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ORIGINAL ARTICLE

A mathematical model for the joint optimization of machining conditions and tool replacement policy with stochastic tool life in the milling process

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Abstract

One major problem in the machining process is the optimization of tool replacement policy and machining condition simultaneously. Although many studies have been developed to optimize the machining process considering the stochastic tool life, they have not optimized these two problems together. Therefore, unlike these investigations, the objective of this study is to develop an integrated mathematical model for joint-optimization of tool replacement policy and machining condition given the dependence between them and various costs in the machining process. In this paper, the dependence of cutting tool life distribution and surface roughness of workpiece to the machining conditions is modeled based on a five-step methodology initially. For this purpose, empirical data of a milling process obtained via design of experiments (DOE) based on Box-Behnken design (BBD) is used. These data, converted by total time on test (TTT), transform and using an optimization process based on golden section search (GSS), the relation between machining conditions and parameters of the tool life distribution is obtained as a full quadratic model. The R^2 values for the surface roughness, shape, and scale parameters in the full quadratic models are 89.61, 92.52, and 96.80% respectively, which confirms the adequacy of the proposed methodology. Then, a mathematical optimization model is proposed for multi-pass machining with considering costs related to tool replacement policies, direct labor costs, machining costs, loading/unloading of workpiece costs, and quality costs in a machining process. The proposed model of this study can optimize both of the tool replacement policy and the machining conditions simultaneously and also it can lead to choosing the optimized policy of the continuous or the discrete tool condition monitoring approaches. This model is implemented on a case study and its result is reported. For solving the mathematical model, the electromagnetism-like mechanism algorithm is used that has the proper performance to optimize the continuous spaces.

Keywords Machining condition · Tool replacement policies · Optimization · Weibull distribution · Electromagnetism-like mechanism

1 Introduction

Nowadays, the producers are looking for improving the quality of products and reducing the net price. The quality of a workpiece and its production costs are directly under the effect of cutting parameters. Thus, optimizing the machining conditions is an important issue. Generally, the machining conditions optimization takes place in two phases. The first phase is to model the relationship between the parameters or inputs of machining process and desired outputs of the decision maker (input-output modeling). The next phase is to determine the optimum or near the optimum conditions of the process [1].

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In this regard, some recent researches [2–7] modeled and optimized the machining conditions. Sen and Shan [8] used the hybrid NN-DF-GA approach for optimal selection of machining conditions. Yoon et al. [9] used Taguchi and RSM to do the geometric optimization. Guu et al. [10] used the Taguchi and FEM for determining the machining conditions in order to optimize the stress concentration. Liang et al. [11], using Taguchi, optimized the tool life and surface roughness. The process variables in this study were spindle speed, feed rate, and coated deposition. In these studies, the main focus was machining conditions optimization, while another factor influencing the quality and the cost of the produced products of machining process was tool wear problem. Dureja et al. [12] in a review paper indicated that there is no general model to define the cutting behavior over the various machining conditions.

In the traditional models, the tool life is assumed deterministic and Taylor equations for tool life modeling can be used. In this regard, the models are used to minimize the production cost of each workpiece or maximize the profit rate [13]. It is also assumed that the efficiency of the machining tools does not change over the time.

For factors such as machining time, cost of loading/unloading, and tool cost, this is a good approximation assumption because these factors are relatively insensitive to time. However, this is not true for qualitative factors such as surface integrity [14]. The capability of a cutting tool is reduced due to wear over the time. The quality of the workpiece may be significantly affected by such deterioration. Therefore, some other studies have considered the stochastic nature for tool life. Overall, in these investigations, the most frequent probability distribution function is the Weibull distribution. However, other distribution functions such as normal, log-normal, inverse Gaussian distribution, and Bernstein are used for tool life estimation as well. In this regards, Cao and Xu [15] presented a tool replacement model for balancing the costs of product quality, tool failure, production capacity loss, and tool replacement considering the Weibull distribution. Wang et al. [16] showed that time of tool replacement is a function of the tool reliability, which is calculated based on tool failure rate. They also used Weibull distribution for the tool life modeling. Vagnorius et al. [17] studied the tool replacement considering the tool reliability. In this research, the optimal replacement time was determined using the total time on test transform on the data obtained from the cutting tool life. In Rodriguez and Souza [18] research, the authors considered the tool replacement scheduling to increase the reliability of the machining process. In this research, the tool replacement time is a function of the tool reliability, which is calculated based on the tool failure rate. They used log-normal distribution for the cutting tool reliability modeling. The relationship between tool replacement policies and product quality was also studied by Pearn and Hsu [19]. The presented strategy by them is maintaining a certain level of process capability. In the study, process capability index was a random variable with the combination of the Chi-square and the non-central chi-square distribution. Hsu and Shu [20] developed a non-homogeneous continuous time Markov process to model the tool wear under a multi-state process. A reliability estimation approach to the cutting tools based on advanced approximation methods is proposed by Salonitis and Konstantinos [21]. Conrads and Scheffer [22] assessed the maintenance strategies for tool replacements based on the simulation methods.

In the following, a general review over the studies in the field of the optimizing variables influencing on the production process by machining and related assumptions by the researchers is presented. Some studies contained papers in which the real data in practical processes were used to model the relationship of machining conditions with the desired output of the researchers. These studies mainly used the methods such as design of experiments (DOE) and neural networks as input-output modeling. They either entirely neglected tools replacement problem or used deterministic approaches for tool life modeling [2–5, 23–25]. Some other investigations studied the optimization of the tool replacement time, assuming that the tool life has been modeled by stochastic approaches. In these studies, the machining conditions optimization has not been considered [15–22].

One important problem in the machining process optimization is the tool deterioration. The speed of deterioration also depends on the machining conditions. Machining conditions not only have a significant effect on tool life but also it affects cutting quality of the tool in its life time duration. The emphasis on the cutting quality of tool highlights tool replacement problem. Hence, to ensure the quality of a cutting process, an efficient mathematical optimization model is necessary. Therefore, some studies [26–29] have used mathematical modeling for the machining process optimization. In these studies, data-based methods such as DOE and neural networks have not been used to model the relationship between machining conditions and the desired outputs of decision maker. Also, in these studies, the deterministic approaches were used for tool life modeling. Meanwhile, Hui et al. [14] presented a complicated mathematical model for a single-pass machining, wherein an additional consideration is that the tool replacement problem has been the optimized machining conditions by assuming the stochastic feature of tool life. However, in their study, the relationship between cutting tool life distribution and machining conditions was not determined using empirical method. Also, they used the exponential distribution that is a special mode of Weibull distribution.

So far, various methods have been used to solve the presented models; meanwhile, meta-heuristic algorithms have been widely used for optimizing machining processes in different studies. Some of them are as follows:

multi-objective particle swarm optimization [3], Cuckoo search algorithm [26], differential evolution algorithm [27], particle swarm optimization [23], firefly algorithm [30], bio-geography-based algorithm [31], genetic algorithm [32], and simulated annealing [33].

Unlike other investigations, in the proposed mathematical model of this study, besides optimizing the machining conditions, the tool replacement time is optimized with regard to two policies of tool condition monitoring. To achieve this target, a mathematical optimization model for multi-pass machining is proposed in which the relationship between the machining conditions with the surface roughness and tool life is determined based on a proposed methodology. Also, the Weibull distribution has been used for tool life modeling. The model objective function minimizes the total costs of machining process as well. Presented model is solved by using electromagnetism-like mechanism algorithm (EM). This algorithm was introduced by Birbil and Feng [34].

The rest of the paper is organized as follows: Section 2 presents the problem definition in the details. In Section 3, assumptions and mathematical optimization model are presented. In Section 4, solution procedure of the presented model based on electromagnetism-like mechanism algorithm is described. Section 5 provides the description of material and experimental planning for a case study. Section 6 is focused on the result and discussion. Sensitivity analysis of some model parameters comes in Section 7 and finally, Section 8 represents the conclusions.

2 Problem definition

In this section, first a mathematical methodology is introduced which models in five steps the tool life distribution parameters that follows Weibull distribution. To this end, Weibull distribution parameters dependence to the machining conditions is identified. Then a mathematical optimization model is developed for multi-pass machining. This model considers the stochastic nature of tool life, also optimizes the tool replacement and inspection policy. This model is analyzed by two types of tool replacement policies which can be implemented either by continuous tool condition monitoring or discrete tool condition monitoring. In this regard, the quality cost of the machining process is considered in the proposed mathematical model as minimizing the deviation from the target surface roughness. In this approach, the economic evaluation regarding the machining quality, tool replacement, and inspection policy is studied, so that two sets of interrelated decisions exist; first, the machining conditions specified by the spindle speed, feed rate, and depth of cut; second, the cutting tool conditions specified by the tool replacement and inspection policy, also scheduling inspection or replacement. The parameters and variables used in this paper are presented in Table 1.

Table 1 Nomenclatures

Parameters	
a	Average extra cost incurred under the tool failure influence
r	Tool replacement cost
b	Tool inspection cost
e	Cost of machine downtime under the tool failure influence per unit time
k	Quality cost per unit deviation per unit time
h	Continuous tool condition monitoring cost per unit time
l	Loading and unloading cost per unit time
w	Direct labor cost per unit time
z	Machining cost per unit time
$C_{\max}$	Maximum allowable value for total costs of machining, direct labor and loading and unloading
D_{total}	Total amount of depth of cut requirement of the working surface
UN	Maximum spindle speed
LN	Minimum spindle speed
UF	Maximum feed rate
LF	Minimum feed rate
UM	Maximum number of pass
LM	Minimum number of pass
UT	Maximum replacement/inspection time
LT	Minimum replacement/inspection time
$R_{z(\text{target})}$	Target surface roughness
t_L	Loading and unloading time
t_{pass}	Time between two consecutive passes
L	workpiece length
D	workpiece width
Vol	Volume of metal removed from the workpiece surface
MRR	Meta removal rate
Variables	
y	$\begin{cases} 1 & \text{If tool replacement policy with continues tool condition monitoring was selected} \\ 0 & \text{If tool replacement policy with discrete tool condition monitoring was selected} \end{cases}$
N	Spindle speed
F	Feed rate
D	Depth of cut per pass
V	Replacement time interval
U	inspection time interval
n_j	$\begin{cases} 1 & \text{If pass } j \text{ is justified} \\ 0 & \text{If pass } j \text{ is rejected} \end{cases}$
$R_Z(N, F, D)$	Average surface roughness under spindle speed N, feed rate F, and depth of cut D
$\lambda(N, F, D)$	Scale parameter of Weibull distribution under spindle speed N, feed rate F, and depth of cut D
$\alpha(N, F, D)$	Shape parameter of Weibull distribution under spindle speed N, feed rate F, and depth of cut D
$f(t, N, F, D)$	Tool life probability density function under spindle speed N, feed rate F, and depth of cut D
$F(t, N, F, D)$	Tool life cumulative distribution function under spindle speed N, feed rate F, and depth of cut D
$R(t, N, F, D)$	Tool reliability function under spindle speed N, feed rate F, and depth of cut D
t_w	Cutting time
$E(S_1)$	Expected time of a cycle in the tool replacement policy with continuous tool condition monitoring
$E(C_{1T})$	Total expected cost of a cycle in the tool replacement policy with continuous tool condition monitoring
$E(S_2)$	Expected time of a cycle in the tool replacement policy with discrete tool condition monitoring
$E(C_{2T})$	Total expected cost of a cycle in the tool replacement policy with discrete tool condition monitoring

2.1 Tool life and surface roughness modeling

In this paper, instead of defining the relationship between tool life and machining conditions based on deterministic equations, the relationship between the distribution parameters of tool life and machining conditions is defined. The tool life distribution is assumed Weibull in this paper; therefore, first, the distribution parameters of the tool life should be modeled under the conditions that the tool life follows the Weibull distribution. To this end, we intend to identify the relationship between Weibull distribution parameters and machining conditions and then investigate various changes of the tool life distribution under different machining conditions. Thus, in this paper, a five-step methodology is proposed to model the relationship between Weibull distribution parameters and machining conditions. Also to identify the effects of machining conditions on the surface roughness, the obtained values for the surface roughness based on Box-Behnken design (BBD) were recorded and used for modeling.

