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ORIGINAL ARTICLE

Calculation of stiffness parameters and vibration analysis of a cold rolling mill stand

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Abstract The elastic deformation of rolling mill elements during the rolling process is important. By knowing the displacement of rolls, the optimum gap between the work rolls can be calculated. In present research, a vibration model with two degrees of freedom is proposed for a cold sheet rolling mill and the stiffness parameters of different mill elements are calculated. A numerical simulation and a finite element analysis are also carried out for the related vibrations. Afterwards, the maximum displacement of rolls is calculated using the vibration transient response of the work roll and backup roll. It is found that the system vibration reaches the critical damped level, and the rolls return to their resting positions quicker, and the effects of oscillations on the sheet being rolled decreases. As a result, precision of reduction in sheet thickness increases. Moreover, due to decrease in the sheet speed, the oscillation amplitude of rolls declines and movements of rolls turn into movements without oscillating. Finally, to verify the effectiveness of the proposed method, the experimental data are compared with calculated stiffness parameters and the rolling force.

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1 Introduction

Due to the elastic deformation of different elements of a rolling mill during the rolling process, the adjusted roll gap changes. Consequently, precision of the reduction in thickness declines. Based on our knowledge in the literature, there is not any report about vibrations of a rolling mill, the elastic deformation of all elements of rolling mill, and their effects on changing the gap between work rolls as studied in this paper. Kimura et al. [1] studied the self-exited vibration in a rolling mill with four rolls. They used a vibration model with five degrees of freedom, and considered a separate mass for structure. They found that mill vibration largely depends on rolling speed and the coefficient of friction between rolls and the sheet. Younes et al. [2] found that to reduce the vibration of rolls, it is necessary to reduce the thickness reduction and increase rolling temperature. Yang et al. [3], for vertical vibration analysis of a cold rolling mill with four rolls, used a single degree of freedom vertical vibration system and for contact between work roll and backup roll they considered a spring. Zhang et al. [4], for analysis of a rolling mill stand with six rolls, considered a dynamic model with seven degrees of freedom. They found that the 3rd and 6th natural frequencies play main role in vertical vibration system. Kapil et al. [5], for dynamic analysis of a rolling mill, modeled the working roll as a beam. They considered spring elements for bearings at the end of the work roll.

In this paper, the elastic deformations of different elements of a cold rolling mill with four rolls (including rolls, chocks, bearings, fixtures, and adjustment screws) on top of the machine were studied. To this end, the stiffness of elements was calculated using elastic relations and by replacing elements with springs. Considering the method used to connect the elements and the share of each element from the rolling force, a vibration model with two degrees of freedom was proposed for vibration analysis of the system. Moreover, through software simulations, the vibration transient response of the system to the impact excitation applied by the sheet was investigated.

2 Vibration modeling

In the proposed vibration model, backup and work rolls are allowed to vibrate separately and independently. The modeling was only carried out on the upper half of the rolling mill due to the symmetric structure of the mill.

The vibration model shown in Fig. 1 is composed of two concentrated masses: m_w (work roll mass) and m_b (backup roll mass). This model consists of several springs including the following: the stiffness of work roll bearingk_{bw}, the stiffness of work roll chock k_{qw}, the stiffness of fixtures k_f, the stiffness of backup roll bearing k_{bb}, the stiffness of backup roll chock k_{qb}, the equivalent stiffness of the screw transmission mechanism on the top of the rolling mill k_s, and the equivalent stiffness of the work roll and backup roll contact k_c.

2.1 Simplification of vibration model

Usually, researchers first propose a primary model of a vibration system and then simplify the model using the fundamentals of dynamics and considering the assumption of symmetry. In the primary model, springs that assumed to be equivalents of chock stiffness, bearing stiffness, and fixture stiffness are arranged with a series connection (i.e., all of them are under the influence of the same amount of force). Moreover, this set is in a parallel connection with the equivalent stiffness of roll contact (i.e., they have equal deformations). Therefore, by defining k_1 andk_2, the Eqs. (1) and (2) could be written as follows based on the rules governing connection of springs [6]:

$$k_2 = 2\left(\frac{1}{\frac{1}{k_{\mathrm{bb}}}+\frac{1}{k_{\mathrm{qb}}}+\frac{1}{k_{\mathrm{bw}}}+\frac{1}{k_{\mathrm{qw}}}+\frac{1}{k_{\mathrm{f}}}}\right)+k_{\mathrm{c}} \tag{1}$$

$$k_1 = 2k_{\mathrm{s}} \tag{2}$$

Assuming the equivalent structural damping parameters for each of the given rolling mill segments, the simplified two degrees of freedom vibration model could be depicted in Fig. 2.

2.2 Vibration equations

In this paper, to obtain the equations describing the system oscillations; Newton's second law is used [6].

$$\sum F_x = m\ddot{x} \tag{3}$$

$$m_w\ddot{x}_w + c_2\dot{x}_w - c_2\dot{x}_b + k_2x_w - k_2x_b = F \tag{4}$$

$$m_b\ddot{x}_b + (c_1+c_2)\dot{x}_b - c_2\dot{x}_w + (k_1+k_2)x_b - k_2x_w = 0 \tag{5}$$

where the w subscript denotes the quantity of the work roll and b stands for the quantity of the backup roll. At $t = 0$, the

