Int J Adv Manuf Technol (2018) 94:701–710
DOI 10.1007/s00170-017-0867-z

ORIGINAL ARTICLE

An optimization method of tool-path parameters for curved surface by construction of cutter location mesh units

Jian-wei Ma [1] · **Wei-wei Su** [1] · **Zhen-yuan Jia** [1] · **De-ning Song** [1] · **Guo-qing Hu** [1]

Received: 10 April 2017 / Accepted: 24 July 2017 / Published online: 17 August 2017

Abstract Curved surface parts are widely used in industrial applications and its machining is still a major problem because of the continuously geometric feature variation. When the processing condition is determined, the machining quality and efficiency of curved surface are affected directly by the generation of tool-path, especially by the selection of tool-path parameters including cutting step length and cutting path interval. However, the common selection method for cutting step length and cutting path interval is usually conservative in the tool-path planning process for curved surface parts, so as to affect the machining efficiency. In this way, an optimization method of tool-path parameters for curved surface by the construction of cutter location mesh units is proposed. Based on the cutter location points obtained by constant scallop-height method, the cutter location mesh units are constructed by optimizing cutting step length and cutting path interval, and then the nodes of the cutter location mesh units are taken as the final cutter location points. Take a saddle surface workpiece as an example, the results of simulation and experiment show that the tool-path with the optimized cutting step length and cutting path interval can not only maintain the machining quality but also improve the machining efficiency obviously. These research achievements are of vital importance in tool-path planning for realizing high-quality and high-efficient machining for the curved surface parts.

Keywords Curved surface · Cutter location point · Tool-path planning · Machining efficiency · Parameter optimization

1 Introduction

Curved surface parts are widely used in aerospace, automobile, and mold manufacturing industry fields for special, functional, or esthetic feature [1]. Through decades of development, the curved surface machining is still a major problem because of the continuous variation of geometric feature, such as the curvature [2, 3]. To improve machining quality and efficiency are the permanent subjects in the machining field of curved surface parts, and the machining quality and efficiency are affected directly by the generation of tool-path when the processing condition is determined. In this way, the tool-path planning, which is the key technology for curved surface machining, becomes a research hotspot [4, 5]. The tool-path planning includes the planning of cutting step length and the planning of cutting path interval. Therefore, it is of great importance for the optimization of these two factors, so as to guarantee both machining quality and machining efficiency.

Many researches have done in this area. Sang et al. [1, 6] constructed an offset cutter location surface for the triangular mesh surface. The offset distance was equal to the cutter radius and the offset direction was the normal vector of the grid vertex, then the cutter location points were the intersection points of the selected plane and the offset surface. Elber and Cohen [7] optimized the iso-parametric method to create the high adaptive tool-path for shortening the machining time and make the machining efficiency higher. Kim and Yang [8] presented a cutter location surface deformation approach for constant scallop-height tool-path generation from the triangular mesh. The triangular mesh model of the stereo lithography

---

✉ Jian-wei Ma
mjw2011@dlut.edu.cn

[1] Key Laboratory for Precision and Non-traditional Machining Technology of the Ministry of Education, School of Mechanical Engineering, Dalian University of Technology, Dalian 116024, China

format was computed according to the slope and the curvature of the cutter location surface. Sun et al. [9] presented a novel approach to iso-parametric tool-path generation for triangular meshes via a harmonic map. The cutter-contact points and the path interval were calculated based on the machining tolerance requirements, and finally the iso-parametric tool-path was generated. Feng and Li [10] presented a new approach for minimizing redundant tool-path in the machining of sculptured surfaces with constraint of scallop height for the machined surface. Yoon et al. [11] used Dupin indicatrices of cutter surface and the designed surface at the contact point to find out the locally millable cutting areas. Then the second order approximations of the machining strip width were applied to generate the optimal cutting positions for cutting directions. Lin and Koren [12] presented an improved version of the iso-scallop method for sculptured surface machining in which the determination of the magnitude of the side step was non-iterative, the Taylorian expansion of the side step was of higher order and a procedure was proposed to find out the optimum tool-path. Teng et al. [13] presented a novel tool-path planning algorithm based on the point cloud data. The primary direction of the generated iso-planar tool-path was derived from the projected boundary of the discrete points. Then a projected cutter location net (CL-net) was created on considering the machining error and the surface roughness, and consequently, the nodes of the CL-net were taken as the cutter location point directly. Chu et al. [14] proposed a new method for generating a spline-curve-constrained tool-path that produced minimized geometrical deviations on the machined surface, maintained satisfactory surface quality, and reduced the dimensionality in the solution space. Li et al. [15] adopted the thought of generating cutter location (CL) data directly from corresponding cutter-contact (CC) data, used a discrete computational model to calculate the single-layer interference-free CL contour. Zhang et al. [16] proposed a spiral tool-path generation method in finish-cut process based on the offset-surface method (also called cutter location method). And then the gouge-free tool-path was generated by organizing path through vertex of the offset-mesh. Chen and Khan [17] proposed a new approach to generating accurate, gouge-and-interference free, and smooth NURBS CL path with the arc length parameter. Moodleah and Makhanov [18] presented the numerical generation of a curvilinear grid adapted to the vector field of optimal directions and the biased space-filling curve. Zhang and Tang [19] used an optimal tool-path generation model for a ball-end tool which strived to globally optimize a tool-path with various objectives and constraints.

From the present research work, it can be found that the main difference in the optimization of tool-path planning method is that the calculation method for cutting step length and cutting path interval is different. However, the tool-path planning method is usually for a certain kind of curved surface to meet the machining requirement. For the curved surface parts becoming more complex and its geometric feature is also complex, the selection of cutting step length and cutting path interval is usually conservative in the tool-path planning process for curved surface parts, and the effect of geometric feature on the selection of cutting step length and cutting path interval is seldom involved. In this way, the low adaptability will be encountered for common tool-path generation algorithm, especially in the machining process of the curved surface parts with complex geometric feature, so as to affect the machining efficiency. Thus, an effective optimizing strategy of the parameters in tool-path planning for complex curved surface is urgently needed to be put forward according to the geometric feature.

