ORIGINAL ARTICLE

Tool posture dependent chatter suppression in five-axis milling of thin-walled workpiece with ball-end cutter

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Abstract In aerospace industry, five-axis ball-end milling is widely used for flexible thin-walled aerospace parts by considering the process mechanics. A new analytical method is proposed to investigate the effects of the change of tool posture, especially lead and tilt angle, on chatter stability in milling. In this method, the dynamic cutter-workpiece engagement regions, which are essential to predict dynamic cutting forces and investigate chatter stability of the thin-walled workpiece, are determined considering the dynamic machining conditions. Then, according to dynamic cutter-workpiece engagement region data, the dynamic cutting force model considering the effects of lead and tilt angle is developed in machining the thin-walled aerospace parts with ball-end cutter. Subsequently, the milling chatter stability model is established based on the dynamic cutting force model, and the milling stability limits are determined in different lead and tilt angles. Finally, the feasibility and effectiveness of the proposed method are verified by experiments with ball-end cutter in five-axis machining center. The results are shown that the predicted values well match with the experimental results.

Keywords Five-axis milling \cdot Tool posture \cdot Chatter suppression . Stability . Process geometry

1 Introduction

Five-axis ball-end milling is a widely used process in manufacturing of complex free surface in aerospace industry.

 \boxtimes Junjin Ma majunjin@gmail.com Its excellent advantages of adjusting the tool orientation in real time considering machining conditions are suitable for machining the thin-walled flexible complex aeroengine parts such as blades, castings, impellers, and blisks. However, these parts have complex surface structure and thin-walled properties, and hard-to-cut materials are used in milling, which lead to excessive machining vibration due to the non-uniform dynamic cutting force. This phenomenon will cause deteriorated surface quality, excessive tool wear, shorten spindle and cutter life, and low production efficiency. Therefore, the objective of this paper is to investigate the method of tool posture dependent chatter suppression for improving the machining accuracy and productivity.

Most of aerospace parts have complex free surface and thin-walled properties. Therefore, in order to obtain the high machining accuracy and surface quality, the ball-end cutter is used due to its obvious advantages compared with the flat and bull-nose end cutter, and the cutter is apt to adjusting the tool posture, which can suppress the machining vibration effectively. During machining, the cutter-workpiece engagement regions (CWEs) vary with tool posture changing, which have close relationship with the dynamic cutting force [\[1,](#page-11-0) [2\]](#page-11-0) due to the complex workpiece geometry and varying tool-axis posture. However, the CWEs are difficult to be determined by the accurate methods in five-axis ball-end milling of thin-walled free surface parts. For overcoming these problems, researchers have developed different methods such as analytical methods $[3-6]$ $[3-6]$ $[3-6]$, discrete modeling methods $[7-10]$ $[7-10]$ $[7-10]$, and solid modeling methods [\[11](#page-11-0)–[14](#page-11-0)] to extract the CWE data. For analytical methods, Gupta et al. [\[3\]](#page-11-0) proposed a geometric algorithm to calculate the continuous closed-form cutter-workpiece engagement functions with the halfspaces for 2.5D milling operations. Then, Kiswanto et al. [\[4](#page-11-0)] presented an analytical method to identify the CWEs during semi-finish milling process by finding the lower engagement point and the upper engagement point. In addition, Tunc et al. [[5\]](#page-11-0) and Budak

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et al. [[6\]](#page-11-0) calculated the complex process geometry and the continuous variation cutter-workpiece engagement region based on computer-aided manufacturing contact point data file by an analytical method. However, these methods are only suitable for milling of simple flat workpiece surface, and these methods can limit the wide application in complex aeroengine workpiece. For the discrete modeling methods, Aras et al. [\[7\]](#page-11-0) presented a parametric method to update the in-process workpiece surfaces by discrete vectors in a virtual environment. A geometric model with discretized mechanistic model was proposed by Zhang [\[8](#page-11-0)] to extract the cutter-workpiece engagement region. Then, Wei et al. [\[9](#page-11-0)] used an improved Z-map method to determine the cutter-workpiece engagement for predicting the dynamic cutting force of the complex workpiece in three-axis ball-end milling. In addition, Kiswanto et al. [\[10](#page-11-0)] proposed a novel hybrid analytical- and discretebased methodology to determine the cutter-workpiece engagement in five-axis milling. However, these methods need large store memory and computational requirements for obtaining high-accuracy computation of cutter-workpiece engagement region and require high-discretized resolution of the workpiece. For solid modeling methods, Boz et al. [\[11\]](#page-11-0) proposed a method using three-orthogonal dexelfield discrete model and parasolid boundary representation kernel solid modeler-based model to extract the cutter-workpiece engagement regions. Then, Aras et al. [[12](#page-11-0)] presented a solid modeling methodology to extract cutter-workpiece regions for prediction cutting force in five-axis milling of free surfaces. In addition, Yang et al. [[13](#page-11-0)] used a simple solid trimming method to determine cutter-workpiece engagement maps for investigating dynamic cutting force, machining errors, and chatter stability in multi-axis milling. Recently, Ju et al. [[14](#page-11-0)] presented a novel and integrated solid-analytical-based approach, which could effectively determine the cutter-workpiece engagement region data for predicting the dynamic cutting force in five-axis milling complex parts with free surfaces. However, the computational efficiency and robustness of these improved methods are limited, and the chatter properties of thin-walled workpiece machining have no investigation in detail by these methods.