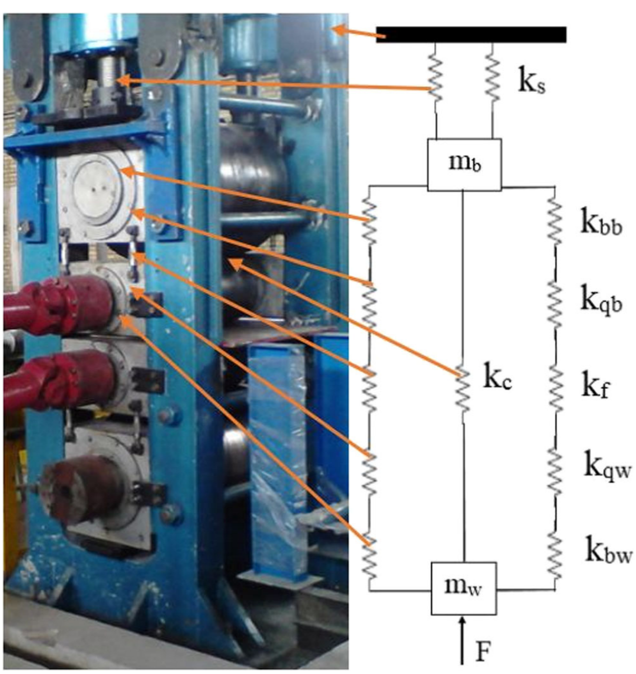


Fig. 1 Primary vibration model for the rolling mill stand

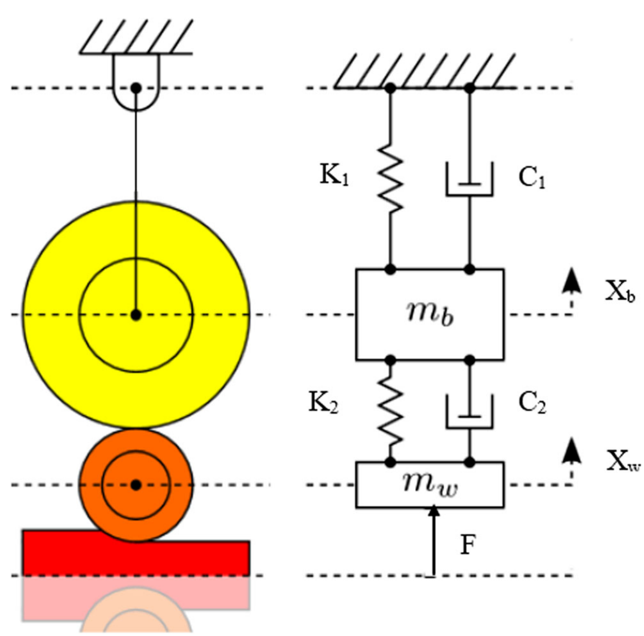


Fig. 2 Simplified 2DOF equivalent vibration model

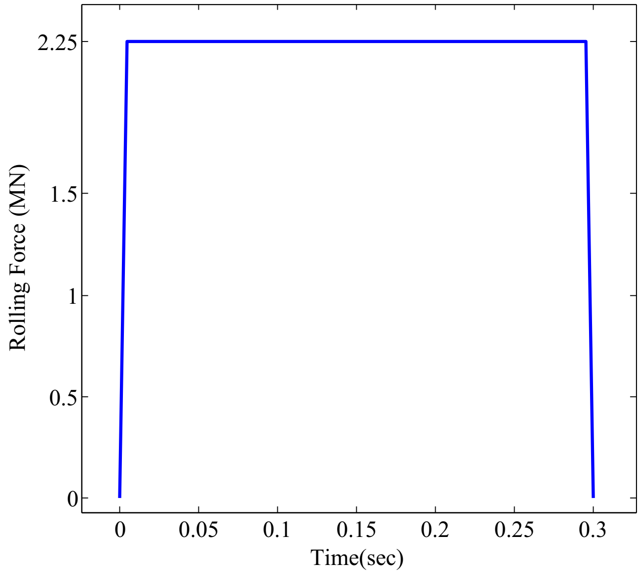


Fig. 3 Rolling force vs. time curve

position and the speed of the rolls are assumed to be zero. The matrix form of the above Eqs. (4) and (5) is as follows:

$$\begin{pmatrix} m_w & 0 \\ 0 & m_b \end{pmatrix}\begin{pmatrix} \ddot{x}_w \\ \ddot{x}_b \end{pmatrix} + \begin{pmatrix} c_2 & -c_2 \\ -c_2 & c_1 + c_2 \end{pmatrix}\begin{pmatrix} \dot{x}_w \\ \dot{x}_b \end{pmatrix} + \begin{pmatrix} k_2 & -k_2 \\ -k_2 & k_1 + k_2 \end{pmatrix}\begin{pmatrix} x_w \\ x_b \end{pmatrix} = \begin{pmatrix} F \\ 0 \end{pmatrix} \tag{6}$$

3 Parameter determination of the system

To carry out the rolling process, a force normal to the sheet surface is exerted on the sheet by the rolls to reduce the thickness of the sheet. Therefore, the reaction of this force, which actually results from the sheet resistance against deformation, is applied to the rolls by the sheet. This reaction force tends to pull the rolls apart from one another. By rolling sheets of limited lengths, the same force is exerted on the rolling mill each time. Moreover, due to the duration of application of this force, it could be considered an excitation of impact normal to the sheet surface. In this paper, the cold rolling force of the sheet is calculated through Eq. (7) [7].

$$\frac{F}{w} = \frac{2}{\sqrt{3}}\frac{1}{\eta}\overline{Y}\sqrt{R\Delta h} \tag{7}$$

where Fdenotes the cold rolling force, $w$stands for the sheet width, η shows the deformation efficiency, $\overline{Y}$ shows the mean yield stress of the sheet material, R is the work roll radius, and Δh is the decrease in sheet thickness. In this research, the rolling force magnitude has been calculated by using Eq. (7) as 2.25 MN for $w = 0.5m$, $\eta = 0.94$, $R = 0.175m$, $\Delta h = 0.001m$, and $\overline{Y} = 145.5\,\text{MPa}$. Time duration of rolling varies depending on the length of the sheet subjected to the rolling process and the rotational speed of work rolls. The rolling mill that examined in this paper was designed for the sheets with short length. Under the operational condition for a rolling mill used for sheets with a length of 1.5 m and speed of 5 m/s, the time history of rolling force has been plotted as shown in Fig. 3.

3.1 The stiffness parameters

Based on the above described problem, one can calculate the following parameters which are needed for simulation.

Bearing stiffness. To place the ends of the rolls in the chocks of the rolling mill, cylindrical roller bearings are used on each side. Similar to the findings by Edwin [8], when a roller bearing is subjected to radial loading, the elastic deformation would be occurred only on a few numbers of the rollers of the bearing. This portion of the bearing is called the load expansion region. In the theoretical approach adopted in this paper, each roller of bearing is replaced with a beam according to Fig. 4a. The stiffness value of the roller bearing is considered as the

