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ORIGINAL ARTICLE

Chatter stability prediction in milling using time-varying uncertainties

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Abstract The chatter stability in milling severely affects productivity and quality of machining. Tool wear causes both the cutting coefficient and the process damping coefficient, but also other parameters to change with cutting time. This variation greatly reduces the accuracy of chatter prediction using conventional methods. To solve this problem, we consider the cutting coefficients of the milling system to be both random and time-varying variables and we use the gamma process to predict cutting coefficients for different cutting times. In this paper, a time-varying reliability analysis is introduced to predict chatter stability and chatter reliability in milling. The relationship between stability and reliability is investigated for given depths and spindle speeds in the milling process. We also study the time-varying chatter stability and time-varying chatter reliability methods theoretically and with experiments. The results of this study show that the proposed method can be used to predict chatter with high accuracy for different cutting times.

Keywords Tool wear · Cutting force coefficients · Random and time-varying variables · Gamma process · Time-varying chatter stability · Time-varying chatter reliability

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1 Introduction

Regenerative chatter is a kind of self-excited vibration involving both the tool and the workpiece. It accelerates tool wear, reduces the quality of the workpiece surface, and decreases the overall machining accuracy of the milling process. It is one of the major factors that limits the metal removal rates of a machine. In recent years, many research have studied regenerative chatter prediction in milling.

Tobias and Fishwick [1] studied the orthogonal cutting model for regenerative chatter and established a chatter model. They used a stability lobe diagram (SLD) to predict chatter stability for the first time. Consistent with the definition, the area below the lobe is stable and the area above the lobe is unstable. The SLD provided an important basis for the prediction of chatter. Subsequently, the SLD became a common method to study regenerative chatter in the cutting process. Many researchers conducted in-depth investigations of the different processing forms. Altintas [2] used the Fourier approximation method to determine the dynamic cutting force and the SLD according to the stability criterion. This method provides a new way to investigate the accurate determination of variable cutting forces, and it became the most influential method for chatter stability prediction in the last 20 years. Later, some more accurate methods were proposed to describe the cutting force in milling at any time. Insperger [3, 4] and Elbeyli [5] described the semi-discretization method, while Ding [6] proposed the full-discretization method. Both methods divided the cutting process into a number of smaller processes. The SLD was obtained by solving time-delayed differential equations of the milling process. Wan [7] studied the stability of the thread milling process by transforming the dynamic cutting process into semi-discrete time domain. Fei [8] used semi-discretization method to predict chatter stability for milling of flexible pocket-structure. To improve the efficiency and validity of chatter prediction Wan and Zhang [9] took multiple modes into account instead of considering only the most flexible mode in milling process chatter stability prediction. Zhang [10] used the numerical differentiation method to predict the chatter stability in high-speed milling. To simplify the complexity of the discretization iteration formula, Li [11] used Runge-Kutta-based completely discretization method to predict chatter stability for milling process. Aiming at the multifunctional tool chatter stability prediction, Wan and Zhang [12, 13] proposed static cutting force and chatter stability prediction models. Ahmadi [14] presented a model for machining chatter simulation in the time domain of flank milling. In addition, Feng [15] try to identify chatter in milling based on cutting force signals and surface topography

Considering the square and cubic polynomial terms related to the cutting forces, structural stiffness, and power-law functions for cutting forces, delayed nonlinear models of the process have been governed [16–18]. Hanna [19] developed a mathematical theory of nonlinear chatter. Moon [20] proposed a new model using hysteresis for complex dynamics in cutting materials. Martınz [21] proposed a weakly nonlinear model with square and cubic terms in both structural stiffness and regenerative terms, using the method of multiple scales to better representation of the nonlinear behavior in machining. Ahmadi and Ismail [22] investigated nonlinear behavior in turning experimentally. Later, they [23] developed the stability lobes analytically with taking into account the effect of nonlinear process. Bayly [24] reported a time finite element method, which divided the displacement by the tool vibration into smaller units in the time domain. He obtained the SLD based on the stability criterion. However, some scholars found that the cutting force coefficients and natural frequency are uncertain during in the cutting process [25, 26]. Several factors contribute to this phenomenon including tool wear, uneven distribution of materials, measurement errors, noise interferences, changes in the processing environment, nonlinear behavior, and the use (or lack of use) of cooling fluid.

These uncertain parameters can be classified as random and time-varying parameters depending on whether they are affected by the cutting time or not. Because the SLD is particularly sensitive to changes of the structure parameters, they greatly affect the chatter stability prediction. Nevertheless, in the above studies, both structure parameters and cutting parameters were regarded as constants.

