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ORIGINAL ARTICLE

# Online machine health prognostics based on modified duration-dependent hidden semi-Markov model and high-order particle filtering

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**Abstract** Accurate online health prognostics is considered as a significant part of the condition-based maintenance (CBM), and it contributes to reduce downtime and achieve the most reliable running condition for machinery equipment. In this paper, an online machine health prognostics approach is proposed based on the modified duration-dependent hidden semi-Markov model (MDD-HSMM) and the high-order particle filter (HOPF) method. In the MDD-HSMM, the health state transition probabilities and the observation probabilities are both defined not only as state dependent like traditional HSMM does, but also as duration dependent, which is more realistic to describe the state space model to model the mechanical failure propagation process. And a new forward-backward algorithm is developed to facilitate the training process and to reduce computational complexity of the proposed MDD-HSMM. Then, the HOPF method with an online update scheme is applied to recognize the health states and predict the residual useful lifetime (RUL) value of machine in real time, which is based on the health state space model established by the MDD-HSMM and the online sensing monitoring data. And, a sliding window with variable length, which represents the relationship between current state and several previous states, is applied to adjust the order of HOPF. Finally, a real case study is used to illustrate the prognostics performance of the proposed approach and the experiment results indicate that the proposed approach has higher effectiveness than conventional HSMM methods.

**Keywords** Online health prognostics · Modified duration-dependent hidden semi-Markov model · Forward-backward algorithm · High-order particle filter · Residual useful lifetime assessment

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## 1 Introduction

The demands for lower environmental risks and higher operation efficiency and reliability of the machinery equipment are increasing with the development of modern manufacturing [1]. Meanwhile, the complexity of machinery equipment and the causes of its failures are also increasing with the technology development, which makes it difficult to monitor and predict the machine health conditions in real time. In modern manufacture systems, the failures of machine may not only cause large economic losses, but also threat to human safety in some cases. Consequently, the effective maintenance is necessary to provide a healthy operating condition for machinery equipment to prevent unexpected downtime and reduce maintenance cost in the manufacturing industry, in which the reliability, maintainability, and safety are considered as important criteria [2, 3]. From the earliest breakdown maintenance to the later plan maintenance, the equipment maintenance strategies have been changing to the current preventive maintenance [4] especially the condition- based maintenance (CBM) [5], which is a decision-making strategy for optimally maintenance schedule based on the current equipment health condition assessment using the real-time condition monitoring data [6].

Health prognostics is considered as an important part of CBM to improve the reliability and lifetime of machine systems. The objective of health prognostics is to predict the future failure and estimate the residual useful life (RUL) values of machines before catastrophic faults occur [7]. Because of involving in large amounts of historical data processing and complex computation, health prognostics is generally complex and difficult, especially when the real-time prognosis is acquired while the machine is in operation. During the last few decades, numerous research efforts, including academic papers and developed prediction models for practical applications, have appeared in the field of machinery equipment health prognostics [8–10].

In general, machine health prognostics methods can be classified into three categories: physical-based methods [11], data-driven methods [12–14] and model-driven methods [15]. With the difficulty of constructing a good physical model to present the degradation process accurately and the higher costs, it is unrealistic that apply the physical-based methods to heavy machine health prognostics, since the fault degradation process is complicated. For the data-driven methods that use the condition monitoring data to analysis and predict the current and future health of a machine, there are unavoidable limitations of slow convergence, local optimal solution, and computational explosion problems with massive online monitoring data. Consequently, the model-driven methods that rely on the mathematical models to describe the degradation process of equipment and predict the health condition based on the mathematical model and monitoring data are considered as more appropriate for heavy machine equipment health management. The hidden Markov model (HMM) [16–18] is utilized as a main method of model-driven health prognostics models. However, there are some inherent limitations of HMM, such as the memoryless assumption of Markov chains, unrealistic exponential state duration distribution. In recent years, the semi-hidden Markov model (HSMM), as an extension and generalization of HMM, has become an effective and widely applied health condition prognostics technology by adding the explicit state duration into its structures [19–21].

In this paper, we will develop a machinery equipment health prognostics approach based on the HSMM. In fact, the HSMM has been applied to machine health management effectively with its mathematical foundation and modeling perspectives. For example, Liu and Dong proposed an online health prognosis method to recognize the health state and predict the residual useful lifetime (RUL) values of equipment based on HSMM and sequential Monte Carlo (SMC) method [22]. Furthermore, they presented an adaptive hidden semi-Markov model (AHSMM) integrated with multi-sensor measurements to predict the health condition of equipment [23]. In addition, Wu et al. [24] also applied HSMM to identify the machine states in real-time based on the acoustic emission (AE) sensor monitoring signal. Although the previous HSMM has been proved as a good model for machinery equipment health prognostics, the deterioration effect within a health state is not considered in this model, which assumes that the state transition probabilities are invariant, while it's not applicable in real world applications. For this limited time-invariant characteristic of HSMM, Peng and Dong [21] integrated three types of aging factors into the HSMM transition matrix to characterize the deterioration of equipment, and the deterioration was assumed as a certain distribution form. Moreover, Tao et al. [25] proposed the duration-dependent state transition probabilities of HSMM, and developed a new forward-backward algorithm to train the modified HSMM, which was used for speech synthesis successfully. Wang et al. [26] also utilized the HSMM with duration-dependent state transition probabilities for equipment health prognostics, and its effectiveness was proved with the comparison of traditional HSMM. However, in the above studies, only the duration-dependent characteristic of state transition probabilities is considered, while the observation probabilities are still duration independent, which ignore the relationship between the observations and the time-varying characteristic of observation sequence. In fact, the sensing monitoring signal of operating machinery equipment is affected by the deterioration effect of health state, as a result, the output observation probabilities of HSMM should also be considered as changing with duration time of each health state. Therefore, this paper presents a modified duration-dependent HSMM (MDD-HSMM) where the transition probability distribution and observation probability distribution are both related to the duration of state.

In addition, the particle filter (PF) method is chosen as the online state recognition and prediction algorithm for heavy duty mechanical equipment based on the complex nonlinear state space model established by MDD-HSMM. The PF method is a sequential Monte Carlo method refers to the recursive Bayesian algorithm, which approximates the state probability density function (PDF) in real time using a set of random particles with associated weights [27]. With the great advantages of decreasing the computational complexity and dealing with complex nonlinear and non-Gaussian problem usefully, PF has been applied to machinery health states estimation and prognosis successfully [28]. High-order particle filter (HOPF) [29, 30] extends the first-order state equation of original PF method to multi-order, which takes the relationship between current state and several previous states into consideration. Therefore, HOPF is more appropriate to describe the relationship between unobservable states and monitored observation sequence of machinery equipment where the health state is not only dependent on the last state, but also on multiple previous states that remaining in the same state.

Motivated by the above demands, in this paper, an improved equipment health prognostics approach based on MDD-HSMM and HOPF is proposed, where the MDD-HSMM with duration-dependent transition probabilities and observation probabilities is used to obtain the nonlinear state space model for modeling the health degradation of machine equipment. In order to facilitate the computation of the proposed MDD-HSMM, the new forward-backward variables and associated algorithm are developed. Then, the HOPF method is adopted to recognize and predict the health states of machine accurately in real time based on the nonlinear state space model constructed by the MDD-HSMM and the online sensing monitoring data.

The remainder of this paper is organized as follows. In Sections 2 and 3, the theories of MDD-HSMM and HOPF are introduced respectively. Section 4 presents the proposed online health prognostics framework. In Section 4, the duration-dependent state space model is introduced first, and then, the integration of the state space model and HOPF for online health state recognition is demonstrated. Next, the corresponding RUL value prediction method is proposed. Section 5 illustrates a case study for the proposed prognostics approach and gives a discussion for its performance. Finally, conclusions are made in Section 6.

## 2 Modified duration-dependent hidden semi-Markov model

### 2.1 Basic theory of general HSMM

The HSMM is an extension of HMM by adding the explicit state duration into its structures, and the state duration is assumed as a random integer variable in the set $D = 1, 2, ..., D$. With the explicit duration, a state in HSMM generates a segment of observations determined by the length of state duration $d$, instead of a single observation in standard HMM [20]. In general, the states in a segmental HSMM are called macro-states and each of which consists of several single states called micro-states, while only the transition between macro-states is the Markov process [31]. Suppose that a macro-state sequence consists of $n$ segments (i.e., has $n$ macro-states), and for the $i$th segment, the end-point time index is $q_i$, and the observations segment is $o_{q_{i-1}+1},\ldots,o_{q_i}$, and the corresponding micro-states is $s_{q_{i-1}+1},\ldots,s_{q_i}$, which have the same macro-state label $h_i$. The general HSMM framework is illustrated in Fig. 1.

