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ORIGINAL ARTICLE

Bi-objective optimization for tolerance allocation in an interchangeable assembly under diverse manufacturing environment

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Abstract This paper presents bi-criteria formulation of a tolerance allocation model for an interchangeable assembly to simultaneously evolve suitable combination of manufacturing facility in multiple facility shaft-hole production environments of medium- and large-scale industries and tolerances to complement the need of customers. An Exhaustive Search Procedure is used to obtain the optimal solution for small and medium size problems and simulated annealing algorithm is used for large size problems. The usefulness of the Pareto front in manufacturing tolerance allocation decisions is demonstrated with three case study problems. The effect of process capability of shaft-hole assembly manufactured from alternative manufacturing machines and the optimality is analyzed in three cases to understand their criticality in decision-making. The models discussed in this paper could be useful for medium- and large-scale manufacturing industries, where there will be a variety of manufacturing facilities (specifications, capabilities, models, and types) for making both shaft-hole assembly and play a key role to meet the tolerance and cost requirements of different customers. This paper further discusses how this formulation and methodologies can be used for two hole and two shaft assemblies and multiple shaft-hole assemblies. Finally, the paper ends with highlighting directions of future research avenues in the shaft-hole assembly.

Keywords Tolerance allocation · Shaft-hole assembly · Quality loss · Positional tolerance · Pareto front · Machine allocation

1 Introduction

In a manufacturing environment, tolerances are specified to control the actual dimensions of processed features within allowable zones for the product functional requirements and manufacturing costs [1]. The functional performance and manufacturing cost of an assembly are directly influenced by its assembly clearance and part tolerances [2, 3]. Tolerance allocation has been performed early in the product development cycle before the parts are produced. In general, a large tolerance makes parts cheaper and is easy to produce. However, it leads to performance degradation (i.e., quality) and more number of rejections. The customer always wants a quality product with minimum cost. The allocation of part tolerance is a crucial factor in the assembly, as it directly affects both cost and quality. Hence, both the cost and quality of a product need to be given parallel importance. As widely recognized by both engineering and manufacturing community, the necessity to specify a reasonable magnitude of the dimensional tolerance of mating parts has now become an importance issue [4]. From the above discussion, it is evident that satisfying the tolerance requirements of the customer in terms of quality and cost is a challenge to the manufacturers. Medium- and large-scale industries have multiple machines to produce parts with various process capabilities. The assembly tolerance is typically based on performance requirements, whereas the part tolerances are closely related to the capabilities of the manufacturing machines. As the process capability of a manufacturing facility influences the cost and quality of an assembly, the selection of a particular manufacturing

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facility is an important consideration in tolerance allocation problems under the manufacturing environments.

This paper proposes a model to simultaneously identify suitable manufacturing processes and tolerances for shaft-hole assemblies. In a shaft-hole assembly, a solid cylindrical member (Shaft) either rotates or reciprocates inside another hollow cylindrical member (Hole). This pair is termed as a shaft-hole assembly in engineering and plays a vital role in many engineering applications such as automobiles, aircraft, and press tools.

In general, the objectives of the tolerance allocation models in shaft-hole assemblies are minimum manufacturing cost of the assembly, minimum clearance variation (an attribute for quality of the assembly), and minimum total cost of assembly. The tolerance allocation approaches can be grouped into four categories as (i) methods focusing on manufacturing, (ii) methods focusing on quality, (iii) methods focusing on manufacturing and quality/total cost, and (iv) methods focusing on non-cost parameters.

The following review investigates how these methodologies have been adopted to allocate tolerance to the parts in an assembly.

1. Methods focusing on manufacturing

Speckhart [5] has proposed an analytical model to find an optimal combination of dimensional tolerances for an assembly. Spotts [6] has allocated tolerance for each part in an assembly by assuming manufacturing cost which varies inversely with the square of the tolerances and thus developed a nonlinear equation to determine the optimal tolerance. Chase et al. [7] have considered more than one manufacturing facility to each of the three parts of an assembly to determine the tolerances of the parts and the manufacturing cost of each manufacturing process and identified the better process for each part based on the assembly tolerance to minimize the manufacturing cost.

Chen and Fischer [8] proposed a model to allocate tolerance of a part with nonlinear constraints and an exponential objective function. Similar model also investigated by Ganesan et al. [9]. The best combination of the process was selected among alternative processes to minimize manufacturing cost. Singh et al. [10] dealt with the tolerance synthesis problem with alternative manufacturing processes to minimize the total cost (setup cost + manufacturing cost) and suitable processes were identified from various available processes for a minimum total cost.

2. Methods focusing on quality

Evans [11] discussed the tolerance allocation method considering the rejection rate and its salvage (cost) value using the statistical approach for assigning part tolerances and the rate of rejection is determined by the normal distribution. Asha et al. [12] and Jeevanantham and Kannan [13] proposed a method to minimize clearance variation using the normal distribution to find the best combination of the groups for an assembly. Cheng and Tsai [14] proposed a comprehensive method for optimal tolerance allocation using Lagrange multiplier for minimum manufacturing cost.

3. Methods focusing on manufacturing and quality

Ye and Salustri [15] developed simultaneous tolerance synthesis method to minimize the total cost (manufacturing cost + quality loss cost) of the assembly (piston and cylinder). Here, the manufacturing cost is a function of process tolerance and quality loss is a function of design tolerance. Rao and Rao [16] described a method of synthesizing tolerances simultaneously for manufacturing cost and quality for a simple linear assembly using two nontraditional approaches and concluded that this method is suited where either high-quality or low-cost products are designed and manufactured. Kumar et al. [17] dealt with tolerance allocation for piston cylinder assembly considering manufacturing cost and quality. Muthu et al. [18] described a nonlinear integral model for tolerance allocation to minimize the total manufacturing cost and quality loss considering alternate processes, and Taguchi quality loss function was used to find the quality loss.

Singh et al. [19] considered more than one manufacturing facility to each of the three parts of an assembly tolerance allocation problem to determine the tolerances of the parts. Sivakumar et al. [20] proposed a tolerance allocation model with the objective functions of tolerance stack up, total manufacturing cost, and quality loss cost. Jawahar et al. [21] developed a bi-objective formulation of a tolerance allocation model for an interchangeable assembly of pin and hole assembly with two objectives which are minimum total cost of an assembly and minimum clearance variation.

4. Methods focusing on non-cost parameters

Swift et al. [22] described knowledge-based statistical approach method for tolerance allocation considering manufacturing risk and assembly risk. Delany and Phelan [23] described a method to find the most suitable process to manufacturing a product based on the process capability in the initial design stage in terms of functional reliability after manufacturing a product.

The salient observations of the different approaches are as follows:

- Manufacturing cost-based approaches addresses the selection of the most preferred process from various alternative processes or machines with varying process capabilities/ manufacturing cost to meet the required tolerances.

- Clearance variation is considered as the objective criterion in quality cost approaches. However, this approach is widely adopted in selective assembly. In interchangeable assembly, it is often addressed as a constraint rather than an objective.
- Though the recent research considers manufacturing cost and quality loss cost simultaneously, very few researchers have concentrated on rejection of the assembly.
- It is generally accepted that the two primary objectives for the allocation of tolerance among the parts of the shaft-hole assembly are minimum clearance variation and least cost, and process capabilities become the principal constraints.

The observations of the previous tolerance allocation models reveal that there is a need for a general tolerance allocation models connecting cost and quality to assist the manufacturer for choosing the proper manufacturing facility and fixing the manufacturing tolerances according to varying needs of the customers in terms of quality and cost. On the above considerations, recently, Jawahar et al. [21] developed a bi-objective formulation of a tolerance allocation model for an interchangeable assembly of pin and hole assembly. In their model, the total cost (TC) is expressed as the summation of the cost of rejections of pin and hole on the assumption that the manufacturing variations follow the normal distribution and quality loss cost using the Taguchi nominal the best quality loss function. Their model is limited to a set of a manufacturing facility, one for the pin and one for the hole, a typical case for small scale industries and neglects the surplus parts that may arise due to variation, which may occur when the number of rejections of pin and hole is different. However, medium- and large-scale industries will have the variety of facilities (different capabilities, models, types) for both shaft and hole manufacturing.

Considering the above points, this paper presents bi-criteria formulation of a tolerance allocation model for an interchangeable assembly to simultaneously evolve suitable combination of a manufacturing facility in multiple facility shaft-hole production environments of medium- and large-scale industries and tolerances to complement the need of customers. The proposed multiple manufacturing environment model of this paper considers the minimum total cost of assembly and minimum clearance variation as the two objectives and is formulated by extending the bi-criteria tolerance allocation model developed for single manufacturing facility for shaft-hole assemblies by Jawahar et al. [21] to multiple manufacturing facility and surplus part considerations. The formulation belongs to multi-objective nonlinear program with exponents in its objective function. Heuristics are suggested to solve this type of problems. This article proposes two methodologies such as Enumerative Search Procedure (ESP) and Simulated Annealing Algorithm (SAA) to obtain optimal Pareto front solutions, i.e., the best combinations of machines and tolerances, for minimum total cost and clearance variation. Besides the model extension, this paper further discusses how this formulation and methodologies can be used for multiple shaft-hole assemblies.

The rest of the paper is organized as follows: Section 2 describes the tolerance allocation problem, Section 3 presents the bi-objective tolerance allocation model formulation for multiple manufacturing scenarios, Section 4 delineates the procedural steps adopted ESP with a numerical illustration, Section 5 illustrates the SAA with an example, and Section 6 discusses (i) the effect of process capability of shaft and hole manufacturing machines, (ii) importance of simultaneous allocation of machine and tolerance in meeting customer requirements on cost and quality in decision-making, and (iii) the application of the proposed model in multiple shaft-hole assemblies. The concluding Section 7 describes how this model is valuable to medium- and large-scale industries.

2 Problem description

This section presents the multiple facility production environments, objectives, and decision parameters of the proposed bi-objective tolerance allocation model.

The manufacturing center has N_s machines for making the shaft and N_h machines for making the hole facilitate the design specification of fit C_d as per the requirements of the customer. Let i (i = 1 to N_s) and j (j = 1 to N_h) the identifiers for shaft and hole manufacturing machines, respectively, and their corresponding per unit manufacturing cost are m_i and m_j. The manufacturer follows a policy that is the entire order of a customer or lot will be done in one shaft manufacturing machine and one hole manufacturing machine (i.e., no splitting of lots).

The standard deviation (σ_i) is based on the chosen shaft manufacturing machine is i and the standard deviation (σ_j) is based on the chosen hole manufacturing machine is j. The two extremes from the mean μ_s at $\pm 3\sigma_i$ become the maximum (U^i) and minimum (L^i) production limits to manufacturing shaft in machine i and the two extremes from mean μ_h' at $\pm 3\sigma_j$ become the maximum (U^j) and minimum (L^j) production limits to manufacturing hole in machine j. The upper tolerance limit $\left(t_i^u\right)$ and the lower tolerance limit $\left(t_i^l\right)$ of the shaft should lie between U^i and L^i, and the upper tolerance limit $\left(t_j^u\right)$ and the lower tolerance limit $\left(t_j^l\right)$ of the hole should lie between U^j and L^j.

Under the above-delineated production environment, the production center needs to meet the customer requirement C_d with minimum deviation and less cost so as to be competitive in the market and get a hold of more profit. On these considerations, the objectives of the problem are set as follows:

- Minimization of the TC of shaft-hole assembly which includes quality loss cost, rejection cost of assembly (including rejections due to surplus parts), and manufacturing cost
- Minimization of the clearance variation (C_v).

Both the objectives depend on the machines i and j chosen and the tolerance limits allocated on those machines (i.e., $t_i^u, t_i^l, t_j^u, t_j^l$). Hence, the choice of facilities becomes an additional decision variable apart from tolerance allocation decisions. The two binary decision variables b_i and b_j indicate the choice of machine i and machine j for shaft and hole, respectively, and the problem can be stated as:

Determination of optimal Pareto front solution set for minimum total cost and minimum clearance variation of a shaft and hole assembly with decision variables of choices of machine **i** and machine **j** to manufacture shaft and hole as denoted by b_i' and $'b_j'$, respectively, and their tolerance limits ($t_i^l, t_i^u, t_j^l, t_j^u$) with the following known data: design specification of fit C_d;, number of machines available for shaft (N_s) and hole (N_h), process capability of the diverse machines defined with their standard deviations of shaft (σ_i ∀i)) and hole(σ_j ∀j), mean diameter of the shaft μ_s;$'$, manufacturing cost of shaft (mc_i ∀ i), and manufacturing cost of hole (mc_j ∀ j).