Figure 1 shows the proposed methodology in this paper. In this methodology, tool life distribution parameters of a cutting tool which follows Weibull distribution based on changes the machining conditions are determined by combining DOE using BBD, total time on test (TTT) transform, and optimization using the golden section search (GSS) method. This methodology, in addition, considers a stochastic approach for specific machining conditions, and analyzes the effect of changes in these conditions on the tool life distribution. In the rest of this section, all steps of the proposed methodology are explained.

2.1.1 Step 1

Generally, DOE based on BBD is a common approach to determine the effect of machining conditions on tool life. BBD not only is more efficient than central composite design but also needs a fewer number of experiments [35]. In this method, the number of experiments could be calculated from Eq. 1.

$$N = 2k(k-1) + C_0 \qquad (1)$$

where N is the number of experiments, k is the number of independent variables, and C_0 is the number of central points in the experiments. In this study, the independent variables are spindle speed, feed rate, and depth of cut. In all studies conducted on using DOE in the machining process, the response variable is a measurable quantitative variable. For example, surface roughness, flank wear, machining time, tool life, and so on are response variables that are often seen in the studies on machining process. However, in this research, the response variables in addition to the surface roughness are the Weibull distribution parameters that are not simply and directly calculable from a machining experiment. In other words, in these experiments, we intend to know how the distribution parameters of tool life is affected by different conditions of machining process with certain values of spindle speed, feed rate, and depth of cut.

2.1.2 Step 2

According to the standard ISO 3685 (1993) [36], tool life is as long as it can produce a workpiece with the desired dimensions and surface roughness or it can cut the workpiece. Practically, flank wear has usually the most critical effect on the workpiece quality. Therefore, VB width is often considered as a criterion for the tool life.

With regard to the case studies in this paper, in each experiment, machining operation on a workpiece is performed, then the insert is separated from the holder, and the flank wear is measured through image of tool tip. This procedure is repeated until the end of the tool life and finalizing an experiment. In these experiments, based on the ISO 3685(1993), VB_{max} = 0.3 mm is considered as the tool life criteria [36]. Also in each experiment and after every machining process, the surface roughness of the workpiece is measured using a surface roughness tester. The average surface roughness measured in

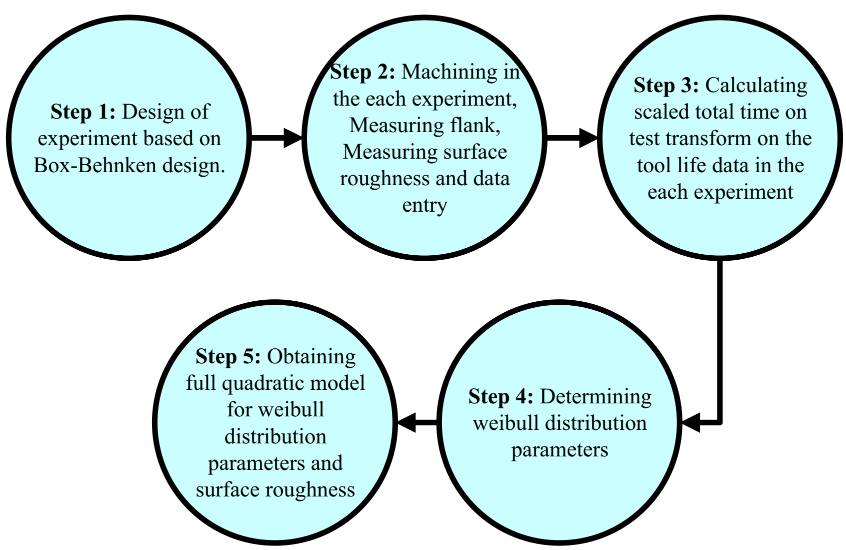


Fig. 1 Proposed methodology steps for tool life distribution parameters and surface roughness modeling

multiple replicates of a level of experiment is considered as the surface roughness of the experiment.

2.1.3 Step 3

Let us design an experiment that each time a cutting tool under the specific machining conditions completes the machining operation on a workpiece with a preset term. This experiment is repeated n times in a constant machining condition and each time, the tool life is recorded. Then the recorded data are arranged in ascending order (t_1, t_2, ..., t_n). Also assume that the tool life follows a continuous distribution with strictly increasing cumulative distribution function and the finite mathematical expectation. In such conditions, the total time on test for ith failure is defined as Eq. 2 [37]. In continuation, for scaling TTT values for ith failure, these values are divided on $T(t_n)$. Now, if a plot is drawn with x axis, including $\left({}^{i}/_{n}\ , i=1,..,n\right)$ and y axis, including $\frac{T(t_i)}{T(t_n)}$ $i=1,\dots,n$. It is called TTT plot.

$$T(t_i) = \sum_{j=1}^{i} t_j + (n{-}i)t_i \tag{2}$$

On the other hand, Rausand and Høyland [14] used TTT transform on $F(t)$ which by $h_f^{-1}(v)$ is defined as Eq. 3.

$$h_F^{-1}(v) = \int_0^{F^{-1}(v)} (1{-}F(u))du \ ; \ 0{\le}v{\le}1 \tag{3}$$

where $F(t)$ is the cumulative distribution function. Also the scaled total time on test transform on $F(t)$ was notated with $G(v)$ and defined as Eq. 4.

$$G(v) = \frac{h_f^{-1}(v)}{h_f^{-1}(1)} \quad ;for\, 0{\le}v{\le}1 \tag{4}$$

Because our focus in this paper is on the Weibull distribution, the cumulative distribution function of this distribution is replaced in Eqs. 3 and 4 and by change of variable $t=(\lambda u)^{\alpha}$ in these equations, we can rewrite them in the form of the Eqs. 5 and 6.

$$h_F^{-1}(v) = \frac{1}{\alpha\lambda}\int_0^{-\ln(1-v)} t^{\left(\frac{1}{\alpha}-1\right)} e^{-t} dt \ ; \ 0{\le}v{\le}1 \tag{5}$$

$$G(v) = \frac{\frac{1}{\alpha}\int_0^{-\ln(1-v)} t^{\left(\frac{1}{\alpha}-1\right)} e^{-t} dt}{\Gamma\left(\frac{1}{\alpha}+1\right)} \tag{6}$$

Application of the function $G(v)$ lies in the estimation of total time on test with respect to different values of v ($0 \le v \le 1$). Equation 7 defines the relationship between $G(v)$ and scaled total time on test.

$$\frac{h_f^{-1}(v)}{h_f^{-1}(1)} = \frac{T(t_i)}{T(t_n)} \tag{7}$$

Equation 7 is true for $i = 1, \dots, n$ when $v = \frac{i}{n}$. In the next section, we explain how by using convex optimization at each level of BBD we can estimate the parameters of the shape and the scale based on TTT transform and finally model the relationship between these parameters and the machining conditions.

2.1.4 Step 4

In the proposed methodology, by DOE based on BBD, the levels of machining variables, including spindle speed, feed rate, and cut depth, are determined in each experiment. Then by considering the preset replicates (n), machining is operated on the workpiece and the tool life (t_i) is recorded. Now the $\frac{T(t_i)}{T(t_n)}$ ratio can be calculated as scaled total time on test using Eq. 3. Then, the function $G(v)$, which is obtained from Eq. 6, should be fitted over the TTT-plot data such that the sum square error (SSE) of this function from $\frac{T(t_i)}{T(t_n)}$ be minimized, namely:

$$Min\, SSE = \sum_{i=1}^{n} \left(G\left(\frac{i}{n}\right) - \frac{T(t_i)}{T(t_n)} \right)^2 \tag{8}$$

Optimization of the above function involves just one variable (α), i.e., the shape parameter of the Weibull distribution for the cutting tool life. As mentioned above, the function $G(v)$ is independent of the parameter λ. Now, if *SSE* is calculated based on different values of α and the obtained points in a diagram be connected, we will have a unimodal function that Fig. 6 shows its image in the case study.

In this paper, the golden section search (GSS) method is used to optimize Eq. 8. Kiefer introduced this method to solve mathematical problems [38]. It can optimize the unimodal functions. In this paper, the target is to find a certain value of α to minimize SSE. The point is shown by α^*. For this purpose, at first the interval [$\alpha_{\min}$, $\alpha_{\max}$] is defined to find the optimum point, then based on Eqs. 10 and 11, α_1 and α_2 are calculated. In these equations, γ is the golden ratio and can be calculated by Eq. 9.

$$\gamma = \frac{-1+\sqrt{5}}{2} \tag{9}$$

$$\alpha_1 = \gamma.\alpha_{\min} + (1{-}\gamma).\alpha_{\max} \tag{10}$$

$$\alpha_2 = \gamma.\alpha_{\max} + (1{-}\gamma).\alpha_{\min} \tag{11}$$

After calculating α_1 and α_2, SSE is determined on each point and nominated them as SSE(α_1) and SSE(α_2). Now if SSE(α_1) < SSE(α_2), then α^* belongs to interval [$\alpha_{\min}$, α_2]; otherwise, α^* belongs to interval [α_1, $\alpha_{\max}$]. In this regard, in each GSS iteration, there are two search intervals, only one of them is selected for next searches; however, it is necessary that the length of these intervals be equal. The above explanation describes the first GSS iteration. In the next iterations, with updating the search intervals, values of α_1 and α_2 are calculated by using Eqs. 10 and 11. Algorithm continues in the same way until it satisfies the stop condition. The stop condition defined in GSS is met when the length of the search interval is less than ε. Figure 2 shows the pseudo-code of GSS algorithm.