Let $s_t$ denote the hidden state at time $t$ in the state set $S = \{1, 2, ..., N\}$ and $o_t$ is the observation value at time $t$ which is usually a feature vector in machine health management. In general, a HSMM is characterized by its parameters: initial state distribution $\pi = \{\pi_i\}(\pi_i = P(s_1 = i), 1 \le i \le N)$; state transition probability distribution $A = \{a_{ij}\}$ $(a_{ij} = P(s_{q_n} = j|s_{q_{n-1}} = i), 1 \le i, j \le N)$; observation probability distribution $B = \{b_i(o_t)\}(b_i(o_t) = P(o_t|s_t = i), 1 \le i \le N, 1 \le t \le T)$, where $T$ is the length of observation sequence; state duration distribution $D = \{p_i(d)\}(p_i(d) = P(t_2 - t_1 = d|s_{t1,\cdots t2} = i, s_{t1-1} \ne i, s_{t2+1} \ne i), 1 \le i \le N, 1 \le d \le D_i)$.

In practical applications, HSMM also has three basic problems that need to be dealt with similar to HMM, including evaluation, decoding, and learning. Furthermore, there also have three basic algorithms for the above three problems, that are, forward-backward algorithm, Viterbi algorithm, and Baum-Welch algorithm.

### 2.2 Inference for modified duration-dependent HSMM

As opposite to the general HSMM, the MDD-HSMM supposes that both the transition probabilities and observation probabilities are related to the duration of state, which means that these probability distributions depend not only on the current state but also on the duration of state. In this

**Fig. 1** A general HSMM framework

| | | | | | |
|---|---|---|---|---|---|
| Time units | $1,\cdots,q_1$ | $q_1+1,\cdots,q_2$ | $\cdots$ | $q_{n-2}+1,\cdots,q_{n-1}$ | $q_n$ |
| Observation | $o_1,\ldots,o_{q_1}$ | $o_{q_1+1},\ldots,o_{q_2}$ | $\cdots$ | $o_{q_{n-2}+1},\ldots,o_{q_{n-1}}$ | $o_{q_n}$ |
| Duration time | $d_1 = q_1$ | $d_2 = q_2$-$q_1$ | $\cdots$ | $d_{n-1} = q_{n-1}$-$q_{n-2}$ | $d_n = \infty$ |
| Micro-state | $s_1,\ldots,s_{q_1}$ | $s_{q_1+1},\ldots,s_{q_2}$ | $\cdots$ | $s_{q_{n-2}+1},\ldots,s_{q_{n-1}}$ | $s_{q_n}$ |
| Macro-state | $h_1$ | $h_2$ | $\cdots$ | $h_{n-1}$ | $h_n$ |
| Segments | 1 | 2 | $\cdots$ | $n$ - 1 | $n$ |
| Transitions | ⇢ | ⇢ | ⇢ | ⇢ | |

paper, we research the Markov model with left-to-right structure where the states are ordered linearly from left to right, and the state can only transit to adjacent right state or stay in itself [32]. This directional structure is suitable to describe the health state degradation of machinery equipment.

The parameters of the MDD-HSMM are defined as follows: the initial state distribution $\pi = \{\pi_i\}$ and the duration distribution $D = \{p_i(d)\}$ are the same as the general HSMM; the transition probability from state $i$ to state $j$ at time $t$ after having stayed in state $i$ for $d_t(i) = d$ is defined as:

$$a_{ij}(d) = P(s_{t+1} = j | s_{t-d} \neq i, s_{t-d+1} = \cdots = s_t = i) \quad (1)$$

where $j = i + 1$ or $j = i$.

The observation probability of state $i$ at time $t$ after having stayed in state $i$ for $d_t(i) = d$ is defined as:

$$b_{i,d}(o_t) = P(o_t | s_{t-d} \neq i, s_{t-d+1} = \cdots = s_t = i) \quad (2)$$

In this study, the observation sequence for MDD-HSMM is consisted of multi-dimensional feature vectors extracted from the machine sensing monitoring data, which may not simply follow a single parametric probability distribution. However, the Gaussian Mixture Model (GMM) has the advantage of simply depicting the distribution of a feature vector in the probability space, thanks to the use of the weighted sum of the probability density functions of multiple Gauss distributions [33], which is often chosen to model the observation probability distribution of HSMM [17, 22, 26]. Thus, the Gaussian mixture distribution is used to represent the duration-dependent observation probabilities in this study.

$$b_{i,d}(o_t) = \sum_{g=1}^{G} \omega_{i,d}(g) N(o_t, \mu_{i,d}(g), \Sigma_{i,d}(g)) \quad (3)$$

where $G$ is the number of Gaussian elements, and $N(o_t, \mu_{i,d}(g), \Sigma_{i,d}(g))$ is the $g$th Gaussian element at state $i$ with $d_t(i) = d$. $\omega_{i,d}$(g), $u_{i,d}$(g), and $\Sigma_{i,d}$(g) are the weight coefficient, mean vector, and covariance matrix of the $g$th Gaussian element, respectively. In order to adapt to the duration-dependent characteristic of the MDD-HSMM, the forward and backward variables are modified and a new forward-backward algorithm is developed to facilitate the model training process.

The forward variable is defined as:

$$\alpha_t(i,d) = P(o_1^t, s_t = i, d_t(i) = d | \lambda) \quad (4)$$

It presents the joint probability of generating the observation sequence $o_1, \cdots, o_t$ and having stayed in state $i$ with duration time $d$ given model $\lambda$. We use the pair process $(s_t, d_t)$ to denote the state and its duration value at time $t$. With the directional property of life-right model structure, the initial value of the forward variable is:

$$\alpha_1(i,d) = \begin{cases} b_{i,d}(o_1), (i,d) = (1,1) \\ 0, \text{else} \end{cases} \quad (5)$$

By analyzing all the possible states that the state $(i, d) = (1, 1)$ can only be the value of $(s_1, d_1)$, and the state $(s_t, d_t) = (i, d)$ is transited either from the state $(s_{t-1}, d_{t-1}) = (i - 1, d')$ when $d = 1$ or $(s_{t-1}, d_{t-1}) = (i, d - 1)$ when $1 < d \leq D_i$. Thus, for time $t = 2, \cdots, T$, the forward recursion formulae are written as follows:

$$\alpha_t(1,1) = 0 \quad (6)$$

$$\alpha_t(i,1) = \sum_{d \geq 1} \alpha_{t-1}(i-1,d) P_{i-1}(d) a_{(i-1)i}(d) b_{i,1}(o_t) \quad (7)$$

$$\alpha_t(i,d) = \alpha_{t-1}(i,d-1) b_{i,d}(o_t) \quad (8)$$

As similar to forward variable, the backward variable is defined as:

$$\beta_t(i,d) = P(o_{t+1}^T | s_t = i, d_t(i) = d, \lambda) \quad (9)$$

The initial value of backward variable is:

$$\beta_T(i,d) = \begin{cases} P_i(d) a_{i,(N)}(d), i = N, 1 \leq d \leq D \\ 0, \text{else} \end{cases} \quad (10)$$

For time $t = 2, \cdots, T$, the state $(s_t, d_t) = (i, d)$ will transit either to $(s_{t+1}, d_{t+1}) = (i, d + 1)$ or $(s_{t+1}, d_{t+1}) = (i + 1, 1)$, and the backward recursion formula is written as below:

$$\begin{aligned} \beta_t(i,d) &= b_{i,d+1}(o_{t+1}) \beta_{t+1}(i,d+1) \\ &\quad + P_i(d) a_{i(i+1)}(d) b_{i+1,1}(o_{t+1}) \beta_{t+1}(i+1,1) \end{aligned} \quad (11)$$

For the purpose of reducing the computational complexity, some variables are defined in the MDD-HSMM for parameters re-estimation. Firstly, given the model $\lambda$ and observation sequence $O_1^T$, the posterior probability for a transition from state $i$ to next state $i+1$ with the duration $d$ at time $t$ can be expressed using the defined forward-backward variables in above.

