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ORIGINAL ARTICLE

Design of an order-picking warehouse factoring vertical travel and space sharing

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Abstract This paper considers the layout design problem of a single block order-picking rack-based warehouse that employs turnover-based storage assignment in both vertical and horizontal dimensions. An analytical expression for vertical travel distance is derived which is incorporated in the pick distance model. The effect of inventory staggering on storage space requirement is considered in arriving at warehouse dimensions. A model that incorporates area cost along with handling cost in optimizing warehouse design is developed and a solution algorithm is presented. The analytical model for vertical travel and the optimization model are applied to data from a real life case. It was found that the model would offer considerable operational cost savings, especially when space costs are high. Computational experiments show that the effect of inventory staggering is quite significant in the estimated storage space. Experiments also demonstrated the importance of segregating products based on turnover in the vertical dimension.

Keywords Order-picking warehouse · Three-dimensional layout design · Vertical travel · Inventory staggering

1 Introduction and relevant literature

Warehouses form important parts in the supply chain of various products. Apart from matching supply with demand fluctuations and consolidating product for smooth logistics, warehouses are increasingly used to implement delayed differentiation and value addition [2]. It is important that warehouses are given due attention as they would impact the efficiency and effectiveness of supply chain of any manufacturing company.

1.1 Warehouse layout design

Design of a warehouse is a crucial task and involves some of the most important decisions in warehouse management that has far reaching cost impacts. Rouwenhorst et al. [39] observe that the operational costs incurred in a warehouse are mostly determined at the stage of design itself. Since design involves sizeable capital expenditure, it would be very difficult to change once the warehouse is actually built. Layout design, which involves determination of length, width, height, aisle width, position of Pickup/Deposit (P/D) point, etc., is an important element of warehouse design and has received attention among researchers for the last few decades. Berry [5] compares two warehouse arrangements—block stacking and pallet racks on volume requirements and handling costs. Bassan et al. [3] develops an analytical model to decide the dimensions of a rectangular unit-load warehouse and a zoned warehouse. Park and Webster [31] develop a model that captures different kinds of costs in designing a three-dimensional warehouse and illustrates how to compare design alternatives using the model. Yoon and Sharp [43] proposes a stage-by-stage procedure for analysis and design of order-picking system using order characteristics and system parameters. Roodbergen and Vis [37] develops analytical expressions for travel length for two different routing heuristics and use the expressions to optimize the layout of a warehouse. Önüt et al. [30] proposes a particle swarm optimization algorithm for designing a multi-level unit-load warehouse employing class-based storage. A detailed review of warehouse design literature is available in Gu et al. [17], and De Koster et al. [10]. Baker and Canessa

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[1] observes that warehouse literature lacks a comprehensive systematic method for designing warehouses. Similar observations are found in other studies also, notably Rouwenhorst et al. [39] and Goetschalckx et al. [14].

Now, we introduce the important elements of warehouse layout design problem considered in this paper. A three-dimensional order-picking warehouse of interest is shown in Fig. 1 in which the pickup/deposit (P/D) point is located at one corner and picking aisles are rectangular. As seen in Fig. 1a, the products are stored in a way that their turnover decreases from aisle to aisle as one moves away from P/D point, similar to the "within-aisle" policy given by Petersen [32].

The traditional within-aisle policy assumes random storage within each aisle. However, this paper does not make such an assumption. The products inside an aisle are further segregated turnover based on the height dimension as shown in Fig. 1b, to achieve potential pick distance benefit in vertical direction as well. Simpler versions of such a storage policy exist in modern warehouses. For example, in "Forward–Reserve" warehouses, high turnover products are placed on the lower levels of a pallet rack (see [2], for example).

There have been several studies describing warehouses that employ turnover-based storage policies in the horizontal direction (e.g., [12, 16, 23]). In addition, we also find studies that describe turnover-based slotting in automated storage/retrieval systems (e.g., [15, 40]). One can find detailed reviews of such location assignment policies in Van Den Berg [42] and Gu et al. [18]. There are no studies on order-picking rack-based warehouses that employ turnover-based storage arrangement in vertical direction.

Accurate estimation of space required to store products is crucial to a good warehouse design. Dedicated storage policy allocates maximum inventory for each product that is to be stored in an aisle. There is potential space saving because of the possibility of staggering in product inflow and hence it may not be necessary to allocate space for maximum inventory for all the products within an aisle. This "space-sharing" effect can be found in some studies that explain class-based storage policy (e.g., [28, 41]). However, this effect has been largely ignored in warehouse literature modeling travel time. A recent study by Yu et al. [44] provides an expression for the average space requirement for each class considering the sharing effect of inventory. It is demonstrated that using more number of classes is not necessarily better as the effect of sharing diminishes with a reduction in the class size. Guo et al. [19] uses similar expression in their study to analyze the impact of storage space performance of different storage policies.

Our review of literature reveals that this sharing effect on warehouse space requirement has not been considered in studies that deal with the design of three-dimensional warehouses. Hence, one objective of this study is to include this in the calculation of 'Required Storage Space', which forms a crucial input to the design algorithm.

