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ORIGINAL ARTICLE

Rectifying inspection for PAOSQLL scheme based on variable repetitive group sampling plan

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Abstract The concept of the quality loss derived by Taguchi has been accepted as the evaluation measure of quality instead of the traditional attribute property such as the proportion of non-conforming items. Since the variable single sampling plan having the desired operating characteristics indexed by the quality loss was proposed in 1997, various kinds of sampling plans indexed by the quality loss have been considered. Among them, there are two kinds of rectifying variable single sampling (RVSS) plans indexed by the quality loss. In the RVSS plans, two inspection schemes called the acceptance quality loss limit (AQLL) scheme and the permissible average outgoing surplus quality loss limit (PAOSQLL) scheme have been formulated. Note that the concepts of the AQLL and PAOSQLL schemes in the RVSS plans indexed by the quality loss are equivalent to those of the lot tolerance percent defective (LTPD) and the average outgoing quality limit (AOQL) schemes in the traditional rectifying attribute single sampling plans, respectively. On the other hand, in the sampling plan having the desired operating characteristics, an attribute repetitive group sampling plan was proposed for the purpose of reducing the average sampling number in the inspection. Recently, by applying the repetitive group sampling to the rectifying variable sampling plan for the AQLL scheme, the rectifying variable repetitive group sampling (RVRGS) plan for the AQLL scheme has been considered for the purpose of reducing the average total inspection (ATI). However, the RVRGS plan for the PAOSQLL scheme has not been investigated yet. Accordingly, the RVRGS plan for the PAOSQLL scheme must be investigated to complete the RVRGS plans indexed by the quality loss. In this article, the RVRGS plan for the PAOSQLL scheme is addressed. Then, the design procedure in the RVRGS plan for the PAOSQLL scheme is proposed for the purpose of reducing ATI. Through some numerical investigations, the effectiveness to reduce ATI by the RVRGS plan for the PAOSQLL scheme is confirmed.

Keywords Average total inspection (ATI) · Patnaik's approximation · Permissible average outgoing surplus quality loss limit (PAOSQLL) inspection scheme · Repetitive group sampling · Quality loss

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1 Introduction

According to the loss function defined by Taguchi [1, 2] as a quadratic function based on variable quality characteristics, the item quality characteristics are evaluated based on the deviation from the target value of item quality. Therefore, the item quality characteristics can be evaluated more strictly based on this loss function in comparison with the evaluation based on the proportion of nonconforming items. Further, the quality loss has been defined as the expectation of the loss function. The concept of the quality loss can be adopted as the new quality evaluation instead of a traditional quality evaluation such as the proportion of nonconforming items. Under the new quality evaluation criterion as the quality loss, Arizono et al. [3] have proposed the variable single sampling inspection plan having the desired operating characteristics (OC) indexed by the quality loss. Since then, various kinds of sampling plans indexed by the quality loss have been considered.

On the other hand, there is a sampling programs called the rectifying inspection proposed by Dodge and Roming [4] separately from the sampling plans having the desired OC. In the rectifying inspection, when lots are rejected, the lots are totally inspected. Recently, by applying the quality loss, two kinds of rectifying variable single sampling (RVSS) plans have been proposed by Arizono et al. [5]. In the RVSS plans, two inspection schemes called the acceptance quality loss limit (AQLL) scheme and permissible average outgoing surplus quality loss limit (PAOSQLL) scheme have been respectively formulated. Note that the concepts of the AQLL and PAOSQLL schemes in the RVSS plans indexed by the quality loss are equivalent to those of the lot tolerance percent defective (LTPD) and average outgoing quality limit (AOQL) schemes in the traditional rectifying attribute single sampling plans, respectively.

By the way, it is also important to reduce the cost in the sampling inspection in addition to the quality assurance by inspection. In regard to the cost reduction, the attribute repetitive group sampling plan for the purpose of reduction of sampling number was proposed by Sherman [6]. Since then, many research results related to the concept of repetitive group sampling have been reported by Balamurali et al. [7], Balamurali and Jun [8] and Jun et al. [9]. Then, Aslam et al. [10] and Tomohiro et al. [11] have applied this concept to the sampling inspection plan indexed by the quality loss mentioned above, and proposed the variable repetitive group sampling plan having the desired OC indexed by the quality loss. Furthermore, Arizono et al. [12] have applied this concept to the RVSS plan for the AQLL scheme, and proposed the rectifying variable repetitive group sampling (RVRGS) plan for the AQLL scheme lately. In this article, Arizono et al. [12] have confirmed that the RVRGS plan for the AQLL scheme reduces the average total inspection (ATI) in comparing to the RVSS plan for the AQLL scheme. However, the design procedure of the RVRGS plan for the PAOSQLL scheme has not been established yet. Accordingly, the RVRGS plan for the PAOSQLL scheme should be considered. Then, under the condition that the quality characteristic of items obeys a normal distribution, the RVRGS plan for the PAOSQLL scheme indexed by quality loss is investigated. Specifically, we establish the design procedure for deriving the sampling plan satisfying the requirement of the PAOSQLL scheme and minimizing ATI. Through some numerical examples, it is confirmed that the RVRGS plan for the PAOSQLL scheme is effective in the reduction of ATI in comparing to the RVSS plan for the PAOSQLL scheme.

2 Brief of quality loss

When the quality characteristic x in individual items obeys $N(\mu, \sigma^2)$, the expected loss per item can be evaluated as

$$k\tau^2 = E\left[k(x-\mu_T)^2\right] = k\left\{(\mu-\mu_T)^2+\sigma^2\right\}, \qquad (1)$$

where k denotes the proportional coefficient based on the functional limit of quality characteristics and gives the monetary loss brought by the item which cannot fulfill its fundamental function, and μ_T indicates the mean in the ideal quality characteristic distribution for items. Without loss of generality, k can be specified as 1, because k is a constant. In consequence, τ^2 can be redefined as the quality loss. Then, the quality loss τ^2 has been accepted as the new variable evaluation criterion of quality instead of the traditional attribute evaluation criterion of quality such as the proportion of nonconforming items. Note that, in the viewpoint of the quality loss, even if the proportion of nonconforming items was same, the quality loss would not always be the same. Furthermore, from Eq. 1, remark that there are innumerable combinations of (μ, σ^2) yielding same τ^2.