After calculating the Weibull distribution shape parameter (α), to calculate scale parameter (λ), the relationship between scale parameter and mathematical expectation of the Weibull distribution is used that equals to $E(t) = \frac{1}{\lambda}\Gamma\left(\frac{1}{\lambda}+1\right)$. On the other hand, with respect to tool life data obtained at each level of BBD with n replicates, $E(t) = \frac{1}{n}\sum_{i=1}^{n} t_i$ where t_i is the lifetime of ith tool. Now, using the shape parameter obtained from GSS, the scale parameter for each level of BBD can be calculated according to Eq. 12.

$$\lambda = \frac{\Gamma\left(\frac{1}{\alpha}+1\right)}{E(t)} \tag{12}$$

2.1.5 Step 5

At this step, by implementing designed experiments in the previous stages, the relationships between Weibull distribution parameters and machining conditions could be modeled. Also, the mathematical relationship between surface roughness and machining conditions is obtained. These relationships are displayed as a full quadratic model (Eq. 13).

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1^2 + \beta_5 x_2^2 + \beta_6 x_3^2 + \beta_7 x_1 x_3 + \beta_8 x_1 x_2 + \beta_9 x_2 x_3, \tag{13}$$

where x_1, x_2, and x_3 are independent variables and Y is the response variable, also β_0 is intercept and β_1 to β_9 are model coefficients. In this research, the independent variables comprised spindle speed, feed rate, and depth of cut, and the dependent variables were surface roughness and distribution parameters of the tool life.

2.2 Tool replacement policies

A variety of scenarios for tool condition monitoring are possible. These scenarios can be implemented based on direct monitoring such as acoustic emission and tool temperature or based on indirect monitoring such as measuring qualitative characteristics of the workpiece, i.e., surface roughness. Dimla [39] in a review paper briefly categorized these methods. Overall, these scenarios to inspect the tool condition can be implemented in discrete or continuous modes. Each has advantages and disadvantages that must be considered in the machining process and the best should be selected. Therefore in this paper, two approaches for tool condition monitoring and tool replacement is proposed and in the following, we analyze them. The critical point about these approaches is that tool life distribution depends on machining conditions. In this paper according to Section 2.1, the distribution parameters of tool life are assumed to be dependent on machining

Fig. 2 Pseudo-code of golden section search algorithm

```
Parameter Setting (ε, α_min, α_max)
γ ← (−1+√5)/2                          % calculating golden section
α_1 ← γ.α_min + (1−γ).α_max
α_2 ← γ.α_max + (1−γ).α_min
While |α_max − α_min| > ε               % checking the stop condition
    if SSE(α_1) < SSE(α_2)
        α_max ← α_2
        α_2 ← α_1
        α_1 ← γ.α_min + (1−γ).α_max
        α* ← α_1          % updating α* by α1
    Else
        α_min ← α_1
        α_1 ← α_2
        α_2 ← γ.α_max + (1−γ).α_min
        α* ← α_2          % updating α* by α2
    End if
End While

Reporting output (α*)
```

conditions. This dependency is modeled by a full quadratic function. Thus, the tool distribution function in addition to time variable has other independent variables such as spindle speed, feed rate, and depth of cut.

2.2.1 Tool replacement policy with continuous tool condition monitoring

In this policy, the tool condition can be assessed continuously. Also the tool is replaced in a fixed time interval if the tool does not fail in the duration of the time interval. However, if between the time intervals the tool breaks or fail, due to continuous tool condition monitoring, the failure is identified immediately, and the tool is replaced. In this situation (failure between the replacement time interval), the costs imposed to the workpiece and the machine must be considered. As regards the costs related to any replacement policy, it must be calculated per unit time. Therefore, the replacement tool problem can be modeled as a renewal reward processes so that each cycle is defined between two consecutive tool replacements. In this model, the total expected costs in each cycle is equal to the ratio of the expected cost of a cycle to the expected time of a cycle [40]. Equation 14 calculates the expected time of a cycle in this policy($E(S_1)$).

$$E(S_1) = VR(V,N,F,D) + \int_0^V tf(t,N,F,D)dt \tag{14}$$

In this policy, three types of costs must be calculated: (1) the tool replacement cost up to the replacement time interval ($E(C_{11})$), (2) the failure cost (including workpiece and machine failure) and tool replacement cost when the tool failed during the replacement time interval ($E(C_{12})$), and (3) the continuous monitoring cost, ($E(C_{13})$). These costs can be calculated by Eqs. 15, 16, and 17.

$$E(C_{11}) = r.R(V,N,F,D) \tag{15}$$

$$E(C_{12}) = (a+r)(1-R(V,N,F,D)) \tag{16}$$

$$E(C_{13}) = h.E(S_1) = h.\left(VR(V,N,F,D) + \int_0^V tf(t,N,F,D)dt\right) \tag{17}$$

where $f(t, N, F, D)$ is tool life probability density function under spindle speed N, feed rate F, and depth of cut D; $R(V, N, F, D)$ is tool reliability function under spindle speed N, feed rate F, and depth of cut D; V is the tool replacement time interval; a is the average extra cost incurred under the tool failure influence; r is the tool replacement cost; and h is continuous tool condition monitoring cost per unit time. Therefore, the total expected cost of a cycle in this policy ($E(C_{1T})$) is calculated as Eq. 18.

$$E(C_{1T}) = \frac{(a + r)(1 - R(V,N,F,D)) + r.R(V,N,F,D) + h.\left(V.R(V,N,F,D) + \int_0^V t.f(t,N,F,D)dt\right)}{V.R(V,N,F,D) + \int_0^V t.f(t,N,F,D)dt} \tag{18}$$

2.2.2 Tool replacement policy with discrete tool condition monitoring

In this policy, the tool condition is inspected at discrete time intervals. So, if the tool fails between two inspections, the tool replacement will not occur until the next inspection, and if no failure happens between the inspections, then machining process is continued. In this policy, like the previous policy, each cycle is defined between two consecutive tool replacements (not between the two consecutive inspections), but unlike the previous policy, there is no continuous monitoring cost, instead there are inspection cost on discrete time intervals and the costs of machine downtime between tool failure and inspection. Also, there are other costs, including the average extra cost incurred under the tool failure influence and tool replacement cost between two consecutive replacements. Thus the total costs between two consecutive replacements ($E(C_{2T})$) are calculated by Eq. 19.

$$E(C_{2T}) = \frac{b.E(I) + e.E(P) + r + a}{E(S_2)} \tag{19}$$

where $b$is the inspection cost, I is number of inspections on every cycle, e is the cost of machine downtime per unit time, P is the duration of machine downtime in each cycle, and S_2 is the duration of each cycle. However, the expected number of inspections, the expected value of machine downtime, and expected time between two replacements in this policy are calculated by Eqs. 20, 21, and 22 [41].

$$E(I) = \sum_{j=1}^{\infty} j(F(jU,N,F,D)-F((j-1)U,N,F,D)) \tag{20}$$

$$E(P) = \sum_{j=1}^{\infty} \left(F(jU,N,F,D) - F((j-1)U,N,F,D)\right) \int_{(j-1)U}^{jU} (jU-t) f(t,N,F,D)dt \quad (21)$$

$$E(S_2) = \sum_{j=1}^{\infty} jU\left(F(jU,N,F,D) - F((j-1)U,N,F,D)\right) \quad (22)$$

where U is the time interval between two consecutive inspections; $f(t, N, F, D)$ is tool life probability density function under spindle speed N, feed rate F, and depth of cut D; and $F(U, N, F, D)$ is tool life cumulative distribution function under spindle speed N, feed rate F, and depth of cut D.

2.3 Costs of labor, loading/unloading, and machine tool

In the previous sections by using a proposed methodology, the relationship between the life distribution and machining condition was obtained. Then the tool replacement policies were analyzed. In this section, the relationship between machining conditions, machining time, and machining costs will be discussed. Generally, in machining processes, cutting time, denoted by t_w, is calculated by the ratio of the volume of metal removed from the workpiece surface (*Vol*) on the metal removal rate (MRR). The possibility of multi-pass machining has been assumed in this paper thus the cutting time is calculated by Eq. 23.

$$t_w = \sum_{j=1}^{UM} n_j \left(\frac{Vol}{MRR}\right) + \left(\left(\sum_{j=1}^{UM} n_j\right) - 1\right) t_{\text{pass}} \quad (23)$$

where n_j is a binary variable, which is 1 if *J*th pass are performed, otherwise 0. *UM* is the upper bound of allowable pass number and t_{pass} is the time between two consecutive passes. In continuation, we discuss the cutting time (t_w) and metal removal rate (MRR) in the presented mathematical optimization model in this research. As in the case study, the milling machine is operated on a cubic piece, and the volume of metal removed from the workpiece surface (Vol) in Eq. 23 is found from the following relation: Vol = $L \times d \times D$, where L is the workpiece length, d is cutting width (workpiece width), and D is depth of cut per pass. Also, MRR for milling processes is calculated by MRR = $d \times D \times N \times F$ [42]. By considering these values in Eq. 23, cutting time (t_w) is rewritten as Eq. 24.

$$t_w = \sum_{j=1}^{UM} n_j \left(\frac{L}{N.F}\right) + \left(\left(\sum_{j=1}^{UM} n_j\right) - 1\right) t_{\text{pass}} \quad (24)$$

The cost of loading and unloading for each workpiece equals $l.t_L$, where l is the loading and unloading cost per unit time and t_L is the loading and unloading time. Labor costs for the machining operations per workpiece is $w.t_w$ and the machining cost for each workpiece equals $z.t_w$. Therefore, the total cost of machining per unit time is calculated as $\frac{(l.t_L+(w+z).t_w)}{t_w+t_L}$.

2.4 Cost of surface roughness deviation

In this section, cost related to the quality of the workpiece is calculated by defining a target surface roughness ($R_{z(\text{target})}$), and then the deviation from this target value is considered as the deviation from the desired quality. For this purpose, the cost of deviation from target roughness is assessed using the Taguchi's quality loss function under continuous assumption [43]. Accordingly, the quality costs caused by roughness is calculated as $k.(R_Z(N, F, D) - R_{z(\text{target})})^2$; where $R_Z(N, F, D)$ is the average of "mean roughness depth" with spindle speed N, feed rate F, and depth of cut D; and k is the quality cost per unit deviation in unit time. The average cost of deviation from the target surface roughness per unit time is calculated as $\frac{k.\left(R_Z(N,F,D)-R_{z(\text{target})}\right)^2}{t_w+t_L}$.

