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ORIGINAL ARTICLE

Mechanistic force model for machining process—theory and application of Bayesian inference

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Abstract This work discusses the Bayesian parameter inference method for a mechanistic force model for machining. Bayesian inference methods have gained popularity recently owing to their intuitiveness and ease with which empirical knowledge may be combined with experimental data considering the uncertainty. The first part of the paper discusses Bayesian parameter inference and Markov Chain Monte Carlo (MCMC) methods. MCMC method effectiveness has been further analyzed by (1) changing the number of particles in MCMC estimation and (2) changing the MCMC move step size. The second part of the paper discusses two example applications as nonlinear mechanistic force model coefficient identification. The Bayesian inference scheme performs prediction of the cutting force coefficients from the training data. Using these coefficients and input parameters to the model, the cutting force is predicted. This prediction is validated using experimental data, and it is demonstrated that with very few parameter updates the predicted force converges with the measured cutting force. The paper is concluded with the discussion of future work.

Keywords Bayesian inference · Machining process model identification · MCMC · Parameter uncertainty

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1 Introduction

This work discusses the uncertainty treatment in machining process model development by applying Bayesian methods for parameter inference. The uncertainty in the machining process can be attributed to the multiphysics nature of a phenomenon, coupled with an unknown variation in material and geometrical variations in tool and workpiece [1]. Conventionally, the machining process models are generated from rigorous experiments. Since the machining process involves effect from multiple inputs, to generate a reliable model, large sets of experiments are needed. For example, considering machining force as an output for the process, depth of cut, feed rate, cutting speed, tool geometry, workpiece geometry, tool wear state, and other machining conditions have to be accounted for as an input [2]. Furthermore, as environment variables change state, the model output may not be reliable. This paper presents an alternative model building approach based on sequential learning of the model parameters with the help of Bayesian probabilistic methods. In recent years, Bayesian methods have gained importance in the area of robust model construction. Bayesian methods are attractive to be applied in manufacturing process modeling because robust and reliable process models can be generated combining the first principle (physics-based) process understanding and process measurements (data).

The paper discusses the basics of Bayesian inference along with the numerical tools necessary to apply these methods to practical parameter inference. The Markov Chain Monte Carlo (MCMC) method is discussed in detail which is used in estimating the parameter distributions. The parameters that affect the accuracy of the estimate from the MCMC methods are discussed with the help of numerical simulations. To demonstrate the use of such inference scheme, two case applications are presented. One of the applications demonstrates the parameter inference scheme for obtaining the cutting force

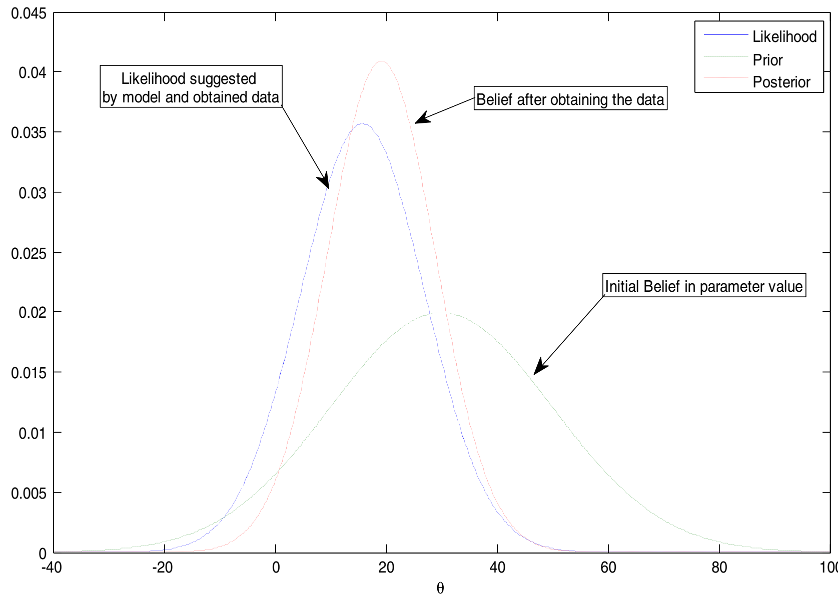


Fig. 1 Central idea of Bayesian update

estimation using a machine spindle current transducer. The parameter values obtained from the inference scheme show significant improvement over least square estimation. The results are verified with the help of machining experiments. The paper also discusses the inference scheme for the mechanistic force model coefficient estimation. A similar scheme is developed, and numerical results are discussed. The paper concludes discussing the contribution of present works and continuing research work.

