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ORIGINAL ARTICLE

A CAD method for tolerance allocation considering manufacturing difficulty based on FMECA tool

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Abstract In a product life cycle, the tolerances present major impacts on the quality and cost of mechanisms. This paper presents an innovative methodology for tolerance allocation. The proposed approach allows tolerance integration in a computer-aided design (CAD) model, while considering functional and manufacturing requirements in an early phase of digital mock-up (DMU). The purpose is to consider the manufacturing process in the tolerance allocation using tools for the study and analysis of reliability of the design or the process, as the Failure Mode, Effects and Criticality Analysis (FMECA) and Ishikawa diagram. The results lead to the broadening tolerance values of difficult machined dimensions while respecting the functional requirements. Thus, the total cost of assembly decreases. The model is validated through a case study of tolerance allocation using various approaches.

Keywords CAD model · Tolerance allocation · Manufacturing difficulty, FMECA

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1 Introduction

In the design of mechanical systems, the tolerance is a critical task since it guarantees the functionality and quality of the mechanism while optimizing the production cost and respecting manufacturing tools. Tolerance is a communication support between the phases of design, manufacturing, and control. This key stage of the life cycle of a product is a subject that interests increasingly industrialists and researchers. Tolerance is the most important requirements in the design, manufacturing, and assembly product phases [1–3]. In practice, the consideration of the two above requirements is performed sequentially in two steps: the first step determines the tolerance (analysis, synthesis, and specification) according to functional requirements. The second consists in the distribution of those computed tolerances on manufacturing operation according to a process plan. These reveals are presented in several researches [4–8].

The digital mock-up (DMU), based on computer-aided design (CAD) model, must consider both the design and manufacturing requirements: assembly functionality and manufacturing process in the tolerance allocation methods. Thus, in this work, a method of tolerance allocation which considers difficulty coefficient deduced from CAD model is proposed.

This paper is organized as follows. In Sect. 2, a review of the literature is presented. Then, an overview of the proposed model is described as well as the methods and approaches used. In Sect. 3, a case study is presented and followed by a comparative study according to the cost criterion. In order to observe the real simulations, the MC simulation is used to analyze the allocated tolerances. To highlight contributions and limitations of the proposed method, a discussion is established. The conclusions and perspectives for this work are presented in Sect. 4.

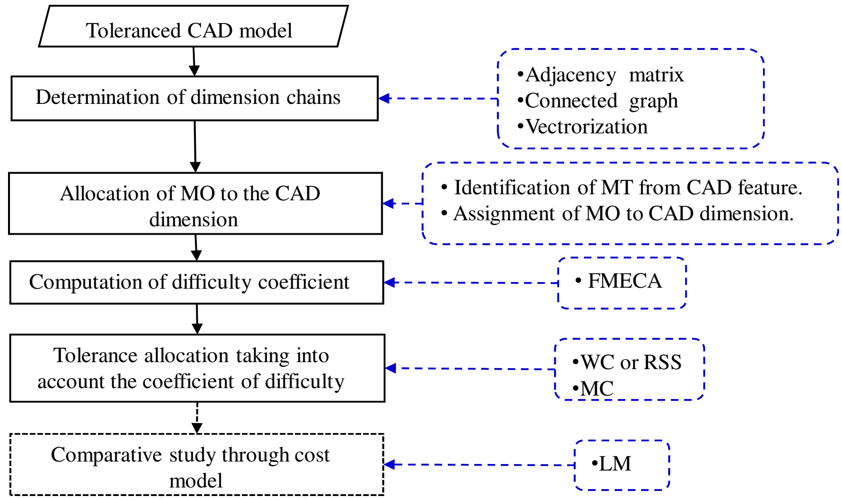


Fig. 1 Flowchart of the CADTM model

2 Related works

2.1 Tolerance analysis and synthesis

Three approaches are mainly used for tolerance, i.e., worst case (WC), statistical as root sum square (RSS), and inertial tolerancing. The rate of non-compliance (RNC) can be used to express quality requirements [9]. Hassani et al. [10] proposed a methodology to evaluate this defect probability for linear analytical expressions of functional characteristics using Monte Carlo (MC) simulation. Beaucaire et al. [11] developed a solution for tolerance analysis and synthesis based on ensuring the RNC by applying reliability methods. Taguchi et al. [12] established a quality loss (QL) function to evaluate the product quality. QL is quantified as a quadratic expression defining the loss of the product deviation. For the tolerance synthesis, called also tolerance allocation, the coupling between the design, manufacturing, and quality is required. Using tolerance allocation tools, a designer may distribute proper tolerances to a part or an assembly. An optimal tolerance allocation is a trade-off between the functional requirements and the manufacturing cost. During the early decades of the twentieth century, various manufacturing cost–tolerance models have been proposed [13–15]. Tolerance requirements impose the selection of machining processes, tools, and fixtures used for a product, required operator skill levels, setup costs, inspection standards, and rework. Several methods of tolerance allocation have been reported in references [16, 17]. The tolerance allocation research can be positioned belonging to two themes: (a) tolerance allocation methods considering the manufacturing process and (b) CAD model considering the tolerances.

