

Interim check and practical accuracy improvement for machine tools with sequential measurements using a double ball-bar on a virtual regular tetrahedron

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Abstract This study proposes a method for a quick, simple interim check and practical accuracy improvement of machine tools using just a double ball-bar. The double ball-bar is used to measure sequentially the length of the six sides of a virtual regular tetrahedron within the workspace of the machine tool. Then, the scale and squareness errors of and between the three linear axes are calculated from the length results, and the measured lengths and the calculated errors can be used as criteria for the interim check. The calculated errors can also be compensated for to improve the accuracy of experimented machine tools practically. A sample machine tool was subjected to experimental interim checks applying the proposed method; it showed primarily large length deviations for the six sides due to geometric errors mainly. To improve the geometric accuracy practically, the calculated errors were compensated for and the measurements were repeated, showing significantly improved length deviations for the six sides. The main advantage of the proposed method is that it requires only a double ball-bar and sequential measurements; thus, it is a simple procedure with a measuring time of ~5 min for a virtual regular tetrahedron. Additionally, the size of the virtual regular tetrahedron can be readily modified by changing the nominal length of the double ball-bar, increasing measurement flexibility. Thus, the proposed method is suitable for quick, simple, cost-effective daily and periodic interim checks, with practical improvement of machine tool accuracy.

Keywords Double ball-bar · Interim check · Machine tools · Scale error · Squareness error · Virtual regular tetrahedron

Nomenclature

c_i	Scale errors of linear axis i ($i = X, Y, Z$), ppm
s_{ij}	Squareness errors of linear axis j around i axis, μrad
L	Nominal length of the sides of a virtual regular tetrahedron, mm
L_{ij}	Measured length between vertices $P_{i,m}$ and $P_{j,m}$, mm
$P_i(x_i, y_i, z_i)$	Nominal coordinate of the vertex of a virtual regular tetrahedron ($i = 1, \dots, 4$), mm
$P_{i,m}(x_{i,m}, y_{i,m}, z_{i,m})$	Measured coordinate of the vertex of a virtual regular tetrahedron ($i = 1, \dots, 4$), mm
EM_i	4×4 matrix of the scale errors for linear axis i
SM_i	4×4 matrix of the squareness errors for linear axis i
TM_i	4×4 matrix of the nominal translation for linear axis i
τ_A^B	4×4 homogeneous transformation matrix from coordinate system $\{B\}$ to coordinate system $\{A\}$

1 Introduction

The geometric accuracy of machine tools (MTs) should be checked, and main factors must be measured and compensated for to improve the form accuracy of the machined part [1]. Geometric errors are one of the main factors in the geometric accuracy of MTs [2]. A test-piece with geometric features is machined and its form accuracy is used to quickly check and improve the accuracy of the MTs [3,

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4]. However, it is difficult to apply the machining-based method to all MTs in a manufacturing factory because the machining and measuring of the workpiece is time-consuming. Therefore, both direct and indirect measurements are recommended [5, 6], and these are used to compensate for individual geometric errors, for simplicity. Recently, highly sophisticated computer numerical control (CNC) supply compensation functions for measured geometric errors have been applied to improve geometric accuracy between the workpiece and tool. However, a cost-effective, easy-to-use method is still required for interim checks of MTs, daily and periodically, to monitor geometric accuracy, to make engineering judgments for measurement and compensation, and to maintain consistent productivity of the manufacturing system.

For interim checks of MTs, a circular test is widely used with a double ball-bar (DBB) [7, 8]; however, the test results are, in general, not geometrically accurate due to dynamic test conditions [9]. In detail, because the dynamic effects depend on the machine tested, it is difficult to determine the criterion for measuring the velocity in the circular test such that the dynamic effects are excluded from the result. Making detailed velocity measurements increases the measurement time, which is unacceptable for interim checks. By way of contrast, there are two methods, using a ball-bar and a three-dimensional (3D) artifact for interim checks of coordinate-measuring machines (CMMs). For the ball-bar method, the fixed length between two balls is measured at several positions, and the measured lengths are used to calculate squareness errors between the three linear axes [10, 11]. Similarly, a telescoping ball-bar can be used to calculate squareness errors by measuring the coordinates of the ball during a circular test [12]. In the case of a 3D artifact, feature points of a dimensionally well-established artifact are measured, and geometric deviations from the nominal values are used to calculate scale and squareness errors of and between the three linear axes [13]. In particular, a tetrahedral artifact is used widely in interim daily checks due to its simplicity [14].

However, it can be difficult to apply these methods to MTs because they require a dimensionally well-established artifact and a 3D touch probe (not found on many MTs) to measure the artifact. Thus, the measurement procedure becomes more complex than those for CMMs, with increases in measurement time and cost. To overcome these limitations partially, a laser ball-bar was developed for sequential trilateration and applied to MTs [15]. However, this is limited to assessing the spindle thermal drift and dynamic evaluation [16, 17].