2 Parameter inference with Bayesian method

In the simplest sense, a Bayesian view of probability indicates the state of knowledge or belief in a certain hypothesis [3]. In the context of parameter identification, let ω be the parameter of interest and D be the data point; the Bayesian theorem can be written as follows:

$$p(\omega|D) = \frac{p(D|\omega)p(\omega)}{\int(D|\omega)p(\omega)d\omega} \tag{1}$$

```
Initialize X_0; set k=0;
set n=large number;
for k=1:n
{
Sample a point Y from q(.|X_k)
Sample a Uniform(0,1) random variable u
If u<=α(X_k,Y) set X_{k+1}= Y
       Else set X_{k+1}= X_k
}
```

Fig. 2 Pseudo-code for Metropolis-Hastings Algorithm

In Eq. (1), $p(\omega)$ is read as the "probability distribution of value of parameter ω based on initial belief" often referred to as a "*prior*." $p(D|\omega)$ is read as the "probability that the data point observed would relate to the parameter value," called the "*likelihood*." Likelihood often relates the data point to the parameter of interest via a model; it is a very important part of the solution as we shall observe in the later sections. And finally, $p(\omega|D)$ is the "probability distribution of value of parameterω, having observed the data point$D$, called a "*posterior*." The denominator is a normalization factor, since the probability distribution must sum to unity.

Figure 1 shows this process graphically. Few points are worth noting; the posterior distribution has much less spread as compared to prior. Also, the definitiveness of both prior and likelihood dictates the variance of the posterior. Furthermore, the prior can be uninformative (uniform distribution), thus showing complete ignorance about the value of the parameter; in that case, it is the likelihood that dominates the posterior behavior.

```
Initialize X_0; set k=0;
set n=large number;
for k=1:n
{
Y=X_k+ normal random(0,Σ);
Sample a Uniform(0,1) random variable u

Calculate α(X_k,Y)={1, p(Y)/p(X_k)}


If u<=α(X_k,Y) set X_{k+1}= Y
       Else set X_{k+1}= X_k
}
```

Fig. 3 Pseudo-code for the random walk Metropolis algorithm

3 Numerical tools: Monte Carlo simulations and Markov chain Conte Carlo methods

Before discussing the numerical procedure, it is important to shed light on the tools that enable one to apply Bayesian techniques in practice. As mentioned in the earlier section, the closed form solutions of the probability densities are scarcely obtained.

MCMC methodology is a very important numerical tool that helps perform numerical integrations that are otherwise intractable in analytical form. This technique is widely used in biostatistics, image and video processing, voice recognition, and machine learning fields. A major applied work in the MCMC area is reported by [4]. It is essentially Monte Carlo integration using Markov chains. One of the principle applications of MCMC is to generate samples from a distribution which is otherwise difficult to generate samples from. It is achieved by strategically constructing Markov chains whose stationary distribution converges to the desired distribution. To deploy this in practice, there are various algorithms which include Gibbs sampling, Metropolis Algorithm, and Metropolis-Hastings Algorithm [4].

In this paper, authors used Metropolis-Hastings Algorithm to generate samples from the posterior distribution of regression coefficients. This algorithm is described in detail in the next section, but in this section, the key points of algorithm are explained [5]. As described earlier, the Markov chain needs to be generated whose stationary distribution is the target distribution we want to sample from $p(Y)$. At each iteration step k, the next state X_{k+1} is generated by sampling a candidate point Y from a proposal distribution $q(Y|X_k)$. We define an additional variable as follows:

$$\alpha = \min\left\{1, \frac{p(Y)q(Y|X_k)}{p(X_k)q(X_k|Y)}\right\} \tag{2}$$

which is known as an acceptance probability; if the candidate point is accepted, then the next state becomes $X_{k+1} = Y$, otherwise the chain does not move and $X_{k+1} = X_k$. The pseudo-code for the Metropolis-Hastings (M-H) sampler is given in Fig. 2.

It is important to make a few observations here. First of all, the candidate generating frequency specifies the M-H algorithm. Secondly, the calculation of α does not involve calculation of the normalizing constant since the probability distributions appear in both the numerator and the denominator. Depending upon the nature of the problem, the calculation of α can be simplified. For example, in cases where the candidate generating distributions are symmetric, $q(Y|X_k) = q(X_k|Y)$, yielding

$$\alpha = \min\left\{1, \frac{p(Y)}{p(X_k)}\right\} \tag{3}$$

This is the algorithm that was proposed by [6]. In our work, we use the random walk Metropolis sampler, initially introduced by [7]. In the following pseudo-code, the algorithm is described; please refer to Fig. 3.

Now, it will be discussed how MCMC methods can be used to sample from posterior distribution, which is almost intractable analytically. It is important to discuss the practical issues that need consideration while implementing a Bayesian inference scheme since MCMC is a numerical scheme. The stability of the mean and variance produced by the method depends upon the number of particles and sample move step size. To study this, the following analyses are performed.