Table 1 Tolerance expressions

Approaches	Formulas
WC	$t_i = \beta_i \times \frac{t_Y}{\sum_i \lvert\alpha_i\rvert \times \beta_i}$
RSS	$t_i = \beta_i \times \frac{t_Y}{\sqrt{\sum_i \alpha_i^2 \times \beta_i^2}}$

2.1.1 Tolerance allocation methods considering the manufacturing process

Advanced optimization techniques such as the genetic algorithm (GA), particle swarm optimization, colony algorithm, teaching-learning-based optimization (TLBO) algorithm, and bat algorithm (BA) are used as an optimization method for both the quality improvement and optimal tolerance allocation in many literatures [18–25]. Zong and Mao [26] proposed a methodology to obtain the least manufacturing cost and to reduce QL for the mechanism of multiple correlation characteristics. Dinesh et al. [27] presented a model dedicated to the concurrent optimization of design and manufacturing tolerances for a prismatic mechanism. Kumar et al. [28] proposed a recent approach using the Lagrange multiplier (LM) method which solves all the drawbacks of the methods reported in references [15, 18, 19, 23, 29–34] and considers the QL and minimization of RSS tolerance as objective functions proposed in references [26, 30, 33]. Wang et al. [34] introduced the variable coefficients reciprocal squared model (VCRSM) into the tolerance allocation process aiming to resolve the multi-constraints of an aircraft mechanism

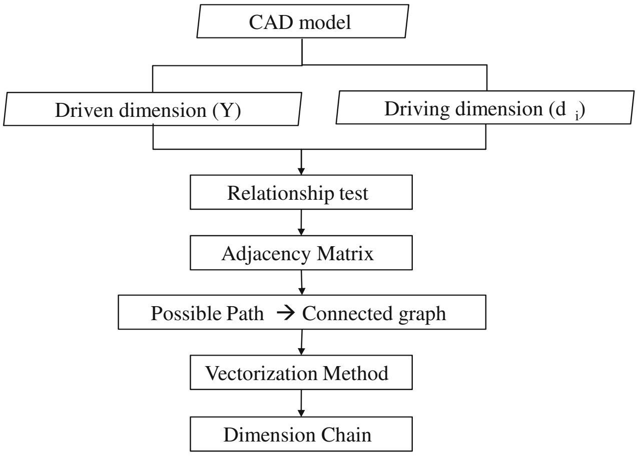


Fig. 2 Flowchart of the dimension chain determination

tolerance allocation problem and guarantee the quality and minimum cost. Based on the analysis of fuzzy factors in the tolerance allocation, different methods have been published in many literatures [36–39] for an optimal tolerance model. Liu et al. presented a method of tolerance grading allocation based on the uncertainty analysis of the remanufacturing assembly [40].

2.1.2 CAD model considering tolerances

Tlija et al. [41] established a method that takes into account the tolerances in the CAD model. The realistic parts are obtained by face displacements (translations and/or rotation). Those realistic faces are determined according to the small displacement torsor deduced from the geometrical tolerance. The model is a tool of the tolerance analysis while considering the assembly process planning and the contact types between parts. Anselmetti et al. [42] developed a "Clearances in Localization with Influence of the Contacts" (CLIC) model which is a computer-aided tolerancing (CAT) software based on three-dimensional computation. To determine the assembly specifications, CLIC model takes into account the assembly process planning specified by the designer.

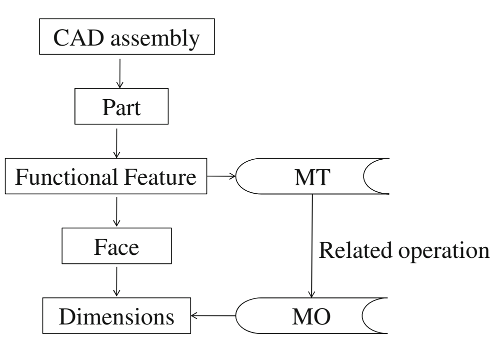


Fig. 3 Flowchart of the influencing MO identification

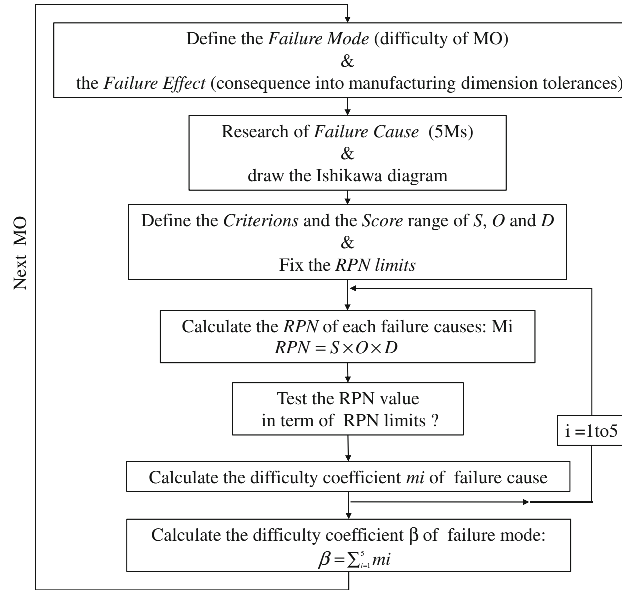


Fig. 4 Flowchart of FMECA principle

2.2 Limitations and research objectives

The major drawback of the general allocation methods considering the manufacturing process [15, 18–40] is that they neglect the difficult cost that reflects the impediment of the manufacturing operation to obtain machined dimensions on the one hand and are not integrated into the CAD model on the other hand. The limitation of previous CAD models [41, 42] that consider tolerances consists in neglecting the manufacturing process. Taking into consideration the improvements and inconveniences of the above methods, this paper proposes a new method of the tolerance allocation by weight factors called difficulty coefficient β. The proposed method is named CAD/tolerance integration based on the manufacturing process (CADTM). The β is computed using FMECA tool to evaluate the manufacturing operation feasibility. A higher β is assigned to manufacturing dimensions that are more difficult to obtain. Moreover, this new tolerance allocation method is integrated into the CAD model to improve the DMU of tolerance analysis and synthesis approaches, considering early