For investigating the chatter stability of thin-walled workpiece in five-axis milling, Ozturk et al. [[15](#page-11-0)–[17](#page-11-0)] investigated the effects of tool posture (such as lead and tilt angles) on the stability of the five-axis ball-end milling. In addition, Shamoto et al. [\[18](#page-11-0)] presented an analytical method to investigate chatter stability of ball-end milling of thin-walled workpiece with tool inclination, then the method was extended by Shamoto et al. [[19](#page-12-0)], novel strategies were presented to optimize tool path/posture to avoid chatter vibration in various machining operations, then the feasibility of the method is validated only in turning. Subsequently, Lazoglu et al. [\[20](#page-12-0)] proposed a new comprehensive mechanic-based strategy to select rational tool postures, which was verified by experiments on 5-axis ballend milling of flexible free-form structures. Geng et al. [\[21\]](#page-12-0) proposed a simulation method to determine tool postures based on cutting force prediction. Recently, Sun et al. [\[22\]](#page-12-0) developed the dynamics of ball-end milling in cutterworkpiece engagement coordinate system and presented a method of automatic adjustment of tool-axis orientations to avoid chatter along the tool path. However, these methods are only applicable to milling of flat workpiece surfaces. Moreover, these methods can not satisfy the machining requirement for the complex workpiece surface such as aeroengine blades. In addition, the workpiece is regarded as rigid part in the experiments and the cutting tool is flexible in recent research. Therefore, the usage of these methods is limited in milling of thin-walled workpiece in aerospace industry.

In this work, we proposed a new analytical method of chatter stability in milling considering the tool posture in five-axis milling process. Firstly, the cutter-workpiece engagement region is extracted based on VERICUT secondary development. Then, the dynamic cutting force is predicted based on the data of cutter-workpiece engagement region. Finally, the effects of tool posture on chatter stability are verified by the experiments, and the results show that the proposed method can overcome the problems such as using the sample flat workpiece, computational efficiency, and robustness.

Henceforth, this paper is organized as follows. The problem formulation of thin-walled workpiece in milling is expressed in Section 2. The cutter-workpiece engagement region is determined in Section [3](#page-2-0). Chatter stability model in milling of thinwalled workpiece is developed in engagement system in Section [4](#page-3-0). Experimental cases are conducted to validate the proposed methods; then, some experimental results are discussed in Section [5](#page-5-0). Finally, some conclusion is drawn in Section [6.](#page-11-0)

2 Problem description

In aerospace industry, the five-axis milling is widely used for high precision and good quality. A general aeroengine blisk workpiece is schematically described as shown in Fig. 1a.

Fig. 1 Typical thin-walled workpiece model. a Blisk. b Equivalent blade model

Without loss of generality, in order to investigate the chatter stability in milling, an equivalent aeroengine blade model with fixture constraints is proposed to substitute a whole blisk workpiece as shown in Fig. [1](#page-1-0)b. The equivalent blade model contains a number of geometric and physical information, and this simplification ignores the interference effect among the adjacent blades of blisk. The blade is the typical cantilever thin-walled structure, large overhanging, complex geometric features, and high precision. In this work, the stability analysis of the blade in milling is thoroughly investigated using a new analytical method. Finally, the analytical method and experiment approach are used to obtain the chatter stability during machining. The detailed analytical procedure will be given in the following sections.