- Impact of number of particles on estimation
- Impact of sample move step size

The goal of the above analyses is to compare the output of the MCMC inference scheme with the maximum likelihood solution. Although in most of the situations only the maximum likelihood estimate of the parameter may be required, the method discussed in this work aims to solve the general problem by re-creating the posterior distribution to be able to

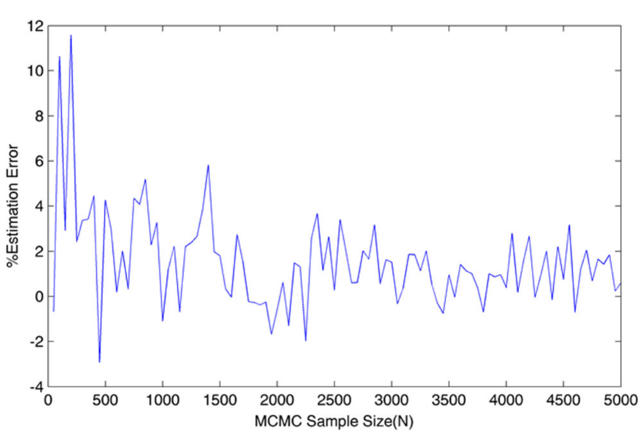


Fig. 4 Estimation error in scheme as a function of number of samples

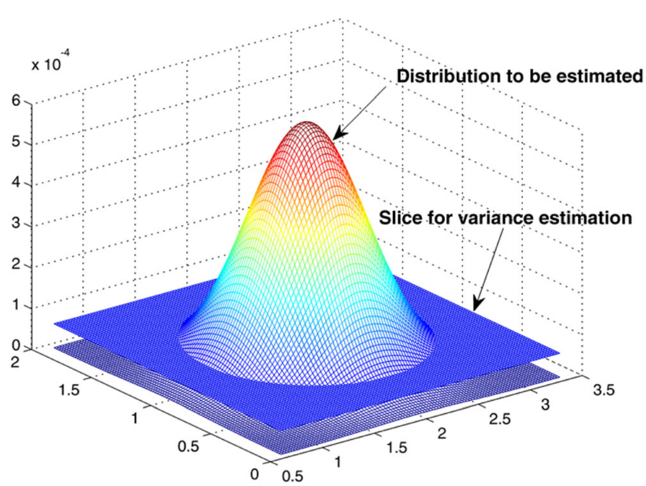


Fig. 5 Variance estimation slicing illustration

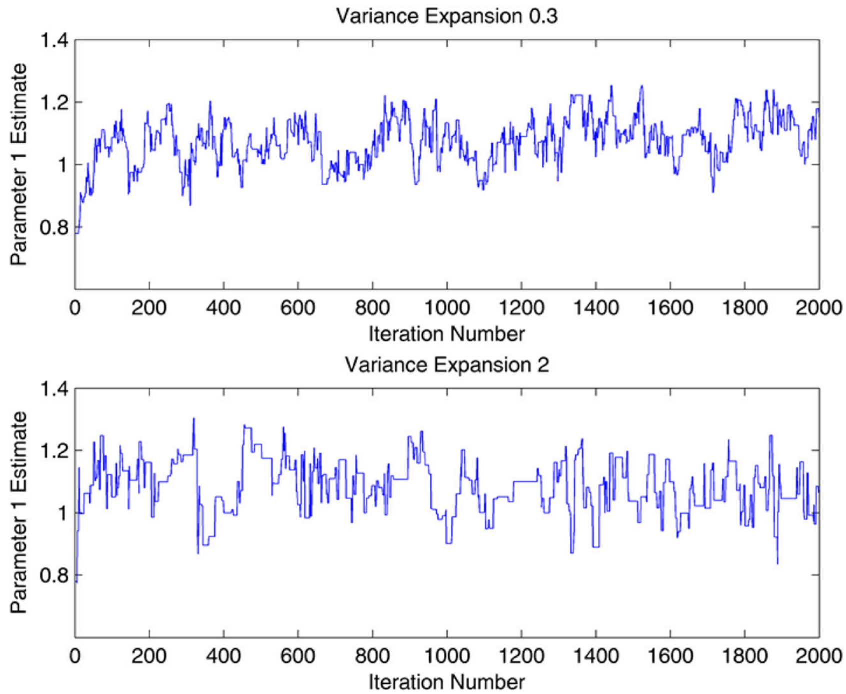


Fig. 6 Effect of variance expansion factor on sample generation

sample from the same. The input to the MCMC scheme is the posterior distribution of the parameter in terms of probabilities associated with each possible value of the parameter within random variable space. Under consideration is a bivariate normal distribution of the parameters, in terms of probabilities. At this point, the analytical expression is not available. The goal of the scheme is to sample from the posterior distribution and return the mean and variance of the posterior distribution. The input to the MCMC scheme is number of samples, input variance of the sample, and posterior distribution. The scheme generates the samples and takes the mean of the last 70% samples (to bypass burn in period). It is obvious that the accuracy of the method is dependent upon the length of samples.

Figure 4 shows a particular numerical simulation where the MCMC scheme is run multiple times with increasing sample sizes. As the sample size increases, the estimation error reduces and oscillates in a ± 2% error. Another factor contributing to estimation accuracy is the variance expansion factor. As explained earlier, in every iteration, a candidate sample is generated from candidate density; in this demonstration, the candidate density was assumed to be bivariate Gaussian. Sample generation requires mean and variance to generate an appropriate sample; the variance was estimated by taking the "slice" of the posterior distribution and calculating the difference between maximum and minimum random variable values associated with the probability. It is worth noting that by doing so, we are representing the target density variance by the variance at that slice.