Therefore, our study proposes a method for a quick, easy-to-use interim check for the practical accuracy improvement of MTs by sequential measurements of a DBB

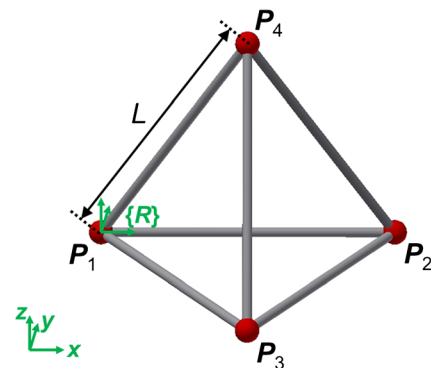


Fig. 1 Regular tetrahedron with nominal length L

to measure the length of the six sides of a virtual regular tetrahedron. It only requires a DBB to complete the interim check for scale and squareness errors, thus simplifying the measurement procedure cost-effectively. In Sect. 2, sequential measurements of a DBB on a virtual regular tetrahedron are proposed, and the relationship between the measured lengths of the six sides and scale and squareness errors are derived using homogeneous transformation matrices under a small-value assumption. In addition, the possible range of the scale and squareness errors are calculated by using main contributors. In Sect. 3, the proposed method is applied to a MT for an interim check and to improve the geometric accuracy via compensation for measured scale and squareness errors. In Sect. 4, the advantages of the proposed method are summarized with a discussion of its inherent limitations.

2 Interim check of machine tools using a double ball-bar

Geometrically, a tetrahedron consists of vertexes P_i ($i = 1, \dots, 4$) and six sides (Fig. 1). The relationships between the

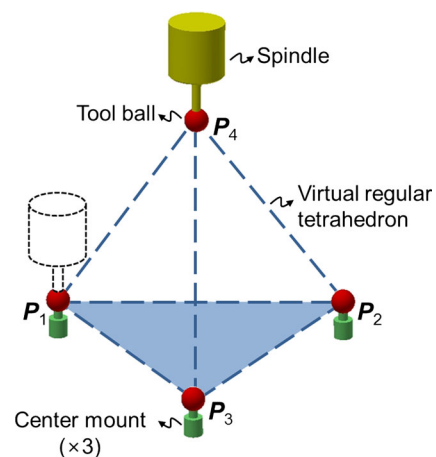


Fig. 2 Formation of the virtual regular tetrahedron using the tool ball and center mounts

coordinates of the vertexes and the lengths of the sides are determined as in Eq. (1).

$$[P_1 \ P_2 \ P_3 \ P_4] = \begin{bmatrix} 0 & L & \frac{1}{2}L & \frac{1}{2}L \\ 0 & 0 & -\frac{\sqrt{3}}{2}L & -\frac{\sqrt{3}}{6}L \\ 0 & 0 & 0 & \frac{\sqrt{6}}{3}L \end{bmatrix} \quad (1)$$

A tetrahedral artifact is commonly used for interim checks of CMMs by comparing the measured and nominal coordinates of the vertexes and calculating scale and squareness errors. However, as mentioned in Sect. 1, this method has an increased measurement cost with MTs as a tetrahedral artifact and 3D touch probe are required to measure vertex coordinates. Additionally, the checking workspace for MTs would be determined by the fixed size of the commercially available tetrahedral artifact. Thus, it would be necessary to prepare several tetrahedrons for various cases. These limitations can be avoided using sequential measurements of a DBB on a virtual regular tetrahedron, as explained in detail below.

2.1 Sequential measurements of a virtual regular tetrahedron

A tool ball, fixed at the tool nose, is commanded sequentially for the nominal vertexes P_i ($i = 1, 2, 3$) of a virtual

Fig. 3 Sequential measurement of a virtual regular tetrahedron using a DBB

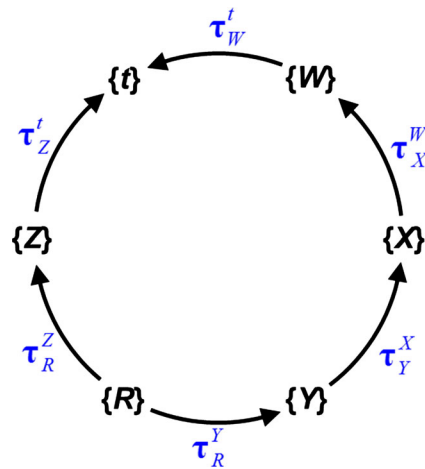
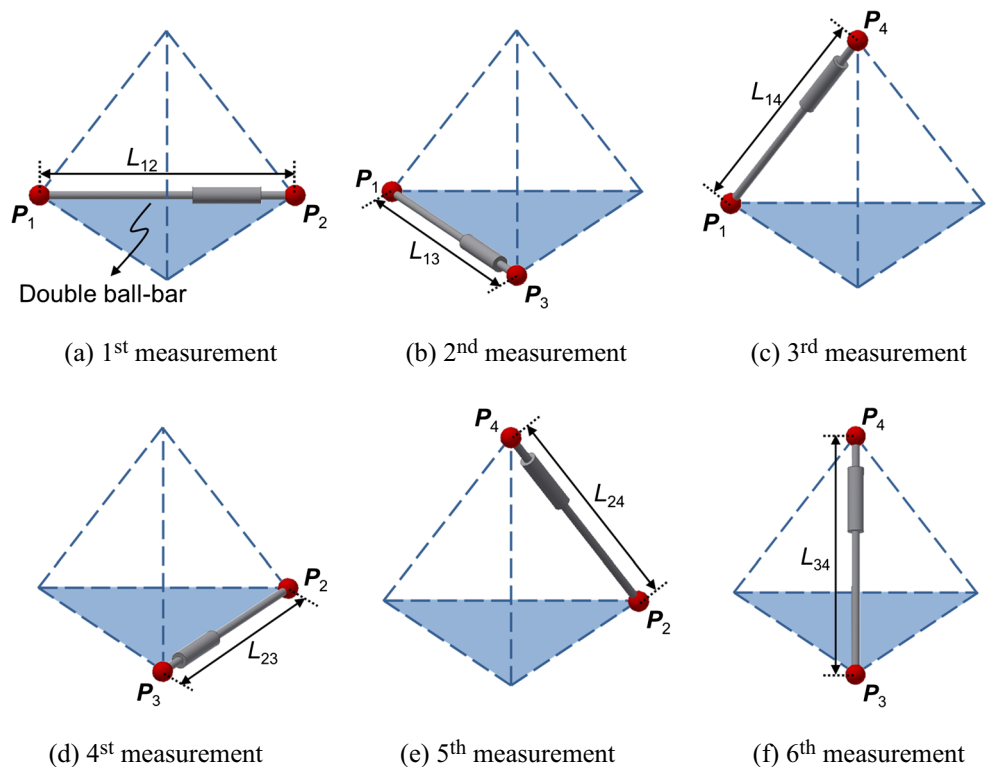