3 Calculation of cutter-workpiece engagement region

Dynamic cutting force is a very important factor, which is used to predict the cutting stations such as cutting vibration. In this model, cutter-workpiece engagement region is regarded as a critical input to calculate cutting force in milling operations. In addition, the cutter-workpiece engagement region varies along the cutter path. In order to accurately calculate the dynamic cutting force, the extraction of the cutterworkpiece engagement region data needs to be done prior to predicting the dynamic cutting force and analyzing chatter stability. Therefore, the calculation of cutter-workpiece engagement (CWE) is required for establishing accurate process model. However, for the workpiece with complicated geometry, the cutter-workpiece engagement region can be influenced by the change of the tool posture (lead and tilt angle). The inhomogeneous change of cutter-workpiece engagement region leads to extensive cutting force, which causes chatter vibration and tool deflection. Especially, when the unreasonable cutter posture is selected, the chatter vibration is obvious and causes low machining productivity and poor surface quality. Therefore, the effects of tool postures on the cutterworkpiece engagement region are investigated firstly.

3.1 Transformation of coordinate system

For defining cutter-workpiece engagement region, three main coordinate systems are defined in five-axis milling process, as shown in Fig. 2. The first one is determined as machine coordinate system (MCS), and the directions are X , Y , and Z axes of machine tool. Then, the process coordinate system (FCN) consisted of feed F, cross-feed C, and surface normal N axes. Thirdly, the tool-axis coordinate system (TCS) is a moving coordinate system; the coordinate system is rotated corresponding to the change direction of the FCN by lead angle α and tilt angle β. In five-axis milling, tool tip position, tool orientation, and milling process parameters can be extracted from the cutter location (CL) file obtained using the commercial software Siemens NX7.5® in MCS coordinate system. In order to investigate the effects of tool posture (lead angle α and tilt angle β) on the chatter stability, an appropriate method is used to transform the CL information in MCS to TCS. Firstly, the origin of the MCS is translated to the origin of FCS by transformation matrix $T_{\text{M-F}}$. Then, the translated coordinates are rotated to the FCS by rotation matrix R_{M-F} . Finally, the tool-axis coordinate system can be obtained by rotating the FCS considering the appropriate lead angle α and tilt angle β with a rotation matrix R_{F-T} . According to Ref. [[14\]](#page-11-0), the translation relation of MCS to TCS can be expressed as

$$
(X, Y, Z, 1)T_{\text{M-F}}R_{\text{M-F}}R_{\text{F-T}} = (x, y, z, 1)
$$
 (1)

where $(X, Y, Z, 1)$ represents the tool orientation vector extracted from the CL file in MCS, and $(x, y, z, 1)$ is the tool orientation vector obtained by a series of transformation process.

In order to determine lead angle α and tilt angle β , the rotation transformation of the unit tool orientation vector (P_i) , P_i , P_k) extracted from CL file is performed. The transformation relationship can be expressed as

$$
(P_{\rm i}, P_{\rm j}, P_{\rm k}, 1)R_{\rm M\text{-}F}R_{\rm F\text{-}T} = (0, 0, 1, 1) \tag{2}
$$

system (MCS), process coordinate system (FCN), and tool-axis coordinate system (TCS). a Blade profile in MCS. b Machining process model

Solving Eq. ([2\)](#page-2-0), lead angle α and tilt angle β can be calculated as

$$
\alpha = \arcsin(P_i F_i + P_j F_j + P_k F_k)
$$

$$
\beta = \arctan\left(-\frac{P_i C_i + P_j C_j + P_k C_k}{P_i N_i + P_j N_j + P_k N_k}\right)
$$
(3)

where (F_i, F_i, F_k) , (C_i, C_i, C_k) , and (N_i, N_i, N_k) are unit orientation vectors in-process coordinate system (FCN).

3.2 Generation of the feasible contact surfaces

In milling process, only partial tool surface contacts the inprocess workpiece in the feed direction. The surface is defined as feasible contact surface (FCS) according to Aras (2008) [\[23\]](#page-12-0). When FCS contacts with the in-process workpiece, the cutter-workpiece engagements (CWEs) are generated since the CWEs are the subsets of the FCS. The boundaries of the FCS are defined by the cutter geometry and the envelop boundary set. For calculating FCS of the ball-end cutter performing five-axis tool motions, the tangency function P is defined by the tool surface normal $N(h, \theta, t)$ and the instantaneous velocity $V(h, \theta, t)$. The tangency function P can be expressed as

$$
P(h, \theta, t) = N(h, \theta, t) \cdot V(h, \theta, t)
$$
\n⁽⁴⁾

where h and θ are the parameters representing tool surfaces. t is the instantaneous cutting time corresponding to specific cutter locations. At any instant, the cutter surface boundary consisted of three sub-boundaries: forward boundary, envelope boundary, and backward boundary, which are illustrated in Fig. 3. The different boundaries can be defined as

in forward boundary, $P(h, \theta, t) > 0$,

in envelope boundary, $P(h, \theta, t) = 0$, and

in backward boundary, $P(h, \theta, t) < 0$.