3 Mathematical optimization model

In this section, the mathematical optimization model is presented according to the previous sections. The assumptions considered in this model would be as follows:

- Multi-pass machining on the workpiece is possible.
- At every pass, spindle speed, feed rate, and the depth of cut are constant.
- The cost and time parameters in the presented problem are definite and specific.
- The cutting tool life is modeled based on the Weibull distribution.
- Machining conditions affect the tool life distribution.
- Tools are unrepairable.
- There are two types of tool replacement policies: the continuous tool condition monitoring and discreet tool condition monitoring.

The mathematical optimization model is presented with above-mentioned assumptions so that Eq. 25 is the model objective function. It includes the costs of replacement and inspection, the direct labor costs, machining costs, the costs of loading/unloading of the workpiece, and the cost of the deviation from target surface roughness. In Eq. 25, y is a binary variable, which determines the replacement policy so that it is 1, when the tool replacement policy is continuous tool condition monitoring; and it is 0, when the tool replacement policy is discrete tool condition monitoring.

$$
\begin{aligned}
MinCost \;=\;& y\left(\frac{(A+r)(1-R(V,N,F,D))+r.R(V,N,F,D)+h.\left(VR(V,N,F,D)+\int_0^V tf(t,N,F,D)dt\right)}{V.R(V,N,F,D)+\int_0^V tf(t,N,F,D)dt}\right)\\
&+(1-y)\left(\frac{1}{\sum_{j=1}^{\infty} jU(F(jU,N,F,D)-F((j-1)U,N,F,D))}\right)\Big\{b.\sum_{j=1}^{\infty} j(F(jU,N,F,D)-F((j-1)U,N,F,D))\\
&+e.\left(\sum_{j=1}^{\infty}(F(jU,N,F,D)-F((j-1)U,N,F,D))\int_{(j-1)U}^{jU}(jU-t)f(t,N,F,D)dt\right)+r+A\Big\}\\
&+\frac{(l.t_L+(w+z).t_w)+k.\left(R_Z(N,F,D)-R_{z(\text{target})}\right)^2}{t_w+t_L}
\end{aligned}
\tag{25}
$$

The objective function 25 in general can be used for all tool life distribution functions. In this paper, the tool life distribution function is Weibull, so in Eq. 26, the reliability function is obtained from the relation $R(V,N,F,D)=e^{-(\lambda(N,F,D).V)^{\alpha(N,F,D)}}$ and probability density function is obtained from the relation $f_{N,F,D}(t)=\alpha(N,F,D).\lambda(N,F,D)(\lambda(N,F,D).t)^{\alpha(N,F,D)-1}e^{-(\lambda(N,F,D).t)^{\alpha(N,F,D)}}$, where $\alpha_{N,\,F,\,D}$ and $\lambda(N,F,D)$ are the shape and scale parameters of Weibull distribution under spindle speed N, feed rate F, and depth of cut D respectively and the five-step methodology is modeled as a full quadratic function. Equations 26 and 27 show the parametric functions related to $\alpha(N,F,D)$ and $\lambda(N,F,D)$.

$$
\begin{aligned}
\alpha(N,F,D)=\;&\beta_o^{\alpha}+\beta_1^{\alpha}.N+\beta_2^{\alpha}.F+\beta_3^{\alpha}.D+\beta_4^{\alpha}.N^2+\beta_5^{\alpha}.F^2+\beta_6^{\alpha}.D^2\\
&+\beta_7^{\alpha}.N.D+\beta_8^{\alpha}.N.F+\beta_9^{\alpha}.D.F
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
\lambda(N,F,D)=\;&\beta_o^{\lambda}+\beta_1^{\lambda}.N+\beta_2^{\lambda}.F+\beta_3^{\lambda}.D+\beta_4^{\lambda}.N^2+\beta_5^{\lambda}.F^2+\beta_6^{\lambda}.D^2\\
&+\beta_7^{\lambda}.N.D+\beta_8^{\lambda}.N.F+\beta_9^{\lambda}.D.F
\end{aligned}
\tag{27}
$$

Likewise, $R_Z(N,F,D)$ is the average of "mean roughness depth" of the workpiece under the spindle speed N, feed rate F, and depth of cut D, which is modeled by the proposed methodology as a full quadratic function. It is shown in Eq. 28.

$$
\begin{aligned}
R_Z(N,F,D)=\;&\beta_o^{r}+\beta_1^{r}.N+\beta_2^{r}.F+\beta_3^{r}.D+\beta_4^{r}.N^2+\beta_5^{r}.F^2\\
&+\beta_6^{r}.D^2+\beta_7^{r}.N.D+\beta_8^{r}.N.F+\beta_9^{r}.D.F
\end{aligned}
\tag{28}
$$

Now the mathematical optimization model is presented in Eqs. 29–44.

$$
\begin{aligned}
MinCost \;=\;& y\left(\frac{1}{Ve^{-(\lambda(N,F,D).V)^{\alpha(N,F,D)}}+\int_0^V t\alpha(N,F,D)\lambda(N,F,D)(\lambda(N,F,D).t)^{\alpha(N,F,D)-1}e^{-(\lambda(N,F,D).t)^{\alpha(N,F,D)}}dt}\right)\\
\times&\left\{\begin{array}{l}(A+r)\left(1-e^{-(\lambda(N,F,D).V)^{\alpha(N,F,D)}}\right)+r.e^{-(\lambda(N,F,D).V)^{\alpha(N,F,D)}}\\ +h.\left(Ve^{-(\lambda(N,F,D).t)^{\alpha(N,F,D)}}+\int_0^V t\alpha(N,F,D).\lambda\left(N,F,D\right)(\lambda(N,F,D).t)^{\alpha(N,F,D)-1}e^{-(\lambda(N,F,D).t)^{\alpha(N,F,D)}}dt\right)\end{array}\right\}+(1-y)\left(\frac{1}{\sum_{j=1}^{\infty} jU\left(e^{-(\lambda(N,F,D).(j-1)U)^{\alpha(N,F,D)}}-e^{-(\lambda(N,F,D).jU)^{\alpha(N,F,D)}}\right)}\right)\\
\times&\left\{b.\sum_{j=1}^{\infty} j\left(e^{-(\lambda(N,F,D).(j-1)U)^{\alpha(N,F,D)}}-e^{-(\lambda(N,F,D).jU)^{\alpha(N,F,D)}}\right)+e.\left(\begin{array}{l}\sum_{j=1}^{\infty}\left(e^{-(\lambda(N,F,D).(j-1)U)^{\alpha(N,F,D)}}-e^{-(\lambda(N,F,D).jU)^{\alpha(N,F,D)}}\right)\\ \times\int_{(j-1)U}^{jU}(jU-t)\alpha(N,F,D)\lambda(N,F,D)(\lambda(N,F,D).t)^{\alpha-1}e^{-(\lambda(N,F,D).t)^{\alpha(N,F,D)}}dt\end{array}\right)+r+A\right\}\\
+&\frac{(l.t_L+(w+z).t_w)+k.\left(R_Z(N,F,D)-R_{z(\text{target})}\right)^2}{t_w+t_L}s.t.
\end{aligned}
\tag{29}
$$

$$t_w+t_L\le T_{Avl} \tag{30}$$

$$\frac{(l.t_L+(w+z).t_w)}{t_w+t_L}\le \mathrm{C}_{\max} \tag{31}$$

$$R_Z(N,F,D)\le R_{\max} \tag{32}$$

Fig. 3 Pseudo-code of electromagnetism-like mechanism

```
Parameter setting Population size, number of iteration, LSITER, α
Lower and upper bound setting:
l1 = LN, u1 = UN; l2 = LF, u2 = UF; l3 = LT, u3 = UT; l4 = LM, u4 = UM; l5 = 0, u5 = 1;

For i = 1 to number of iteration do
    For i = 1 to Population size do
        For k = 1 to NOV        %Initialization
            s_ik ~ Uniform (l_k, u_k)
            Calculate f(s_i)
        End For
    End For
For it = 1 to number of iteration
    For i = 1 to Population size do
        C=1                     % Local search
        While counter ≤ LSITER do
            For k = 1 to NOV do
                g ~ Normal(0, 1)
                w = tanh (α.g)
                if w > 0
                    s_new,k = s_ik + w(u_k − s_ik)
                Else
                    s_new,k = s_ik + w(s_ik − l_k)
                End if
            End For
            Calculate f(s_new)
            if f(s_new) < f(s_i) then
                s_i = s_new
                C = LSITER
            End if
            C = C + 1
        End While
    End For
            .
            .
            .
            .
            .
            .
For i = 1 to Population size do
    Calculate (q_i)
    F_i = 0
End For
For i = 1 to Population size do
    For j = 1 to Population size do
        if f(s_j) < f(s_i) then      % Total force calculation
            F_i = F_i + (s_j − s_i) q_i q_j / ||s_j − s_i||^2 ; {Atraction}
        Else
            F_i = F_i − (s_j − s_i) q_i q_j / ||s_j − s_i||^2 ; {repulsion}
        End if
    End For
End For
For i = 1 to Population size do
    F_i = F_i / ||F_i||
    For k=1 to 5 do                  %Moving
        z ~ Uniform(0, 1)
        if F_ik > 0 then
            s_ik = s_ik + z.F_ik(u_k − s_ik)
        Else
            s_ik = s_ik + z.F_ik(s_ik − l_k)
        End if
    End For
    Calculate f(s_i)
End For
s_best = argmin {f(s_i), ∀i }
End For
```

$$R_Z(N,F,D) = \beta_o^r + \beta_1^r.N + \beta_2^r.F + \beta_3^r.D + \beta_4^r.N^2 + \beta_5^r.F^2 + \beta_6^r.D^2 + \beta_7^r.N.D + \beta_8^r.N.F + \beta_9^r.D.F \quad (33)$$

$$\alpha(N,F,D) = \beta_o^\alpha + \beta_1^\alpha.N + \beta_2^\alpha.F + \beta_3^\alpha.D + \beta_4^\alpha.N^2 + \beta_5^\alpha.F^2 + \beta_6^\alpha.D^2 + \beta_7^\alpha.N.D + \beta_8^\alpha.N.F + \beta_9^\alpha.D.F \quad (34)$$