$$\begin{aligned} \xi_t(i,i+1,d) &= P(s_t = i, s_{t+1} = i+1, d_t(i) = d | O_1^T, \lambda) \\ &= \frac{1}{P(O_1^T|\lambda)} \alpha_t(i,d) p_i(d) a_{i(i+1)}(d) \\ &\quad \cdot b_{(i+1),1}(o_{t+1}) \beta_{t+1}(i+1,1), 1 \leq i < N \end{aligned} \quad (12)$$

Then, the posterior probability that state $i$ is visited at time $t$ and has stayed for $d$ time units can be calculated as follow:

$$\begin{aligned} \gamma_t(i,d) &= P(s_t = i, d_t(i) = d | O_1^T, \lambda) \\ &= \frac{\alpha_t(i,d)\beta_t(i,d)}{P(O_1^T|\lambda)} \end{aligned} \quad (13)$$

The posterior probability that state $i$ transits to itself with the duration $d$ at time $t$ can be obtained as follow, especially $\xi_t(N,N,d) = 1, 1 \leq d \leq D_N$.

$$\begin{aligned}\xi_t(i,i,d) &= P(s_t = i, s_{t+1} = i, d_t(i) = d|O_1^T, \lambda) \\ &= \gamma_t(i,d) - \xi_t(i,i+1,d)\end{aligned} \tag{14}$$

Given the observation sequence, the parameters of this duration-dependent model can be re-estimated using the variables defined above. Firstly, the initial state probability distribution is the probability that state $i$ visited at time $t = 1$ with the duration $d = 1$:

$$\bar{\pi}_i = \gamma_1(i, 1) \tag{15}$$

Next, the re-estimate formula of the transition probability distribution is written as follows:

$$\overline{a_{i(i+1)}}(d) = \frac{\sum_{t=1}^{T} \xi_t(i,i+1,d)}{\sum_{t=1}^{T} \gamma_t(i,d)} \tag{16}$$

Then, the observation probability distribution $b_{i,d}(o_t)$ represented by Gaussian mixture distribution is updated. The probability that state $i$ is visited at time $t$ with the duration $d_t(i) = d$ and the observation $o_t$ belong to the $g$th Gaussian element is derived as follow:

$$\gamma_t(i,d,g) = \frac{\gamma_t(i,d) \cdot w_{i,d}(\mathrm{g}) N(o_t, \mu_{i,d}(g), \Sigma_{i,d}(g))}{\sum_{g=1}^{G} w_{i,d}(\mathrm{g}) N(o_t, \mu_{i,d}(g), \Sigma_{i,d}(g))} \tag{17}$$

Then, the weight $\overline{w_{i,d}}(\mathrm{g})$, mean vector $\overline{\mu_{i,d}}(\mathrm{g})$ and variance matrix $\overline{\Sigma_{i,d}}(\mathrm{g})$ of the $g$th Gaussian element are calculated.

$$\overline{w_{i,d}}(\mathrm{g}) = \frac{\sum_{t=1}^{T} \gamma_t(i,d,g)}{\sum_{g=1}^{G}\sum_{t=1}^{T} \gamma_t(i,d,g)} \tag{18}$$

$$\overline{\mu_{i,d}}(\mathrm{g}) = \frac{\sum_{t=1}^{T} \gamma_t(i,d,g) \cdot o_t}{\sum_{t=1}^{T} \gamma_t(i,d,g)} \tag{19}$$

$$\overline{\Sigma_{i,d}}(\mathrm{g}) = \frac{\sum_{t=1}^{T} \gamma_t(i,d,g)(\mathrm{o_t} - \mu_{i,d}(\mathrm{g}))(\mathrm{o_t} - \mu_{i,d}(\mathrm{g}))^T}{\sum_{t=1}^{T} \gamma_t(i,d,g)} \tag{20}$$

Finally, the state duration probability with single Gaussian distributions is re-estimated, which can be expressed as the probability that a state has remained for $d$ time units, then transits to the next state. The mean and variance of which can be calculated as below:

$$\overline{\mu}(i) = \frac{\sum_{t=1}^{T}\sum_{d=1}^{D} \xi_t(i,i+1,d) \cdot d}{\sum_{t=1}^{T}\sum_{d=1}^{D} \xi_t(i,i+1,d)} \tag{21}$$

$$\overline{\sigma}(i) = \frac{\sum_{t=1}^{T}\sum_{d=1}^{D} \xi_t(i,i+1,d) \cdot d^2}{\sum_{t=1}^{T}\sum_{d=1}^{D} \xi_t(i,i+1,d)} - [\overline{\mu}(i)]^2 \tag{22}$$

Based on the defined variables, the MDD-HSMM is trained iteratively, and all the parameters are modified until the model is convergent with the maximum value of $P(O|\lambda)$.

## 3 High-order particle filter method

The particle filter method has been proposed in the 1990s, which is a significant and effective state estimation methodology based on the Monte Carlo simulations for implementing the recursive Bayesian filter [34]. The Bayesian theorems construct the posterior probability density function of the states based on the observation data in specific space model. In this paper, the HOPF method is researched, and the state space model of the machine health dynamic propagation system is modeled by the MDD-HSMM introduced above. For the HOPF with $m$th-order, the current health state evolution depends on a group of $m$-steps-before states instead of only on the previous state in traditional first-order PF model. The $m$th-order HOFP model is written as:

$$x_k = f_k(x_{k-1}, x_{k-2}, ..., x_{k-m}, w_{k-1}) \tag{23}$$

$$y_k = h_k(x_k, v_k) \tag{24}$$

where $\{x_k, k \in N\}$ is the state sequence, $x_k$ denotes the state at time $k$, and $\{y_k, k \in N\}$ is the observation sequence, $y_k$ denotes the observation value at time $k$. $f_k$ is state evolution function, and $h_k$ is measurement function that denotes the nonlinear mapping relationship between the model states and the observation sequence, and the functions $f_k$ and $h_k$ are corresponding to the states transition and observation probability function of MDD-HSMM, respectively [30]. $\{u_k, k \in N\}$ and $\{v_k, k \in N\}$ are independent identically distributed process noise and observation noise, respectively.

The key idea of the PF method is to calculate the posterior density function $p(x_k|y_{1:k-1})$ recursively using the sample set, which is obtained iteratively through prediction and update operation. In this paper, we denote the set of random weighted samples as $\{x^i_{0:k}, w^i_{0:k}\}^N_{i=1}$, where the $x^i_k$ is a particle with the weight $w^i_k$, and $N$ is the number of particles for the computation. Then, the posterior probability $p(x_{0:k}|y_{1:k})$ can be approximated as follows [35]:

$$p(x_{0:k}|y_{1:k}) \approx \sum_{i=1}^{N} w^i_k \delta\left(x_{0:k} - x^i_{0:k}\right) \tag{25}$$

where $w^i_k \propto \frac{p\left(x^i_{0:k}|y_{1:k}\right)}{q\left(x^i_{0:k}|y_{1:k}\right)}$.

$q\,(x_{0:k}|y_{1:k})$ is the importance density function, which is chosen with a special structure as the factorization.

$$q\,(x_{0:k}|y_{1:k}) = q\,(x_k|x_{k-m:k-1}, y_k)\, q\,(x_{0:k-1}|y_{1:k-1}) \tag{26}$$

In general, the state transition probability is taken as the importance density function for brevity of calculation:

$$q\left(x^i_k|x^i_{k-m:k-1}, y_k\right) = p\left(x^i_k|x^i_{k-m:k-1}\right) \tag{27}$$

and the factorization of $p\,(x_{0:k}|y_{1:k})$ can also be derived using Bayesian criterion:

$$\begin{aligned} p(x_{0:k}|y_{1:k}) &= \frac{p(y_k|x_{0:k}, y_{1:k})\, p(x_{0:k}|y_{1:k-1})}{p(y_k|y_{1:k-1})} \\ &= \frac{p(y_k|x_{k-n+1:k})\, p(x_{0:k}|y_{1:k-1})}{p(y_k|y_{1:k-1})} \\ &\propto p\,(y_k|x_{k-n+1:k})\; p\,(x_k|x_{k-\mathrm{m}:k-1}) \\ &\quad \cdot p\,(x_{0:k-1}|y_{1:k-1}) \end{aligned} \tag{28}$$

Consequently, the weight of particle $x^i_k$ updates by:

$$w^i_k = w^i_{k-1} p\left(y_k|x^i_{k-n+1:k}\right) \tag{29}$$

and $w^i_k = w^i_k / \sum\limits_{i=1}^{N} w^i_k$.