Therefore, the accurate CWEs should meet the inequality $P(h, \theta, t) \geq 0$.

Fig. 3 Boundary partition of ball-end cutter

3.3 Generation of cutter-workpiece engagement maps

For simple workpiece in fixed axis machining, actual engagement status can be easily calculated by uncomplicated mathematical algorithm. However, for complex geometric workpiece, condition parameters (such as tool orientation, axial depth of cut, radial depth of cut, feed rate, and spindle speed) will be real-time changes in machining. These make the extraction of cutter-workpiece engagement region difficult. To overcome these problems, spatio-temporal mapping method (STMM) based on VERICUT is proposed in this work. Spatio-temporal mapping method is defined as the process conditions at a specified moment, which is calculated by simulation considering the actual cutter geometries; then, these process conditions is mapped to the actual machining signals to the projected plane and analyzed the actual machining conditions. In this process, the accurate cutter-workpiece engagement region data are obtained by performing VERICUT secondary development. These data are conducive to predict the physics properties of the cutting process such as cutting force and chatter stability. The schematic diagram of obtaining cutter-workpiece engagement maps was based on VERICUT secondary development, which is shown in Fig. [4.](#page-4-0)

In VERICUT secondary development, cutter-workpiece engagement projective region is introduced to calculate realtime cutting condition. The projective region is the projection of cutter-workpiece engagement region in vertical direction of the feed plane. VERICUT secondary development technology can easily obtain real-time machining conditions. Then, some relative functions will be used to acquire condition data such as opapi_get_grid_map (SOPAPI_MAP*map) for cutterworkpiece engagement data. Finally, the dynamic cutterworkpiece engagement data are extracted and stored in SOPAPI_MAP for cutting force prediction.

4 Chatter stability model of the thin-walled workpiece dynamic milling

4.1 Formulation of the chatter stability problem

The instantaneous cutter-workpiece engagement region can be affected by the change of tool posture in five-axis ballend milling, which can cause extensive dynamic cutting force. In different engagement regions, the magnitudes and direction of dynamic cutting forces are changing in different tool postures, which affect the chatter stability of thin-walled flexible workpiece in milling. Therefore, tool posture parameters (such as lead and tilt angles) should be considered in modeling dynamic cutting forces in five-axis ball-end milling.

A cutting force model is developed to predict the dynamic cutting force at each cutter location point in five-axis ball-end milling. The schematic diagram of the cutting force model

Fig. 4 Schematic diagram of obtaining cutter-workpiece engagement maps

with a typical ball-end mill is illustrated in Fig. 5. The dynamic cutting forces on tooth i in tangential, radial, and axial directions are defined as follows [\[24,](#page-12-0) [25\]](#page-12-0):

$$
\begin{cases}\ndF_{\rm ii} = K_{\rm te} ds + K_{\rm tc} h(\phi_{\rm i}, \kappa) db \\
dF_{\rm ri} = K_{\rm re} ds + K_{\rm rc} h(\phi_{\rm i}, \kappa) db \\
dF_{\rm ai} = K_{\rm ae} ds + K_{\rm ac} h(\phi_{\rm i}, \kappa) db\n\end{cases} \tag{5}
$$

where K_{te} , K_{re} , K_{ae} and K_{tc} , K_{rc} , K_{ac} are the cutting force coefficients considering the cutter edge, shear interaction in tangential, radial, and axial directions, respectively, which depend on the cutter-workpiece engagement region. κ, ϕ_i, ds , and db are the axial and radial immersion angular of tooth i , the cutting edge length, and the projected length of cutting flute in cutting region along the cutting velocity, respectively. $h(\phi_i, \kappa)$ is the dynamic uncut chip thickness normal to the cutting edge calculated by [\[26](#page-12-0), [27\]](#page-12-0).

