate section used for round-off error compensation is always guaranteed to exist although the actual maximum feedrate might be smaller than the command feedrate. Then, in the stage of compensation parameters calculation, the round-off error can be obtained based on the proposed time rounding principle and the scheduled parameters should be updated. In order to maintain the continuity of the acceleration profile, the improved trapezoidal ACC/DEC algorithm is introduced to conduct the error compensation and the crucial parameters can be calculated based on its special properties. In addition, the feedrate look-ahead strategy is also tweaked to enhance the reliability of feedrate scheduling. Finally, a series of simulations and practical experiments with two non-uniform rational B-spline (NURBS) curves are conducted to verify the good performance and applicability of the proposed method.

Keywords Feedrate scheduling · Round-off error compensation · S-shaped ACC/DEC algorithm · Feedrate look-ahead

1 Introduction

The feedrate scheduling which determines the smoothness, accuracy, and stability of the machining process is one of the most important factors in computer numerical control (CNC) systems [1]. An appropriate feedrate profile can enhance the machining accuracy and surface finish significantly, while the kinematic and dynamic constraints of machine tools can be satisfied. Therefore, in order to achieve high-speed and high-accuracy machining, feedrate scheduling for both the linear and parametric tool paths becomes a crucial research problem.

A lot of acceleration/deceleration (ACC/DEC) control schemes have been developed for feedrate scheduling. The linear ACC/DEC algorithm [2, 3] is simple to implement. But it results in various oscillations on the feedrate and acceleration profiles. Hence, various jerk-limited ACC/DEC algorithms, such as Sine-shaped algorithm [4], S-shaped algorithm [5], and other polynomial algorithms [6, 7] have been proposed to generate smooth feedrate profile. Because of the simplicity and smoothness, S-shaped ACC/DEC algorithm with seven sections is wildly used. Du et al. [8] and Dong et al. [9] proposed a similar adaptive feedrate scheduling method that the feedrate was adjusted adaptively based on S-shaped ACC/DEC algorithm at the sensitive areas. Lin et al. [10] proposed a real-time look-ahead algorithm to generate smooth and jerk-limited feedrate profile based on the curvatures of the curves and the confined chord errors. Liu et al. [11] developed a non-uniform rational B-spline

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(NURBS) interpolator with feedrate scheduling method. However, none of these methods refers to the round-off error, which is defined as the difference between the desired arc length and the sum of the feed length calculated by the scheduled feedrate and the interpolation period.

Theoretical motion parameters, such as the section time $\{[T_1, T_2, T_3, T_4, T_5, T_6, T_7]\}$ and the start feedrate $\{[v_1, v_2, v_3, v_4, v_5, v_6, v_7]\}$ of each section can be obtained by feedrate scheduling with standard S-shaped ACC/DEC algorithm. But in practical interpolation, neither each scheduled section time nor the total time can be discretized exactly in an integer multiple of the interpolation period because the interpolation length, start and end feedrates, and other conditions are given arbitrarily. Therefore, the remnant length less than one interpolation period is always existing and will cause position errors and feedrate fluctuations; although it might be small.

To cope with this problem, some round-off error compensation methods have been developed. Li et al. [12] proposed a variable-period feedrate scheduling method to compensate the round-off error for linear ACC/DEC profile. In addition, Wang et al. [13] presented an optimized feedrate control method with various constraints to make sure that the sum of the feed length in each interpolation period should be equal to the machining segment length. However, the acceleration profiles of these methods are discontinuous, which will cause vibration and shock to machine tools. Cao et al. [14] introduced a feedrate control method by integrating the look-ahead strategy into S-shaped ACC/DEC algorithm, which can deal with the low feedrate zone called “tail” in deceleration zone. However, each section time of acceleration zone and deceleration zone are assumed to be equal to each other, which might not be suitable for some cases. Du et al. [15] and Luo et al. [16] proposed a similar round-off error compensation scheme. However, every section time was rounded and the round-off error was compensated with constant acceleration. Thus, the acceleration profile after compensation was discontinuous. Liu et al. [17] proposed a feedrate scheduling method based on a cubic polynomial feedrate profile with similar time rounding scheme as [15, 16].

Because every section time might need to be adjusted by the time rounding method given in [15–17], the motion parameters scheduled by standard ACC/DEC algorithms, such as the start feedrate of each section and the end feedrate of the interpolating curve will be changed. However, the time rounding during feedrate scheduling is not considered in feedrate look-ahead which is conducted earlier. When the actual end feedrate v'_{ei} of i-th curve is larger than the theoretical feedrate v_{ei} obtained by look-ahead, the feedrate might not be able to slow down to the end feedrate v_{ei+1} of next curve where $v_{ei} > v_{ei+1}$, especially for the short curves. Hence, the time rounding method given in [15–17] affects the reliability of feedrate scheduling for the following curves. Furthermore, this time rounding method increases the round-off time and the corresponding errors. The larger the amount of round-off error, the more easily the motion parameters, such as the feedrate, acceleration, and jerk are beyond the constraint ranges, the greater the adverse effects to interpolation accuracy and machine tools.

To cope with these problems, this paper proposes an optimized feedrate scheduling method based on S-shaped ACC/DEC algorithm with two stages namely initial feedrate scheduling and parameters calculation of round-off error compensation. In the stage of initial feedrate scheduling, a novel time rounding principle is proposed. Meanwhile, the interpolation time without rounding and the corresponding motion parameters can be obtained by the optimized feedrate scheduling method. The constant feedrate (CF) section used for round-off error compensation is always guaranteed to exist with at least one interpolation period although the actual maximum feedrate might be smaller than the command feedrate. Then, in the stage of compensation parameters calculation, the round-off error can be calculated according to the proposed time rounding principle. Correspondingly, the motion parameters obtained in initial feedrate scheduling stage should be updated. After that, the improved trapezoidal ACC/DEC algorithm [18] is introduced to conduct the round-off error compensation, and the crucial parameters will be calculated based on its special properties. During the interpolation, the feed length consists of two parts: the length obtained by S-shaped ACC/DEC algorithm and the compensation length calculated by the improved trapezoidal profile, which can maintain the continuity of the acceleration profile. In addition, the look-ahead strategy is also tweaked to match the proposed feedrate scheduling method. Therefore, the actual end feedrate of each curve is always consistent with the look-ahead result.

The remainder of this paper is organized as follows. In Section 2, preliminaries including the typical S-shaped ACC/DEC algorithm and the improved trapezoidal algorithm are introduced briefly. Section 3 develops the proposed feedrate scheduling method. Section 4 presents the tweaked look-ahead strategy corresponding to the feedrate scheduling. The simulation and experimental results are analyzed and compared with previous research works in Section 5. The conclusions are given in Section 6.

2 Preliminaries

In this section, two kinds of ACC/DEC algorithms including the typical S-shaped profile and the improved trapezoidal profile will be introduced briefly.

