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ORIGINAL ARTICLE

# Linking product quality and customer behavior for performance analysis and optimization of make-to-order manufacturing systems

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**Abstract** A queueing network model is developed for understanding the way product quality may affect the profitability of production systems, when the consumers base their future demand patterns on the quality of the products they have recently purchased. We examine a multistage make-to-order system which receives orders from regular and occasional customers, the former having a higher mean demand rate. Each outgoing item undergoes inspection and quality grading to decide whether it will be discarded as nonconforming or shipped to a customer. In the latter case, the customer who purchases the item will subsequently become a regular or occasional customer with complementary probabilities which depend on the quality level of that item. The solutions of simple test cases with dynamic programming show that the optimal policy is state-dependent, complex, and computationally intensive. A much simpler, threshold-type policy is proposed, whose performance evaluation and optimization uses closed queueing network formulas and has minimal computational requirements. Numerical results indicate that the proposed policy performs almost as well as the optimal policy.

**Keywords** Production control · Quality control · Markov decision processes · Closed queueing networks

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## 1 Introduction

Customer satisfaction is considered to provide companies with a competitive advantage for sustaining and increasing their market shares, sales, and revenue rates. The nature of customer satisfaction, its key determinants, and consequences have been examined in [1, 2]. Even though it is hard to deny its benefits, the idea of incorporating customer satisfaction in the company strategy was confronted at first with much of reservedness from managers. This was due to a lack of tangible and measurable evidence of its impact on financial returns. According to an opinion survey among major US firms, only 28% of them could relate their customer satisfaction measures to accounting returns and only 27% to stock returns [3]. According to [1], a failure of establishing a link between customer satisfaction and economic performance may discourage firms from investing on product quality and customer satisfaction. Research concerning the interplay between customer satisfaction and marketing strategy suggests that keeping an existing customer may be a better strategy than attracting a new one [4, 5].

A method to assess the effect of satisfaction on customer retention and market share was provided for the first time in [6]. A relevant study [7] identified factors affecting customer satisfaction, studied the contribution of customer satisfaction to the financial performance of a firm, and observed asymmetries in the effects of opposite variations (satisfaction, dissatisfaction) on profit.

Although numerous studies have examined certain factors linking customer satisfaction and financial corporate results, no universal approach has been proposed, mainly due to several other factors that may significantly affect this linkage (e.g., product or firm-related characteristics, market or other environmental factors). However, it is widely accepted that a higher customer satisfaction leads to higher levels of repurchase intention and, in turn, higher levels of revenue and profitability (see [8] for a detailed discussion). In addition, other intermediate factors (e.g., customer retention, customer loyalty) may increase the complexity of the previous linkage.

For manufacturing systems, customer satisfaction is a complex concept which depends on many factors, such as product quality, price, warranty terms, and lead time in filling customer orders. Although customer satisfaction has received considerable attention by marketing researchers, it has not yet been studied to the same extent jointly with the problems of production and quality control. The majority of studies concerning customer satisfaction focus on product pricing and lead-time decisions, even though product quality is a key factor in customer satisfaction. Models for studying the behavior of stockout-averse customers of a multiperiod inventory system with partially observable, service-dependent demand are proposed in [9]. A somehow dual situation is considered in [10], where a single customer chooses randomly between two suppliers depending on credibility factors, which are decreasing functions of the stockouts experienced in the past when ordering from each firm. The firms make dynamic inventory decisions by observing the current inventories and credibilities of each other.

In the literature, quality is usually specified by the deviations of a product's principal functional characteristics from the specified target value of the product design specification. The economic losses caused by these deviations are called quality loss. These costs may include loss of sales and loss of producer's goodwill. A common quality loss function is the quadratic function, which has been proposed by Taguchi et al. [11]. This approach is a rather simple approximation and it does not describe the complex dynamics between production decisions, product quality, customer satisfaction, and market shares.

Recently, there has been a growing interest in the interconnection of production control and quality control. Towards this end, problems in which production design or control decisions are coupled with quality control strategies have received considerable attention [12–16]. A recent literature review on this topic can be found in [17].

A common quality control practice is the design of complete inspection plans, also known as screening or 100%-inspection procedures. Detailed literature reviews on the design of screening procedures can be found in [18, 19]. The problem of coordinated production and quality control in manufacturing systems when screening is applied has been studied in detail [20–23]. In these works, it has been clearly shown that a joint consideration of production and quality control problems results in a considerable improvement of manufacturing systems performance.

Our goals in this work are to examine how product specification decisions interact with customer satisfaction and market share and to propose simple and efficient policies, in order to increase manufacturing systems profitability.

A recent study [24] reported on the impact of quality control on customer satisfaction and market size for single-stage production systems. This work extends the analysis to complex multistage manufacturing systems. In Sections 2 and 3, we describe a multistage, make-to-order manufacturing system with quality-dependent customer behaviors. We formulate a quality control problem for maximizing the average profit rate of the system. It turns out that the optimal policy requires exact knowledge of the current market state (numbers of regular and occasional customers) and the current backlog (number of unfilled orders). In Section 4, we propose a threshold-type heuristic quality control policy, which is easily computable and implementable and does not require any state information. In Section 5 we present a numerical comparison between the optimal and heuristic policies. Concluding remarks are presented in Section 6.