3 Mathematical model

The mathematical formulation employs the following indices, notations, and decision variables.

Indices

i	Shaft manufacturing machine identifier (i = 1 to N_s)
j	Hole manufacturing machine identifier (j = 1 to N_h)
N_s	Numbers of machines available for shaft manufacturing
N_h	Numbers of machines available for hole manufacturing

Notations

A	Manufacturing cost of an assembly
b_i	A binary variable that indicate the choice of the shaft manufacturing machine
b_j	A binary variable that indicate the choice of the hole manufacturing machine
C_a	Average clearance of an assembly
C_d	Design specification fit (clearance/interference)
C_v	Clearance variation of an assembly
F_i	Fraction of shaft accepted
F_j	Fraction of hole accepted
M_h	Maximum material condition of the hole
M_s	Maximum material condition of the shaft
mc_i	Manufacturing cost of the shaft in ith machine
mc_j	Manufacturing cost of the hole in jth machine
QLC	Quality loss cost of an assembly
r_i	Cost for fraction of defective of shaft manufacturing in ith machine
r_j	Cost for fraction of defective of hole manufacturing in jth machine
RC	Rejection cost of set of shaft-hole assembly
t_i^l	Lower tolerance limit of the shaft in ith machine
t_i^u	Upper tolerance limit of the shaft in ith machine
t_j^l	Lower tolerance limit of the hole in jth machine
t_j^u	Upper tolerance limit of the hole in jth machine
t_p	Position tolerance value
TC	Total cost of an assembly
U^i	Maximum size of shaft at + 3σ$_i$ limit in ith machine
L^i	Minimum size of shaft at − $3\sigma_i$ limit in ith machine
U^j	Maximum size of hole at + $3\sigma_j$ limit in jth machine
L^j	Minimum size of hole at − $3\sigma_j$ limit in jth machine
σ_i	Standard deviation of the shaft in ith machine
σ_j	Standard deviation of the hole in jth machine
μ_s	Mean dimension of shaft
μ_h	Mean dimension of hole
Δx	Precision level of shaft manufacturing machine
Δy	Precision level of hole manufacturing machine
휈	Binary control parameter used to find number of rejections of assembly

Objective function 1 (minimization of the TC of shaft and hole assembly):

$$Min\ TC = (A)\{1-[\delta(F_i)+(1-\delta)(F_j)]\} + \left\{\frac{A}{(\Delta)^2}(C-C_d)^2\right\} + (A) \quad (1)$$

First termSecond termThird term

Objective function 2 (minimization of the C_v

$$Min\ C_v = max\left\{mod\left(\left[\sum_{i=1}^{N_s} t_i^u - t_i^l\right] - C_d\right), mod\left(\left[\sum_{j=1}^{N_h} t_j^u - t_j^l\right] - C_d\right)\right\} \quad (2)$$

Subject to

$$\sum_{i=1}^{N_s} b_i = 1 \quad (3)$$

$$\sum_{j=1}^{N_h} b_j = 1 \quad (4)$$

Where,

$$t_i^l > L^i = \mu_s - 3\sigma_i \forall i \quad (5)$$

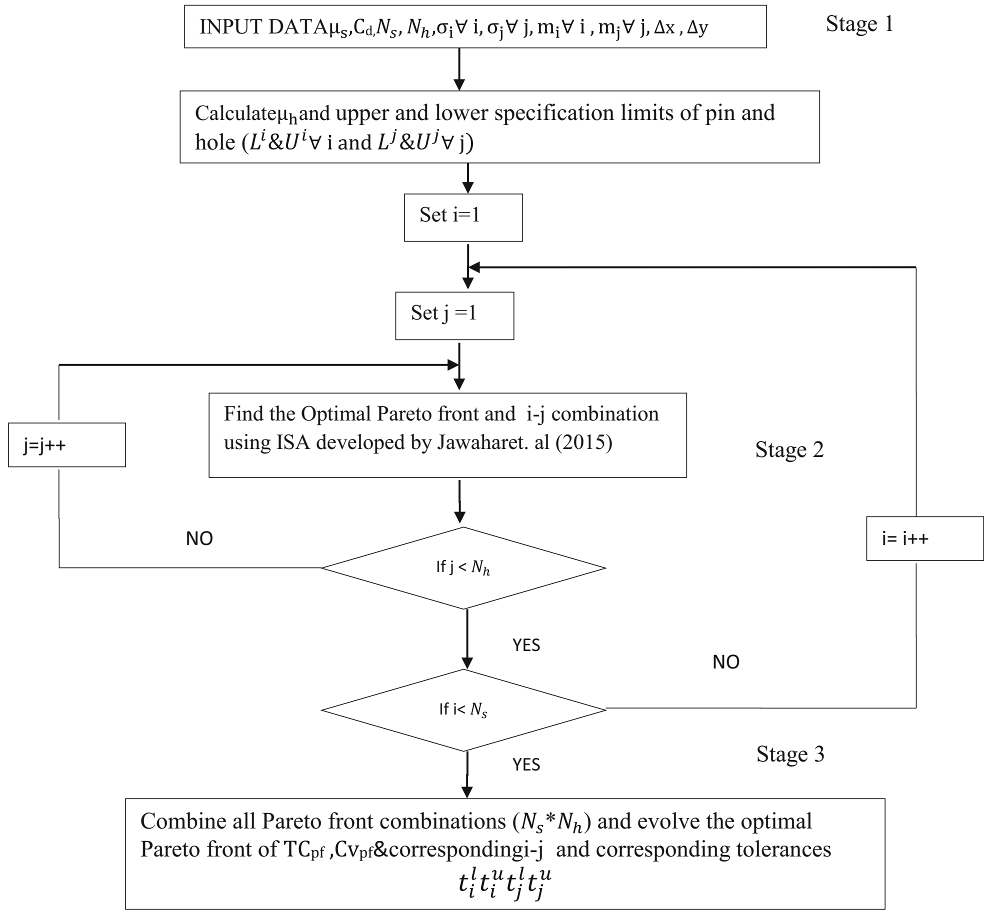
$$t_i^u < U^i = \mu_s + 3\sigma_i \forall i \quad (6)$$

$$t_i^l < t_i^u \forall i \quad (7)$$

$$t_j^l > L^j = \mu_h - 3\sigma_j \forall \mathrm{j} \quad (8)$$

$$t_j^u < U^j = \mu_h + 3\sigma_j \forall j \quad (9)$$

Fig. 1 Framework of ESP



$$t_j^l < t_j^u \forall j \tag{10}$$

$$\left[\boldsymbol{\delta}\left(\boldsymbol{F_i}-\boldsymbol{F_j}\right)\right]-\left[(1-\boldsymbol{\delta})\left(F_i-F_j\right)\right] > 0 \tag{11}$$

Where,

$$\mu_h = \mu_s + C_d$$

$$\delta = \left\{ \begin{matrix} 1 & if\ F_i < F_j \\ 0 & otherwise \end{matrix} \right\}$$

Decision variables

b_i– Binary variable that indicates the choice of the shaft manufacturing machine

i.e., $b_i = \left\{ \begin{matrix} 1 & if\ shaft\ is\ manufactured\ in\ ith\ machine \\ 0 & otherwise \end{matrix} \right\}$

b_j –Binary variable that indicate the choice of the hole manufacturing machine

i.e., $b_j = \left\{ \begin{matrix} 1 & if\ hole\ is\ manufactured\ in\ jth\ machine \\ 0 & otherwise \end{matrix} \right\}$

Table 1 Process capability and manufacturing cost of shaft and hole manufacturing machines

Machine (*i/j*)	Shaft manufacturing		Hole manufacturing	
	σ_i (mm)	m_i (INR)	σ_j (mm)	m_j (INR)
1	0.007	30	0.01	20
2	0.015	25	0.015	15

Table 2 Initial parameters of the problem illustrated

Parameter	Value (mm)
μ_s	29.9 (given)
C_d	0.05 (given)
μ_h	29.95
i	1
j	1
L^i	29.879[29.9 − (3 × 0.007)]
U^i	29.921[29.9 + (3 × 0.007)]
U^j	29.98 [29.95 + (3 × 0.01)]
L^j	29.92 [29.95 − (3 × 0.01)]
t_i^u	29.921
t_i^l	29.916[29.921 − 0.005]
t_j^l	29.98
t_j^u	29.985 [29.98 + 0.005]

Table 3 Iteration steps and result for $i1ji$ (0.007, 0.01) combination

t_i^u	t_j^l	t_j^u	t_i^l	C_v	TC
29.921	29.925	29.930	29.916	0.046	107.454
			29.911	0.046	105.091
			.	.	.
			.	.	.
			.	.	.
			29.881	0.046	72.463
		29.935	29.916	0.046	105.617
			29.911	0.046	103.315
			.	.	.
			.	.	.
			.	.	.
			29.881	0.046	71.045
		.	.	.	.
		.	.	.	.
		.	.	.	.
		29.98	29.916	0.046	81.088
			29.911	0.046	79.326
			.	.	.
			.	.	.
			.	.	.
			29.881	0.046	50.302
29.921	29.930	29.935	29.916	0.041	105.054
			29.911	0.041	108.821
			.	.	.
			.	.	.
			.	.	.
			29.881	0.041	70.903
		.			
		.			
		.			
		.			
		29.98	29.916	0.036	102.487
			29.911	0.036	100.365
			.	.	.
			.	.	.
			.	.	.
			29.881	0.036	52.364
	.				
	.				
	.				
	29.975	29.98	29.911	0.014	99.875
			29.906	0.019	95.365
			.	.	.
			.	.	.
			.	.	.
			29.881	0.049	73.417
29.916	29.975	29.98	29.881	0.049	99.398
.					
.					
.					
29.886	29.975	29.98	29.881	0.049	108.625

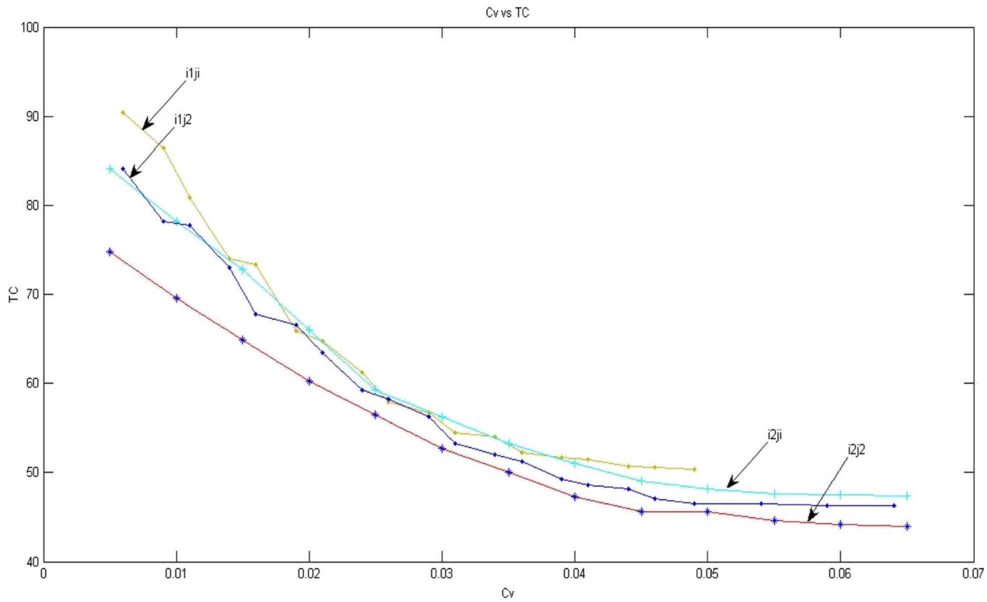


Fig. 2 Pareto fronts for the various combinations of two machines for shaft and two for hole ($i1j1$, $i1j2$, $i2j1$, and $i2j2$)

Equation 1 expresses the three cost elements of a shaft-hole assembly. The first term in Eq. 1 represents the rejection cost of a set of shaft-hole assembly (RC). It is formulated based on the logic that the cost of rejection is the multiplication of fraction defective of shaft-hole assembly given in Eq. 12 and manufacturing cost of a assembly set (m_i + m_j) .The fraction defective of the assembly is the minimum of the fraction acceptance of shaft (Eq. 13) and fraction acceptance of hole (Eq. 14) under the selected machine and tolerance allocations. A binary control parameter δ is used to find the minimum of fraction of acceptance from the shaft and hole.