The purpose of this study is to provide a high adaptability parameter optimization method for the tool-path planning on considering the geometric feature of complex curved surface parts. Based on the cutter location points obtained by constant scallop-height method, the optimization method for cutting step length and cutting path interval is proposed by the construction of cutter location mesh units, and then the nodes of the cutter location mesh units are taken as the final cutter location points, so as to make the tool-path shorter than before. For the size of a cutter location mesh unit is determined by the scallop-height requirement, the processing error requirement and the corresponding initial cutter location points within the cutter location mesh unit, the tool-path in any direction based on the final cutter location points can get the uniform scallop height. On account of the high adaptability parameter optimization method for tool-path planning, high-quality and high-efficiency machining for the curved surface parts can be realized.

The rest of this paper is organized as follows. Section 2 explains basic concept and calculation for the tool-path planning. Section 3 presents the optimization process of cutting step length and cutting path interval by the construction of cutter location mesh units, and finally the cutter location mesh units are obtained. Section 4 conducts the simulation and the experiment and makes the comparison. Conclusions are summarized in Section 5.

2 Basic concept and calculation for tool-path planning

Before carrying out the optimization of the tool-path parameters, some correlated concepts and calculation methods that are necessary for the tool-path planning need to be put forward at first.

Cutter-contact (CC) point CC point is the contact point between cutting tool and machined surface in the machining process. CC point path is the path by connecting all the CC points.

Cutter location (CL) point CL point is usually defined as the center point of the cutting tool in the machining process. CL point path is the path by connecting all CL points.

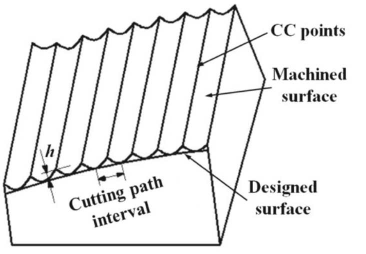
Scallop height The height of the remained material between adjacent CL point paths after finish machining is called scallop height, expressed by h.

Cutting step length The tool-path can be divided into a series of CL Points along the cutting direction. The linear distance between adjacent CL Points is called the cutting step length, as shown in Fig. 1. The cutting step length is calculated by the limit of machining tolerance.

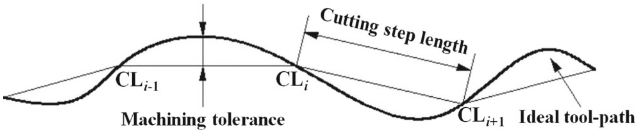
Cutting path interval The linear distance between adjacent CL point paths calls the cutting path interval, as shown in Fig. 1 and expressed by L.

The curved surface is usually machined by the ball-end milling cutter. For any curved surface in terms of differentials, the machining of curved surface can be regarded as the combining of convex surface machining and concave surface machining according to the geometric characteristic of the curved surface. In this way, the calculation of cutting path interval for the curved surface can be classified as calculation of path interval for convex surface and calculation of path interval for concave surface. The calculation method of cutting path interval for the ball-end milling cutter is as following.

The principle of the convex surface machining with the ball-end milling cutter is shown in Fig. 2. In order to simplify the calculation, the curve near the CC point is assumed to be a circular arc. The radius curvature of the machined convex surface at a CC point P is R_b and the milling cutter radius is R. In order to facilitate identification, let q is $R_b + R$. To establish a local coordinate system shown in Fig. 2, the angle between OP and Y-axis is θ, then the equation for the profile of the ball-end milling cutter is as follows,



(a)



(b)

Fig. 1 Schematic diagram of correlated concepts. (**a**) Schematic diagram of cutting path interval, (**b**) Schematic diagram of cutting step length

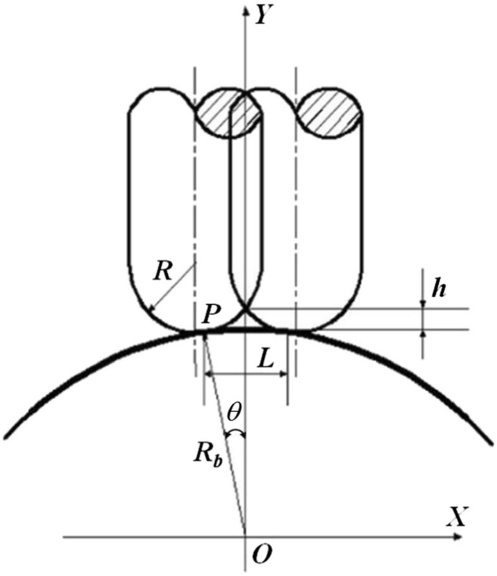


Fig. 2 Convex surface machining with ball-end milling cutter

$$(x - q\cos\theta)^2 + (y - q\sin\theta)^2 = R^2 \tag{1}$$

In which,

$$\begin{cases} q = R_b + R \\ \cos\theta = \sqrt{1 - \left(\dfrac{L}{2R_b}\right)^2} \\ \sin\theta = \dfrac{L}{2R_b} \end{cases} \tag{2}$$

The coordinate for the CC point $P(x_P, y_P)$ can be solved by the following equation,

$$\begin{cases} \left(x - q\sqrt{1 - \left(\dfrac{L}{2R_b}\right)^2}\right)^2 + \left(y - q\dfrac{L}{2R_b}\right)^2 = R^2 \\ y = 0 \end{cases} \tag{3}$$

As a result, the solution can be expressed as,

$$x_P = q\sqrt{1 - \left(\frac{L}{2R_b}\right)^2} - \sqrt{R^2 - \left(\frac{qL}{2R_b}\right)^2} \tag{4}$$