1.2 Travel distance/time

Travel distance depends upon warehouse physical characteristics such as aisle structure, position of P/D point, and operation policies such as storage and picking policies. Travel time/distance models in warehouses are found in several studies considering various storage policies, routing policies, and warehouse configurations. The earlier studies in modeling pick travel in order-picking warehouses assume randomized storage (for examples, see, [13, 20, 37]). Some studies (see, [8, 12, 26]) consider order batching and estimate travel time. Caron et al. [7] presents close form expressions for travel distances under full turnover policy with return and largest gap routing methods, for low-level picker to part systems. Rao and Adil [35] gives an exact analytical expression for travel distance in the case of low-level picker to part systems when the storage policy is turnover based and routing policy is

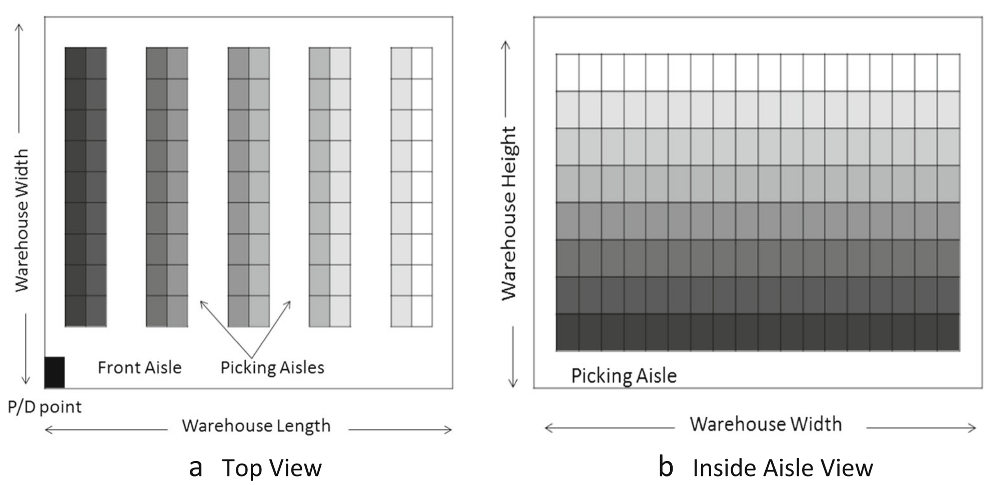

Fig. 1 A three-dimensional layout of a warehouse which employs turnover-based storage policy in horizontal and vertical dimensions. Progressively *darker shades* represent locations that are accessed more frequently

traversal. Similar studies in low-level systems have been found for class-based storage policy either using rectilinear distance measures [4, 25] or Chebyshev distance measures in the case of AS/RS systems [21, 36, 38]. In a recent paper, Khojasteh and Son [24] analyze an S/R system with independent vertical and horizontal movements to develop a heuristic to minimize the travel time.

Interaction between storage, routing, batching and order-picking efficiency has been of interest to researchers. Petersen and Aase [33] use simulation to understand the interaction between different storage and routing policies and conclude that class-based policy improves pick distances by a significant amount. For other examples, see Manzini et al. [27] and Hsieh and Tsai [22].

Most studies, barring the ones that consider AS/RS systems, are concentrated on low-level systems where the vertical travel is ignored. However, in reality, warehouses can be more than several stories high where the picker spends considerable time traveling in the vertical dimension as well. The existing travel distance/time models are however inadequate to explain situations when the vertical component of pick travel is not negligible, as is the case in some order-picking systems. This research gap is bridged in this paper, and a suitable model is proposed.

1.3 Space cost in warehouse layout design

Most papers consider only handling cost in the warehouse design, particularly when order picking is considered. Muppani and Adil [29] argue that storage space cost needs to be considered along with picking cost to arrive at class partitions in designing a warehouse with given stacking height. Such an approach would be even more relevant for designing a three-dimensional warehouse. Increasing the height of the warehouse would mean lesser floor area consumed and therefore lesser rent paid. However, this comes at the cost of increased vertical travel time. This means that warehouses located in locations with higher area cost, like metropolitan cities, would have to build taller racks than usual. Warehouse located in regions with higher labor cost would have to build shorter racks than in other areas (assuming vertical travel is slower than horizontal). In addition, staggering of product inflow changes the space required for storing the same number of pallets when one switches between different storage policies. For example, if one were to follow a shared storage policy, the amount of space required would be far less than what is required for turnover-based storage policy. But the latter would reduce the average pick distance because products that are demanded more frequently are placed closer to the retrieval point. Thus there are some complex interactions between these factors that must be taken into account and modeled while deciding the layout of the warehouse.

Rakesh and Adil [34] consider area cost and labor cost in determining the optimal dimensions of a warehouse that employs random policy. This paper extends the concept to warehouses that arrange products based on turnover both in the horizontal as well as vertical directions. This study also considers the effect of space sharing to arrive at an ideal design.

In short, the following points describe the key literature gaps identified and our contributions in this paper towards bridging the identified gaps.

1. Most design studies in order-picking warehouse consider only handling cost as objective. This paper brings the area cost explicitly as a part of objective function along with material handling cost in deciding the layout dimensions.
2. The papers that describe turnover-based storage policy in the case of a rack-based warehouse do not consider turnover-based product arrangement in vertical dimension. In this paper, we explore such a possibility in the light of reduction in material handling cost.
3. There are no analytical models available when an S-shaped product routing is followed and the warehouse has products segregated on the basis of turnover in the vertical dimension. This paper presents the derivation of an analytical model that incorporates such a vertical travel as well.
4. The existing design studies ignore the finiteness of number of products in estimating space, which lead to underestimation. This study incorporates the space-sharing factor in the design model.

The remainder of the paper is organized as follows. The mathematical model including different cost components is developed in Sect. 2, followed by a solution algorithm for the model in Sect. 3. Section 4 applies the model to real-life case data. Computational experiments are presented in Sect. 5, along with designer insights. Section 6 gives the summary and conclusions.

2 Problem description and mathematical model

The problem involves designing a three-dimensional order-picking rectangular warehouse, with picking aisles parallel to each other and perpendicular to the front aisle. The P/D point is located at the front left corner of the warehouse. The warehouse has a single block with no cross aisles.

The warehouse employs a turnover-based arrangement of products both in the horizontal as well as vertical dimensions. The products are first arranged according to turnover in the horizontal dimension following within-aisle storage policy (see, Fig. 1a). Later, within each aisle, the products are further arranged in the decreasing order of turnover in the height dimension (see, Fig. 1b). Horizontal travel employs traversal routing (Fig. 2a) and vertical travel employs return routing (see Fig. 2b).

2.1 Assumptions

The main assumptions made in this paper about the warehouse operating conditions are as follows.