2.1 S-shaped ACC/DEC algorithm

As shown in Fig. 1, the typical S-shaped ACC/DEC profile consists of seven sections. According to the specified motion parameters, including the maximum jerk $J_{\max}$, maximum

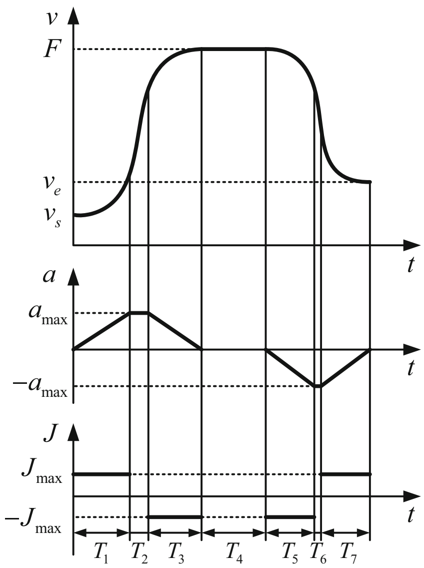

Fig. 1 The S-shaped ACC/DEC profile with seven sections

acceleration a_{max}, command feedrate F, curve length L, start feedrate v_s, and end feedrate v_e, the section time $\{[T_1, T_2, T_3, T_4, T_5, T_6, T_7]\}$ can be calculated by analytical or numerical methods, while some of them might be zero. Then, based on the section time and the kinematic characteristics of S-shaped ACC/DEC algorithm, the start feedrate $\{[v_1, v_2, v_3, v_4, v_5, v_6, v_7]\}$ and the cumulative displacement $\{[S_1, S_2, S_3, S_4, S_5, S_6, S_7]\}$ of each section can be obtained. Therefore, the feed length in each interpolation period can be calculated easily. Furthermore, because of the arbitrarily given parameters and numerical calculation methods, neither the durations $T_i (i = 1, 2, \ldots, 7)$ nor the total time $T_{total} = \sum_{i=1}^{7} T_i$ might be discretized exactly in an integer multiple of the interpolation period T_s, which causes the round-off error.

2.2 The improved trapezoidal ACC/DEC algorithm

In this paper, the improved trapezoidal ACC/DEC algorithm shown in Fig. 2 is introduced to achieve the compensation of round-off error. Its acceleration profile is continuous and the equation can be expressed as follows:

$$a(t) = \begin{cases} a_{\mathrm{Tmax}} \sin\left(\frac{4\pi}{T} t\right) & 0 \le t \le \frac{1}{8}T \\ a_{\mathrm{Tmax}} & \frac{1}{8}T < t \le \frac{3}{8}T \\ a_{\mathrm{Tmax}} \cos\left(\frac{4\pi}{T}\left(t - \frac{3}{8}T\right)\right) & \frac{3}{8}T < t \le \frac{5}{8}T \\ -a_{\mathrm{Tmax}} & \frac{5}{8}T < t \le \frac{7}{8}T \\ -a_{\mathrm{Tmax}} \cos\left(\frac{4\pi}{T}\left(t - \frac{7}{8}T\right)\right) & \frac{7}{8}T < t \le T \end{cases} \quad (1)$$

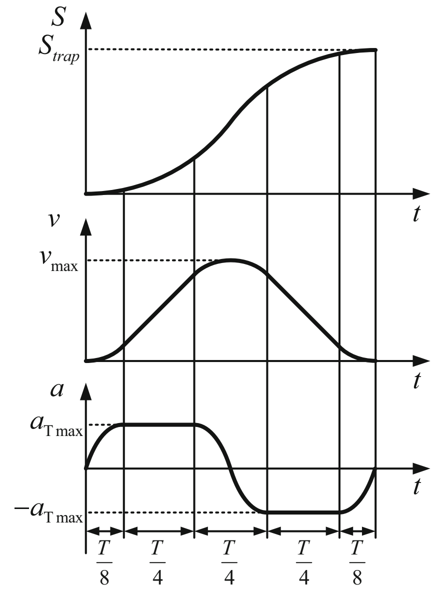

Fig. 2 The improved trapezoidal ACC/DEC profile

where T is the total motion time and a_{Tmax} is the maximum acceleration of the improved trapezoidal profile. Integrating Eq. 1 yields the feedrate and displacement equations with the same time partitioning rule as follows:

$$v(t) = \begin{cases} -\frac{T}{4\pi} a_{\mathrm{Tmax}} \cdot \cos\left(\frac{4\pi}{T} t\right) + \frac{1}{4\pi} a_{\mathrm{Tmax}} T \\ a_{\mathrm{Tmax}} t + \left(\frac{1}{4\pi} - \frac{1}{8}\right) a_{\mathrm{Tmax}} T \\ \frac{T}{4\pi} a_{\mathrm{Tmax}} \cdot \sin\left[\frac{4\pi}{T}\left(t - \frac{3}{8}T\right)\right] + \left(\frac{1}{4\pi} + \frac{1}{4}\right) a_{\mathrm{Tmax}} T \\ -a_{\mathrm{Tmax}} t + \left(\frac{1}{4\pi} + \frac{7}{8}\right) a_{\mathrm{Tmax}} T \\ -\frac{T}{4\pi} a_{\mathrm{Tmax}} \cdot \sin\left[\frac{4\pi}{T}\left(t - \frac{7}{8}T\right)\right] + \frac{1}{4\pi} a_{\mathrm{Tmax}} T \end{cases} \quad (2)$$

$$S(t) = \begin{cases} -\left(\frac{T}{4\pi}\right)^2 a_{\mathrm{Tmax}} \sin\left(\frac{4\pi}{T}\right) + \frac{1}{4\pi} a_{\mathrm{Tmax}} Tt \\ \frac{1}{2} a_{\mathrm{Tmax}} t^2 + \left(\frac{1}{4\pi} - \frac{1}{8}\right) a_{\mathrm{Tmax}} Tt + \left(\frac{1}{128} - \frac{1}{16\pi^2}\right) a_{\mathrm{Tmax}} T^2 \\ -\left(\frac{T}{4\pi}\right)^2 a_{\mathrm{Tmax}} \cos\left(\frac{4\pi}{T}\left(t - \frac{3}{8}T\right)\right) + \left(\frac{1}{4\pi} + \frac{1}{4}\right) a_{\mathrm{Tmax}} Tt - \frac{1}{16} a_{\mathrm{Tmax}} T^2 \\ -\frac{1}{2} a_{\mathrm{Tmax}} t^2 + \left(\frac{1}{4\pi} + \frac{7}{8}\right) a_{\mathrm{Tmax}} Tt + \left(\frac{1}{16\pi^2} - \frac{33}{128}\right) a_{\mathrm{Tmax}} T^2 \\ \left(\frac{T}{4\pi}\right)^2 a_{\mathrm{Tmax}} \cos\left(\frac{4\pi}{T}\left(t - \frac{7}{8}T\right)\right) + \frac{1}{4\pi} a_{\mathrm{Tmax}} Tt + \frac{1}{8} a_{\mathrm{Tmax}} T^2 \end{cases} \quad (3)$$

where the start and end feedrates are always set to zero. Because the total time T is evenly divided into eight parts, which constitute five sections in definite proportion, the special relation between the whole displacement S_{trap}, T, and a_{Tmax} can be derived as follows:

$$T = \sqrt{\frac{S_{\text{trap}}}{\left(\frac{1}{4\pi}+\frac{1}{8}\right)a_{\text{Tmax}}}} \tag{4}$$

Therefore, according to the given displacement S_{trap}, motion time T, and Eq. (4), a_{Tmax} and other parameters in Eqs. (1), (2), and (3) can be obtained.

3 The proposed feedrate scheduling method with round-off error compensation

In order to reduce the round-off error and achieve the compensation with continuous acceleration profile, an optimized feedrate scheduling method with two stages namely initial feedrate scheduling and parameters calculation of round-off error compensation based on S-shaped ACC/DEC algorithm is introduced in detail.

3.1 Initial feedrate scheduling based on S-shaped ACC/DEC algorithm

In traditional methods, the feedrate scheduling and round-off error compensation are completely independent. In addition, every scheduled duration T_i is rounded to be an integer multiple of interpolation period T_s, which affects the reliability of feedrate scheduling and increases the round-off error. Hence, in order to reduce the round-off error, a new time rounding principle is proposed as follows. As can be seen from Fig. 1, the acceleration and feedrate profiles are continuous during the scheduled time. The target feedrate and the corresponding feed length of each interpolation period can be calculated by periodic sampling shown in Fig. 3. Therefore, it only needs to make sure that the total scheduled time T_{total} is an integer multiple of T_s, but the each section time T_i is not required, which is different from the traditional time rounding principle. If T_{total} is an integer multiple of T_s, no time rounding is needed for both T_{total} and T_i. Correspondingly, the round-off error is zero. However, in traditional methods, the time rounding and error compensation will still be conducted in this situation. On the contrary, if T_{total} is not an integer multiple of T_s, it should be rounded and the round-off time is smaller than T_s, which can reduce the round-off error significantly.