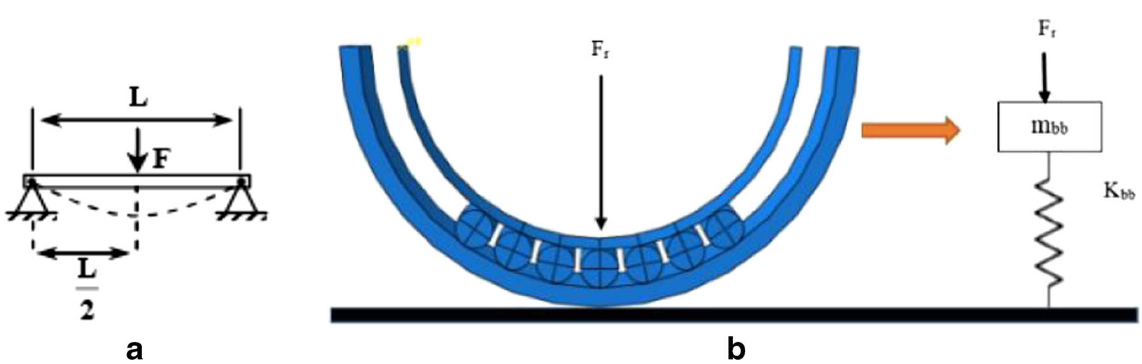


Fig. 4 **a** Modeling each roller of bearing with a beam and **b** mass-spring model of the bearing under the radial impact

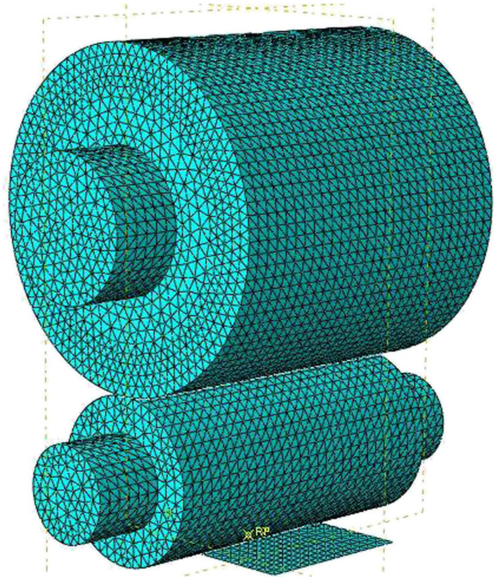


Fig. 5 Finite element model of the rolls

is transformed into a mass-spring model without damping as shown in Fig. 4b.

When a force suddenly acts on a vibration system without damping, the maximum displacement is almost twice the static displacement [6].

$$X_{\text{Max}} = \frac{2F_0}{K} \tag{10}$$

where X_{Max} denotes the maximum displacement of the bearing inner ring as a result of the radial impact, $F_0$is the magnitude of the applied radial force, and K is the equivalent stiffness of the bearing.

The equivalent stiffness of the work roll to the backup roll contact. This parameter mostly demonstrates a non-linear behavior for the contact between rolls. Hence, the stiffness between rolls is calculated by modeling the finite elements of rolls as shown in Fig. 5. In this modeling procedure, rolls are modeled as deformable elements and the sheet is modeled as a rigid element because of neglected thickness reduction comparing to displacement magnitude of the rolls. The model is assumed to be static and the backup roll is loaded in two steps.

Using the displacement of roll axes with respect to each another in the force variations range and by using Eq. (11), it is possible to calculate the stiffness parameter in the variation range of the force [6].

$$K = \frac{\Delta F}{\Delta X} \tag{11}$$

where K shows the spring stiffness, ΔF is magnitude of the force variation and ΔX is the roll displacement variation of the corresponding force variation.

The stiffness of chocks and of fixtures. To connect each roll to the rolling mill stand, two chocks are used at both sides. In the rolling mill which studied in this research, the chocks are made of steel. Inside the chocks, there are spaces for placement of bearings. According to Fig. 6a, to connect the chocks to each other, two fixtures are used at each side.

approximate stiffness of a few numbers of rollers in the load expansion region [9].

$$K_j = \frac{48E_jI_j}{L^3{}_j} \qquad j = 1, 2, \ldots, \mathrm{n} \tag{8}$$

$$I_j = \frac{\pi r_j^4}{4} \tag{9}$$

where E_j denotes the modulus of elasticity of jth roller of bearing, I_j is the area moment of inertia of jth roller of bearing about of its natural axis, L_j is the length of jth roller of bearing, r_j is the radius of jth roller of bearing, and n is the number of the rollers of bearing which suffer the radial force. In the software simulation, a finite element model of the bearing include of rollers is developed. Then, a force is applied to the bearing inner ring as a radial impact force, and the model

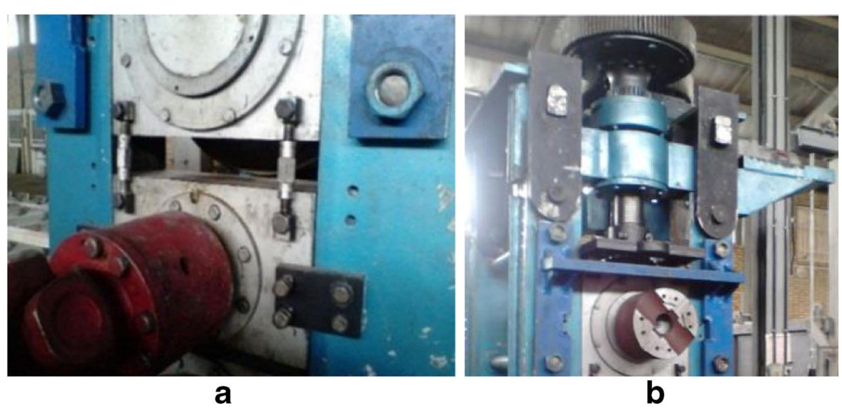


Fig. 6 **a** Position of chocks and fixtures and **b** position of the screw in the upper part of the mill

Assuming that the chocks and fixtures are under axial loading, the stiffness of each of these elements is calculated using Eq. (12) [9].

$$K = \frac{AE}{L} \tag{12}$$

where E is the modulus of elasticity, A is the cross-sectional area, and L denotes the length of fixture which is under tension.

The equivalent stiffness of the screw transmission mechanism on the top of the rolling mill. Since the upper section of the rolling mill is separable, the screw transmission mechanism is used to transfer strength and adjust the gap between work rolls. Moreover, the screw is connected to a steel plate which acts similar to a nut. According to Fig. 6b, this nut pushes the backup roll to the rolled sheet by pressuring the chock.

The nut is compacted between the chock and screw, and thus it is under an axial loading. According to Eq. (12), the stiffness of this nut is approximated. Moreover, the stiffness of the screw which is a strength transmission screw is calculated by Eq. (13) [10].

$$K_{\text{screw}} = \frac{A_d A_t E}{A_d L_t + A_t L_d} \tag{13}$$

Where $A_d$is the non-threaded cross-sectional area, A_t is the corresponding tensile stress area, $L_d$is the non-threaded length, L_t is the threaded portion length, and E is the modulus of elasticity of screw.