$$\textit{Fraction defective of assembly} = \{\mathbf{1} - [\boldsymbol{\delta}(\mathrm{F}_i) + (\mathbf{1}-\boldsymbol{\delta})(F_i)]\} \quad (12)$$

$$\textit{Fraction of shaft accepted } F_i = \left[\sum_{i=1}^{N_s} b_i \left[\frac{1}{\sigma_i\sqrt{2\pi}} \int_{t_i^l}^{t_i^u} e^{-\left(\frac{(x_i-\mu_s)^2}{2\sigma_i^2}\right)} \mathrm{d}x_i\right]\right] \quad (13)$$

$$\textit{Fraction of hole accepted } F_j = \left[\sum_{j=1}^{N_h} b_j \left[\frac{1}{\sigma_j\sqrt{2\pi}} \int_{t_j^l}^{t_j^u} e^{-\left(\frac{(x_j-\mu_h)^2}{2\sigma_j^2}\right)} \mathrm{d}x_j\right]\right] \quad (14)$$

The second term of Eq. 1 represents the quality loss cost (QLC). The QLC equation is formulated based on the C_v, defined as variation of the actual value of fit C from

Table 4 ESP Pareto solution set of the illustrative problem

Pareto points	t_i^u	t_j^l	t_j^u	t_i^l	C_v	TC
A_{22}	29.905	29.95	29.955	29.9	0.005	74.778
B_{22}	29.905	29.945	29.955	29.895	0.01	69.555
C_{22}	29.91	29.945	29.96	29.895	0.015	64.877
D_{22}	29.91	29.94	29.96	29.89	0.02	60.199
E_{12}	29.906	29.935	29.965	29.891	0.024	59.285
F_{22}	29.915	29.94	29.965	29.89	0.025	56.446
G_{12}	29.911	29.935	29.97	29.891	0.029	56.25
H_{22}	29.915	29.935	29.965	29.885	0.03	52.692
I_{12}	29.911	29.93	29.975	29.891	0.034	52.085
J_{22}	29.92	29.935	29.97	29.885	0.035	49.995
K_{12}	29.911	29.925	29.975	29.886	0.039	49.307
L_{22}	29.92	29.93	29.97	29.88	0.04	47.297
M_{22}	29.925	29.93	29.975	29.88	0.045	45.56
N_{22}	29.925	29.93	29.98	29.875	0.055	44.589
O_{22}	29.925	29.93	29.985	29.875	0.06	44.111
P_{22}	29.925	29.93	29.99	29.875	0.065	43.947

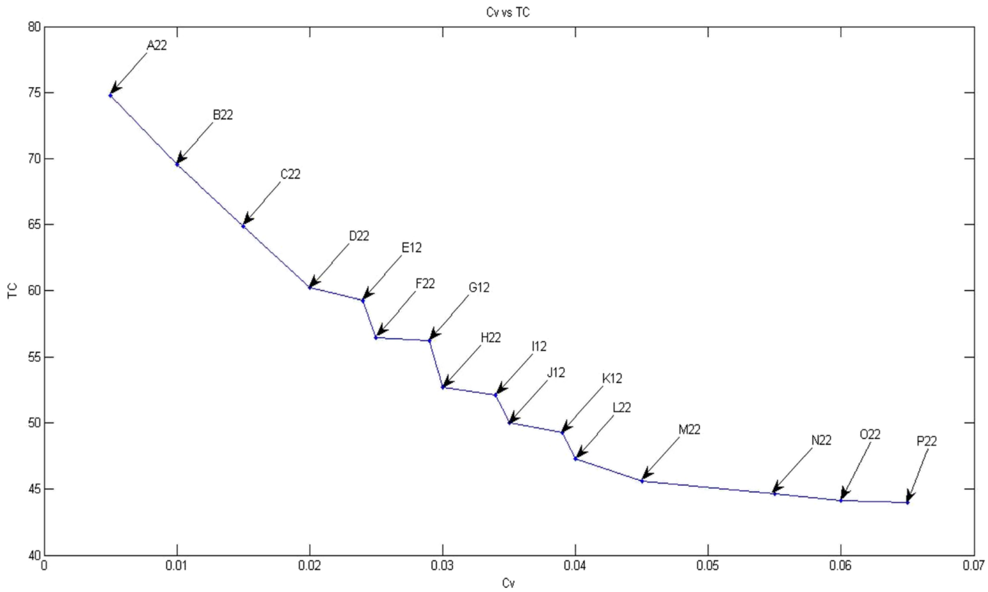


Fig. 3 ESP Pareto plot of the illustrative problem

design clearance C_d.The value C depends on the manufacturing processes and the resultant dimensions of the shaft and hole. Hence, C_v is inevitable and leads to weakening the quality of assembly. The associated cost is addressed as QLC. It is formulated based on the general Taguchi quality loss function to the nominal the best product/system as given in Eq. 15.

$$L(Y) = K(Y - m)^2 \tag{15}$$

Where,

$L(Y)$	Quality Loss for the performance Y
Y	Performance output value
m	Target performance of Y
K	Proportionality constant, which depends on financial criticality of Y (relating the performance deviation and cost)

With respect to the shaft-hole assembly, QLC for the shaft-hole assembly can be expressed as the function of actual clearance C and design specification C_d. The proposed QLC function for the shaft-hole assembly is given in Eq. 16.

$$QLC = K(C - C_d)^2 \tag{16}$$

Table 5 ESP Pareto solutions under different precision levels

Parameters	Experiment 1	Experiment 2	Experiment 3
Precision levels (mm)	$\Delta X = 0.005$ $\Delta Y = 0.005$	$\Delta X = 0.003$ $\Delta Y = 0.003$	$\Delta X = 0.0025$ $\Delta Y = 0.0025$
No. of Pareto solutions	21	24	36
Computational time (s)	180	11,200	21,000

However, the shafts and holes are manufactured in bulk and accepted within certain specific tolerance limits in interchangeable assembly. Under this circumstance, the value of C is not a constant and varies from pair to pair. Here, C can be assumed as an average value, which is defined as the difference between the mid values of hole and shaft and is given in Eq. 17.

$$C = \sum_{j=1}^{N_h} b_j \left(\frac{t_j^u + t_j^l}{2} \right) - \sum_{i=1}^{N_s} b_i \left(\frac{t_j^u + t_j^l}{2} \right) \tag{17}$$

The manufacturing cost is affected significantly by the selection of the machine due to its age, maintenance, and process capability. A variable K is defined as a ratio of cost of assembly (A) and maximum permissible clearance variation (Δ) as described in Eqs. 18, 19, and 20.

$$K = \frac{A}{\Delta^2} \tag{18}$$

$$A = \left\{ \sum_{i=1}^{N_s} b_i mc_i + \sum_{j=1}^{N_h} b_j mc_j \right\} \tag{19}$$

$$\Delta = \sum_{j=1}^{N_h} b_j \left(U^j - L^j \right) + \sum_{i=1}^{N_s} b_i \left(U^i - L^i \right) \tag{20}$$

The third term of the Eq. 1 represents manufacturing cost of an assembly (A).

The second objective which is minimizing of clearance variation is expressed in Eq. 2. This is based on C_v, which decides the fit that would be resulted with the allocated tolerances on the shaft and hole from the design clearance (C_d). The C_v is maximum among the two cases: (i) difference between hole diameter at upper tolerance limit and shaft diameter at lower tolerance limit

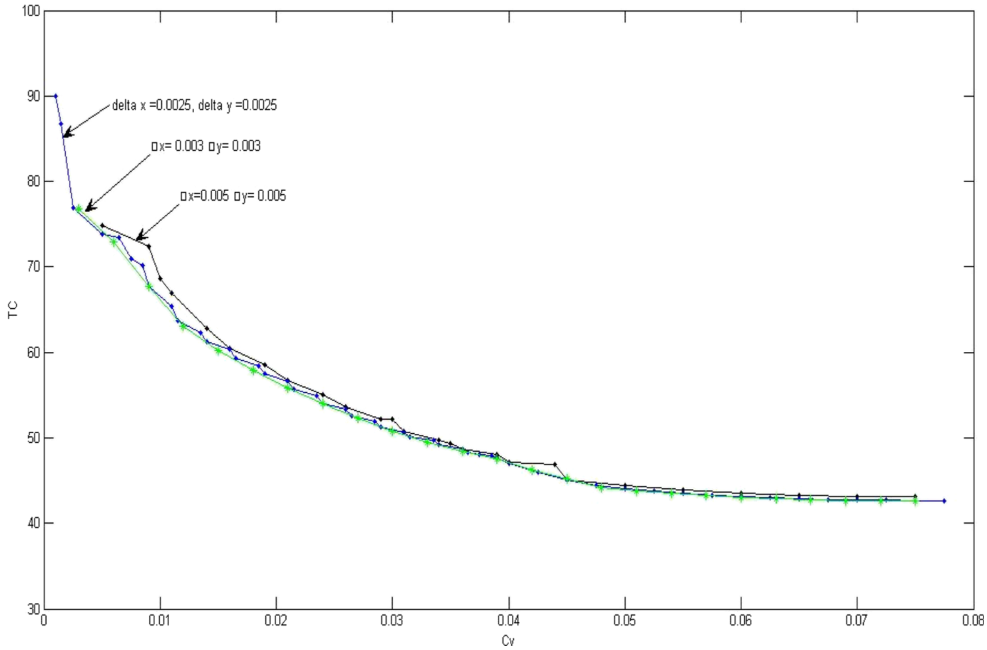


Fig. 4 ESP Pareto plot for the various precision levels

from C_d and (ii) difference between hole diameter at lower tolerance limit and shaft diameter at upper tolerance limit from C_d.

Equation 3 ensures that only one machine (i) is chosen for shaft manufacturing. Equation 4 ensures that only one machine (j) is chosen for hole manufacturing. Constraint 5 assures that the lower tolerance limit of the shaft should be higher than that of minimum size of shaft at − 3 σ_i *limit* of the chosen machine i. Constraint 6 assures that the upper tolerance limits of the shaft should be lesser than that of maximum size of shaft at + 3σ_i *limit* of the chosen machine i. Constraint 7 assures that the lower tolerance limit of the shaft is always lesser than the upper tolerance limit of the shaft of the chosen machine i. Similarly, constraints 8–10 ascertain the allocation limits for hole are within the limits for the chosen machine j. A binary controlling parameter (δ) is used to find the minimum of fraction of acceptance either in shaft or hole in constraint 11 and assures that the fraction of rejection should be in a positive value.

Table 6 shows the process capabilities and manufacturing costs of all machines for analyzing the computational time and performance of ESP

Machine (i/j)	Shaft manufacturing		Hole manufacturing	
	σ_i (mm)	m_i (INR)	σ_j (mm)	m_j (INR)
1	0.007	30	0.01	20
2	0.01	28	0.015	15
3	0.015	25	0.018	13
4	0.02	15	0.02	10

The tolerance allocation model formulated has integrals in its objective function and two kinds of the decision variables, binary (machine choice) and continuous (tolerance allocation). This model belongs to the category of Non-Linear Mixed Integer Programming (NLMIP) problem. These kinds of problems are difficult to solve using conventional optimization techniques. Hence, this paper proposes two search methodologies to provide solutions to the problem and they are:

- An ESP to find optimal solutions for problems of small size by exploring the entire solution space in a reasonably computational time
- SAA to obtain solution closer to the optimal for the larger size of the problem, as an alternate to ESP that would require more computational efforts for big size problems

The following Sections 4 and 5 delineate the proposed ESP and SAA.