Based on the proposed time rounding principle, the total period number of interpolation N_{total} can be calculated as follows:

$$N_{\text{total}} = \left[\frac{T_{\text{total}}}{T_s}\right] \tag{5}$$

where the operator "[]" denotes rounding down. Therefore, the round-off time Δt can be expressed as follows:

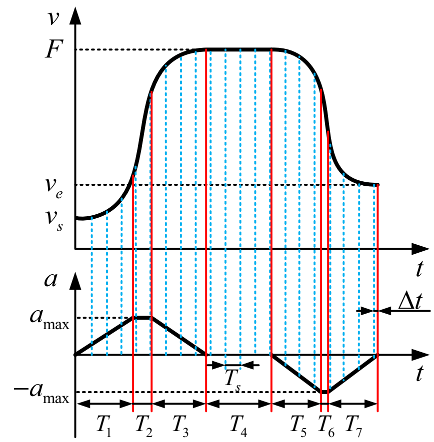

Fig. 3 Periodic sampling of feedrate and acceleration in S-shaped ACC/DEC algorithm

$$\Delta t = T_{\text{total}} - N_{\text{total}} T_s < T_s \tag{6}$$

The corresponding round-off error can be assumed to be ΔS. If Δt and ΔS are taken from the acceleration or deceleration sections, the initial scheduled results of other sections, such as the start feedrate and time also need to be adjusted to maintain the continuity of the acceleration profile and position accuracy, which is very complicated. In contrast, if Δt and ΔS are taken from the CF section, the parameters in other sections are not affected except the cumulative displacements of deceleration sections. Meanwhile, the actual end feedrate v_e' will be equal to v_e obtained by look-ahead. After the calculation of round-off error, the cumulative displacements of deceleration sections can be updated easily. Therefore, the CF section is selected to be adjusted and the corresponding time T_4 should be larger than Δt. However, using the traditional feedrate scheduling method, the CF section might not exist under the given conditions. Therefore, an optimized feedrate scheduling method is proposed and its flowchart is shown in Fig. 4, where $v_{\max}$ is assumed to be the actual maximum feedrate and $0 \le v_s \le v_e < F$. Meanwhile, $v_{\max}^{(1)}$, $v_{\max}^{(2)}$, and $v_{\max}^{(3)}$ are the hypothetical maximum feedrates in three situations respectively, where $v_{\max}^{(1)}$ is assumed to be equal to F, $v_{\max}^{(2)}$ is obtained by accelerating from v_e without constant acceleration section and $v_{\max}^{(3)}$ is calculated by accelerating from v_s without constant acceleration section.

As can be seen, the feedrate scheduling consists of no more than three steps. The key is to calculate the maximum feedrate $v_{\max}$ based on different assumptions. Once $v_{\max}$ is determined, the section time $\{[T_1, T_2, T_3, T_4, T_5, T_6, T_7]\}$ and the corresponding start feedrate $\{[v_1, v_2, v_3, v_4, v_5, v_6, v_7]\}$ and cumulative displacement $\{[S_1, S_2, S_3, S_4, S_5, S_6, S_7]\}$ can be calculated by analytical or numerical methods. Compared with traditional scheduling method, the main improvement is the introduction of S_{con} where $S_{\text{con}} = v_{\max} T_s$. It can make sure that the CF

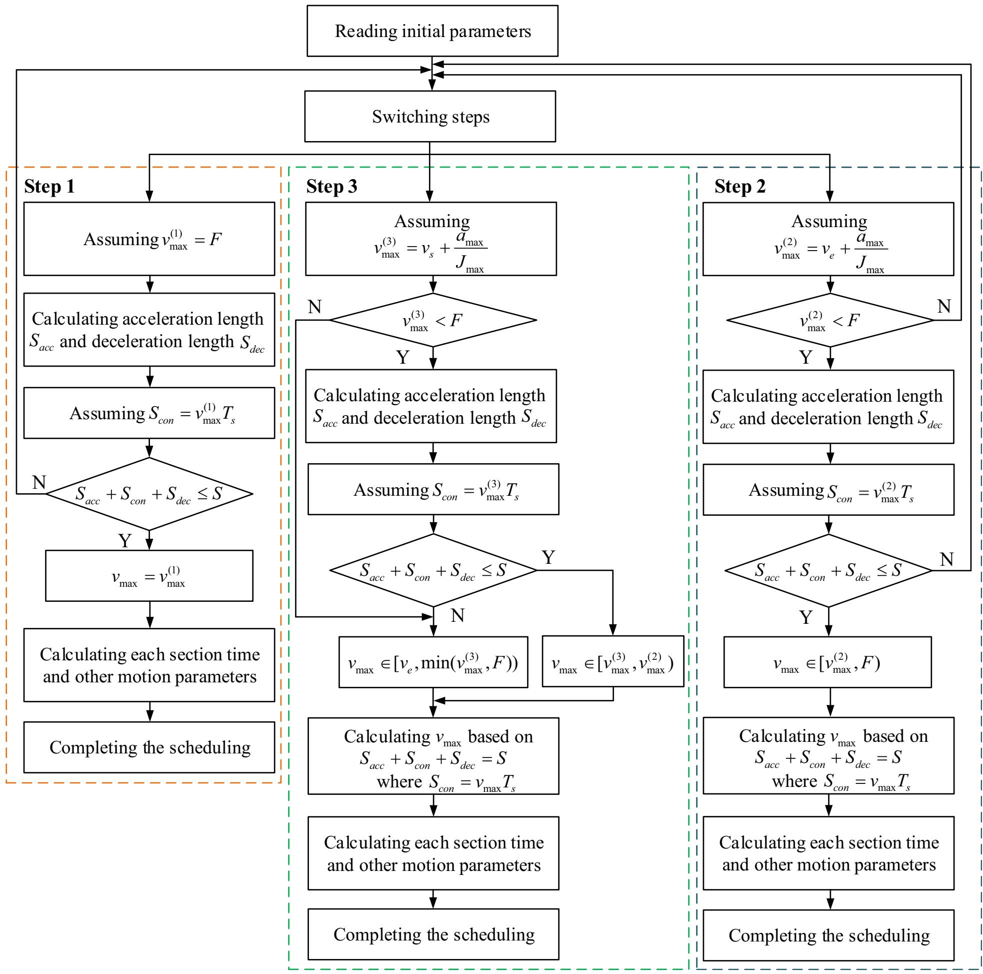

Fig. 4 The flowchart of the proposed optimized feedrate scheduling method

section is always existing with at least one interpolation period although $v_{\max}$ might be smaller than F. Therefore, the conditions of round-off error compensation can be satisfied whether the curve is long or short.