During milling process, the edge forces did not contribute to the regenerative chatter mechanism [\[16](#page-11-0)]. Therefore, for analyzing the chatter stability, Eq. (5) can be rewritten as

$$
\begin{cases}\n dF_{\text{ti}} = K_{\text{tc}}h(\phi_{\text{i}}, \kappa)db \\
 dF_{\text{ri}} = K_{\text{rc}}h(\phi_{\text{i}}, \kappa)db \\
 dF_{\text{ai}} = K_{\text{ac}}h(\phi_{\text{i}}, \kappa)db\n\end{cases}
$$

where K_{tc} , K_{rc} , K_{ac} , $h(\phi_i, \kappa)$ and db depending on the cutterworkpiece engagement region are determined by the tool posture (lead angle α and tilt angle β) and are calibrated by slotting milling experiments considering the change of the tool posture.

Then, for the sake of analysis, those forces are mapped into Cartesian coordinate system:

$$
\begin{bmatrix} dF_{\rm xi} \\ dF_{\rm yi} \\ dF_{\rm zi} \end{bmatrix} = \begin{bmatrix} -\sin(\kappa)\sin(\phi_{\rm i}) & -\cos(\kappa)\sin(\phi_{\rm i}) & -\cos(\phi_{\rm i}) \\ -\sin(\kappa)\cos(\phi_{\rm i}) & -\cos(\kappa)\cos(\phi_{\rm i}) & \sin(\phi_{\rm i}) \\ \cos(\kappa) & -\sin(\kappa) & 0 \end{bmatrix} \begin{bmatrix} dF_{\rm ti} \\ dF_{\rm ri} \\ dF_{\rm ai} \end{bmatrix}
$$

For obtaining the data of cutter-workpiece engagement \sqrt{t} . gion, the cutter-workpiece engagement region is discretized by the steps of Δz , and the corresponding axial depth of cut is Δa , which is shown in Fig. [6](#page-5-0). Therefore, the cutting width db can be calculated by

$$
db = \frac{\Delta a}{\cos \gamma \sin \kappa} \tag{8}
$$

where the inclination angle γ is defined by the angle between the tool axis (z) , and the surface normal (N) due to the lead and tilt angle is defined using the form

$$
\gamma = \arccos[\cos(\alpha)\cos(\beta)]\tag{9}
$$

Substituting Eqs. (8) and (9) into Eq. (7) and summing the cutting force generated by all the flutes, the total dynamic cutting force can be obtained as follows:

$$
\begin{bmatrix} F_{\mathbf{x}}(t) \\ F_{\mathbf{y}}(t) \\ F_{\mathbf{z}}(t) \end{bmatrix} = a \left(\sum_{i=1}^{m} A_0^i \right) \begin{bmatrix} \Delta x_d \\ \Delta y_d \\ \Delta z_d \end{bmatrix}
$$
 (10)

with

 (6)

$$
A_0^i = \frac{1}{T} \int_{0}^{T} A^i(t) dt
$$

$$
A^{i}(t) = \sum_{r=-\infty}^{r=\infty} \left(\sum_{j=1}^{N} \frac{1}{\cos\gamma \sin\kappa} \begin{bmatrix} -\sin(\kappa)\sin(\phi_{i}) & -\cos(\kappa)\sin(\phi_{i}) & -\cos(\phi_{i})\\ -\sin(\kappa)\cos(\phi_{i}) & -\cos(\kappa)\cos(\phi_{i}) & \sin(\phi_{i})\\ \cos(\kappa) & -\sin(\kappa) & 0 \end{bmatrix} \begin{bmatrix} K_{\text{tc}}\\ K_{\text{rc}}\\ K_{\text{ac}} \end{bmatrix} \right) e^{ir\omega_{i}t}
$$

where a is the axis depth of cut, T is the tooth passing periods, and ω -represents the tool passing frequency. N is the number of flutes on the cutter.

4.2 Solution of chatter stability problem

Using single frequency solution method, the dynamic response of the milling system at chatter frequency is considered in frequency domain. Then, the dynamic displacements are calculated using the frequency response function matrix (FRF) of the cutter-workpiece structure and can be expressed as

$$
\begin{bmatrix}\n\Delta x_{\rm d} \\
\Delta y_{\rm d} \\
\Delta z_{\rm d}\n\end{bmatrix} = (1 - e^{-i\omega_{\rm c}T}) \Phi(i\omega_{\rm c}) \begin{bmatrix}\nF_x(i\omega_{\rm c}) \\
F_y(i\omega_{\rm c}) \\
F_z(i\omega_{\rm c})\n\end{bmatrix} e^{i\omega_{\rm c}t}
$$
\n(11)

where ω_c is the chatter frequency. $\Phi(i\omega_c) = \Phi_c(i\omega_c) + \Phi_w(i\omega_c)$ is the frequency response function determined by the cutterworkpiece contact zone in tool coordinate system.