4 Enumerative search procedure

4.1 Framework

Enumerative search is a very simple and efficient technique for small or moderate search space [24]. The decision variables are continuous in its solution space constrained with upper and lower limits. The ESP is structured to search the entire decision space with tolerances at certain precision levels (Δx, Δy) acceptable by the user

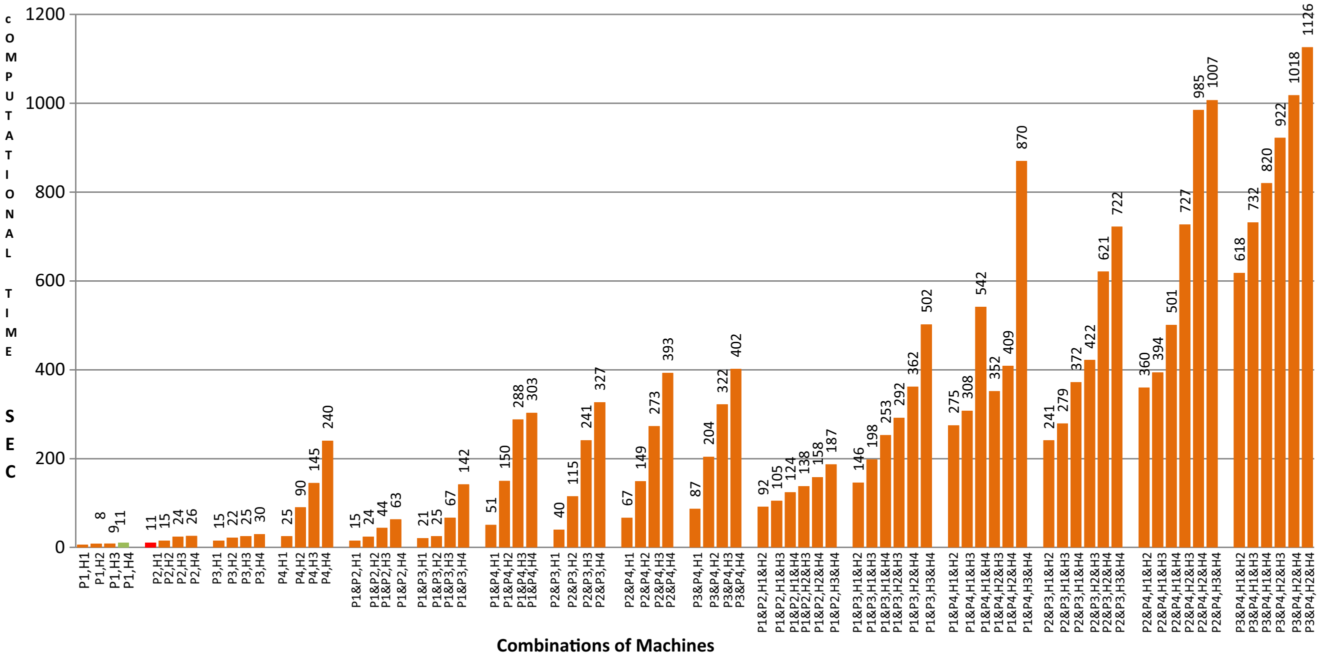


Fig. 5 Computational experience of ES

and to evolve optimal Pareto front for the bi-objective tolerance allocation model described in Section 3. The decision space is then transferred to the objective space so as to evaluate and obtain the *Pareto* optimal front. In this methodology, all enumerated solutions are stored and the Pareto optimal front is obtained using the principle of dominance. Flow chart given in Fig. 1 outlines the various stages of the proposed ESP.

4.2 Illustration

The ESP is illustrated with a sample problem of a two shaft and two hole manufacturing environment. The data of the sample problem are as follows: nominal/mean dimension of the shaft μ_s = 29.9 mm, design specification of the fit C_d = 0.05 mm, number of machines available for shaft $N_s = 2$; , number of machines available for hole $N_h = 2$; , precision level for shaft Δx = 0.005, and precision level for hole Δy = 0.005. The process capability is expressed as standard deviation for all the machines and their corresponding manufacturing costs as given in Table 1. The above data are given as the input to the ESP in the first stage of ESP.

The second stage is the selection of shaft manufacturing machine by using identifier i and hole manufacturing machine by using identifier j, then the tolerance limits of the shaft (t_i^l, t_i^u) and hole (t_j^l, t_j^u) are determined as initial values of the iteration by Eqs. 21 to 24 and shown in Table 2.

$$t_j^l = L^j \tag{21}$$

$$t_i^u = U^i \tag{22}$$

$$t_j^u = t_j^l + \Delta x \text{ (incremented with} \Delta x) \tag{23}$$

$$t_i^l = t_i^u - \Delta y \text{ (decremented} \Delta y) \tag{24}$$

The third stage of ESP finds the Pareto fronts of all possible combinations of shaft-hole manufacturing machines, taking one combination of machines at a time. After that, the entire design space is explored by incrementing the hole dimension and decrementing the shaft dimension. The shaft dimension is decremented by Δy which is the precision level of the shaft manufacturing machine. The hole dimension is incremented by Δx which is the precision level of the hole manufacturing machine. The decision variables are defined in this space using the specified levels of precision (Δx and Δy). The decision space is then transferred to the objective space so as to evaluate and obtain the *Pareto* optimal front. In this stage, TC is calculated (using Eq. 1) and C_v (using Eq. 2) by substituting the input data given in Table 1 and the current shaft and hole sizes. The calculated TC, C_v, and the corresponding t_j^u , t_j^l , $t_{i,}^u$ t_i^l are stored and then proceeded further by updating the shaft and hole sizes. Table 3 shows the t_j^u , t_j^l , $t_{i,}^u$ t_i^l , C_v, and TC at different iterations. The fourth and last stages of ESP evolve the final Pareto solution set by applying the dominance principle on the combined Pareto solution sets obtained in stage 3.

Figure 2 shows the Pareto front plots of the various machine combinations. Table 4 and Fig. 3 show the final

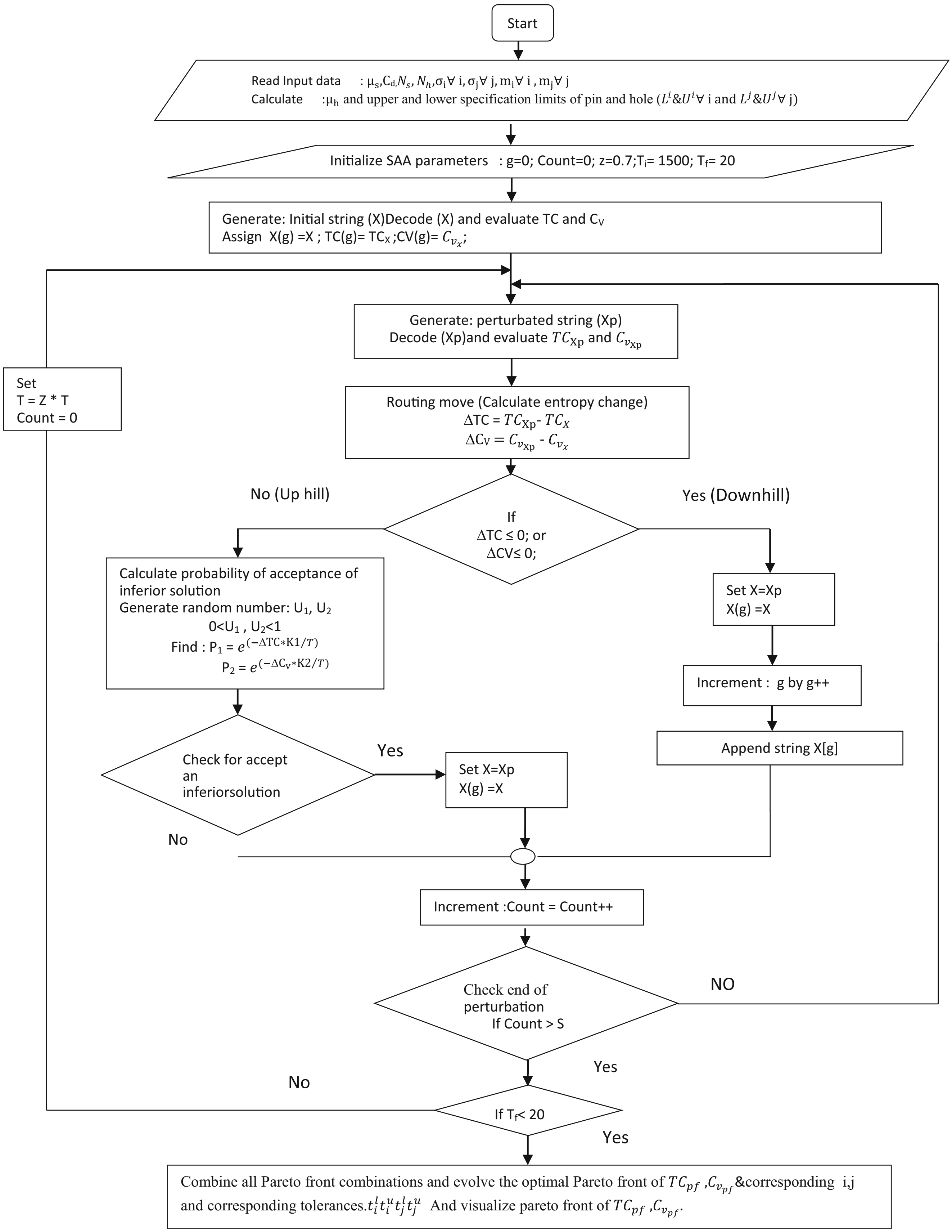


Fig. 6 Structure of simulated annealing algorithm

Table 7 Notations used for SAA

Notations	Information
Count	Index for perturbation
g	Number of perturbed solutions accepted
n	Length of the string
R	Random positive integer number
S	Number of perturbations in each temperature
X	Initial string
T	Current temperature
Z	Temperature reduction factor
K1	Multiplication factor 1
K2	Multiplication factor 2
P_1	Probability of inferior solution 1
P_2	Probability of inferior solution 2
U1	Random number 1
U2	Random number 2
T_i	Initial temperature
T_f	Final temperature
$X(g)$	Global best string
$TC(g)$	Total cost in global best string $X(g)$
$C_v(g)$	Clearance variation in global best string $X(g)$
X_p	Perturbed string
TC_X	Total cost in string X
TC_{Xp}	Total cost in Perturbed string X_p
C_{Vx}	Clearance variation in string X
C_{Vxp}	Clearance variation in perturbed string X_p
$C_{v_{pf}}$	Clearance variation in Pareto front
TC_{pf}	Total cost in Pareto front
ΔTC	Difference in total cost
ΔC_V	Difference in clearance variation

Pareto solution and the plot of the illustration problem, respectively.

4.3 Performance analysis

The solution quality and the solution time of the ESP depend on the required precision levels (Δx, Δy), number of machines (N_s, N_h), and their standard deviations (σ_i, σ_j). They are discussed in this section.

4.3.1 Effect of precision level (Δx, Δy)

Table 5 compares the number of Pareto solutions and computational time of the illustrative problem experienced with three precision levels. Figure 4 presents the Pareto plot of the above settings.

Figure 4 concludes that the higher precision level (Δx, Δy) leads to higher solution quality with considerable increase in the number of Pareto points, however at the expense of higher computational time.

4.3.2 Effect of process capability on computational time

The effect of process capability on computational time (CT) is studied by the illustrative problem by including two more machines in each category. Table 6 shows the process capabilities and costs of all machines. Figure 5 shows the pictorial representation of computational time of different production environments.

The observations from Fig. 5 are denoted as follows: an increase in standard deviation of the machine (σ_i, σ_j) results in increased computational time due to wider search space and an increase in size of the problem, due to number of machines, creates a larger problem search space which leads to increased computational time.

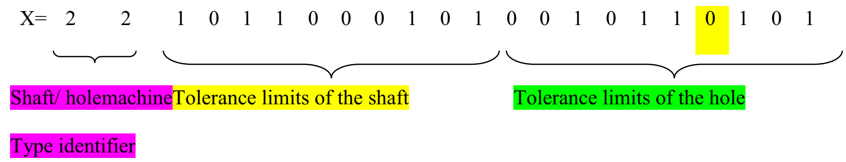
4.4 Performance review of ESP

The performance study reveals that the computational time and the number of Pareto points increase with a decrease in process capability, an increase in number of machines in the manufacturing center, and an increase in precision level. Hence, the ESP can be used to get optimal Pareto front with reasonable CT for low precision levels and limited number of machine environments. On the other hand, ESP would require huge computational effort to

Table 8 Parameter values used in SAA with rationale

Parameter	Value	Rationale
Initial temperature T_i	1500	Probability of acceptance at the beginning is about 0.9
Final temperature T_f	20	Probability of acceptance at the end near 0.1
Temperature reduction factor Z	0.8	Z from 0.6 to 0.99 used by previous researchers
Length of the string n	22	For accuracy, each tolerance limit has 5 bits and 1 bit for machine identifier for shaft i and another 1 bit for hole j
Number of perturbation in each temperature S	88 (2 × 2 × 22)	It $S = N_s \times N_h \times n$ Solution search space depends on problem size and accuracy
K1	10	Multiplication factor for TC to get the probability in acceptable order
K2	10^4	Multiplication factor for C_v to get the probability in acceptable order

Fig. 7 Initial string



evolve optimal front for higher precision levels and environments with more number of machines.

As the Pareto front does not change much (Fig. 4) with change in precision levels, the computational advantage can be derived by adopting heuristics to provide solutions in a reasonable CT by compromising the solution quality.