1. All storage slots of the warehouse are equal in height, width, and depth.
2. All shelves in the warehouse are of same height and width and are constrained by physical limitations of the building.
3. The picking aisles are equal in width.
4. The forklifts have their forks in resting position while moving horizontally and rise only when it reaches the desired location. Such lifts are considered by various studies; the latest to our knowledge being Cardona et al. [6].
5. Within a particular aisle and at a particular vertical level, the products are placed randomly on the available lanes. This is because once an aisle is entered by a picker, it needs to be traversed fully. Segregating products based on turnover in this direction, thus, would not lead to additional reduction in pick distance.
6. The warehouse follows an inventory policy wherein each product is stocked up to a certain number of days of demand.
7. Space-sharing effect due to inventory staggering for products stored in different aisles is negligible. Further, for this purpose turnover variation in vertical dimension is assumed to have negligible effect on storage space sharing.

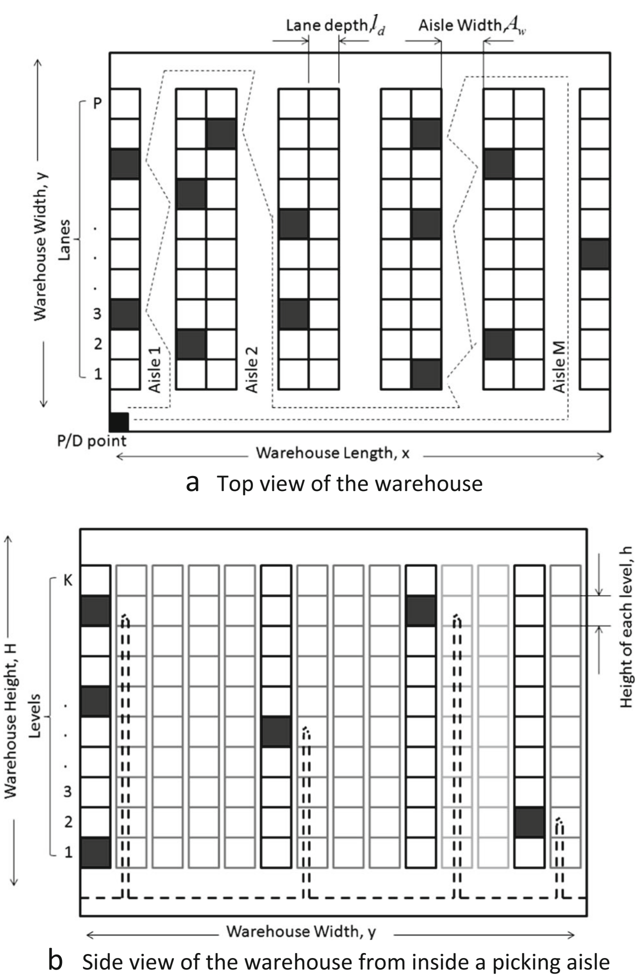

Fig. 2 Organization of picking aisles and racks of the warehouse

The notations used in the mathematical model are given in Table 1.

2.2 Horizontal travel cost

For a within-aisle storage policy assumed in this study, the cumulative of fraction of picks till a particular aisle m can be expressed as in Eq. (1), similar to Eynan and Rosenblatt [11],

$$G(m) = \left(\frac{m}{M}\right)^s \tag{1}$$

Using Eq. (1), the probability of a pick occurring in a particular aisle m can be expressed as

$$p(m) = G(m) - G(m-1) = \left(\frac{m}{M}\right)^s - \left(\frac{m-1}{M}\right)^s \tag{2}$$

The expected traversal distance (as in y direction in Fig. 2a) can be expressed following approach similar to Rao and Adil [35], as

$$\mathrm{HD}_{\mathrm{trav}} = P\,w \sum_{m=1}^{m=M} \left(1 - \left(1 - \left(\left(\frac{m}{M}\right)^s - \left(\frac{m-1}{M}\right)^s\right)\right)\right) \tag{3}$$

The quantity under summation in Eq. (3) represents the expected number of different aisles visited. It is to be noted that if an aisle has to be entered even for picking just one product, it needs to be traversed entirely as per the traversal routing policy.

Similarly, the expected longitudinal travel distance (as in x direction in Fig. 2a) can be expressed as

$$\mathrm{HD}_{\mathrm{long}} = 2(2l_d + A_w) \sum_{m=1}^{M} m\left(\left(\frac{m}{M}\right)^{sN_p} - \left(\frac{m-1}{M}\right)^{sN_p}\right) \tag{4}$$

Table 1 Notations used in the model

Indices	
$i = \{1, 2, 3 \ldots P_g)$	Product index for each SKU
$i' = (0, 1]$	Normalized product index for each SKU
$m = \{1, 2, 3 \ldots M\}$	Index for aisle number
$k = \{1, 2, 3 \ldots K\}$	Index for vertical storage levels
Parameters	
s	Demand skew factor
C_l	Cost of labor per hour per person
C_r	Rent per square feet per hour
V_h	Horizontal velocity of order pickers
V_v	Vertical velocity of order pickers
P_g	Number of SKUs
D_i	Demand of a product i
Q_i	Order quantity of a product i
w	Width of each storage location
h	Height of each level
l_d	Depth of each shelf
A_w	Aisle width
N_p	Average number of stops per order-picking tour
ε	Space-sharing factor
$P_{\min}$ and $P_{\max}$	Minimum and maximum allowable values for number of lanes per aisle
$K_{\min}$ and $K_{\max}$	Minimum and maximum allowable values for number of vertical storage levels
Decision variables	
x	Length of the warehouse
y	Width of the warehouse
H	Height of the warehouse
M	Number of picking aisles
P	Number of lanes in each aisle
K	Number of vertical storage levels
Other variables	
$G(m)$	Cumulative fraction of picks till a particular aisle m
$G(i')$	Cumulative fraction of demand till a product index i'
A_{Total} and AC	Total floor area and area cost
HD and HDC	Horizontal travel distance and cost
VD and VTC	Vertical travel distance and cost

The quantity inside parentheses in Eq. (4) represents the probability of each aisle being the farthest picking aisle. The correction for odd number of aisles in the analytical model for traversal routing is ignored. Such correction term exists in studies like Roodbergen and Vis [37], Chew and Tang [8], and Rao and Adil [35]. This is ignored to reduce the complexity of the model. However, the solution algorithm would be unaffected if one wishes to include this factor as well, for more accurate results.