However, some researchers described chatter stability prediction methods treating structure parameters and cutting parameters as random parameters. Duncan [25] first suspected that parameter uncertainty affects the SLD. He suggested that the structural modal parameters and the cutting force coefficients are random parameters in the cutting process. Park [27] introduced the robust stability method, which considers the natural frequencies and the cutting force coefficient to be uncertain parameters with regard to chatter prediction. They obtained the SLD based on the robust stability theory of a dynamic system. Sims [28] introduced a fuzzy algorithm into chatter prediction and obtained a fuzzy stable lobe. He used the transformation method with nine membership levels to represent the range of parameter variation. Totis [29] introduced the probability algorithm into the cutting process and established the dynamic milling model to include uncertainty. Liu [30] introduced the dynamic reliability analysis of the structure as part of the chatter prediction for a turning system. He obtained a series of different reliable chatter stability lobes and defined the chatter reliability to represent the probability of stability (no chatter occurs) of the turning process. Huang [31] studied the probability characteristic of the regenerative chatter stability in turning using the Monte Carlo simulation (MCS) method and advanced first order second moment (FOSM) method. Although the above chatter prediction methods consider random parameters, none of them take into account the influence of the time-varying characteristic of the cutting force coefficients on the chatter prediction due to tool wear.

The cutting force coefficient changes with cutting time due to tool wear. Therefore, chatter stability varies with the cutting time. The rapid tool wear in the process has been recognized as a difficult problem for a long time [32]. However, research on tool wear mainly focuses on tool wear monitoring [33–37] and the processing environments [38]. Studies about the impact of tool wear on chatter stability prediction do not exist in the literature.

To address this problem, we combine the ideas of time-varying reliability of a dynamic system with the structural analysis of the milling process. We use the gamma process to describe the time-variation of the cutting force coefficient resulting from tool wear. Time-varying chatter reliability is defined to represent the probability of stability (no chatter occurs) in the milling process using time-varying cutting coefficients. The relationships between the cutting time and both chatter stability and reliability are identified. We define the relationship as time-varying chatter stability and time-varying chatter reliability, respectively. They are described using time-varying stability lobe diagrams as well as time-varying reliability curves.

The dynamical model of milling and the time-varying cutting force coefficient model are established in section 1. The time-varying chatter reliability model is introduced to predict milling chatter vibration, in which cutting coefficients are time-varying parameters—see section 2. In section 3, a case study is provided to verify stability changes that occur with tool wear. The discussion is presented in section 4, followed by a conclusion in section 5.

2 Dynamic modeling of milling chatter

2.1 Dynamic cutting model of the milling process

We define m_x, m_y, k_x, k_y, c_x, c_y as mass, stiffness, and damping coefficients in the X and Y directions, respectively, for a two degrees of freedom (2DOF) model of milling—see Fig. 1.

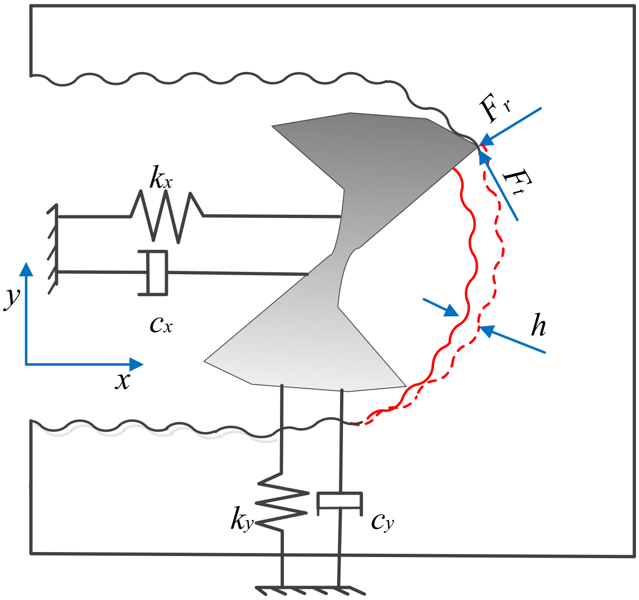

Fig. 1 Dynamic modeling of the milling process

The dynamic chip thickness is represented by h, where h is the difference between $x(t)$ and $x(t\text{-}T)$. Moreover, $x(t)$ is the current amplitude of the vibration at the tool tip and $x(t\text{-}T)$ is the previous tooth period amplitude of the vibration at the tool tip. The general equations to describe the motion of the 2DOF milling system are the following:

$$m_x\ddot{x} + c_x\dot{x} + k_x x = \sum_{i=1}^{N}\left(-F_{\mathrm{t},i}\cos\phi_i - F_{\mathrm{r},i}\sin\phi_i\right) \quad (1)$$

$$m_y\ddot{y} + c_y\dot{y} + k_y y = \sum_{i=1}^{N}\left(F_{\mathrm{t},i}\sin\phi_i - F_{\mathrm{r},i}\cos\phi_i\right) \quad (2)$$

where N is the number of the teeth of the milling tool, $F_{\mathrm{t},i}$, $F_{\mathrm{r},i}$ are the ith teeth cutting force in the tangential and radial directions, respectively.