Therefore, Eq. [\(10](#page-4-0)) can be rewritten as in frequency domain:

$$
\begin{bmatrix} F_x(i\omega_c) \\ F_y(i\omega_c) \\ F_z(i\omega_c) \end{bmatrix} e^{i\omega_c t} = a \left(\sum_{i=1}^m A_0^i \right) \left(1 - e^{-i\omega_c T} \right) \Phi(i\omega_c) \begin{bmatrix} F_x(i\omega_c) \\ F_y(i\omega_c) \\ F_z(i\omega_c) \end{bmatrix} e^{i\omega_c t} \tag{12}
$$

In the chatter frequency ω_c , the system is critically stable; therefore, the characteristic equation of the closed-loop dynamic milling system is approximated with single frequency solution, and its non-trivial solution can be calculated as

Fig. 6 Cutter-workpiece engagement region analysis. a Dynamic cutting force in x, y, and z directions. **b** Inclination angle on the *i*th disk element

$$
\det \left[I - a \left(\sum_{i=1}^{m} A_0^{i} \right) \left(1 - e^{-i\omega_c T} \right) \Phi(i\omega_c) \right] = 0 \tag{13}
$$

Then, in order to improve the surface quality and machining productivity of the thin-walled workpiece, the method of chatter suppression considering the lead and tilt angles is investigated. Consequently, taking the tool posture into consideration in milling, the identified frequency response function matrix $\Phi(i\omega_c)$ should be oriented by a transformation matrix T_{G} as follows [\[28](#page-12-0)]:

$$
\Phi_0(i\omega_{\rm c}) = T'_{\rm G}\Phi(i\omega_{\rm c})\boldsymbol{T}_{\rm G}
$$
\n(14)

with

$$
\boldsymbol{T}_{\mathrm{G}} = \begin{bmatrix} f_{X} & c_{X} & n_{X} \\ f_{Y} & c_{X} & n_{X} \\ f_{Z} & c_{X} & n_{X} \end{bmatrix} \begin{bmatrix} -\sin(\kappa)\sin(\phi_{i}) & -\cos(\kappa)\sin(\phi_{i}) & -\cos(\phi_{i}) \\ -\sin(\kappa)\cos(\phi_{i}) & -\cos(\kappa)\cos(\phi_{i}) & \sin(\phi_{i}) \\ \cos(\kappa) & -\sin(\kappa) & 0 \end{bmatrix}
$$

where superscript ' is the transpose operation. f_X , f_Y , f_Z ; c_X , c_Y , c_Z ; and n_X , n_Y , n_Z are the unit component of feed, cross-feed, and surface normal vector in MCS, respectively. Therefore, Eq. (13) can be simplified as the form

$$
\det[I + A\Phi_0(i\omega_c)] = 0 \tag{15}
$$

with

$$
\Lambda = -a \left(\sum_{i=1}^{m} A_0^i \right) \left(1 - e^{-i\omega_c T} \right)
$$

In five-axis milling process, the axis depth of cut a_{lim} varies with the lead and tilt angle changing, which affects the stability limits of the milling process considerably. Therefore, the critical chatter-free axis depth of cut a_{lim} and the corresponding spindle speeds n are calculated and the system stability was obtained and analyzed according to Altintas Y (2000) Ref. [\[26](#page-12-0)].

5 Experimental results and discussion

The effects of tool posture on the dynamic cutting force and chatter stability are verified by experiments in this section. The experimental setup is shown in Fig. [7.](#page-6-0) The cutting forces are measured using a Kistler 9255B dynamometer. All of the simulation and experimental cases are designed for GH4169

Fig. 7 Experimental setup of blade in five-axis milling center

block with $86 \times 46 \times 20$ mm dimension. A cemented carbide milling cutter with the diameter of 8 mm and four flutes and the helical angle 40° was used in all simulation and experimental cases. Experimental tests are carried out in MIKRON UCP 1350 five-axis milling center.