Fig. 4 Kinematic chain of the experimental MT structure

regular tetrahedron. Center mounts, fixtures to keep the ball position stationary during measurements, are then used to retain each position of the tool ball (Fig. 2). The tool ball is then commanded to locate at the vertex P_4 to completely form a virtual regular tetrahedron. The actual vertexes $P_{i,m}$ ($i = 1, \dots, 4$) might deviate from the nominal position mainly due to geometric errors, including scale and squareness errors. The virtual regular tetrahedron is formed using the MT, so there are no alignment errors of the tetrahedron, in comparison with the existing approach [14]; this ensures consistent measurement results.

Then, the lengths L_{ij} between the actual vertexes $P_{i,m}$ ($i = 1, \dots, 4$) are measured using a DBB sequentially (Fig. 3). The relationship between the lengths L_{ij} and the actual vertexes $P_{i,m}$ ($i = 1, \dots, 4$) is defined as in Eq. (2).

$$\begin{aligned}
 & [P_{1,m} \quad P_{2,m} \quad P_{3,m} \quad P_{4,m}] \\
 &= \begin{bmatrix} 0 & L_{12} & L_{13}\cos\theta_{312} & \frac{L_{14}\cos\theta_{412}}{L_{13}^2 + L_{14}^2 - 2x_{3,m}x_{4,m} - L_{34}^2} \\ 0 & 0 & -L_{13}\sin\theta_{312} & \frac{2y_{3,m}}{\sqrt{L_{14}^2 - x_{4,m}^2 - y_{4,m}^2}} \\ 0 & 0 & 0 & \end{bmatrix} \tag{2}
 \end{aligned}$$

where

$$\begin{aligned}
 \cos\theta_{312} &= \frac{L_{12}^2 + L_{13}^2 - L_{23}^2}{2L_{12}L_{13}}, \\
 \cos\theta_{412} &= \frac{L_{12}^2 + L_{14}^2 - L_{24}^2}{2L_{12}L_{14}}
 \end{aligned}$$

In this case, the length L_{ij} may deviate from the nominal length L due mainly to geometric errors of the three linear axes. Generally, there are 21 geometric errors for the three linear axes [18]; however, only three scale errors and three squareness errors of and between the three linear axes are modeled and calculated from the lengths L_{ij} for the purpose of a quick interim check and practical accuracy improvement.

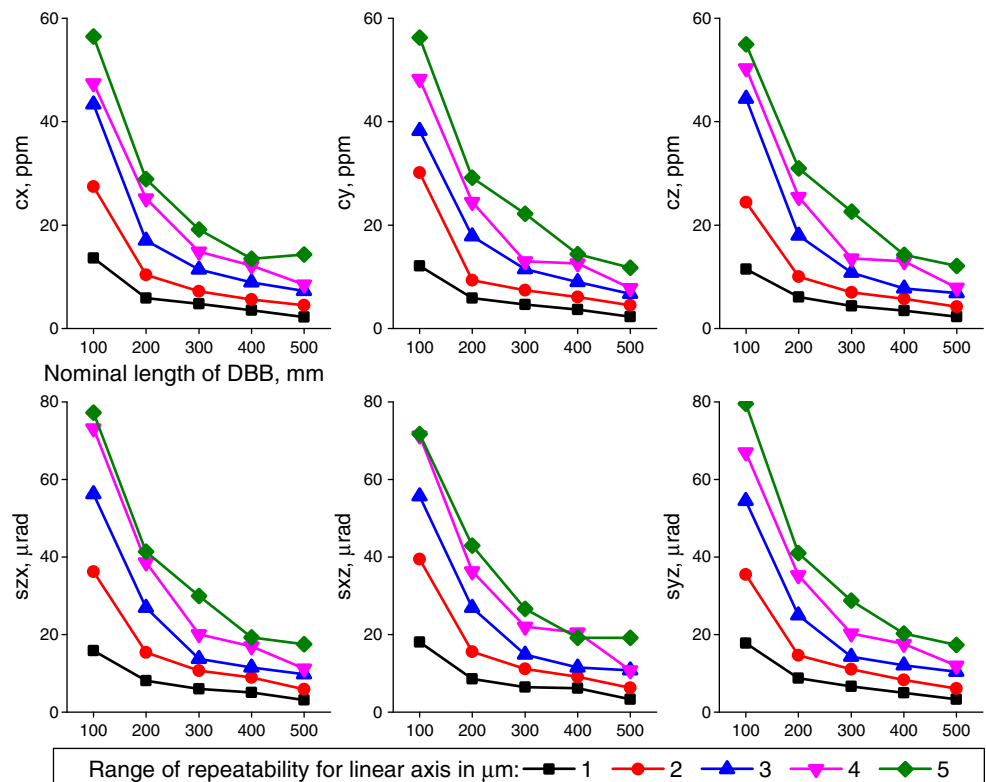
2.2 Linear relationships between the measured lengths of the six sides and geometric errors