3.2 Parameters calculation of round-off error compensation

In this stage, the crucial parameters of round-off error compensation can be calculated to prepare for the real-time interpolation. Based on the proposed time rounding principle given in Subsection 3.1, the round-off error can be obtained as follows:

$$\Delta S = S_{\mathrm{trap}} = v_{\max}\Delta t \tag{7}$$

Then the cumulative displacement $\{[S_1, S_2, S_3, S_4, S_5, S_6, S_7]\}$ should be updated corresponding to ΔS where $S_j = S_j - \Delta S (j = 4, 5, 6, 7)$. In order to reduce the compensation length in each interpolation period, ΔS should be compensated throughout the interpolation process of current curve. Thus, according to the improved trapezoidal ACC/DEC algorithm, the total period number of compensation can be expressed as follows:

$$N_{\mathrm{com}} = \left[\frac{N_{\mathrm{total}}}{8}\right] 8 \tag{8}$$

Then, the maximum acceleration of round-off error compensation can be calculated according to Eq. (4) as follows:

$$a_{\mathrm{Tmax}} = \frac{S_{\mathrm{trap}}}{\left(\frac{1}{4\pi} + \frac{1}{8}\right) N_{\mathrm{com}}^2 T_s^2} \tag{9}$$

Therefore, the compensation feedrate and the corresponding compensation feed length can be obtained by combining Eqs. (1)–(3) with Eqs. (7)–(9). During the real-time interpolation, the feed length Δl can be calculated as follows:

$$\Delta l = \Delta l_S + \Delta l_T \tag{10}$$

where Δl_S is calculated by S-shaped ACC/DEC algorithm and Δl_T is the compensation length obtained by the improved trapezoidal algorithm.

Benefiting from the improved trapezoidal ACC/DEC algorithm for round-off error compensation, the continuity of the acceleration profile can be maintained and the jerk is limited. Meanwhile, the start and end feedrates of the interpolating curve can be guaranteed to equal to the look-ahead results, which improves the reliability of feedrate scheduling for the following curves.

4 The tweaked look-ahead strategy corresponding to the proposed feedrate scheduling method

In order to match the proposed feedrate scheduling method, the tweaked look-ahead strategy will be introduced in this section. Although there are a lot of look-ahead methods for different types of curves, the main step is to calculate the maximum endpoint feedrate from the other side of the curve in forward or backward direction. Therefore, the bidirectional look-ahead method [19] which consists of back-trace module and forward module for NURBS curves is selected as an example to introduce the tweaked strategy.

In general, the NURBS curves are split into several blocks and segments at the breakpoints and the critical points before the feedrate look-ahead. Meanwhile, the restricted endpoints feedrates of each segment should be obtained. The procedures of the traditional bidirectional look-ahead method can be introduced briefly as follows. Firstly, the endpoint feedrate of each block is assumed to be 0 mm/s. And the back-trace module is performed to calculate the start feedrate of each segment from the last segment to the start segment of each block using standard S-shaped ACC/DEC algorithm. Secondly, the forward module is also applied to recalculate the end feedrate of each segment from the start segment to the last segment of each block. Through these two steps, it can make sure that the start feedrate can speed up or slow down to end feedrate smoothly in each segment. In the proposed feedrate look-ahead strategy, the bidirectional scanning is also applied. The i-th segment with length L_i is taken as an example and the look-ahead procedures are described as follows.

4.1 Back-trace scanning

As shown in Fig. 5, before the back-trace scanning of L_i, the restricted start feedrate v_{si}^r has been obtained according to the confined chord error, centripetal acceleration, and other constraints. Meanwhile, the end feedrate v_{ei} is obtained by back-trace scanning of the (i + 1)-th segment. In order to guarantee that the CF section is always existing with at least one interpolation period, v_{ei} should be restricted especially when L_i is small or T_s is large and updated as follows:

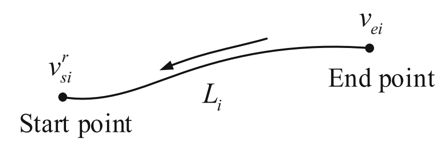

Fig. 5 Back-trace scanning for the i-th segment

$$v_{\mathrm{ei}} = \min\left(v_{\mathrm{ei}}, \frac{L_i}{T_s}\right) \tag{11}$$

Based on L_i and the specified motion parameters $J_{\max}$, $a_{\max}$, and F, the maximum start feedrate $v_{s\max}$ through acceleration from the end point can be calculated by the three assumptions given in Fig. 4. Different from the traditional methods, the calculation process of $v_{s\max}$ should satisfy the following equation:

$$S_{\mathrm{acc}}^{'} + S_{\mathrm{con}}^{'} = L_i \tag{12}$$

where $S_{\mathrm{acc}}^{'}$ is the acceleration length and $S_{\mathrm{con}}^{'} = v_{s\max} T_s$. However, $v_{s\max}$ might be smaller than F. According to Eq. (12) and the relation between v_{ei}, v_{si}^r, and $v_{s\max}$, there are three different situations described as follows:

(1) $v_{\mathrm{ei}} \le v_{\mathrm{si}}^r$ and $v_{s\max} \ge v_{\mathrm{si}}^r$.
In this situation, the start feedrate v_{si} can be set to v_{si}^r. Meanwhile, the CF section must exist based on the proposed feedrate scheduling method with v_{si}^r and v_{ei}.

(2) $v_{\mathrm{ei}} \le v_{\mathrm{si}}^r$ and $v_{s\max} < v_{\mathrm{si}}^r$.
v_{si} should be set to $v_{s\max}$. Meanwhile, the CF section have only one interpolation period exactly.

(3) $v_{\mathrm{ei}} > v_{\mathrm{si}}^r$ and $v_{s\max} \ge v_{\mathrm{si}}^r$.

In this situation, the start feedrate should be v_{si}^r as (1). However, it cannot be ensured that the CF section is existing in feedrate scheduling, which will be further corrected in forward scanning.

4.2 Forward scanning

As shown in Fig. 6, v_{ei} is the end feedrate obtained by back-trace scanning. And v_{si} is the start feedrate obtained by forward scanning of (i-1)-th segment. Similar to the back-trace scanning, v_{si} should also be restricted by L_i and T_s as follows:

$$v_{\mathrm{si}} = \min\left(v_{\mathrm{si}}, \frac{L_i}{T_s}\right) \tag{13}$$

Then, the maximum end feedrate $v_{e\max}$ can be obtained based on Eq. (12) where $S_{\mathrm{con}}^{'} = v_{e\max} T_s$. Therefore, there are also three situations:

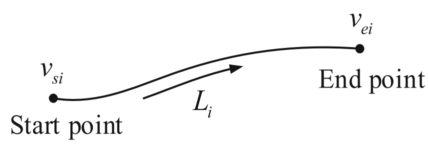

Fig. 6 Forward scanning for i-th segment

(1) $v_{si} < v_{ei}$ and $v_{emax} \geq v_{ei}$.
In this situation, the end feedrate should be maintained at v_{ei}. The existing of CF section can be guaranteed.

(2) $v_{si} < v_{ei}$ and $v_{emax} < v_{ei}$.
The end feedrate should be updated by v_{emax}. And the CF section has one interpolation period exactly with v_{si} and v_{emax}.

(3) $v_{si} \geq v_{ei}$ and $v_{emax} \geq v_{ei}$.

The end feedrate should be maintained at v_{ei}. Based on the results of back-trace scanning, feedrate can reach v_{si} from v_{ei} through acceleration in backward direction with at least one interpolation period of CF section. Therefore, it can make sure that the CF section is existing in this situation.

Through (1)–(2) in Subsection 4.2, the existing of CF section can be guaranteed when $v_{si} < v_{ei}$, which is not sure in (3) of Subsection 4.1. Therefore, owing to the tweaked bidirectional scanning look-ahead method, the CF section is always existing with at least one interpolation period. Meanwhile, the proposed feedrate scheduling method can obtain the same end feedrate of each segment as the result of look-ahead without the aforementioned problems. In addition, this strategy is also suitable for the feedrate look-ahead of small line segments interpolation, while the forward scanning is only employed for the current segment and the back-trace scanning is for the subsequent segments.