The expected horizontal travel distance per order given by Eq. (5) would be the sum of Eqs. (3) and (4) above.

$$\mathrm{HD}_{\text{order}} = \mathrm{HD}_{\text{trav}} + \mathrm{HD}_{\text{long}} \tag{5}$$

The total horizontal distance is obtained by multiplying the quantity obtained in Eq. (5) with the average number of orders during a specific time period, i.e.,

$$\mathrm{HD} = \mathrm{HD}_{\text{order}} \times \frac{\sum_{i=1}^{P_g} D_i}{N_p} \tag{6}$$

The horizontal travel cost is the total cost of labor incurred for the movement of order picker vehicles on the floor of the warehouse and it can expressed as in (7) which is the product of (6) and travel cost rate.

$$\mathrm{HTC} = \frac{C_l}{V_{\mathrm{hor}}} \times \mathrm{HD} = \frac{C_l}{V_{\mathrm{hor}}} \times \frac{\sum_{i=1}^{P_g} D_i}{N_p} \left(P\,w \sum_{m=1}^{m=M} \left(1 - \left(1 - \left(\left(\frac{m}{M}\right)^s - \left(\frac{m-1}{M}\right)^s \right) \right) \right) + 2(2l_d + A_w) \sum_{m=1}^{M} m \left(\left(\frac{m}{M}\right)^{sN_p} - \left(\frac{m-1}{M}\right)^{sN_p} \right) \right) \tag{7}$$

2.3 Vertical travel cost

Vertical travel in warehouses can happen either sequential or parallel with the horizontal movement. Clark and Meller [9] classify the kinds of picker movement scenarios as (i) rectilinear scenario, where the lift truck can move only in one direction, either vertical or horizontal, at a time; (ii) Chebyshev scenario, where the lift truck can raise its forks and travel horizontally simultaneously; and (iii) a compromise scenario. Where the Chebyshev motion is restricted to one aisle.

In this paper, the rectilinear scenario is considered similar to the assumption in Cardona et al. [6]. The warehouse employs order picking with man on board order pickers that do not have simultaneous horizontal/vertical movements due to technical characteristic of vehicle and safety considerations. When an item needs to be picked from higher level, the picker moves along the aisles to the floor position below the location of the item. Thereafter, it rises up the level from which the product needs to be picked. The same model can be used for warehouses that use manual order picking where the worker moves to the appropriate floor position and uses an implement like stacker to pick products in the vertical dimension. We are aware of at least one warehouse that employs this kind of order picking.

Travel models for estimating vertical distance for order pickers are not available in literature for turnover-based policy. However, this is a crucial component in understanding the operational cost of the warehouse. We derive the vertical travel of an order-picking warehouse as below. For the kind of picking system assumed in this paper, vertical travel can be well approximated by 'return routing' as shown in Fig. 2b.

Given that there are N_p stops in an order-picking tour, the average number of stops occurring in a particular aisle m and N_p^m, using Eq. (2) can be expressed as

$$N_p^m = \sum_{i=1}^{N_p} i \times {}^{\mathrm{Np}}C_i \times p(m)^i \times (1-p(m))^{N_p-i} \tag{8}$$

where ${}^{\mathrm{Np}}C_i$, the combination function, is the number of ways of selecting i stops from a total of N_p stops.

Within each aisle, the products are segregated based on turnover in the vertical dimension and in a random fashion in the width dimension. This implies that each of the P lanes within an aisle would have an equal probability of pick. The average number stops (T^m) at lanes within an aisle m can be expressed as

$$T^m = P\left(1-\left(1-\frac{1}{P}\right)\right)^{N_p^m} \tag{9}$$

The average number of picks at any particular stop within aisle m can be expressed as

$$N_{\text{per stop}}^m = \frac{N_p^m}{T^m} \tag{10}$$

Since the travel in the vertical dimension involves return routeing and the products are arranged in the order of non-increasing turnover in the dimension of height, the average distance in the vertical dimension for each stop in aisle m can be expressed as in Eq. (11)

$$\mathrm{VD}_{\mathrm{stop}}^m = h \times \sum_{i=1}^{i=K} i \times \left(\left(\frac{i}{K}\right)^{N_{\text{per stop}}^m} - \left(\frac{i-1}{K}\right)^{N_{\text{per stop}}^m} \right) \tag{11}$$

The quantity given in Eq. (11) above gives the average vertical distance traversed per stop, which happens for an average of T^m stops per aisle as given in Eq. (9). This has to be summed across each aisle. Hence expected vertical distance for the total order-picking tour can be expressed as follows

$$\mathrm{VD}_{\mathrm{order}} = \sum_{m=1}^{M} T^m \times \mathrm{VD}_{\mathrm{stop}}^m \tag{12}$$

Similar to Eqs. (6) and (7), the expected vertical distance through the entire order-picking tour and the total cost of vertical travel can be expressed respectively as

$$\mathrm{VD} = \mathrm{VD}_{\mathrm{order}} \times \frac{\sum_{i=1}^{P_g} D_i}{N_p} \tag{13}$$

$$\mathrm{VTC} = \frac{C_l}{V_v} \times \frac{\sum_{i=1}^{P_g} D_i}{N_p} \times \sum_{m=1}^{M} \left(T^m \times h \times \sum_{i=1}^{i=K} i \times \left(\left(\frac{i}{K}\right)^{N^m_{\text{per stop}}} - \left(\frac{i-1}{K}\right)^{N^m_{\text{per stop}}} \right) \right) \tag{14}$$

The total pick cost during the planning horizon is the sum of horizontal and vertical travel costs expressed in equations.