The actual vertexes $P_{i,m}$ ($i = 1, \dots, 4$) deviate from the nominal vertexes P_i ($i = 1, \dots, 4$), because of geometric errors, including scale and squareness errors. Therefore, it is necessary to derive the relationship between the actual vertexes $P_{i,m}$ ($i = 1, \dots, 4$), nominal vertexes P_i ($i = 1, \dots, 4$), and geometric errors under a small-value assumption for high-order terms of the geometric errors [19]. The relationship is integrated to calculate the geometric errors by applying a least squares method. As an example, the experimental MT structure in Sect. 3 is used to derive the relationship by applying the kinematic chain shown in Fig. 4.

In this case, the coordinate system $\{Y\}$ can be derived from the reference coordinate system $\{R\}$ as follows:

$$\begin{aligned}
 \tau_R^Y &= \mathbf{EM}_y \quad \mathbf{TM}_y \\
 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & c_y y_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -y_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -y_i + c_y y_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}
 \end{aligned}$$

Fig. 5 Range of the calculated geometric errors according to the repeatability of the linear axes and the nominal length of the DBB



Then, the coordinate system $\{X\}$ is defined from the coordinate system $\{Y\}$ by defining τ_Y^X in Eq. (4).

$$\begin{aligned} \tau_Y^X &= \mathbf{SM}_x \mathbf{EM}_x \mathbf{TM}_x \\ &= \begin{bmatrix} 1 & -s_{zx} & 0 & 0 \\ s_{zx} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & c_x x_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -x_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4) \\ &= \begin{bmatrix} 1 & -s_{zx} & 0 & -x_i + c_x x_i \\ s_{zx} & 1 & 0 & -s_{zx} x_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

The coordinate system $\{Z\}$ from the coordinate system $\{R\}$ is defined as shown in Eq. (5).

$$\begin{aligned} \tau_R^Z &= \mathbf{SM}_z \mathbf{EM}_z \mathbf{TM}_z \\ &= \begin{bmatrix} 1 & 0 & s_{yz} & 0 \\ 0 & 1 & -s_{xz} & 0 \\ -s_{yz} & s_{xz} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c_z z_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5) \\ &= \begin{bmatrix} 1 & 0 & s_{yz} & s_{yz} z_i \\ 0 & 1 & -s_{xz} & -s_{xz} z_i \\ -s_{yz} & s_{xz} & 1 & z_i + c_z z_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

To complete the kinematic chain of the MT, it is also necessary to define the workpiece coordinate system $\{W\}$ from coordinate system $\{X\}$ and the tool position $\{t\}$ from coordinate system $\{Z\}$, as shown in Eq. (6).

$$\tau_X^W = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \tau_Z^t = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (6)$$

Finally, the actual vertices $P_{i,m}$ ($i = 1, \dots, 4$) are derived by using τ_W^t defined in Eq. (7).

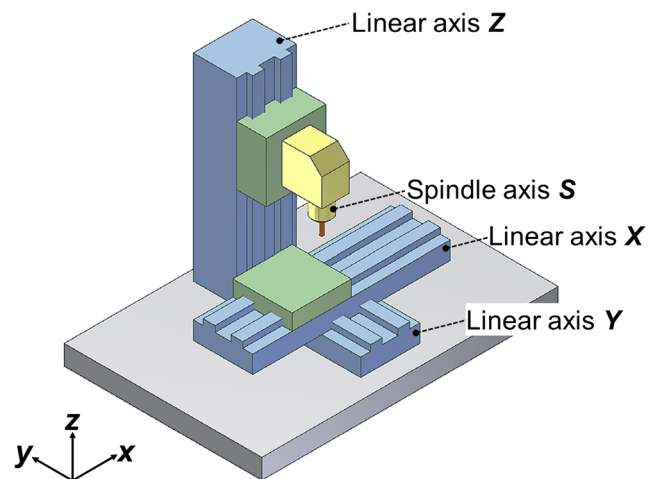
$$\begin{aligned} \begin{bmatrix} x_{i,m} \\ y_{i,m} \\ z_{i,m} \\ 1 \end{bmatrix} &= \tau_W^t = (\tau_R^W)^{-1} \tau_R^t = (\tau_R^Y \tau_Y^X \tau_X^W)^{-1} \tau_R^Z \tau_Z^t \\ &= \begin{bmatrix} x_i - c_x x_i + s_{zx} y_i + s_{yz} z_i \\ y_i - c_y y_i - s_{xz} z_i \\ z_i + c_z z_i \\ 1 \end{bmatrix} \quad (7) \end{aligned}$$

By integrating the relation in Eq. (7) for vertexes, a relationship between the actual vertexes $P_{i,m}$ ($i = 1, \dots, 4$) nominal

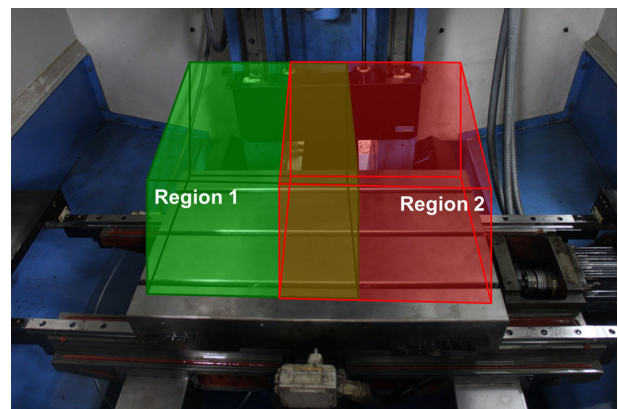
vertexes P_i ($i = 1, \dots, 4$) and geometric errors is determined as in Eq. (8). Then, the geometric errors are calculated by applying a least squares method to Eq. (8).