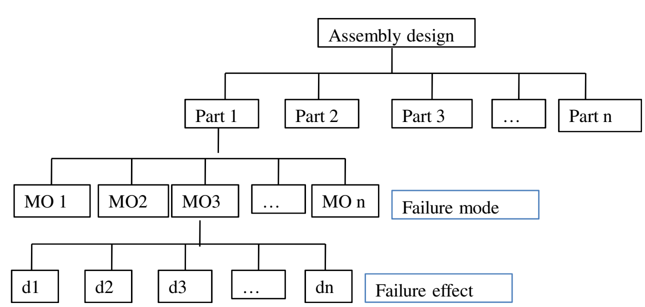


Fig. 5 Failure mode and effect

Table 2 Score range criterions

Score	Criterions	
Severity (S)		
Score 1	Minor	The tolerance interval is off 0.1%
Score 2	Average	The tolerance interval is off 0.5%
Score 3	Major	The tolerance interval is off 1%
Score 4	Catastrophic	The tolerance interval is off 5%
Occurrence (O)		
Score 1	Very unlikely	Less than once per year
Score 2	Remote	Less than once per month
Score 3	Frequent	Less than once per week
Score 4	Certain	More than once per week
Detection (D)		
Score 1	Obvious	Effective detection for preventive action
Score 2	Possible	The risk of non-detection in some cases
Score 3	Unlikely	Unreliable detection
Score 4	Impossible	No detection

the influence of the manufacturing process. Therefore, the originality and novelty of the CADTM model are the use of the FMECA tool to quantify the difficulty of manufacturing dimensions in a spirit of co-design: process-product-quality. Indeed, the main advantage of this model is that the designer considers, early in the CAD model, the important factors on the tolerance allocation. Previously, those factors were neglected or difficult to be evaluated quantitatively.

3 Proposed approach

3.1 Overview of the CADTM model

The essential steps of the proposed CADTM flowchart are reported in Fig. 1. In this figure, the approaches and tools used are indicated in the right side of the corresponding step. The CADTM steps are briefly defined below (additional detailed discussions are presented in the identified sections):

1. *Determination of dimension chains* (Sect. 3.2): The dimension chain is automatically determined. The dimension relationships between the functional requirement and component dimensions are obtained using a method based on the adjacency matrix and connected graph tools. The dimension chain is determined by a vectorization method.
2. *Allocation of MO to the CAD dimension* (Sect. 3.3): The manufacturing types (MT) are recognized from CAD feature. According to MT, manufacturing operations (MO) are assigned to the corresponding manufacturing dimensions.
3. *Computation of difficulty coefficient based on the FMECA procedure* (Sect. 2.4): The β is computed using the FMECA procedure for the difficulty evaluating of MO.
4. *Tolerance allocation taking into account the coefficient of difficulty*: The tolerance allocation is performed using WC and RSS approaches [43]. The dimension tolerances are

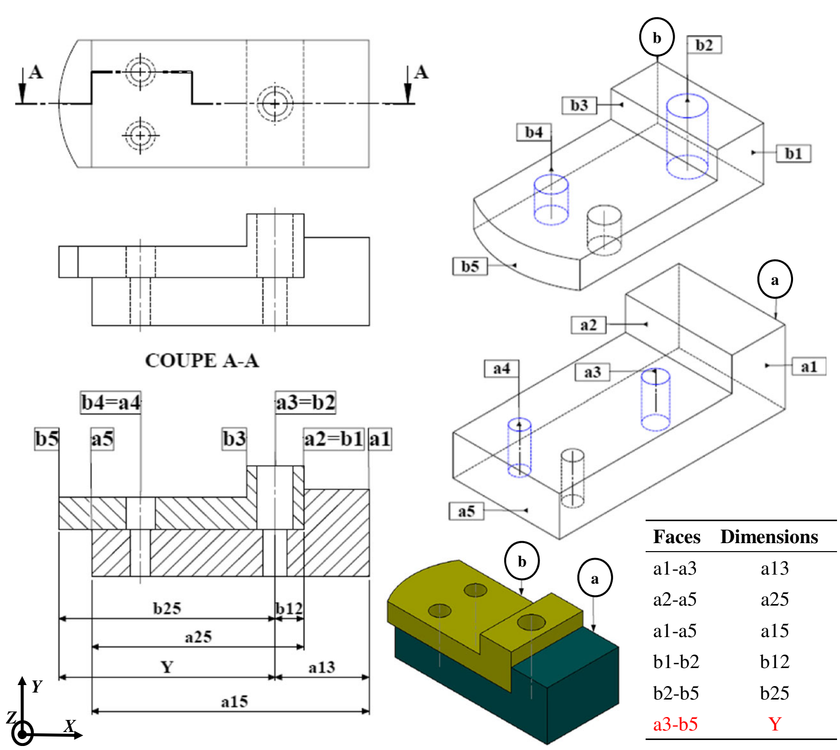


Fig. 6 Three-dimensional rotor key base assembly

	$a1$	$a2$	$a3$	$a4$	$a5$	$b1$	$b2$	$b3$	$b4$	$b5$
$a1$	0	0	1	0	1	0	1	0	0	0
$a2$	0	0	1	0	1	0	1	0	0	0
$a3$	1	1	0	0	0	1	0	0	0	2
$a4$	0	0	0	0	0	0	0	0	0	0
$a5$	1	1	0	0	0	1	0	0	0	0
$b1$	0	0	1	0	1	0	1	0	0	0
$b2$	1	1	0	0	0	1	0	0	0	1
$b3$	0	0	0	0	0	0	0	0	0	0
$b4$	0	0	0	0	0	0	0	0	0	0
$b5$	0	0	2	0	0	0	1	0	0	0

Fig. 7 Adjacency matrix of the rotor key base assembly

calculated in Table 1, where t_i is the dimension tolerance, t_Y is the tolerance of the functional requirement, α_i is the influence coefficient, and β_i is the difficulty coefficient. The MC method is used to simulate the allocated tolerance. In this paper, the sample number is supposed equal to 1000 iterations since it is sufficient to model the manufacturing dimension variations [45, 46] (Table 1).