5 Simulation and experiment

In this section, analytical simulations of two NURBS curves are conducted to evaluate the performance of the proposed feedrate scheduling method. In 5.1, analysis and comparisons are performed with the representative methods given in [15, 17]. In addition, the experimental results of the test NURBS curves are given in 5.2 to validate the feasibility and applicability of the proposed method.

5.1 Simulation results and analysis

5.1.1 Test cases and simulation environment

As shown in Fig. 7, two NURBS curves which are ∞-shaped curve and butterfly-shaped curve are selected as the case studies. The degrees, control points, knot vectors, and weight vectors of the test curves are given in Appendix 1 and 2, respectively.

According to the proposed feedrate scheduling method and the interpolation procedures given in [20], the flowchart of NURBS interpolator is shown in Fig. 8. There are two main stages namely pre-processing and real-time interpolation. In the stage of pre-processing, the necessary features can be obtained through five modules, such as curve splitting at breakpoints, arc length and curvature calculation of each block, partitioning segments at critical points, feedrate look-ahead, and feedrate scheduling by the proposed method. In the stage of real-time interpolation, the feedrate and the corresponding feed length with round-off error compensation can be obtained in each interpolation period. Then, the predictor-corrector method [21] and de Boor-Cox algorithm [22] are employed to calculate the interpolation point position.

The simulations are conducted on a personal computer with Intel(R) Core(TM) i7-6700HQ 2.60GHz CPU, 8.00GB SDRAM, and Windows 7 operating system. And all the algorithms for simulations are developed and implemented on Microsoft visual studio 2008 using C++ language. The interpolation parameters are listed in Table 1.

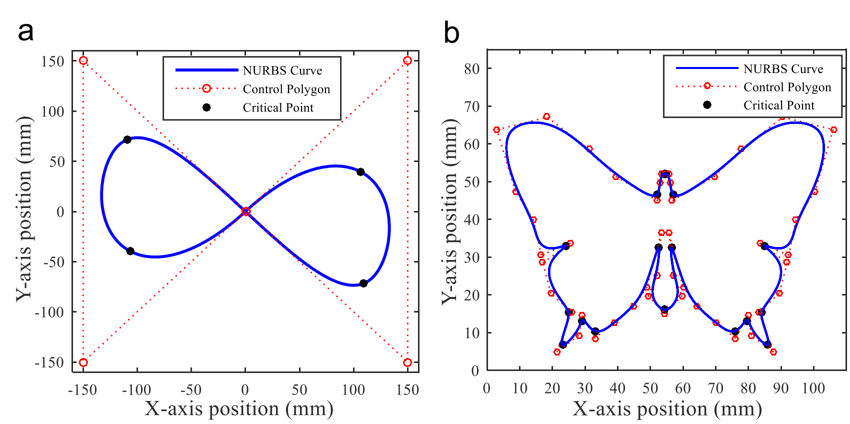

Fig. 7 Test curves. **a** ∞-shaped curve. **b** Butterfly-shaped curve

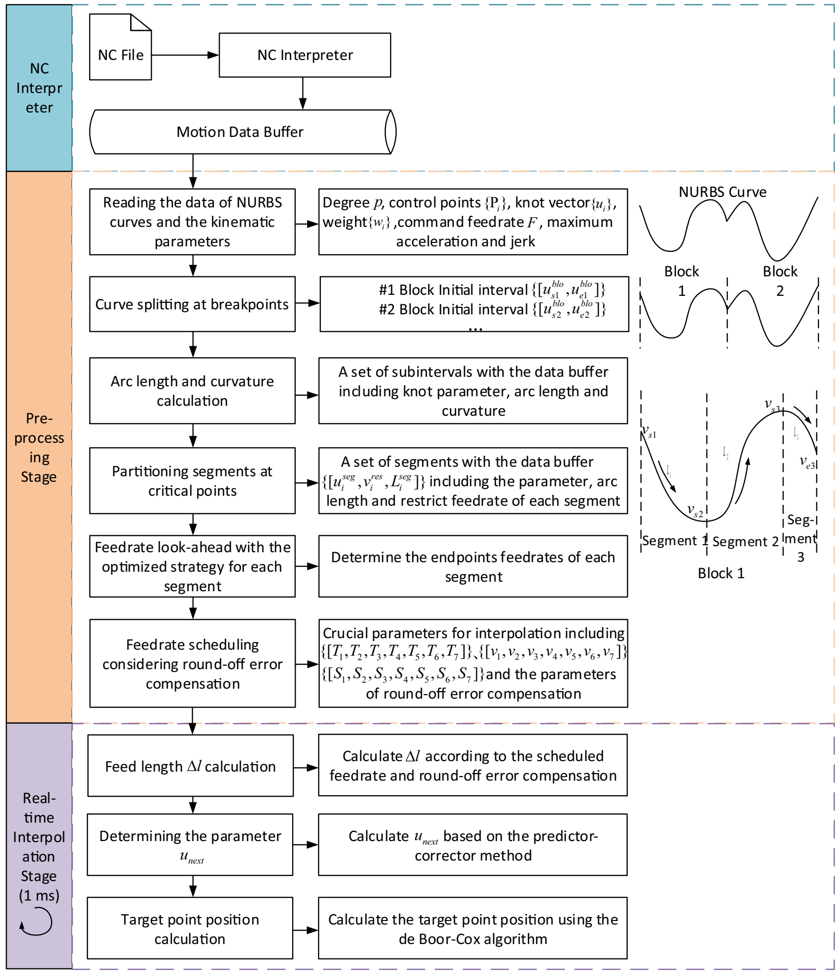

Fig. 8 The flowchart of the NURBS interpolator

5.1.2 Analysis and comparisons of ∞-shaped curve

∞-shaped curve is divided into four segments and each of them is long enough to have CF section with F.The simulation results obtained by the proposed feedrate scheduling method are shown in Fig. 9a–c. As can be seen, the acceleration profile is continuous, while the feedrate profile is smooth. Meanwhile, the actual feedrates at the endpoints of each segment during the real-time interpolation are always consistent with the feedrate look-ahead results. However, the jerk profile slightly exceeds the confined range in some interpolating points due to the rounding-off error compensation but still keeps in a certain extent.

The simulation results obtained by the methods given in [15, 17] are shown in Figs. 10a–c and 11a–c, respectively. As can be seen, the acceleration profile shown in Fig. 10b has abrupt changes in some interpolating points because the acceleration of round-off error compensation is discontinuous. Meanwhile,

Table 1 Interpolation parameters

Parameters	∞-shaped curve	Butterfly-shaped curve
Interpolation period	$T_s = 1$ ms	$T_s = 1$ ms
Arc length tolerance	$\delta_{\mathrm{arc}} = 10^{-6}$ mm	$\delta_{\mathrm{arc}} = 10^{-6}$ mm
Maximum chord error	$\delta_{\mathrm{cho}} = 1$ μm	$\delta_{\mathrm{cho}} = 1$ μm
Feedrate fluctuation tolerance	$\delta_{\mathrm{flu}} = 10^{-8}$	$\delta_{\mathrm{flu}} = 10^{-8}$
Numerical method tolerance	$\delta_{\mathrm{num}} = 10^{-4}$	$\delta_{\mathrm{num}} = 10^{-4}$
Command feedrate	$F = 500$ mm/s	$F = 100$ mm/s
Maximum centripetal acceleration	$a_{\max c} = 4600$ mm/s^2	$a_{\max c} = 3000$ mm/s^2
Maximum tangential acceleration	$a_{\max t} = 4600$ mm/s^2	$a_{\max t} = 3000$ mm/s^2
Maximum jerk	$J_{\max} = 51{,}000$ mm/s^3	$J_{\max} = 55{,}000$ mm/s^3