In order to eliminate the weights degeneracy problem, the approximated effective sample size $N_{eff}$ is introduced.

$$N_{eff} = \left(\sum_{i=1}^{N} \left(w^i_k\right)^2\right)^{-1} \tag{30}$$

Furthermore, the resampling method is applied to prevent the degeneracy by eliminating the particles with small weights and replicating those having large weights when the effective samples number $N_{eff}$ is smaller than the fixed threshold $N_{thres}$ [36], and after this, the weights of all samples are reset to be equal.

## 4 The proposed MDD-HSMM and HOPF based online machine health prognostics approach

Health prognostics procedure fuses and utilizes the historical information with the objective of assessing and determining the state transition relations and duration information, and further predicting the health condition or estimating the RUL values. The proposed machine health prognostics approach is presented in this section.

Firstly, The MDD-HSMM is trained offline based on the historical lifetime health data to obtain the duration-dependent probability distributions and the state duration information, and then a nonlinear state space model is constructed. Next, the HOPF model is established for modeling dynamic health changes of machine based on the nonlinear state space model. With the online sensing monitoring data, the health state will be recognized using the HOPF model, and then the corresponding RUL values can be estimated in real time. The framework of online health prognostics approach based on the MDD-HSMM and HOFP model is shown in Fig. 2.

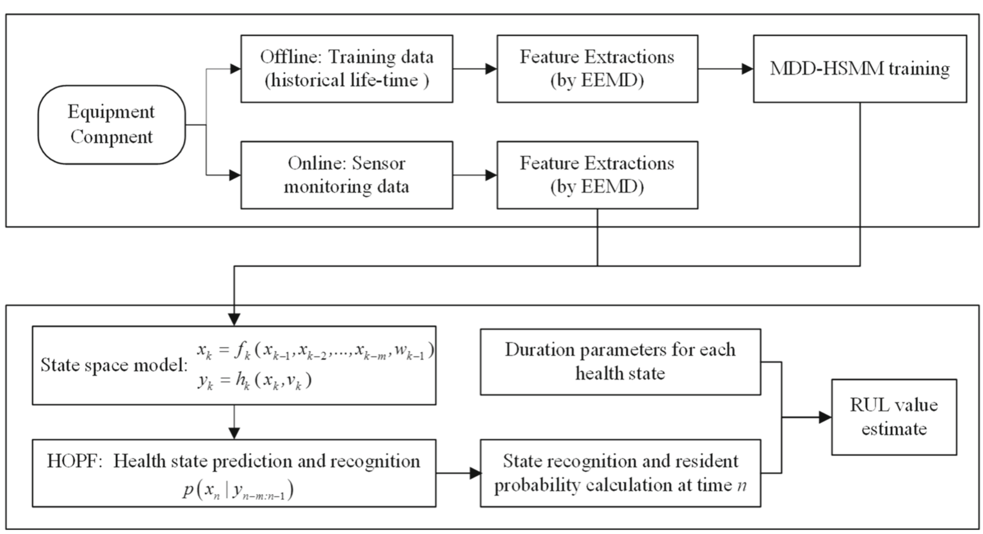


**Fig. 2** Health prognostics framework based on HSMM and HOFP model

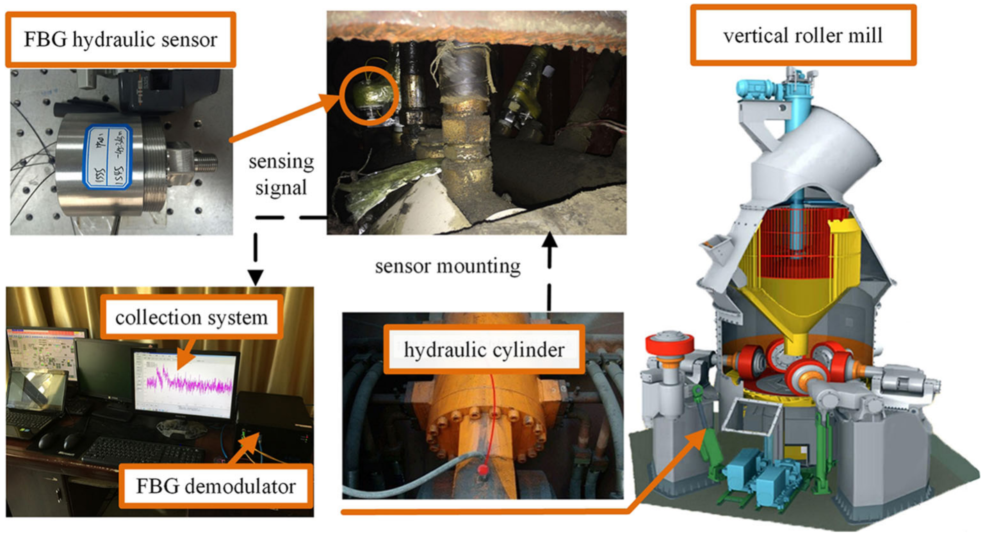


**Fig. 3** Diagram of the experimentation platform

### 4.1 Integration of MDD-HSMM in HOPF for online machine health state recognition

The proposed online health prognostics framework integrates the MDD-HSMM and $m$th-order HOPF model, which aims to take advantages of the robust mathematical foundation of MDD-HSMM and the real-time characteristic of HOPF. Thus, the state space model defined in Eq. 23 and 24 for HOPF need to be constructed firstly based on the trained MDD-HSMM. Furthermore, since the machine health state evolution is no longer only state-dependent, but also duration-dependent, as defined in MDD-HSMM, a variable-order state space model related to the duration of state is explored for HOPF. In this paper, we propose a sliding window with variable length to present the relationship between current state and several-steps-before states that remaining in the same state, and the length of this window, ranging from $l = 1$ to $l = D_i$, is determined by the duration of the last state $i$, where $D_i$ is the maximal duration of state $i$ defined in MDD-HSMM. Finally, the health state probability distribution can be calculated based on the HOPF model and the sensor monitoring data in real time, and the probability change trend of each health state can be obtained.

For the HOPF model with variable order, the order $m$ is equal to the length of the sliding window $l$, and the $m$th-order state equation is expressed as follow:

$$x_k = g_k(x_{k-1}, x_{k-2}, ..., x_{k-m}) \tag{31}$$

where $g_k(x_{k-1}, x_{k-2}, ..., x_{k-m})$ is the nonlinear function determined by duration-dependent state transition probability of the MDD-HSMM (i.e. $a_{ii}(m)$ or $a_{i(i+1)}(m)$ ), and $x_{k-1}, x_{k-2}, ..., x_{k-m}$ are the states sequences which have stayed in the same state for $m$ time units. This online health recognition algorithm aims to calculate the state probability density function using the modified $m$th-order state equation with random samples set. Let $\{x^i_{0:k}, w^i_{0:k}\}^N_{i=1}$ approximate the state PDF $p(x_{0:k}|y_{1:k})$. The procedure of the one-step-ahead health recognition algorithm can be carried out as follows:

Step 1: one-step-ahead state PDF prediction:

$$p(x_k|y_{1:k-1}) \approx \sum_{i=1}^{N} w^i_{k-1}\delta\left(x_k - x^i_k\right)$$

where particle $x_k$ is emitted from previous several states according to the state duration and the nonlinear function in Eq. 31.

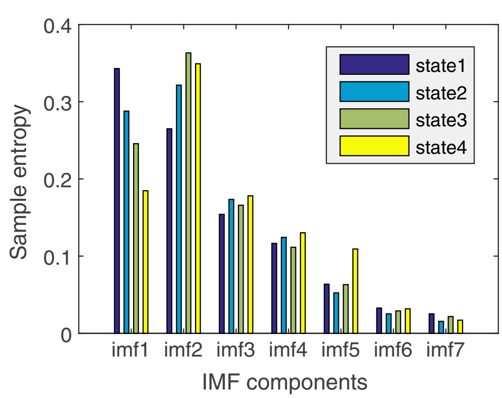


**Fig. 4** Comparison of sample entropy distribution of the four health states

**Table 1** Initial weight coefficients of observation probability

| Health states | State1 | State2 | State3 | State4 |
|---|---|---|---|---|
| Gaussian1 | 0.3333 | 0.2857 | 0.5714 | 0.4285 |
| Gaussian2 | 0.3840 | 0.3840 | 0.2381 | 0.3840 |
| Gaussian3 | 0.2857 | 0.3333 | 0.1905 | 0.1905 |

**Table 2** Initial means of observation probability

| Health states | State1 | State2 | State3 | State4 |
|---|---|---|---|---|
| Gaussian1 | [−0.1322, 0.0378] | [ 0.0088, 0.0702] | [0.0334,−0.0065] | [0.1153,−0.0435] |
| Gaussian2 | [−0.1915,−0.0459] | [−0.0282,−0.0129] | [0.0627, 0.0326] | [0.1077, 0.0066] |
| Gaussian3 | [−0.0758,−0.0318] | [−0.0568, 0.0590] | [0.0131,−0.0510] | [0.2038, 0.0052] |

Step 2: Update the weight with the new measurement $y_k$ :

$$w_k^i = w_{k-1}^i p\left(y_k|x_{k-m+1:k}^i\right)$$

where $p\left(y_k|x_{k-n+1:k}^i\right)$ is the duration-dependent observation probability (i.e.$b_{x_k,n}(y_k)$) that the output probability of $y_k$ is determined by all the $n$ steps previous particles which stayed in the same state as $x_k$.