5. *Comparative study through cost model* (Sects. 4.4 and 4.5): The computation of the assembly cost is a comparative step to mount the benefits of the proposed model. The LM method is proposed in this paper to the tolerance allocation by combining the cost minimization function and the functional requirements. This optional step is shown through the case study.

3.2 Determination of dimension chains

The CAD software uses two types of dimension: driving and driven dimensions. The tolerance of the functional dimension Y is defined on a driven one. The dimensions d_i forming the dimension chain are the driving dimensions. A method based on the adjacency matrix and the connected graph is implemented to automatically detect the relationship between dimensions using Matlab algorithm and the SolidWorks© Application Programming Interface (API). The flowchart in Fig. 2 illustrates steps of the dimension chain determination. An adjacency matrix [M], which represents the result of the relationship test step (Fig. 2), is expressed as follows: the [M] is a symmetric matrix with $(N \times N)$ size, where N is the number of nodes. The $M_{(i,j)}$ element represents a relationship between the i^{th} face and the j^{th} one. $M_{(i,j)}$ can have three possible attributes as follows:

- $M_{(i,j)} = 1$ if a driving dimension is between i and j,
- $M_{(i,j)} = 0$ if there is no relationship between i and j,
- $M_{(i,j)} = 2$ if a functional requirement is between i and j.

The main Brep items, faces, edges, and vertices, constitute graph nodes. The graph arcs are the driving and driven dimensions. Thus, the graph paths represent the relationships between dimensions. Based on the adjacency matrix, the connected graph which contains the Y-dimension is determined. The dimensions, which constitute the above connected graph, are modeled by vectors (Eq. 1). The relation in Eq. 1 is projected on the axes of the coordinate system. The dimension chain is expressed as Eq. 2. The influence coefficients α_i of the driving dimensions are deducted according to Eq. 3.

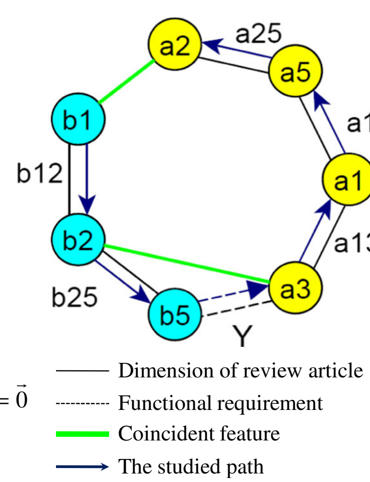
$$\overrightarrow{Y} = \sum_{i=1}^{n} \overrightarrow{d_i} \tag{1}$$

$$Y = \sum_{i}^{n} \alpha_i \times d_i \tag{2}$$

$$\alpha_i = \frac{\partial d_i}{\partial Y} \tag{3}$$

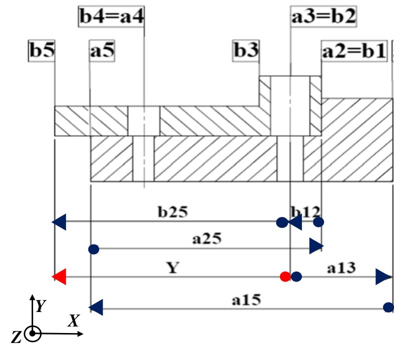
Fig. 8 Connected graph deduction

	$a1$	$a2$	$a3$	$a5$	$b1$	$b2$	$b5$
$a1$	0	0	1	1	0	1	0
$a2$	0	0	1	1	0	1	0
$a3$	1	1	0	0	1	0	2
$a5$	1	1	0	0	1	0	0
$b1$	0	0	1	1	0	1	0
$b2$	1	1	0	0	1	0	1
$b5$	0	0	2	0	0	1	0



$$\overrightarrow{b5a3} + \overrightarrow{a3a1} + \overrightarrow{a1a5} + \overrightarrow{a5a2} + \overrightarrow{b1b2} + \overrightarrow{b2b5} = \vec{0}$$

$$\overrightarrow{a3b5} = \overrightarrow{a3a1} + \overrightarrow{a1a5} + \overrightarrow{a5a2} + \overrightarrow{b1b2} + \overrightarrow{b2b5}$$



Projection on $\begin{cases} \vec{X}: Y = a15 + b12 + b25 - a25 - a13 \\ \vec{Y}: 0 = 0 \\ \vec{Z}: 0 = 0 \end{cases}$

Dimension chain: $Y = a15 + b12 + b25 - a25 - a13$

Dimensions	a13	a25	a15	b12	b25
α	-1	-1	1	1	1

Fig. 9 Dimension chain determination

3.3 Allocation of MO to the CAD dimension

Once the dimension chain of Y is determined, the sub-algorithm of MO allocation to the CAD feature begins. The feature recognition of the assembled parts is very important in design and manufacturing since this information is required for many decisions. Several CAD methods are established to identify the optimum assembly planning as in references [45, 46]. The recognition method depends on the modeling purpose. Thus, a new method for the automatic identification of MO from CAD, using SolidWorks© API, is established (Fig. 3). In the CAD model, machined face dimensions are identified and assigned automatically to MO which are supposed to have an effect on the tolerances' dimensions. Each function feature of a part is related to an MT. Then, the face dimensions are linked to the related MO of MT.

3.4 Computation of the difficulty coefficient based on the FMECA tool

The developed methodology quantifies the level of the manufacturing difficulty using a coefficient value β. A β value is assigned to each MO. This coefficient affects the corresponding component' dimension according to the FMECA method.