2.4 Area cost

There is a possibility of sharing storage space between different products whose inflow is staggered. Yu et al. [44] gives an expression for the average space required for a product within a particular class, as

$$\alpha_i(\mathrm{Num}_i) = 0.5\left(1 + \mathrm{Num}_i^{-\varepsilon}\right) Q(i) \tag{15}$$

Where $\alpha_i(\mathrm{Num}_i)$ is the space required to stock product 'i', given there are a total of Num_i items in the class which contains this particular product i. $Q(i)$ is the order quantity of the product and ε is the space-sharing factor which would take a value of 1, if the inventory replenishments are perfectly coordinated and staggered. However, for all practical purposes, Yu et al. [44] observe that the value of ε can be considered to vary between 0.15 and 0.25.

In the model considered in this paper, each aisle is treated as a class. The available storage space in any aisle, $2 \times P \times K$, must match with the quantity of products stored in the aisle, n_m, as given by Eq. (16)

$$2 \times P \times K = 0.5\left(1 + n_m^{-\varepsilon}\right) \sum_{i=1}^{n_m} Q(m,i) \tag{16}$$

Where $Q(m,i)$ is the order quantity of the ith product in the aisle m and where number of products within the aisle,n_m.

Thus, the total space requirement for placing all the products is obtained by summing Eq. (16) across all aisles

$$I = \sum_{m=1}^{M} \left(0.5\left(1 + n_m^{-\varepsilon}\right) \sum_{i=1}^{n_m} Q(m,i) \right) \tag{17}$$

The warehouse needs to accommodate the entire inventory as obtained in Eq. (17). Therefore, dimensions of the warehouse should be such that the total storage volume of the warehouse equals the total inventory storage requirement as shown in Eq. (18).

$$2 \times M \times P \times K = \sum_{m=1}^{M} \left(0.5\left(1 + n_m^{-\varepsilon}\right) \sum_{i=1}^{n_m} Q(m,i) \right) \tag{18}$$

Also, all the products would have to be placed in the warehouse

$$\sum_{m=1}^{M} n_m = P_g \tag{19}$$

The total floor area required for the above configuration can be given as

$$A_{\text{Total}} = P \times M \times w(2l_d + A_w) \tag{20}$$

And the resultant area cost would be

$$\mathrm{AC} = C_r \times P \times M \times w(2l_d + A_w) \tag{21}$$

2.5 Total cost of operations

The total cost of operations, which is a function of M, P, and K can be expressed as a sum of horizontal travel cost (HTC) (Eq. 7), vertical travel cost (VTC) (Eq. 14), and area cost (AC) (Eq. 21)

$$\begin{aligned} f(M,P,K) = {} & \frac{C_l}{V_{\text{hor}}} \times \frac{\sum_{i=1}^{P_g} D_i}{N_p} \Bigg(P\, w \sum_{m=1}^{m=M} \left(1 - \left(1 - \left(\left(\frac{m}{M}\right)^{s} - \left(\frac{m-1}{M}\right)^{s} \right) \right) \right) \\ & + 2(2l_d + A_w) \sum_{m=1}^{M} m \left(\left(\frac{m}{M}\right)^{sN_p} - \left(\frac{m-1}{M}\right)^{sN_p} \right) \Bigg) \\ & + \frac{C_l}{V_{\text{ver}}} \times \frac{\sum_{i=1}^{P_g} D_i}{N_p} \times \sum_{m=1}^{M} \Bigg(St^m \times h \times \sum_{i=1}^{i=K} i \\ & \times \left(\left(\frac{i}{K}\right)^{N^m_{\text{per stop}}} - \left(\frac{i-1}{K}\right)^{N^m_{\text{per stop}}} \right) \Bigg) \\ & + C_r \times P \times w \times M(2l_d + A_w) \end{aligned} \tag{22}$$

2.6 Optimization model

Using the developments of the previous sections, an optimization model, "OPTDIM," can be formulated with the objective to minimize the total cost of operations. The decision variables are

- The number of aisles required (M)
- The number of lanes within each aisle (P), and
- The number of vertical storage levels in each aisle (K)

(Model OPTDIM)

$$\text{Min } f(M, P, K) \tag{23}$$

Subject to Eqs. (16) $\forall m$, (18), (19), and

$$P_{\min} \le P \le P_{\max} \tag{24}$$

$$K_{\min} \le K \le K_{\max} \tag{25}$$

$$M, P, K \in Z^{+} \tag{26}$$

The model OPTDIM minimizes the expected cost of warehouse operations. Constraint (16) ensures that each aisle has sufficient space to contain all the products. Constraint (18) ensures that the designed warehouse has sufficient capacity to accommodate the inventory requirements of all products. Constraint (19) ensures that all products are placed in the warehouse. Constraints (24) and (25) place realistic bounds on warehouse dimensions depending on business situations like size of building, technology, ergonomics, etc. (26c) ensures that the dimensions take positive integral values.

3 Solution procedure

The model OPTDIM is a non-linear model involving integer variables. We propose a procedure based on fixing values of warehouse dimensions P and K in the required limits (constraints 24–26) and thereby the available storage space per aisle. Products are then assigned in non-increasing turnover order to fill up each aisle, honoring constraint (16). This leads to computation of total number of aisles required, M, as per Eq. (18) and total cost (Eq. 22). The solution algorithm is outlined using a pseudo-code as shown in Table 2.