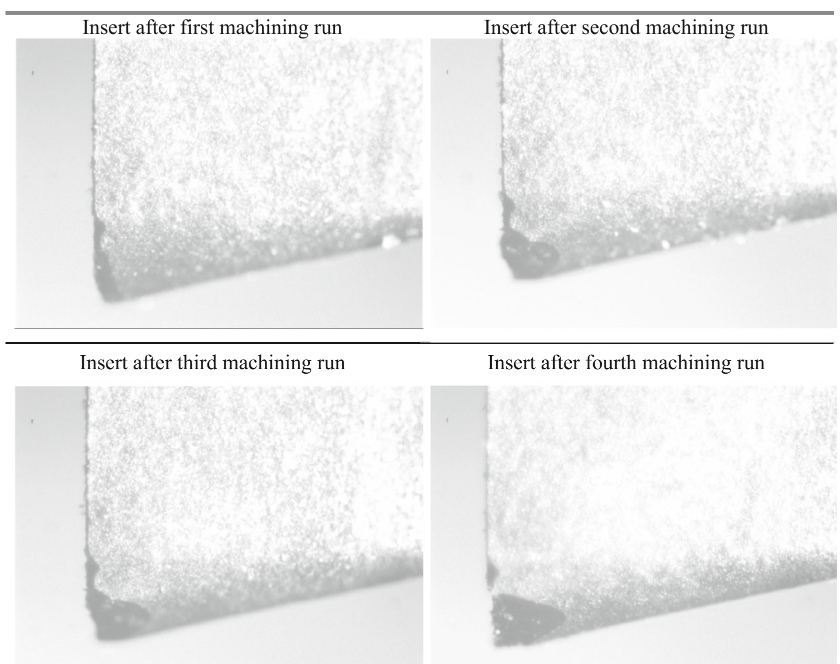


Fig. 4 Flank wear progress for the fourth failed insert in experiment number 13 (N = 1500 rpm, F = 0.2 mm/rev, D = 0.15 mm)

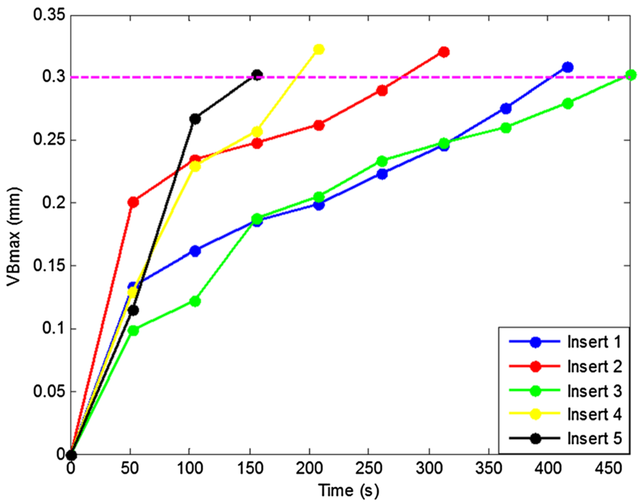


Fig. 5 Flank wear progress for five failed inserts in experiments number 13

$$\lambda(N,F,D) = \beta_o^\lambda + \beta_1^\lambda.N + \beta_2^\lambda.F + \beta_3^\lambda.D + \beta_4^\lambda.N^2 + \beta_5^\lambda.F^2 + \beta_6^\lambda.D^2 + \beta_7^\lambda.N.D + \beta_8^\lambda.N.F + \beta_9^\lambda.D.F \quad (35)$$

$$t_w = \sum_{j=1}^{UM} n_j\left(\frac{L}{N.F}\right) + \left(\left(\sum_{j=1}^{UM} n_j\right) - 1\right) t_{pass} \quad (36)$$

$$\sum_{j=1}^{UM} n_j D = D_{\text{total}} \quad (37)$$

$$n_j = 1 \qquad \forall j = 1,..,LM \quad (38)$$

$$n_{j-1} - n_j \geq 0, \qquad \forall j = 2,..,UM \quad (39)$$

$$LN \leq N \leq UN \quad (40)$$

$$LF \leq F \leq UF \quad (41)$$

$$LT \leq V \leq UT \quad (42)$$

$$LT \leq U \leq UT \quad (43)$$

$$n_j \in \{0,1\} \qquad \forall j = 1,..,UM \quad y \in \{0,1\} \quad (44)$$

In the above model, Eq. 29 is the objective function. Constraint 29 limits the maximum available time to produce a workpiece. Constraint 30 ensures that the total cost of direct labor, machining, and loading/unloading do not exceed the maximum permissible which is determined by the decision maker. Constraint 31 ensures that the surface roughness does not exceed the maximum permissible determined by the decision maker. Constraints of 32 to 34 are full quadratic functions for surface roughness and the Weibull distribution parameters obtained from five-step methodology presented in this paper, and model the relation between machining conditions and each variable. Constraint 35 is a calculative constraint which calculates the cutting time in machining process. Constraint 36 ensures that the total depth of cut in different passes of machining equals D_{total}. Constraint 37 ensures minimum required passes. Constraint 38 indicates the relation of the consecutive passes on machining process. Constraints 39 and 40 ensure that the values of spindle speed and feed rate do not exceed the minimum and maximum allowed limits. Constraints 41 and 42 guarantee that the period of inspection or replacement do not exceed the lower and upper bounds. Constraint 43 indicates that n_j and y are binary variables.

4 Solution procedure

Electromagnetism-like mechanism algorithm is applicable to solve the mathematical optimization problems. The

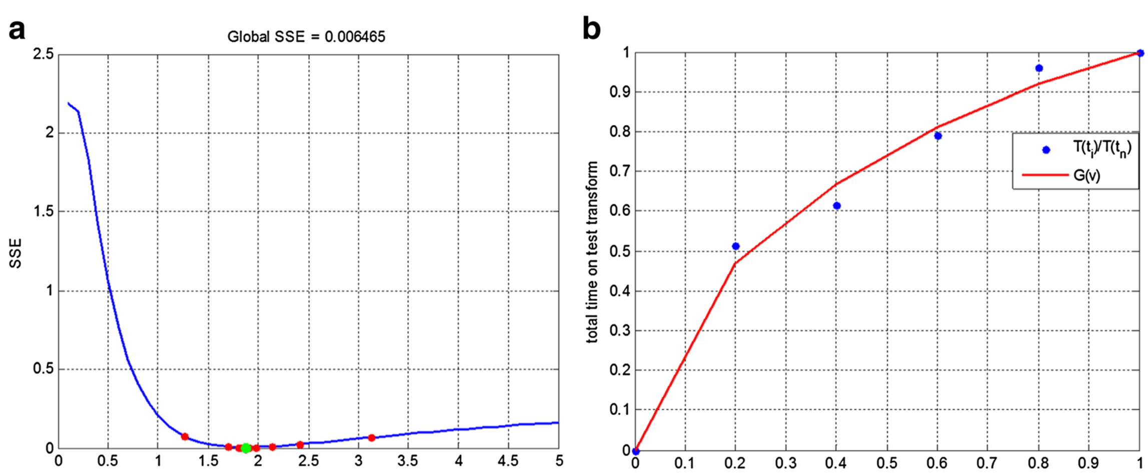


Fig. 6 **a** SSE function and the solutions in each iterations of golden section search. **b** TTT plot for five failed inserts for the experiments number 13

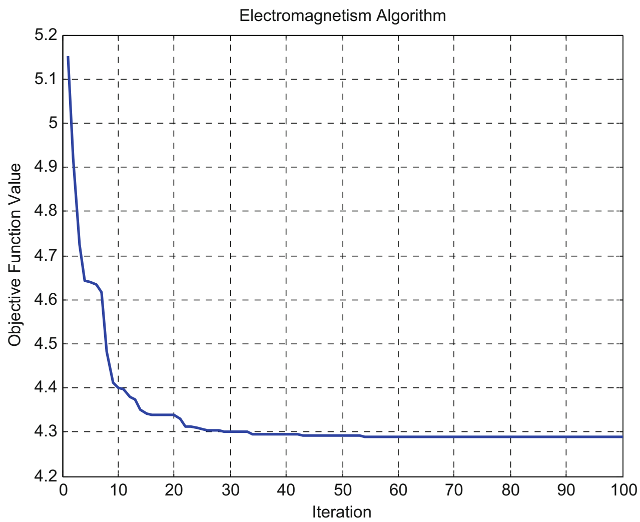


Fig. 7 Convergence diagram of electromagnetic algorithm

algorithm uses the feature of attract–repulse for charged particles to solve the problems. In this algorithm, any solution is considered as a charged particle. Now the particles with more value evaluation have higher charges and can attract other particles. Particles with less value evaluation repulse other particles. Electromagnetism-like mechanism consists of four phases as follows: (1) initialization, (2) local search to exploit local optimum, (3) total force calculation exerted on the particles, and (4) moving particles along the direction of the force.

In the first phase, according to the population size, the solutions are generated randomly. The initial values are chosen between a lower and upper limit range. Then evaluation values of solutions are calculated. In this research, every solution is encoded as a vector with five elements ($S=[N \quad F \quad T \quad n \quad y]$), in which the first to fifth elements refer to spindle speed, feed rate, tool replacement/inspection time interval, number of passes, and tool replacement policy respectively. The variables V and U are determined based on T and y in vector S, so that if the value of y in S is equal to 1, then $V=T$; otherwise, $U=T$. Other variables of the model are the variables dependent on the vector values of S. For example, D can be calculated from the relation $D=\frac{D_{\text{total}}}{n}$. In the initialization phase, elements of the vector S are generated randomly based on the continuous uniform distribution between lower and upper bounds which are determined in the mathematical model. Because n and y are binary variables, their related elements are rounded before the calculation of evaluation value. The evaluation value of each solution, ($f(s_i)$), is obtained from Eq. 45.

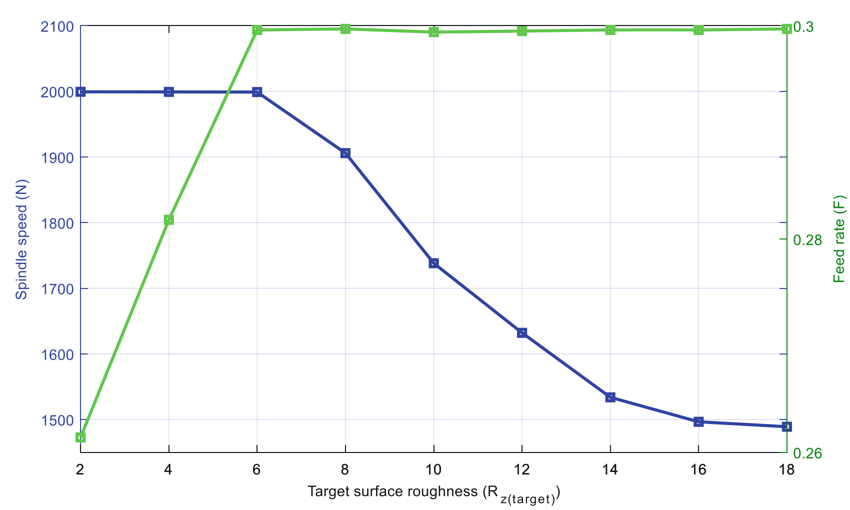
$$f(s_i) = \text{cost} \times \Bigg(1 + \max\{t_w+t_L-T_{Avl},0\}+ \max\left\{\frac{(l.t_L+(w+z).t_w)}{t_w+t_L}-C_{\max},0\right\} + \max\{R_Z(N,F,D)-R_{\max},0\}\Bigg) \tag{45}$$

where cost is calculated by Eq. 29. Also in Eq. 45, the violation from the constraints of 29, 30, and 31 is considered as penalties.