## 2 Inspection planning with quality-dependent customer satisfaction and classification

Consider a make-to-order production system that manufactures a single type of product. The system serves a market comprising a total of $M$ customers, where $M$ is constant. At each time instant, a fraction of these customers have recently placed orders which are still outstanding. Each of the remaining customers belongs to one of two distinct classes: the class of regular or satisfied customers, denoted $i = 1$, and the class of occasional or dissatisfied customers, $i = 2$. Regular customers have a higher loyalty and a tendency to make more frequent purchases in the future than occasional customers. The system receives orders from customers of either class who currently have no orders outstanding. Thus, $M$ is the sum of three time-varying state variables:

| | |
|---|---|
| $n_1$ | number of regular customers who currently have no outstanding orders, |
| $n_2$ | number of occasional customers who currently have no outstanding orders, |
| $M - n_1 - n_2$ | number of customers awaiting fulfillment of outstanding orders. |

Customer demands occur according to independent Poisson processes. Each customer requests one unit of product and has a class-dependent mean demand rate, $\lambda_i$, $i = 1, 2$, where $\lambda_1 > \lambda_2$. Thus, the mean demand rate in state $(n_1, n_2)$ is $\lambda_1 n_1 + \lambda_2 n_2$. When a customer places an order, a raw item is released into the first production stage of the system, and when a product is finished, its quality is inspected and a decision is made as to whether this item will be scrapped or shipped to fill an outstanding order. Each outgoing product has acquired a certain quality level[1] $q$ with corresponding probability $p_q$, where $q = 0, 1, \ldots, L$, $p_0 + \ldots + p_L = 1$, and the product quality decreases with increasing $q$. If a product of quality level $q$ is sold to a customer, then this customer will eventually be satisfied with (conditional) probability $s_q$ or dissatisfied with the complementary probability. Satisfied customers join the regular class while dissatisfied ones become occasional customers. High-quality items have higher satisfaction probabilities; thus, $s_0 > s_1 > \ldots > s_L$.

The production system is assumed to be modeled as a network of single-machine, queueing nodes of the Jackson type. This assumption is adopted here in order to keep the formulation as simple as possible and for computational convenience. The analysis that follows can be extended to more general Markovian production networks.

For notational convenience, we assume that the system has a production facility with $N-2$ machines denoted $i = 3, 4, \ldots, N$, where machine 3 processes the raw material corresponding to each placed order and machine $N$ makes the final product, as shown in Fig. 1. Each machine is fed by a buffer of unlimited capacity, in which items from other machines are temporarily stored. The processing times at machine $i$ are independent random variables from an exponential distribution with mean $1/\mu_i$.

The flow of items in the system is described by a matrix $\Pi = [p_{i,j}]$, where $p_{i,j}$ is the routing probability from machine $i$ to machine $j$. For example, if machines $i$ and $i+1$ are consecutive in a production line then we have $p_{i,i+1} = 1$, whereas for more general geometries we have $p_{i,j} \geq 0$ and $\sum_j p_{i,j} = 1$ for all $i$. If a quality $q$ item goes out of machine $N$ and is sold to a customer, then either this customer will be satisfied and join the pool of class 1 customers with corresponding routing probability $p_{N,1} = s_q$ or will be dissatisfied and become a class 2 customer with corresponding routing probability $p_{N,2} = 1 - s_q$.

The objective is to find an optimal control policy, which dictates whether to sell or scrap an outgoing product of a given quality level $q$ so as to maximize the mean profit rate of the system. The mean profit rate depends on the sales rate (system throughput), the costs of rejected items, and the holding costs, with corresponding parameters.

$r$ profit per unit of product sold,

$c$ unit rejection cost (cost for scrapping and/or reworking an item),

$b_i$ per item and pending order holding cost rate at node (buffer, machine) $i$, $i = 3, \ldots, N$.

The parameters $b_i$ accumulate the unit inventory cost (cost of holding one item for one-time unit) and the unit backlog cost (cost of delaying an order by one-time unit) of node $i$, $i = 3, 4, \ldots, N$.

The state of the system is described by the vector $n = (n_1, n_2, n_3, \ldots, n_N)$ with state space $Z = \{n \mid n_1 + n_2 + n_3 + \ldots + n_N = M$ and $n_i \geq 0$ for all $i = 1, \ldots, N\}$. Whenever machine $N$ produces an item, we observe its quality level $q$ and the system state $n$, and then we make a selling decision $\pi(n, q)$, where $\pi(n, q) = 1$ if the item will be shipped to a waiting customer and $\pi(n, q) = 0$ if it will be scrapped (discarded). In the latter case, a new raw item is released into the buffer of machine 3; thus, $n_N$ is reduced by one and $n_3$ is increased by one. When we sell an item, $n_N$ also decreases by one and either $n_1$ increases by one if the customer is satisfied with the item's quality or $n_2$ increases by one. Analogous changes in the state of the system take place when an entity (item or customer order) moves from one node of the system to another: a move from $i$ to $j$ causes $n_i$ to decrease by one and $n_j$ to increase by one. The new state is expressed in vector notation as $n - e_i + e_j$ where $e_i$ is a vector having unity in the $i$th place and zeros elsewhere.

Three types of events are defined:

- order placement by a type $i$ customer ($i = 1$ for regular customers, $i = 2$ for occasional customers),
- completion of a semi-finished item at machine $i = 3, \ldots, N-1$,
- production of a quality $q$ finished item at machine $N$ and a selling or scrapping decision.

Because the interevent times are exponentially distributed, we can formulate the optimization problem as a Markov decision process. We do this using the uniformization technique [25] and a homogeneous Poissson process with rate $\nu = M\lambda_1 + \mu_3 + \mu_4 + \ldots + \mu_N$. Let $V_k(n)$ denote the value of the maximum expected profit over the first $k$ events of the Poisson process when the initial state is $n$. The dynamic programming equations for the optimal expected profit over the first $k+1$ events are

$$\begin{aligned} V_{k+1}(n) = \frac{1}{\nu}\cdot\{ & n_1\lambda_1 V_k(n-e_1+e_3) + n_2\lambda_2 V_k(n-e_2+e_3) \\ & + \sum_{i=3}^{N-1}\left[\mu_i I_{n_i>0}\sum_{j=3}^{N} p_{i,j} V_k\left(n-e_i+e_j\right)\right] \\ & + \mu_N I_{n_N>0}\sum_{q=0}^{L} p_q \max\left[s_q V_k(n-e_N+e_1) + \left(1-s_q\right)V_k(n-e_N+e_2) + r,\right. \\ & \left. V_k(n-e_N+e_3)-c\right] \\ & + \left(\nu - n_1\lambda_1 - n_2\lambda_2 - \sum_{i=3}^{N}\mu_i I_{n_i>0}\right)V_k(n) - \sum_{i=3}^{N} b_i n_i \} \end{aligned} \quad (1)$$

for every $n \in Z$, where $V_0(n) \equiv 0$ and

$$I_C = \begin{cases} 1 & \textit{condition } C \textit{ is satisfied} \\ 0 & \textit{otherwise} \end{cases}$$

The terms inside the braces in Eq. 1 correspond to the arrival of a regular or an occasional customer (first two terms),

[1] Assessing a set of discrete quality levels is consistent with the quality management literature (e.g., six sigma approach).

