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ORIGINAL ARTICLE

5-Axis CNC incremental forming toolpath generation based on formability

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Abstract To the problem of the existing CNC incremental forming that mostly adopts the 3-axis CNC incremental forming method and cannot form the complex sheet metal part containing the surface by one-stage forming, which cannot be visible in the positive and negative directions of the *Z*-axis and whose forming angle is larger than 90°, a multi-directional slant and multi-directional pressing toolpath generation method for the 5-axis CNC incremental forming was proposed to accomplish the forming of the complex sheet metal parts by fully considering the differences of the forming angles at each position of the surface and the accessibility of the extrusion tool to the surface. In this method, the posture of the forming toolpath is adjusted individually according to the forming angle at each position of the surface so that the posture of the sheet metal is constantly adjusted in the forming process, which ensures all the forming angles of the surface to be less than the forming limit angle. Moreover, the posture of the extrusion tool is also adjusted according to the accessibility of the extrusion tool to the surface and the features of the surface, which makes the extrusion tool to press the sheet metal along the needed directions so as to control the deformation state and material flow efficiently, and then accomplish the forming of the complex sheet metal part. The results of the research show that the 5-axis CNC incremental toolpath generated based on the method mentioned earlier can form the complex sheet metal part containing the surfaces whose forming angle is larger than 90° and that is invisible from the positive and negative directions of *Z*-axis.

Keywords Sheet metal forming · 5-axis CNC incremental forming · Multi-directional slant toolpath · Multi-directional pressing toolpath

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1 Introduction

The sheet metal CNC incremental forming is a flexible dieless forming technology [1] that can form sheet metal parts rapidly and economically from CAD data files without expensive molds. This technology can form more complicated and more difficult-to-form sheet metal parts compared to the traditional stamping forming, and can be used in prototyping and small batch production, which has a wide application prospect [2].

At present, multi-stage incremental forming is always applied to the forming of the sheet metal parts with straight-wall surface in the incremental forming process [3, 4]. But, this technology has the problems of bottom sinking and material hardening [5, 6] and is limited to the forming of simple models such as cylinder, because the middle transition model and its toolpath are difficult to be generated. Moreover, the sheet metal parts with the surfaces that are not visible in the positive and negative directions of the *Z*-axis and whose forming angle is larger than 90° cannot be formed because the existing CNC incremental forming technology mostly adopts the 3-axis CNC incremental forming process (the extrusion directions are parallel to those of *Z*-axis) [7]. Compared to the 3-axis CNC incremental forming, the extrusion tool of the 5-axis CNC incremental forming has more degrees of freedom, which can press the sheet metal part in arbitrary directions.

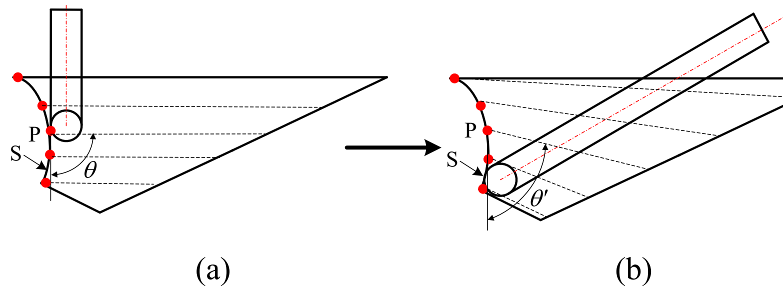


Fig. 1 Forming method. **a** Existing 3-axis CNC incremental forming. **b** Proposed 5-axis CNC incremental forming

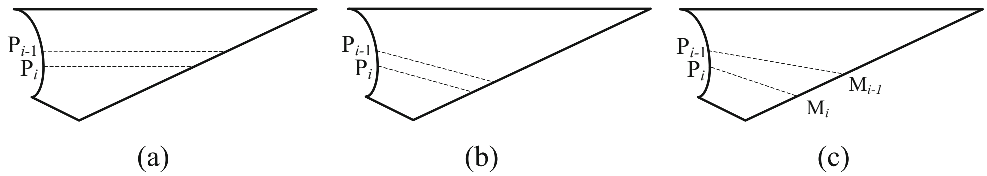


Fig. 2 Toolpath. **a** Horizontal toolpath. **b** Single-directional slant toolpath. **c** Multi-directional slant toolpath

Therefore, it is possible that the extrusion force is added to the needed directions to control the deformation effectively, and then accomplish the forming of the complicated sheet metal parts [8]. The key to make full use of the advantages of the 5-axis CNC incremental forming is to reasonably plan and generate the toolpath according to the formability.

The toolpath that is used for controlling the movement of the extrusion tool has great influence on the dimensional accuracy, shape complexity, and forming time of the sheet metal part, because the CNC incremental forming is accomplished by the extrusion movement of the extrusion tool around the profile of the sheet metal part to be formed [9]. Just because of this, there are many researches on the generation of the toolpath. But, the current researches are mainly focus on the 3-axis CNC incremental forming toolpath [10], such as contour toolpath [11], spiral toolpath [12, 13], multi-stage toolpath [14], and double-sides toolpath [15]. Because these researches are aimed at the approaches for the toolpath generation of the 3-axis CNC incremental forming, they cannot be adapted to the 5-axix CNC incremental forming toolpath generation.

In recent years, although there were some CNC incremental forming researches on the multi-degree of freedom based on robots [16] and parallel mechanism [17], these researches always adopt the method of 3-axis CNC incremental forming method (extruding the sheet along a single direction, such as *Z*-axis) and has not deal with the problem of the 5-axis CNC incremental forming toolpath planning and generation. Zhu [18] proposed a spiral toolpath generation method for the 5-axis CNC incremental forming based on a stereolithography (STL) model by considering the factors of sheet thickness variation, surface smoothness, and extruding direction. However, the method does not involve the forming of the sheet metal parts with a straight-wall surface whose forming angle is larger than 90° or that cannot be visible in the positive and negative directions of *Z*-axis.