The above mentioned nut and screw suffer the same amount of force at the time of application of rolling force. As a result, based on the law of connection of springs, they can be considered linear springs with series connections [10].

$$\frac{1}{K_s} = \frac{1}{K_{\text{screw}}} + \frac{1}{K_{\text{nut}}} \tag{14}$$

where $K_{\text{nut}}$is the nut stiffness, K_{screw} is the screw stiffness, and $K_s$is the equivalent stiffness of the screw transmission mechanism on the top of the mill stand. Therefore, the equivalent stiffness of the nut and screw connection is calculated approximately. Table 1 shows calculated stiffness of elements of rolling mill (see Appendix A).

Table 1 Calculated stiffness values of different parts of the mill stand

Element	Stiffness symbol	Stiffness value (MN/m)
Work roll bearing	k_{bw}	3233
Backup roll bearing	k_{bb}	6049
Work roll chock	k_{qw}	28,571
Backup roll chock	k_{qb}	20,000
Fixtures	k_f	1047
Screw	k_s	6880
Roll contact	k_c	50,435

Table 2 Different values of damping constants

2DOF system stiffness parameters (MN/m)	$\beta = \frac{1}{4000}$	$\beta = \frac{1}{2000}$	$\beta = \frac{1}{1000}$
$k_1 = 13,760$	$C_1 = 3$	$C_1 = 6$	$C_1 = 13$
$k_2 = 51,755$	$C_2 = 12$	$C_2 = 25$	$C_2 = 51$

3.2 System damping

Damping of this vibration system is structural. In this case, loss or dissipation of energy results from the material properties and occurs as a result of the friction between inner layers of the body. One of the methods for simulating the structural damping is proportional damping as stated in [9].

$$[\mathrm{C}] = \alpha[\mathrm{M}] + \beta[\mathrm{K}] \tag{15}$$

whereαandβ are modal damping ratios, $[C]$ is damping matrix, $[M]$ is mass matrix, and $[K]$ is stiffness matrix. In this paper, the damping matrix is considered to be proportional to the stiffness matrix. That is, only the β ratio is available.

$$[C] = \beta[K] \tag{16}$$

$$\beta = \frac{2\zeta}{\omega_n} \tag{17}$$

whereζdenotes the damping ratio and $\omega_n$shows the natural frequency of free vibrations.

By selecting different values forβ, different damping constant values are obtained for the system. Table 2 shows these values.

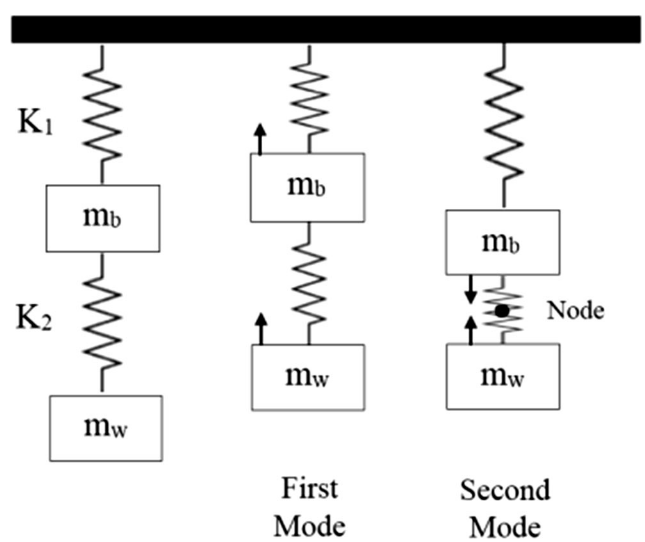


Fig. 7 Vibration modes in the rolling mill vibration model

To calculate the natural frequencies, it is necessary to examine the free vibration. In this case, system damping values are neglected. Eq. (18) describes the free vibration and the natural frequencies, and mode shapes are obtained by solving Eq. (19) [9].

$$[M]\{\ddot{x}\} + [K]\{x\} = \{0\} \tag{18}$$

$$\left|[K]-\omega^2[M]\right| = 0 \tag{19}$$

According to Fig. 7, the two masses in this vibration model have the same phase in the first mode and the natural frequency in this mode is 2042 rad/s. In the second mode, the two masses have opposing phases. Therefore, a node is formed in the middle of the spring connecting the two masses (K_2). This point lacks motion. In the second mode, natural frequency is higher and about 9541 rad/s.

4 Vibration transient response of rolls

In this section, vibration transient response of rolls to the impact force applied by the sheet is simulated numerically. First, the response of system is considered without damping. In this case, since no loss occurs in the oscillations, its vibrations should be in the form of uniform oscillation amplitudes. No decrease should be observed in the oscillation amplitudes over time [9]. The vibration transient response of work roll $X_w$with zero damping is shown in Fig. 8.

In Fig. 8, from ($t = 0$ s) until ($t = 0.3$ s), the rolling process takes place and from ($t = 0.3$ s) onward, the sheet leaves the space between rolls. As seen, while the sheet is not between the rolls and there is no excitation in the system, the oscillations continue. Then, assuming structural damping for equivalent 2DOF system, the transient response of the system is investigated. Assuming damping values of the system as ($C = K/4000$), the vibration diagrams including the transient response of work and backup rolls are plotted as shown in Fig. 9. The main objective is to study the maximum displacement of rolls.

In Fig. 9, two small oscillation ranges can be seen for the rolls. One of them occurred when the rolling starts. In this case, the sheet enters into the gap between the rolls and pushes the rolls toward the top of the mill. As a result, the system experiences very short-range oscillations for a few seconds.