3.4.1 FMECA concept

The FMECA methodology is a widely recognized tool for the study and reliability analysis of a design or process. The FMECA is a bottom-up method to identify and analyze all potential failure modes of the various system parts, as well as the effects produced by failures and possible solutions. This method quantifies important parameters, identifies critical elements, and defines certain intervention priorities. The FMECA considers three parameters which are evaluated through interpreted linguistic expressions: (1) severity (S) which indicates the gravity of the effects of a failure mode, (2) occurrence (O) that denotes the probability of a failure occurring, and (3) detection (D) which measures a failure's visibility that is the attitude of a failure mode to be identified by controls or inspections. Each parameter is correlated to a score range (minimum of 1 to a maximum of 4 in this paper). The risk priority number (RPN) is defined as the product of these three parameters (Eq. 4).

$$RPN = S \times O \times D \tag{4}$$

The RPN assigns a weight to each considered failure mode. The higher value of RPN corresponds to the worst product reliability. In this case, modifications are required

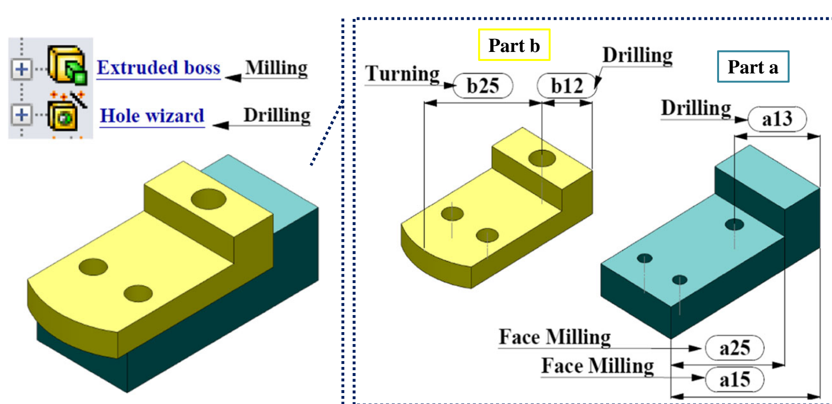


Fig. 10 MT recognition and extraction influencing MO

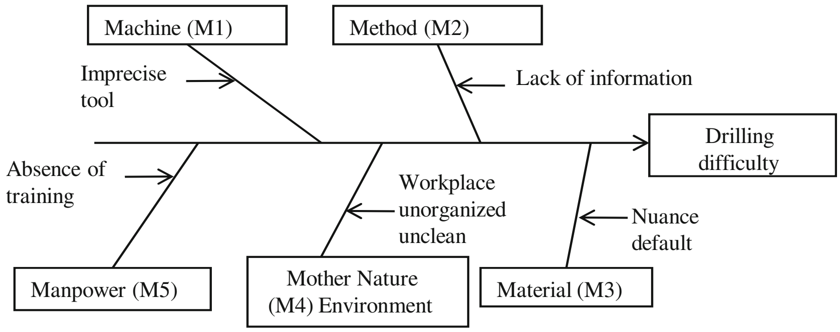


Fig. 11 Ishikawa diagram of the drilling difficulties

in order to avoid a failure's repetition. Concerning types of FMECA, the design and process FMECA are carried out, in this work, to focus on the problems stemming from difficulty of MOs that affect the driving dimension. The difficulty coefficient is estimated based on the RPN which is the criticality index for the failure mode. The most critical failure effect in this case is a looser tolerance of the machined dimensions. Hence, the possible corrective action is to consider the difficulty coefficient in the tolerance evaluation.

3.4.2 Procedure of the difficulty coefficient calculation

An overview of the proposed method to determine the difficulty coefficient using the FMECA approach is elucidated in Fig. 4 and described in the following:

1. Failure mode and effect: The failure mode is the difficulty of MO, and the failure effect is the consequences on the tolerance dimensions as shown in Fig. 5.
2. Failure cause: The different sources of the manufacturing defects are shown in the diagram of Ishikawa. The Ishikawa diagram, called also fishbone diagram, is a cause and effect diagram. This brainstorming tool is used to identify the potential causes for a performance problem: the MO difficulty. The main endogenous causes of the machined dimension failure are the 5 Ms: machine (M1), method (M2), material (M3), mother nature (M4), and manpower (M5).
3. FMECA parameters' elicitation: The proposed criteria and the score range of S, O, and D are illustrated in Table 2.
4. Computation of the β coefficient: The β is the aggregation of five parameters (m_i). Those parameters are assumed to be dependent to the RPN value and limits. The FMECA workgroup fix the RPN limits according to the manufacturing requirements. In the case of the studied example, the RPN limits are chosen equal to the following:

- Limit 1 = Score 2 × Score 2 × Score 2 = 8,
- Limit 2 = Score 2 × Score 3 + Score 2 × Score 4 = 14,
- Limit 3 = Score 2 × Score 3 × Score 4 = 24,
- Limit 4 = Score 4 × Score 4 × Score 4 = 64.