Thus, the scallop-height h, which is the crest height of the remained material between adjacent CL point paths after finished machining, is,

$$h = x_P - R_b = q\sqrt{1 - \left(\frac{L}{2R_b}\right)^2} - \sqrt{R^2 - \left(\frac{qL}{2R_b}\right)^2} - R_b \tag{5}$$

Conversely, if the scallop-height h is given, the cutting path interval L can be calculated as,

$$L = \frac{R_b}{(R_b + h)q}\sqrt{2\left(q^2 + R^2\right)(R_b + h)^2 - \left(q^2 - R^2\right) - (R_b + h)^4} \tag{6}$$

In practical application, as R is much larger than h, Eq. (6) can be simplified as,

$$L \approx 2\sqrt{2hR}\frac{R_b}{(R_b + R)} \tag{7}$$

The principle of concave surface machining by the ball-end milling cutter is shown in Fig. 3, and the calculation of cutting path interval is the same with that of the convex surface machining. The result is expressed as,

$$L \approx 2\sqrt{2hR}\frac{R_b}{(R_b - R)} \tag{8}$$

3 Tool-path parameter optimization by construction of cutter location mesh units

For the continuous change of the geometric feature for curved surface parts, the cutting step length and the cutting path interval among machining progress are changing with the geometric feature of the curved surface, and appropriate cutting step length and cutting path interval can guarantee both machining efficiency and machining quality. Based on the cutter location points obtained by constant scallop-height method, the optimization method for cutting step length and cutting path interval is proposed in this section by the construction of cutter location mesh units.

The CAD/CAM integration module software provides a useful method to generate the NC program. In this study, the saddle surface is taken as an example. Based on the unidirectional parallel cutting method, the initial CL points are obtained by constant scallop-height method in UG software, as shown in Fig. 4. Then the coordinates of all the CL points are stored in the cell array and used to optimize the tool-path parameters.

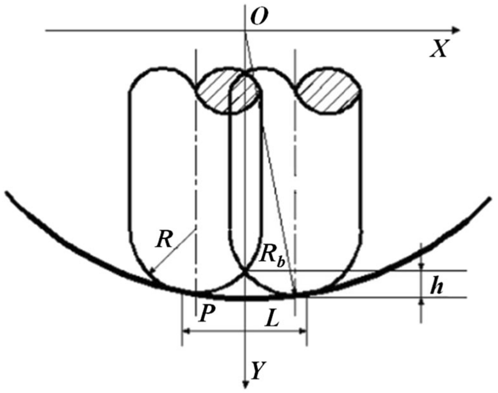


Fig. 3 Concave surface machining with ball-end milling cutter

3.1 Size determining for the first cutter location mesh unit

Because the optimization for tool-path parameters in this study is carried out by the construction of cutter location mesh units, the initial CL points should be divided into groups with the optimal cutting step length and cutting path interval that are related to the geometric feature of the curved surface parts, so as to construct the cutter location mesh units by the least square fitting. In this way, the size of the cutter location mesh units needs to be determined so as to find the corresponding CL points involved. Due to the change of the geometric feature for the curved surface parts, the fitted cutter location mesh unit can be regarded as an oblique plane with an angle α to the XY plane (shown in Fig. 5). Thus, the size of the cutter location mesh unit can be calculated according to the cutting path interval of the oblique plane by ball-end milling.

The path interval calculation of oblique plane by using the ball-end milling cutter is shown in Fig. 5, and in this study, the side step length in row direction for the cutter location mesh unit is equal to the cutting path interval of oblique plane machining by using the ball-end cutter. Set the mathematical expression of the oblique plane for the cutter location mesh unit as,

$$Z = aX + bY + c \tag{9}$$

where a, b, and c are the coefficients of the least squares fitting. The normal side step length s' can be obtained by,

$$s' = 2\sqrt{R^2 - (R - h)^2} = 2\sqrt{2Rh - h^2} \tag{10}$$

where R represents the radius of ball-end mill, h is the scallop height.

As the angle between the XY plane and the cutter location mesh unit is unknown before fitting, the exact CL points involved cannot be obtained for fitting of the cutter location mesh unit. In this study, choose the CL points whose projection are located in the cutter location mesh unit on the XY plane to do the least square planar fitting. Under the condition of the constant scallop height, the normal step length of all cutter location mesh unit is almost the same, but the position change of each fitted oblique plane leads to the difference of the projected side step length. Set the angle between oblique plane and YZ plane as β, and the angle between oblique plane and XY plane as α. It can be seen that,