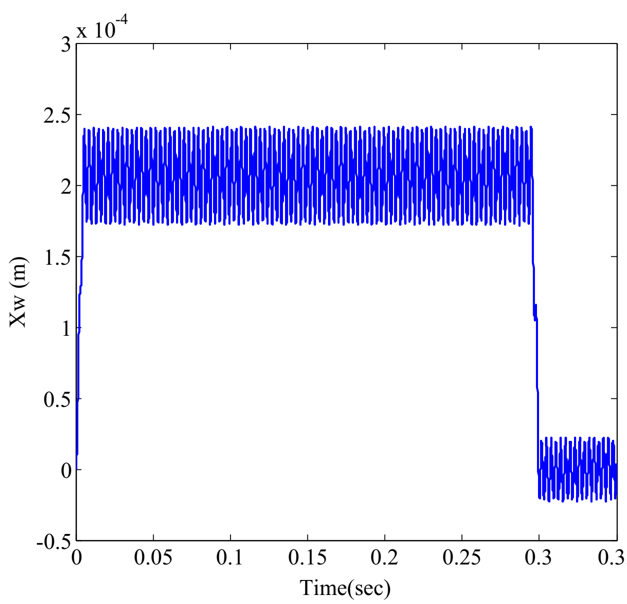


Fig. 8 Vibration transient response of the work roll without damping

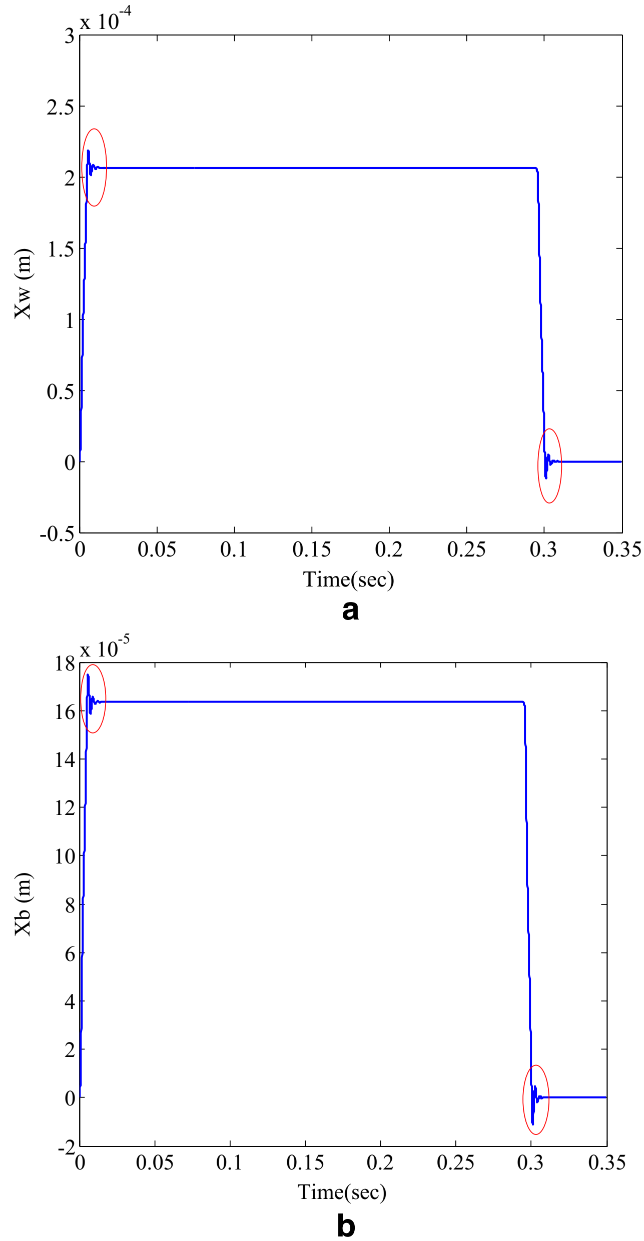


Fig. 9 **a** Vibration transient response of the work roll and **b** backup roll

These oscillations may diminish the precision of the process of reducing sheet thickness, especially in the first parts.

The other oscillation range covers the moment the sheet leaves the gap between rolls and the rolls return to their main resting position. Oscillations in this region are important because if the time taken by the next sheet to reach the mill is short, it affects the oscillation amplitude resulted from the passage of the next sheet and increases the oscillation amplitude. Hence, the time interval between rolling of consecutive sheets should be adjusted such that the system loses its oscillation before the next sheet reaches the mill. Due to the significance of oscillations of work and backup rolls in the two above mentioned ranges, these oscillations were studied for the sheet speed variations and different values of system damping as follows:

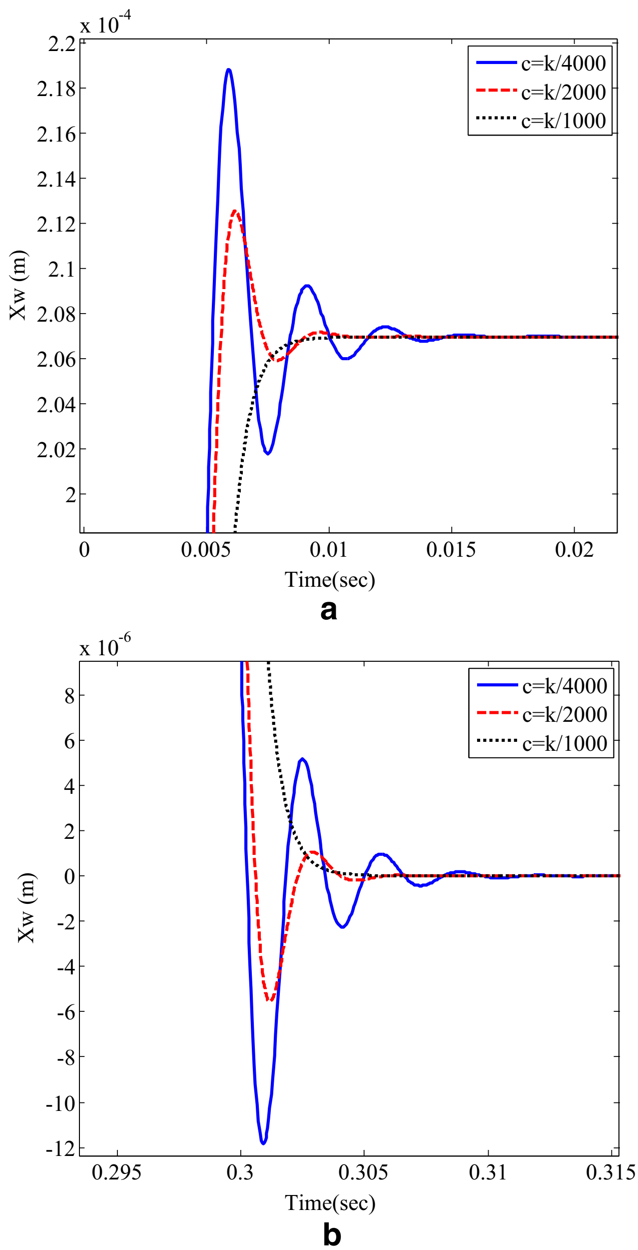


Fig. 10 **a** Vibration transient response of the work roll at the onset of rolling and **b** at the moment the sheet leaves the gap between rolls with different damping values

4.1 Vibration transient response of the work roll with different damping values

In this section, the transient response of the work roll at the onset of rolling and end of rolling is simulated with different damping values and the results are shown in Fig. 10.