In the second phase, the local search is executed for particles to find local optimization. At this phase, we can use any local search method to increase the efficiency of the algorithm. The local search used in this study is shown in pseudo-code of algorithm EM (Fig. 3), where *tanh* is the hyperbolic tangent function used to confine g between 0 and 1.

The third phase is the total force calculation exerted on the particles. The electrostatic force between two charge points is directly proportional to the amount of charge and inversely proportional to the square of distance between charges. Charge of particle i (q_i)

Fig. 8 Effect of target surface roughness on the spindle speed and feed rate



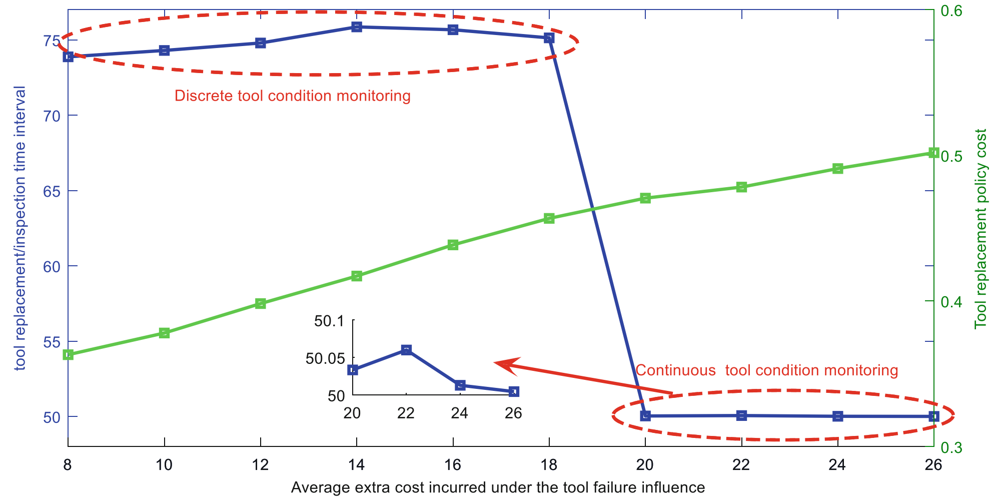


Fig. 9 Effect of a on the tool replacement policy

determines the attractive or repulsive power of ith particle. This charge is calculated through Eq. 46 as follows:

$$q_i = \exp\left(-NOV\frac{f(s_i)-f(s^{\text{best}})}{\sum_{k=1}^{P}\left(f(s_k)-f(s^{\text{best}})\right)}\right), \forall i \qquad (46)$$

where q_i is the charge of particle i and also $f(s_i)$, $f(s_k)$, and $f(s_{\text{best}})$ are the evaluation values of ith particle, kth particle, and the best solution of the population respectively. P is the population size and NOV are the number of independent variables that is five in this research. Using Eq. 46, the solutions with less objective function value have higher charges. After calculating the charge of particles, the total force exerted on particle i (F_i) is calculated by Eq. 47.

$$F_i = \sum_{j\neq i}^{P}\left\{\begin{array}{lll}(s_j-s_i)\frac{q_i q_j}{\|s_j-s_i\|^2} & if & f(s_j)<f(s_i)\\ (s_i-s_j)\frac{q_i q_j}{\|s_j-s_i\|^2} & if & f(s_j)\geq f(s_i)\end{array}\right\}, \forall i \qquad (47)$$

After calculating the total force exerted on a particle, the final phase is moving along the force direction. Based on Eq. 48, particle i moves along the force direction with a random walk length. The random walk length is assumed as z generated between 0 and 1 with the continuous uniform distribution. The force exerted on the particle is normalized.

$$s_{ik} = s_{ik} + z\frac{F_i}{\|F_i\|}(allowable\ range) \qquad (48)$$

The second to fourth phase are repeated until a stop criterion is satisfied. In this research, EM algorithm stops after a number of iterations. In continuation, Fig. 3 shows the pseudo-code of the EM algorithm.

Table 2 Machining process characteristics

Workpiece	Material: steel 304 Length (L): 260 mm Width (d): 40 mm
Machine tool	Emco-PC MILL 100
Tool	Holder: T114-D042-16 Z03 TP16 Insert: Sandvik T-PUN-16-03-04-H13A
Spindle speed (N)	Lower bound: 1000 rpm Upper bound: 2000 rpm
Feed rate (F)	Lower bound: 0.1 mm/rev Upper bound: 0.3 mm/rev
Depth of cut (D)	Lower bound: 0.1 mm Upper bound: 0.2 mm
Coolant	Air

5 Case study and experimental planning

In this paper, for implementing the proposed mathematical model, a milling process is used. This machining process is implemented on a rectangle workpiece made of steel AISI304 (surface dimension: 260 mm and 40 mm). In this study, three-edge cemented carbide inserts as the cutting tools were used. A three-axis CNC milling machine was used to do the experiments. Table 2 presents the information about the machining process.

In each experiment, after machining operation on the workpiece, the insert is separated from the holder and the flank wear is measured. In this study, the flank wear as the criteria of tool life was measured by the captured image of cutting tool using a machine vision because the vision systems are suitable

Table 3 Trend of the tool wear progress in the experiments

Experiment no.	Insert no.	Tool wear progress
1	1	0.0782, 0.1521, 0.2113, 0.2155, 0.2197, 0.2239, 0.2577, 0.3014
	2	0.1085, 0.1507, 0.1634, 0.1789, 0.1803, 0.1817, 0.2028, 0.2183, 0.2394, 0.2606, 0.2676, 0.3014
	3	0.0592, 0.0634, 0.1197, 0.2296, 0.2352, 0.2507, 0.2563, 0.2577, 0.2634, 0.2761, 0.2803, 0.2958, 0.3028
	4	0.0944, 0.131, 0.138, 0.1521, 0.1549, 0.1662, 0.1859, 0.1901, 0.1972, 0.2056, 0.2282, 0.2394, 0.2408, 0.2549, 0.2873, 0.3732
	5	0.1507, 0.2535, 0.2634, 0.2761, 0.3324
2	1	0.2972, 0.4901
	2	0.1746, 0.238, 0.3099
	3	0.1648, 0.2324, 0.293, 0.4268
	4	0.1718, 0.2521, 0.3704
	5	0.1634, 0.2225, 0.362
3	1	0.1042, 0.1085, 0.1563, 0.1606, 0.1718, 0.1901, 0.2183, 0.2352, 0.2718, 0.276, 0.2775
	2	0.0746, 0.1366, 0.4085
	3	0.0831, 0.0958, 0.1113, 0.1225, 0.138 0.1803, 0.2225, 0.2282
	4	0.0732, 0.1394, 0.1648, 0.4775
	5	0.0718, 0.1944, 0.2099, 0.2507, 0.2634, 0.269, 0.293, 0.3085
4	1	0.1761, 0.2451, 0.2789, 0.3408
	2	0.1408, 0.1775, 0.2141, 0.269, 0.3338
	3	0.1239, 0.2028, 0.2296, 0.2887, 0.3718
	4	0.1845, 0.2113, 0.4268
	5	0.1746, 0.231, 0.307
5	1	0.069, 0.0859, 0.1732, 0.2028, 0.2127, 0.2211, 0.231, 0.2338, 0.2451, 0.262, 0.269, 0.2775, 0.2901, 0.3577
	2	0.1218, 0.1479, 0.1563, 0.1662, 0.2831, 0.3549
	3	0.0887, 0.1218, 0.138, 0.1901, 0.1986, 0.2141, 0.2183, 0.2423, 0.2746, 0.2873, 0.293, 0.3028
	4	0.0873, 0.1042, 0.1549, 0.1549, 0.1676, 0.1901, 0.193, 0.1958, 0.1972, 0.2014, 0.2028, 0.2056, 0.207, 0.2606, 0.2648, 0.3606,
	5	0.0901, 0.1451, 0.1972, 0.2577, 0.2887, 0.2958, 0.3042
6	1	0.1789, 0.2408, 0.2915, 0.4141
	2	0.1972, 0.2465, 0.3408
	3	0.2056, 0.269, 0.362
	4	0.1437, 0.2239, 0.2817, 0.3563
	5	0.1437, 0.2239, 0.3197
7	1	0.0704, 0.1225, 0.2141, 0.269, 0.2958, 0.407
	2	0.1718, 0.2775, 0.3563
	3	0.1634, 0.1915, 0.2014, 0.2254, 0.2324, 0.2704, 0.2873, 0.2887, 0.3028
	4	0.1225, 0.1746, 0.2, 0.2662, 0.2718, 0.2873, 0.3183
	5	0.1225, 0.131, 0.2056, 0.2169, 0.2225, 0.2282, 0.2338, 0.2394, 0.2465, 0.2563, 0.262, 0.2803, 0.3085
8	1	0.162, 0.3676
	2	0.2324, 0.3507
	3	0.2042, 0.2901, 0.3606
	4	0.1789, 0.2986, 0.3915
	5	0.1352, 0.2254, 0.2958, 0.3901
9	1	0.1141, 0.1352, 0.1408, 0.1507, 0.162, 0.1662, 0.1775, 0.2085, 0.2127, 0.2225, 0.2282, 0.231, 0.2338, 0.2352, 0.238, 0.2606, 0.2831, 0.3
	2	0.1662, 0.2268, 0.2451, 0.2662, 0.269, 0.2789, 0.2859, 0.2901, 0.3042
	3	0.1662, 0.1817, 0.1901, 0.1944, 0.1986, 0.2042, 0.2056, 0.2831, 0.2859, 0.2887, 0.3014
	4	0.1338, 0.1423, 0.1451, 0.1563, 0.1606, 0.1732, 0.1775, 0.1873, 0.1901, 0.1986, 0.2169, 0.2718, 0.2831, 0.2859, 0.2915, 0.293, 0.3113
	5	

Table 3 (continued)