Step 3: Correct the state PDF with new weight:

$$p(x_k|y_{1:k}) = \sum_{i=1}^{N} w_k^i \delta\left(x_k - x_k^i\right)$$

Then, the state PDF can be calculated iteratively with the process of probability prediction and weight update.

### 4.2 Integration of MDD-HSMM in HOPF for online machine RUL prognostics

The purpose of online machine health prognostics is to predict the degradation condition and estimate the RUL values using the result of health recognition algorithm. Suppose that the whole machine life can be characterized by normal stage, degradation stage and failure stage, and the machine will go through health states $s_i(i = 1, 2, \cdots, N-1)$ before entering failure state $s_N$. Let $D(s_i)$ denote the expected duration of health state $s_i$, which is calculated as below:

$$D(s_i) = \mu(s_i)+\rho\sigma^2(s_i) \tag{32}$$

where $\mu(s_i)$ and $\sigma^2(s_i)$ are duration mean and variance of state $s_i$, respectively. And $\rho$ is denoted as:

$$\rho = (T - \sum_{i=1}^{N} \mu(s_i))/\sum_{i=1}^{N} \sigma^2(s_i) \tag{33}$$

there has a constraint that $T = \sum_{i=1}^{N} D(s_i)$.

The RUL values can be calculated as the summation of the expected residual useful life of current health state $s_i$ and the duration of total future health states before failure [22]. The expected residual useful life of current health state $s_i$ can be denoted as $\overline{D}({s_i}^{(d)})$ at the $d$th time point, which is determined by the probability of staying at health state $s_i$.

$$\overline{D}({s_i}^{(d)}) = p(s_d = s_i|y_{1:d})D(s_i) \tag{34}$$

Thus, the RUL of equipment at the $d$th time point can be calculated using the following formula.

$$RUL^{(d)}=\overline{D}({s_i}^{(d)}) + \sum_{j=i+1}^{N-1} D(s_j) \tag{35}$$

By integrating the MDD-HSMM with HOPF, the duration of each health state and the probability of staying at current health state can be obtained, and then the RUL values also can be calculated.

### 4.3 Application procedure of the proposed prognostics approach

The detailed application procedure for the proposed online health prognostics approach can be described as follows:

Step 1: Collect the health monitoring signals from the machinery equipment, and then the signal denoising and feature extraction methods are applied to obtain the input observation feature vectors sequence for MDD-HSMM.

Step 2: The MDD-HSMM is trained offline based on the modified forward-backward algorithm, then the duration dependent transition probabilities among health states

**Table 3** Initial covariances of observation probability (*$10^{-3}$)

| Health states | State1 | State2 | State3 | State4 |
|---|---|---|---|---|
| Gaussian1 | $\begin{pmatrix} 0.9175 & 0.2012 \\ 0.2012 & 0.5908 \end{pmatrix}$ | $\begin{pmatrix} 0.5857 & -0.2044 \\ -0.2044 & 0.2056 \end{pmatrix}$ | $\begin{pmatrix} 0.5677 & -0.1707 \\ -0.1707 & 0.3243 \end{pmatrix}$ | $\begin{pmatrix} 0.4153 & 0.0022 \\ 0.0022 & 0.2936 \end{pmatrix}$ |
| Gaussian2 | $\begin{pmatrix} 0.7009 & -0.4695 \\ -0.4695 & 1.4737 \end{pmatrix}$ | $\begin{pmatrix} 0.2702 & 0.1291 \\ 0.1291 & 0.5511 \end{pmatrix}$ | $\begin{pmatrix} 0.2592 & 0.0747 \\ 0.0747 & 0.1758 \end{pmatrix}$ | $\begin{pmatrix} 0.3257 & 0.1209 \\ 0.1209 & 0.6799 \end{pmatrix}$ |
| Gaussian3 | $\begin{pmatrix} 0.7146 & 0.2616 \\ 0.2616 & 0.6220 \end{pmatrix}$ | $\begin{pmatrix} 0.6011 & 0.0512 \\ 0.0512 & 0.1533 \end{pmatrix}$ | $\begin{pmatrix} 0.3343 & 0.0622 \\ 0.0622 & 0.4602 \end{pmatrix}$ | $\begin{pmatrix} 0.5698 & -0.5396 \\ -0.5396 & 1.6586 \end{pmatrix}$ |

**Table 4** Expected duration values of each health state

| Health states | State1 | State2 | State3 | State4 |
|---|---|---|---|---|
| Resident Mean | 19.5929 | 19.5147 | 19.6278 | 17.9070 |
| Resident Variance | 1.3793 | 1.6064 | 1.2740 | 3.5629 |
| Expected duration | 20.1849 | 20.2042 | 20.1746 | 19.4363 |

and output observation probabilities and the expected state duration are obtained.

Step 3: Based on the duration-dependent transition probabilities and observation probabilities of the MDD-HSMM, the nonlinear state space model is constructed for modeling the fault propagation of machine.

Step 4: A variable-order HOPF model is applied with the nonlinear state space model. Based on the random sampling particles, the health state probability distribution is calculated iteratively with the online sensing monitored data.

Step 5: Based on the health state probability distribution and change trend, the health condition change point can be determined, and then the current health state is recognized.

Step 6: The residual duration of current health state and the RUL of machinery equipment are calculated with Eqs. 34 and 35.

## 5 Case study and result analysis

In this section, the proposed online health prognostics approach was applied to a real case study for the vertical roller mill (VRM) based on the dynamic pressure signal of the hydraulic supply system, and a detailed analysis for the experiment result is described to evaluate the application performance of the proposed method.

### 5.1 Experimental setup

In this case study, the hydraulic cylinder of the long-term working VRM, which is a widely used heavy rotating machine equipment in cement production industry, was researched. As the important supply and drive unit of VRM, the hydraulic cylinder is mainly working for driving the rocker arm movement and providing working power for the grinding roller through transforming the hydraulic energy into mechanical energy. In the experiment, the internal leakage fault of the hydraulic fluid has been studied, which is one of the most serious faults regarding to the hydraulic cylinder. The hydraulic pressure signal was selected as the data source to detect the corresponding fault feature information. And there are some advantages of acquiring the pressure signal easily in the operating hydraulic cylinder, because of the low-cost and convenient mounts and non-intrusive sensors. The pressure signal is mainly composed of the low-frequency pressure ripples, the high-frequency transient fluctuation, and the environment noise. With the internal leakage increasing, the flow rate of the hydraulic cylinder will decrease, which results in the slowly rise of pressure signal and the disorder movement of the oil fluid. Since the pressure signal indicates hydraulic cylinders health state, the internal leakage stages corresponding to different degrees of flow loss and pressure signal transient fluctuation were defined as the health states of the hydraulic cylinder in this study.