Therefore, the following equation (Eq. 5) expresses the β formulation.

$$\beta = \sum_{i=1}^{5} mi \quad \text{where} \quad m_i = \left\{ \begin{array}{l} \frac{1}{5} \ \text{if } 1 \leq RPN \leq 8 \\ \frac{1}{4} \ \text{if } 8 < RPN \leq 14 \\ \frac{1}{3} \ \text{if } 14 < RPN \leq 24 \\ \frac{1}{2} \ \text{if } 24 < RPN \leq 64 \end{array} \right\} \quad (5)$$

Operation	Failure mode	Effect	Failure cause		Evaluation				Action	mi	βi
					G	F	D	C			
Drilling	Difficulty of Drilling	Looser tolerance	*Machine*	Imprecise tool	4	1	2	8	*Calculate the difficulty level and suitable tolerance allocation*	1/5	
			Method	Lack of information	3	1	1	3		1/5	
			Materiel	Nuance default	4	2	1	8		1/5	
			Environment	Workplace unclean	2	3	2	12		1/4	
			Manpower	Absence of formation	3	2	2	12		1/4	1,10

RPN limit	8

Fig. 12 Proposed FMECA worksheet

Table 3 Influencing MO and related β

Driving dimensions	Tolerance notation	MO	β
a15	t_{a15}	Face milling	1.48
a25	t_{a25}	Face milling	1.48
a13	t_{a13}	Drilling	1.10
b25	t_{b25}	Turning	1.62
b12	t_{b12}	Drilling	1.10

These above steps are the data of the FMECA table (worksheet): A suitable FMECA worksheet for the analysis has to be decided in this work and to be updated for each MO. During the implementation of FMECA, the operators, leaders, and officials must be solicited as much as possible to collect the maximum information and circumvent any possible problems.

4 Case study

In this paper, the rotor key base assembly given by Sampath et al. in references [47, 48] is considered as the case study problem. This example is taken to use the proposed cost model. It is a simple mechanism of two components *a* and *b* as shown in Fig. 6. The functional requirement *Y* is between the axis of the cylindrical face *a3* and the face *b3* of parts *a* and *b*, respectively. A tolerance of 1.016 mm is required: $t_Y = 1.016$ mm. The contact between *a* and *b* is between *a2* and *b1* (*a2* = *b1*) which is not indicated in Sampath's article. In the CAD mechanism with the nominal configuration, if a coincident mate is defined between two faces as *a3* and *b2* in Fig. 6, the two faces are considered as the same dimension reference (*a3* = *b2*, *a4* = *b4*). However, this is not the real case.

4.1 Completing the FMECA table according to the CADTM model

4.1.1 Dimension chains of Y

According to references [47, 48], the five dimensions, *a13*, *a25*, *a15*, *b12*, and *b25* drawn in Fig. 8, constitute the dimension chains of *Y*. The adjacency matrix of rotor key base assembly is performed as shown in Fig. 7.

To simplify this adjacency matrix, the columns and lines (4, 8, and 9) are removed because they are null. Then, a possible path is obtained according to the simplified adjacency matrix using Matlab program as shown in Fig. 9: $m_{37} \rightarrow m_{31} \rightarrow m_{41} \rightarrow m_{42} \rightarrow m_{32} \rightarrow m_{37}$

Hence, this path is illustrated using the connected graph and a vectorial equation which models the driving and driven dimensions (Fig. 8).

The above relationship in Fig. 8 is projected on the axes (*X*, *Y*, and *Z*) to obtain the dimension chain of *Y* and α_i values as in Fig. 9.

4.1.2 Allocation of MO to CAD feature

The MOs which are face milling, drilling, turning, and drilling are associated with *a13*, *a25*, *a15*, *b12*, and *b25*, respectively. Here, the choice of Sampath in references [47, 48] is adopted to the design rotor key base assembly. For the CAD feature, the extruded boss of the parts is obtained by milling. A face milling is affected to the dimension *a15* and *a25* of part *a* which is obtained by an extruded boss (Fig. 10).

4.1.3 Computation of β

The example of the drilling operation that affects the dimensions *a13* and *b12* in the Fig. 10 is treated:

- Failure mode: The failure mode is the difficulty of the drilling operation.
- Failure cause: Fig. 11 shows the cause of the drilling difficulty problem.
- FMECA table (worksheet):

The FMECA worksheet of the drilling operation is elucidated in Fig. 12.

The same worksheet is completed for other influencing MO. Hence, Table 3 recapitulates the influencing MO of the driving dimensions and the related coefficients of difficulty after completing their worksheets.

4.1.4 Cost calculation

In this paper, the exponential form of cost function is used (Eq. 6). This formulation presents an easier manipulation and realistic results.

$$C(t) = C_0 \times \exp(-C_1 \times t) \quad (6)$$

where C_0 and C_1 are the two constants determined from the test data given as in Table 4 and based on the cost model given by Sampath et al. [47, 48].

Table 4 Values of cost-tolerance model

Cost-tolerance model	C_0	C_1
t_{a15}	431.5	17.64
t_{a25}	431.5	17.64
t_{a13}	27.84	3.661
t_{b25}	66.34	2.738
t_{b12}	27.84	3.661

Table 5 Tolerance and total cost results

Method\item	±Tolerance							
	Uniform allocation		LM		GA		Proposed	
	WC	RSS	WC	RSS	WC	RSS	WC	RSS
ta25	0.2032	0.4544	0.0399	0.2417	0.2273	0.2281	0.2218	0.4898
ta15	0.2032	0.4544	0.0399	0.2417	0.2272	0.2275	0.2218	0.4898
ta13	0.2032	0.4544	0.1855	0.1385	0.2591	0.2690	0.1648	0.3640
tb25	0.2032	0.4544	0.2492	0.2229	0.0647	0.0658	0.2428	0.5361
tb12	0.2032	0.4544	0.1855	0.1385	0.0569	0.0581	0.1648	0.3640
Total cost (€)	132.4719	74.1550	493.126	87.9397	110.0622	109.442	**88.2614**	**61.2558**

Thus, the assembly manufacturing cost C_m can be expressed as the summation of the driving dimension cost (Eq. 7).

$$C_m = \sum_{i=1}^{n} C_i(t_i) \tag{7}$$

Nevertheless, for the same tolerance allocation, i.e., all tolerances are equal. Equation 7 neglects the difficulty cost generated by the difficulty of influencing MO. Thus, β is assumed to represent the difficulty cost. Indeed, β allows that the dimension, which is more difficult to be machined, generates a higher manufacturing cost. So, in the case of the same tolerance allocation, the assembly manufacturing cost is defined by the equation (Eq. 8).