Experiment no.	Insert no.	Tool wear progress
		0.1338, 0.1465, 0.162, 0.1648, 0.1789, 0.1915, 0.1944, 0.2028, 0.2183, 0.2211, 0.2296, 0.2324, 0.2366, 0.269, 0.2761, 0.2887, 0.3282
10	1	0.1014, 0.1155, 0.1338, 0.1521, 0.162, 0.169, 0.1732, 0.1845, 0.1887, 0.1915, 0.1944, 0.2225, 0.2394, 0.2704, 0.3155
	2	0.1352, 0.1521, 0.1577, 0.1634, 0.1746, 0.1789, 0.1873, 0.1958, 0.2028, 0.2211, 0.2549, 0.269, 0.2915, 0.3085
	3	0.1169, 0.1493, 0.169, 0.2, 0.238, 0.2507, 0.2662, 0.2873, 0.2887, 0.3268
	4	0.0986, 0.1282, 0.131, 0.1507, 0.1535, 0.1577, 0.1648, 0.1746, 0.1859, 0.2169, 0.2338, 0.2746, 0.2873, 0.2958, 0.3113
	5	0.1141, 0.1282, 0.1366, 0.1535, 0.1634, 0.1775, 0.1803, 0.1817, 0.1944, 0.2, 0.2042, 0.2085, 0.2127, 0.2211, 0.238, 0.2577, 0.3282
11	1	0.1423, 0.1563, 0.1704, 0.1761, 0.1901, 0.1958, 0.2028, 0.3211
	2	0.1197, 0.1324, 0.193, 0.2028, 0.2408, 0.2606, 0.2676, 0.2732, 0.2775, 0.2789, 0.2817, 0.2831, 0.2887, 0.3479
	3	0.1155, 0.1324, 0.1451, 0.1775, 0.1986, 0.2155, 0.2183, 0.2211, 0.2225, 0.231, 0.2338, 0.2423, 0.2761, 0.2958, 0.3085
	4	0.131, 0.1366, 0.1549, 0.1746, 0.1845, 0.1944, 0.2042, 0.2127, 0.2183, 0.2225, 0.231, 0.2465, 0.2549, 0.2563, 0.2592, 0.262, 0.269, 0.3197
	5	0.1, 0.1254, 0.1352, 0.1648, 0.2113, 0.2761, 0.3366
12	1	0.2352, 0.3986
	2	0.1901, 0.2155, 0.2211, 0.2732, 0.4493
	3	0.1732, 0.2451, 0.307
	4	0.1761, 0.3183
	5	0.1887, 0.231, 0.2549, 0.3451
13	1	0.1338, 0.162, 0.1859, 0.1986, 0.2239, 0.2465, 0.2761, 0.3085
	2	0.2014, 0.2352, 0.2479, 0.262, 0.2901, 0.3211
	3	0.0986, 0.1225, 0.1873, 0.2056, 0.2338, 0.2479, 0.2606, 0.2803, 0.3028
	4	0.1296, 0.2296, 0.2577, 0.3225
	5	0.1155, 0.2676, 0.3028

Table 4 Experimental plan and results for tool life

Number of experiment	Machining conditions			Life time for failed inserts					$\overline{R}_z$
	Spindle speed (N)	Feed rate (F)	Depth of cut (D)	t_1	t_2	t_3	t_4	t_5	
1	1000	0.1	0.15	1243.00	1865.54	1965.60	2363.06	690.22	1.00200
2	2000	0.1	0.15	79.13	223.26	238.08	187.58	199.33	0.24450
3	1000	0.3	0.15	1352.00	135.25	1071.02	178.48	387.48	7.04500
4	2000	0.3	0.15	86.86	116.44	107.54	62.70	75.61	0.94867
5	1000	0.2	0.10	1025.42	408.36	913.71	1198.66	507.00	1.36719
6	2000	0.2	0.10	119.70	100.13	91.00	126.57	108.98	0.49767
7	1000	0.2	0.20	392.95	178.27	686.51	499.95	990.49	1.29078
8	2000	0.2	0.20	65.18	61.29	83.48	78.59	118.74	0.41350
9	1500	0.1	0.10	1872.00	905.02	1132.54	1703.78	1693.75	0.21700
10	1500	0.3	0.10	508.09	468.00	322.28	494.73	575.47	2.38960
11	1500	0.1	0.20	813.45	1371.85	1490.39	1831.59	665.08	0.32011
12	1500	0.3	0.20	48.41	143.94	100.08	64.87	121.33	2.54533
13	1500	0.2	0.15	402.36	276.61	461.53	189.94	151.86	0.59517

Table 5 Tool life data and TTT estimates for experiment number 13

i	t_i	$T(t_i)$	v	$\frac{T(t_i)}{T(t_n)}$
1	151.86	759.32	0.2000	0.5123
2	189.94	911.64	0.4000	0.6150
3	276.61	1171.63	0.6000	0.7904
4	402.36	1423.13	0.8000	0.9601
5	461.53	1482.30	1.0000	1.0000

for assessing the tool wear [6]. Also the workpiece surface roughness is measured by a roughness tester. This procedure continues until the end of tool life. Based on the standard ISO 3685 [36], $VB_{max} = 0.3$ mm has been considered as the tool life criterion [44].

6 Results and discussion

The experiments were designed using BBD, according to step 1 of the proposed methodology in this study (Section 2.1). The number of experiments can be calculated through Eq. 1, so that if the $k = 3$ and $C_0 = 1$, then the number of designed experiments is equal to 13. In this cubic design which is defined by a number of hypothetical points, containing midpoints of every side of a multidimensional cube, and a central point, the machining condition values are determined and based on them every experiment with preset repetition are done. In this research, five replicates are considered for each experiment. In this research, based on design of experiment method, 13 experiments which involved 5 replicates (totally 65 experiments) were performed in a milling process. The machining process was continued up to flank wear criteria ($VB_C = 0.3$, based on ISO 3685). Trend of the tool wear progress in the experiments is represented in the Table 3. Table 4 presents the machining condition values in each experiment, the tool life presented in seconds, and the average level of surface roughness obtained in each experiment was shown as well.

Figure 4 shows the flank wear progress for the fourth insert in experiment 13. Then, according to the descriptions in Section 2.1, $T(t_i)$ values must be calculated for each experiment. For this purpose, first, the data obtained from the tool life (t_i) are sorted in ascending order, then by using Eq. 2, the total time on test is calculated. Finally, these values are divided on $T(t_n)$. Table 5 presents the TTT transforms for experiment 13.

Figure 5 shows the flank wear progress for five inserts in experiment 13. The values of the machining conditions in this experiment are presented in Table 4.

The Weibull distribution parameters at any level of designed experiments should be determined. For this purpose, according to the descriptions provided in Section 2.1, the functions $G(v)$ and SSE are obtained for each experiment based on Eqs. 6 and 8. Then for minimizing SSE by using golden section search, the shape parameters of Weibull distribution for each experiment, which is designed base on BBD, can be determined. For example, Fig. 6 shows the point to point implementation process of the golden section search in experiment 13. In this figure, the red points are the found solutions in each iteration of the GSS algorithm, and the green point is the optimal point obtained out of this procedure. Figure 6b shows TTT plot for experiment 13 according to Section 2.1, in which $\frac{T(t_i)}{T(t_n)}$ values are shown with blue points and function $G(v)$ in red. After obtaining the shape parameter, the scale parameter can be found through Eq. 12.

After obtaining the values of the shape and scale parameters of Weibull distribution at each level of designed

Table 6 Shape and scale parameters and SSE value in the experimental plan

Number of experiment	Machining conditions			α	λ	SSE
	Spindle speed (N)	Feed rate (F)	Depth of cut (D)			
1	1000	0.1	0.15	1.98924	0.0005453	0.0162
2	2000	0.1	0.15	2.44646	0.0047813	0.0499
3	1000	0.3	0.15	0.89286	0.0016913	0.0357
4	2000	0.3	0.15	3.44661	0.0100082	0.0009
5	1000	0.2	0.10	1.97024	0.0010936	0.0145
6	2000	0.2	0.10	6.41528	0.0085209	0.0002
7	1000	0.2	0.20	1.50150	0.0016422	0.0052
8	2000	0.2	0.20	3.69836	0.0110792	0.0080
9	1500	0.1	0.10	2.87611	0.0006099	0.0101
10	1500	0.3	0.10	4.15211	0.0019175	0.0093
11	1500	0.1	0.20	2.12786	0.0007174	0.0105
12	1500	0.3	0.20	2.00489	0.0092573	0.0043
13	1500	0.2	0.15	1.87706	0.0029944	0.0065

Table 7 Analysis of variance for α

Source	DF	Seq SS	Adj SS	Adj MS	*F*	*P* value
Regression	9	22.9286	22.9286	2.5476	4.12	0.135
Linear	3	16.4094	16.4094	5.4698	8.85	0.053
N	1	11.6472	11.6472	11.6472	18.85	0.023
F	1	0.1396	0.1396	0.1396	0.23	0.667
D	1	4.6225	4.6225	4.6225	7.48	0.072
Square	3	3.6675	3.6675	1.2225	1.98	0.295
N × N	1	0.1081	0.4866	0.4866	0.79	0.440
F × F	1	1.0015	0.0478	0.0478	0.08	0.799
D × D	1	2.5579	2.5579	2.5579	4.14	0.135
Interaction	3	2.8517	2.8517	0.9506	1.54	0.366
N × F	1	1.0989	1.0989	1.0989	1.78	0.275
N × D	1	1.2636	1.2636	1.2636	2.05	0.248
F × D	1	0.4893	0.4893	0.4893	0.79	0.439
Residual error	3	1.8535	1.8535	0.6178		
Total	12	24.7821				
$R^2 = 92.52\%$						

experiments by BBD, it is possible to model the relationship between tool life parameters and the machining conditions with a full quadratic model. Table 6 presents the shape and scale parameters of the Weibull distribution at each level of the experiments; it also reports the values of function SSE which were found in the optimization process by using the GSS algorithm.

To investigate the impact of the machining conditions on Weibull distribution parameters and surface roughness of the workpiece, the tables of analysis of variance (ANOVA) are used. ANOVA was obtained using Minitab software in this study. ANOVA tables for the shape and scale parameters and surface roughness are separately presented in Tables 7, 8, and 9. In the tables, the degrees of freedom (DF) is the number of changeable values in a statistic and its value is found by subtracting the number of estimated parameters from the number of independent observations. In this study, there are 13 levels, so the degree of freedom will be 12. Seq SS is the sequential sums of squares and depends on the number of factors entered into the model. Adj SS is the adjusted sums of squares and it is independent of the number of factors entered into the model. Adj MS is the adjusted mean squares and

Table 8 Analysis of variance for λ

Source	DF	Seq SS	Adj SS	Adj MS	*F*	*P* value
Regression	9	0.000187	0.000187	0.000021	10.07	0.042
Linear	3	0.000155	0.000155	0.000052	25.05	0.013
N	1	0.000108	0.000108	0.000108	52.45	0.005
F	1	0.000033	0.000033	0.000033	15.95	0.028
D	1	0.000014	0.000014	0.000014	6.75	0.080
Square	3	0.000014	0.000014	0.000005	2.22	0.265
N × N	1	0.000010	0.000008	0.000008	3.84	0.145
F × F	1	0.000002	0.000001	0.000001	0.40	0.574
D × D	1	0.000001	0.000001	0.000001	0.59	0.499
Interaction	3	0.000018	0.000018	0.000006	2.95	0.199
N × F	1	0.000004	0.000004	0.000004	2.02	0.250
N × D	1	0.000001	0.000001	0.000001	0.49	0.534
F × D	1	0.000013	0.000013	0.000013	6.34	0.086
Residual error	3	0.000006	0.000006	0.000002		
Total	12	0.000193				
$R^2 = 96.80\%$						

Table 9 Analysis of variance for R^z

Source	DF	Seq SS	Adj SS	Adj MS	*F*	*P* value
Regression	9	766.513	766.513	85.168	2.87	0.208
Linear	3	505.500	505.500	168.500	5.69	0.094
N	1	186.464	186.464	186.464	6.29	0.087
F	1	317.763	317.763	317.763	10.72	0.047
D	1	1.273	1.273	1.273	0.04	0.849
Square	3	104.256	104.256	34.752	1.17	0.449
N × N	1	5.543	14.841	14.841	0.50	0.530
F × F	1	95.057	63.832	63.832	2.15	0.238
D × D	1	3.656	3.656	3.656	0.12	0.749
Interaction	3	156.757	156.757	52.252	1.76	0.326
N × F	1	154.517	154.517	154.517	5.21	0.107
N × D	1	0.186	0.186	0.186	0.01	0.942
F × D	1	2.054	2.054	2.054	0.07	0.809
Residual error	3	88.902	88.902	29.634		
Total	12	855.415				
R^2 = 89.61%						

found from dividing Adj SS on degree of freedom. *F* is found by dividing Adj MS for each factor (in any row of the table) on mean square error (MSE). *P* value represents the significance of a factor.