If the variance input for the MCMC scheme is used directly from the slicing method illustrated in Fig. 5, the samples generated will refer to the distribution whose base corresponds to that slice. To counter this problem, the variance expansion factor is introduced, which simply expands the estimated variance to account for the error introduced because of slicing. In a numerical simulation, this variance expansion factor was varied from 0.001 to 2 (0.1% expansion to 100% expansion). The results indicate that for variance expansion ratios greater than 0.5, the estimation error is contained within ±3%. It should also be noted that the variance also represents how much the candidate sample moves from iteration to iteration. For the low variance expansion factors, the sample move will be smaller, while higher variance expansion factors will yield large jumps in sample generation. This is illustrated in Fig. 6. The effect of the variance expansion factor on the estimation error is shown in Fig. 7.

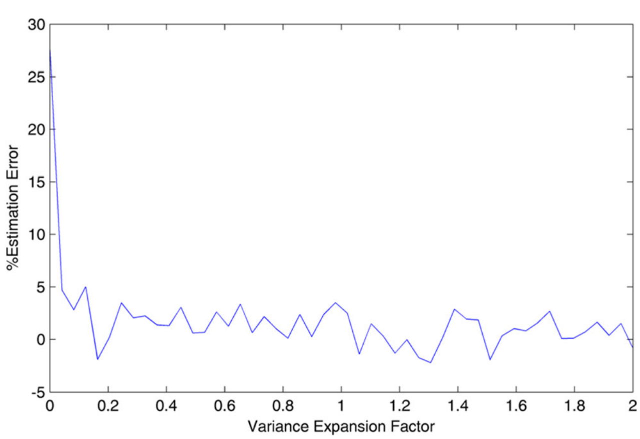


Fig. 7 Effect of variance expansion factor on estimation error

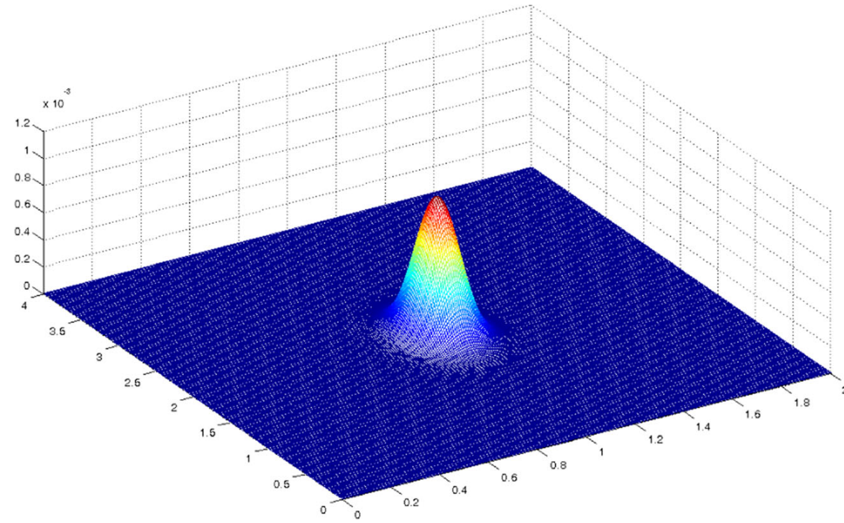


Fig. 8 Prior establishment for coefficients

4 Application: mechanistic force model estimation

4.1 Literature review for the machining cutting force models

The cutting force models descend majorly following the classic Merchant's orthogonal two-dimensional force model [8]. This model describes the relationship between measurable forces in thrust and tangential directions to derived forces along the idealized shear plane and tool rake face, under the assumption that shear angle is the same as the angle of grain elongation. Rao et al. modeled the tool-workpiece dynamic system and predicted chatter condition in turning operation [9]. Guo et al. presented the work on the cutting force model for contour generation for gear indexing cam with flat end milling [10]. The dynamic cutting force model for milling was applied to wave the removing process by Wu [11]. The Bayesian inference has been applied to tool wear coefficient estimation for Taylor's tool wear model [2] in the past by Schmitz [12], but the distributions have been assumed to be Gaussian and the MCMC approach has not been shown.

What is important to note is that though many extensions have been applied to the quintessential Merchant's orthogonal force model, it still remains to be the most general model for use. The goal of this presented work is to introduce users to understand how Bayesian inference may be applied to such a complex set of equations. It is important to note that Bayesian inference provides a theoretical construct to estimate posterior densities of model parameters, and MCMC provides an efficient numerical recipe for the same. The aim of this paper is to