5 Simulated annealing algorithm

Meta-heuristic has been largely used in approximate optimization technique in the last two decades. A trajectory-based technique starts with a single initial solution and, at each step of the search, the current solution is replaced by another (often the best) solution found in its neighborhood. It is usual that trajectory-based meta-heuristics allow quickly finding a locally optimal solution. When the search space has a few peaks and valleys, trajectory search is more efficient.

The algorithms have been applied to tolerance allocation problems: genetic algorithm (GA) by Bai et al. [25], Ming and Mak [26], Rao and Rao [26], Geetha et al. [27], and Kumar et al. [28]; particle swarm optimization (PSO) by Muthu et al. [18] and Kannan et al. [29]; pattern search algorithm (PSA) by Zhang and Wang [2] and Kumar et al. [28]; SAA by Cagan and Kurfees [30], Zhang and Wang [2], Singh et al. [10], Ganesan et al. [9], and Malaichamy et al.[]; and ant colony algorithm by Prabhaharan et al. [4]. The methodology used to solve the problem is based on the number of solutions in the search space and the nature of the problem. Duh and Brown [] used SAA-mentioned Pareto-simulated annealing for multi-objective spatial allocation. In a multi-objective problem, SA with a composite energy clearly converges to the true Pareto front solutions [31]. The big advantage of SAA is its capability to move to states of higher energy [32]. On these points, this paper presents SAA for the bi-objective tolerance allocation problem.

5.1 Structure of simulated annealing algorithm

This section describes the various modules of the proposed SAA. The flow chart presented in Fig. 6 describes the structure of SAA with the aid of using the notations given in Table 7.

5.2 Illustration of SAA

5.2.1 Step 1: input and derived data

The following data relevant to the problem are given as the input: C_d; μ_s; ($\sigma_i \forall i$), i = 1 to N_s; ($\sigma_j \forall j$), j = 1 to N_h; mc_i, i = 1 to N_s; and mc_j, j = 1 to N_h.

The following values that govern the solution space and solution fitness are derived from the input data: μ_h; U^i, for i = 1 to N_s; L^i, for i = 1 to N_s; U^j, for j = 1 to N_h; and L^j, for j = 1 to N_h;.

For illustration purpose, the input data taken for SAA are the same data as taken for ESP. They are given in Table 1 in Section 4.2.

5.2.2 Step 2: initialization of SAA parameters

This step initializes the following parameters that influence the performance of SAA with the certain criterion as

Table 9 Initial string decoded values

Bit number in X	Representation in X	Decoded value	Corresponding decision variables
1	2	2	Shaft manufacturing machine (i) = 2
2	2	2	Hole manufacturing machine (j) = 2
3–7 8–12	1 0 1 1 0 0 0 1 0 1	29.954 29.853	Lower tolerance limit of the shaft (t_i^l) = 29.863 mm Min (29.863 mm, 29.954 mm) Upper tolerance limit of the shaft (t_i^u) = 29.923 mm Max (29.853 mm, 29.923 mm)
13–17 18–22	0 0 1 0 1 1 0 1 0 1	29.932 29.957	Lower tolerance limit of the hole (t_j^l) = 29.932 mm Min (29.932 mm, 29.957 mm) Upper tolerance limit of the hole (t_j^u) = 29.957 mm Max (29.932 mm, 29.957 mm)Max (29.925 mm, 29.954 mm)

```
{
Generate a random positive integer number 'R' below 22;
        If R = 0
            {
Generate a positive integer number between1 and Ns
                Assign as machine type 'i'
Break;
            }
If  R = 1
            {
Generate a positive integer number between1 and Nh
                Assign as machine type  'j'
Break;
            }
        If 2 ≤ R < 22
            {
Rth bit of  string X is flipped either as 0 to1 or 1 to 0
            }
    }
```

Fig. 8 Pseudo-code for perturbation

initial temperature T_i that decides the probability of acceptance of inferior (uphill) solution with maximum distance (i.e., differences in cost function or clearance variation); final temperature T_f that verifies the solution convergence with accuracy requirement; temperature reduction factor Z leads to set current temperature; and the number of perturbation in each temperature S to capture local optima with the exploration space ($= N_s \times N_h \times n$;). The multiplication factors K1 and K2 are introduced to get a normalized fitness function for TC and C_v in order to get the probability of acceptance of about 0.9 at the initial stages of annealing process (i.e., higher temperature) and reduced probabilities as the temperature reduces. This is necessary because each objective has various ranges in their distance/evaluation functions. Table 8 provides the parameters used for illustration with values and reason for taking the values.

5.2.3 Step 3: generation of initial solution strings and performance parameters

Each solution string needs to represent the shaft and hole manufacturing machines among the available machines in the manufacturing center, and their corresponding lower and upper tolerance values. A 22-bit string represents a solution set. The first two bits are the phenotype number: the first one represents the shaft manufacturing machine and second represents hole manufacturing machine and the both lie between the number of available manufacturing machines (i.e., 1 to N_p and to N_h) and 1 to N_h. Next 10 bits represent the lower and upper tolerance values of shaft with binary numbers. The last 10 bits represent the lower and upper tolerance values of hole with binary numbers. These binary representations are genotype and they are decoded on the basis of maximum and minimum size of the shaft and hole with respect to manufacturing machines addressed in first two bits. A string is generated randomly as delineated above and is set as initial string X. The performance parameters of total cost TC_X'and and clearance variation C_{v_x} are calculated using input and derived data and the values i,j, t_i^l, t_i^u, t_j^l, t_j^u are decoded from X, when they are applied in Eqs. 1 and 2, respectively.

The decoding procedure for string X is as follows:

1. First bit number is fixed as shaft manufacturing machine identifier i
2. Second bit is set as hole manufacturing machine identifier j
3. The five binary bits 3 to 7 and 8 to 12 are converted as real numbers by interpolation corresponding to the shaft manufacturing machine i according to the first bit using the expression 25.

$$\left[\frac{(U^i - L^i) \times Real\ value\ of\ 5\ binary\ bits\ as\ decoded\ from\ 3\ to\ 7\ or\ 8\ to\ 12}{Maximum\ real\ value\ for\ five\ bit\ binary\ number}\right] + L^i \tag{25}$$

4. The larger of these two tolerance values is fixed as the upper tolerance limit (t_i^u) and smaller is taken as the lower tolerance limit (t_i^l) of the shaft.
5. The five binary bits 13 to 17 and 18 to 22 are converted as real numbers by interpolation corresponding to the hole manufacturing machine j according to the second bit using the expression 26.

Xp	2	2	1	0	1	1	0	0	0	1	0	1	0	0	1	0	1	1	1	1	0	1

Fig. 9 Perturbed string (X_p)

Table 10 Perturbed string decoded values

	i	j	t_i^l	t_i^u	t_j^l	t_j^u	$C_{v_{X\mathrm{p}}}$	$TC_{X\mathrm{p}}$
X_p	2	2	29.853 mm	29.954 mm	29.932 mm	29.984 mm	0.065 mm	52.052 INR

$$\left[\frac{(U^j-L^j)\times Real\ value\ of\ 5\ binary\ bits\ \text{as}\ decoded\ from\ 13\ to\ 17\ or\ 18\ to\ 22}{Maximum\ real\ value\ for\ five\ \text{bit}\ binary\ number}\right]+L^j \tag{26}$$

6. The larger of these two tolerance values is fixed as the upper tolerance limit (t_j^u) and the smaller is taken as the lower tolerance limit ((t_j^l) of the hole.
7. Finally the TC and clearance variation of the string X are found out by using Eqs. 1 and 2 which represent TC_X and C_{v_x}, respectively.

The initial string is generated for the illustrative problem as shown in Fig. 7.

The decoded values are given in Table 9.

For the illustrative problem, the total cost and clearance variation of string X are as follows:

TC_X = 54.735 INR; C_{v_x} = 0.049 mm.

5.2.4 Step 4: initiation of global solution set X(g)

Initially, X is the first solution set of $X(g)$, which is subsequently appended during the course of SAA. C_v and TC are assigned as $C_v(g)$ and $TC(g)$, respectively.

$TC(g)$ = 54.735 INR; $C_v(g)$ = 0.049 mm

5.2.5 Step 5: perturbation and its evaluation

The perturbed string X_p is derived from the current string X using single bit random mechanism. The pseudo-code is given in Fig. 8.

Figure 9 shows the perturbed string X_p of illustrative problem with 19th bit and is perturbed from 0 to 1.

The decoding process is same as decoding procedure outlined for X narrated in step 3. Table 10 shows the decoded value of perturbed string X_p with their corresponding values of TC and C_v and are denoted as $TC_{X\mathrm{p}}$ and $C_{v_{X\mathrm{p}}}$, respectively.

The change is in t_j^u, as a result change of 19th bit during perturbation.

5.2.6 Step 6: routing move

Whenever the performance of one of the objective ($TC_{XP} \geq TC_X$ or $C_{v_{X\mathrm{p}}} - C_{v_x}$)$X_\mathrm{p}$ outperforms other solution X in either TC or C_v, the objective values of total cost and clearance variation corresponding to perturbed string X_p are subtracted to the objectives corresponding to the string X (i.e., $\Delta TC = TC_{XP} - TC_X$ and $\Delta C_v = C_{v_{X\mathrm{p}}} - C_{v_x}$). If $\Delta TC < 0$ or $\Delta C_v < 0$, then the algorithm proceeds to downhill move, otherwise go to uphill move and set $X = X_\mathrm{p}$.

For this illustrative problem,

ΔTC = 52.052–54.735 = −2.683 INR

ΔC_v = 0.065–0.049 = 0.016 mm

In this illustrative problem, as ΔTC < 0, it moves to downhill move (step 7).

5.2.7 Step 7: downhill move

The solution X_p is included in the solution space of the Pareto set $X(g)$. Thus, the global solution set $X(g)$ is the composition of solutions X_p included during every downhill move of the SAA. In the downhill move, perturbed string X_p is appended in $X(g)$ and the number of perturbed solution g is incremented by 1. Then, X is set with X_p and proceeds to step 9.

In this illustrative problem, Fig. 10 shows the perturbed string X_p is appended in $X(g)$.

Fig. 10 Appended string $X(g)$

X=	2	2	1	0	1	1	0	0	0	0	0	1	0	0	1	1	1	1	0	0	0	1
$X(g)_1$	2	2	1	0	1	1	0	0	0	0	0	1	0	0	1	1	1	1	0	0	0	1
$X(g)_2$	2	2	1	0	1	1	0	0	0	0	0	1	0	0	1	1	1	1	1	0	0	1

Table 11 Optimal Pareto solution set of machines $i = 1,2$ and $j = 1,2$ ($N_s = 2, N_h = 2$)

Pareto points (g)	Machine (i)	Machine (j)	Decoded values of $X(g)$					
			t_i^l	t_i^u	t_j^l	t_j^u	$C_{v_{pf}}$	TC_{pf}
1	2	1	29.922	29.925	29.972	29.972	0.002	89.427
2	2	1	29.922	29.928	29.972	29.972	0.005	89.008
3	2	1	29.916	29.919	29.963	29.97	0.006	87.522
4	2	1	29.913	29.916	29.963	29.97	0.007	87.278
5	2	1	29.887	29.904	29.943	29.943	0.011	79.708
6	2	2	29.887	29.904	29.94	29.943	0.015	68.7
7	1	2	29.897	29.91	29.951	29.966	0.019	67.445
8	1	1	29.895	29.913	29.943	29.97	0.025	63.443
9	1	1	29.886	29.917	29.941	29.955	0.026	60.833
10	1	1	29.89	29.907	29.932	29.966	0.027	58.9
11	1	1	29.89	29.914	29.941	29.968	0.029	57.443
12	1	1	29.879	29.909	29.93	29.959	0.03	57.17
13	1	1	29.886	29.907	29.926	29.965	0.032	56.965
14	2	1	29.884	29.916	29.932	29.966	0.034	53.972
15	2	1	29.881	29.916	29.932	29.966	0.035	52.97
16	1	1	29.879	29.914	29.93	29.966	0.037	52.285
17	1	2	29.89	29.914	29.931	29.978	0.038	50.02
18	2	1	29.87	29.922	29.926	29.965	0.046	48.85
19	1	2	29.882	29.913	29.92	29.98	0.049	46.823
20	1	2	29.882	29.914	29.92	29.983	0.052	46.418
21	2	2	29.864	29.916	29.911	29.975	0.061	44.581
22	2	2	29.858	29.942	29.931	29.972	0.064	42.736
23	2	2	29.867	29.925	29.905	29.983	0.07	41.776
24	2	2	29.875	29.933	29.911	29.992	0.073	41.666
25	2	2	29.855	29.925	29.905	29.983	0.078	41.47