$$\cos\beta = \frac{a}{\sqrt{a^2 + b^2 + 1}} \tag{11}$$

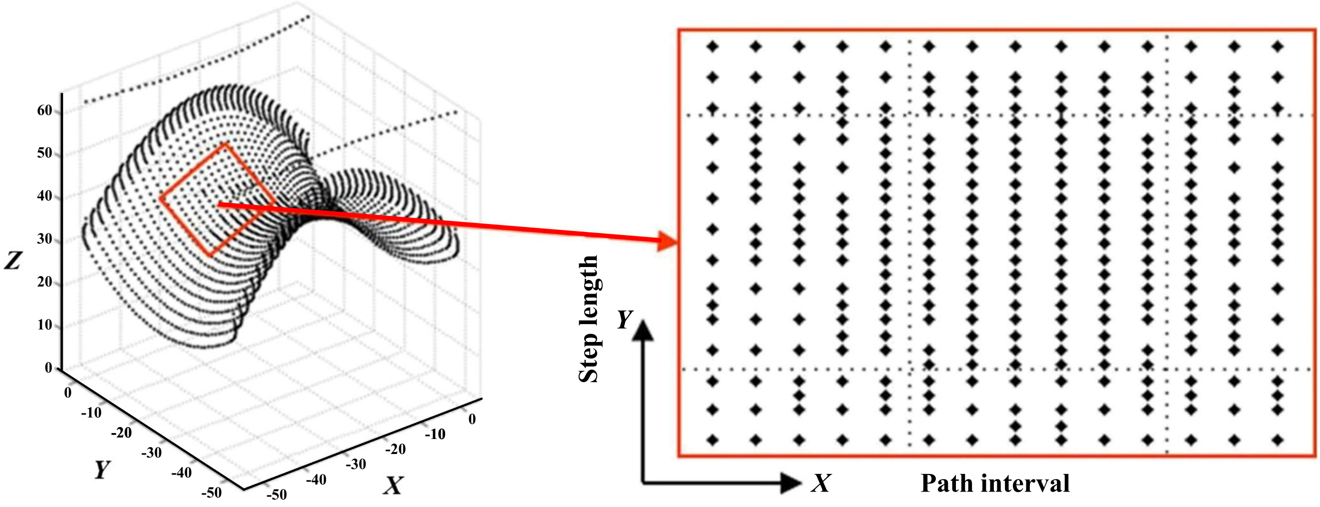


Fig. 4 Initial CL points of saddle surface

As shown in Fig. 5, the geometric relationship between the projected side step length s and the normal side step length $s^{'}$ is,

$$s = s^{'}\cdot\sin\beta = s^{'}\cdot\cos\alpha \tag{12}$$

According to Eq. (9) to Eq. (12), it can be seen that,

$$s = 2\sqrt{\frac{b^2+1}{a^2+b^2+1}}\cdot\sqrt{2Rh-h^2} \tag{13}$$

Based on Eq. (10), let the initial projected side step length of the cutter location mesh unit s_0 as,

$$s_0 = 2\sqrt{2Rh-h^2} \tag{14}$$

Then set the initial forward step length f_0 equals to s_0, and the size of the first cutter location mesh unit can be determined.

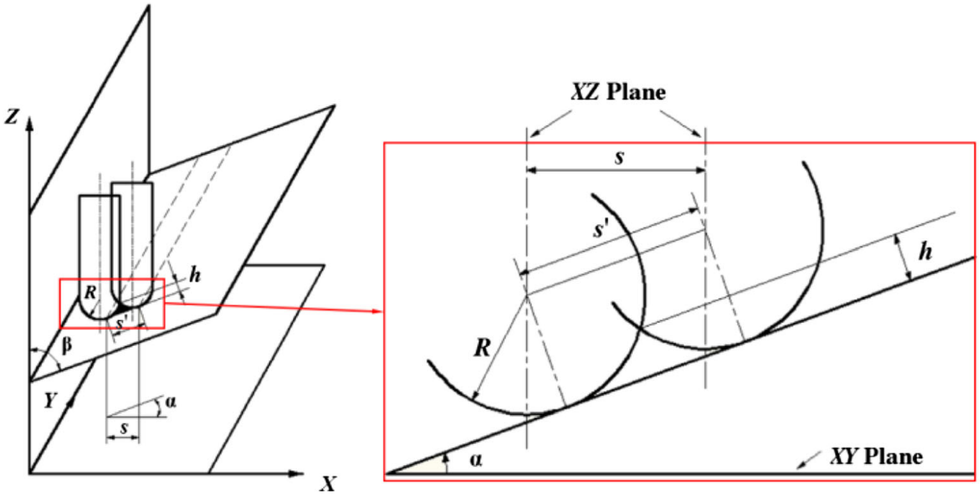
3.2 Iterative process for construction of cutter location mesh units

After obtaining the size of the first cutter location mesh unit, the cutter location mesh unit plane with a certain angle with XY plane can be obtained by using the CL points projected in it. The least square fitting coefficient a, b, c can be obtained by Eq. (15) in which (x_i, y_i, z_i) is the coordinate values of the corresponding CL points. The fitted cutter location mesh unit and its projection are shown in Fig. 6.

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum x_i^2 & \sum x_i y_i & \sum x_i \\ \sum x_i y_i & \sum y_i^2 & \sum y_i \\ \sum x_i & \sum y_i & 1 \end{bmatrix} = \begin{bmatrix} \sum x_i y_i \\ \sum x_i z_i \\ \sum z_i \end{bmatrix} \tag{15}$$

Then the following value of forward step length f can be obtained by the iterative method. The increase or decrease of the f value is determined by the comparison between RSM and the machining error requirement ε. Here, RSM servers as the

Fig. 5 Oblique plane machining by ball-end cutter



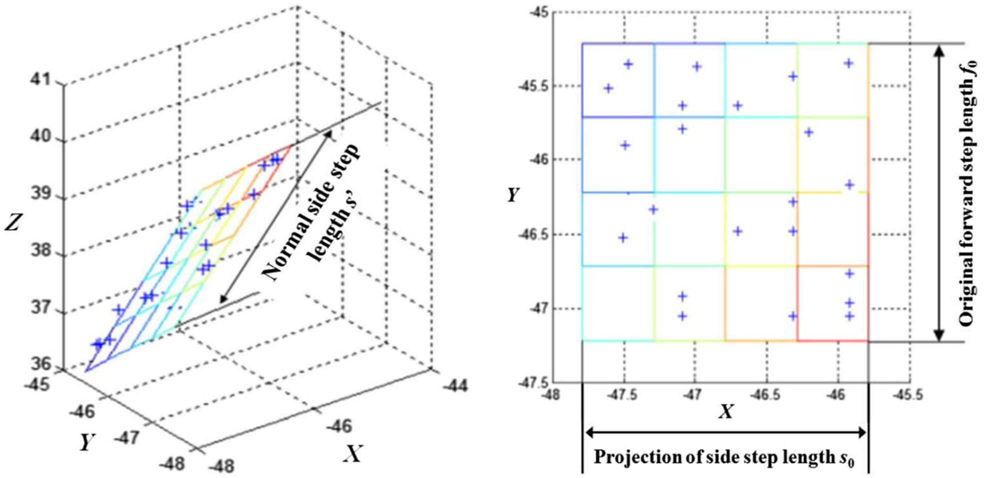


Fig. 6 Cutter location mesh unit and its projection

deviation root mean square of least squares fitting and it is as follows,

$$\mathrm{RMS}=\left(\frac{1}{n}\sum_{i=1}^{n}\left(\frac{|ax_i+by_i-z_i+c|}{\sqrt{a_i^2+b_i^2+1}}\right)^2\right)^{\frac{1}{2}} \tag{16}$$