2.2 Modeling of the cutting force coefficient with characteristics of time-variation and uncertainty

Li [39] and Orak [40] found that tool wear is a typical stochastic process with continuous time and a continuous state. Because the tool cannot repair itself, the process has independent non-negative increments. The gamma process is a stochastic process with independent non-negative increments having a gamma distribution with identical scale parameters. In this paper, we use the Gamma process to describe the relationship between cutting force coefficients and cutting time. We define that $K_\mathrm{t}(t)$, where the unit of t is minute, is the increment cutting force coefficient during the cutting time from zero to t. The gamma process probability density function is the following:

$$f_{K_t(t)}(x) = G_a(x|v(t),\mu) = \frac{\mu^{v(t)}x^{v(t)\text{-}1}\exp(-\mu x)}{\Gamma[v(t)]}I_{(0,\infty)} \quad (3)$$

where $G_a(\bullet)$ is the gamma distribution function, $\Gamma(\bullet)$ is the gamma function, and μ, $v(t)$ are scale parameters and shape parameters, respectively,

where $I(x) = \begin{cases} 1 & \text{if} \quad x\in(0,\infty) \\ 0 & \text{if} \quad x\notin(0,\infty) \end{cases}$.

The mean and variance of $K_t\,(t)$ are the following:

$$E[x(t)] = \frac{v(t)}{\mu} \quad (4)$$

$$E\left[\left\{x(t) - E\left[x(t)\right\}^2\right]\right] = \frac{v(t)}{\mu^2} \quad (5)$$

Moriwaki [41] found that the deterioration of the expected value is proportional to the energy law. Hence, Eq. (4) can be expressed as follows:

$$E[x(t)] = \frac{ct^b}{\mu} \quad (6)$$

where μ, b, and c are positive real numbers. Now, the shape parameters for the gamma process can be written as follows:

$$v(t) = ct^b \quad (7)$$

The time-varying cutting force coefficient and its average for a given cutting time are the following:

$$K_t(t) = \frac{\mu^{ct^b}x^{ct^b\text{-}1}\exp(-\mu x)}{\Gamma[ct^b]}I_{(0,\infty)} + K_{\mathrm{t}0} \quad (8)$$

$$E[K_t(t)] = \frac{ct^b}{\mu} + K_{\mathrm{t}0} \quad (9)$$

where $K_{\mathrm{t}0}$ is the cutting force coefficient with a new tool.

3 Modeling of chatter reliability with time-varying and random parameters

3.1 Chatter stability modeling with time-varying parameter based on ZOA

The time-varying chatter stability is the chatter stability that takes into account time effects. From Schmitz's [41] study, the critical depth using the ZOA method for milling process can be formulated as the following:

$$b_{\mathrm{lim}} = \frac{2\pi\cdot\mathrm{Re}(\lambda)}{N_{\mathrm{t}}K_{\mathrm{t}}\left(\mathrm{Re}(\lambda)^2+\mathrm{Im}(\lambda)^2\right)}\left(1+\left(\frac{\mathrm{Im}(\lambda)^2}{\mathrm{Re}(\lambda)^2}\right)\right) \tag{10}$$

where b_{lim} is the critical depth, N_{t} is the number of teeth, Re (λ) and Im (λ) are real and imaginary parts of eigenvalues, and K_{t} is the tangential cutting force coefficient.

$$\Omega = \frac{\omega_n}{N_{\mathrm{t}}}\frac{1}{(c+2\pi\cdot\mathrm{N})} \tag{11}$$

where ω_{n} is the chatter frequency and N *is* the number of lobes. We can formulate c as the following:

$$c = \pi - 2\cdot\tan^{-1}\left(\frac{\mathrm{Im}(\lambda)}{\mathrm{Re}(\lambda)}\right) \tag{12}$$

The critical depth b_{lim} can be rewritten by substituting Eq. (8) in Eq. (10) to yield:

$$b_{\mathrm{lim}}(t) = \frac{1}{\frac{\mu^{v(t)}x^{v(t)-1}\exp(-\mu x)}{\Gamma[v(t)]}+K_{\mathrm{t0}}}\frac{2\pi\cdot\mathrm{Re}(\lambda)}{N_{\mathrm{t}}\left(\mathrm{Re}(\lambda)^2+\mathrm{Im}(\lambda)^2\right)} \times\left(1+\left(\frac{\mathrm{Im}(\lambda)^2}{\mathrm{Re}(\lambda)^2}\right)\right) \tag{13}$$

In order to obtain the SLD for a given time t, we use the mean of the cutting force coefficients instead of the one at the cutting time t. The expression for the critical depth at the time t can now be rewritten as follows:

$$b_{\mathrm{lim}}(t) = \frac{1}{\frac{ct^b}{\mu}+K_{\mathrm{t0}}}\frac{2\pi\cdot\mathrm{Re}(\lambda)}{N_{\mathrm{t}}\left(\mathrm{Re}(\lambda)^2+\mathrm{Im}(\lambda)^2\right)}\left(1+\left(\frac{\mathrm{Im}(\lambda)^2}{\mathrm{Re}(\lambda)^2}\right)\right) \tag{14}$$

The tangential cutting force coefficient increases with cutting time, while the critical depth shows an opposite trend when both the axial depth of the cut and the spindle speed are given.