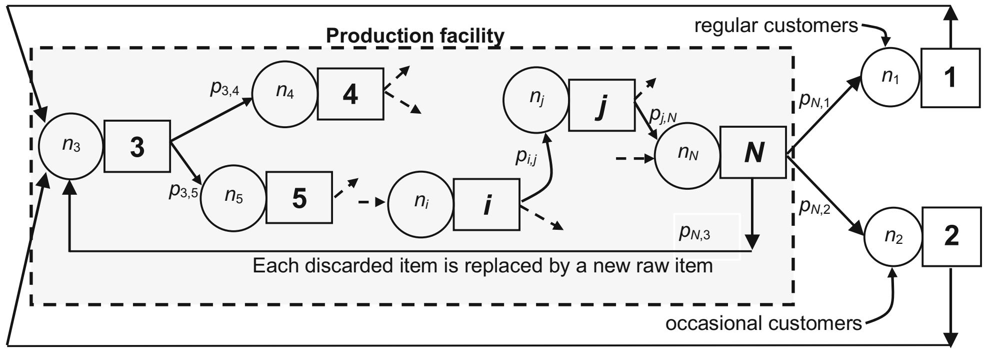


**Fig. 1** Make-to-order production system with two customer classes and scrapping of products

completion of a semi-finished part (the two nested summations), production of a final product followed by inspection and the most profitable decision (fourth term with max operation), a self-transition (dummy event, fifth term) and, finally, the overall holding cost in state $n$.

Because all states have self-transitions (i.e., $\nu$ is greater than the total transition rate out of any state), they are aperiodic. Also, the number of states in $Z$ and the number of decisions in each state are finite. As a result ([26] Theorem 9.4.5), the optimal long-run average profit $J^*$ of the system is given by

$$J^* = \nu \lim_{k\to\infty} [V_k(n) - V_{k-1}(n)] \quad (2)$$

for every $n \in Z$ and can be approximated numerically by value iteration (Eq. 1) on $V_k(n)$ for sufficiently large $k$.

## 3 Simple systems, complex policies

We consider here a few test cases of a single-machine system (i.e., $N = 3$) serving regular and occasional customers. By performing several dynamic programming iterations, we compute $J^*$ using Eq. 2 and the optimal decision at each state $n$ (the maximizer term in Eq. 1). It turns out that even for such simple production systems the optimal policy has a rather elaborate structure.

Standard parameter values of the test cases are $M = 50$, $\lambda_1 = 1$, $\lambda_2 = 0.1$, $\mu_3 = 60$, $r = 4$, $c = 3.5$, and $b_3 = 0.3$. Next we make a few additional assumptions only for establishing some plausible values for the quality level production probabilities $p_q$ and the corresponding probabilities of satisfaction $s_q$. The quality of every produced item is determined by the absolute deviation of the value of a certain quality characteristic $Y$ from a target value $t = 10$. We assume that $Y$ follows a normal distribution with a mean value equal to the target 10 and variance $\sigma^2 = 1$. This is a common assumption in the quality control literature. For each outgoing item, the characteristic $Y$ is screened and assigned one of the eight quality levels $q = 0, \ldots, 7$. For the experimental results reported herein the quality levels are determined as follows. We divide the interval $[t - 3\sigma, t + 3\sigma]$ containing the 0.997 probability mass of $Y$ values into 16 segments of equal length $3\sigma/8$, which are pairwise symmetric around $t$. The symmetric segments $[t - 3(q + 1)\sigma/8, t - 3q\sigma/8]$ and $[t + 3q\sigma/8, t + 3(q + 1)\sigma/8]$ are both assigned the quality level $q$, $q = 0, \ldots, 7$. The two symmetric intervals corresponding to $q = 7$ are modified to $(-\infty, t - 21\sigma/8]$ and $[t + 21\sigma/8, \infty)$ so as to cover all possible $Y$ values. Using normal probability approximations, we calculate the probability $p_q$ that $Y$ falls in one of the two symmetric intervals corresponding to quality level $q$. Finally, we assume that the probability of customer satisfaction $s_q$ is a sigmoid function of $q$. Sigmoid functions are commonly used in the literature to model different relationships between product quality and customer satisfaction (see, e.g., Grigoroudis and Siskos; 2010). The customer satisfaction and product quality probabilities for the test cases investigated are shown in Table 1.

Figure 2 shows the optimal policy as a function of $n_1$ (regular customers) and $n_3$ (pending orders) when the item produced is of quality level 4 (recall that quality decreases as level $q$ increases and that $n_2 = M - n_1 - n_3$). For quality levels 0 to 3 the optimal decision is $\pi(n, q) = 1$ for all system states $n$, while for quality levels 5 to 7 the optimal decision is $\pi(n, q) = 0$ for all states. This result agrees with intuition: it is worth scrapping products only if their quality is poor. In Fig. 2 we see the regions of acceptance and scrapping decisions within the state space for quality level $q = 4$. It becomes clear that in those quality levels where there are two competing decisions, it is optimal to sell a product of moderate quality (here $q = 4$) either when $n_1$ is large, i.e., there are already many regular customers in the market, or when $n_3$, the number of customers with pending orders, is large and the system incurs a high backlog cost rate $b_3n_3$. For the above parameters, the value iterations of Eq. 1 returned an optimal mean profit rate $J^* = 77.44$.