Aiming at the complex sheet metal parts with the straight-wall surface whose forming angle is larger than 90° and that is not visible in the positive and negative directions of *Z*-axis, a method for the toolpath generation of the 5-axis CNC incremental forming that is multi-directional slant and multi-directional pressing was proposed based on the formability in this paper.

2 Overall methods

The 5-axis CNC incremental forming method shown in Fig. 1b (multi-directional slant and multi-directional pressing toolpath) was proposed based on the individual adjustment of the toolpath posture and the extrusion directions at each position of the surfaces, which are different from those of the 3-axis CNC incremental forming method shown in Fig. 1a (the toolpath is perpendicular to *Z*-axis, and the extrusion direction is always parallel to *Z*-axis). As shown in Fig. 1, the forming angle θ is defined as the included angle between the sheet and the line that is parallel to the *Z*-axis. According to the forming angle at each position of the surfaces, the posture of the toolpath was adjusted individually by fully considering the effects of the differences of the forming

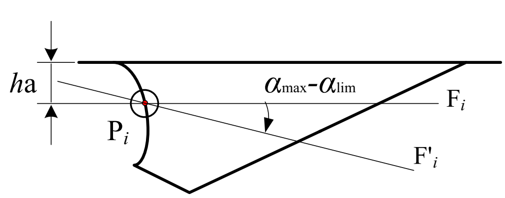


Fig. 3 Adjusting the cutting planes individually

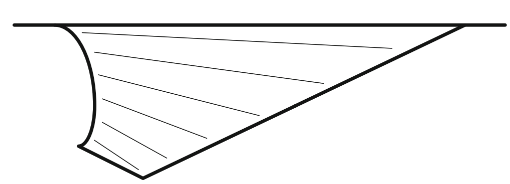


Fig. 4 Multi-directional slant toolpath

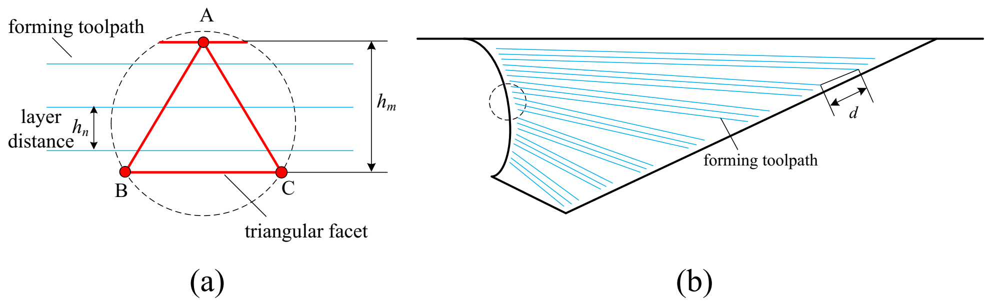


Fig. 5 Toolpath. **a** Locally enlarged view of Fig. 5b. **b** Toolpath before interpolation

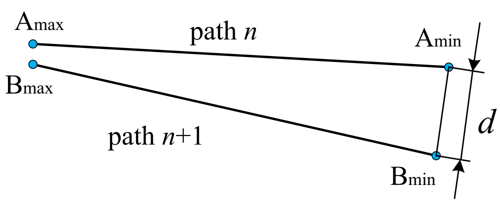


Fig. 6 Distance between the adjacent toolpaths

angle at each position of the surface on the formability, so that the posture of the sheet metal is adjusted continuously in the forming process and ensure the forming angles at each position to be less than the forming limit angle. For example, because the forming angle θ at the point P of the surface shown in Fig. 1a closes to 90°, the surface is not easy formed. But when the posture of the toolpath at the point P is adjusted to the posture shown in Fig. 1b, the forming angle will be changed to θ' that is far less than 90° and can be formed easily. The postures of the toolpaths at each position of the surface are different with each other, which are determined according to the forming angles individually. Besides, according to the accessibility of extrusion tool to surfaces (or interference condition) and the characteristics of the surfaces, adjusting the posture of the extrusion tool constantly makes the extrusion tool to press the sheet metal along the direction of perpendicular to the surface in each position, which controls the deformation state and material flow efficiently, and then realizes the complicated forming of the complex sheet metal parts. For example, the extrusion tool cannot access the surface indicated by S in Fig. 1a, but can in Fig. 1b.

3 Generation of toolpath

3.1 Multi-directional slant toolpath generation algorithm

In the CNC incremental forming process, the contour lines are always taken as toolpath obtained through cutting the model to be formed with a series of equal distance horizontal planes [7]. Although the contour toolpath has the advantages of simple algorithm and easy implementation, it is difficult to achieve a desired forming effect, because the contour toolpath generated by cutting the model to be formed with horizontal planes along the direction perpendicular to the *Z*-axis without the characteristics of the part to be formed is considered. Lu [19] introduced a feature-based approach by constructing the toolpath employing the equipotential lines in order to achieve local geometrical features accurately. Better forming quality can be obtained at the critical edges and for the non-symmetrical parts by the approach. However, the toolpath based on the equipotential lines obtained by the size of side wall cannot be used for the forming process with optimum forming angle at all position of the surfaces.

According to the forming angles in each position of the surfaces of the sheet metal parts, the toolpath generation plane was adjusted individually, and then the sheet model was cut with different posture planes to obtain multi-directional slant paths with different posture, which will make the posture of the sheet metal to be adjusted constantly in the forming process, to ensure the forming angles in all position of the surfaces are less than the forming limit angle.

As shown in Fig. 2c, the multi-directional slant toolpath with different postures was obtained by cutting the model using the plane M_{i-1} and M_i with different postures at different points P_{i-1} and P_i on the surface. The multi-directional slant tool path can more well meet the requirements of the different forming angles in each position of the surfaces compared to the horizontal toolpath shown in Fig. 2a and the single-directional slant toolpath shown in Fig. 2b.

The detailed algorithm is to compare the maximum forming angle $a_{\max}$ at each forming height position of the sheet surfaces (at the height of h_a) with the forming limit angle $a_{\lim}$ ($a_{\lim}$ is decided by the user), as shown in Fig. 3.