$$\begin{bmatrix} x_{1,m} - x_1 \\ y_{1,m} - y_1 \\ z_{1,m} - z_1 \\ \vdots \\ x_{4,m} - x_4 \\ y_{4,m} - y_4 \\ z_{4,m} - z_4 \end{bmatrix} = \begin{bmatrix} -x_1 & 0 & 0 & y_1 & 0 & z_1 \\ 0 & -y_1 & 0 & 0 & -z_1 & 0 \\ 0 & 0 & z_1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -x_4 & 0 & 0 & y_4 & 0 & z_4 \\ 0 & -y_4 & 0 & 0 & -z_4 & 0 \\ 0 & 0 & z_4 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_x \\ c_y \\ c_z \\ s_{zx} \\ s_{xz} \\ s_{yz} \end{bmatrix} \quad (8)$$

In addition, it is essential to check the possible range of the calculated geometric errors in Eq. (8) corresponding to the possible range of the main contributors [20]. This means that the calculated geometric errors are affected by the repeatability of the linear axes controlled for the measurement, and by the accuracy of the DBB used to measure length L_{ij} for the virtual regular tetrahedron. In



(a) Kinematic structure of the subject MT



(b) Regions 1 and 2 for interim checks

Fig. 6 Interim check volumes for the overall workspace of the subject MT

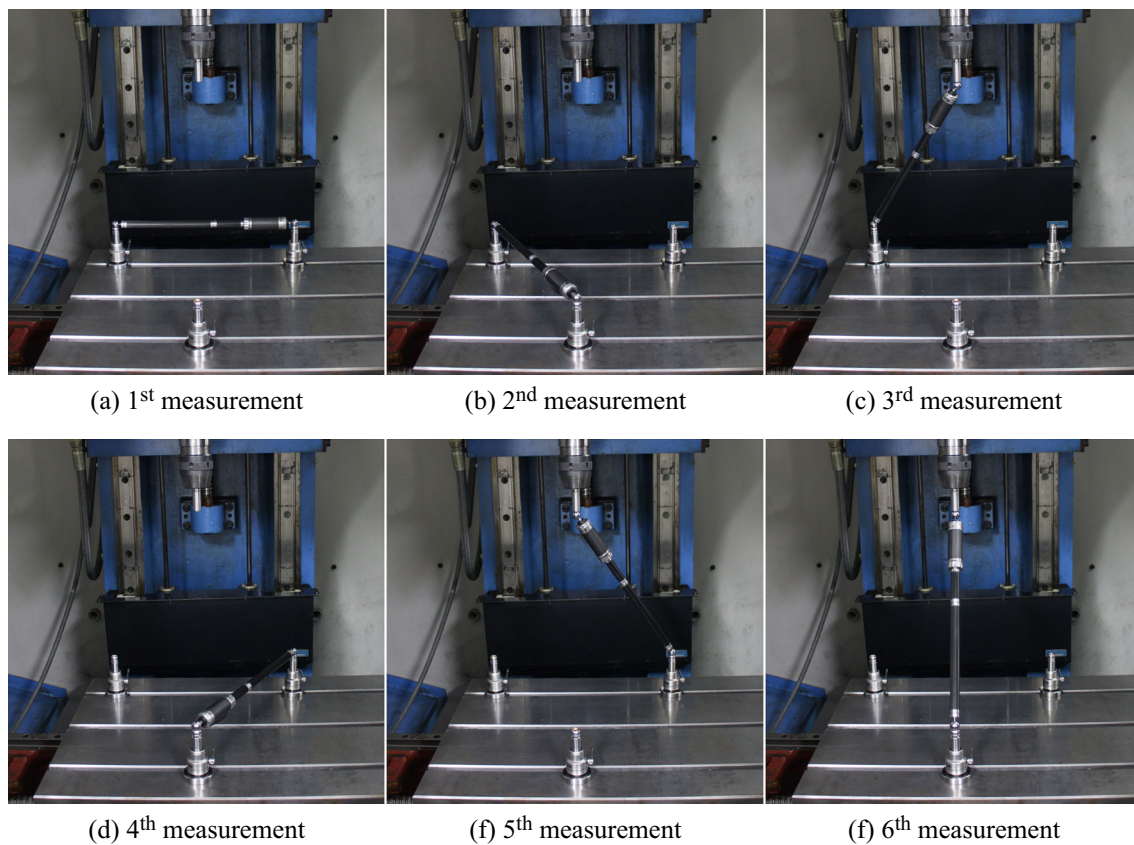


Fig. 7 Sequential measurements of a DBB for an interim check in region 1

detail, the repeatability and accuracy of the DBB affect the actual vertexes $P_{i,m}$ ($i = 1, \dots, 4$) and the measured length L_{ij} , respectively. In addition, the effects of the repeatability and the DBB accuracy on the measurement result are weighted in combination with the nominal length L . Thus, the effects of these contributors on the calculated geometric errors are investigated using a Monte Carlo simulation. It should be noted that there are few commercial DBBs, and the accuracy of the commercial DBB used on the experiment in Sect. 3 is just $\pm 0.2 \mu\text{m}$. In comparison, the repeatability and nominal length L have various practical values and their effects on the measurement result are investigated. For the simulation, random numbers following a normal distribution are first generated within the possible ranges for the repeatability and the DBB accuracy. Then, the numbers generated for the repeatability and DBB accuracy are added to the nominal values. Finally, the geometric errors are calculated using Eq. (8). This process is repeated 10,000 times and the range of the calculated geometric errors is obtained to investigate the effect of the main contributor on the measurement result. The range of the calculated geometric errors is shown in Fig. 5 according to the repeatability of the linear axes and the nominal length L .