We now examine how sensitive the optimal policy is to changes in parameter values. First we use three different values for the revenue parameter $r$. The left graph of Fig. 3 shows the switching curves of the optimal policy

**Table 1** Product quality and customer satisfaction probabilities

| $q$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $p_q$ | 0.261 | 0.234 | 0.188 | 0.135 | 0.087 | 0.050 | 0.026 | 0.019 |
| $s_q$ | 0.950 | 0.938 | 0.868 | 0.692 | 0.458 | 0.282 | 0.212 | 0.200 |

for quality level 4. The regions on the left of each switching curve comprise all states ($n_1$, $n_3$) for which a scrapping decision is optimal and those on the right correspond to optimal selling decisions for quality 4 items. As the unit revenue $r$ increases, the number of states for which scapping is optimal within a certain quality level increases as well. For the same quality level we investigate how sensitive the optimal policy is to the changes of rejection cost value $c$. The right graph of Fig. 3 suggests that as the rejection cost decreases scrapping becomes more appealing, a result that agrees with intuition.

Next, we investigate the sensitivity of the optimal policy to the unit backlog cost $b_3$ and the order arrival rate $\lambda_1$ of regular customers. We use three different values for each parameter. Figure 4 shows the optimal policies returned at quality level 4. We see that when $b$ decreases, scrapping decisions are more often optimal than not, which is something we expect since a small backlog cost gives the opportunity to prolong the order lead times by scrapping items of relatively low quality, while higher backlog costs tend to make it more urgent to fill orders as soon as possible with quality becoming a secondary concern. The optimal policy is also sensitive to variations of $\lambda_1$. The smaller the arrival rate of regular customers (and yet higher than $\lambda_2$), the smaller the backlog and the easier it is for the system to scrap items so as to have as many satisfied customers (and regular) as possible, while avoiding excessive delays in filling customer orders.

We have also numerically examined the impacts of changing the production rate $\mu_3$ and the order arrival rate $\lambda_2$ on the optimal policy. In all cases, there was only one quality level in which both decisions are optimal, separated by a switching curve. However, there exist problem instances in which both decisions are present in several quality levels, as we shall see later.

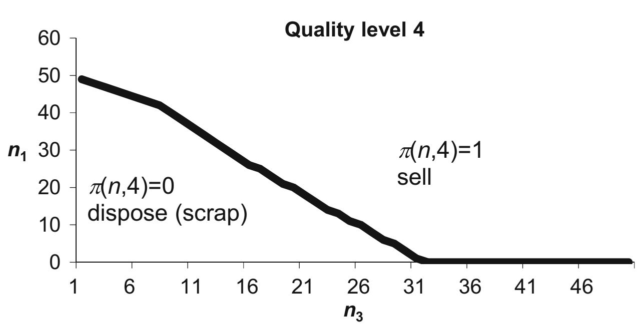


**Fig. 2** Optimal policy $\pi(n, q)$ for quality level $q = 4$

## 4 A heuristic threshold policy

### 4.1 Policy description

The numerical investigation of the previous section indicates that the optimal quality control policy does not have a simple structure because, in addition to the item's quality level, it depends on the state vector $n$ as well. This makes the optimal policy unappealing for practical use since human operators prefer using simple rules of thumb to complex computer generated solutions. Nevertheless there are some values of $q$ for which the optimal policy is independent of $n$. In the previous example, we always sell outgoing items of quality levels $q = 0$, 1, 2, 3 and always dispose items having $q = 5$, 6, 7. The only exception is when $q = 4$ where both decisions are applicable and the optimal policy has the form of a switching surface, as shown in Figs. 2, 3, and 4.

In this section, we consider a heuristic, threshold-type policy for ease in the implementation, as an alternative to the optimal policy. We assume that $Q$ is a quality threshold ($Q \leq L$) such that all items having $q > Q$ are rejected, otherwise they are shipped to the waiting customers. Then, the problem becomes one of finding the optimal $Q$ so as to maximize the mean profit rate. The major advantage of the proposed policy is its computational efficiency. For complex production networks the optimal policy cannot be computed due to the explosion of the state space. The derivation of the optimal policy is computationally demanding even for a two-stage production line. On the other hand, the heuristic policy requires fewer computations and may be applied to production systems with increased complexity. Apart from its computational advantage, the proposed threshold policy is applicable when the manufacturing firm has only partial information about the market status, e.g., when $M$ is known but the market shares $n_1$ and $n_2$ corresponding to regular and occasional customers are unknown.

### 4.2 Performance evaluation

When the system operates under the threshold-type quality control policy, it can be modeled as a closed queueing network (CQN) with $N$ nodes and $M$ jobs. Each node $i$, $i = 3, \ldots, N$, of the CQN has a single exponential server with mean service rate $\mu_i$ equal to the mean production rate of machine $i$ of the production facility. Nodes 1 and 2 represent, respectively, the mechanisms that generate arrivals of regular and occasional customers and at each time instant they have as many active servers as the number of jobs in their queues. The servers in these two nodes have exponentially distributed processing times with means $1/\lambda_i$, for $i = 1, 2$. The CQN and the production system have the same schematic representation shown in Fig. 1.