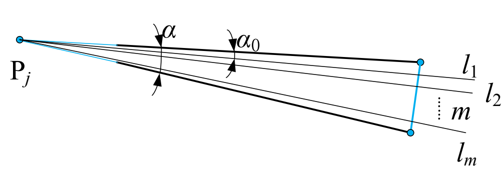


Fig. 7 Interpolation of the toolpath

Fig. 8 Sheet metal part

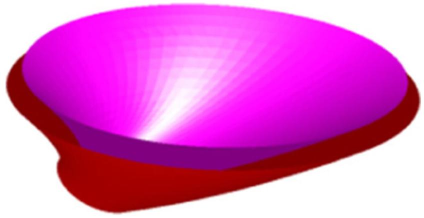


Fig. 9 Equidistance offset surface

If the maximum forming angle $a_{\max}$ is larger than the forming limit angle $a_{\lim}$, the cutting plane for toolpath generation at this position should be rotated and adjusted to make the forming angle less than the forming limit angle; If the maximum forming angle $a_{\max}$ is less than or equal to the forming limit angle $a_{\lim}$, the cutting plane will be not adjusted. The detailed algorithm based on the STL model is as follows:

1. Offsetting the inner facet of the STL model of the sheet metal part inward with a distance of extrusion tool radius based on Qu's equidistance offset algorithm [20] to obtain the surface for the cutter location points (the equidistance facet of the inner facet of the STL model).
2. Cutting the STL model to be formed from the top using the horizontal plane F_i at a certain layer distance to generate the contour line.
3. Searching the maximum forming angle $\alpha_{\max}$ at the position of the contour line on the internal surface of the model that is to say, searching the triangular facets whose Z coordinates of the normal vector is the smallest at the height of the contour line. The forming angle at the point P_i on the distance of h_a to the top of the model is the maximum.
4. Comparing the maximum forming angle $\alpha_{\max}$ at the position of the contour line (at the height of h_a) with the forming limit angle $\alpha_{\lim}$
5. If $\alpha_{\max} > \alpha_{\lim}$, rotating the cutting plane F_i around the P_i to ensure the forming angle to be less than the forming limit angle $\alpha_{\lim}$. That is to say, rotating F_i by the angle $\alpha_{\max}$-$\alpha_{\lim}$ around the P_i to generate a new cutting plane F'_i.

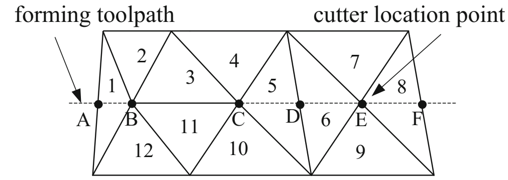


Fig. 11 Calculation of the normal vectors of the cutter location points

6. Cutting the surface for the cutter location points by using the new cutting plane F'_i to generate the multi-directional slant toolpath.
7. If $\alpha_{\max} < \alpha_{\lim}$, the cutting plane F_i will not be adjusted.
8. By the same methods, cutting the whole model from top to down to generates the multi-directional slant toolpath as shown in Fig. 4.

3.2 Interpolation of the toolpaths

The process of cutting the triangular facets using the toolpath generation plane is shown in Fig. 5, among them h_n is the minimum layer distance, h_m is the distance from vertex A of the triangular facets to the bottom line, and d is the distance between the lowest points on the adjacent slant toolpaths. The triangular facet that is made up of A, B, and C contains two or more toolpath generation planes whose layer distance is h_n (that is, $2h_n < h_m$), thus the adjusted angles of the toolpath generation planes contained in the triangular facet are equal, making some adjustment cutting planes parallel to each other, and then the distance d between the lowest points on the adjacent slant toolpaths is increased.

Because the size of the triangular facets in the inner surface of the STL model is bigger than that of the double layer distance, the adjusted angles of some toolpath generation planes are the same, which leads to the spacing of the toolpaths too large on the surface with gentle forming angles, and affects the forming quality seriously.

To the abovementioned problem, a method for the toolpath interpolation based on the distance d and included angles between the adjacent toolpaths is proposed and the detailed algorithm is as follows:

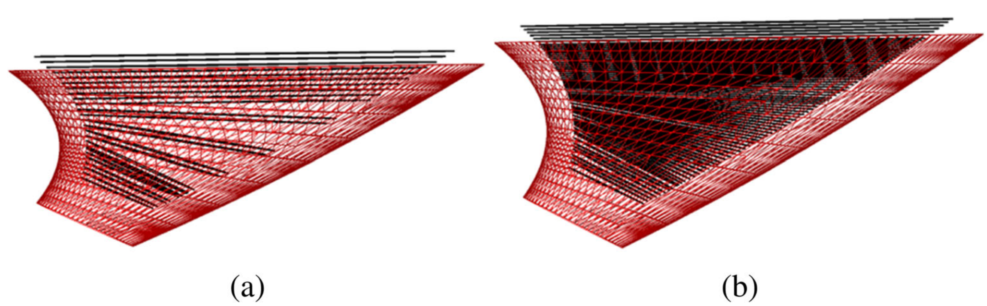


Fig. 10 Multi-directional slant toolpath. **a** Toolpaths before insertion. b Toolpaths after insertion

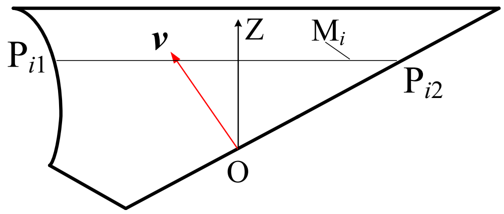


Fig. 12 Calculation of the interference point

1. Globally searching the slant toolpaths obtained by individually adjusting the toolpath generation planes according to the forming angle of the straight-wall and then, calculate the distance d between the adjacent toolpaths by using Eq. (1). As shown in Fig. 6, $A_{\max}$ is the highest point on the slant toolpath n, $A_{\min}$ is the lowest point on the slant toolpath n, $B_{\max}$ is the highest point on the slant toolpath $n + 1$, $B_{\min}$ is the lowest point on the slant toolpath $n + 1$, d is the distance between the lowest points located on the slant toolpath n, *a*nd the slant toolpath $n + 1$, respectively.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \tag{1}$$

Where (x_2, y_2, z_2) and (x_1, y_1, z_1) are the coordinates of $B_{\min}$ and $A_{\min}$, respectively.