The diagram of the experimentation setup is shown in Fig. 3. The test was performed on the VRM VME46.4, and the studied hydraulic cylinder is working at normal condition with the hydraulic pressure of 14 MPa. The FBG pressure sensors are mounted on the hydraulic cylinder that connected with the rocker arm to measure the hydraulic pressure in real time. The FBG pressure sensor used in this study is a high hydraulic pressure transducer suitable for the wide-range measurement requirement, which is characterized with the measuring range of 0 to 130 MPa and the sensitivity of 10 pm/MPa. The pressure sensing signals were processed with the high performance FBG demodulator and stored in data servers, and the A/D conversion scheme of this FBG demodulator is 8-bit, and the sampling rate is set

**Table 6** Health states transition probability matrix with $d_t(i) = 10$

| Health states | State1 | State2 | State3 | State4 |
|---|---|---|---|---|
| State1 | 0.8622 | 0.1378 | 0 | 0 |
| State2 | 0 | 0.7707 | 0.2293 | 0 |
| State3 | 0 | 0 | 0.8229 | 0.1959 |
| State4 | 0 | 0 | 0 | 1 |

**Table 5** Health states transition probability matrix with $d_t(i) = 1$

| Health states | State1 | State2 | State3 | State4 |
|---|---|---|---|---|
| State1 | 0.9374 | 0.0626 | 0 | 0 |
| State2 | 0 | 0.9626 | 0.0374 | 0 |
| State3 | 0 | 0 | 0.9661 | 0.0339 |
| State4 | 0 | 0 | 0 | 1 |

**Table 7** Health states transition probability matrix with $d_t(i) = D_i$

| Health states | State1 | State2 | State3 | State4 |
|---|---|---|---|---|
| State1 | 0 | 1 | 0 | 0 |
| State2 | 0 | 0 | 1 | 0 |
| State3 | 0 | 0 | 0 | 1 |
| State4 | 0 | 0 | 0 | 1 |

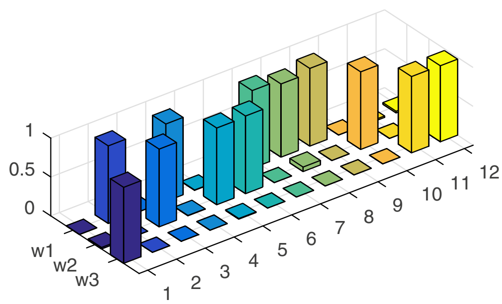


**Fig. 5** Duration-dependent weight coefficients of observation probability

to 4 kHz. The long-term signal collection is conducted to obtain the run-to-failure hydraulic pressure data.

### 5.2 Data processing

According to the failure experience knowledge of hydraulic cylinder, there are four hidden health states divided with the change of pressure, including the normal (denoted by state1), deterioration1 (denoted by state2), deterioration 2 (denoted by state3) and failure (denoted by state4), respectively. For the convenience of this experiment, we took the data filtering and sampling process for the massive long-term sensing monitoring signals, and got the whole life data of hydraulic cylinder for the internal leakage fault.

In general, there is a lot of noise in the raw sensing signals which needs to be minimized before the feature information extraction phase. It is known that the wavelet transform technology has great capability in non-stationary signals de-noising because of its extraordinary time-frequency representation performance, which decompose signals into separate frequency bands orthogonally [37]. In this study, the wavelet packet with Symlets wavelet 5 (sym5) was applied to decompose the noisy signals into two layers, then the heuristic threshold and soft threshold function were chosen to process the wavelet decomposition coefficients to eliminate the redundant noisy information. Finally, the de-noised signals were reconstructed from the threshold wavelet coefficients based on the inverse wavelet transform.

After the wavelet decomposition and reconstruction process, the ensemble empirical mode decomposition (EEMD) method was used to extract the fault feature information from the de-noised signals, which is an effective self-adaptive analysis method for decomposing the time series signal into a series of different frequency bands, termed the intrinsic mode function (IMFs) [38]. The ensemble number and the white noise amplitude of the EEMD were set as 100 and 0.3 times standard deviation of the de-noised signal respectively in this study, then the IMFs relating to the fault information were obtained, and the sample entropy of each IMF component was selected to compose the feature vectors. The procedure of the fault feature extraction method based on EEMD is described as follows:

Step 1: The EEMD is used to decompose the signals into several IMFs, and the first $k$ IMFs which contain the main failure information are extracted.

Step 2: The sample entropy [39] $E_i$ of $IMF_i$ is computed as the composition of fault feature vector T.

Step 3: Construct and normalize feature vector $T = T/E$, where $E = \sum_{j=1}^{k} E_j$.

The sample entropy feature vectors of four health states were constructed, as displayed in Fig. 4.

Figure 4 shows that the sample entropy is mainly concentrated in the first two bands, and the distribution is changed with the health state degradation. Moreover, a principal component analysis (PCA) method was used to capture the major variance, and the variances explained by these seven IMFs are 69.41, 20.74, 4.93, 3.17, 1.43, 0.31, 0.01%. The sum variance of the first two principal components PC1 and PC2 is 90.15%, which contains most of the variance. Thus, for the propose of reducing the dimensionality of feature vectors for lower computational complexity while ensuring the major feature information, the new feature vectors consist of the first two principal components was selected as the observation sequence for the MDD-HSMM.

### 5.3 MDD-HSMM training

The MDD-HSMM with all health states is trained offline using the processed observation sequence consists of the first two principal components PC1 and PC2 from PCA. A left-to-right Markov structure with four states is chosen in this study, and the initial state distribution is $\pi_0 =$ [1, 0, 0, 0], and the initial health states transition probability is set as equiprobability and duration invariant, i.e. $a_{ii}(d) =$

**Table 8** Means and variances of observation probability with $d_t(i) = 1$

| Health states | State1 | State2 | State3 | State4 |
|---|---|---|---|---|
| Mean | [−0.2242, −0.0058] | [−0.0892, 0.0645] | [0.0044, −0.0790] | [0.0959, −0.0516] |
| Variance (*$10^{-4}$) | $\begin{pmatrix} 1.7392 & -2.0135 \\ -2.0135 & 2.7359 \end{pmatrix}$ | $\begin{pmatrix} 1.0572 & -0.1780 \\ -0.1780 & 0.0312 \end{pmatrix}$ | $\begin{pmatrix} 0.0746 & 0.2416 \\ 0.2416 & 0.7837 \end{pmatrix}$ | $\begin{pmatrix} 0.3785 & 0.1558 \\ 0.1558 & 0.0679 \end{pmatrix}$ |

**Table 9** Means and variances of observation probability with $d_t(i) = 10$

| Health states | State1 | State2 | State3 | State4 |
|---|---|---|---|---|
| Mean | [−0.1081, −0.0403] | [−0.0440, 0.0617] | [0.0305, −0.0035] | [0.1178, −0.0267] |
| Variance (*10$^{-4}$) | $\begin{pmatrix} 1.7027 & 0.4676 \\ 0.4676 & 0.2884 \end{pmatrix}$ | $\begin{pmatrix} 0.1634 & 0.0344 \\ 0.0344 & 0.0072 \end{pmatrix}$ | $\begin{pmatrix} 0.0089 & -0.0092 \\ -0.0092 & 0.0095 \end{pmatrix}$ | $\begin{pmatrix} 0.0065 & 0.0431 \\ 0.0431 & 0.2887 \end{pmatrix}$ |

$a_{i(i+1)}(d)$ and $a_{ij}(1) = \cdots = a_{ij}(D_i)$. The initial value of state duration distribution is denoted as a Gaussian distribution $P(i, d) = N(d|\mu_i, \sigma_i)$ with means and variances $(\mu_i, \sigma_i) = (T/N, 2)$, where $i = 1, 2, 3, 4$. The observation probability distribution is represented as Gaussian mixture distribution with three elements, and the initial values are also set as duration invariant. The weight coefficient $w_{i,d}$; mean vector $\mu_{i,d}$; and covariance matrix $\Sigma_{i,d}$ are initialized with k-means clustering algorithm and the initial values are shown in Tables 1, 2, and 3, respectively.

The initial parameters indicate the necessity of taking the Gaussian mixture distribution to represent the observation probability, since the weight coefficients distribution is nearly uniform and the distance between each mean vector within a state is obvious.

Based on the initial parameters, the MDD-HSMM is trained using the modified forward-backward and parameter re-estimation algorithm mentioned in Section 2.2. The log-likelihood is used to indicate the iterative training process, and the maximum iteration steps is set to 40 and the convergence error is 0.0001. The final estimated values of the MDD-HSMM are presented below. The initial probability is $\pi = [1, 0, 0, 0]$, which means that the machine must start from the normal state. The expected duration values of each health state are obtained in Table 4.

The estimated values for the health states transition probability matrixes with duration time $d_t(i) = 1$, $d_t(i) = 10$, $d_t(i) = D_i$, where $i = 1, 2, 3, 4$, are shown in Tables 5, 6, and 7, respectively.

The estimated values of duration-dependent weight coefficients of the Gaussian mixture distribution for the four states with duration time $d_t(i) = 1$, $d_t(i) = 10$ $d_t(i) = D_i$, where $i = 1, 2, 3, 4$, are given in Fig. 5 as all 12 groups of weight data. It is noticed on Fig. 5 that convergent weight-coefficients have large variance for every certain state and-duration time, which means that the observation probability focus on a major single Gaussian element at each certain duration time.