Figure 10 shows the system transition from the under damping state to the over damping state.

- In the case of $C = K/4000$, the work roll oscillates with the amplitude gradually decreasing to zero. It is called under damped phase ($\zeta_w < 1$).
- In the case of $C = K/2000$, the work roll returns to equilibrium quickly with slight oscillating. It is called the critically damped phase ($\zeta_w \approx 1$).
- In the case of $C = K/1000$, the displacement of work roll has no oscillation. It is called the over damped phase ($\zeta_w > 1$).

With an increase in damping values, oscillations of the work and backup rolls decrease at the start of rolling and at the time the sheet leaves the gap between rolls. Moreover, the time for the loss of oscillations declines and the rolls return to their resting positions quicker. As the system damping values escalate and reach the critically damped level, the effect of oscillations on the sheet being rolled decreases. As a result, precision of reduction in sheet thickness increases.

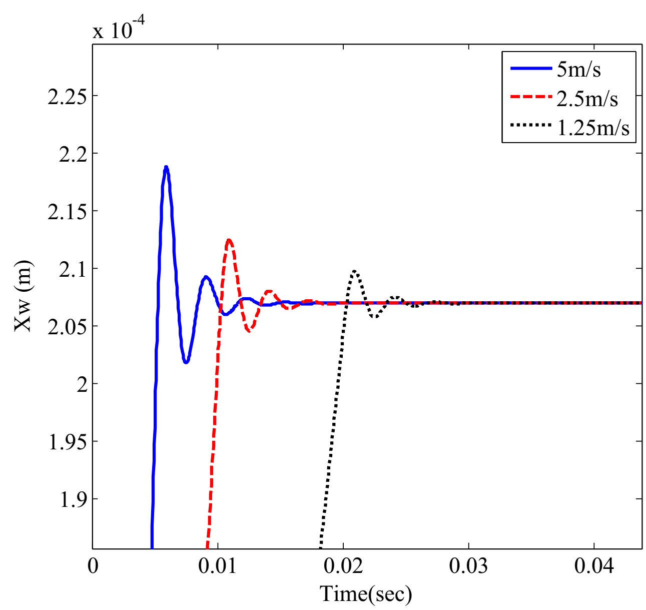


Fig. 11 Vibration transient response of the work roll for three different speeds of sheet

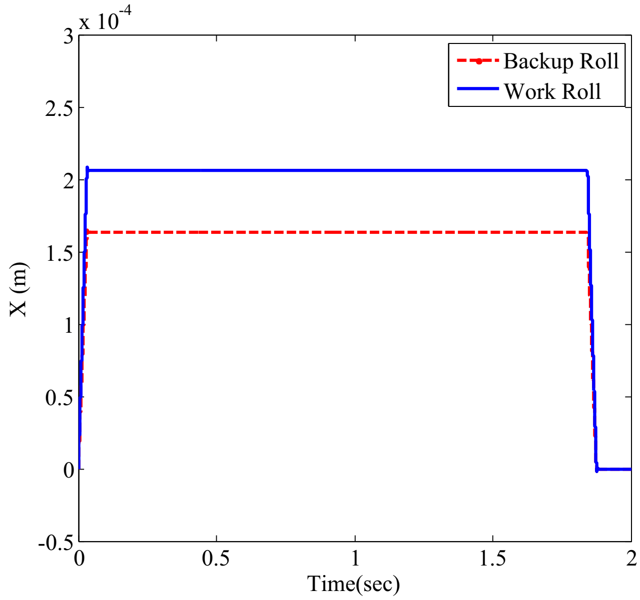


Fig. 12 Vibration transient response of the work and backup rolls at a sheet speed of 0.8 m/s

4.2 Vibration transient response of the work roll with different sheet speeds

In this part of the paper, the vibration transient responses are compared numerically with three different speeds of the sheet between work rolls. Figure 11 shows vibration transient response of the work roll for different values of sheet speed of 1.25, 2.5, and 5 m/s.

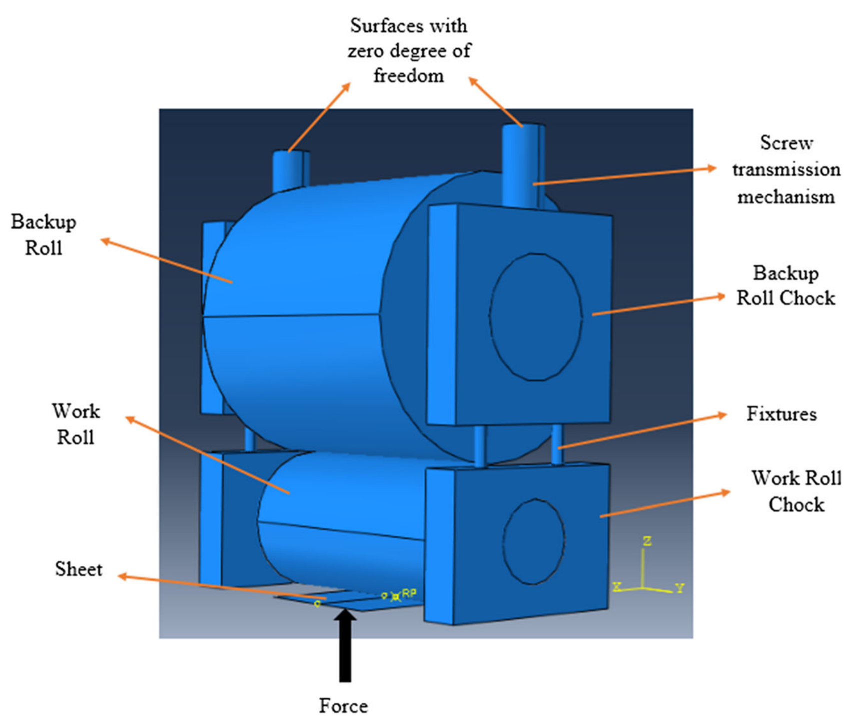
With a decrease in sheet speed or, in other words, with an increase in duration of impact, oscillations of work roll decline. For example, oscillations of the work roll are slight when the sheet speed is 1.25 m/s. Moreover, in all of the three above states for speed of the sheet, after a short while the oscillations are damped. Therefore, to reduce oscillations of rolls in this range, a decrease in sheet speed may be effective. However, it should be noted that a decrease in sheet speed may have positive effects due to the decrease in oscillation amplitudes of rolls, but it is impossible to assume a very low sheet speed due to the importance of time in the production process.