Table 2 Solution algorithm for model "OPTDIM"

I. Rank all products in the non-increasing order of turnover.
II. Select a value of P between $P_{\min}$ and $P_{\max}$
III. Select a value of K between $K_{\min}$ and $K_{\max}$
IV. Compute required storage space for the particular combination of P and K
a. Unit block space is $P \times K$
b. Allot 1st product to 1st aisle
c. Calculate the space required to keep the product using (17)
d. Check if space within the current aisle is sufficient. If no, add one more aisle and repeat till the space become sufficient for the particular product
e. Check if all the products are exhausted. If no, then add the next product and repeat steps (d) and (e). This would ensure that all products are exhausted after several iterations.
f. Compute the number of aisles required, M, as per (18) for combination of P and K.
V. Compute operational cost for the particular combination of P, K, and M
a. Calculate HTC using (7)
b. Calculate VTC using (14)
c. Calculate area cost using (21)
d. Sum of (a), (b), and (c) above would be the total operational cost for the particular combination of M, P, and K
VI. Repeat II to IV exhaustively for different possible values of P and K such that all possible combinations of values are enumerated. The combination which gives minimum value in IV is the optimal combination of M, P, and K.

4 Illustrative case

Two main contributions of this paper are in developing an analytical model for finding average vertical travel distance (see, Sect. 2.3) and in developing analytical model that incorporates area cost and handling cost in finding the optimal warehouse dimensions (see, Sect. 2.6). In this section, we apply these analytical models to real life industry demand data comprising 174 SKUs from a company in Mumbai (India) that sells home appliances. The company uses an inventory policy wherein every SKU has an order quantity worth 10 days of demand. The pallets are arranged in turnover-based slotting in both horizontal and vertical dimensions while applying to analytical models.

Similar to Eq. (1), the cumulative fraction of total demand till a product i can be expressed as a function of fraction of products, as

$$G(i') = i'^{s} \tag{27}$$

where $G(i')$ is the cumulative demand fraction till a product i', the cumulative product index. The demand data is obtained from the case company and is fitted onto a plot of $G(i')$ to i', as shown in Fig. 73. Using curve fitting, the best fit value of s is extracted as 0.12.

4.1 Validation of vertical travel model

An experiment is designed to simulate order-picking process in the warehouse and validate the newly proposed vertical travel model. To understand the validity of vertical travel model, the following operating conditions were tested. The number of vertical storage levels (K) was selected from $\{3, 5, 10\}$. As the exact value of stops per order-picking tour (N_p) is unknown at this stage, it was selected from $\{10, 50\}$. The other parameters were fixed as $M = 10$, $P = 50$, and $h = 3$.

The process of Simulation involved generating instances of pick lists with size N_p in such a way that the picks originate in aisles with probability given by Eq. (2). This was achieved by generating N_p random numbers from a uniform distribution $U[0, 1]$ and obtaining the pick aisle m for each using equation $m = U[0, 1]^{\frac{1}{s}} \times M$. Similarly, the level k from which a pick

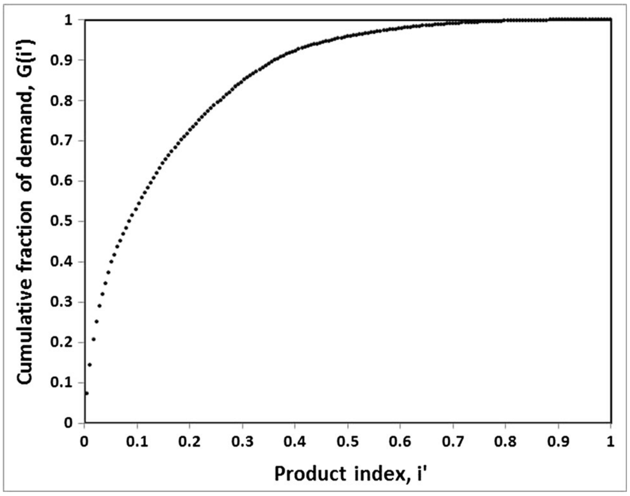

Fig. 3 Plot of $G(i')$ to i' from case data

needs to be performed was generated using $k = U[0,1]^{\frac{1}{s}} \times K$. The lane, p within each aisle were selected in a random fashion using $p = U[0,1] \times P$

In the vertical travel, the highest pick at each location was recorded and total vertical distance thus obtained in the entire tour was measured. The total vertical travel distance for an order was calculated and averaged over 100 instances of orders. Table 3 provides comparison of vertical travel distance obtained from case simulation and that with analytical model using expression (12).

The average error recorded was 1.50% with 3.78% as the largest error. The analytical model is thus accurate over a wide range of operating conditions.

4.2 Effect of area cost in design

Our model OPTDIM explicitly brings in area cost as one of the components of the objective function. To understand the effectiveness of the model, it is applied to case data. The total warehouse cost, which is the sum of handling and area costs, is compared for two types of warehouse designs. In the first type, the warehouse is designed using OPTDIM model where the warehouse dimensions are optimized for total cost of operations (which is the sum of area cost and material handling cost). In the second type, the warehouse dimensions are optimized for handling cost alone, as is done in some conventional literature mentioned in Sect. 1.

Table 3 Accuracy of vertical travel model compared with simulation based on case data

K	N_p	Vertical travel distance		
		Analytical	Case simulation	% difference
3	10	64.78	63.66	1.72%
	50	239.09	232.14	2.91%
5	10	76.36	77.88	−1.99%
	50	294.23	289.26	1.69%
10	10	106.69	105.72	0.91%
	50	437.62	421.08	3.78%

The design exercise is repeated for three combinations of $\{C_l\ (\mathrm{Rs/h}), C_r\ (\mathrm{Rs/sqft\text{-}day})\}$, namely $\{50, 2.5\}$, $\{100, 0.5\}$ and $\{150, 0.1\}$. The other parameters have values $V_v = 1000$ ft/min, $V_h = 3000$ ft/min, $h = 3$ ft, $w = 3$ ft, $l_d = 3$ ft, $A_w = 15$ ft, and $\varepsilon = 0.22$. The results are shown in Table 4. The time for execution of the model is averaged over four instances and is given with each use case. The execution time is around 10–13 s, which is not quite large considering the fact that warehouse design decision is fairly less frequent and applies to a longer planning horizon. It is seen that OPTDIM is significantly better than conventional models over the range of realistic operating conditions studied. The model reduces the cost by as much as 84.46% when the rent is on the higher end and labor cost is on the lower end. Also, the design varies according to the combination of cost parameters C_l and C_r.