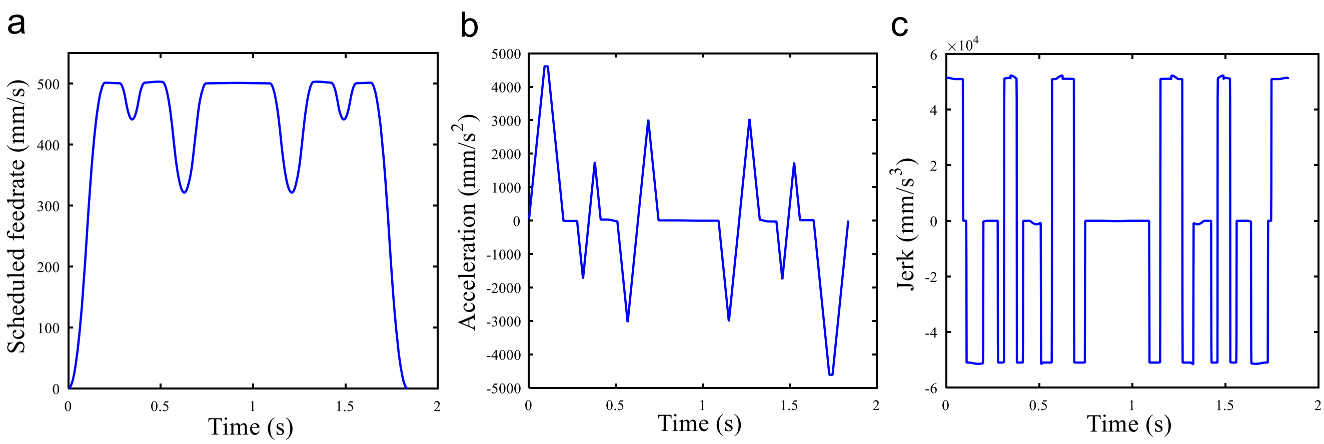

Fig. 9 Simulation results of ∞-shaped curve by the proposed feedrate scheduling method. **a** Scheduled feedrate. **b** Acceleration. **c** Jerk

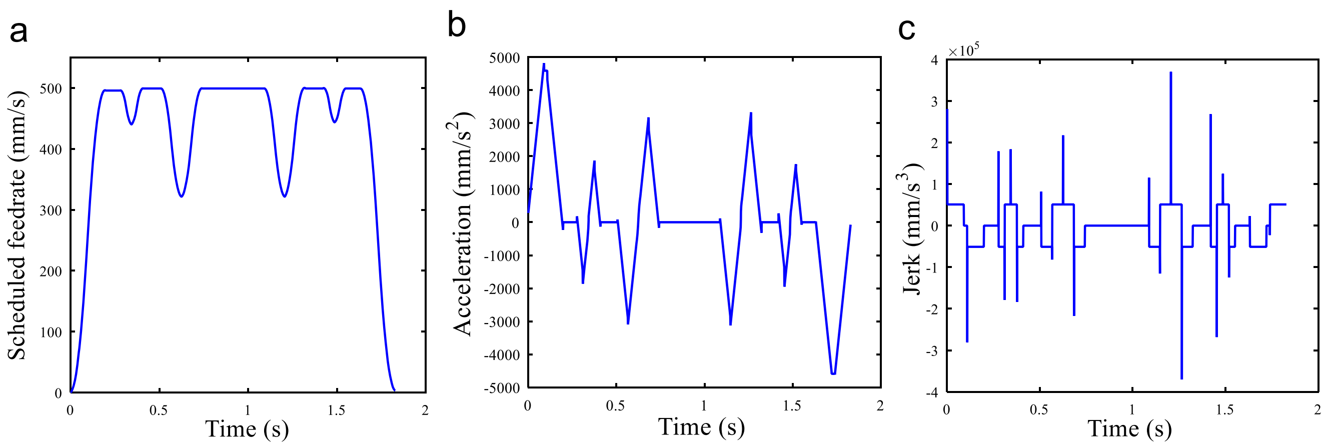

Fig. 10 Simulation results of ∞-shaped curve by the feedrate scheduling method given in [15]. **a** Scheduled feedrate. **b** Acceleration. **c** Jerk

the jerk profile is far beyond the constraint range. Therefore, it will cause vibration and shock to machine tools and affects the surface finish. In contrast, the acceleration profile shown in Fig. 11b is continuous owing to the acceleration-continuous ACC/DEC algorithm in round-off error compensation. However, as shown in Table 2, the compensation length obtained by traditional methods is larger than the proposed method. Therefore, the acceleration and jerk can easily exceed the constraints. In addition, the end feedrates in the last segment obtained by [15, 17] are equal to 3.417 mm/s and cannot slow down to zero, which will affects the motion smoothness and machining accuracy. As can be seen from Table 2, the proposed

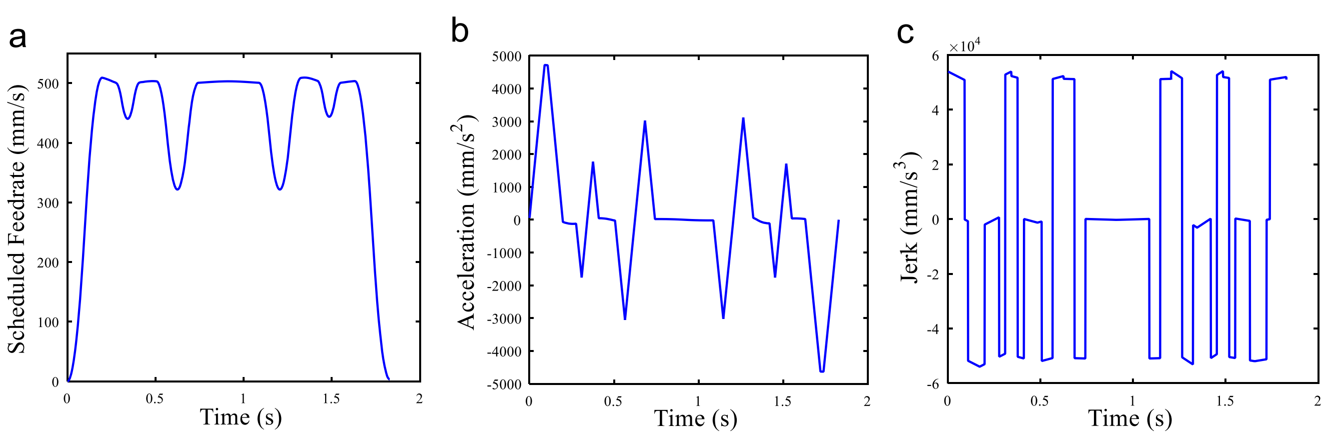

Fig. 11 Simulation results of ∞-shaped curve by the feedrate scheduling method given in [17]. **a** Scheduled feedrate. **b** Acceleration. **c** Jerk

Table 2 Static comparison of ∞-shaped curve simulation results

Test methods	Round-off error	Actual maximum acceleration	Actual maximum jerk	Total interpolation time
Proposed method	1.841 mm	4612.29 mm/s^2	52,259.62 mm/s^3	1.835 s
Method in [15]	6.416 mm	4819.89 mm/s^2	370,233.58 mm/s^3	1.828 s
Method in [17]	6.416 mm	4715.67 mm/s^2	54,103.46 mm/s^3	1.828 s

time rounding method has the smallest round-off error as well as the actual maximum acceleration and jerk. The total interpolation time of the proposed method is little longer than two other methods for two reasons. Firstly, every scheduled section time is rounded in [15, 17], which shortens the machining time and increases the round-off error. Secondly, the existing of CF section in the proposed method limits the maximum feedrate and acceleration especially for the short segment.