2. Comparing the distance d of the two lowest points between the adjacent slant toolpaths and the maximum layer distance $h_{\max}$. If d is larger than $h_{\max}$, connecting the point with maximum Z coordinate and the point with minimum Z coordinate on the adjacent toolpaths, respectively, generates two straight lines. As shown in Fig. 7, the two lines are projected onto the XOZ plane, and the projection lines are extended to intersect at point P_j.

3. Taking the point P_j as the rotation center, inserting m (m is the number of toolpaths inserted) toolpaths into between the two adjacent toolpaths when the distance d is larger than the maximum layer distance $h_{\max}$ at an angle intervals of α_0 equably, which are calculated by Eq. (2). As shown in Fig. 7, α is the included angle of the extended lines of the projection lines of the adjacent toolpaths (calculated by Eq. (3)); α_0 is the angle intervals of the toolpaths inserted (calculated by formula (4)); l_1, l_2......l_m are the toolpath generation planes inserted.

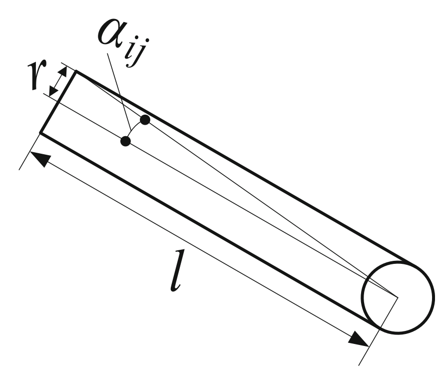


Fig. 14 Correction angle calculation

$$m = \frac{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}{h_{\max}} \tag{2}$$

$$\begin{cases} \alpha = across\left(\dfrac{a^2 + b^2 - c^2}{2ab}\right) \\ a = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \\ b = \sqrt{(x_1 - x_4)^2 + (y_1 - y_4)^2 + (z_1 - z_4)^2} \\ c = \sqrt{(x_1 - x_4 + x_3 - x_2)^2 + (y_1 - y_4 + y_3 - y_2)^2 + (z_1 - z_4 + z_3 - z_2)^2} \end{cases} \tag{3}$$

Where (x_3, y_3, z_3) and (x_4, y_4, z_4) are the coordinates of $A_{\max}$ and $B_{\max}$, respectively.

$$\alpha_0 = \frac{\alpha}{m} \tag{4}$$

3.3 Case studies

The software system of the toolpath generation algorithm for adjusting the toolpath individually according to the forming angles had been implemented using C++, OpenGL graphics library and VC++6.0 on the circumstance of Windows 7. A case study for the generation of the multi-directional slant toolpath was given by taking the sheet metal part shown in

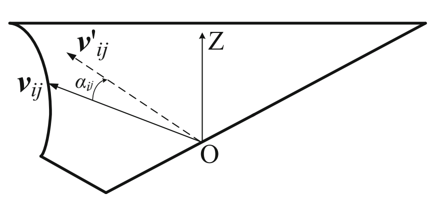


Fig. 13 Vectors correction

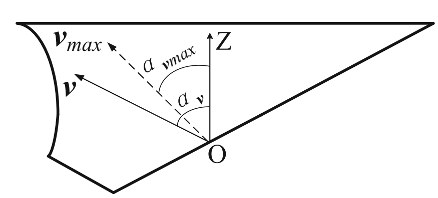


Fig. 15 Interference detection and correction

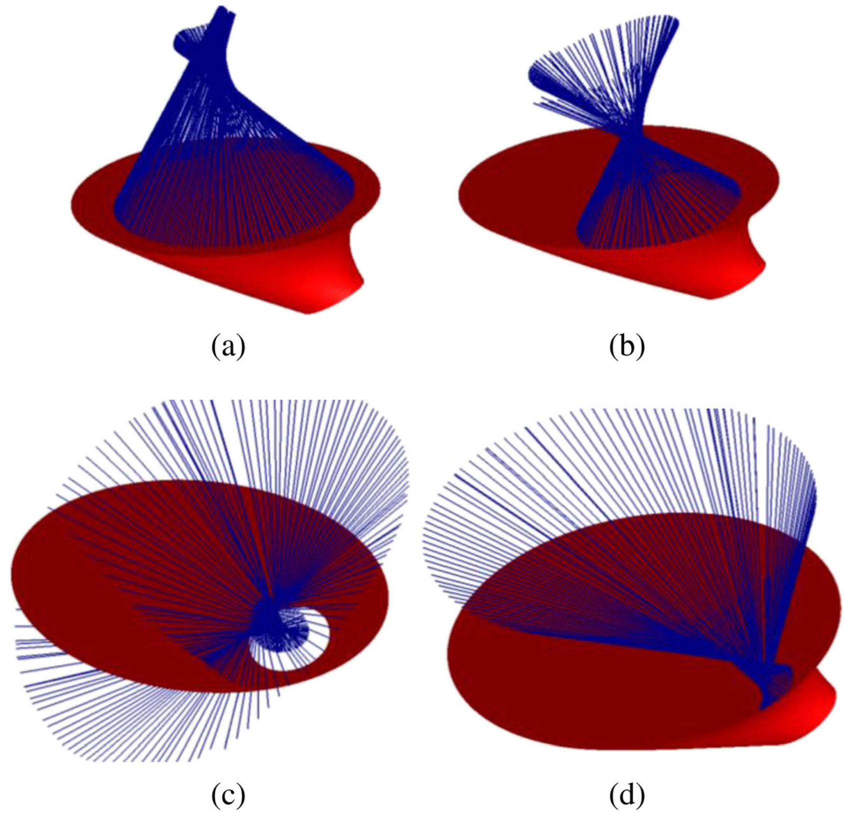


Fig. 16 Interference detection and correction. **a** The 10th layer toolpath (with interference). **b** The 100th layer toolpath (with interference). **c** The 150th layer toolpath (with interference). **d** The 150th layer toolpath (interference corrected)

Fig. 8 as a test model, which could not be visible in the positive and negative directions of *Z*-axis and whose forming angle was larger than 90 .