$$C_m = \sum_{i=1}^{n} \beta_i \times C_i(t_i) \text{ Where } \beta_i \text{ is the } t_i \text{ difficuly cost.} \tag{8}$$

The total product cost is the summation of the manufacturing cost C_m and QL (Eq. 9).

$$C_T = C_m + QL \tag{9}$$

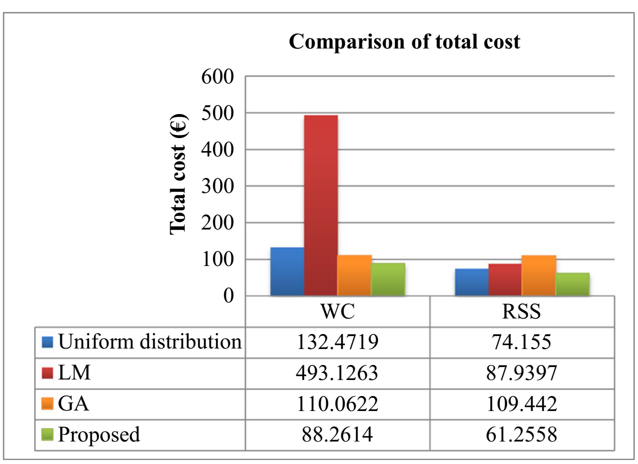


Fig. 13 Comparison of the total assembly cost

The QL is calculated according to Noorul et al. [49] as Eq. 10.

$$QL = \frac{A}{9t_y^2} \sum_{i=1}^{n} t_i^2 \tag{10}$$

4.1.5 LM method for the case study of the tolerance allocation

LM is a mathematical method for solving the optimization problems (Eq. 11), where ψ is the Lagrange's multiplier.

$$\frac{\partial}{\partial T_i}(\text{cost function}) + \psi \frac{\partial}{\partial T_i}(\text{constraints}) = 0 \tag{11}$$

Each individual tolerance, which provides a minimum exponential cost tolerance, can be determined in terms of t_1 in the following form.

- Statistical model

$$t_i = \frac{1}{C_{1i}} \text{lambertw}\left[\left(\frac{C_{1i}^2 \times C_{0i}}{C_{11} \times C_{01}}\right) \times t_1 \times \exp(C_{11} \times t_1)\right] \tag{12}$$