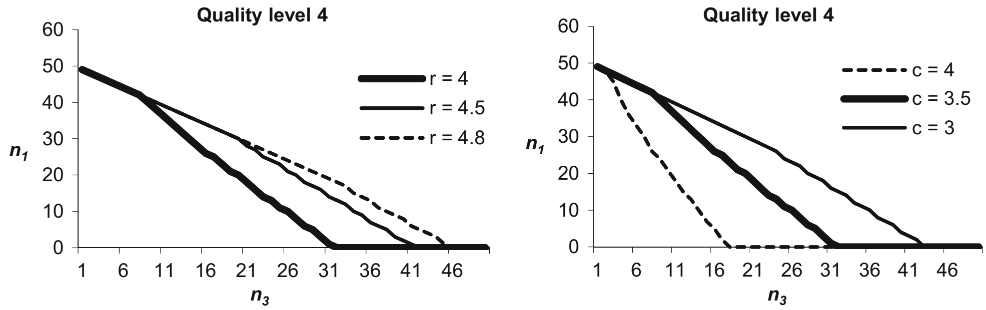


**Fig. 3** Optimal policies for $q = 4$ versus unit revenue and rejection costs

At any time instant, the state vector $n$ of the production system is the same as the vector of the number jobs in the nodes of the CQN. The routing probabilities of node $N$ depend on the quality of the corresponding outgoing item and the decision made in the production system. Under the threshold policy, the end item's quality $q$ is identified by inspection and if $q > Q$ (low quality), then this item is rejected and a new raw item is released into the first machine of the production facility. In essence, this is the same as a job being routed form node $N$ back to node 3, as shown in Fig. 1. The corresponding routing probability is given by $p_{N,3} = p_{Q+1} + \ldots + p_L$. Moreover, if the end item is of acceptable quality level ($q \leq Q$), then it will be purchased by a customer. With probability $p_{N,1} = p_0 s_0 + \ldots + p_Q s_Q$ the customer who purchases the item will be satisfied. In the equivalent CQN, a job departs from node $N$ and is routed to node 1 (satisfied customer becomes a regular customer). However, with probability $p_{N,2} = p_0(1 - s_0) + \ldots + p_Q(1 - s_Q)$ the customer will be dissatisfied and in the equivalent CQN the job will go to node 2 (dissatisfied customer becomes an occasional customer). The other routing probabilities of the CQN are the same as the ones of the production facility. Finally, all jobs coming from nodes 1 and 2 are routed to node 3 of the CQN, i.e., $p_{1,3} = p_{2,3} = 1$.

Next we summarize a known algorithm from queueing theory which permits the computation of the equilibrium probabilities $P(n) = P(n_1, \ldots, n_N)$ and all the components of the mean profit rate of the system.

Let $\mathrm{TH}_N$ denote the average outflow rate (throughput) of node $N$, $B_i = E(n_i)$ the average inventory/backlog of node $i$, $\Pi = [p_{i,j}]$ the matrix of routing probabilities, and $u = [u_1 \ldots u_N]$ *any* nonnegative solution of the system of linear equations $u = u\Pi$. The vector $u$ is determined only within a multiplicative constant, so we are free to choose a normalization scheme for the $u_i$ (e.g., $u_N = 1$ or $u_1 + \ldots + u_N = 1$). Let $\alpha_i(n_i)$ be the number of occupied servers in node $i$. We have that $\alpha_i(n_i) = 1$ for all $i \geq 3$ and $n_i \geq 1$, since each of these nodes has only one server, while for nodes 1 and 2 we have $\alpha_1(n_1) = n_1$ and $\alpha_2(n_2) = n_2$. Next, we recursively calculate the sequences $\beta_i(n_i) = \alpha_i(n_i)\beta_i(n_i - 1)$ using $\beta_i(0) = 1$ as a boundary value. It turns out that $\beta_i(n_i) = 1$ for $i \geq 3$ and $\beta_i(n_i) = n_i!$ for $i = 1, 2$. The equilibrium probabilities of the system are given by (see e.g. [27])

$$P(n) = \frac{1}{G(M)} \prod_{i=1}^{N} \frac{\rho_i^{n_k}}{\beta_i(n_i)}$$

where $\rho_1 = u_1/\lambda_1$, $\rho_2 = u_2/\lambda_2$, $\rho_i = u_i/\mu_i$, and $G(M)$ is is a normalization constant given by

$$G(M) = \sum_{n_1+\ldots+n_N=M} \left[ \prod_{i=1}^{N} \frac{\rho_i^{n_k}}{\beta_i(n_i)} \right]$$

The remaining performance indices of the system are given by.

$\mathrm{TH}_i = \frac{1}{G(M)} \rho_i G(M-1)$ and $B_i = \frac{1}{G(M)} \sum_{m=1}^{M} G(M-m)\rho_i^m$ for $i \geq 3$.and the overall mean profit rate

$$J = r(1-p_{N,3})\mathrm{TH}_N - c p_{N,3}\mathrm{TH}_N - \sum_{i=3}^{N} b_i B_i$$