Figure 9 shows the equidistance offset surface obtained by equidistantly offsetting the inner surface of the sheet metal part inward a distance of the extrusion tool radius 5 mm.

The multi-directional slant toolpath was generated by slicing the equidistance offset surface using the individually adjustment plane at an interval of 1 mm (shown in Fig. 10a). Then, the final toolpaths shown in Fig. 10b were generated by inserting some toolpaths into the multi-directional slant toolpaths using the toolpaths insertion algorithm.

4 Extrusion tool posture calculation and interference correction

4.1 Extrusion tool posture calculation

In order to obtain better forming quality, the extrusion direction should be perpendicular to the surface of the model to be formed [18] under the condition of non-interference between the extrusion tool and the sheet metal part as well as the support. That is to say, taking the normal direction of each cutter location point as the axis direction of the extrusion tool. The normal vector of each cutter location point of the model to be formed can be calculated by averaging the normal vectors of all triangles sharing the cutter location point as shown in Fig. 11. For example, the direction vector of the cutter location point B is the average of the normal vectors whose triangle facet numbers are 1, 2, 3, 11, and 12; the direction vector of the cutter location point C is the average of the normal vectors whose triangle facet numbers are 3, 4, 5, 10, and 11; the direction vector of the cutter location point D is the average of the normal vectors whose triangle facet numbers are 5 and 6; the direction vector of the cutter location point E is the average of the normal vectors whose triangle facet numbers are 6, 7, 8, and 9.

Fig. 17 Test model

4.2 Interference detection and correction

In the research, in order to extrude the sheet metal part with the extrusion tool along the direction perpendicular to the model surface to be formed, the posture of the extrusion tool should to be adjusted constantly at each cutter location point, which will lead to the interference between the extrusion tool and the part, so the interference detection and correction are needed between the extrusion tool and the part.

Table 1 Material mechanics property parameter

	Density (kg m^{-3})	Elastic modulus (Gpa)	Poisson's ration	Tangent stress (Mpa)	Tangent modulus (Gpa)	Harding parameter
Sheet	2700	55.94	0.324	153.6	2.9	0.19775
Extrusion tool	8160	218	0.30	–	–	–
Support die	7810	212	0.29	–	–	–

Ho [21] dispersed discrete point-cloud in triangular facets based on the stochastic method, then enveloped the discrete points model with the bounding box, and made a rough interference detection according to the intersection operation of the bounding box, then found the exact interference position by traversing the bounding hierarchy tree. Wang [22] used AABB to approximately envelope the parts and tools and crudely judged the interference position, and then used two binary trees to detect the position of interference accurately. This method has the advantage of quick judgment, but when the interference occurs, it is needed to traverse the whole data to find the accurate position of the interference, which leads to lower the judgment speed and increase the calculation amount and the number of division when the one object closes to the other without interference. Chang [23] proposed an algorithm for conducting the intersections calculation between the triangular facets quickly in the interference detection based on an oriented bounding box (OBB). Chang [24] made a collision detection between objects using OBB and the dual OBB-sphere bounding volume tree. In order to improve the efficiency of the collision detection, the bounding spheres were used to detect the interference between the objects with longer distance and the OBB was used to detect the interference between objects with closer distance. Ilushin [25] induced the high precise algorithm for calculating the intersection between polygon/surface and tool using the space subdivision techniques and ray-tracing, which solved the interference collision problem in the multi-axis NC machining. Tang [26] detected

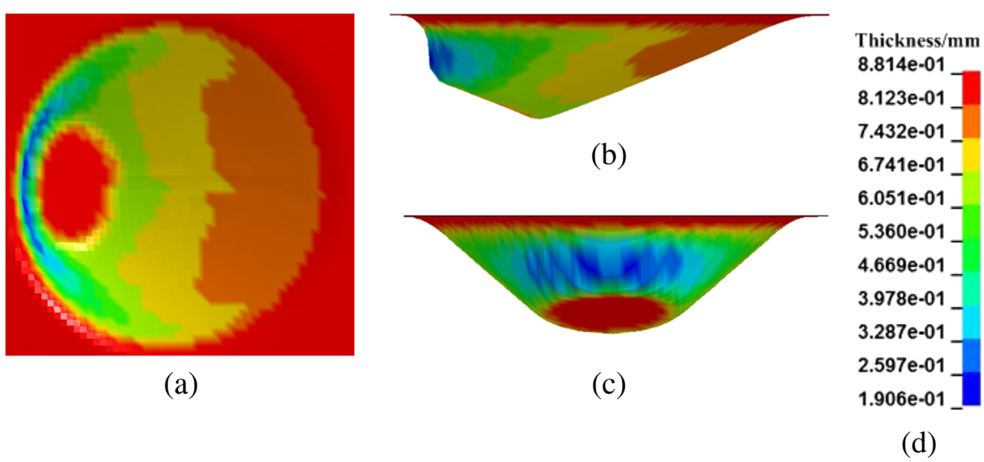


Fig. 18 Thickness distribution nephogram. **a** Top view. **b** Main view. **c** Left view. **d** The thickness legend