5.1.3 Analysis and comparisons of butterfly-shaped curve

The butterfly-shaped curve is divided into 18 segments by segment partitioning module. But some of them are short and their actual maximum feedrates cannot reach *F*. The simulation results obtained by the proposed method are shown in Fig. 12a–c with continuous acceleration and smooth feedrate profiles. Meanwhile, the jerk profile keeps in a certain extent, although it exceeds the confined range in some areas resulted from the round-off error compensation.

The simulation results of [15, 17] are shown in Figs. 13a–c and 14a–c, respectively. However, only the first 17 segments are interpolated. The last segment is short where the length is only 0.33 mm. According to the look-ahead results based on the standard S-shaped ACC/DEC algorithm, the start feedrate of the last segment should be 18.07 mm/s, which can decrease to 0 mm/s. However, because of the traditional time rounding method, the actual end feedrate of the 17-th segment is rounded to 21.67 mm/s, which is also the start feedrate of last

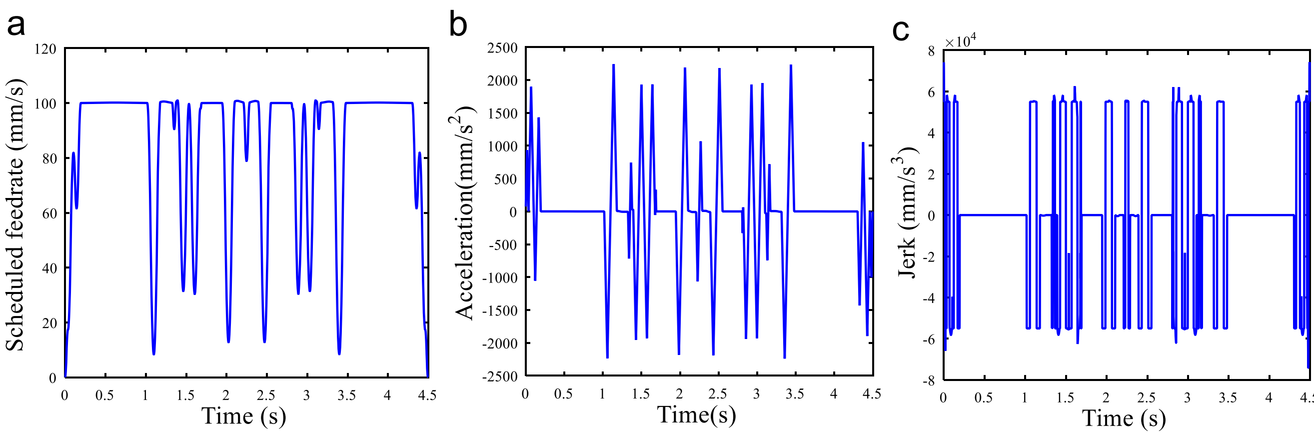

Fig. 12 Simulation results of butterfly-shaped curve by the proposed feedrate scheduling method. **a** Scheduled feedrate. **b** Acceleration. **c** Jerk

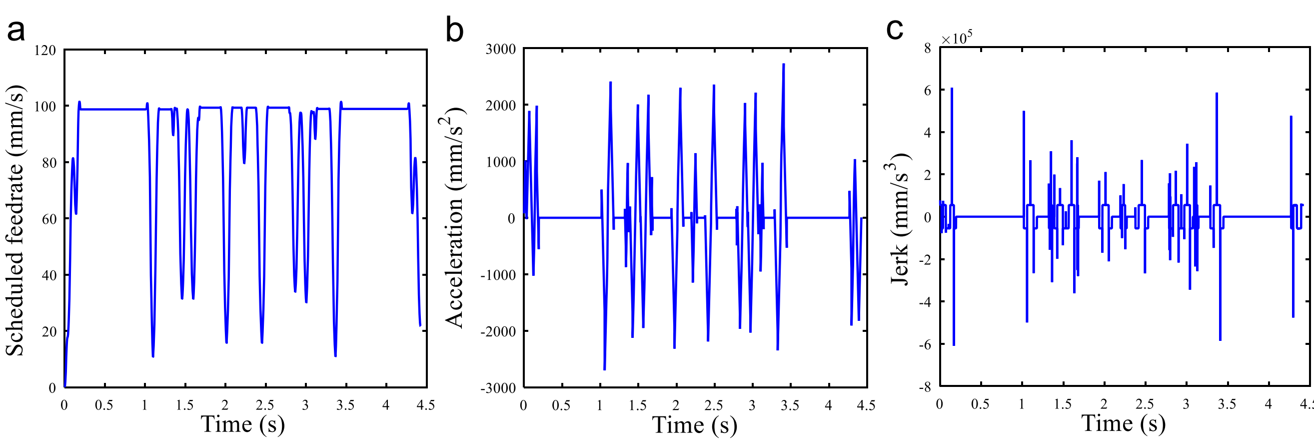

Fig. 13 Simulation results of the first 17 segments on the butterfly-shaped curve by the feedrate scheduling method given in [15]. **a** Scheduled feedrate. **b** Acceleration. **c** Jerk

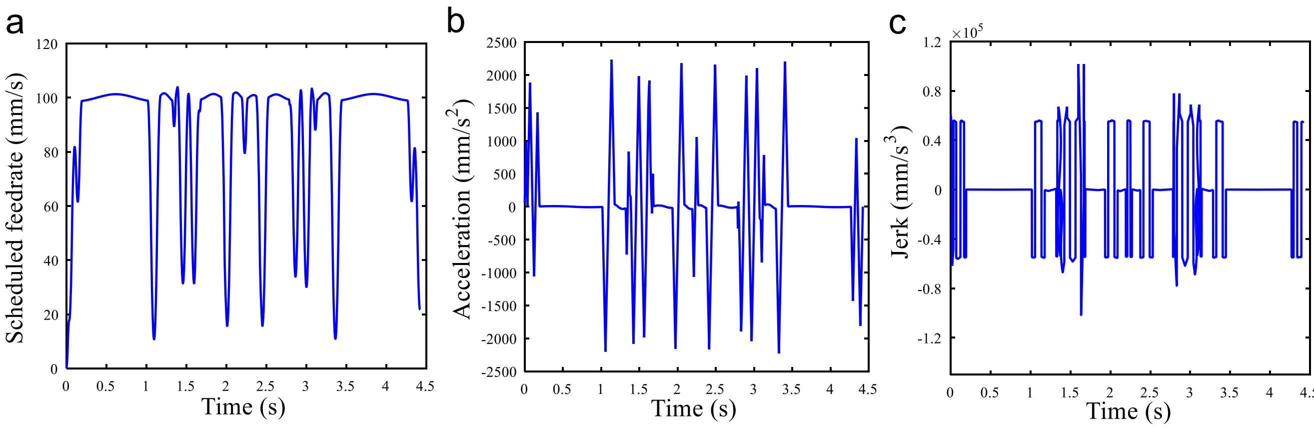

Fig. 14 Simulation results of the first 17 segments on the butterfly-shaped curve by the feedrate scheduling method given in [17]. **a** Scheduled feedrate. **b** Acceleration. **c** Jerk

segment. Hence, the feedrate cannot slow down to zero with the short length. It explains that the traditional time rounding method affects the reliability of feedrate scheduling for the following curves. Meanwhile, the look-ahead algorithm based on standard S-shaped ACC/DEC algorithm cannot match the feedrate scheduling results. The acceleration profile shown in Fig. 13b obtained by [15] is also discontinuous. In addition, as shown in Table 3, both [15, 17] have larger round-off error. The machining efficiency of the proposed method is slightly lower than two other methods for the same reasons given in 5.1.2.