The range of the calculated geometric errors increases with the repeatability of the linear axes and decreases as the nominal length L increases. The range of the squareness errors exceeds the range of the scale errors because the squareness errors are defined and calculated using relative values between

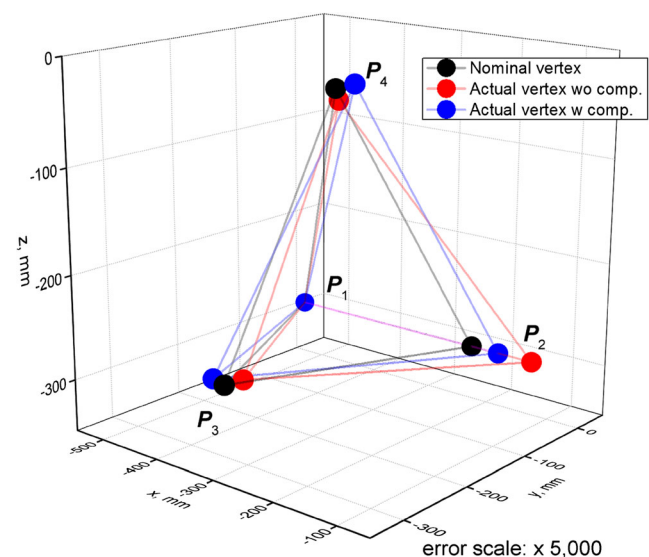


Fig. 8 Nominal vertex and actual vertex without/with compensation at region 1

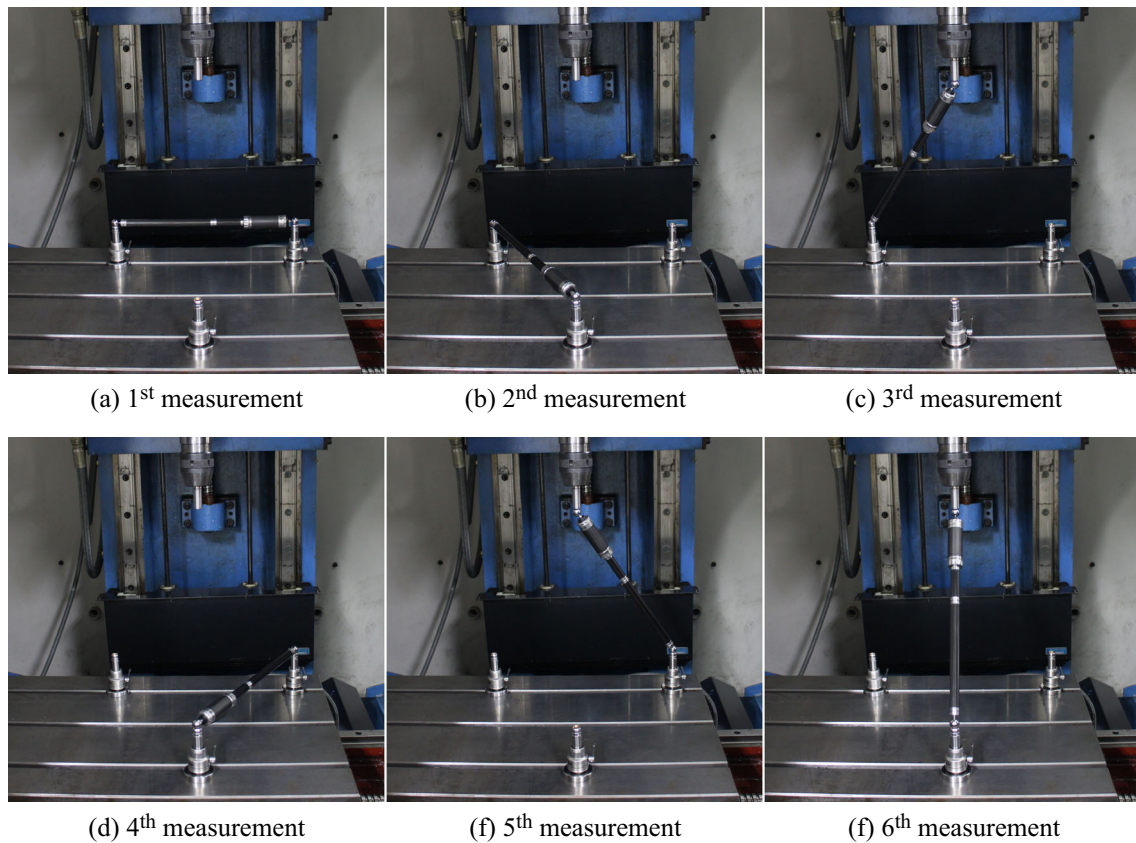


Fig. 9 Sequential measurements of a DBB for an interim check in region 2

the measured coordinate in Eq. (2). In general, the repeatability is more significant for the measurement result than the nominal length L . Thus, it is critical to use a MT with high repeatability to obtain more accurate results with the method proposed here.