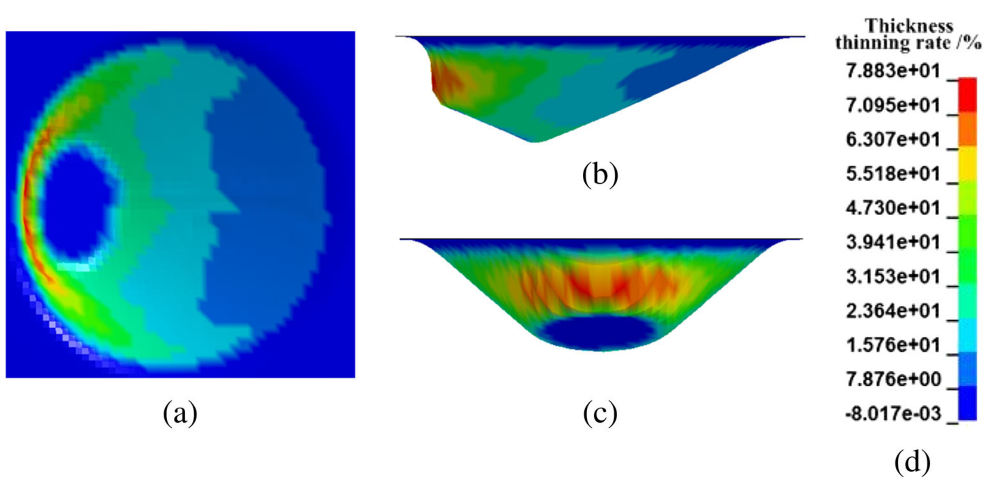
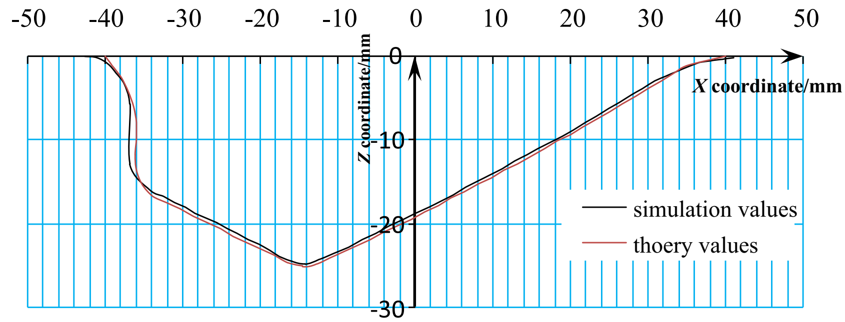


Fig. 19 Thinning rate distribution nephogram. **a** Top view. **b** Main view. **c** Left view. **d** The thinning rate legend

Fig. 20 Profile at $y = 0$ section



the interference of the tool, part, hilt, and fixture based on the sweep plane algorithm using the parallel slices obtained by equidistantly cutting the part and tool, in which the interference detection precision depends on the distance between slices. The smaller the distance of slices is, the larger the amount of the computation is. Zhu [27] proposed an interference detection method of CNC incremental forming between the extrusion tool and part based on the octree segmentation model of STL-format models with index sequence but without recursive search. However, when the extrusion tool fully enters the workpiece, the interference between the extrusion tool and workpiece cannot be avoided with only one interference correction, in contrary, multiple interference detections and corrections should be done, which leads to lots of computation and time. Chen [28] used visibility pyramid to detect the interference between the extrusion tool and workpiece at each cutter location point from top to bottom, but this method cannot be applied to the interference detection and correction in the 5-axis CNC incremental forming process.

Aiming at the extrusion tool that extrudes the sheet metal part in the direction perpendicular to the surface, the following method was proposed to detect and correct the interference between the extrusion tool and the workpiece, the detailed algorithm is as follows:

1. Extracting all vertexes of the triangular facets on the side wall surface of the cutter location surface, and deduplication and sorting the vertexes according to their Z coordinates.
2. Cutting the cutter location surface with horizontal planes containing the vertexes of the triangular facets to obtain a series of polygons M_i (i is the number of layer).
3. The abovementioned polygons M_i and the plane with normal vector $\boldsymbol{v}$ of the cutter location points and perpendicular to the horizontal plane (that is, the plane depended on the Z-axis through cutter location point O and the normal vector of the cutter location point O, as shown in Fig. 12) intersect at point P_{ij} (i is the number of layer, j is the intersection number of each layer polygon).
4. Connecting the cutter location points and the intersection points to obtain a series of vectors $\boldsymbol{v}_{ij}$ with the cutter location point as the origin. In the process of the interference detection, the vectors $\boldsymbol{v}_{ij}$ should be rotated α_{ij} to the Z-axis in the cutting planes to obtain the correction vectors $\boldsymbol{v}'_{ij}$, as shown in Fig. 13. The α_{ij} could be calculated by Eq. (5), as shown in Fig. 14. r and l are the radius and the length of the extrusion tool, respectively.

$$\alpha_{ij} = \arcsin(r/l) \tag{5}$$

5. Finding the vector $\boldsymbol{v}_{\max}$ among the vectors $\boldsymbol{v}'_{ij}$, which the included angle between the $\boldsymbol{v}_{\max}$ and Z-axis is minimum. If the included angle $\alpha_{\boldsymbol{v}\max}$ between the $\boldsymbol{v}_{\max}$ and Z-axis is larger than the included angle $\alpha_{\boldsymbol{v}}$ between the vector $\boldsymbol{v}$ of the cutter location point and Z-axis, it is can be judged that

Fig. 21 Thickness at $y = 0$ section profile

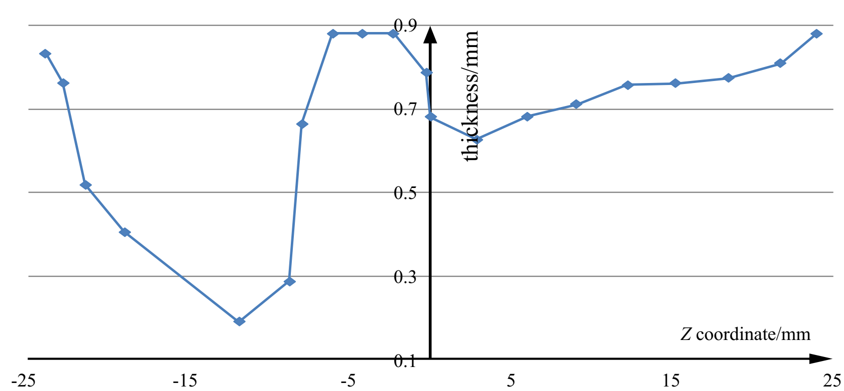


Fig. 22 Sheet metal part to be formed

there will be no interference between the rod of the extrusion tool and the workpiece; If $\alpha_{v\max}$ is smaller than $\alpha_{\boldsymbol{v}}$, it can be judged that there will be interference between the rod of the extrusion tool and the workpiece, and then it is needed to take the $\boldsymbol{v}_{\max}$ as the axis direction vector at the cutter location point, as shown in Fig. 15.