5.2 Experiment results

In this paper, the experiments are conducted on a three-axis engraving machine with Panasonic MBDH series servo derives and MHMD series motors. The layout of the experimental system is shown in Fig. 15. And the proposed feedrate scheduling method is implemented on a PC-based motion controller developed by our team with Intel(R) Core(TM) i5-4460 3.2GHz CPU, 4.00GB SDRAM, and Windows 7 operating system. In order to improve the real-time performance of the controller, the Kithara real-time suite (KRTS) [23], which is a modular real-time extension software for Windows range of operating systems is installed to the controller. Hence, the OS is divided into two sub-systems: the non-real-time operating system (non-RTOS) and the KRTS-Kernel, which is a real-time system. Therefore, the pre-processing task and real-time interpolation task can be implemented in one controller. The control data of each axis can be sent to the servo derives by EtherCAT.

Two NURBS curves given in 5.1 are tested in real experiments. The machining results is shown in Fig. 16. As can be seen, smooth trajectories can be obtained by the proposed feedrate scheduling method with round-off error compensation. Therefore, the feasibility and applicability of the proposed method can be validated.

6 Conclusions

In this paper, an optimized feedrate scheduling method based on S-shaped ACC/DEC algorithm is proposed with two stages namely initial feedrate scheduling and parameters calculation of round-off error compensation. In the stage of initial feedrate scheduling, a novel time rounding principle and an optimized feedrate scheduling method are proposed to obtain the motion parameters of each section, which can reduce the round-off error and ensure that the CF section is always existing. In the stage of compensation parameters calculation, the improved trapezoidal ACC/DEC algorithm which possesses continuous acceleration profile is introduced to achieve the round-off error compensation and the corresponding crucial parameters can be calculated. Therefore, the continuity of the acceleration profile after compensation can be guaranteed. Furthermore, the look-ahead strategy is also tweaked to match the proposed feedrate scheduling method. Finally, the simulations for two NURBS curves are conducted to validate the good

Table 3 Static comparison of the first 17 segments simulation results of the butterfly-shaped curve

Test methods	Round-off error	Actual maximum acceleration	Actual maximum jerk	Total interpolation time
Proposed method	0.872 mm	2241.46 mm/s^2	74,177.83 mm/s^3	4.460 s
Method in [15]	6.092 mm	2731.16 mm/s^2	609,339.94 mm/s^3	4.423 s
Method in [17]	6.092 mm	2230.36 mm/s^2	101,726.4 mm/s^3	4.423 s

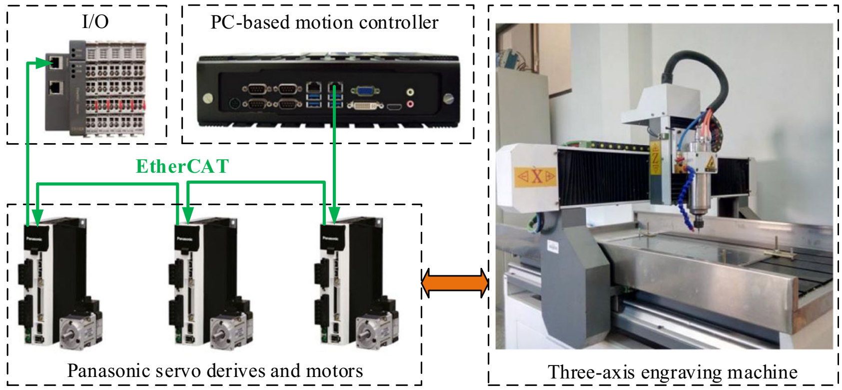

Fig. 15 Layout of the experimental system

performance of the proposed method, which brings the smallest round-off error and high reliability compared with the traditional methods. Moreover, the test curves are machined by a self-developed motion controller and a three-axis engraving machine to verify the feasibility and applicability of the proposed method.

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Appendix 1. Parameters of ∞-shaped curve

The degree:p=3.

The control point (mm):P= [(0, 0), (− 150, − 150), (− 150, 150), (0, 0), (150, − 150), (150, 150), (0, 0)].

The knot vector:U= [0, 0, 0, 0, 0.25, 0.5, 0.75, 1.0, 1.0, 1.0, 1.0].

The weight vector:W= [1.0, 0.6, 0.85, 1, 0.85, 0.6, 1.0].

Appendix 2. Parameters of butterfly-shaped curve

The degree:p=3.

The control point (mm):P=[(54.493, 52.139), (55.507, 52.139), (56.082, 49.615), (56.780, 44.971), (69.575, 51.358), (77.786, 58.573), (90.526, 67.081), (105.973, 63.801), (100.400, 47.326), (94.567, 39.913), (92.369, 30.485), (83.440, 33.757), (91.892, 28.509), (89.444, 20.393), (83.218, 15.446), (87.621, 4.830), (80.945, 9.267), (79.834, 14.535), (76.074, 8.522), (70.183, 12.550), (64.171, 16.865), (59.993, 22.122), (55.680, 36.359), (56.925, 24.995), (59.765, 19.828), (54.493, 14.940), (49.220, 19.828), (52.060, 24.994), (53.305, 36.359), (48.992, 22.122), (44.814, 16.865), (38.802, 12.551), (32.911, 8.521), (29.152, 14.535), (28.040, 9.267), (21.364, 4.830), (25.768, 15.447), (19.539, 20.391), (17.097, 28.512), (25.537, 33.750), (16.602, 30.496), (14.199, 39.803), (8.668, 47.408), (3.000, 63.794), (18.465, 67.084), (31.197, 58.572), (39.411, 51.358), (52.204, 44.971), (52.904, 49.614), (53.478, 52.139), (54.492, 52.139)].

The knot vector:U= [0, 0, 0, 0, 0.0083, 0.015, 0.0361, 0.0855, 0.1293, 0.1509, 0.1931, 0.2273, 0.2435, 0.2561, 0.2692, 0.2889, 0.3170, 0.3316, 0.3482, 0.3553, 0.3649, 0.3837, 0.4005, 0.4269, 0.4510, 0.4660, 0.4891, 0.5000, 0.5109, 0.5340, 0.5489, 0.5731, 0.5994, 0.6163, 0.6351, 0.6447, 0.6518, 0.6683, 0.6830, 0.7111, 0.7307, 0.7439, 0.7565, 0.7729, 0.8069, 0.8491, 0.8707, 0.9145, 0.9639, 0.9850, 0.9917, 1.0, 1.0, 1.0, 1.0].

The weight vector:W= [1.0, 1.0, 1.0, 1.2, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1, 2, 1.0, 1.0, 5.0, 3.0, 1.0, 1.1, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.1, 1.0, 3.0, 5.0, 1.0, 1.0, 2.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.2, 1.0, 1.0, 1.0].

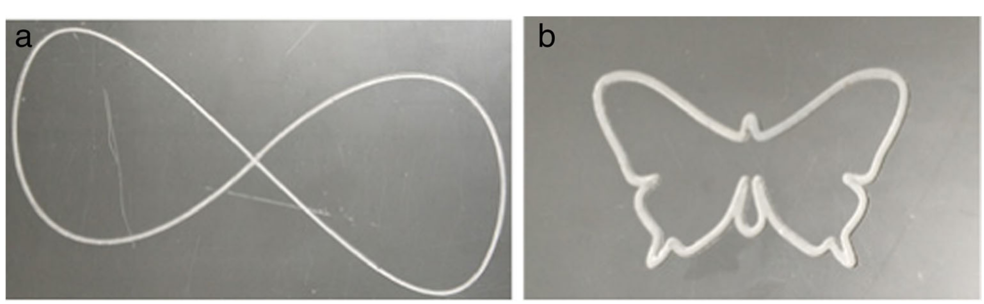

Fig. 16 Machining results of the two NURBS curves by the proposed method. **a** ∞-shaped curve. **b** Butterfly-shaped curve

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