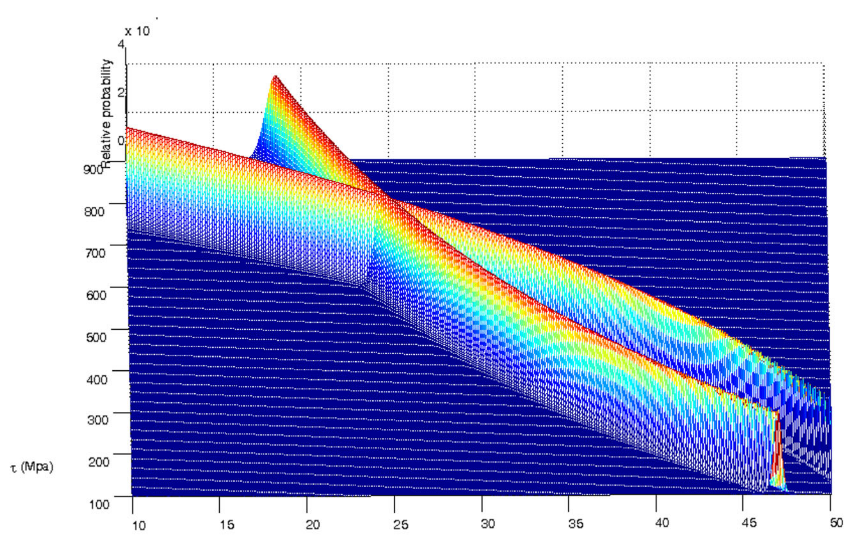


Fig. 9 Data likelihood for forces

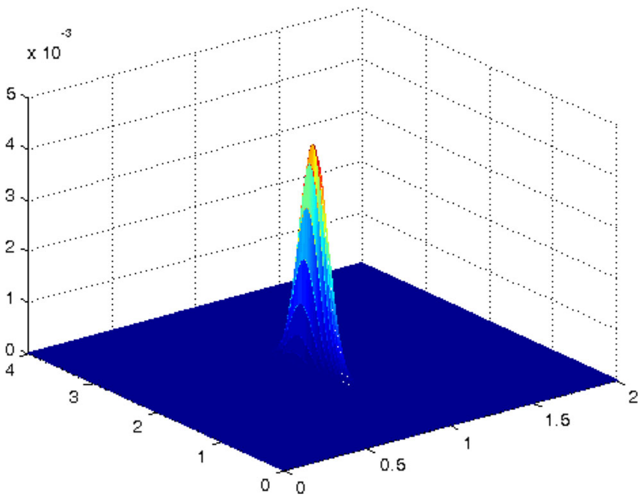


Fig. 10 Posterior distribution of coefficients

provide an example of such regression as is applied in manufacturing process model parameter estimation. Thus, for the discussion that follows, the mechanistic force model given by Merchant has been considered.

4.2 Bayesian inference approach for mechanistic force model

In this section, the step-by-step procedure of performing Bayesian inference for the mechanistic force model has been described. At a high level, the procedure can be summarized in the following steps.

1. Establishment of priors for coefficients
2. Data likelihood for the force
3. Generating the posterior distribution using MCMC scheme and updating the coefficient distributions

4.2.1 Establishment of priors

The mechanistic force model is given as follows:

$$\begin{aligned} F_c &= K_c bh + \varepsilon \\ F_t &= K_t bh + \Psi \end{aligned} \tag{4}$$

where b is depth of cut and h is the feed per revolution. ε and ψ represent the uncertainty in measurement of the cutting force because of variation in force coefficients.

The force coefficients are given as [12] following the orthogonal machining theory,

$$\begin{aligned} K_c &= \frac{\tau\cos(\beta - \alpha)}{\sin(\phi)\cos(\phi + \beta - \alpha)} \\ K_t &= \frac{\tau\sin(\beta - \alpha)}{\sin(\phi)\cos(\phi + \beta - \alpha)} \end{aligned} \tag{5}$$

where τ is the shear stress during the cutting (assuming orthogonal machining model), β is the friction angle, and ϕ is