5 Computational experiments

5.1 Effectiveness of turnover-based storage policy in vertical dimension

Several authors have studied the effect of turnover-based storage policy in the horizontal dimension ([32], [35], [7], etc). The policy of segregating products based on turnover in the vertical dimension is studied in this section. Table 5 contains the values for the set of parameters used for this experiment.

To understand the contribution of turnover-based product placement in vertical dimension, the total travel distance is obtained for the following two kinds of storage policies:

- Policy 1—Within-aisle turnover-based arrangement in horizontal dimension, random arrangement in vertical dimension.
- Policy 2—Within-aisle turnover-based arrangement in horizontal dimension, turnover-based arrangement in vertical dimension

Table 6 gives the results comparing the two policies over the range of parameters. It is seen that policy 2 (segregating products based on turnover in the vertical dimension as well) would give significant savings when the warehouse is taller, when the number of picks per order is higher or when the demand ABC curve is more skewed. When all the three

Table 4 Comparison of OPTDIM with conventional model

Cost parameters		Warehouse optimized for handling cost alone (conventional)				Warehouse optimized for total cost (OPTDIM)					
Labor cost in Rs/h (C_l)	Rent in Rs/sqft-day (C_r)	Dimensions			Cost (Rs)	Dimensions			Cost (Rs)	Cost saving (%)	Average execution time (s)
		Aisles	Lanes	Levels		Aisles	Lanes	Levels			
50	2.5	11	17	3	295,178.4	1	28	15	45,861.23	84.46%	12.49
100	0.5	11	17	3	60,211.71	1	28	15	12,342.45	79.50%	11.23
150	0.1	11	17	3	13,741.07	8	5	14	5787.46	57.88%	12.28

factors are favorable, the savings was as high as 18.05% in our experiment. On an average, policy 2 gave 5.5% additional savings across the range of parameters considered.

5.2 Effect of space sharing

The model discussed in this paper uses the effect of staggering of product inflow to adjust the volume required for storage in a three-dimensional rack employing within-aisle storage policy. Yu et al. [44] and Guo et al. [19] examine the effect of space sharing on the travel distance in a unit-load warehouse. Not incorporating the space-sharing effect in calculation of storage space required can lead to certain under estimation in space calculations. In this section, we quantify this degree of under estimation under certain operating conditions.

Given the rack space within an aisle, the algorithm calculates the number of aisles required for storage. The number of aisles thus obtained is compared with the number of aisles obtained without considering the effect of space sharing over a range of parameters shown in Table 7, and results are shown in Table 8.

As can be seen from Table 8, ignoring the space-sharing effect will lead to large underestimation in most operating scenarios studied. Further, the following observations can be made from Table 8.

Table 5 Parameters used for the experiment 5.1

Parameter	Values
Number of vertical storage levels (K)	5, 10
Average number of stops per order (N_p)	10, 50
Skewness of ABC curve (s)	0.139 (20/80), 0.318 (20/60), 1 (20/20)
Number of storage aisles (M)	10
Number of lanes per aisle (P)	50
Height of each level (h)	3

i. As the skewness of ABC curve decreases (in other words, as s increases), the warehouse would require lesser number of aisles than otherwise. This is because as the order quantity becomes skewed across various products, the first few products would consume more space reducing the quantity n_m in some of the aisles.

ii. As the space within each aisle increases, the space underestimation becomes less pronounced. This is because the increased block space can accommodate more products, increasing the value of n_m for each aisle.

iii. As the number of products increases (and total demand is a constant), the degree of underestimation is slightly lesser. This is due to similar reason mentioned in (ii).

5.3 Sensitivity of the estimated space to ε

Yu et al. [44] show that under normal circumstances of uniform batch sizes and evenly distributed replenishment schedules, ε can be expected to vary between 0.17 and 0.25, and the space required is less sensitive in this range. However, in real world conditions, the value of ε might vary outside this range because of non-uniformities in batch sizes and replenishment schedules. Since it is difficult to know the range of variation of ε under real world conditions, one might consider using a particular value of ε, say 0.22, instead of an exact value. The following limited experiment was conducted to understand the accuracy of this approach through a sensitivity analysis of ε.

The parameters of the warehouse was varied as in Table 7, except for ε which was varied in the range $[0.1, 0.5]$. The exact space required for the warehouse under each of these operating conditions is estimated using the solution algorithm in section 3. In addition, the space estimated using traditional approach (perfect sharing assumption) is also recorded. Yu et al. [44] suggests using an average value of $\varepsilon = 0.22$ for space calculations. In this light, two kinds of errors are calculated for each set of operating conditions.

Table 6 Effectiveness of turnover-based arrangement in the vertical dimension

Number of vertical levels (K)	Average number of picks per order (N_p)	S	Total pick travel distance		Savings (%) of policy 2 over policy 1
			Policy 1	Policy 2	
5	10	0.139	1972.06	1809.44	8.25%
		0.318	2560.02	2590.08	−1.17%
		1	3264.8	3254.2	0.32%
	50	0.139	4741.6	4327.4	8.74%
		0.318	6060.92	5737.42	5.34%
		1	7121.88	7151.24	−0.41%
10	10	0.139	2125.52	1927.3	9.33%
		0.318	2733.34	2574	5.83%
		1	3454.28	3447.2	0.20%
	50	0.139	5505.1	4511.3	18.05%
		0.318	6775.12	6037.3	10.89%
		1	7917.18	7864.4	0.67%

$$\text{Error 1} = \frac{\text{Space estimated using constant } \varepsilon = 0.22}{\text{Space estimated at a particular } \varepsilon} - 1$$

$$\text{Error 2} = \frac{\text{Space estimated using traditional approach with assumption of perfect sharing}}{\text{Space estimated at a particular } \varepsilon} - 1$$