Suppose that the quality characteristic in each item obeys a normal distribution $N(\mu, \sigma^2)$, where μ and σ^2 are unknown parameters, respectively. Then, let $x_i, i = 1, 2, \cdots, n$, be observations obtained from random samples from a lot. In this situation, we have the estimator $\hat{\tau}^2$ of the quality loss τ^2 as follows:

$$\hat{\tau}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu_T)^2 = (\bar{x}-\mu_T)^2 + s^2, \qquad (2)$$

where $\bar{x}$ and s^2 denote the maximum likelihood estimators of μ and σ^2 calculated as follows:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n}x_i, \ \ s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2. \qquad (3)$$

And then, the estimator $\hat{\tau}^2$ is the maximum likelihood estimator and the unbiased estimator of τ^2. Further, the statistic $n\hat{\tau}^2/\sigma^2$ obeys the non-central chi-square distribution with n degrees of freedom and non-central parameter $n\xi$, where ξ is defined as

$$\xi = \frac{(\mu-\mu_T)^2}{\sigma^2}. \qquad (4)$$

Since the non-central chi-square distribution is complicated in the stochastic analysis, Arizono et al. [3, 5] and Morita et al. [13] have employed an approximate distribution for $\hat{\tau}^2$ based on the approximation technique proposed by Patnaik [14]. Then, Arizono et al. [3, 5] have considered the following statistic ρ:

$$\rho = \frac{1+\xi}{1+2\xi}\frac{n\hat{\tau}^2}{\sigma^2}. \tag{5}$$

The mean and variance of the statistic ρ can be obtained as follows:

$$E[\rho] = \frac{1+\xi}{1+2\xi}E\left[\frac{n\hat{\tau}^2}{\sigma^2}\right] = \frac{n(1+\xi)^2}{1+2\xi}, \tag{6}$$

$$V[\rho] = \left(\frac{1+\xi}{1+2\xi}\right)^2 V\left[\frac{n\hat{\tau}^2}{\sigma^2}\right] = \frac{2n(1+\xi)^2}{1+2\xi}. \tag{7}$$

It is found that the mean and variance of the statistic ρ coincide with those of the central chi-square distribution with ϕ degrees of freedom, where

$$\phi = \frac{n(1+\xi)^2}{1+2\xi}. \tag{8}$$

Accordingly, the central chi-square distribution with ϕ degrees of freedom in Eq. 8 can be employed as the approximate distribution of ρ. Note that the function ϕ is a monotonically increasing function in ξ. It can be easily seen that the minimum value $\phi_{\min} = n$ is given by the condition of $\xi = 0$.

From Eqs. 2–8, the statistic ρ can be rewritten as $\rho = \phi\hat{\tau}^2/\tau^2$. Hereby, the distribution of the estimator $\hat{\tau}^2$ is specified approximately as follows:

$$\hat{\tau}^2 \sim \frac{\tau^2}{\phi}\chi^2_\phi, \tag{9}$$

where χ^2_ϕ means the central chi-square distribution with ϕ degrees of freedom. Note that ϕ is given by a function consisting of μ and σ^2. Hence, the distribution of $\hat{\tau}^2$ is not unique even if the value of τ^2 is identical.

3 Outline of RVSS plan for PAOSQLL scheme

Arizono et al. [5] have proposed the RVSS plan for the PAOSQLL scheme based on the concept of the quality loss. In the RVSS plan for the PAOSQLL scheme, the permissible average outgoing surplus quality loss limit S_{limit} is prescribed. And, it is necessary to determine the inspection plan satisfying the constraint for the limitation S_{limit} under the arbitrary combination (μ, σ^2) yielding any τ^2. Let c be the acceptance criterion. Further, $P_A(\tau^2)$ and $P_R(\tau^2)$ are described as the acceptance probability and rejection probability of the lot with the quality loss τ^2, respectively.

Usually, there is a feasible minimum variance σ_T^2 in the comfortable manufacturing environment. From this fact, the state described by the combination (μ_T, σ_T^2) can be defined as the ideal state about the item quality. And then, we express the quality loss yielded by the combination (μ_T, σ_T^2) in the ideal state as $\tau_T^2 (= \sigma_T^2)$.

In the rectifying sampling inspection, the items in the rejected lot are inspected totally. Then, the following screening rule is adopted in order to guarantee that the quality loss of items after screening and replacing is less than τ_T^2:

$$(x - \mu_T)^2 \le \tau_T^2. \tag{10}$$

That is, if the quality characteristic x of the item does not satisfy (10), the item is replaced by the item of the quality characteristic satisfying (10). Based on this screening rule, the expected quality loss L per item after screening and replacing is represented as

$$L = \tau^2 P_A\left(\tau^2\right) + T^2 P_R\left(\tau^2\right), \tag{11}$$

where T^2 is the expected loss per item on the screened lot, and then it is seen that $T^2 \le \tau_T^2$. However, since it is difficult to evaluate the value of T^2 exactly, it is also difficult to obtain the value of L.

Meanwhile, under the screening rule of Eq. 10, the upper limit of L can be obtained as follows:

$$L_{\text{upper}} = \tau^2 P_A\left(\tau^2\right) + \tau_T^2 P_R\left(\tau^2\right), \tag{12}$$

where L_{upper} denotes the upper limit of L. Furthermore, Eq. 12 is transformed into

$$L_{\text{upper}} - \tau_T^2 = \left(\tau^2 - \tau_T^2\right) P_A\left(\tau^2\right) \equiv S. \tag{13}$$

Then, the value of S in Eq. 13 represents the maximum value of the surplus expected quality loss. Based on the concept of the PAOSQLL scheme, the rectifying sampling inspection plan is designed in order to guarantee that the value of S is less than the specified value S_{limit}. This is represented as follows:

$$\left(\tau^2 - \tau_T^2\right) P_A\left(\tau^2\right) \le S_{\text{limit}}. \tag{14}$$

As mentioned previously, note that there are innumerable combinations of (μ, σ^2) yielding same τ^2 due to the relation of $\tau^2 = (\mu - \mu_T)^2 + \sigma^2$. Then, the condition for satisfying the requirement of the PAOSQLL scheme is

$$\max_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2} P_A\left(\tau^2\right) \le \frac{S_{\text{limit}}}{\tau^2 - \tau_T^2} \equiv \varepsilon, \tag{15}$$

where $\Omega(\tau^2)$ represents the set consisting of the combination in (μ, σ^2) satisfying the quality loss τ^2.