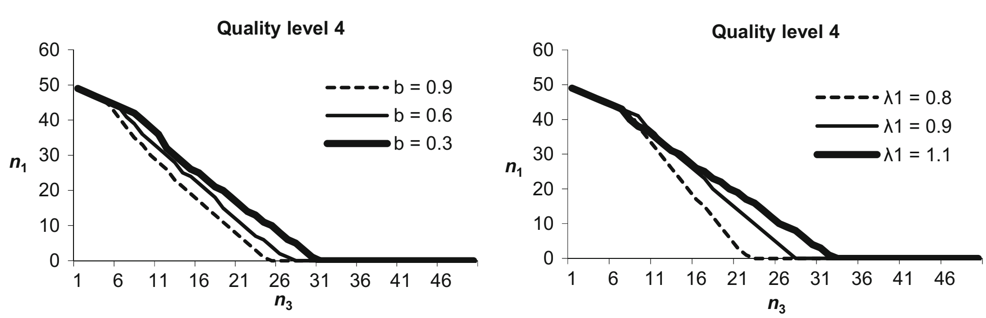


**Fig. 4** Optimal policies for $q = 4$ versus $b_3$ an $\lambda_1$

**Table 2** Comparison of policies for a single-stage system and various parameter values

| Parameter values | | | | | | Optimal policy | | Threshold policy | |
|---|---|---|---|---|---|---|---|---|---|
| $\lambda_1$ | $\lambda_2$ | $\mu$ | $c$ | $r$ | $b$ | Policy structure | Average profit | Average profit | Quality threshold |
| 1 | 0.1 | 60 | 3.5 | 4 | 0.3 | 1111M000 | 77.44 | 77.44 | Level 3 |
| 1 | 0.1 | 60 | 3.5 | 4 | 0.4 | 1111M000 | 77.15 | 77.15 | Level 3 |
| 1 | 0.1 | 60 | 3.5 | 4 | 0.9 | 1111M000 | 76.87 | 76.87 | Level 3 |
| 1 | 0.1 | 60 | 3.5 | 4 | 1.2 | 1111MM00 | 76.59 | 76.59 | Level 3 |
| 1 | 0.1 | 60 | 3.5 | 4 | 0.3 | 1111M000 | 77.44 | 77.44 | Level 3 |
| 1 | 0.1 | 60 | 3.5 | 3 | 0.3 | 11111M00 | 53.92 | 53.92 | Level 4 |
| 1 | 0.1 | 60 | 3.5 | 4.8 | 0.3 | 1111M000 | 96.75 | 96.75 | Level 3 |
| 1 | 0.1 | 60 | 3.5 | 4.5 | 0.3 | 1111M000 | 89.51 | 89.51 | Level 3 |
| 1 | 0.1 | 60 | 3.5 | 4 | 0.3 | 1111M000 | 77.44 | 77.44 | Level 3 |
| 1 | 0.1 | 60 | 3 | 4 | 0.3 | 1111M000 | 80.13 | 80.13 | Level 3 |
| 1 | 0.1 | 60 | 4 | 4 | 0.3 | 1111M000 | 74.74 | 74.74 | Level 3 |
| 1 | 0.1 | 60 | 2.5 | 4 | 0.3 | 11110000 | 82.82 | 82.82 | Level 3 |
| 1 | 0.1 | 60 | 3.5 | 4 | 0.3 | 1111M000 | 77.44 | 77.44 | Level 3 |
| 0.9 | 0.1 | 60 | 3.5 | 4 | 0.3 | 1111M000 | 73.98 | 73.98 | Level 3 |
| 1.1 | 0.1 | 60 | 3.5 | 4 | 0.3 | 1111M000 | 80.50 | 80.50 | Level 3 |
| 0.8 | 0.1 | 60 | 3.5 | 4 | 0.3 | 1111M000 | 70.07 | 70.07 | Level 3 |
| 1 | 0.1 | 60 | 3.5 | 4 | 0.3 | 1111M000 | 77.44 | 77.44 | Level 3 |
| 1 | 0.08 | 60 | 3.5 | 4 | 0.3 | 1111M000 | 68.15 | 68.15 | Level 3 |
| 1 | 0.15 | 60 | 3.5 | 4 | 0.3 | 1111MMM0 | 94.59 | 94.59 | Level 3 |
| 1 | 0.2 | 60 | 3.5 | 4 | 0.3 | 11111MMM | 108.83 | 108.83 | Level 4 |
| 1 | 0.1 | 60 | 3.5 | 4 | 0.3 | 1111M000 | 77.44 | 77.44 | Level 3 |
| 1 | 0.1 | 55 | 3.5 | 4 | 0.3 | 1111M000 | 77.12 | 77.12 | Level 3 |
| 1 | 0.1 | 65 | 3.5 | 4 | 0.3 | 1111M000 | 77.67 | 77.67 | Level 3 |
| 1 | 0.1 | 50 | 3.5 | 4 | 0.3 | 1111M000 | 76.69 | 76.69 | Level 3 |

The value $J$ corresponds to the choice of the quality threshold $Q = 0, \ldots, L$, which determines the routing probabilities $p_{N,1}$, $p_{N,2}$ and $p_{N,3}$, and, eventually, the equilibrium probabilities of the system. To find the optimal $Q$ we perform an exhaustive search. We initialize the optimal policy, setting $J^* = -\infty$ and the quality threshold $Q = 0$. For any given $Q$, we compute the corresponding routing probabilities $p_{N,1}$, $p_{N,2}$, and $p_{N,3}$, and equilibrium probabilities $P(n)$ of the CQN and the overall mean profit rate $J$ of the system. If $J < J^*$, then we set $J^* = J$ and $Q^* = Q$. We increase $Q$ by one and repeat the above steps until all $Q \leq L$ have been evaluated. The optimal pair is $(Q^*, J^*)$.

## 5 Numerical experiments

In this section, we numerically compare the proposed threshold policy and the optimal policy to see if the former can be applied as an easily implementable alternative of the latter at a small cost in performance.

### 5.1 Single-stage test case

First we examine a single-stage manufacturing system as the one described in Section 2. We use $M = 50$ and the same quality level probabilities $p_q$ and satisfaction probabilities $s_q$ as in Section 3 (Table 1). Table 2 shows the remaining parameter values and the corresponding optimal policies. The column "Policy structure" describes the optimal policy for each problem instance. With only a few exceptions, for quality levels close to 0 and close to 7, the optimal decision is, respectively, to accept (1) and reject (0) the item regardless of the system state $n$. The quality levels for which the optimal decision depends on $n$ are indicated in Table 2 with the letter M. For example, the entry 1111M000 in the first row of the table indicates that a selling decision (1) is optimal for $q = 0, \ldots, 3$ and scrapping (0) is optimal for $q = 5, 6, 7$ both regardless of $n$, while for $q = 4$ we have two separate regions of the state space where each decision is optimal. We observe that the threshold $Q$ of the optimal heuristic policy always equals *one less* the number of 1's of the optimal decision.