In which, n represents the number of CL points in the cutter location mesh unit. By comparing *RSM* value and machining error value ε, the iterative process is conducted for determining the forward shift or the backward shift of the cutter location mesh unit boundary, as shown in Fig. 7. Let the iteration step length equal to 0.1 according to the engineering experience and the iterative process will not be stopped until f gets its maximum value under the condition that the *RSM* value is less than or equal to the ε value. If f cannot get a certain value, a small value should be selected for the iteration step length. For the ith cutting row, the side step length keeps constant and its calculation method is shown in Section 2. After obtaining the accurate forward step length f, the final projected side step length can be determined.

According to the above method, the cutter location mesh unit can be constructed for the ith cutting row, and the projected side step length s_i can be obtained. In this study, the smallest one among s_i values is taken as the final cutting path interval. Based on this, the cutter location mesh unit can be fitted by to the other cutting rows until the whole curved surface is covered by the projection zones.

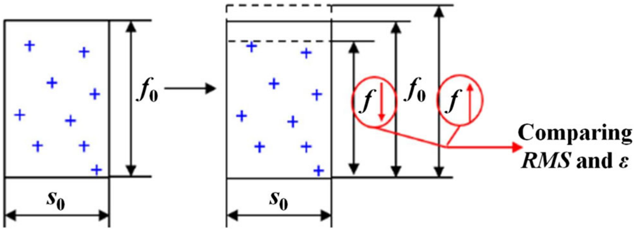


Fig. 7 Iterative process for step length optimizing

When the cutting path interval is determined for each cutting row, the cutter location mesh unit is constructed in the direction of cutting path interval. Then, adopt the determined f value as the forward step length, and use the same method to divide the cutter location mesh unit in the cutting step length direction. In this way, the division of the cutter location mesh unit for the whole saddle surface is realized, as shown in Fig. 8. It can be found that when the nodes of the cutter location mesh unit are used as the cutter location points directly, the cutter location points have the optimal forward step length and side step length, and the tool-path in any direction based on the final cutter location points can get the uniform scallop height. On the other hand, the projection of the cutter location mesh units in the *XY* plane reveals that the step length and the path interval are obviously influenced by the geometric feature of the curved surface.

3.3 Determination for nodes of cutter location mesh unit

The nodes of cutter location mesh units are shown in Fig. 9. When the nodes of the cutter location mesh units are used as the final cutter location points, the coordinate of the nodes should be determined.

Define the direction of cutting path interval as X direction and the direction of cutting step length as Y direction. X coordinate and Y coordinate for the nodes of the cutter location mesh unit can be expressed by side step length and forward step length of the mesh unit separately, while Z coordinate needs to be calculated through the least square fitting method (as Eq. (9)) which is related to the node number around this node. For the convenience of finding a certain node, define the node number in cutting path interval direction (X direction) as m (m = 1,2,3…) and that in cutting step direction (Y direction) as n (n = 1,2,3…), as shown in Fig. 10.

The X coordinate of the mesh node is stored in the array *x_row*, the Y coordinate of the mesh node is stored in the array