The results can be seen in Fig. 4 which shows a box plot of the above mentioned errors. It can be seen that the space required is not really sensitive to ε in the range of operating parameters mentioned. Using a constant ε of 0.22 gives us a really good approximation of space required (Error 1 interquartile range narrowly spread around 0%), similar to the suggestion in Yu et al. [44]. Such an approach would be much better to use instead of a traditional approach which gives a severe under estimation in almost all the cases (Error 2 interquartile range is spread around −45%). A more robust approach, needless to say, would involve estimation of space using exact value of ε which would be a function of several factors including staggering, batch sizes, etc., which requires a larger theoretical study.

Table 7 Parameters used for experiment 5.2

Parameter	Values
Skewness of ABC curve (s)	0.139 (20/80), 0.318 (20/60), 1 (20/20)
Space within each aisle in (2 P × K)	100, 500, 1000
Space-sharing factor (ε)	0.22
Number of products (P_g)	500, 2000
Total demand	10,000

5.4 Insights to the warehouse manager

The results obtained in the previous sections offer the following crucial insights to the warehouse manager that can become valuable in several operating conditions.

i. In an already existing warehouse, if the number of vertical levels is comparable with the number of aisles and the number of picks is large, it would make sense to arrange the products based on non-increasing turnover in the vertical dimension as well. This is also true if the skewness of demand curve is large.
ii. The effect of inventory sharing on the total number of aisles required cannot be ignored at the design stage. This is particularly important when the number of products are large, the space inside an aisle is large, and when the demand is highly skewed. Ignoring this effect can lead to severe underestimation of space required.
iii. In case an exact value of ε is unknown (which presumably is true in most practical scenarios), one can use an $\varepsilon = 0.22$ in the model to get a reasonably good design.

Table 8 Under estimation of space required in absence of space-sharing considerations

s	Space within aisle	Number of aisles required as per conventional method	Percentage of under estimation	
			P_g = 500	P_g = 2000
s = 0.139 (20/80)	100	50	46.24%	43.18%
	500	10	44.44%	41.18%
	1000	5	44.44%	37.50%
s = 0.318(20/60)	100	50	43.82%	38.27%
	500	10	37.50%	33.33%
	1000	5	37.50%	28.57%
s = 1 (20/20)	100	50	41.18%	33.33%
	500	10	33.33%	28.57%
	1000	5	28.57%	28.57%

iv. Area cost needs to be factored in the design of warehouses. This becomes particularly important in places where there are high costs associated with renting or leasing space.

6 Summary and conclusion

A model to design an order-picking warehouse was developed. This incorporated an analytical model for vertical travel component which was found to represent the vertical travel component accurately. The optimization model OPTDIM explicitly incorporates area cost with handling cost in the objective function for warehouse design. A solution algorithm is presented which incorporates the space-sharing effect, vertical and horizontal travel costs, and space cost to arrive at an optimum design of warehouse dimensions. Validation using real life case data suggests that OPTDIM gives considerable advantage over conventional design approaches, with savings as high as 84% when space costs are high. The analytical vertical travel model was found to be close to case simulation with the maximum error of around 3.78% when applied to case data. Computational experiments over a range of operating conditions showed benefits of up to 18% in travel time by segregating products based on turnover in the vertical dimension when the demand skewness or number of picks per order is high. The space-sharing effect due to inventory staggering and its effect on total warehouse volume is considered and found to be significant, which, if ignored, might lead to space underestimation as much as 46% in certain cases. It was seen that in the absence of an exact value of ε, using a value of 0.22 would give a reasonably good model.

The limitations of the study are as follows. The warehouse considered in this paper uses order pickers which allow only one kind of motion, either vertical or horizontal, at a time.

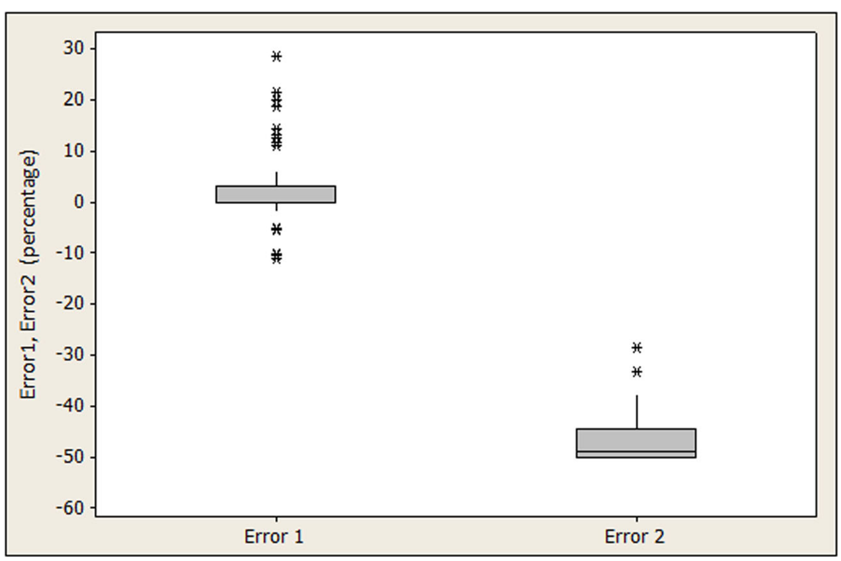

Fig. 4 Box plot Error 1 and Error 2

Advances in technology have brought sophisticated order pickers that can move both vertically and horizontally at the same time in parallel fashion. A travel model for such a situation would be one avenue of furthering this research. A model for a more accurate estimation of ε would be another interesting study that can go a long way in making warehouse design models more industry friendly.

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