Then, under the quality loss τ^2, $ATI_S(\tau^2)$ describing ATI in the RVSS plan is given as

$$ATI_S\left(\tau^2\right) = nP_A\left(\tau^2\right) + NP_R\left(\tau^2\right). \tag{16}$$

Consequently, among the feasible sampling plans (n, c) satisfying (15), the sampling plan (n, c) minimizing ATI is decided as the RVSS plan for the PAOSQLL scheme.

4 Proposal of RVRGS plan for PAOSQLL scheme in consideration of ATI

In this section, we formulate the RVRGS plan for the PAOSQLL scheme. In this plan, we intend to reduce the cost of sampling inspection in comparison with the existing RVSS plan for the PAOSQLL scheme.

Let c_0 and c_1 be acceptance and rejection criteria in the RVRGS plan, respectively. Then, the acceptance rule in the RVRGS plan for the PAOSQLL scheme is constructed as

$$\begin{cases} \text{if } \hat{\tau}^2 \leq c_0, \text{ then accept the lot,} \\ \text{if } c_0 < \hat{\tau}^2 \leq c_1, \text{ then continue the inspection,} \\ \text{otherwise, reject the lot.} \end{cases} \tag{17}$$

Then, the rejected lot is totally inspected. Note that, if the inspection is continued, the judgment of each inspection stage is not affected by the sampling result of previous inspection stage. This is a unique feature of the repetitive group sampling plan introduced by Sherman [6].

For developing the design procedure, we define the following probabilities $P_a(\tau^2)$ and $P_r(\tau^2)$:

$$P_a\left(\tau^2\right) = \Pr\left\{\hat{\tau}^2 \leq c_0 \mid \tau^2\right\}, \tag{18}$$

$$P_r\left(\tau^2\right) = \Pr\left\{\hat{\tau}^2 > c_1 \mid \tau^2\right\}, \tag{19}$$

where $P_a(\tau^2)$ and $P_r(\tau^2)$ are defined as the acceptance probability and rejection probability of the lot with the quality loss τ^2 at each inspection stage by sampling plan (n, c_0, c_1), respectively. Based on $P_a(\tau^2)$ and $P_r(\tau^2)$, the probabilities $P_A(\tau^2)$ and $P_R(\tau^2)$ that the lot with the quality loss τ^2 is eventually accepted and rejected are derived as follows:

$$\begin{aligned} P_A\left(\tau^2\right) &= \sum_{k=1}^{\infty} P_a\left(\tau^2\right)\left\{1 - P_a\left(\tau^2\right) - P_r\left(\tau^2\right)\right\}^{k-1} \\ &= \frac{P_a\left(\tau^2\right)}{P_a\left(\tau^2\right) + P_r\left(\tau^2\right)}, \end{aligned} \tag{20}$$

$$\begin{aligned} P_R\left(\tau^2\right) &= \sum_{k=1}^{\infty} P_r\left(\tau^2\right)\left\{1 - P_a\left(\tau^2\right) - P_r\left(\tau^2\right)\right\}^{k-1} \\ &= \frac{P_r\left(\tau^2\right)}{P_a\left(\tau^2\right) + P_r\left(\tau^2\right)}. \end{aligned} \tag{21}$$

Furthermore, by expressing ATI in the RVRGS plan under the quality loss τ^2 as $ATI(\tau^2)$, $ATI(\tau^2)$ is given as

$$\begin{aligned} ATI\left(\tau^2\right) &= n\sum_{k=1}^{\infty} kP_a\left(\tau^2\right)\left\{1 - P_a\left(\tau^2\right) - P_r\left(\tau^2\right)\right\}^{k-1} \\ &\quad + N\sum_{k=1}^{\infty} P_r\left(\tau^2\right)\left\{1 - P_a\left(\tau^2\right) - P_r\left(\tau^2\right)\right\}^{k-1} \\ &= P_A\left(\tau^2\right) ASN\left(\tau^2\right) + P_R\left(\tau^2\right) N, \end{aligned} \tag{22}$$

where N and $ASN(\tau^2)$ denote the lot size and the average sample number in the repetitive group sampling. In this connection, $ASN(\tau^2)$ is derived as

$$ASN(\tau^2) = \frac{n}{P_a\left(\tau^2\right) + P_r\left(\tau^2\right)}. \tag{23}$$

In the same manner as the RVSS plan, the condition for satisfying the requirement of the PAOSQLL inspection scheme is represented as

$$\max_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2} P_A\left(\tau^2\right) \leq \frac{S_{\text{limit}}}{\tau^2 - \tau_T^2} \equiv \varepsilon, \tag{24}$$

Then, the feasible sampling plans (n, c_0, c_1) should satisfy (24). Note that there are many sampling plans (n, c_0, c_1) satisfying the requirement of the PAOSQLL inspection scheme prescribed by Eq. 24. In this article, we define the RVRGS plan for the PAOSQLL scheme for minimizing ATI based on Eq. 22 in the case of the ideal state (μ_T, σ_T^2).