**Table 3** Comparison of policies for a two-stage system and various parameter values

| Parameter values | | | | | | | | Optimal policy | | Threshold policy | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\lambda_1$ | $\lambda_2$ | $\mu_3$ | $\mu_4$ | $c$ | $r$ | $b_3$ | $b_4$ | Policy structure | Average profit | Average profit | Quality threshold |
| 0.12 | 0.015 | 6.5 | 5.5 | 3.5 | 4 | 0.25 | 0.4 | 1111MMMM | 9.31 | 9.31 | Level 4 |
| 0.12 | 0.015 | 6.5 | 5.5 | 2.5 | 4 | 0.25 | 0.4 | 1111MMMM | 9.68 | 9.60 | Level 4 |
| 0.12 | 0.015 | 6.5 | 5.5 | 3 | 4 | 0.25 | 0.4 | 1111MMMM | 9.46 | 9.45 | Level 4 |
| 0.12 | 0.015 | 6.5 | 5.5 | 4 | 4 | 0.25 | 0.4 | 11111MMM | 9.17 | 9.16 | Level 4 |
| 0.12 | 0.015 | 6.5 | 5.5 | 3.5 | 3 | 0.25 | 0.4 | 11111MMM | 6.64 | 6.64 | Level 5 |
| 0.12 | 0.015 | 6.5 | 5.5 | 3.5 | 3.5 | 0.25 | 0.4 | 11111MMM | 7.95 | 7.93 | Level 4 |
| 0.12 | 0.015 | 6.5 | 5.5 | 3.5 | 5 | 0.25 | 0.4 | 1111MMMM | 12.13 | 12.06 | Level 4 |
| 0.12 | 0.015 | 6.5 | 5.5 | 3.5 | 4 | 0.2 | 0.4 | 1111MMMM | 9.35 | 9.35 | Level 4 |
| 0.12 | 0.015 | 6.5 | 5.5 | 3.5 | 4 | 0.3 | 0.4 | 1111MMMM | 9.27 | 9.26 | Level 4 |
| 0.12 | 0.015 | 6.5 | 5.5 | 3.5 | 4 | 0.35 | 0.4 | 1111MMMM | 9.23 | 9.22 | Level 4 |
| 0.12 | 0.015 | 6.5 | 5.5 | 3.5 | 4 | 0.25 | 0.3 | 1111MMMM | 9.43 | 9.43 | Level 4 |
| 0.12 | 0.015 | 6.5 | 5.5 | 3.5 | 4 | 0.25 | 0.5 | 1111MMMM | 9.19 | 9.19 | Level 4 |
| 0.12 | 0.015 | 6.5 | 5.5 | 3.5 | 4 | 0.25 | 0.55 | 11111MMM | 9.14 | 9.13 | Level 4 |
| 0.1 | 0.015 | 6.5 | 5.5 | 3.5 | 4 | 0.25 | 0.4 | 11111MMM | 8.70 | 8.70 | Level 4 |
| 0.15 | 0.015 | 6.5 | 5.5 | 3.5 | 4 | 0.25 | 0.4 | 1111MMMM | 9.99 | 9.99 | Level 4 |
| 0.18 | 0.015 | 6.5 | 5.5 | 3.5 | 4 | 0.25 | 0.4 | 1111MMMM | 10.49 | 10.48 | Level 4 |
| 0.12 | 0.01 | 6.5 | 5.5 | 3.5 | 4 | 0.25 | 0.4 | 1111MMMM | 7.48 | 7.46 | Level 3 |
| 0.12 | 0.02 | 6.5 | 5.5 | 3.5 | 4 | 0.25 | 0.4 | 11111MMM | 10.72 | 10.70 | Level 5 |
| 0.12 | 0.025 | 6.5 | 5.5 | 3.5 | 4 | 0.25 | 0.4 | 111111MM | 11.93 | 11.93 | Level 7 |
| 0.12 | 0.015 | 5.5 | 5.5 | 3.5 | 4 | 0.25 | 0.4 | 1111MMMM | 9.18 | 9.17 | Level 4 |
| 0.12 | 0.015 | 6.0 | 5.5 | 3.5 | 4 | 0.25 | 0.4 | 1111MMMM | 9.25 | 9.25 | Level 4 |
| 0.12 | 0.015 | 7.0 | 5.5 | 3.5 | 4 | 0.25 | 0.4 | 1111MMMM | 9.35 | 9.35 | Level 4 |
| 0.12 | 0.015 | 6.5 | 4.5 | 3.5 | 4 | 0.25 | 0.4 | 11111MMM | 8.97 | 8.92 | Level 4 |
| 0.12 | 0.015 | 6.5 | 5 | 3.5 | 4 | 0.25 | 0.4 | 1111MMMM | 9.16 | 9.15 | Level 4 |
| 0.12 | 0.015 | 6.5 | 6.5 | 3.5 | 4 | 0.25 | 0.4 | 1111MMMM | 9.50 | 9.50 | Level 4 |

Recall that the first bit 1 corresponds to $q = 0$. This means that $Q$ coincides with the last quality level for which the decision to sell is always optimal. Moreover, we see that the mean profit rates of the two policies are equal to two decimal places. Thus, for the datasets used in this study, the proposed policy seems to be a very good approximation of the optimal policy. We also observe that the acceptable quality thresholds in both policies are rather insensitive to parameter variations except for $\lambda_2$. As $\lambda_2$ increases, occasional customers tend to have a similar purchasing behavior as the satisfied ones and, therefore, a high level of sales rate can be maintained even with a large share of occasional customers by selling products of lower quality and avoiding scrapping costs.

### 5.2 Production lines with multiple machines

We now test the threshold policy in multi-machine production lines. Two cases are examined: a two-stage production line, and a three-stage line. As previously, we use eight quality levels and the same probabilities $p_q$ and $s_q$ (Table 1). To avoid the problem of state-space explosion we assume that the market comprises a total of $M = 25$ customers for the three-stage line. In the case of the two-stage system we have used $M = 50$ as in the first test case. Table 3 shows the other parameter values and the corresponding optimal policy parameters and profit rates for the two-stage system. Results and parameter values for the three-stage system are presented in Table 4. As the number of stages increases, one may observe that the complexity of the optimal policy increases too. In multi-stage test cases it is never optimal to fully scrap items of very poor quality. Even for the worst quality level, scrapping decision depends on systems state. This seems quite reasonable as in the case of item rejection, we may have a significant increase of customers waiting times, especially when the number of pending orders in early production stages is high. The complexity of the optimal policy does not seem to have an effect on the performance of the heuristic policy. As previously, compared to the optimal