3.2 Chatter reliability modeling using time-varying parameters

The chatter reliability of the milling process is defined as the probability of no chatter occurring in a dynamic system with time-varying parameters K_{t0}. The performance function of the milling process system in a critical stable state can be expressed as the following:

$$g(X) = b_{\mathrm{lim}} - b_0 \tag{15}$$

where b_0 is the axial depth of the cut in the milling process and X is a time-varying parameter. The time-varying reliability is the following:

$$P_r(X) = P(g(X)>0) = \int_{X_R} f_x(x)dx \tag{16}$$

where X_R is the stable region $g(X) > 0$ within the X_R region.

3.3 Chatter reliability calculation with a time-varying parameter

The performance function and time-varying chatter reliability that contains time-varying parameters can be obtained by substituting Eq.(13) into Eqs. (15)–(16):

$$g(X) = \frac{1}{\frac{\mu^{v(t)}x^{v(t)-1}\exp(-\mu x)}{\Gamma[v(t)]}+K_{\mathrm{t0}}}\frac{2\pi\cdot\mathrm{Re}(\lambda)}{N_{\mathrm{t}}I_{(0,\infty)}\left(\mathrm{Re}(\lambda)^2+\mathrm{Im}(\lambda)^2\right)}\left(1+\left(\frac{\mathrm{Im}(\lambda)^2}{\mathrm{Re}(\lambda)^2}\right)\right) - b_0 \tag{17}$$

$$p_r(X) = P\left\{\frac{1}{\frac{\mu^{v(t)}x^{v(t)-1}\exp(-\mu x)}{\Gamma[v(t)]}+K_{\mathrm{t0}}}\frac{2\pi\cdot\mathrm{Re}(\lambda)}{N_{\mathrm{t}}\left(\mathrm{Re}(\lambda)^2+\mathrm{Im}(\lambda)^2\right)}\left(1+\left(\frac{\mathrm{Im}(\lambda)^2}{\mathrm{Re}(\lambda)^2}\right)\right) - b_0 \geq 0\right\} \tag{18}$$

Using the ZOA method, we find that the cutting force coefficient is the only factor that affects the size of the critical cutting depth when the machining parameters and spindle speed are known. Therefore, the probability distribution of the cutting force coefficient is the only factor that determines directly the chatter reliability in a milling system, which can be simplified to:

$$p_r(X) == \int_0^{K_{tb}} \left[\frac{\mu^{v(t)} x^{v(t)-1} \exp(-\mu x)}{\Gamma[v(t)]} \right] dx \tag{19}$$

where 0-K_{tb} is the integral area X_R and K_{tb} is the tangential cutting force coefficient at a given critical depth and spindle speed. We can now obtain the performance function in 0-K_{tb} by integration:

$$\begin{aligned} p_r(X) &= \left[-\frac{\left(x^{\nu(t)} (-\mu)^{\nu(t)} \times \gamma(\nu(t), -\mu x) \right)}{\Gamma(\nu(t))(\mu x)^{\nu(t)}} \right]_0^{K_{tb}} \\ &= \left[-\frac{\left(x^{ct^b} (-\mu)^{ct^b} \times \gamma\left(ct^b, -\mu x\right) \right)}{\Gamma(ct^b), (\mu x)^{ct^b}} \right]_0^{K_{tb}} \end{aligned} \tag{20}$$

where γ is the incomplete Gamma function as formulated in Eq.(21) and the specific expression for K_{tb} is given by Eq. (22):

$$\gamma(\eta, z) = \int_z^{\infty} t^{a-1} \exp(-t) dt \tag{21}$$

where a, z, and η are known values.

$$K_{\mathrm{tb}} = \frac{1}{b_0} \frac{2\pi \cdot \mathrm{Re}(\lambda)}{N_{\mathrm{t}}\left(\mathrm{Re}(\lambda)^2 + \mathrm{Im}(\lambda)^2\right)} \left(1 + \left(\frac{\mathrm{Im}(\lambda)^2}{\mathrm{Re}(\lambda)^2}\right)\right) - K_{\mathrm{t0}} \tag{22}$$

3.4 Drawing the time-varying chatter reliability curve

The time-varying chatter reliability is used to describe the possibility of a process without chatter at a given condition. The calculation process is shown in Fig. 2.