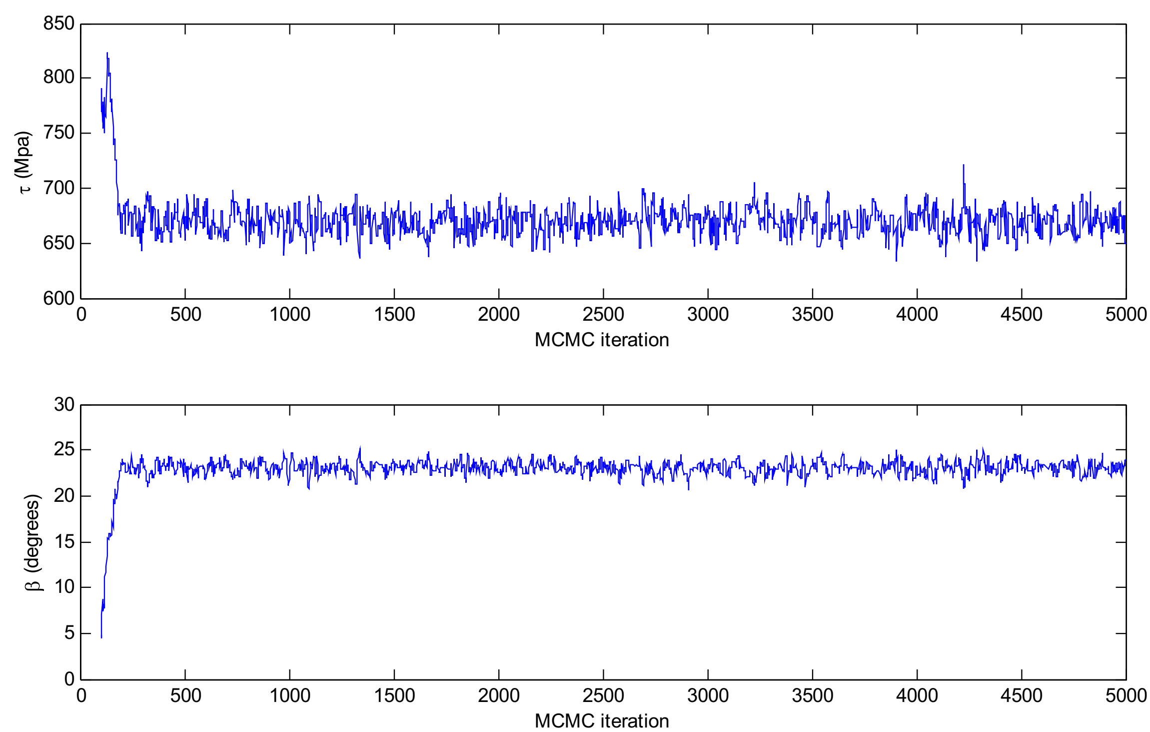


Fig. 11 Markov Chain Monte Carlo simulations to generate samples from posterior distribution of coefficients

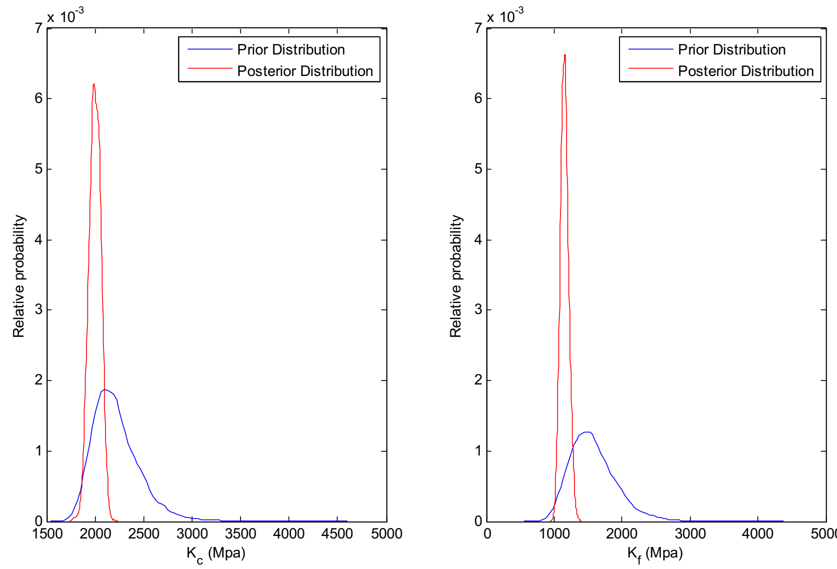


Fig. 12 Force coefficient update—reduced variability

the shear plane angle. Now, the variability in force is directly proportional to the variability in force coefficient since depth of cut and feed are machine parameters usually known and controlled.

$$p(K) \propto p(\tau, \beta, \phi) \tag{6}$$

where $p(K)$ indicates the probability distribution of the force coefficient and $p(\tau, \beta, \phi)$ is the joint probability distribution of shear stress, friction angle, and shear plane angle. Shear plane angle is independent of shear stress and friction angle. Shear plane angle is a function of chip thickness and tool rake angle. Thus, Eq. (6) can be reduced to

$$\begin{aligned} p(K) &\propto p(\tau, \beta) p(\phi) \\ p(K) &\propto p(\tau, \beta) \end{aligned} \tag{7}$$

Thus, variability in the force coefficient is directly proportional to variability in shear stress and friction angle. Therefore, for the estimation of the forces, it is necessary to observe the joint variability (or joint probability distribution) of $\tau$and β. It is important to note here that for the accurate update of the force coefficient, it is necessary to have values of τ,β, and ϕ. However, the (online) measureable quantities here are only F_c and F_f(cutting and feed force). For the update of the shear plane angle, with the knowledge of the chip thickness, the following relation can be used.

$$\phi = \frac{r_c \cos\alpha}{1 - r_c \sin\alpha}; r_c = \frac{t_c}{t_{un}} \tag{8}$$

This equation produces an initial belief in the shear plane angle which can be updated after every cut. In the scenario