5 Design procedure for RVRGS plan for PAOSQLL scheme in consideration of ATI

In this section, the design procedure for the RVRGS plan for the PAOSQLL scheme defined in the previous section is developed. Since $P_A(\tau^2)$ is a function composed of $P_a(\tau^2)$ and $P_r(\tau^2)$, it is difficult to find the condition satisfying (24) analytically under the arbitrary combination of (μ, σ^2) yielding any τ^2. However, we can derive the following relation about (24):

$$\begin{aligned} &\max_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2} P_A\left(\tau^2\right) \\ &\quad = \max_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2} \frac{P_a\left(\tau^2\right)}{P_a\left(\tau^2\right) + P_r\left(\tau^2\right)} \\ &\quad = \max_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2} \frac{1}{\dfrac{P_r\left(\tau^2\right)}{P_a\left(\tau^2\right)} + 1} \\ &\quad \leq \frac{1}{\dfrac{\min_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2} P_r\left(\tau^2\right)}{\max_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2} P_a\left(\tau^2\right)} + 1}. \end{aligned} \tag{25}$$

From Eq. 25, the required condition of Eq. 24 is always satisfied if the following inequality is established:

$$\frac{1}{\dfrac{\min\limits_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2} P_r\left(\tau^2\right)}{\max\limits_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2} P_a\left(\tau^2\right)}+1} \le \varepsilon. \tag{26}$$

Therefore, we consider the combinations (μ, σ^2) maximizing $P_a(\tau^2)$ and minimizing $P_r(\tau^2)$ individually. Note that it is not necessary that $P_a(\tau^2)$ and $P_r(\tau^2)$ are evaluated under the same combinations (μ, σ^2) on guaranteeing the establishment of Eq. 24.

At first, we consider the maximization of $P_a(\tau^2)$. Then, we define

$$\max_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2} P_a\left(\tau^2\right) \equiv \varepsilon^{\dagger} \left(0 < \varepsilon^{\dagger} \le \varepsilon\right), \tag{27}$$

Note that the relation of $P_a(\tau^2) \le P_A(\tau^2)$ is satisfied from Eq. 20. Here, we define the acceptance probability of the lot with the ideal quality loss $\tau^2 = \tau_T^2$ as α and represent (μ_T, τ_T^2) as (μ, σ^2) composing this quality loss. Then, this probability α is represented as

$$\begin{aligned} \alpha &= \Pr\left\{\hat{\tau}^2 \le c_0 \mid \tau_T^2\right\} \\ &= \Pr\left\{\hat{\tau}^2 \le \frac{\chi_n^2(1-\alpha)}{n}\tau_T^2\right\}, \end{aligned} \tag{28}$$

where $\chi_n^2(1-\alpha)$ denotes the $100(1-\alpha)$ percentile of chi-square distribution with n degrees of freedom. From Eq. 28, the acceptance criterion c_0 is derived as

$$c_0 \equiv \frac{\chi_n^2(1-\alpha)}{n}\tau_T^2. \tag{29}$$

Next, we derive $\varepsilon^{\dagger}$ by using the calculated c_0. Then, as mentioned previously, there are innumerable combinations of (μ, σ^2) yielding the arbitrary quality loss τ^2. Accordingly, we need to obtain the combination of (μ, σ^2) giving $\varepsilon^{\dagger}$, that is, maximizing $P_a(\tau^2)$. Then, we define (μ, σ^2) maximizing $P_a(\tau^2)$ as (μ^*, σ^{2*}), and investigate the combination of (μ^*, σ^{2*}).

In this situation, the relation between c_0 and $\varepsilon^{\dagger}$ is derived as follows:

$$c_0 = \frac{\chi_{\phi^*}^2\left(1-\varepsilon^{\dagger}\right)}{\phi^*}\tau^2, \tag{30}$$

where ϕ^* is given as follows:

$$\phi^* = \frac{n(1+\xi^*)^2}{1+2\xi^*}, \tag{31}$$

$$\xi^* = \frac{(\mu^*-\mu_T)^2}{\sigma^{2*}}, \tag{32}$$

in conformity to Eqs. 4 and 8.

On the other hand, by applying the Wilson-Hilferty approximation [15], the behavior of $\chi_\phi^2(1-\varepsilon^{\dagger})/\phi$ in τ^2 against ϕ can be specified approximately as follows:

$$\frac{\chi_\phi^2(1-\varepsilon^{\dagger})}{\phi} = \left(1-\frac{2}{9\phi}+u_{1-\varepsilon^{\dagger}}\sqrt{\frac{2}{9\phi}}\right)^3, \tag{33}$$

where $u_{1-\varepsilon^{\dagger}}$ denotes the upper $100(1-\varepsilon^{\dagger})$ percentile of standard normal distribution. By transforming (33), the following equation is obtained:

$$\begin{aligned} u_{1-\varepsilon^{\dagger}} &= \sqrt{\frac{9\phi}{2}}\left(\sqrt[3]{\frac{\chi_\phi^2(1-\varepsilon^{\dagger})}{\phi}}+\frac{2}{9\phi}-1\right) \\ &= \sqrt{\frac{9\phi}{2}}\left(\sqrt[3]{\frac{c_0}{\tau^2}}+\frac{2}{9\phi}-1\right). \end{aligned} \tag{34}$$

Further, the differential coefficient (primary derivative) for ϕ can be derived as

$$\frac{du_{1-\varepsilon^{\dagger}}(\phi)}{d\phi} = \sqrt{\frac{9}{2}}\phi^{-\frac{1}{2}}\left\{\frac{1}{2}\left(\sqrt[3]{\frac{c_0}{\tau^2}}-1\right)-\frac{1}{9}\phi^{-1}\right\}. \tag{35}$$

If $\sqrt[3]{c_0/\tau^2}-1 \le 0$, Eq. 35 is always negative regardless of the value of ϕ. Therefore, Eq. 34 is a monotonically decreasing function in ϕ, and maximized in $\phi = \phi_{\min}$, where $\phi_{\min}$ is the minimum of ϕ. Furthermore, ϕ is specified as a function related to ξ, and $\xi \ge 0$. By differentiating ϕ with respect to ξ, the following relation is obtained:

$$\frac{d\phi}{d\xi} = \frac{2n\xi(1+\xi)}{(1+2\xi)^2} \ge 0. \tag{36}$$

Based on Eq. 36, it is obvious that ϕ is a monotonically increasing function in ξ and minimized when ξ is minimized. So, ϕ is minimized in $\xi = 0$ because $\xi \ge 0$. Based on this logic, under the condition of $(\mu^*, \sigma^{2*}) = (\mu_T, \tau^2)$ yielding $\xi^* = 0$, it is seen that (34) is maximized and then $P_a(\tau^2)$ is maximized.