4.3 Case studies

The software system for the abovementioned algorithms of the posture calculation of extrusion tool and interference correction was implemented using C++, OpenGL graphics library and VC++6.0 on the circumstance of Windows 7. The multi-directional slant toolpath was generated by taking the sheet metal part shown in Fig. 8 as an object. The posture of the extrusion tool was calculated layer by layer and point by point, and then the interference was detected and the posture was corrected. Figure 16a, b shows the posture of the extrusion tool at some position of the cutter location point in the first layer and the 100th layer, respectively, and the extrusion direction is always perpendicular to the surface to be formed without interference. Figure 16c shows the posture of the extrusion tool, which the extrusion tool interferences with the workpiece at some cutter location points on the 150th layer, and the posture of the extrusion tool after the interference correction is shown in Fig. 16d.

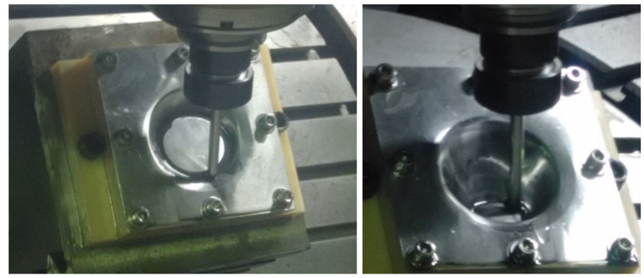
5 The finite element analysis

5.1 The establishment of the finite element analysis model

In this paper, the finite element analysis model that is consist of the extrusion tool, sheet, and support was established, and

(a) (b)

Fig. 23 Support die. **a** Top view of support die. **b** Bottom view of support die



(a) (b)

Fig. 24 5-axis CNC incremental forming experiment. **a** Upper part forming. **b** Lower part forming

took the model shown in Fig. 17 as an analytic object to do the numerical simulation of the forming process—the model cannot be visible in the positive and negative directions of Z-axis and whose forming angle is larger than 90°. The SHELL 163 and Belytschko-Tsay algorithm were used to define the sheet, and the SOLID 164 and full integration algorithm were used to define the support die and extrusion tool. Bearing steel (GCr15) and high-speed steel (W6Mo5Cr4V2) were used for the support die and extrusion tool, respectively, and 1060 aluminum sheet whose thickness is 1 mm is used for sheet. The tangent modulus of 1060 aluminum refers to Hu Zhu's research result [29]. The material mechanics property parameters are shown in Table 1.

The sheet was meshed by the mapping mesh method whose size is 1 mm, and the support die and extrusion tool were meshed by free mesh method whose size were 4 and 1.5 mm, respectively. The contact between extrusion tool and sheet was defined as point to facet contact, the contact between the sheet and support die was defined as facet to facet contact, and the penalty function algorithm was used for contact problems. The multi-directional slant toolpath whose maximum layer distance is 1 mm was used in the analysis, which was generated based on the proposed algorithm.

5.2 Simulation result and analysis

The thickness distribution nephogram of the simulated parts is shown in Fig. 18. The thickness thinning rate nephogram of the simulated parts is shown in Fig. 19. The thickness distribution is between 0.7432 and 0.3287 mm. The thinning rate distribution is between 15.76 and 63.07%. The thickness of the side wall in the area with larger forming angle is between 0.3287 and 0.1906 mm, and the thinning rate distribution is between 78.83 and 63.07%. The minimum thickness is 0.190595 mm, and the maximum thinning rate is 78.8338.

As shown in Fig. 20, extract of the coordinates of the element nodes at the profile was obtained by cutting the numerical simulation model with the section of $y = 0$ and drawing the profile at $y = 0$ section. As can be seen from Fig. 20, in the interval of − 40 to − 30 mm of X coordinates,

Fig. 25 Forming part. **a** Main view. **b** Side view

with the decrease of Z coordinate, the X coordinates first increase and then decrease.

The thickness distribution curve of the sheet part had been drawn as shown in Fig. 21 by extracting the thickness at the section of $y = 0$ of the numerical simulation model. As can be seen from Fig. 21:

1. The thickness of the left side wall of the numerical simulation model is thinner than that of the right side wall of the numerical simulation model;
2. From the left side wall and top of the numerical simulation model, the thickness is decreased with the increase of the forming angle of the side wall, the minimum thickness occurs at the area whose forming angle is 90°, and the thickness value is 0.190595 mm;
3. When the height is decreased to the area whose forming angle is below 90°, the thickness is increased with the decrease of the height. There is no change at the bottom of the formed part that has not been pressed by the extrusion tool.

6 Forming experiments

In order to investigate the feasibility of the toolpath planning and generation algorithm of the 5-axis CNC incremental forming based on the formability, the actual forming experiment had been conducted by taking the sheet metal part shown in Fig. 22, which is not visible in the positive and negative directions of Z-axis and whose forming angle is larger than 90°.

The forming experiments had been done using 5-axis machining center (DMU 65 mono BLOCK). The sheet metal used in the experiment was 1060 aluminum whose thickness is 0.88 mm. The extrusion tool used in the experiment was the hemispherical headed cylindrical forming tool whose head radius and length were 10 and 150 mm, respectively.

Considering the problems to take the formed part out from the support after forming, the partial support die (i.e., half support die) was used. The support die was made with the material of nylon as shown in Fig. 23. The aluminum sheet was clamped on the support die and fixed on the worktable of the 5-axis machining center.