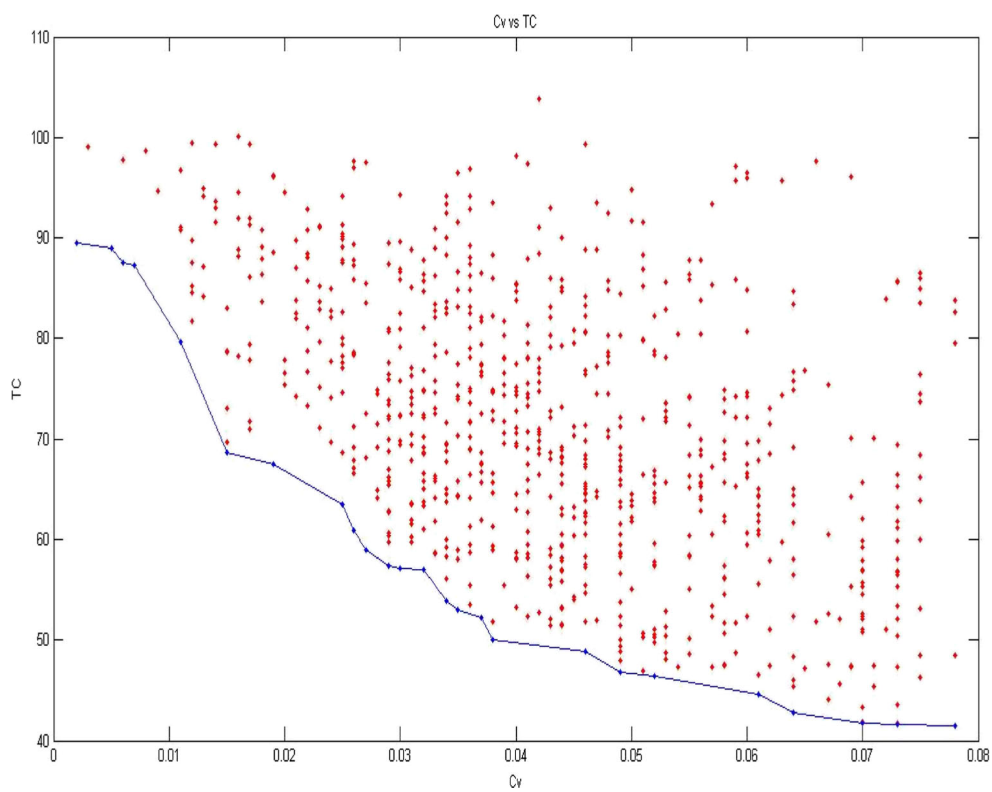


Fig. 11 Optimal Pareto plot of machines $i = 1,2$ and $j = 1,2$ ($N_s = 2, N_h = 2$)

Table 12 Comparison of Z and number of Pareto front points

S. no.	Temperature reduction factor Z	Number of Pareto front points	Computational time (s)
1	0.6	16	18
2	0.65	18	20
3	0.7	19	21
4	0.75	21	23
5	0.8	22	25
6	0.85	24	27
7	0.9	25	30
8	0.95	25	36

5.2.8 Step 8: uphill move

The solution corresponds to perturbed string (X_p) is inferior. Then, it is accepted with a probability (P1 and P2) determined using Eqs. 27 and 28. Initially, the probability of acceptance is generally high (say 0.9) and decreases as temperature reduces and is low (i.e., around 0.1 or less) at end temperatures.

$$P1 = e^{-\Delta TC \times K1 / T} \quad (27)$$

$$P2 = e^{-\Delta Cv \times K2 / T} \quad (28)$$

Generate two random numbers U1 and U2 between 0 and 1 such that 0 < (U1 and U2) < 1 U1 < P1 or U2 < P2

P1 is greater than U1 or P2 is greater than U2. Then, the solution is accepted and the string is set as $X = X_p$. On the other hand, the solution is rejected and the original X is retained, as X goes to step 9.

In this illustrative problem, K1 = 10 and K2 = 10^4.

5.2.9 Step 9: solution set updating in current temperature

The "Count" is the index for number of perturbation in each temperature. After the perturbation, the count is incremented by 1 (i.e., Count = Count + 1). After every perturbation, the downhill or uphill move is to be checked.

5.2.10 Step 10: checking the termination of perturbations at temp T and updating T

Check the number of perturbed solution in each temperature, and if it exceeds the expressed value, go to step 11; otherwise, the steps will continue from 5 to 9, until the count is reached S.

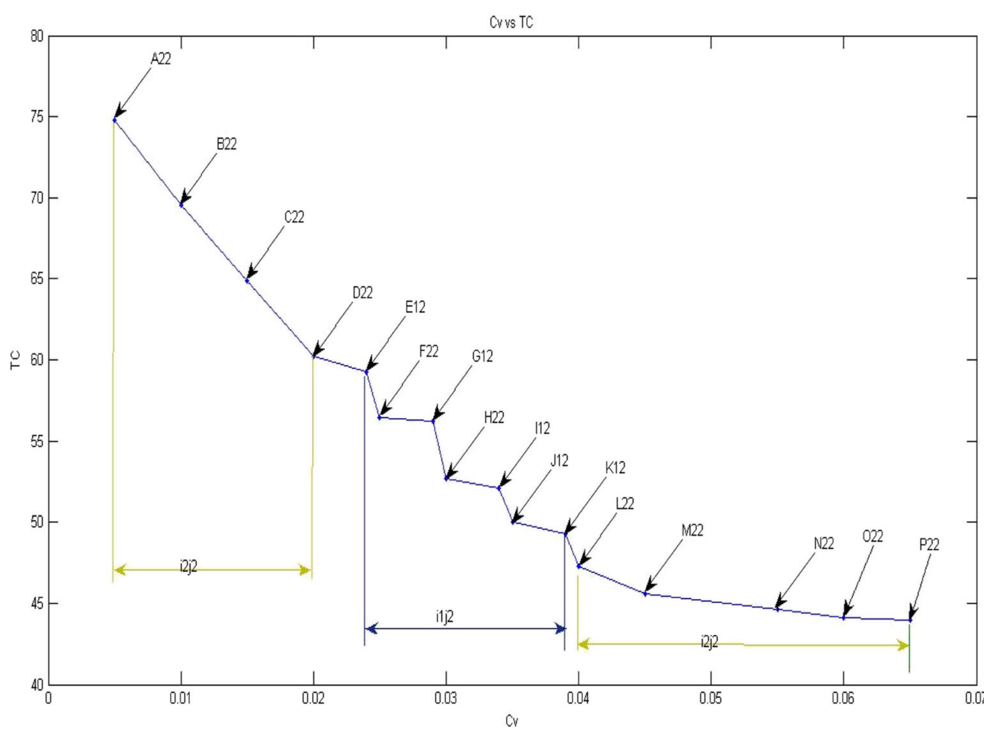


Fig. 12 Optimal Pareto plot of machines $i = 1,2$ and $j = 1,2$ ($N_s = 2, N_h = 2$)

Table 13 Process capability and cost of shaft and hole manufacturing machines

Machine	Shaft manufacturing (i)		Hole manufacturing (j)	
	σ_i (mm)	m_i (INR)	σ_j (mm)	m_j (INR)
1	0.007	30	0.007	25
2	0.01	28	0.01	20
3	0.015	25	0.015	15
4	0.018	20	0.018	13
5	0.02	15	0.02	10
6	0.022	13	0.022	08

In this illustrative problem, the number of perturbation is to be reached 88. It moves to step 11 otherwise continues steps from 5 to 9 until the count reached 88.

5.2.11 Step 11: updating temperature reduction factor and termination

This step checks the termination criteria by checking the final temperature value ($T_f \leq 20$). The condition is not satisfied, and the current temperature is reduced by temperature reduction factor (Z) and further continues step 5 to step 10 till the termination criteria is satisfied.

For this problem, initial temperature is taken as 450 and the final temperature is 20. If $T_f \leq 20$ go to step 12. Otherwise, set $T = Z \times T$ and Count = 0 and proceed further from steps 5 to step 10.

5.2.12 Step 12: Pareto front result

Combine all the appended values, sort based on the clearance variation and total cost and evolve the optimal Pareto front of TC_{pf}, $C_{v_{pf}}$ for corresponding machines i,j with lower and upper tolerance limits of the shaft (t_i^l, t_i^u) and hole (t_j^l, t_j^u) by using the principle of dominance.

Table 11 shows the optimal Pareto front solutions of the illustrative problem, as the number of shaft manufacturing machines is two and number of hole manufacturing machines is two (i.e., N_s=2, N_h = 2). Figure 11 shows the Pareto front plot of the illustrative problem.

5.3 Performance study

Initially, the behavior of SAA is analyzed under various temperatures and reduction factor *(Z)* is a key parameter that decides the accuracy of the solution to fix optimal Z. Next, the computational effort of SAA under different production environment is studied.

5.4 Tuning of temperature reduction factor *Z*

For tuning of temperature reduction factor Z, it is varied from 0.6 to 0.95 with the increment of 0.05 for the illustrative problem. Table 12 shows the number of Pareto points and the computational effort corresponding to Z. This reveals that the number optimal Pareto front point is increased, due to

Table 14 Average computational experience for ESP and SAA

S. no.	Number of possible combinations	Average computational time (s)	
		ESP	SAA
1	2	150	11
2	3	270	15
3	4	370	29
4	5	510	36
5	6	600	36
6	8	760	44
7	9	1900	59
8	10	2300	68
9	12	2980	82
10	15	3930	110
11	16	4640	170
12	18	5300	190
13	20	5700	178
14	24	6280	190
15	25	7200	202
16	30	12,200	230
17	36	15,400	250

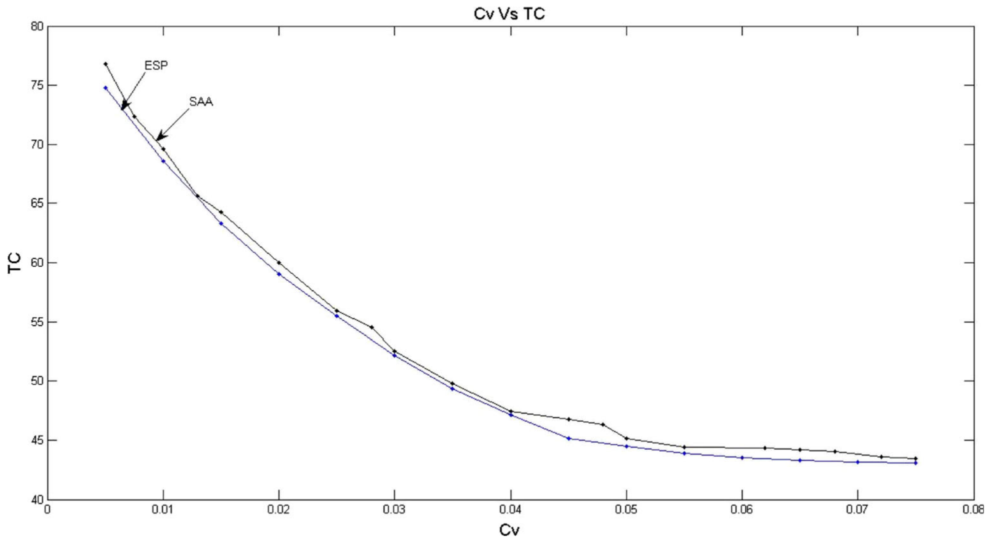


Fig. 13 Pareto front plot of one machine for shaft and two for hole (*i*2 and *j*1*j*2)

the increase of *Z*. $Z = 0.9$ which shows quite as good as fine number of Pareto points gathered in less computational time.

6 Discussions

Tolerance allocation highly influences the total cost of the assembly. A high clearance variation reduces the manufacturing cost by giving greater choice in selecting the machine available in a production center. The data from Table 14 and Figs. 2, 3, 4, 5, and 6 are taken for this analysis of model utility. This section describes the utility of the models and performance comparison of ESP and SAA.

6.1 Model utility

This model utility helps the manufacturer to decide the best and better combination of the machines to cater the need of the customer. In the manufacturing, three different cases are taken for analysis based on the availability of the machines. The following are the reasons to analyze the combinations of the machines.

- Few customers demand products in shorter time duration.
- Due to certain uncertainties like machine break down, other unforeseen circumferences, the production should not be affected.
- In case of a need arising to choose an alternative machine, due to the one of the machines in the best combination being engaged in completion of previous order.