On the other hand, if $\sqrt[3]{c_0/\tau^2}-1 > 0$, Eq. 34 is convex in ϕ, and maximized when ϕ is minimized or maximized. In this case, we compare the value of $\phi_{\min}$ or $\phi_{\max}$ and $\phi^{\dagger}$, where $\phi_{\max}$ is the maximum of ϕ and $\phi^{\dagger}$ is ϕ yielding the following relation:

$$\phi^{\dagger} = \left\{\frac{9}{2}\left(\sqrt[3]{\frac{c_0}{\tau^2}}-1\right)\right\}^{-1}. \tag{37}$$

If $\phi^{\dagger} \le \phi_{\min}$, Eq. 34 is maximized in $\phi = \phi_{\max}$. Since ϕ is a monotonically increasing function in ξ from Eq. 8, Eq. 34 is maximized when ξ is maximized. In addition, because ξ is a monotonically decreasing function in σ^2 from Eq. 4, ξ is maximized in $\sigma^2 = \sigma_T^2$, where σ_T^2 is the minimum of σ^2. Therefore, under the condition of

$(\mu^*, \sigma^{2*}) = \left(\mu_T \pm \sqrt{\tau^2 - \sigma_T^2}, \sigma_T^2\right)$, Eq. 34 is maximized. On one hand, if $\phi^\dagger \geq \phi_{\max}$, Eq. 34 is maximized in $\phi = \phi_{\min}$. Therefore, Eq. 34 is maximized under the condition of $(\mu^*, \sigma^{2*}) = (\mu_T, \tau^2)$. Further if $\phi_{\min} < \phi^\dagger < \phi_{\max}$, Eq. 34 is maximized in $\phi = \phi_{\min}$ or $\phi = \phi_{\max}$. In this case, we have the condition of $(\mu^*, \sigma^{2*}) = (\mu_T, \tau^2)$ in $\phi = \phi_{\min}$, and $(\mu^*, \sigma^{2*}) = \left(\mu_T \pm \sqrt{\tau^2 - \sigma_T^2}, \sigma_T^2\right)$ in $\phi = \phi_{\max}$.

Next, we consider the minimization of $P_r(\tau^2)$ under the condition of $\max_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2} P_a(\tau^2) = \varepsilon^\dagger$. From Eq. 26, the following relation is established:

$$\frac{1}{\dfrac{\min_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2} P_r\left(\tau^2\right)}{\max_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2} P_a\left(\tau^2\right)} + 1} = \frac{1}{\dfrac{\min_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2} P_r\left(\tau^2\right)}{\varepsilon^\dagger} + 1} \leq \varepsilon. \tag{38}$$

By transforming (38), we obtain the following relation:

$$\min_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2} P_r\left(\tau^2\right) \geq \frac{1-\varepsilon}{\varepsilon}\varepsilon^\dagger. \tag{39}$$

Consequently, we consider the following inequality:

$$\max_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2} \left\{1 - P_r\left(\tau^2\right)\right\} \leq 1 - \frac{1-\varepsilon}{\varepsilon}\varepsilon^\dagger, \tag{40}$$

where we define $1 - (1-\varepsilon)\varepsilon^\dagger/\varepsilon \equiv \varepsilon^\ddagger (\varepsilon \leq \varepsilon^\ddagger < 1)$, and represent (μ^{**}, σ^{2**}) as (μ, σ^2) yielding the situation of $\max_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2}\{1 - P_r(\tau^2)\}$. Successively, the relation of $\max_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2}\{1 - P_r(\tau^2)\} = \varepsilon^\ddagger$ is represented as

$$\begin{aligned}
\varepsilon^\ddagger &= \max_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2} \left\{1 - P_r\left(\tau^2\right)\right\} \\
&= \max_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2} \Pr\left\{\hat{\tau}^2 \leq c_1 \mid \tau^2\right\} \\
&= \Pr\left\{\hat{\tau}^2 \leq \min_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2} \frac{\chi_\phi^2(1-\varepsilon^\ddagger)}{\phi}\tau^2\right\} \\
&= \Pr\left\{\hat{\tau}^2 \leq \frac{\chi_{\phi^{**}}^2(1-\varepsilon^\ddagger)}{\phi^{**}}\tau^2 \,\middle|\, (\mu^{**}, \sigma^{2**})\right\},
\end{aligned} \tag{41}$$

where ϕ^{**} is given by

$$\phi^{**} = \frac{n(1+\xi^{**})^2}{1+2\xi^{**}}, \tag{42}$$

$$\xi^{**} = \frac{(\mu^{**} - \mu_T)^2}{\sigma^{2**}}, \tag{43}$$

in conformity to Eqs. 4 and 8.

Based on Eq. 41, the rejection criterion c_1 is derived as

$$c_1 = \min_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2} \frac{\chi_\phi^2(1-\varepsilon^\ddagger)}{\phi}\tau^2. \tag{44}$$

By applying the Wilson-Hilferty approximation, the behavior of $\chi_\phi^2(1-\varepsilon^\ddagger)/\phi$ in τ^2 against ϕ can be specified approximately as follows:

$$\frac{\chi_\phi^2(1-\varepsilon^\ddagger)}{\phi} = \left(1 - \frac{2}{9\phi} + u_{1-\varepsilon^\ddagger}\sqrt{\frac{2}{9\phi}}\right)^3. \tag{45}$$

Further, the differential coefficient (primary derivative) for ϕ can be derived as

$$\begin{aligned}
\frac{d}{d\phi}\left(\frac{\chi_\phi^2(1-\varepsilon^\ddagger)}{\phi}\right) &= \sqrt{\frac{1}{2\phi^3}} \\
&\quad \times \left(1 - \frac{2}{9\phi} + u_{1-\varepsilon^\ddagger}\sqrt{\frac{2}{9\phi}}\right)^2 \\
&\quad \times \left(\sqrt{\frac{8}{9\phi}} - u_{1-\varepsilon^\ddagger}\right).
\end{aligned} \tag{46}$$

By investigating (46), we obtain (μ^{**}, σ^{2**}) as follows:

1. If $1 - \varepsilon^\ddagger > 0.5$, namely $\varepsilon^\ddagger < 0.5$, Eq. 45 is a monotonically increasing function in ϕ and minimized when ϕ is minimized. So, $P_r(\tau^2)$ is minimized under the condition of $(\mu^{**}, \sigma^{2**}) = (\mu_T, \tau^2)$.
2. If $1 - \varepsilon^\ddagger \leq 0.5$, we consider based on the comparison of $\varepsilon^\ddagger$ and γ yielding $u_\gamma = \sqrt{8/9n}$, where $\sqrt{8/9n}$ is the maximum of $\sqrt{8/9\phi}$.
   (a) If $0 \leq 1 - \varepsilon^\ddagger \leq \gamma$, namely $1 - \gamma \leq \varepsilon^\ddagger < 1.0$, $u_{1-\varepsilon^\ddagger}$ is more than $\sqrt{8/9n}\,(\geq \sqrt{8/9\phi})$. In this case, Eq. 46 is negative regardless of the value of ϕ. Hereby, Eq. 45 is a monotonically decreasing function in ϕ and minimized in $\phi = \phi_{\max}$. As a result, Eq. 45 is minimized under the condition of $(\mu^{**}, \sigma^{2**}) = \left(\mu_T \pm \sqrt{\tau^2 - \sigma_T^2}, \sigma_T^2\right)$
   (b) If $\gamma < 1 - \varepsilon^\ddagger \leq 0.5$, that is, $0.5 \leq \varepsilon^\ddagger < 1 - \gamma$, Eq. 45 is minimized when ϕ is maximized or minimized, because $\chi_\phi^2(1-\varepsilon^\ddagger)/\phi$ is concave in ϕ.
      i If $\sqrt{8/9\phi_{\max}} - u_{1-\varepsilon^\ddagger} \geq 0$, Eq. 45 is minimized when ϕ is $\phi_{\min}$.
      ii If both relations of $\sqrt{8/9\phi_{\min}} - u_{1-\varepsilon^\ddagger} > 0$ and $\sqrt{8/9\phi_{\max}} - u_{1-\varepsilon^\ddagger} < 0$ are satisfied simultaneously, Eq. 45 is minimized when ϕ is $\phi_{\max}$ or $\phi_{\min}$. Hereby, we have the condition of $(\mu^{**}, \sigma^{2**}) = (\mu_T, \tau^2)$ when Eq. 45 is minimized at $\phi_{\min}$. On

one hand, the condition of $(\mu^{**}, \sigma^{2**}) = \left(\mu_T \pm \sqrt{\tau^2 - \sigma_T^2}, \sigma_T^2\right)$ is derived in the case that (45) is minimized at $\phi_{\max}$.

By the argument so far, the rejection criterion under the prescribed sample size n and quality loss τ^2 can be defined as

$$c_1 = \min_{(\mu,\sigma^2)\in\Omega(\tau^2)|\forall\tau^2} \frac{\chi^2_{\phi}(1-\varepsilon^{\ddagger})}{\phi}\tau^2$$
$$\equiv \min_{\forall\tau^2} \frac{\chi^2_{\phi^{**}}(1-\varepsilon^{\ddagger})}{\phi^{**}}\tau^2, \quad (47)$$

Then, we define $c_1^{\dagger}$ as the candidate for the rejection criterion c_1 as follows:

$$c_1^{\dagger} = \frac{\chi^2_{\phi^{**}}(1-\varepsilon^{\ddagger})}{\phi^{**}}\tau^2, \quad (48)$$

where $\varepsilon^{\ddagger}$ is defined as

$$\varepsilon^{\ddagger} = 1 - \frac{1-\varepsilon}{\varepsilon}\varepsilon^{\dagger}, \quad (49)$$

and ϕ^{**} is represented as follows:

$$\phi^{**} = \begin{cases} \phi_{\min} = n, & (0 \le \varepsilon^{\ddagger} < 0.5) \\ \phi_{\max} = \dfrac{n\left(\dfrac{\tau^2}{\sigma_T^2}\right)^2}{2\dfrac{\tau^2}{\sigma_T^2} - 1}, & (1-\gamma \le \varepsilon^{\ddagger} < 1.0) \\ \phi_{\min} \text{ or } \phi_{\max}, & (0.5 \le \varepsilon^{\ddagger} < 1-\gamma) \end{cases} \quad (50)$$

where ϕ^{**} making (48) smaller is adopted in the case of $0.5 \le \varepsilon^{\ddagger} < 1-\gamma$. Furthermore, we adopt the smallest $c_1^{\dagger}$ against the arbitrary quality loss τ^2 as the rejection criterion c_1 in the RVRGS plan for the PAOSQLL scheme.

6 Algorithm for designing optimal RVRGS plan for PAOSQLL scheme

From the results mentioned above, we can provide the following algorithm for the purpose of specifying the RVRGS plan (n, c_0, c_1) in the PAOSQLL scheme:

(i) Set the initial value $n = 2$.
(ii) Set the initial value $\alpha = 0.50$, where α is the acceptance probability of lots with the quality loss $\tau^2 = \tau_T^2$.
(iii) Derive the acceptance criterion c_0 based on Eq. 29 using the values of α, n, and τ_T^2.
(iv) Set the initial value $\tau^2 = \tau_T^2 + S_{\text{limit}} + \delta$, where δ is the small value such as $\delta = 0.01$.
(v) Derive $u_{1-\varepsilon^{\dagger}}$ based on Eq. 34 and obtain $\varepsilon^{\dagger}$.