The mean vector $\mu_{i,d}$ and covariance matrix $\Sigma_{i,d}$ of the major single Gaussian observation probability distribution for the four states with duration time $d_t(i) = 1$, $d_t(i) = 10$ $d_t(i) = D_i$, where $i = 1, 2, 3, 4$, are presented in Tables 8, 9 and 10, respectively.

### 5.4 Health prognostics based on MDD-HSMM and HOPF

Based on the trained MDD-HSMM, the HOPF method is executed to predict the health state and RUL values of equipment. According to the state duration values calculated in the Sub-section 5.3, we set the maximal order of the HOPF as $D(i)$, and the order of HOPF is changed dynamically with time. Firstly, the probability distribution of staying at health state $s_i$ at the $d$th observation point can be computed as $p(s_d = s_i | y_{1:d}), i = 1, 2, 3, 4$ and the results are shown as Fig. 6.

The Fig. 6 shows the changing trend of state probability with the time points, and the health state change points are obtained when the state probability reaches the maximum value. As shown in Fig. 6, the health state is changing from the state1 to state4 gradually, and the start point of each state is assumed as the corresponding state change point. From the 1st time point to the 20th time point, the probability of health state1 is decreasing with the time points, while the probability of health state2 is increasing, which means that the health state1 is remaining from the 1st time point to the 19th time point and the health state2 is start from the 20th with the maximum state probability. According to the analysis of the state transition process, it can be seen from the Fig. 6 that the state3 is start from the 40th observation time point, and once the machine equipment enters the state4 i.e. failure state, it always stays at that state. Thus, the

**Table 10** Means and variances of observation probability with $d_t(i) = D_i$

| Health states | State1 | State2 | State3 | State4 |
|---|---|---|---|---|
| Mean | [−0.0640, −0.0025] | [0.0320, 0.0555] | [0.0849, 0.0430] | [0.1912, 0.0388] |
| Variance (*10$^{-4}$) | $\begin{pmatrix} 0.4122 & 0.5714 \\ 0.5714 & 1.4149 \end{pmatrix}$ | $\begin{pmatrix} 0.5359 & -0.3393 \\ -0.3393 & 0.2148 \end{pmatrix}$ | $\begin{pmatrix} 0.4935 & 0.2309 \\ 0.2309 & 0.1081 \end{pmatrix}$ | $\begin{pmatrix} 0.1611 & -0.4267 \\ -0.4267 & 1.1314 \end{pmatrix}$ |

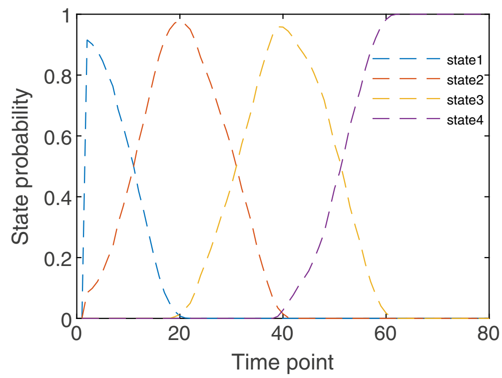


**Fig. 6** Changing trend of the state probability

next machine RUL assessment only relates to the expected residual duration of health state 1 to state 3. Based on Eq. (34), the residual useful life of current health state $\overline{D}(s_i{}^{(d)})$ $(i = 1, 2, 3)$ can be obtained, and then according to the Eq. (35), the RUL values for all observation time points are calculated. The prognostic results of MDD-HSMM and the actual RUL and the relative error are listed in Table 11, and the comparison between the prognostic results and actual RUL is shown in Fig. 7, which indicates that the predicted RUL can fit with the real RUL quite well. Therefore, the proposed method based on the MDD-HSMM is effective for equipment health prognostics.

### 5.5 Discussions and performance evaluation

In order to further illustrate the better performance of the proposed method for online machine health prognostics, some HSMM based modified models are conducted as comparative studies for health prognosis. Firstly, the modified HSMM with duration-dependent transition probability (DD-HSMM) [26] which considered that the state transition probabilities are varying with the duration of each state, is used as the comparison to illustrate the necessary of considering the deterioration effect within a health state and prove the superiority of the proposed method with additional duration-dependent observation probabilities. Moreover, the DD-HSMM using HOPF method is also selected as comparison to show the strength of the HOPF method. In addition, the age-dependent HSMM [21] integrated with aging factors is used as another comparative study.And in this paper, the aging factor with multiple form is selected and the hazard rate method is also performed for the RUL computation. Similarly to Peng and Dong [21] the aging factors $\hat{\beta}$ in this paper is calculated with $step\_length = 0.001$ and iteration number $k = 24$, and $\hat{\beta} = k \times step\_length = 0.024$. Finally, the pure HSMM [19] is introduced for comparation, in which the RUL computation is only related to current state and the residual duration of each health state is assumed as the all duration of this state. The comparison results of the five prognostics methods are shown in Table 12 and Fig. 8, which illustrate that the proposed method based on the MDD-HSMM with HOPF has a better performance for machine health prognostics than DD-HSMM with HOPF, DD-HSMM, age-dependent HSMM and pure HSMM.

In generally, there are some error criteria that used to evaluate the prognostics performance of proposed approach. In this study, both the absolute and relative error criteria are used to illustrate the prognostics performance from different perspectives. The absolute error criteria concentrate on root mean square error (RMSE), mean absolute error (MAE) and variance absolute error (VAE) that are often used in most experiments. The mean absolute relative error (MARE) and variance relative error (VRE) are the relative

**Table 11** Comparison between prognostics results and actual URL

| Actual RUL | Predicted RUL | Relative error (%) | Actual RUL | Predicted RUL | Relative error (%) |
|---|---|---|---|---|---|
| 58.00 | 57.3935 | 1.0458 | 29.00 | 29.3011 | 1.0382 |
| 57.00 | 56.5865 | 0.7254 | 28.00 | 27.8741 | 0.4495 |
| 56.00 | 56.2097 | 0.3745 | 27.00 | 26.7056 | 1.0904 |
| 55.00 | 55.6884 | 1.2516 | 26.00 | 25.4645 | 2.0597 |
| 54.00 | 54.9831 | 1.8205 | 25.00 | 23.9034 | 4.3863 |
| 53.00 | 54.3623 | 2.5703 | 24.00 | 22.6767 | 5.5136 |
| 52.00 | 53.0690 | 2.0558 | 23.00 | 21.4886 | 6.5711 |
| 51.00 | 52.1773 | 2.3085 | 22.00 | 20.4367 | 7.1060 |
| 50.00 | 51.1736 | 2.3471 | 21.00 | 19.7173 | 6.1082 |
| 49.00 | 50.1280 | 2.3021 | 20.00 | 18.1212 | 9.3939 |
| 48.00 | 48.6817 | 1.4202 | 19.00 | 17.8869 | 5.8585 |
| 47.00 | 47.5432 | 1.1558 | 18.00 | 17.4815 | 2.8806 |
| 46.00 | 46.2054 | 0.4465 | 17.00 | 16.9310 | 0.4058 |
| 45.00 | 44.7777 | 0.4940 | 16.00 | 16.4872 | 3.0450 |
| 44.00 | 43.3806 | 1.4078 | 15.00 | 16.0084 | 6.7228 |
| 43.00 | 42.1532 | 1.9694 | 14.00 | 15.3173 | 9.4092 |
| 42.00 | 41.2057 | 1.8912 | 13.00 | 14.5376 | 11.8278 |
| 41.00 | 40.3921 | 1.4828 | 12.00 | 13.6677 | 13.8978 |
| 40.00 | 38.8452 | 2.8869 | 11.00 | 12.3449 | 12.2260 |
| 39.00 | 38.7430 | 0.6591 | 10.00 | 10.9812 | 9.8125 |
| 38.00 | 38.4491 | 1.1818 | 9.00 | 9.9745 | 10.8275 |
| 37.00 | 37.6594 | 1.7821 | 8.00 | 8.6273 | 7.8409 |
| 36.00 | 36.5934 | 1.6484 | 7.00 | 7.1234 | 1.7628 |
| 35.00 | 35.7297 | 2.0849 | 6.00 | 5.9768 | 0.3872 |
| 34.00 | 34.8213 | 2.4155 | 5.00 | 4.8885 | 2.2300 |
| 33.00 | 33.7845 | 2.3772 | 4.00 | 3.5563 | 11.0914 |
| 32.00 | 32.6899 | 2.1561 | 3.00 | 2.4397 | 18.6765 |
| 31.00 | 31.7290 | 2.3517 | 2.00 | 1.5751 | 21.2456 |
| 30.00 | 30.6261 | 2.0869 | 1.00 | 0.8312 | 16.8842 |