The algorithm requires knowing the cutting dynamics at the tool tip, the initial tangential cutting force coefficient, the spindle speed, and the axial depth of the cut. We then obtain the curve fitting parameters of the gamma process and the time-varying tangential cutting force coefficient. Now, we can use the real part Re (λ) and the imaginary part Im (λ) of the eigenvalues and Eq. (22) to calculate K_{tb}. After identifying the cutting time t, we substitute K_{tb} into Eq.(20) and the time-varying chatter reliability is obtained.

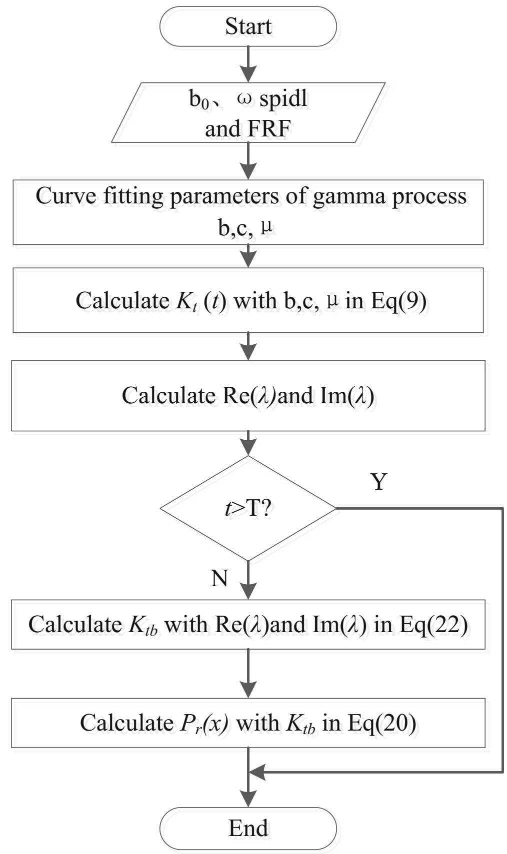

Fig. 2 Flow chart of the time-varying chatter reliability

4 A case study

4.1 Curve fitting cutting force coefficients with the gamma process

The cutting force coefficients are very important factors in chatter prediction. However, they tend to change continuously during the cutting time due to regular tool wear in the machining process. Because the tangential and radial edge coefficients do not contribute to chatter, we only consider the tool wear for the tangential and radial cutting coefficient for the chatter prediction. We use both the Gamma process description and the tangential cutting coefficient to solve the time-varying chatter stability and chatter time-varying reliability of the milling system. Furthermore, we consider the ratio of radial cutting force and tangential cutting force coefficient to be constant.

In order to verify the accuracy of the model, we compared it with the model of the ratio of radial cutting force and tangential cutting force coefficient is time-varying with respect to chatter stability prediction. The results of this study show that the maximum volatility of the critical depth obtained by the two methods at identical spindle speeds, and cutting times is 4.57%. To reflect this important variation accurately, this paper introduces the gamma process model and uses the experimental data of J. Karandikar [42] to curve fit the parameters of the gamma process. In the experiment, the feed length per tooth while using the tool was kept constant at 0.06 mm/tooth. The radial and axial depths of the cut were 9.5 mm (50% radial immersion) and 2 mm. An example result for a spindle speed, Ω, is 2500 rpm. The cutting tool is 19-mm diameter inserted endmill (one square uncoated Kennametal 107,888,126 C9 JC carbide insert; zero rake and helix angles, 15 deg. relief angle, 9.53 mm square × 3.18 mm), which was used to machine 1018 steel.

In this paper, we use the curve fitting toolbox of Matlab (2014a) to fit the data of the increment of the mean of the cutting force coefficients for different cutting moments and set the user-defined fitting function as Eq. (6). For better fitting, the magnitude of the cutting force coefficient was removed during the fitting process. Then, we changed the start point of μ, b, and c to 0 and we set the value of lower as 0.01.

In this paper, according to the standard deviation formula for gamma process as Eq. (23)

$$std = \sqrt{\left(\frac{cx^b}{\mu^2}\right)} \tag{23}$$

Through small-scale adjustment, the starting point of μ needs to be fitted until one obtains a proper standard deviation for the cutting force coefficients. If the standard deviation obtained from fitting is larger than the experimental value, then we increase the starting point value of μ and fit again. The curve fitting results are shown in Fig. 3.

Figure 3 shows that the mean value of the forecast lies in the fluctuation range of the experimental data. It shows that both theoretical predictions and the experimental result of the tangential cutting force coefficient are increased with the cutting time. The deviation between the theoretical value and the experimental data may be due to measurement errors. The values for the curve fitting parameters are listed in Table 1:

Hence, the shape parameter of the gamma process can be expressed as the following:

$$v(t) = 0.1895t^{0.659} \tag{24}$$

Table 1 Fitting parameters for the tangential cutting force coefficient

b	c	μ
0.659	0.1895	0.9133

4.2 Prediction of time-varying chatter stability in milling

Figure 4 shows the differences between SLDs for different cutting times and different colors. It is known that the critical depth decreases gradually with increasing cutting time.