Table 1 The relation of α and $ATI(\tau_T^2)$ in PAOSQLL scheme under $n = 35$ in the case of $S_{\text{limit}} = 0.25$, $N = 500$ and $\tau_T^2 = 1.00$

α	c_0	c_1	$ATI(\tau_T^2)$
0.84	1.235	1.658	45.87
0.85	1.247	1.649	45.68
0.86	1.259	1.639	45.53
0.87	1.272	1.628	45.44
0.88	1.285	1.617	45.41
0.89	1.300	1.605	45.45
0.90	1.316	1.592	45.58
0.91	1.333	1.579	45.80
0.92	1.351	1.564	46.18
0.93	1.372	1.548	46.73

(vi) Calculate $\varepsilon^{\ddagger}$ and obtain the candidate of the rejection criterion $c_1^{\dagger}$ based on Eq. 48 using the values of $\varepsilon^{\ddagger}$, n, and τ^2.
(vii) Reset τ^2 to $\tau^2 + 0.01$, if $\tau^2 < \tau^2_{\max}$. Then, go to (v). Otherwise, determine the rejection criterion c_1 satisfying (47) from the candidates.
(viii) If $c_0 \ge c_1$, go to (ix). Otherwise, go to (x).
(ix) Reset n to $n+1$, if $n < N$. Then go to (ii). Otherwise, go to (xiii).
(x) Evaluate the value of $ATI(\tau_T^2)$ in Eq. 22 for the sampling plan (n, c_0, c_1).
(xi) Reset α to $\alpha + 0.01$. If $\alpha < 1.00$, then, go to (iii). Otherwise, go to (xii).
(xii) Reset n to $n + 1$, if $n < N$. Then, go to (ii). Otherwise, go to (xiii).
(xiii) Specify (n, c_0, c_1) minimizing the value of $ATI(\tau_T^2)$ in the plans obtained by (i)–(xii) as the optimal RVRGS plan for the PAOSQLL scheme.

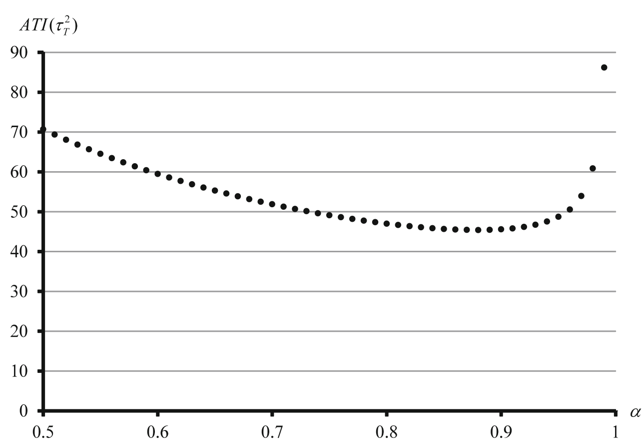

Fig. 1 Fluctuation of $ATI(\tau_T^2)$ for each α under $n = 35$ in the case of $S_{\text{limit}} = 0.25$, $N = 500$ and $\tau_T^2 = 1.00$ in PAOSQLL inspection scheme

Table 2 Sampling plans in PAOSQLL scheme in the case of $S_{\text{limit}} = 0.25$, $N = 500$ and $\tau_T^2 = 1.00$

n	α	c_0	c_1	$ATI(\tau_T^2)$
14	0.53	0.981	1.955	40.54
15	0.56	1.011	1.919	40.08
16	0.59	1.039	1.885	39.77
17	0.61	1.058	1.864	39.57
18	0.64	1.085	1.833	39.47
19	0.66	1.102	1.814	39.45
20	0.68	1.119	1.795	39.50
21	0.70	1.136	1.776	39.62
22	0.72	1.153	1.757	39.80
23	0.74	1.169	1.739	40.03

7 Numerical examples

In this section, we show some numerical examples of the RVRGS plan for the PAOSQLL scheme, and verify the effectiveness of the RVRGS plan for the PAOSQLL scheme proposed in this article.

At first, we illustrate an example for the design process based on the algorithm in the previous section. Let $S_{\text{limit}} = 0.25$, $N = 500$ and $\tau_T^2 = 1.00$. Then, we show the relation of α and $ATI(\tau_T^2)$ under $n = 35$ as an example. Table 1 shows a part of calculated results of α, c_0, c_1 and $ATI(\tau_T^2)$ under $n = 35$. Then, Fig. 1 illustrates the relation of α and $ATI(\tau_T^2)$ under $n = 35$. In this case, ATI is minimized in $\alpha = 0.88$ and c_0, c_1 and $ATI(\tau_T^2)$ obtain 1.285, 1.617 and 45.41.

Then, Table 2 shows a part of results of α, c_0, c_1 and minimum value of $ATI(\tau_T^2)$ for each n, and Fig. 2 illustrates a part of results of n and $ATI(\tau_T^2)$. Consequently, $(n, c_0, c_1) = (19, 1.102, 1.814)$ and $ATI(\tau_T^2) = 39.45$ are

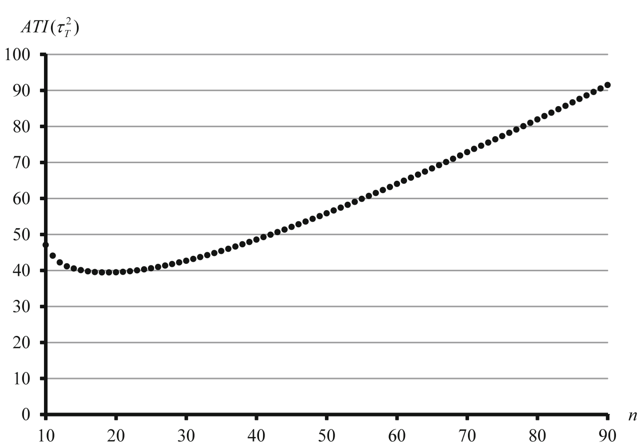

Fig. 2 The relation of n and $ATI(\tau_T^2)$ in the case of $S_{\text{limit}} = 0.25$, $N = 500$ and $\tau_T^2 = 1.00$ in PAOSQLL scheme

Table 3 The values of quality characteristic x_i in the 1st inspection stage (sample size $n = 19$)

0.228	−1.596	1.896	0.611	1.474
1.140	0.692	−0.297	0.506	1.187
0.416	−1.672	−0.792	0.548	−0.593
0.906	−0.530	0.391	1.937	

obtained as the optimal sampling plan and minimum value of $ATI(\tau_T^2)$ through the above numerical results.