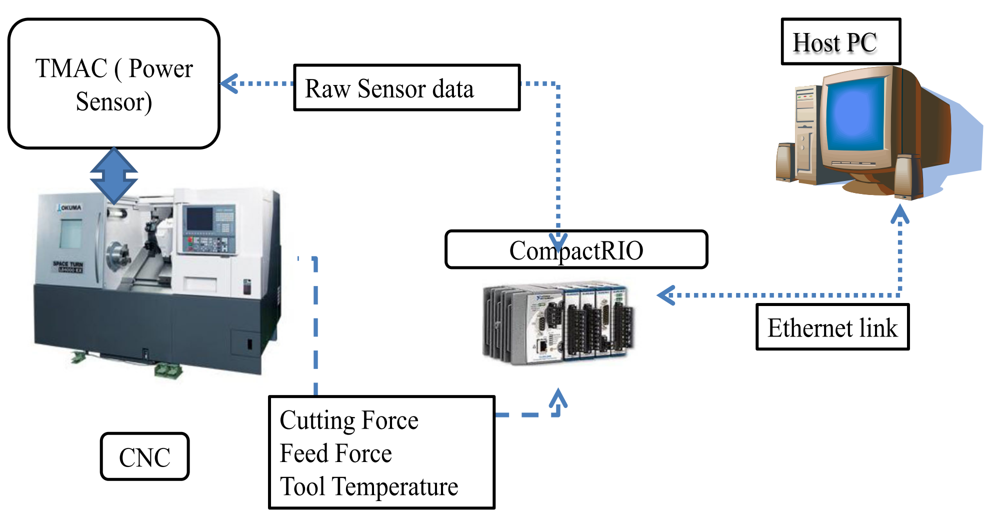


Fig. 13 Experimental setup of current sensor calibration

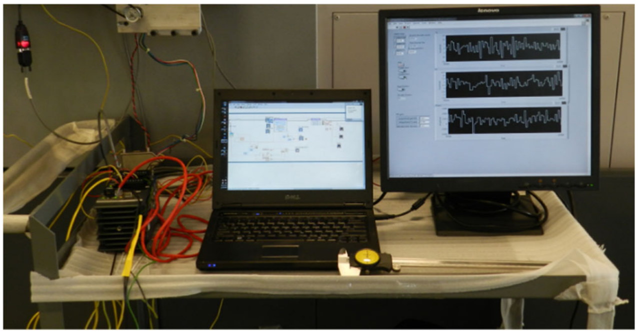


Fig. 14 Data acquisition setup

where the dynamic update of force coefficients has to be made, one needs to resort to the empirical relationships; one of the popular ones is given as follows [2]:

$$\phi = 45^{\circ} - \frac{\beta}{2} + \frac{\alpha}{2} \tag{9}$$

Based on some primary literature search [13–15], for alloy Ti6-Al4V, shear stress and friction angle joint distribution can be represented by

$$p(\tau, \beta) \sim N\left(\begin{bmatrix} 500 \\ 30 \end{bmatrix}, \begin{bmatrix} 200 & 0 \\ 0 & 5 \end{bmatrix}\right) \tag{10}$$

This is a bivariate Gaussian distribution with no covariance. Figure 8 shows the two-dimensional probability distribution of coefficients.

It is important to mention that the convergence to true force coefficient values depends upon the selection of prior distribution. That is, if the prior is chosen close to the actual value of the force coefficient, the convergence will be faster. Though this demonstration assumes a Gaussian prior centered on the literature-reported values, the scheme is also valid for a uniform distribution (noninformative prior).

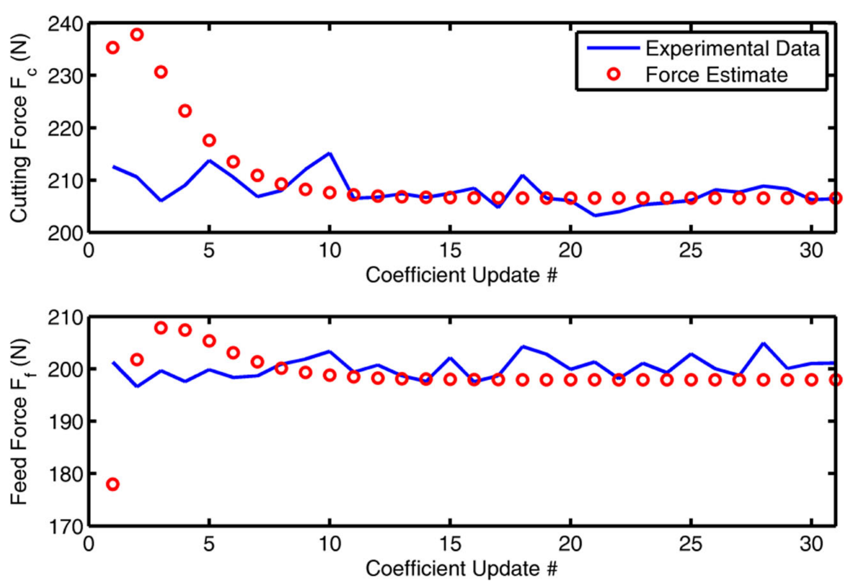
4.2.2 Data likelihood for the force

An update in the force coefficient is made whenever a new data point is made available. Since shear stress and friction angle contribute to cutting and feed forces, both cutting and feed forces help update the force coefficient value. This is done by using Eq. (4). The method deployed here is called discrete grid method [12]. First, the shear stress and friction angle values are divided in a finite grid, and then, with the measured force value, the probability of all possible values of shear stress and friction angles is calculated which will produce that force. To introduce uncertainty, the measured value of torque is assumed to have some measurement noise (2–5%). This way, we get the likelihood function which solves the inverse problem of "given the data point and my model, what is the probability that estimated coefficients (parameters) produce the observed data." And that selected value of shear stress and angle will give the measured value of force using a deterministic model in the presence of uncertainty. The data likelihood is shown in Fig. 9. The calculation of the posterior follows from point to point multiplication of the prior density with the data likelihood.