The different combinations are as follows:

Case 1a: One machine for shaft manufacturing two machines for hole manufacturing; combinations are *i*1*j*1 and *i*1*j*2.

Case 1b: One machine for shaft manufacturing two machines for hole manufacturing; combinations are *i*2*j*1 and *i*2*j*2.

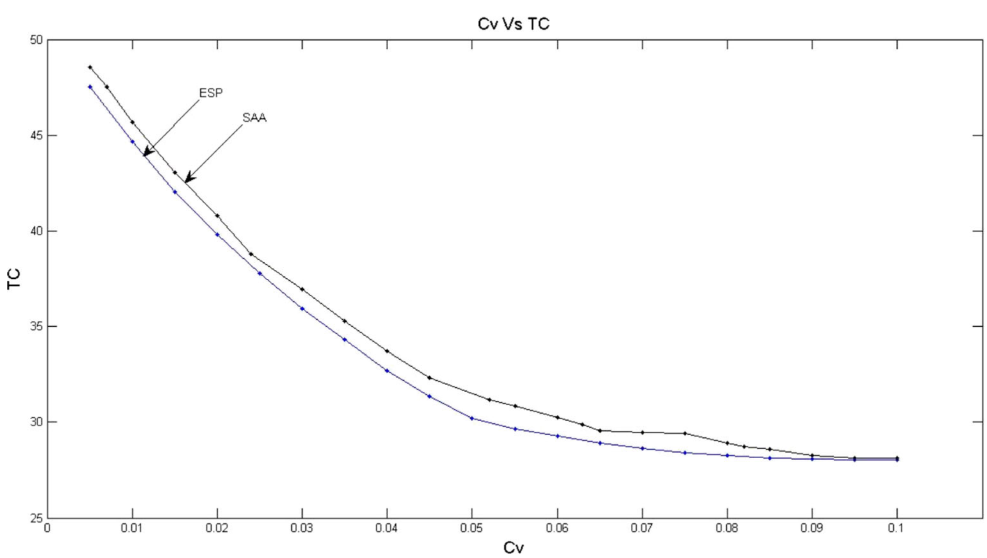


Fig. 14 Pareto front plot of four machines for shafts and four machine for hole (*i*1*i*2*i*3*i*4 and *j*1*j*2*j*3*j*4)

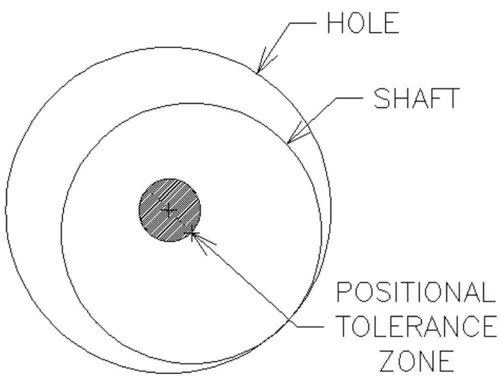


Fig. 15 Positional tolerance zone for ideal geometry [36]

Case 2a: One machine for hole manufacturing and two machines for shaft manufacturing; combinations are $i1j1$ and $i2j1$.

Case 2b: One machine for hole manufacturing and two machines for shaft manufacturing; combinations are $i1j2$ and $i2j2$.

Case 3: Two machines for shaft manufacturing and two machines for hole manufacturing; combinations are $i1j1$, $i2j1$, $i1j2$, and $i2j2$.

Figure 3 shows the Pareto front plot of all possible combinations of illustrative problem. Figure 12 shows the non-dominated optimal Pareto plot of the illustrative problem by considering that the manufacturing environment has two shafts and two hole manufacturing machines with the best combination of the machine with clearance variation and total cost.

When cases 1a and 1b are considered, the machines are used with single shaft manufacturing machine. It is observed that one set of Pareto front point of the machine combination dominates the other set of Pareto front point of the machine combination. For instance, $i1j2$ dominates $i1j1$ in certain range of clearance variation but $i2j2$ dominates $i2j1$ everywhere.

In case 2, the machines used for a combination of one hole manufacturing machine and two shaft manufacturing machines, the results shows that the shaft manufacturing machines influence the tolerance allocation. In case 2, it is observed from Fig. 12 that within the range of clearance variation 0.04 to 0.065, the only option is $i2j2$ combination, and the clearance variation in between 0.02 to 0.04, the $i1j2$ combination is optimal. The range of clearance variation from 0.005 to 0.02 mm, and the $i2j2$ combination is optimal in manufacturing a part.

Table 15 Input parameter positional tolerance for illustration

Parameter	Value
μ_s	29.9 mm
C_d	0.05 mm
σ_i	0.007 mm
σ_j	0.01 mm
m_i	30
m_j	20
Δx	0.005 mm
Δy	0.005 mm
ΔP	0.047 mm

From the plot 12, it is observed that different shaft and hole manufacturing machines are used, for the set of Pareto front points interfering with each other. When all the shaft manufacturing and hole manufacturing machines were considered simultaneously, the non-dominated Pareto front result is obtained.

At certain ranges of clearance variation, one of the hole and shaft manufacturing machine combination is distinctly desirable as mentioned in Fig. 12. The effective Pareto front obtained in Fig. 3 represents different combinations of machine at different clearance variation ranges. Such a plot helps the manufacturer to assign the best combinations of machine that suite the customer requirements as classified in the three cases.

Thus, it is desirable for medium-scale manufacturing industries to have a single hole manufacturing machine and a number of shaft manufacturing machines to make an assembly in order to satisfy the customer's demand fulfill.

6.2 Comparison of SAA with ESP

6.2.1 Computational experience

The average computational time of ESP and SAA for various production environments is calculated by using Table 13. The computational efforts of ESP and SAA are compared and depicted in Table 14. ESP takes more time compared to SAA. SAA can be used to find the best combination of the machine and the cost to satisfy the customer in time.

6.2.2 Similarity of the ESP and SAA

Figures 13 and 14 show the non-dominated Pareto front plot of Cv versus TC for two different combinations of the machines using ESP and SAA. The input data used to analyze the Pareto front plot of ESP and SAA are depicted in Table 6. The plots describe that the variation of result arrived by ESP and SAA is very minimum. But the computational time of SAA is very less compared to ESP. Hence, if the manufacturing company has more number of machines, SAA is enough to find optimal solution.

6.3 Model extensions

6.3.1 Positional tolerance constraint

The developed bi-objective tolerance allocation model would be useful in a manufacturing environment where multiple facilities are available for the manufacturing of shaft and hole

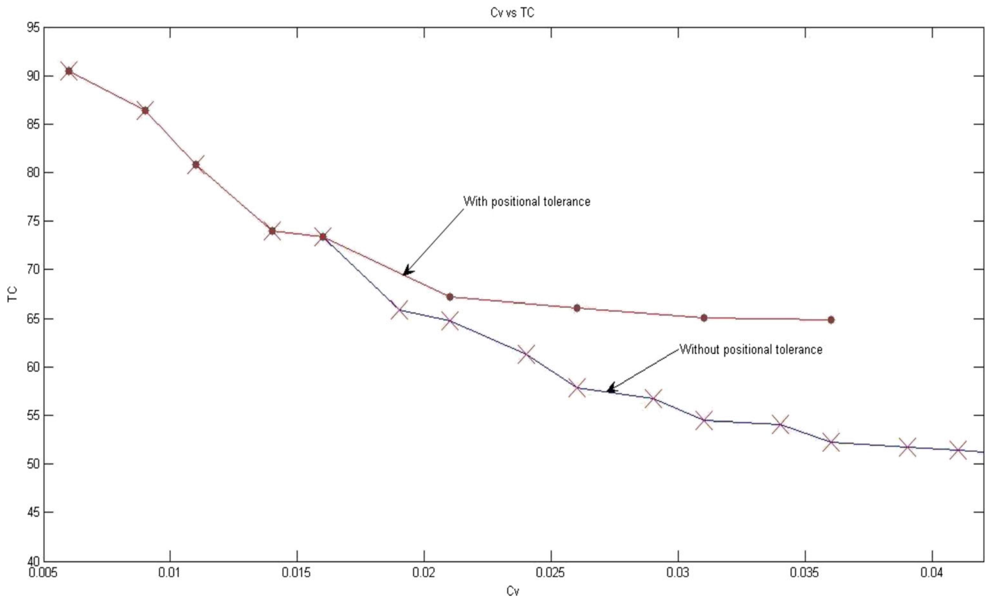


Fig. 16 Pareto front plots with and without consideration of positional tolerance

for the products such as piston-cylinder assembly, connecting rod big end and crank pin of a crank shaft. This section presents the extension of the proposed tolerance allocation model with positional tolerance consideration. As per ASME Y 14.5M [33], form and orientation tolerances that are critical to function and interchangeability are to be specified where the tolerances of size and location do not provide sufficient control. A form or orientation tolerance specifies a zone within which the considered feature, its line elements, its axis, or its center plane must be contained. Certain designs require control over a limited area or length. In case of proper shaft-hole assemblies, the center distance between them must be controlled with in certain limit. The position tolerance defines a zone, within which the center of one part (hole) is permitted to deviate from the center of the other part (shaft) and is dependent on the diameters of the shaft and hole [34]. Figure 15 illustrates the positional tolerance zone.

Under the consideration of positional tolerance, the distance between the centers of the shaft and hole depends on the sizes of them and it is equal to the difference upper size of the hole and lower size of shaft. The positional tolerance ΔP restricts the maximum allowable center distance between the

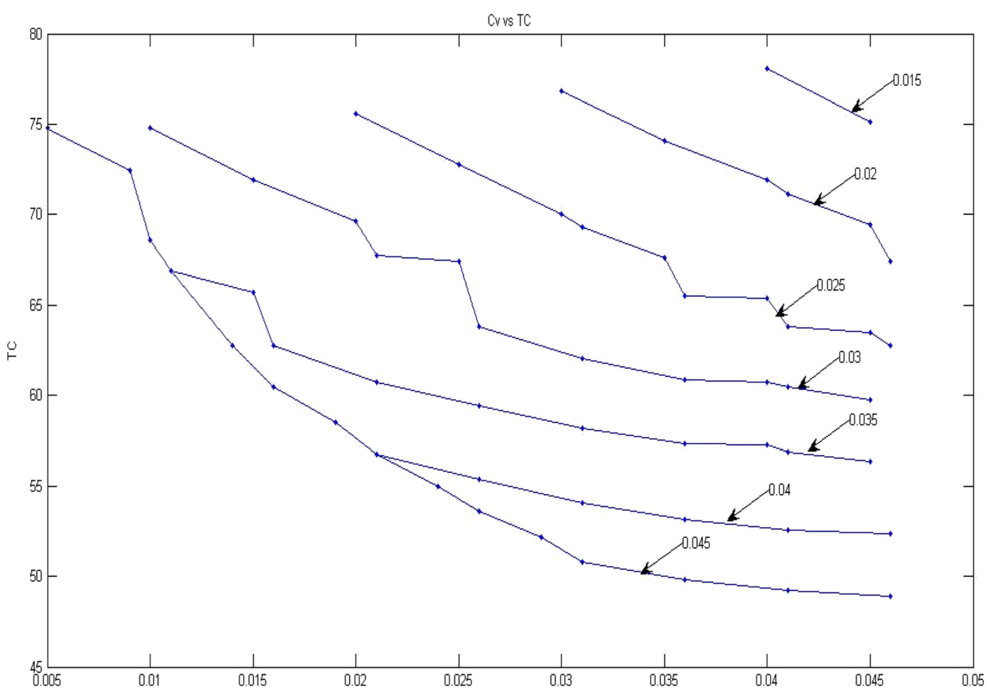
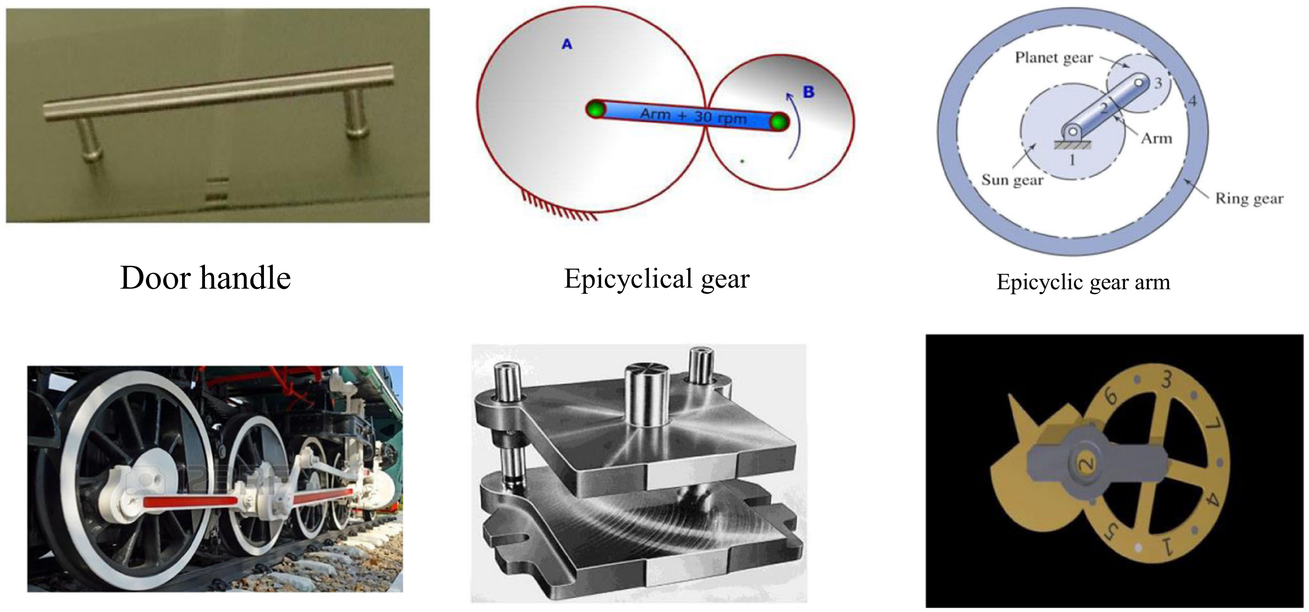