In this paper, by changing the time parameter, the maximum speed for the operation conditions of this mill has been obtained when the system demonstrated the lowest level of oscillations. This optimum value was obtained to be 0.8 m/s for this rolling mill. That is to say, if the sheet speed exceeds this magnitude, oscillations of rolls will be fully evident. Figure 12 shows the vibration transient response of work and backup rolls at a sheet speed of 0.8 m/s. As it can be seen in this case, the rolls move without oscillating.

4.3 Optimum roll gap

Before a rolling mill starts working, the gap between rolls is adjusted to create adequate space for the passage of sheet between the work rolls. This gap depends on the thickness of the input sheet and the required decrease in sheet thickness. The size of this gap changes to some extent during the rolling

Fig. 13 Finite element model of the rolling mill



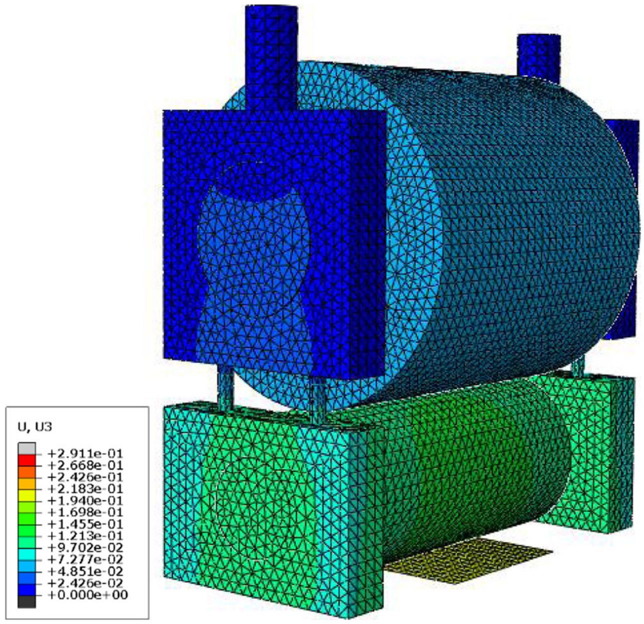


Fig. 14 Displacement of elements normal to the sheet

process due to the elastic deformations of mill elements and slight displacement of rolls normal to the sheet. Hence, after determining the displacement of rolls during the sheet rolling process, it is necessary to re-adjust the roll gap.

In the rolling mill of this research, the gap required for the work rolls was 5 mm. According to the research results, the maximum displacement of work rolls at an input sheet speed of 5 m/s reaches about 0.2069 mm. If the rolling mill is assumed to be almost symmetrical, the gap should be adjusted for 4.5860 mm during the process to obtain a 5 mm gap with the displacement of rolls.

5 Finite element study for the rolling mill

Finite element method is important to study the amount of deformation. Del Pozo et al. [11] for study the deformation in the press process and strain rate introduced a finite element model. In this paper, to compare the results of numerical simulations with results of the finite element method, a model has been created dynamically in FEM software for the upper half of the mill as shown in Fig. 13. For some of the segments simpler, elements were used, which showed behaviors similar to their corresponding elements. In this model, all of the elements are deformable except for the sheet, which is assumed to be rigid. The reason was explained in previous section.

The mill elements were made of a material with a modulus of elasticity of 200 GPa, Poisson's coefficient of 0.3, and density of 7860 Kg/m^3. The solution was determined to be dynamic because the force is applied to the system as an impact (see Fig. 3), and its value changes over time. The required time for solving varies depending on the sheet speed or, in other words, on the duration of application of the impact. In this paper, the quadratic tetrahedron elements have been used for FEM analysis. All of the elements have a degree of freedom perpendicular to the sheet and end of the screws mechanism on top of the mill are fixed.

Figure 14 presents a view of displacement of elements perpendicular to the sheet. The position of bearings and fixtures is exposed to more displacements. Finally, these displacements are transferred to the screw transmission mechanism on top of the mill. The end of screws experience the minimum displacement as it is connected to the mill structure.

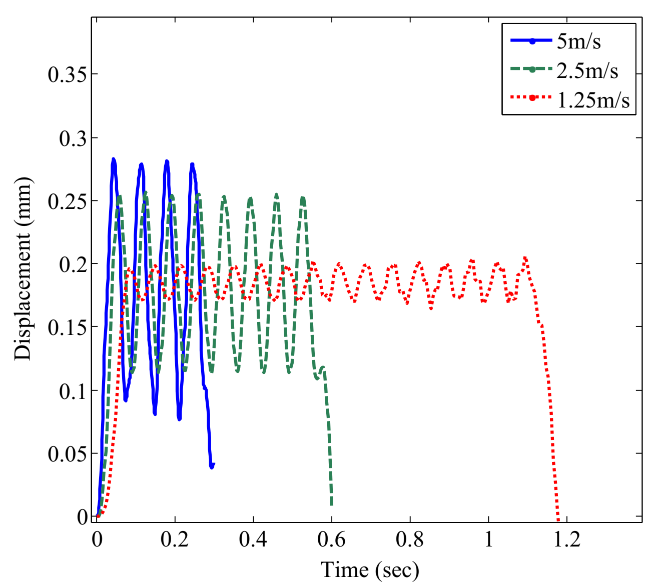


Fig. 15 Displacement of work roll for different sheet speed values in the finite element method

Table 3 Comparing the displacements of the work roll in the numerical simulation and FEM

Sheet speed (m/s)	Max. disp. of the work roll in num. simulation (mm)	Max. disp. of the work roll in FEM (mm)	Percent of the difference of the results (%)
5	0.2180	0.2833	29
2.5	0.2125	0.2545	19
1.25	0.2069	0.1958	5

Figure 15 shows displacements of the work roll normal to the sheet for three different sheet speed values.

According to Fig. 15, the following two phenomena are demonstrated with a decrease in the sheet speed or an increase in the duration of application of the impact to the system.