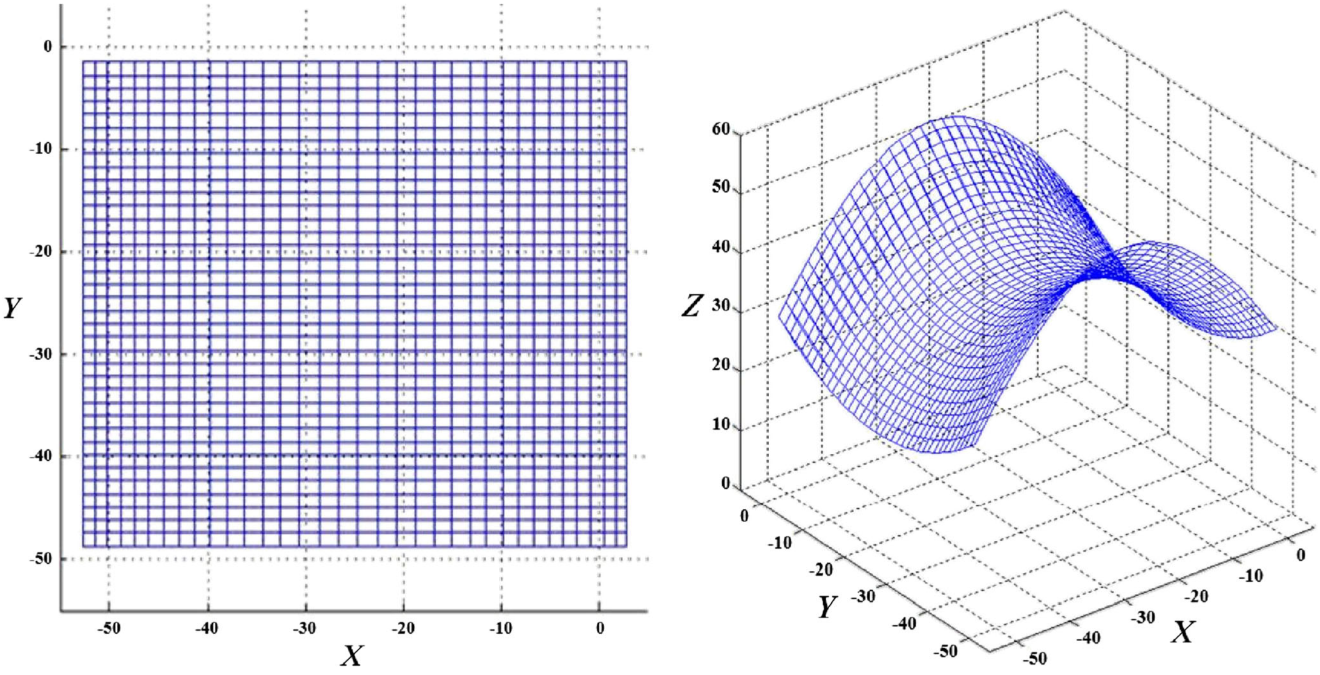


Fig. 8 Cutter location mesh unit for the whole curved surface

y_row, and the final least squares fitting coefficients *a*, *b*, and *c* by using the optimized forward step length and side step length are stored in the array of *net_plane*. Then the *Z* coordinate of the mesh node is obtained by using the fitting plane of cutter location mesh unit. As the number of corresponding mesh units for nodes at different positions are not the same, the fitting plane for calculation is different, as shown in Fig. 10. When the node is located at the corner (node M), there is only one cutter location mesh unit associated with the node; when the node is located on the border (node N), there is two cutter location mesh units associated with the node; when the node is located at other position (node P), there is four cutter location mesh units associated with the node. Thus, the location of the node needs to be determined firstly when calculating the *Z* coordinate of the node. Then, calculate each *Z* coordinate value of the node separately by using the least squares fitting coefficients of the associated cutter location mesh units, and the average value is taken as the final *Z* coordinate value of the node. After obtaining *X*-*Y*-*Z* coordinate of the nodes, the nodes of the cutter location mesh unit can be used as the final cutter

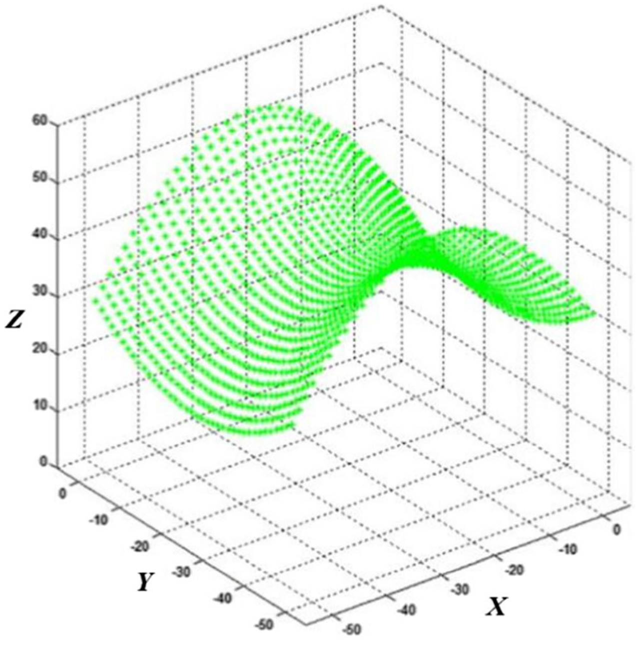


Fig. 9 Nodes of cutter location mesh units

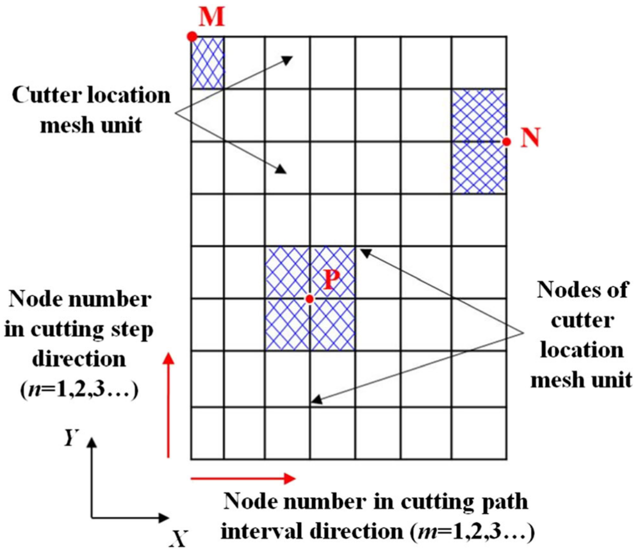


Fig. 10 Corresponding cutter location mesh units for nodes at different positions