The axial depth of the cut is 2 mm, the feed rate is 0.06 mm/flute, and the changes of the critical depths with cutting time are depicted in Fig.5 at spindle speeds 3000, 4500, and 6000 rpm.

Figure 5 shows that the critical depths decrease significantly with cutting time. Therefore, the accuracy of the chatter prediction decreases after looking at the SLDs of milling using conventional methods. Hence, it is necessary to take the parameters' time-varying characteristic into consideration to improve the accuracy of chatter prediction.

4.3 Calculating the time-varying chatter reliability at a given spindle speed and axial cut depth

In the milling process, when the axial depth of cut b_0, spindle speed Ω, and feed rate υ are known, the time-varying chatter reliability can be obtained with Fig.2.

The time-varying chatter reliability as a function of cutting time is shown in Fig. 6.

It shows that the chatter reliability P_r is almost equal to 1 at the beginning of the milling process when the spindle speed is 6000 rpm. This means chatter does not occur. Then, it starts to decrease with cutting time. Notably, chatter reliability drops to 0.5 when the cutting time approaches 35 min. In other words, there is a 50% probability for chatter.

Figure 7 shows that both the critical depth and chatter reliability decrease when the cutting time increases for a spindle speed of 4500 rpm. The critical depth is greater than the axial

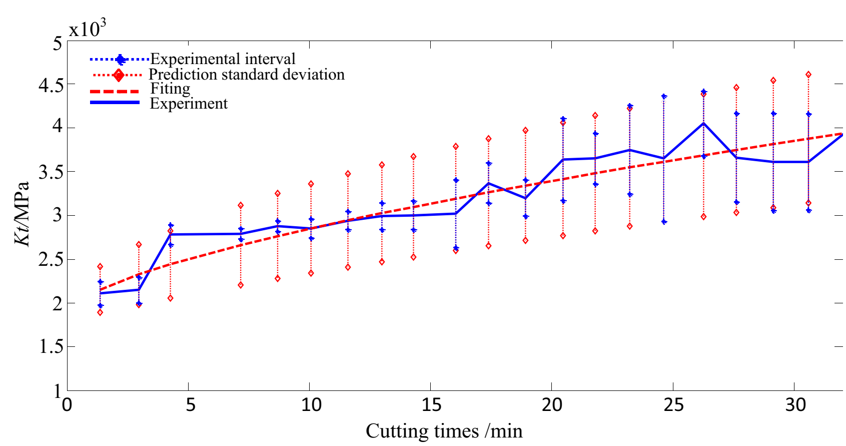

Fig. 3 Comparison of the tangential cutting force coefficient between experiment and calculation

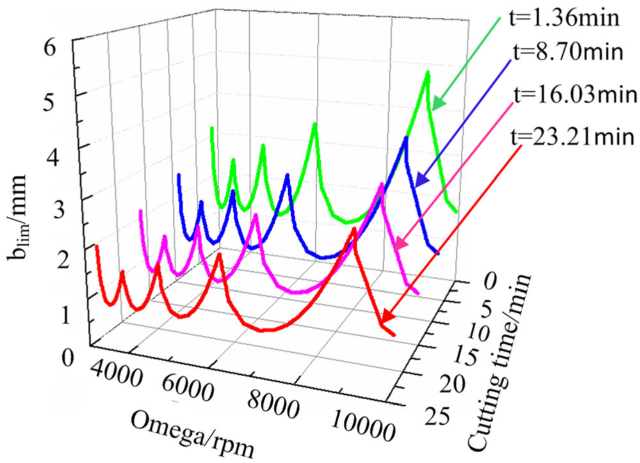

Fig. 4 Time-varying chatter stability lobe diagram

depth of the cut b_0 at the beginning, but smaller in the end. It is noteworthy that, although the critical depth is larger than the axial depth of the cut b_0 in the first 9 min, it is not completely stable. In other words, chatter does not occur with certainty but with a small possibility, when the critical depth becomes larger than the axial depth of the cut.

4.4 Comparative analysis of ZOA and time-varying chatter stability prediction

Our results of the comparison between ZOA and time-varying chatter stability prediction are shown in Fig.8 for our experiment. We chose the spindle speeds 3000, 4500, and 6000 rpm and selected the axial depth of the cut to be 0.5, 1.0, 1.5, 2.0, and 2.5 mm.