In Table 7, R^2 value for the shape parameter is 92.52%, representing a good correlation between the quadratic model and data obtained from the experiments. Table 10 presents the model obtained for the shape parameter of the Weibull distribution (Eq. 49). As shown in the last column of Table 7, *p*-values for *N* and *D* are the lowest compared to other factors indicating the high impact of spindle speed and depth of cut on the shape parameter of the Weibull distribution. Out of these two, spindle speed with a *p* value of 0.023 has more impact than the depth of cut with a *p* value of 0.072. Also considering Eq. 48, *N* and *D* coefficients are negative numbers indicating an inverse relationship between these machining variables and shape parameter of the Weibull distribution. In Table 8, R^2 value for the scale parameter is 96.80% that represents a good correlation between the quadratic model and data obtained from the experiments. The model obtained for the scale parameter of the Weibull distribution is presented in Table 10 (Eq. 50). According to Table 8, spindle speed with a *p* value of 0.005 has the highest impact on the scale parameter. The high impact of speed spindle or cutting speed on the tool life has been reported and confirmed in other research that considered tool life as a deterministic variable. However, the effects of feed rate with a *p* value of 0.028 and depth of cut with a *p* value of 0.080 on the scale parameter are significant. In addition among the interaction factors, $F \times D$ with a *p* value of 0.086 has a significant impact on the scale parameter. With regard to the negative coefficients of *N*, *F*, and *D* in Eq. 50, spindle speed, feed rate, and depth of cut have inverse relationships with scale parameter of the Weibull distribution.

`The model obtained for the surface roughness value is presented in Table 10 (Eq. 51). The obtained results of the surface roughness show a good correlation between quadratic model for the surface roughness and the data obtained from the experiments as the R^2 value with 89.61% represents the fact. Also according to Table 9, feed rate with a *p* value of 0.047 has the most effect on the surface roughness. The impact of spindle speed with a *p* value of 0.087 on the surface roughness is also significant.

Other related parameters of the proposed mathematical optimization model are presented in Table 11.

Table 10 Regression models for parameters of Weibull distribution and surface roughness

$$\alpha(N,F,D) = 9.35623-0.00184806 \times N + 1.87674 \times F-94.4347 \times D + 1.84567e-006 \times N \times N -14.4686 \times F \times F + 423.147 \times D \times D-0.0224818 \times N \times D + 0.0104826 \times N \times F -69.9483 \times D \times F \quad (49)$$

$$\lambda(N,F,D) = 0.0263548-2.20641e-005 \times N-0.0406462 \times F-0.163598 \times D + 7.44103e-009 \times N \times N -0.0598173 \times F \times F + 0.291717 \times D \times D + 2.00967e-005 \times N \times D + 2.0404e-005 \times N \times F +0.361617 \times D \times F \quad (50)$$

$$R_Z(N,F,D) = 5.47529-0.0166661 \times N + 16.5991 \times F + 118.138 \times D + 1.01925e-005 \times N \times N +528.455 \times F \times F-505.883 \times D \times D + 0.00862569 \times N \times D-0.124305 \times N \times F +143.335 \times D \times F \quad (51)$$

Table 11 Parameters of mathematical model for the numerical example

Parameter	Value	Parameter	Value
a	8	C_{max}	10
r	5	D_{total}	0.4
h	0.25	LM	2
b	5	UM	4
e	1	t_{pass}	10
l	0.1	t_L	20
w	0.1	T_{avl}	100
z	5	$R_{z(target)}$	8
k	2	R_{max}	20

To solve the problem based on the provided information, the electromagnetic-like mechanism algorithm was used so that the algorithm parameters included population size, number of iterations, LSITER, and α, which were substituted by 10, 100, 5, and 0.05, respectively. Figure 7 shows the convergence diagram of the EM algorithm for solving the example. Also Table 12 presents all variable values of the mathematical model. According to the results, after 100 iterations, the algorithm finally converged to 4.2885 for the objective function. The machining process was performed in two passes and in each pass, the depth of cut was 0.2 mm. Furthermore, as is shown in Table 12 in this example, the tool replacement policy with discrete tool condition monitoring has been selected as the tool replacement policy in which the tool is inspected after 73.8706 time units successively.

In the proposed methodology, in addition of modeling the workpiece surface roughness based on machining conditions, the relationship between Weibull distribution parameters to machining conditions is determined as a full quadratic model. Then, a mathematical optimization model with consideration of the various machining process costs and output functions of the proposed methodology is developed which evaluates the machining process economically. In this mathematical model, one of two approaches (continuous tool condition monitoring and discrete tool condition monitoring) for replacement and inspection of cutting tool can be chosen based on a binary variable. The proposed methodology can optimize the tool replacement policy and the machining conditions simultaneously. The five-step methodology based on the obtained results of Section 6 is valid.

Table 12 Solution of mathematical model for the numerical example

Total cost	4.2885
Tool replacement policy cost	0.3631
Costs of labor, loading/unloading, and machine	3.9182
Cost of surface roughness deviation	0.0072
N	1905.90
F	0.2997
D	0.2
N	2
Y	0
V	0
U	73.8706
$\alpha(N,F,D)$	3.0655
$\lambda(N,F,D)$	0.0137
$R_z(N, F, D)$	7.4478
MRR	4569.96
t_w	64.6176

7 Sensitivity Analysis

In this section, the impact of the proposed mathematical model parameters on the problem variables is discussed. In this regard, first, the relationship between the target surface roughness with spindle speed and feed rate is reviewed using the model. As shown in Fig. 8, if the target is decrease of surface roughness value ($R_{z(target)}$), when the feed rate is fix (0.3 mm/rev), it is essential to increase the spindle speed which indicates the inverse effect of spindle speed to surface roughness. Also, if the target is decreasing the surface roughness value ($R_{z(target)}$), when the feed rate is not fix (less than 0.3 mm/rev), the spindle speed should be kept fix (2000 rpm). Therefore, to achieve a lower surface roughness value, the feed rate should be decreased. This results about of effect of spindle speed and feed rate on the surface roughness are supported by Kumar et al. [45] that is confirmed the validity of proposed model in this study.

Figure 9 shows the effect of increase of average extra cost incurred under the tool failure influence (a) on the tool replacement policy. So that the blue line represents tool replacement/inspection time interval and green line represents the tool replacement policy cost. As Fig. 9 shows, for values of a between 8 and 18, the discrete tool condition monitoring policy and for the value of a more than 18, the continuous tool condition monitoring policy should be selected. This indicates that when the average extra cost incurred under the tool failure influence increases, the tendency to use continuous tool condition monitoring despite continuous tool condition monitoring cost is increased. In Fig. 9, a values between 20 and 26 is highlighted separately by using a different scale to get a better view of a impact on the tool replacement/inspection time interval.

8 Conclusions

There are some studies that developed the policies to optimize the tool replacement time using the Weibull distribution. The machining conditions in these studies are supposed to be fixed. However, in the metal cutting industry, the machining condition is changed to do the suitable machining process. Therefore, this study aims to develop a mathematical model to optimize the tool replacement policy and the machining conditions simultaneously. On the other hand, the studies in the field of machining process optimization have focused on modeling the dependence of the outputs and the inputs of the machining process (input-output modeling) or optimization of the machining conditions by presenting mathematical models. The combination of these two approaches has some advantages that are discussed in this paper. Its important advantage is joint-optimization of tool replacement policy and machining conditions considering dependency of tool life distribution to the machining conditions. Therefore, in this research, a five-step methodology was introduced to satisfy two targets. The first target is estimation of Weibull distribution parameters at a particular machining condition. The second target is identification of the effect of machining conditions on the tool life distribution. For this purpose, in the proposed methodology in addition of modeling the workpiece surface roughness based on machining conditions, the relationship between Weibull distribution parameters to machining conditions is determined as a full quadratic models. The results show that the values of R^2 for the shape and the scale parameters in the quadratic models are 92.52 and 96.80% respectively. Also based on the proposed methodology, R^2 value in the full quadratic model of surface roughness is 89.61%, which shows the accuracy of this model. Therefore, there is the appropriate correlation between the Weibull distribution parameters and the obtained data from cutting tool life under the various machining conditions in the full quadratic models using the proposed methodology. Also, in this study, a mathematical optimization model is developed based on the various machining process costs and the above full quadratic functions as the output of the proposed methodology. The mathematical optimization model is used to evaluate the machining process economically. In this mathematical model, one of two methods (continuous or discrete tool condition monitoring) for inspection of the cutting tool can be chosen based on a binary variable. According to the results obtained for the case study presented in this paper, for low values of average extra cost incurred under the tool failure influence (a), the discrete tool condition monitoring policy was suggested and by increasing the a value, the policy based on the continuous tool condition monitoring was suggested. The sensitivity analysis of variables using the proposed model indicates the inverse effect of spindle speed to surface roughness, and also clarifies that decreasing the surface roughness needs to decrease the feed rate proportionally. This finding is supported by other investigators that confirmed the validity of the proposed mathematical model of this study. The proposed model is a mix integer non-linear programming in a non-convex space. Thus, the electromagnetic-like mechanism algorithm is used to solve the proposed model. Finally, a CNC milling process is used as a case study, and the implementation results of the five-step methodology and the proposed mathematical optimization model were reported. Developing the single-objective mathematical model of this study to multi-objective model could be considered in the future study. Also, presenting a similar study for multi-stage machining could be an attractive investigation in the future study.

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