4.2.3 Sampling from posterior: MCMC scheme

Once the data likelihood is established, the posterior distribution of the shear stress and friction angle is generated by point by point multiplication of the prior distribution and the likelihood function. Since at this point, we do not have the analytical expression that represents posterior distribution, we use

Fig. 15 Force coefficient inference and comparison with experimental data



MCMC methods discussed in earlier sections to generate samples that represent the posterior distribution.

The mean of the posterior distribution indicates the updated shear stress and friction angle values. These values are then used in equation to generate updated force coefficient values.

As shown in Figs. 10, 11, and 12, reconstruction for the force coefficient distribution reveals much reduced variability before and after the update. The prior distribution is indicated with the blue solid line, and the posterior distribution is indicated with the red dotted line. This validates the numerical scheme accuracy and stability.

4.2.4 Experimental setup and data analysis

This section describes the experimental setup to validate the numerical scheme. The tests were taken on Okuma Lb4000 EX CNC lathe. The lathe was instrumented with a commercially available current transducer-based power monitoring unit. This commercial product is typically used to give alarms about the tool breakage, excessive wear, etc. The schematic of the experimental set up is shown in Fig. 13.

The output from the current transducer [16] is an analog signal (0–10 V) which represents the power measured in HP. This signal is acquired with an NI CompactRIO (cRIO-9023) control prototyping module for signal processing and data storage. In the same setup, there are additional sensors measuring cutting and feed force and temperatures near the cutting edge. For this study, temperature measurements were not included in the model. But in the experimental setup, there are two K type Omega® thermocouples cemented near the cutting insert seat.

Experimental data was generated for three cutting speeds (30, 75, and 120 m/min), and three different feed rates (0.05, 0.15, 0.25 mm/min), while keeping the depth of cut constant at 1.5 mm. These values were chosen for the machining of the Ti6-Al4V alloy using an uncoated carbide tool. It is worth noting that while generating the data, all the cuts were made using a new tool (new cutting edge).

The data obtained from the current transducer while cutting the metal was post-processed using MATLAB®.

For the experimental verification of the force coefficient inference scheme, an online estimation of force coefficient machining Ti6-Al4-V (grade 5 titanium alloy) was performed. For a particular machining condition (2 mm depth of cut, 0.3 mm/rev feed rate, 30 m/min cutting speed), Figs. 14 and 15 show the data of cutting force and feed force (solid blue line). Using the Bayesian inference scheme discussed in the previous section, the force estimates are shown (red circles). After about 10 posterior updates, the force estimates show a good agreement with the experimental data. This can also be used in an online control scenario—force coefficients of the workpiece material can be identified online to perform constant force control. Also, should force coefficient change because of hardness change in the material, the inference scheme will be able to track the values of the coefficient to ensure that the cutting force control model is updated.

It is worth mentioning how this method is novel from the other nonmodel-based (purely feedback-based) methods. Though the force coefficient values are known to be constants, they often vary for different speed and feed regimes. If a deterministic mechanistic model is chosen, it is quite possible that the prediction of forces might be accurate in a particular regime, but not across the entire range. This method not only provides a means for a prediction of forces from a mechanistic point of view but also provides understanding in the distribution of friction values, shear stress, and shear plane angles and how it varies across different cutting load and speed regimes. From the control theory point of view, it provides an automatic tuning feature. In the continuing work, authors are investigating treatment of outliers and in-process identification of shear plane angles.

5 Conclusion

In the machining process model identification, an extensive amount of experiments are required to converge to a reliable model. The idea discussed in this paper helps overcome this limitation by performing a minimal amount of tests (to establish priors), and the learning process is continuous. Bayesian methods offer mathematically robust solutions that treat uncertainty in deterministic models. MCMC numerical schemes are proven methods to sample from a distribution it is difficult to sample from. The MCMC scheme deployed in this work was able to generate samples from general distribution (Gaussian and non-Gaussian). And finally, Bayesian updates provide more accurate models that can be used for the current torque (or force) estimations and control. Experimental data shows the validity of the scheme and agreement with the observed data. A similar approach has been applied to a simpler application of linear model identification for power prediction in machining [17]. Since Bayesian inference can be extended to classification problems as well, application of this can also be found for condition-based maintenance [18].

As the next step, authors wish to integrate the Bayesian update scheme in a closed loop control. Possibility of including the wear estimation is also one of the future directions. The framework will be similar to the one that involves model learning using recursive least square (RLS) methods of parameter estimation in the case of linear Gaussian dynamic models for cutting force control or power control in the turning or milling process.

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