- The maximum displacement of the work roll decreases with a decrease in sheet speed.
- With a decrease in sheet speed, displacement of the work roll turns into movement without oscillating. As it can be seen, at a speed of 1.25 m/s, the oscillation amplitude is slight.

6 Comparing the results of finite element study with numerical simulation

In both analyses, with a decrease in sheet speed, the maximum oscillation amplitudes decreased, and movements of the system gradually turn into non-oscillatory movements. In Table 3, for three different sheet speed values, the maximum oscillation amplitude of the work roll in the numerical simulation is compared to the results of finite element method.

In most cases, displacement of the work roll in the finite element modeling is larger than that of in numerical simulation. Moreover, proportional to the decrease in the sheet speed, results of finite element modeling become closer to results of numerical simulation.

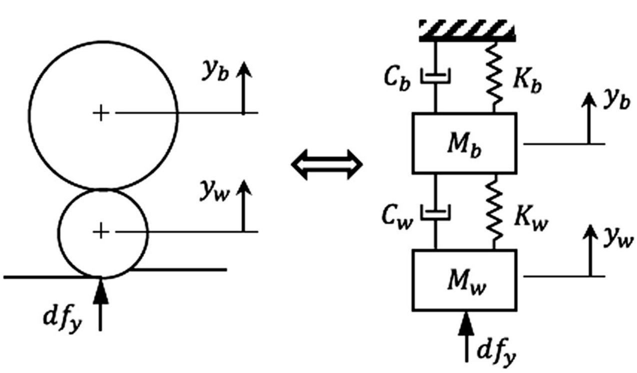


Fig. 16 The rolling mill stand vibration model in [12, 13]

Table 4 Specifications of two-stand tandem mill unit of [12, 13] and present research mill stand

Parameter	Stand 1 [12, 13]	Stand 2 [12, 13]	Present research
Work roll mass, M_w, (Kg)	14,000	14,000	750
Backup roll mass, M_b, (Kg)	38,000	38,000	2500
Work roll radius (m)	0.245	0.245	0.175
Backup roll radius (m)	0.675	0.675	0.350
Strip width (m)	0.762	0.762	0.50
Sheet yield stress in the plane stress condition, $\overline{Y}$, (MPa)	896	953	145.5
Entry thickness, (mm)	0.409	0.317	6
Exit thickness, (mm)	0.317	0.210	5
Work-hardening effect	Yes	Yes	No

7 Numerical verification

In this section, all of the extracted results for the rolling mill stand as rolling force and stiffness parameters are compared with the experimental and theoretical results of [12, 13]. They have studied theoretically and experimentally about chattering in cold strip tandem mill unit (stand 1 and stand 2) of Mobarakeh Steel Company (MSC). It should be noted that the case study of the two aforementioned references are different from the case of this research structurally. However, in view point of the order of magnitudes, the stiffness parameters and the rolling forces may be compared.

The equivalent 2DOF vibrating model which has been made by [12, 13] is shown in Fig. 16. According to Figs. 2 and 16, physical and geometrical specifications of the mill stands are summarized in Table 4.

The measured rolling force and stiffness parameters and damping constants from experimental test in [12, 13] and also their corresponding computed values for the mill stand of present paper are summarized in Table 5. By considering the Table 4 values and comparing the Table 5 values, the calculated parameters in this research can be verify. The calculated parameters in this research. It is worth noting that in [12, 13], the work-hardening effect has been existed while in present research that effect has not to be considered.

8 Proposed solutions to improve performance

To improve the performance of rolling mills and increase precision of rolling, it is possible to take appropriate actions. For instance, using the screw transmission mechanism on top of the mill and by exerting adequate pressure on the whole set, it is possible to partly compensate displacements of the work rolls during rolling. In this research, due to the mill specifications and working conditions, at a sheet speed of 5 m/s, the

Table 5 Experimental results of two-stand tandem mill unit of [12, 13] and computed results of present paper

Parameter	Stand 1 [12, 13]	Stand 2 [12, 13]	Present research
Work roll-damping coefficient, C_w or C_2, (N.s/m)	0	0	12e6 to 51e6
Backup roll-damping coefficient, $C_b$or C_1, (N.s/m)	1.254e6	1.254e6	3e6 to 13e6
Work roll spring constant, $K_w$or K_2, (N/m)	4.97e10	4.97e10	5.1755e10
Backup roll spring constant, $K_b$or K_1, (N/m)	1.22e10	1.22e10	1.3760e10
Rolling Force (MN)	6.1	6.8	2.25

displacement was calculated to be 0.2069 mm in numerical simulation. Moreover, by adding dampers to the system, it is possible to increase the system damping to the critically damped level. In this case, rolls return to their previous resting positions as soon as possible. Moreover, the rolling speed decreases such that the oscillation amplitude declines in the oscillation range. In this research, the optimum speed was calculated to be 0.8 m/s considering the working conditions.

9 Conclusions

In the rolling process, due to the elastic deformation of different elements of a rolling mill, the rolls move and the gap between the work rolls changes. When the sheet leaves the space between rolls, the rolls return to their primary positions. By understanding the displacement of rolls, the optimum gap between the work rolls was calculated. In that case, two oscillation ranges were observed in the displacement of the rolls. One of the oscillation ranges starts when rolling starts, and the other oscillation range forms when the sheet leaves the gap between rolls. In both ranges, oscillations are damped gradually due to the structural damping. With an increase in the damping ratio, duration of oscillating movement of rolls declines and the rolls return to their former positions quicker. As the damping ratio reaches to critical level and exceeds it, displacement of the work roll turns into movement without oscillating. Hence, perhaps it is possible to improve rolling conditions by increasing system damping values. Moreover, with a decrease in sheet speed, the amplitude of oscillation of the rolls decreases. The sheet speed parameter was also studied with the finite element method, and it was found that with a reduction in sheet speed, the maximum oscillation amplitude of the rolls decreases and the movements gradually become non-oscillatory motions. To verify the proposed method of calculation of the rolling force and stiffness parameters numerically, the results were compared with the experimental and theoretical results.

Appendix

Geometrical specifications of some parts of mill stand as work roll chock, backup roll chock, and fixtures are summarized in Table 6.

Table 6 Features of the chocks and the fixtures

Elements	Length (m)	Cross-sectional area (m^2)
Work roll chock	0.35	0.05
Backup roll chock	0.5	0.05
Fixtures	0.12	0.0003

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