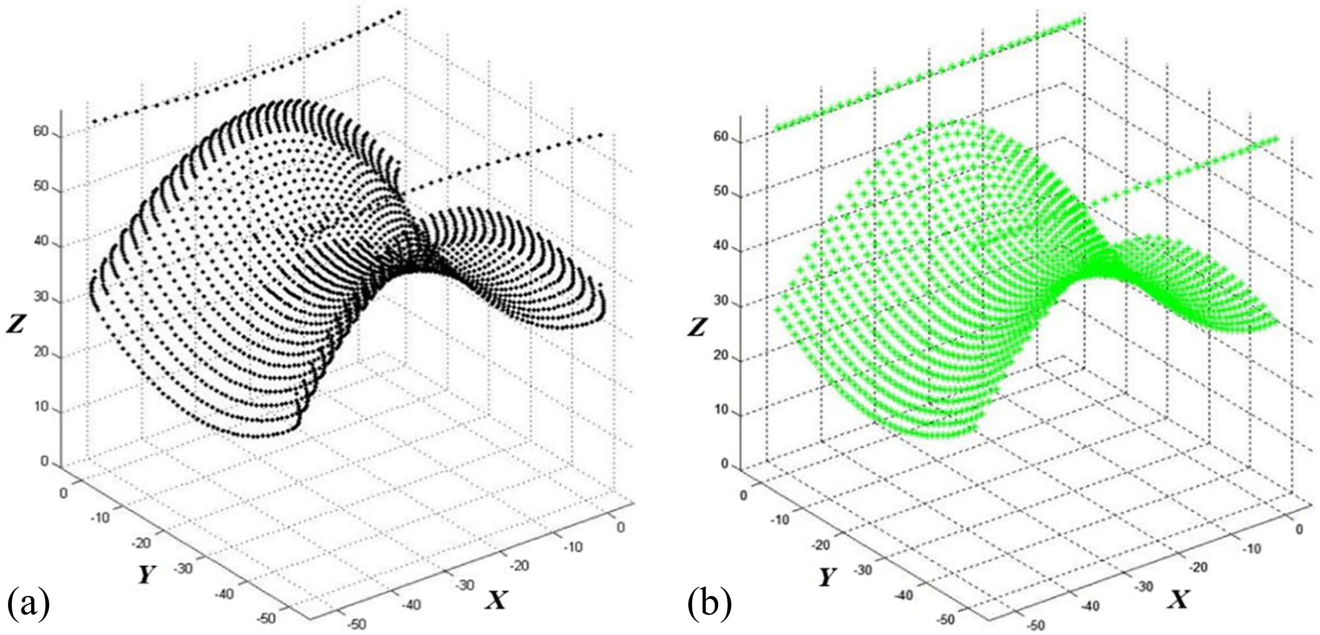


Fig. 11 Tool-path obtained by conventional method and proposed method. (**a**) Conventional method, (**b**) Proposed method

location points to do the machining, which can make the tool-path shorter than before.

4 Simulation and experiment

In order to prove the effectiveness of the proposed optimization method for cutting step length and cutting path interval in tool-path planning for curved surface parts by construction of cutter location mesh units, simulation and experiment are conducted on a saddle surface workpiece whose material is aluminum alloy 6061 with the size of 30 × 30 × 60 mm, as shown in Fig. 11. By utilizing the sample of curved surface, the effective optimizing strategy of the parameters in tool-path planning for complex curved surface is carried out.

4.1 Simulation results

In simulating process by taking the saddle surface as an example, the ball-end milling cutter is used whose diameter is 6 mm, the scallop height is set as 0.02 mm, and the unidirectional parallel cutting method is adopted for machining. The tool-path obtained by the conventional method and the proposed method are shown in Fig. 11. It can be seen that the number of CL points obtained by the conventional method is more than that obtained by the proposed method, especially at the ends of each tool-path.

For the selection of the parameters in tool-path planning for complex curved surface affects the tool-path length and the machining efficiency directly, the comparison between the tool-path obtained by conventional method the tool-path with the optimized cutting step length and cutting path interval is shown in Table 1.

According to the simulation results, it can be seen that the number of CL points is reduced through the optimization of cutting step length and cutting path interval. Moreover, the total length of the tool-path can be shortened. As a result, the machining efficiency can be improved accordingly.

4.2 Verification experiment

In this section, the verification experiment is conducted on the saddle surface of test piece whose material is aluminum alloy 6061, as shown in Fig. 12.

The experiment details are listed as the following:

Machine tool High-speed three-axis vertical milling machine tool MIKEON HSM500 with HEIDENHAIN iTNC530 NC system is adopted to conduct the experiments. The spindle speed can reach up to 54,000 rpm. Its positioning accuracy is 5 μm and the power of the spindle motor is 16 kW.

Cutting tool NTM UZB060 whose diameter is 6 mm with two flutes is adopted. Its helix angle is 30° and rake angle is −10°. The coated material is AlTiN.

Table 1 Comparison of CL points number and total tool-path length for before and after optimization

	CL points number	Total tool-path length
Before optimization	2227	4911.9
After optimization	1400	4234.1

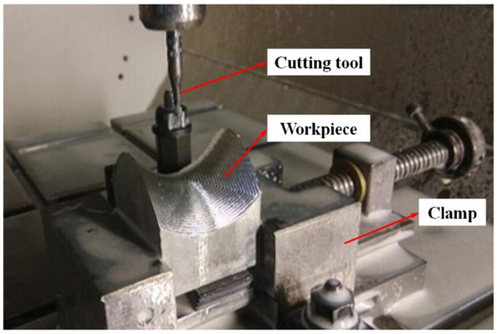


Fig. 12 Saddle surface of test piece

Machining parameters The experimental tests are carried out on the Mikron HSM500 machining center. The workpiece with saddle surface is fixed on the workbench. The cutting depth a_p is 0.5 mm, the feed per tooth f_z is 0.2 mm/z, and the spindle speed n is 4000 r/min.

In the whole machining process of the saddle surface workpiece, the machining time is recorded. After machining by using the above machining parameters, the surface roughness for the machined workpiece is analyzed. Talyrond Hobson surface roughness and profile tester is used to measure the surface roughness, as shown in Fig. 13. In order to guarantee the accuracy of measuring, the average value of three times measuring at the same position is adopted as the final surface roughness.

The final results of the machining time and the surface roughness are shown in Table 2. From the surface roughness values, it can be seen that the machining quality can be maintained when adopting the tool-path with the optimized cutting step length and cutting path interval, while the machining time is shortened for 15.83%. In this way, the machining efficiency for curved surface is improved obviously which proves the effectiveness of the proposed optimization method of cutting step length and cutting path interval in tool-path planning for curved surface parts in this study.