In Fig. 8, the black curve is the prediction using the ZOA method. The red curve below it is the prediction using time-varying chatter stability prediction. The blue triangles represent the simulation points at different axial depths of the cut. In the

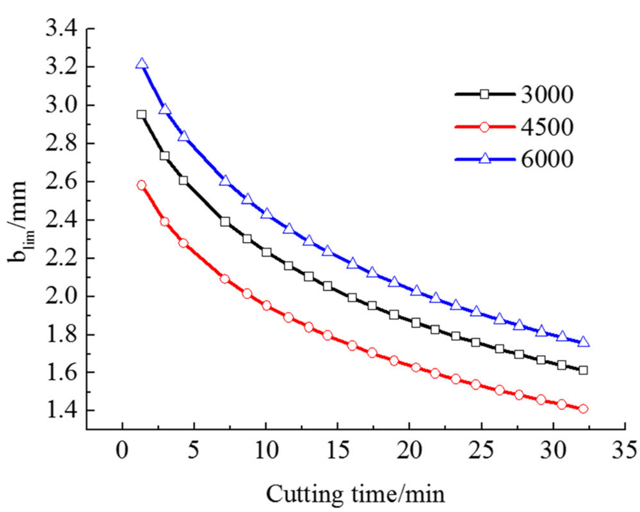

Fig. 5 The relationship between the critical depth and cutting time for three different spindle speeds

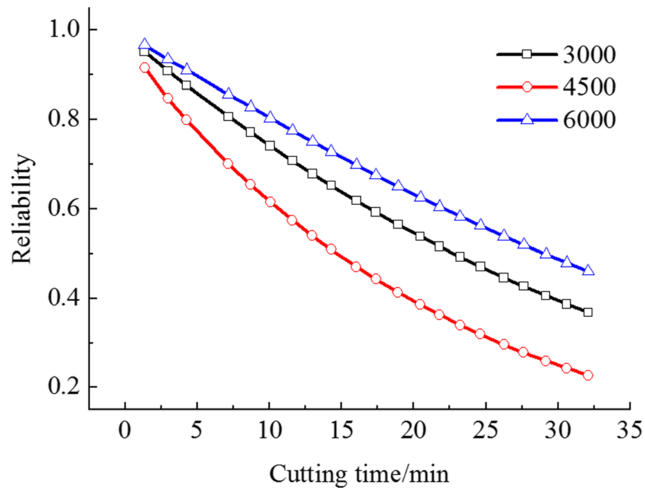

Fig. 6 Time-varying chatter reliability

ZOA method, stability lobes provide the critical depths of cut for the rotational speeds of the spindle: the area below the lobe is stable, but the area above the lobe is unstable. While in the time-varying chatter stability prediction method, the probability of chatter is represented by the degree of reliability. The higher the probability is, the less chatter is likely to occur. The smaller the probability is, the more chatter the system will experience. Figure 8 shows that the critical depth using time-varying chatter stability prediction is almost half of the depth obtained using the ZOA method for a cutting time of 23.2 min. When the axial depth of the cut is 2.5 mm, the spindle speed is 5000 rpm, the prediction result is “chatter” using the ZOA method, and also “chatter” using the time-varying chatter stability prediction. When the axial depth of cut is reduced to 2 mm, the result with the ZOA method is “critical,” but it is clearly “chatter” using the time-varying chatter stability prediction. When the axial depth of the cut is 1 mm, the result of the ZOA method is “stable.” However, there is a 70% possibility for chatter to occur using the time-varying chatter stability prediction method. The detailed data for the chatter reliability values predicted by the chatter time-

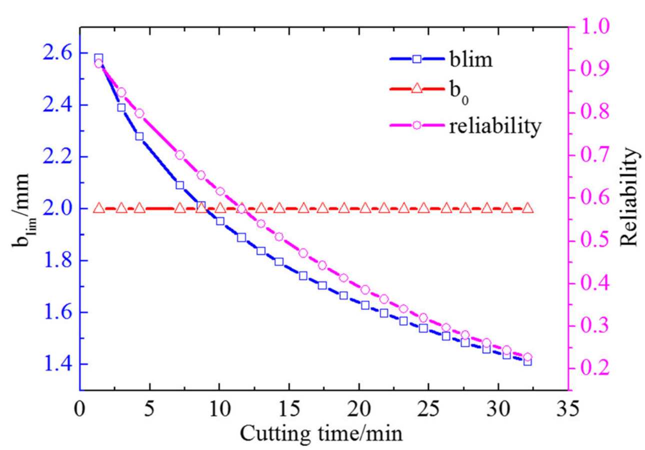

Fig. 7 Critical depth and time-varying chatter reliability for full immersion and a spindle speed of 4500 rpm