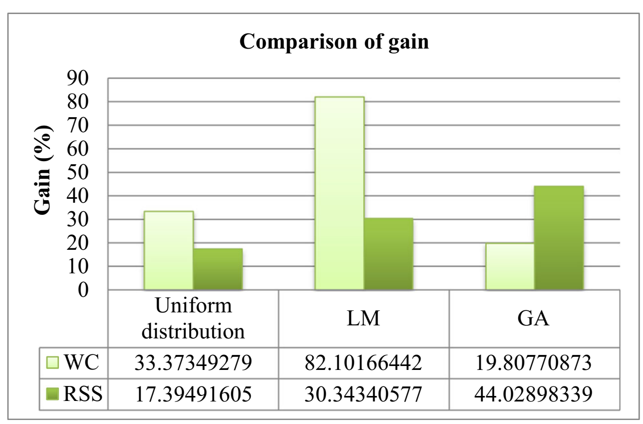


Fig. 14 Comparison of the gain

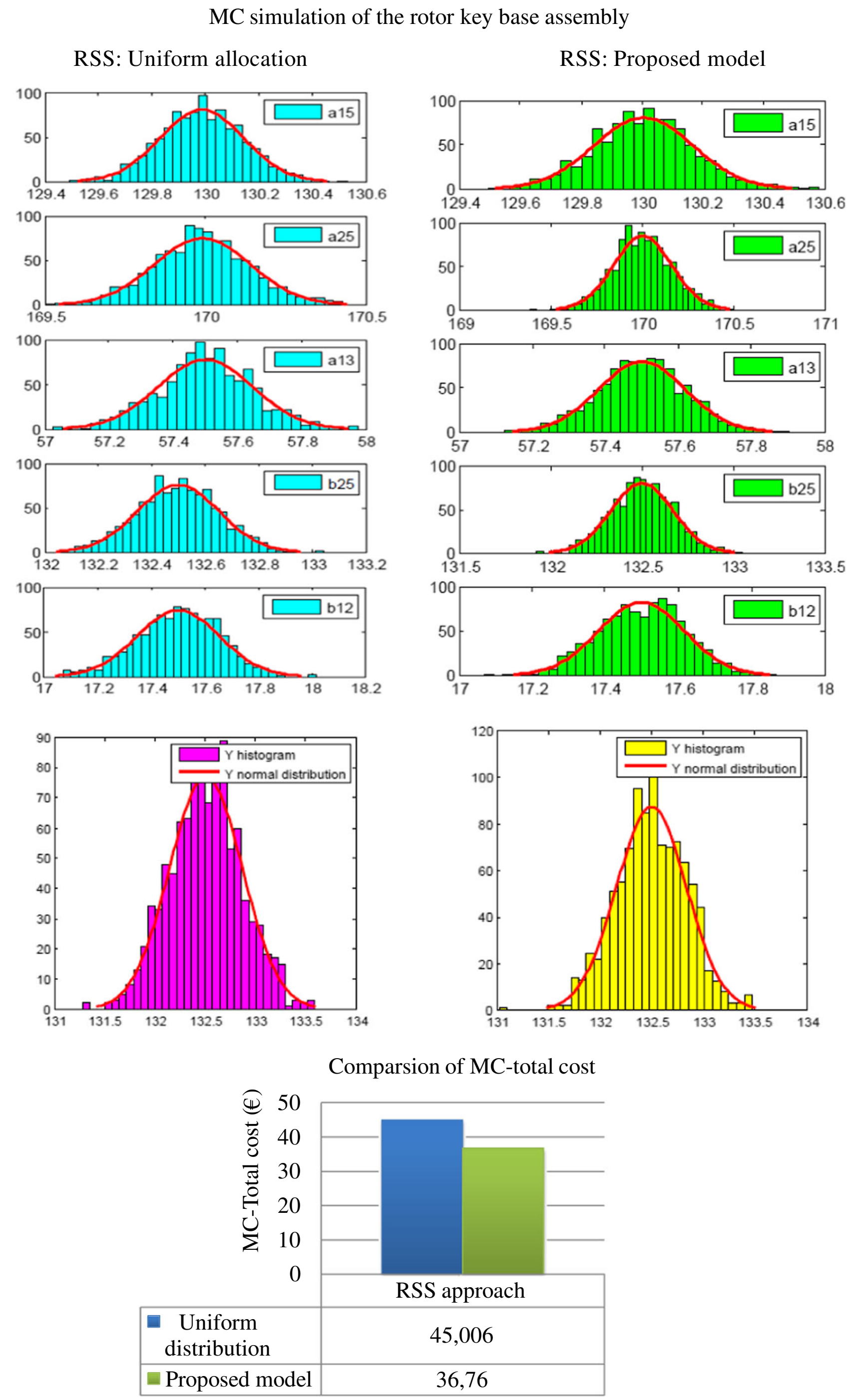


Fig. 15 MC simulation and total cost of the rotor key base assembly

where lambertw(x) is the Lambert's W function at the value x [50]. Consequently, the accumulated tolerance of the functional requirement can be calculated as Eq. 13.

$$t_Y^2 = t_1^2 + \sum \frac{1}{C_{1i}} \text{lambertw}\left[\left(\frac{C_i^2 \times C_{0i}}{C_{11} \times C_{01}}\right) \times t_1 \times \exp(C_{11} \times t_1)\right]; \text{i} = 1 \text{ to n} \quad (13)$$

- WC model

$$t_i = \frac{1}{C_{1i}} \times \left[C_{11} \times t_1 + \ln\left(\frac{C_{1i} \times C_{0i}}{C_{11} \times C_{01}}\right)\right] \quad (14)$$

$$t_Y = t_1 + \sum \frac{1}{C_{1i}} \left[C_{11} \times t_1 + \ln\left(\frac{C_i \times C_{0i}}{C_{11} \times C_{01}} \right) \right]; \mathrm{i} = 1 \text{ to n} \quad (15)$$

In the studied case, the tolerance t_{a25} is taken as a pivot parameter for assembly to solve the problem and to be substitute into the equations (Eqs. 13 and 15). For the WC and RSS study, t_{a25} is used to obtain the expression of other tolerances according to the equation (Eqs. 12 and 14) where the parameters of cost values are shown in Table 4.

4.2 Results and discussion

The tolerance values and total assembly cost obtained by RSS and WC approaches for the driving dimensions of the rotor key base assembly are shown in Table 5. Those results are computed in the case of the uniform tolerance allocation (β = 1), LM, and proposed (CADTM) methods. The tolerance results using the GA optimization techniques are given according to references [48, 49]. The proposed method allowed to obtain optimal tolerances that satisfy the assembly functional requirement and widen the difficult manufacturing dimensions. For example, *a25*, which has β = 1.48, is more difficult than *a13* which has β = 1.10. So, t_{a25} is upper than t_{a13} (t_{a25} = 0.2218 > t_{a13} = 0.1648) using WC approach as illustrated in Table 5. Therefore, the manufacturing dimensions obtained become easier, thus respecting the functional requirement tolerance which improves both the cost and quality.

In fact, compared to the uniform allocation, LM, and GA methods, the proposed model based on β tolerance allocation reduces significantly the total cost. Figure 13, which compares the total assembly cost for the above methods, proves this verdict. The total assembly costs achieved using the CADTM model are 88.261 € and 61.256 € for WC and RSS approaches, respectively. Those cost values are more economical than cost results obtained using uniform, LM, and GA allocation methods. Therefore, the proposed method guarantees an important gain: for example, a gain of 44.029% is obtained compared with GA results as shown in Fig. 14.

Figure 15 shows two results of MC simulation based on the uniform allocation and proposed methods. MC total cost results confirm that the CADTM model ensures a total cost reduction as given in Fig. 15.

Therefore, taking into account the manufacturing dimensions, difficulty for the tolerance allocation and CAD model simultaneously provides an important gain in terms of cost and quality in the framework of co-design.

5 Conclusions

This paper presents an original CAT model considering the manufacturing process. The proposed CADTM model solves the tolerance allocation problems with regard to the manufacturing dimension difficulty. The later is quantified by a difficulty coefficient calculated using the FMECA method. The dimension chain of the mechanism that is functional requirement is obtained automatically in the CAD model using a graph tool and a dimension vectorization method. The types and the difficulty levels of MOs are determined from the CAD model. A case study shows that this methodology takes into account the manufacturing process in an early stage of the product life cycle in order to widen the manufacturing dimension tolerances which pose difficulty. Based on analysis results, the tolerance allocation using the CADTM model reduces total assembly cost considering quality loss. Ultimately, the CADTM model is about the tolerance allocation coupling between the CAD integration and difficulty quantification of the manufacturing dimensions. The model is based on the FMECA tool in the context of co-design. But, the FMECA integration on the field is too long which penalizes the CADTM methodology.

Future works will focus on the FMECA incorporation on the diverse industrial products for a comparative study coupled with experimental results and consideration for geometrical tolerances.

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