3 Experimental study of the proposed method

The proposed method was applied to an example MT (SPT-T30, Komatec Co. Ltd., Republic of Korea) using a DBB (QC20-W, Renishaw PLC, UK) for an interim check. The workspace of the MT was $500 \times 300 \times 300$ mm; thus, a regular tetrahedron could not cover all of the workspace. Two regular tetrahedrons with a nominal side length $L = 300$ mm were planned to cover the workpiece (Fig. 6).

In this case, a virtual regular tetrahedron was formed in region 1 and the lengths L_{ij} of the six sides were measured sequentially using the DBB (Fig. 7). The coordinates of the nominal vertices P_i ($i = 1, \dots, 4$) and the actual vertices $P_{i,m}$ ($i = 1, \dots, 4$) without/with compensation, derived using Eq. (2), are shown in Fig. 8.

This procedure was then repeated for region 2 (Fig. 9), and the nominal vertex P_i ($i = 1, \dots, 4$) and actual vertex

$P_{i,m}$ ($i = 1, \dots, 4$) without/with compensation as shown in Fig. 10. The tetrahedron measurements took ~ 5 min. The length deviation, which is the measured length L_{ij} minus the nominal length L , had maximum values of 19.4 and -

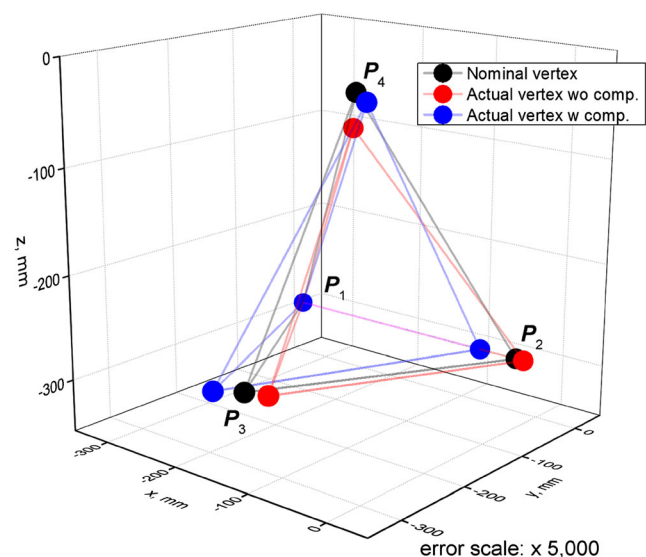


Fig. 10 Nominal vertex and actual vertex without/with compensation at region 2

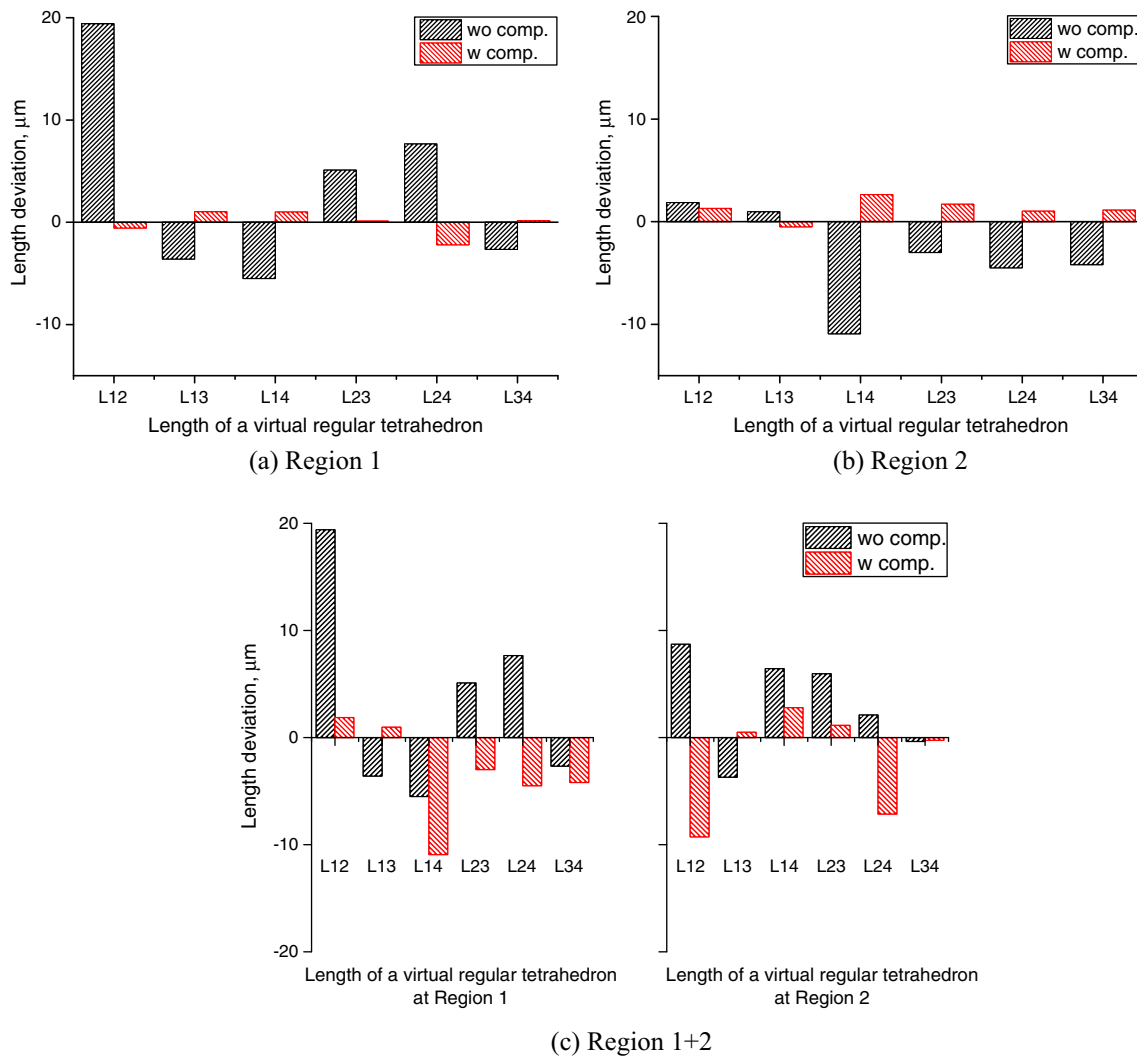


Fig. 11 Measured length deviations in regions 1, 2, and 1 + 2

10.9 μm for regions 1 and 2, respectively (Fig. 11). These findings show that the geometric errors of the subject MT should be measured and compensated for to improve its geometric accuracy. Additionally, the measured lengths in regions 1 and 2 were not the same. Thus, the geometric errors of the subject MT apparently had non-linear characteristics within the workspace.

The scale and squareness errors of and between the three linear axes were calculated using Eq. (8) for regions 1 and 2, individually, and for regions 1 + 2 (Fig. 12). Table 1 lists the possible range of the main contributors and the calculated geometric errors in this case.

There were large values and large deviations between them due to the non-linear characteristics mentioned above. The measurements were repeated after compensating for the calculated geometric errors in Fig. 12 for each case. The measured length deviations without/with compensation are listed in Fig. 11. With compensation for

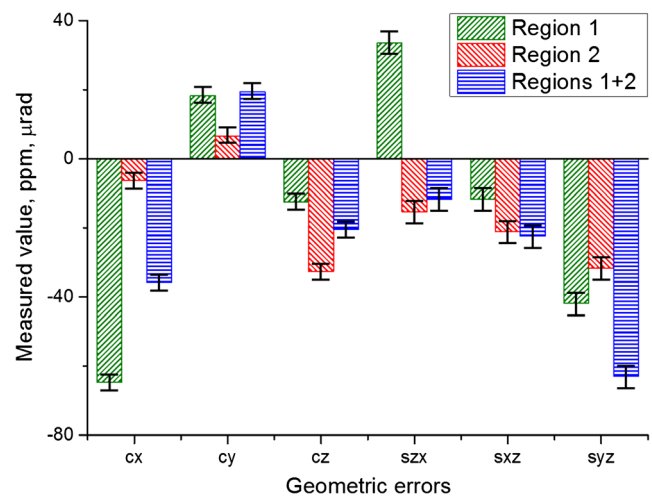


Fig. 12 Calculated geometric errors in regions 1, 2, and 1 + 2

Table 1 Possible range of the main contributors and calculated geometric errors

	Item	Unit	Possible range
Contributor	Repeatability of the linear axes	μm	1.0
	Accuracy of the DBB	μm	0.4
Calculated geometric errors	c_x	ppm	4.7
	c_y	ppm	4.6
	c_z	ppm	4.4
	s_{zx}	μrad	6.0