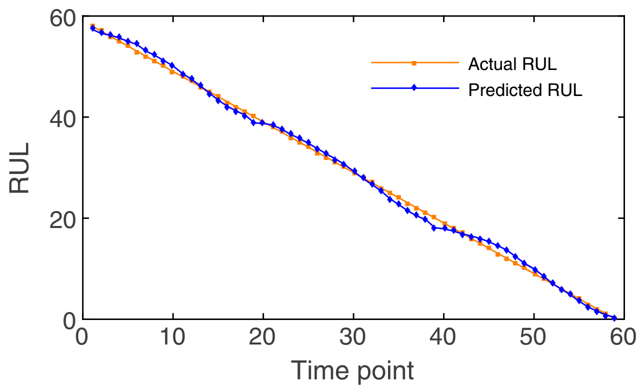


**Fig. 7** Comparison between prognostics results and actual URL

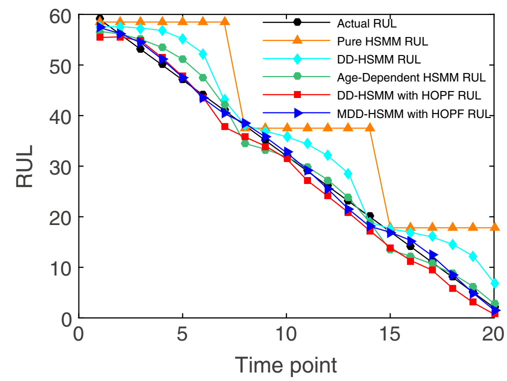


**Fig. 8** The RUL prognostic results comparison of different methods

error criteria which represent the prediction accuracy relative to the original actual value. Based on the selected five error criteria, the prognostic performance evaluation of MDD-HSMM with HOPF, DD-HSMM, with HOPF, age-dependent HSMM, DD-HSMM and pure HSMM is shown in Table 13. It can be seen from Table 13 that the prognostic performance of the proposed method based on the MDD-HSMM with HOPF is superior to the DD-HSMM with HOPF, age-dependent HSMM, DD-HSMM, and pure HSMM when both the absolute error and relative error are considered.

Through the overall case study, the online machine health prognosis is implemented based on the proposed method with the sensing monitoring observation data, and the results illustrate that the proposed methods has a better performance for machine health prognostic than compared methods. For these compared methods, the pure HSMM [19] assumes that state transition probability is fixed, and the residual useful life of current state is assumed to be the total state duration, then the prognostic result is shown as a step signal related to each health state. The DD-HSMM [26] takes duration dependent transition probability into construct, the probability that transits to itself is a decreasing function of duration time $d_t(i)$, so that the RUL prognostic result of DD-HSMM is not only state dependency but

**Table 12** Prognostics results of the different methods

| Actual RUL | MDD-HSMM with HOPF RUL | DD-HSMM with HOPF RUL | Age-HSMM | DD-HSMM RUL | Pure HSMM RUL |
|---|---|---|---|---|---|
| 58.00 | 57.3935 | 55.4330 | 56.6527 | 57.5305 | 58.4945 |
| 56.00 | 56.2097 | 55.5641 | 56.1271 | 57.5865 | 58.4945 |
| 53.00 | 54.3623 | 56.7478 | 55.0568 | 57.2927 | 58.4945 |
| 50.00 | 51.1736 | 51.4811 | 53.4911 | 56.8384 | 58.4945 |
| 47.00 | 47.5432 | 47.7387 | 51.1532 | 55.1804 | 58.4945 |
| 44.00 | 43.3806 | 43.5200 | 47.5843 | 52.2504 | 58.4945 |
| 41.00 | 40.3921 | 37.7389 | 42.0045 | 43.0750 | 58.4945 |
| 38.00 | 38.4491 | 35.7708 | 34.4818 | 38.1278 | 37.5081 |
| 35.00 | 35.7297 | 33.9597 | 33.3319 | 36.8941 | 37.5081 |
| 32.00 | 32.6899 | 31.5008 | 31.8398 | 35.8416 | 37.5081 |
| 29.00 | 29.3011 | 27.1870 | 29.8806 | 34.3644 | 37.5081 |
| 26.00 | 25.4645 | 24.0396 | 27.2749 | 32.0517 | 37.5081 |
| 23.00 | 21.4886 | 20.7837 | 23.7618 | 28.5057 | 37.5081 |
| 20.00 | 18.1212 | 17.3076 | 18.9558 | 18.9560 | 37.5081 |
| 17.00 | 16.9310 | 13.9173 | 13.3803 | 17.6192 | 17.8222 |
| 14.00 | 15.3173 | 11.3331 | 12.1968 | 16.8950 | 17.8222 |
| 11.00 | 12.3449 | 9.4864 | 10.6868 | 16.0019 | 17.8222 |
| 8.00 | 8.6273 | 5.8959 | 8.7394 | 14.5743 | 17.8222 |
| 5.00 | 4.8885 | 3.0213 | 6.1987 | 12.2121 | 17.8222 |
| 2.00 | 1.5751 | 0.7517 | 2.8428 | 6.9563 | 17.8222 |

**Table 13** Prediction performance evaluation results of different methods

| Prognostic methods | RSME | MAE | VAE | MARE | VRE |
|---|---|---|---|---|---|
| MDD-HSMM with HOPF | 0.8841 | 0.0933 | 0.7729 | 0.0462 | 0.0027 |
| DD-HSMM with HOPF | 1.9873 | −1.3580 | 2.1052 | 0.1316 | 0.0242 |
| Age-HSMM | 2.2351 | 0.2931 | 4.9098 | 0.0927 | 0.0094 |
| DD-HSMM | 4.9757 | 4.2100 | 7.0337 | 0.3718 | 0.3915 |
| Pure HSMM | 10.4807 | 8.8768 | 31.0487 | 0.9581 | 5.6854 |

also changing with the observation time point. However, without the effective RUL calculation method, the RUL of DD-HSMM is only determined by the duration dependent transition probability, thus, there are lager prediction error. The DD-HSMM with HOPF method introduces the online iterative and updating process into the RUL values estimation, which is effective for RUL prediction because of the self-updating scheme with observation data. The age-dependent HSMM [21] assumes the transition probability is changed with the multiple aging factors, and the hazard rate is also combined for RUL computation, which is effective for health prognostics. From the prognostic performance evaluation results, we can see that the MDD-HSMM with HOPF method is superior for prognosis than these comparative studies, which indicates that the duration dependent characters of both the transition probabilities and the observation probabilities are significant to model the failure propagation of machine, and the duration-dependent variable-order HOPF is effective for online machine health prognostics.

## 6 Conclusions

In this paper, a machine health prognostics approach based on the MDD-HSMM and HOPF is proposed. In order to describe the degradation of machinery equipment more appropriately, a MDD-HSMM is developed, in which the state transition and output observation probabilities are defined that depend not only on the current state, but also on the duration of state. The forward and backward variables are modified in this paper, and a forward-backward algorithm is also developed to facilitate the training of the proposed MDD-HSMM. Moreover, due to the difficulty for online health recognition and prognostics, the HOPF method with better online features is introduced, and the HOPF is improved with a sliding window to adjust the order of HOPF dynamically according to duration of previous state. Based on the nonlinear state space model of the MDD-HSMM and the online prediction and update scheme of the HOPF, the probability distribution of machine health state is obtained and then the probability distribution change trend is used to recognize the health state. Furthermore, the corresponding RUL prediction algorithm is developed. The integration of MDD-HSMM and HOPF takes the full advantages of the robust mathematical foundation of MDD-HSMM and online characteristic of HOPF, which is proved to obtain better prognostics performance. The real experimental results for the hydraulic cylinder of vertical roller mill equipment indicate the effectiveness of the proposed prognostics method, which has higher prediction accuracy and online performance than several methods, such as DD-HSMM with HOPF, age-dependent HSMM, DD-HSMM, and pure HSMM method. Consequently, the proposed approach in this paper can be applied to online machine health prognostics to improve the process reliability during the manufacturing processing.

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