**Table 4** Comparison of policies for a three-stage system and various parameter values

| Parameter values | | | | | | | | | | Optimal policy | | Threshold policy | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\lambda_1$ | $\lambda_2$ | $\mu_3$ | $\mu_4$ | $\mu_5$ | $c$ | $r$ | $b_3$ | $b_4$ | $b_5$ | Policy structure | Average profit | Average profit | Quality threshold |
| 0.25 | 0.03 | 6 | 5.5 | 5 | 3.5 | 4 | 0.25 | 0.3 | 0.3 | 11111MMM | 8.50 | 8.48 | Level 5 |
| 0.25 | 0.03 | 6 | 5.5 | 5 | 3 | 4 | 0.25 | 0.3 | 0.3 | 11111MMM | 8.59 | 8.57 | Level 4 |
| 0.25 | 0.03 | 6 | 5.5 | 5 | 4 | 4 | 0.25 | 0.3 | 0.3 | 11111MMM | 8.44 | 8.42 | Level 5 |
| 0.25 | 0.03 | 6 | 5.5 | 5 | 3.5 | 3.5 | 0.25 | 0.3 | 0.3 | 11111MMM | 7.30 | 7.28 | Level 6 |
| 0.25 | 0.03 | 6 | 5.5 | 5 | 3.5 | 4.5 | 0.25 | 0.3 | 0.3 | 11111MMM | 9.74 | 9.72 | Level 4 |
| 0.25 | 0.03 | 6 | 5.5 | 5 | 3.5 | 4 | 0.2 | 0.3 | 0.3 | 11111MMM | 8.54 | 8.51 | Level 5 |
| 0.25 | 0.03 | 6 | 5.5 | 5 | 3.5 | 4 | 0.3 | 0.3 | 0.3 | 11111MMM | 8.46 | 8.44 | Level 5 |
| 0.25 | 0.03 | 6 | 5.5 | 5 | 3.5 | 4 | 0.25 | 0.35 | 0.3 | 11111MMM | 8.46 | 8.44 | Level 5 |
| 0.25 | 0.03 | 6 | 5.5 | 5 | 3.5 | 4 | 0.25 | 0.4 | 0.3 | 11111MMM | 8.42 | 8.40 | Level 5 |
| 0.25 | 0.03 | 6 | 5.5 | 5 | 3.5 | 4 | 0.25 | 0.3 | 0.35 | 11111MMM | 8.45 | 8.43 | Level 5 |
| 0.25 | 0.03 | 6 | 5.5 | 5 | 3.5 | 4 | 0.25 | 0.3 | 0.4 | 11111MMM | 8.40 | 8.38 | Level 5 |
| 0.2 | 0.03 | 6 | 5.5 | 5 | 3.5 | 4 | 0.25 | 0.3 | 0.3 | 11111MMM | 7.94 | 7.93 | Level 5 |
| 0.3 | 0.03 | 6 | 5.5 | 5 | 3.5 | 4 | 0.25 | 0.3 | 0.3 | 11111MMM | 8.91 | 8.88 | Level 5 |
| 0.25 | 0.02 | 6 | 5.5 | 5 | 3.5 | 4 | 0.25 | 0.3 | 0.3 | 1111MMMM | 6.88 | 6.87 | Level 4 |
| 0.25 | 0.04 | 6 | 5.5 | 5 | 3.5 | 4 | 0.25 | 0.3 | 0.3 | 11111MMM | 9.79 | 9.79 | Level 7 |
| 0.25 | 0.03 | 5.5 | 5.5 | 5 | 3.5 | 4 | 0.25 | 0.3 | 0.3 | 11111MMM | 8.44 | 8.42 | Level 5 |
| 0.25 | 0.03 | 6.5 | 5.5 | 5 | 3.5 | 4 | 0.25 | 0.3 | 0.3 | 11111MMM | 8.55 | 8.52 | Level 5 |
| 0.25 | 0.03 | 6 | 5 | 5 | 3.5 | 4 | 0.25 | 0.3 | 0.3 | 11111MMM | 8.41 | 8.39 | Level 5 |
| 0.25 | 0.03 | 6 | 6 | 5 | 3.5 | 4 | 0.25 | 0.3 | 0.3 | 11111MMM | 8.57 | 8.54 | Level 5 |
| 0.25 | 0.03 | 6 | 5.5 | 5.5 | 3.5 | 4 | 0.25 | 0.3 | 0.3 | 11111MMM | 8.59 | 8.57 | Level 5 |
| 0.25 | 0.03 | 6 | 5.5 | 6 | 3.5 | 4 | 0.25 | 0.3 | 0.3 | 11111MMM | 8.67 | 8.64 | Level 4 |

policy obtained by value iteration, the proposed threshold-type policy achieves almost the same profit rates with minimal computational cost. The maximum deviation from the optimal profit rate is less than 0.9% and only in one or two occasions this deviation exceeds 0.5%. As in the single-machine case, when $\lambda_2$ increases, it is optimal for the system to sell products of inferior quality, thus avoiding a fraction of the scrapping costs and increasing the population of occasional customers without jeopardizing future sales.

## 6 Conclusions

In this paper, we have used dynamic programming and queueing theory to model the impact of product quality variations on customer satisfaction, market share and profitability in a class of make-to-order production systems. It was shown that the disposable quality levels which maximize profitability are not constant but depend on the congestion in the production facility as well as on the market share (regular customers) of the company. For situations in which it is desirable to maintain a constant quality level for the products or when there is partial market information a simple threshold-type control policy is proposed, which has reasonable computational requirements and appears to be a very good approximation of the optimal policy for single-stage as well as multistage manufacturing systems. The inclusion of past purchasing experience, as well as order lead time performance (customer delays) in the modeling of customer satisfaction will be the subjects for further research. Such extensions pose both computational and theoretical challenges due to the enormity of state space and the violation of the memoryless property, which allows for efficient solution algorithms of queueing problems. Other more straightforward extensions of the model would be the inclusion of more than two customer satisfaction states, the analysis of different customer behaviors and alternative functions for linking $p_q$ and $s_q$ with $q$, and the consideration of market competition and the study of different competition intensities (e.g., different patterns of customer satisfaction, different combinations of mean demand rates). In addition, more complex systems with both final and intermediate inspection stations subject to inspection errors and locally repairable nonconformities in addition to the scrapping of items can also be modeled by extending the models presented herein and in [28]. Finally, another research direction to be considered in further work is the application of the models using historical data from real-world companies.

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