	s_{xz}	μrad	6.4
	s_{yz}	μrad	6.7

regions 1 and 2, individually, there were significant improvements in the length deviation to a maximum of only 2.6 μm . This result may have been affected by the repeatability of the compensation and the possible range of measured geometric errors, which are listed in Table 1. The length deviation is amplified by the measuring sensitivity direction compared to the *X*, *Y*, and *Z* directions of the MT. However, in the case of compensating for regions 1 + 2 simultaneously, the maximum length deviations were 8.7 and -9.3 μm for regions 1 and 2, respectively. This may have been caused by an average effect of the geometric errors in regions 1 and 2; however, they were still smaller than the values without compensation.

Thus, experimentally, we can conclude that if the size of the machined part is within the single tetrahedron volume in region 1 or 2, the form accuracy of the machined part can be improved practically and significantly by compensating for the calculated geometric errors. However, if the size of the machined part is beyond that of a single tetrahedron volume, it is recommended to measure the main geometric errors for the MT and compensate for them to improve the geometric accuracy of the overall workspace.

4 Conclusions

In this study, a method is proposed and demonstrated experimentally for a quick, easy-to-use cost-effective interim check and practical accuracy improvement of MTs. Only a DBB is required to complete the interim check via sequential measurements of a virtual regular tetrahedron. Scale and squareness errors of and between the three linear axes were calculated using the measured lengths of the six sides of the tetrahedron. Thus, the geometric accuracy of the MT can be checked readily and improved upon by

compensating for the measured errors, daily and periodically.

The advantages of the proposed method include (1) a simple measurement procedure, using only a DBB; (2) consistent measurement results by forming a virtual regular tetrahedron using the MT; and (3) high measurement flexibility, by simply changing the DBB length and the size of the tetrahedron. Thus, the proposed method is suitable for cost-effective, quick interim checks of MTs and practical accuracy improvement. In addition, the proposed method may be extended to CMMs due to these advantages.

It should be noted that the proposed method is effective for interim checks of geometric accuracy, daily and periodically, and practical accuracy improvement by measuring and compensating the measure scale and squareness errors. However, it does not address all possible inherent geometric errors of MTs.

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