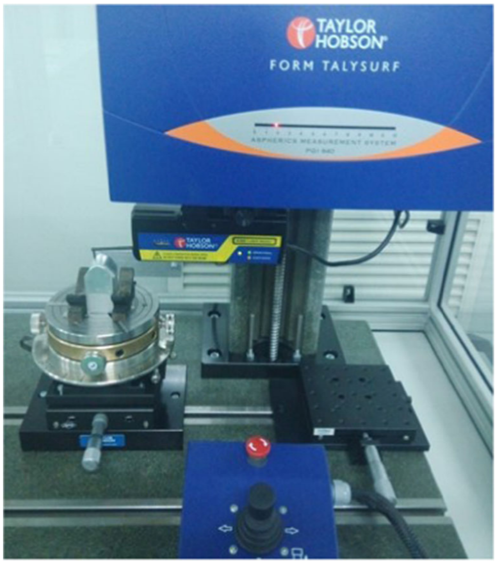


Fig. 13 Measuring process of surface roughness

Table 2 Comparison of machining time and surface roughness for the saddle surface workpiece

	Machining time	Surface roughness
Before optimization	13 min 54 s	Ra = 5.2898 μm; Rz = 21.6064 μm
After optimization	11 min 42 s	Ra = 5.1754 μm; Rz = 21.7918 μm

5 Conclusion

In this study, an optimization method of tool-path parameters for curved surface by construction of cutter location mesh units is proposed. On considering the geometric feature of curved surface parts, the cutter location mesh units are constructed by optimizing cutting step length and cutting path interval, and then the nodes of the cutter location mesh units are taken as the final cutter location points, so as to make the tool-path shorter. The results of the simulation and experiment show that the tool-path with the optimized cutting step length and cutting path interval can not only maintain the machining quality but also improve the machining efficiency obviously, which proves the effectiveness of the proposed optimization method of tool-path parameters for the curved surface.

This research provides guidance for the tool-path parameter optimization of complex surface parts. The optimization strategy has good applicability and can be generalized to any of the complex curved surface parts used in engineering. These research achievements are of vital importance in tool-path planning for realizing the high-quality and high-efficient machining for the curved surface parts.

Acknowledgements This work was supported by the National Natural Science Foundation of China (Grant No. 51575087 and No. 51675081), National Science and Technology Major Project of China (Grant No. 2016ZX04001-002), Innovation Project for Supporting High-level Talent in Dalian (Grant No. 2016RQ012), Science Fund for Creative Research Groups (Grant No. 51621064) and the Fundamental Research Funds for the Central Universities (Grant No. DUT17LAB13). The authors wish to thank the anonymous reviewers for their comments which led to improvements of this paper.

References

1. Sang CP (2004) Sculptured surface machining using triangular mesh slicing. Comput Aided Des 36(3):279–288

2. Neto HK, Diniz AE, Pederiva R (2016) Tool life and surface roughness in the milling of curved hardened-steel surfaces. Int J Adv Manuf Technol 87(9):2983–2995
3. He Y, Chen ZT, Xu RF (2016) Research on five-axis flank milling of convex edge surface with a concave cutter. Int J Adv Manuf Technol 86(9):2401–2409
4. Sun YW, Xu JT, Jin CN, Guo DM (2016) Smooth tool path generation for 5-axis machining of triangular mesh surface with nonzero genus. Comput Aided Des 79:60–74
5. Zhang X, Zhang J, Zhao WH (2016) A new method for cutting force prediction in peripheral milling of complex curved surface. Int J Adv Manuf Technol 86(1):117–128
6. Bolaños GS, Bedi S, Mann S (2014) A topological-free method for three-axis tool path planning for generalized radiused end milled cutting of a triangular mesh surface. Int J Adv Manuf Technol 70(9):1813–1825
7. Elber G, Cohen E (1994) Tool path generation for freeform surface models. Comput Aided Des 26(6):490–496
8. Kim SJ, Yang MY (2006) A CL surface deformation approach for constant scallop height tool path generation from triangular mesh. Int J Adv Manuf Technol 28(3):314–320
9. Sun YW, Guo DM, Jia ZY, Wang HX (2006) Iso-parametric tool path generation from triangular meshes for free-form surface machining. Int J Adv Manuf Technol 28(7):721–726
10. Feng HY, Li HW (2002) Constant scallop-height tool path generation for three-axis sculptured surface machining. Comput Aided Des 34(9):647–654
11. Yoon JH, Pottmann H, Lee YS (2003) Locally optimal cutting positions for 5-axis sculptured surface machining. Comput Aided Des 35(1):69–81
12. Lin RS, Koren Y (1996) Efficient tool path planning for machining free-form surfaces. J Eng Ind 118(1):20–28
13. Teng Z, Feng HY, Azeem A (2006) Generating efficient tool paths from point cloud data via machining area segmentation. Int J Adv Manuf Technol 30(3):254–260
14. Chu CH, Hsieh HT, Lee CH, Yan CY (2015) Spline-constrained tool-path planning in five-axis flank machining of ruled surfaces. Int J Adv Manuf Technol 80(9):2097–2104
15. Li M, Zhang LC, Mo JH, Lu Y (2012) Tool-path generation for sheet metal incremental forming based on STL model with defects. Int J Adv Manuf Technol 63(5):535–547
16. Zhang ZX, Feng YX, Ren BY, Hagiwara I (2016) Exploratory study of spiral NC tool path generation on triangular mesh based on local subdivision. Int J Adv Manuf Technol 83(5):835–845
17. Chen ZZ, Khan MA (2014) A new approach to generating arc length parameterized NURBS tool paths for efficient three-axis machining of smooth, accurate sculptured surfaces. Int J Adv Manuf Technol 70(5):1355–1368
18. Moodleah S, Makhanov SS (2015) 5-axis machining using a curvilinear tool path aligned with the direction of the maximum removal rate. Int J Adv Manuf Technol 80(1):65–90
19. Zhang K, Tang K (2016) Optimal five-axis tool path generation algorithm based on double scalar fields for freeform surfaces. Int J Adv Manuf Technol 83(9):1503–1514