Fig. 26 Profile at $y = 0$ section

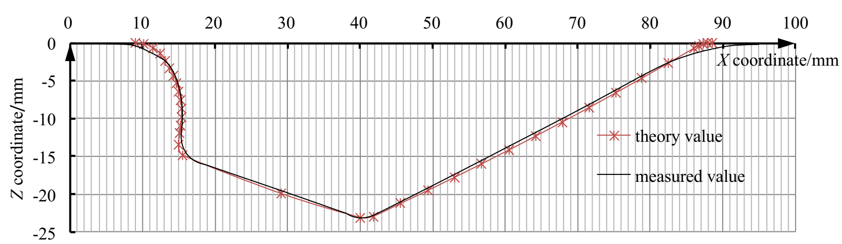


Fig. 27 Deviations in the Z direction at the profile of $y = 0$ section

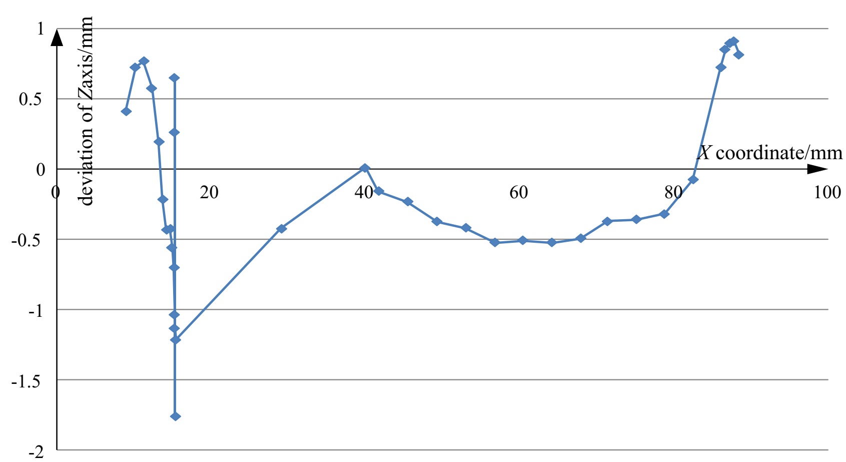


Fig. 28 Measurement. **a** Height measurement. **b** Thickness measurement

In the CNC incremental forming experiment, spindle rotation was stopped, 68# engine oil was used to lubricate the forming tool and sheet metal, the feed speed was set as 800 mm/min, and the multi-directional slant toolpath was employed whose layer distance was 0.2 mm. The 5-axis CNC incremental forming experiment is showed in Fig. 24.

As shown in Fig. 25, the complex sheet metal part which is not visible in the positive and negative directions of *Z*-axis and whose forming angle is larger than 90° had been formed by the way of 5-axis CNC incremental forming.

In order to evaluate the forming effect, the point coordinates of the actual formed part at the profile of $y = 0$ section were measured with 0.2-mm interval using three coordinates measuring machine of hexagon, meanwhile the corresponding point coordinates of the theory model to be formed were extracted by the same way, and the profile curve diagrams of the actual formed part and theory model at the profile of $y = 0$ section were drawn in Fig. 26. Figure 27 shows the deviation in the direction of *Z*-axis between theory model and actual formed part at the profile of $y = 0$ section. As can be seen from Figs. 26 and 27:

1. There is larger dimension error at the edge of the sheet metal part (*Z* coordinates at the range of 0 to −5 mm in Fig. 26), the maximum deviation in *X* direction is 4.37402 mm, and the maximum deviation in *Z* direction is 0.9091 mm, which were caused by the sheet sinking phenomenon due to the spacing between the sheet metal part and support die;
2. The coincidence extent of the profile curves between the theory model and actual formed part on the side wall with negative forming angle (*X* coordinates at the range of 10 to 20 mm in Fig. 26) is well than that on the other side wall (*X* coordinates at the range of 40 to 90 mm in Fig. 26).
3. There is a slight bump phenomenon at the bottom of the formed part, which is a common phenomenon in the CNC incremental forming process.
4. There are no sinking phenomena at the bottom of the sheet metal part, which is different from the multi-stage CNC incremental forming.

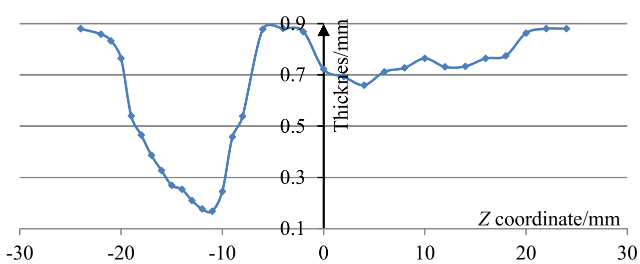


Fig. 29 The thickness of the formed sheet metal part

In order to measure the thickness of the formed sheet metal part, the sheet metal part was cut at $y = 0$ section, and the marked points were made at the left and right of the formed part with 2-mm interval and 1-mm interval, respectively, using the height-measuring apparatus firstly. Second, the thickness of the sheet metal part was measured at the marked points one by one using a point micrometer (as shown in Fig. 28), and then the thickness distribution curve was drawn, as shown in Fig. 29.

As can be seen in the thickness distribution curve:

1. Thickness of the left side wall is smaller than that of the right side wall;
2. From the left side wall and top of the formed part, the thickness is decreased with the increase of the forming angle of the side wall, the minimum thickness occurs at the area whose forming angle is 90°, and the thickness value is 0.168 mm;
3. When the height is decreased to the area whose forming angle is below 90°, the thickness is increased with the decrease of the height. There is no change at the bottom of the formed part that has not been pressed by the pressing tool.

7 Conclusion

The 5-axis CNC incremental forming toolpath that is slant and can press the sheet metal with multi-directions had been developed in this paper, which can individually adjust the postures of the toolpath at each position of the sheet surfaces and the pressing direction according to the influence of the forming angles at each position of the sheet surfaces and the accessibility of the extrusion tool on the formability.

The complex sheet metal part that is not visible in the positive and negative directions of *Z*-axis and whose forming angle is larger than 90° can be formed by using the proposed 5-axis CNC incremental forming toolpath with only one-stage forming. The proposed method can overcome the problems that exist in the multi-stage CNC incremental forming, such as the bottom sinking and the material hardening, which has great practical application values.

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