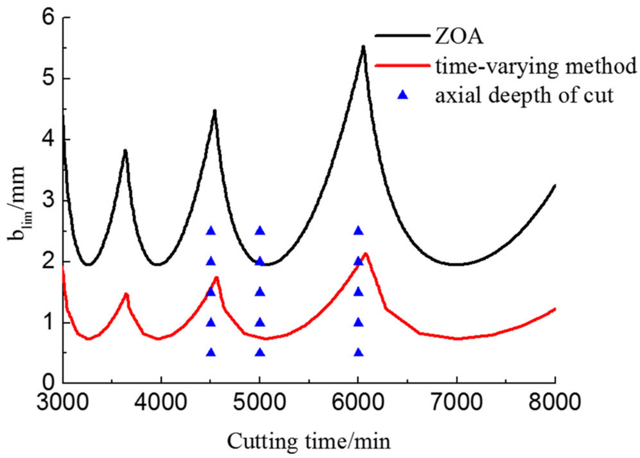

Fig. 8 ZOA prediction and time-varying chatter stability prediction

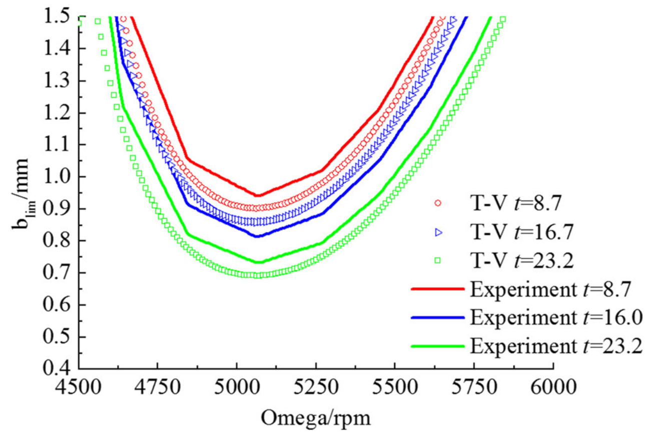

Fig. 9 Comparison between experiment and time-varying chatter stability prediction for different cutting times

varying reliability method at different spindle speeds and axial cutting depths are listed in Table 2.

The time-varying chatter stability prediction is significantly different from the ZOA method. It uses different reliability values to describe the probability for chatter, rather than simply dividing the machining state into "chatter" or "no chatter." Hence, time-varying chatter stability prediction is more accurate than ZOA prediction. In addition, it considers the changes of the processing parameters caused by tool wear. The problem that chatter prediction accuracy decreases gradually during the cutting process could be avoided.

4.5 Comparison between experiment and time-varying chatter stability prediction

We compared the prediction result between the experiment and the time-varying chatter stability prediction at different cutting times in Fig.9.

In Fig. 9, the curve is the prediction using the ZOA method. The symbol is the prediction using time-varying chatter stability prediction. In this figure, the letter "t" represents the cutting times and the letters "T-V" represent the time-varying chatter stability prediction method. For the experiment, the chatter prediction for ZOA and the cutting force coefficients for different cutting times were obtained from the experiment in T.L. Schmitz's paper.

Table 2 Chatter reliability for different spindle speeds and axial depths of the cut

Axial depth of cut/mm	Stability $\omega_{spindle}$ = 4500	Stability $\omega_{spindle}$ = 5000	Stability $\omega_{spindle}$ = 6000
0.5	0.9992	0.9034	0.9999
1.0	0.9167	0.3015	0.9727
1.5	0.6505	0	0.8204
2.0	0.3403	0	0.5829
2.5	0.1006	0	0.3356

Figure 3 shows that the chatter prediction result with experiment date for ZOA and time-varying chatter stability prediction method are very similar to different cutting times.

5 Discussion

Regenerative chatter severely affects both efficiency and productivity of the machining process. It also leads to a poor quality of the surface finish. Therefore, being able to predict accurately, the chatter stability in milling is very important. In practice, tool wear causes the cutting force coefficients to increase gradually. This makes the traditional chatter prediction method inaccurate. To improve the accuracy of the prediction, we introduce and investigate the time-varying chatter stability prediction method. Both the critical depth and the chatter reliability of chatter prediction are constant for the conventional chatter stability prediction, while they are changing with cutting time for a given machining condition using our proposed method.

The time-varying chatter stability prediction method can be applied to the milling process with relatively simple machining conditions. For example, the axial cutting depth and spindle speed are constant. By predicting the time-varying chatter stability of the system for given machining conditions, we can change the axial cutting depth gradually to ensure high machining quality and improve the utilization of the tool.

6 Conclusion

(1) To address the problem that cutting force coefficients are variables with characteristics described as uncertain and random, we use the gamma process to predict the tangential cutting coefficients for different cutting times. The theoretical values are consistent with the obtained experimental data.

(2) Considering the effect of the time-varying character of the cutting force coefficients on chatter prediction, we introduced a time-varying reliability analysis method and the time-varying chatter stability curve. The curve accurately describes the relationship between critical depth and cutting time for a given condition.

(3) A chatter reliability model is developed in which we use the time-varying chatter reliability curve to describe the effect of the coefficient on the chatter reliability prediction. The time-varying chatter reliability model appears more consistent with practical manufacturing than conventional chatter prediction methods.

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