Fig. 17 Pareto front plot for various positional tolerance values



Door handle Epicyclical gear Epicyclic gear arm

Locomotive engine wheel crank Die set with two alignment guide post Inverted Geneva mechanism

Fig. 18 Typical examples of parts with two shafts aligned with two holes

shaft and hole. Hence, the inclusion of positional tolerance in the proposed model ΔP restricts the upper tolerance limit of hole (t_j^u) and lower tolerance limit of shaft (t_i^l). The constraint given in Eq. 29 takes care of the above.

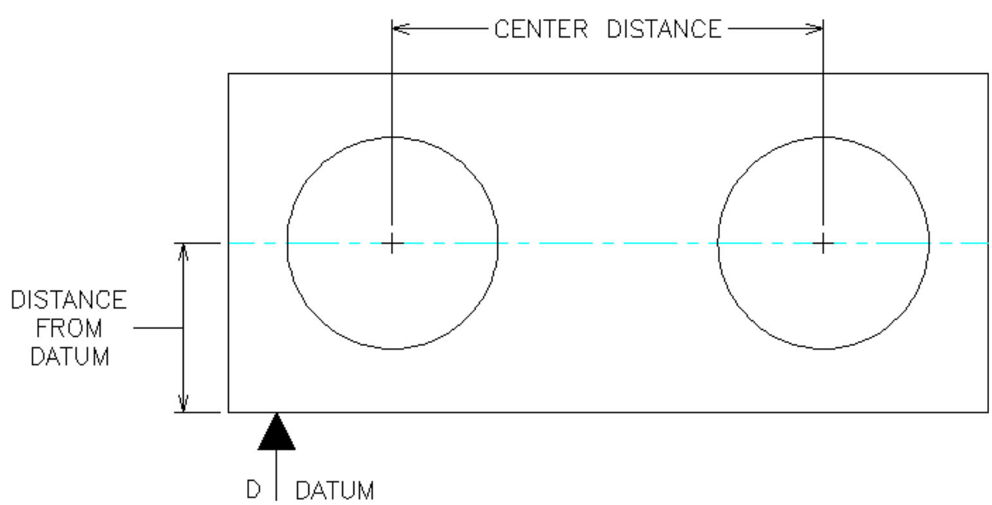
$$t_j^u - t_i^l \leq \Delta P \tag{29}$$

The positional tolerance should be less than the design specification. The effect of including positional error in the developed model is investigated with an example problem. Table 15 provides input parameter values of the example problem used for the illustration purpose. Figure 16 shows difference in Pareto front resulted with and without consideration of positional tolerance. It reveals that the inclusion of the positional tolerance leads to a decrease in a number of Pareto solutions, besides increasing the total cost of an assembly set. Figure 17 shows the Pareto front for various positional tolerance values. The number of Pareto front point reduces with the decrease in positional tolerance zone.

6.3.2 Linear constraint two shaft-two hole assemblies

There are applications in which two or more holes and shafts have to be aligned. Figure 18 shows the typical examples of parts with two shafts aligned with two holes. This section illustrates how the proposed model can be extended to such linearly constrained two shaft-two hole assemblies. Two features that control the two shafts-two holes assemblies are (i)

Fig. 19 Two shaft-two hole assembly with two features



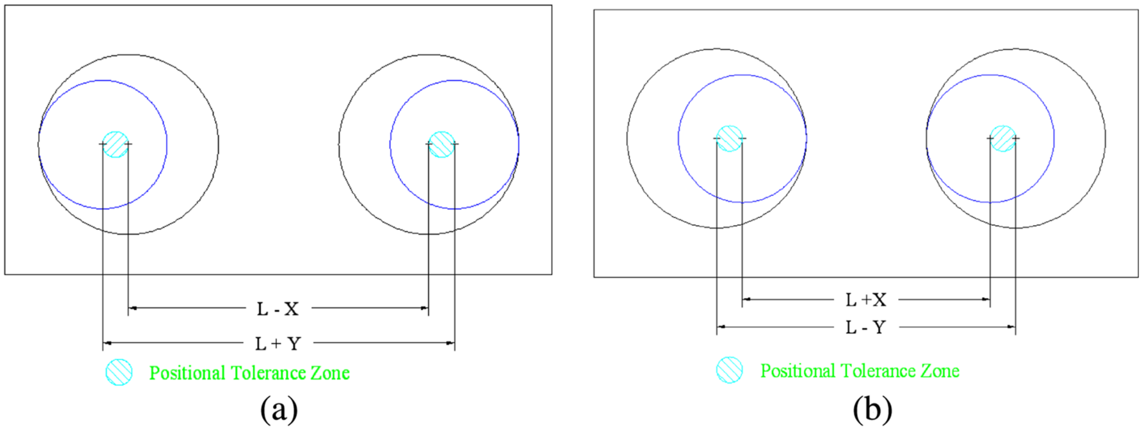


Fig. 20 **a**, **b** Two extremes of tolerance zone in a linear assembly

center distance and (ii) distance from datum reference. Figure 19 shows the above features.

Consider a two shaft-two hole linear assembly with:

- Nominal center to center distance for both hole and shaft (L)
- Allowable variation (i.e., center distance tolerance) between the centers of the holes ($\pm X$) and allowable variation between the centers of shafts ($\pm Y$)
- Allowable deviation of the centers of the hole-shaft or the axis from one reference surface/datum (ΔD)

Figure 20a, b shows the two extremes of the tolerance zone (linear tolerance zone) under the above allowable center distance tolerances of $\pm X$ and $\pm Y$. Hence, the linear tolerance zone ΔL due to center to center tolerances is the half of the sum of X and Y and is given in Eq. 30. This linear tolerance zone ΔL is restricted by the positional tolerance ΔP, which depends on the upper limit of the hole (t_j^u) and lower limit of the shaft (t_i^l). The maximum value of ΔP (= $t_j^u - t_i^l$) should necessary be less than ΔL. This restriction is an additional constraint imposed by the tolerance allocations and is given in Eq. 31. The linear tolerance is actually the total positional tolerance and its excess than positional tolerance is the bonus tolerance. Higher the tolerance values of center distances results in larger bonus tolerances.

$$\Delta L = (X + Y)/2 \tag{30}$$

$$t_j^u - t_i^l \leq \Delta \mathrm{L} \tag{31}$$

Besides, the axes have to be within a specified distance from one surface, i.e., the reference datum. Figure 21a, b shows the two extreme positions of the axis, which occur with lower limit of the hole t_j^l (i.e., Maximum Material Condition of the hole M_h) and the upper limit of the shaft t_i^u (i.e., Maximum Material Condition of the shaft M_s). Figure 21a, b shows that the deviation ΔD depends upon the maximum material conditions of the hole and the shaft. ΔD is equal to the position tolerance value ΔP. Conventionally, the maximum deviation ΔD is represented with circled modifier M [, 35]. The limitation on this allowance ΔD thus becomes the other constraint on the tolerances of the lower limit of the hole t_j^l and upper limit of the shaft t_i^u and it is governed by Eq. 32. Equation 33 shows the difference between maximum size of the hole and the minimum size of the shaft should be less than ΔD. Equations 31 and 32 or 33 are the additional constraints for applying the proposed model to the two shaft-two hole assemblies.

$$t_j^l - t_i^u \leq \Delta D \ or \ \ \Delta P \tag{32}$$

$$M_h - M_s \leq \Delta D \ or \ \ \Delta P \tag{33}$$

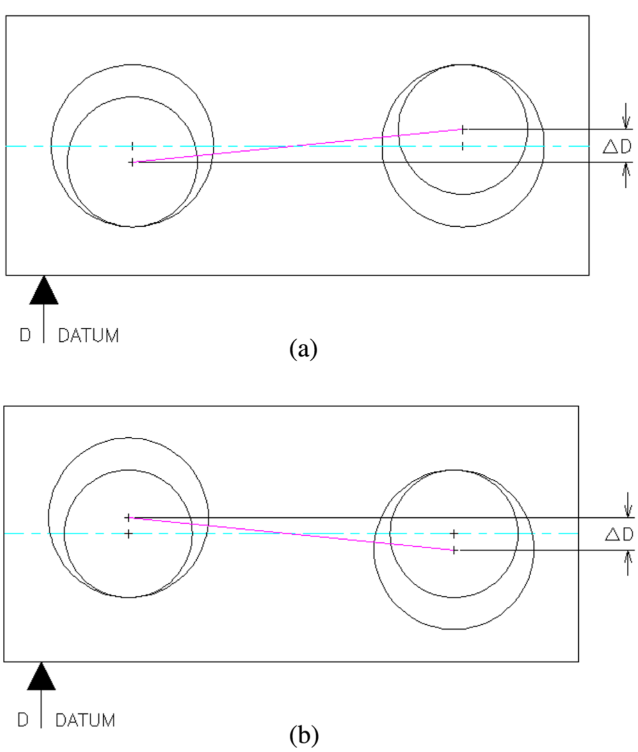


Fig. 21 **a**, **b** Two extreme position of the axis with respect to datum

7 Conclusion

This paper emphasizes the newly developed tolerance allocation model based on the process capabilities of the machine. It is denoted in the form of standard deviation, when the tolerance allocation plays vital role in an assembly. The selection of combination of machines is very significantly influencing the quality and total cost. In this paper, a bi-objective "minimum total cost and minimum clearance variation" for manufacturing tolerance allocation model for an interchangeable assembly of shaft and hole considering surplus part is formulated to balance the need of medium- and large-scale industries where exist more number of machines for shaft manufacturing and hole manufacturing. ESP SAA are proposed to obtain a Pareto optimal tolerance set, the non-dominated minimum total cost and minimum clearance variation, corresponding to various cases. The results are illustrated with a sample data set.

The *Pareto* front may assist the customer to know the clearance variation that is possible to get for the cost he could invest, whereas the non-dominated Pareto front also helps the manufacturer to select the suitable machine for manufacturing an assembly. The manufacturer chooses the machines for manufacturing shaft and hole among the availability of machines based on the requirements of the customer. The two shaft-two hole assembly model has discussed with positional tolerance considerations and the two extreme conditions of shaft-hole assembly considering the center distance between the shaft and hole. The addition bonus tolerance is also discussed. Besides the model extension, this paper further discussed how this formulation and methodologies can be used for multiple shaft-hole assemblies.

This study can be further worked on by relaxing the assumptions for known manufacturing cost of shaft and hole, instead the manufacturing cost can be derived from more realistic tolerance-cost functions such as linear, reciprocal, reciprocal squared, reciprocal power, exponential, and root sum square. Furthermore, by developing tolerance allocation model for parts having multiple operations (e.g., shaft has operation like rough tuning, finish turning, grinding) and including machining time is another interesting area for research. Finally, solving these kinds of problems by other machine learning-based mechanisms and newly developed metaheuristics would be another interesting and important research area for future research.

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