Secondly, for the purpose of promoting the understanding of the operating procedure of the RVRGS plan for the PAOSQLL scheme, a typical operating example is demonstrated. Assume that the sampling plan $(n, c_0, c_1) = (19, 1.102, 1.814)$ is designed as the RVRGS plan for the PAOSQLL scheme and let $\mu_T = 0.0$. The typical operation based on the designed RVRGS plan for the PAOSQLL $(n, c_0, c_1) = (19, 1.102, 1.814)$ is exemplified as follows:

Case: Then, assume that the quality characteristic values x_i in the 1st sampling stage have been obtained as data in Table 3. Based on values in Table 3, $\hat{\tau}^2$ is calculated as follows:

$$\begin{aligned}\hat{\tau}^2 &= \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu_T)^2 \\ &= \frac{1}{19}\left\{(0.228 - 0.0)^2 + \cdots + (1.937 - 0.0)^2\right\} \\ &= 1.133.\end{aligned}$$

Since $c_0 = 1.102 < \hat{\tau}^2 = 1.133 \le c_1 = 1.814$, the inspection has been continued. Accordingly, in the 2nd sampling stage, new samples of size $n = 19$ have been drawn from the identical lot again. The quality characteristic values x_i in the 2nd inspection stage are shown in Table 4. Based on Eq. 2, the estimate $\hat{\tau}^2$ is evaluated as follows:

$$\begin{aligned}\hat{\tau}^2 &= \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu_T)^2 \\ &= \frac{1}{19}\left\{(0.025 - 0.0)^2 + \cdots + (-0.635 - 0.0)^2\right\} \\ &= 0.775.\end{aligned}$$

Table 4 The values of quality characteristic x_i in the 2nd inspection stage (sample size $n = 19$).

0.025	−1.204	−0.083	0.647	1.219
−0.176	0.047	−2.134	−0.721	−1.390
−0.155	−0.940	−0.951	0.399	0.908
−0.846	0.446	−0.459	−0.635	

Table 5 The reduction rate in ATI for each S_{limit} in the case of $N = 500$ and $\tau_T^2 = 1.00$

S_{limit}	n	c_0	c_1	$ATI(\tau_T^2)$	n_S	c	$ATI_S(\tau_T^2)$	Reduction rate (%)
0.15	29	1.037	1.527	71.50	53	1.283	88.82	19.50
0.20	23	1.070	1.672	51.45	42	1.378	65.98	22.02
0.25	19	1.102	1.814	39.45	34	1.470	51.69	23.67
0.30	16	1.129	1.957	31.61	29	1.561	42.06	24.84
0.35	14	1.158	2.095	26.18	25	1.651	35.22	25.66
0.40	12	1.180	2.239	22.21	22	1.740	30.15	26.32

In this case, the estimate $\hat{\tau}^2 = 0.775$ is less than acceptance criterion $c_0 = 1.102$. Therefore, the lot has been accepted at the 2nd sampling stage.

As mentioned previously in Section 4, note that the estimate $\hat{\tau}^2$ at the 2nd inspection stage in Case has been calculated based on the new data in Table 4. Therefore, the judgment at the 2nd inspection stage has not been affected by the observations in the 1st inspection stage in Table 3.

Finally, we consider the proportion of the reduction of ATI in the RVRGS plan for the PAOSQLL scheme from ATI in the RVSS plan proposed by Arizono et al. [5] under the same condition. Table 5 shows the reduction rates in ATI under the proposed RVRGS plan and RVSS plan for each S_{limit} under $N = 500$ and $\tau_T^2 = 1.00$, where the value of reduction rate is given as $\frac{ATI_S(\tau_T^2) - ATI(\tau_T^2)}{ATI_S(\tau_T^2)}$. In the same manner, Table 6 shows the reduction rates in ATI under the proposed RVRGS plan and RVSS plan for each S_{limit} under $N = 1000$ and $\tau_T^2 = 1.00$. Then, n_S, c and $ATI_S(\tau_T^2)$ indicate the sample size, acceptance criterion and ATI in the RVSS plan for the PAOSQLL scheme under the same conditions. From the results in Tables 5 and 6, it can be seen that the effectiveness of the RVRGS plan to the reduction of ATI is confirmed.

As mentioned already, Arizono et al. [5] have considered the RVRGS plan for the AQLL scheme. In their investigation, it has been shown that the proportion of the reduction of ATI in the RVRGS plan for the AQLL scheme has been aroud 30% in comparing to the RVSS plan for the AQLL scheme. On one hand, we have also evaluated the proportion of the reduction of ATI in the RVRGS plan for the PAOSQLL scheme in comparing to the RVSS plan for the PAOSQLL scheme as about 30%. Therefore, the effectiveness of the repetitive group sampling technique on the standpoint for reducing ATI has been reconfirmed.

8 Concluding remarks

As sampling procedure for assuring the quality of the outgoing lot, two kinds of the RVSS plans for the AOLL scheme and PAOSQLL scheme proposed. Recently, by applying the repetitive group sampling to the rectifying variable sampling plan for the purpose of reducing ATI, the RVRGS plan for the AQLL scheme was presented.

Successively, in this article, we have designed the RVRGS plan for the PAOSQLL scheme in order to reduce ATI in comparing to the RVSS plan for the PAOSQLL scheme under the same condition. Then, we have established the design procedure for deriving the RVRGS plan satisfying the conditional requirement of the PAOSQLL scheme and minimizing ATI. Through some numerical results, the feature for the reduction of ATI in the proposed plan has been investigated in comparison with the RVSS plan indexed by quality loss. Therefore, the economic program of the RVRGS plan for the PAOSQLL scheme indexed by quality loss has been constructed. In consequence of the success in this article, two kinds of the RVRGS plans for the AQLL scheme and PAOSQLL scheme have been completly established.

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Table 6 The reduction rate in ATI for each S_{limit} in the case of $N = 1000$ and $\tau_T^2 = 1.00$

S_{limit}	n	c_0	c_1	$ATI(\tau_T^2)$	n_S	c	$ATI_S(\tau_T^2)$	Reduction rate (%)
0.15	42	1.058	1.506	91.64	76	1.287	119.05	23.02
0.20	31	1.086	1.651	64.18	57	1.379	85.65	25.07
0.25	24	1.103	1.807	48.26	45	1.470	65.67	26.51
0.30	20	1.129	1.949	38.14	37	1.560	52.60	27.49
0.35	17	1.158	2.086	31.24	31	1.650	43.51	28.20
0.40	15	1.190	2.217	26.30	27	1.738	36.86	28.65

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