5.1 Effect of tool posture on cutter-workpiece engagement region

In order to investigate the effects of tool posture on cutterworkpiece engagement region in milling, the proposed method is used to extract the instantaneous cutter-workpiece engagement region. The machining tool path (CL files) is obtained using the commercial CAM soft Siemens NX7.5®. Then, the virtual machining process is simulated using the CL files by VERICUT software to obtain the cutterworkpiece engagement region based on the proposed method.

Without loss of generality, the semi-finishing method is considered in machining. In milling process, the cutterworkpiece engagement regions in the 58th CL point on the fifth cut tool path in different tool postures are extracted using the proposed method. And the projection of the engagement regions is mapped on the projected plane perpendicular to the feed direction and the tool-axis direction, which are shown in Fig. [8.](#page-7-0) When the other cutting parameters are constant, it can be seen that different tool postures with different lead and tilt angles can generate different cutter-workpiece engagement regions. In Fig. [8](#page-7-0)a, the lead angle 10° and tilt angle 0° are selected; the engagement angle on the engagement region is determined by the cutter entry angles (−55° to −9°) and the cutter exit angles (1.5° to 237.2°) in the current condition. Under the lead angle 15° and tilt angle 5° in Fig. [8b](#page-7-0), the cutter entry angles and the cutter exit angles are from −45.4° to −8.3° and 1.8° to 241.2°, while the magnitude of cutter entry angles and cutter exit angles is symmetrical distribution in the lead angle 30° and tilt angle 10°, as shown in Fig. [8](#page-7-0)c. In Fig. [8](#page-7-0)d, the engagement region is symmetrical distribution along the y direction in the lead angle 45° and tilt angle 0° approximately.

In Fig. [8,](#page-7-0) the different engagement regions in different tool postures are obtained, which lead to the variation of dynamic cutting force according to Eq. [\(6\)](#page-4-0). Therefore, in machining, the rational tool posture is chosen for improving the machining accuracy because different engagement regions have critical effects on dynamic cutting force and chatter stability.

5.2 Effect of tool posture on dynamic cutting force

In order to investigate the effects of tool posture (lead and tilt angle) on dynamic cutting force, experimental and simulation validations were conducted on different combinations of lead and tilt angles. Firstly, dynamic cutting force coefficients in different tool posture are calibrated by slot milling in small depth of cut and cutting width. Partial cutting force coefficients are listed in Table [1](#page-8-0). Then, the dynamic cutting force is predicted based on the proposed model with lead angle 45° and tilt angle 0°; the predicted results coincide with the measured results, which is shown in Fig. [9.](#page-8-0)

In Fig. [10,](#page-9-0) it is noted that the variations of the dynamic cutting force is simulated; the direction of which is normal to the machined surface in different tool postures and feed per tooth. For all of the cases, the spindle speed is 2000 rmp, the depth of cut is 0.2 mm, and the cutting width is 0.2 mm, respectively. In the experiments, the effects of the interference factors such as machine tool thermal character and dynamic characteristic on the dynamic cutting force are ignored; the effects of tool posture on the dynamic cutting force as the main objective is investigated only. Then, in different feed per tooth (0.04, 0.05, and 0.06 mm/z), the maximum resulting cutting force is selected for analyzing the change characteristics of dynamic cutting force in different tool postures. It can be seen that the maximum cutting force appears in the small lead angle and large tilt angle as shown in Fig. [10](#page-9-0). When the lead angle is large, the maximum cutting force is small. In addition, the effects of lead angle are more obvious than that of tilt angle. When the lead angle is large, the effects of tilt angle changes on the cutting force are small and vice versa. Therefore, in order to obtain stable milling process, the rational tool posture should be chosen to reduce the dynamic cutting force.

5.3 Effect of tool posture on chatter stability

In order to investigate the effects of tool posture on the chatter stability, the stability lobe diagrams with different combinations of lead and tilt angle for five-axis milling of the blade can be calculated based on the proposed method, which is shown in Fig. [11.](#page-9-0)

In Fig. [11,](#page-9-0) it can be noted that the chatter stability boundaries enlarge in vertical direction, while the boundaries almost have no changes in the horizontal direction. Therefore, for a five-axis milling system, different tool postures have obviously different chatter stability limits. This is why the cutter-

Fig. 8 Cutter-workpiece engagement region for the same CL point in different tool-axis postures. $\mathbf{a} \alpha = 10^{\circ}$, $\beta = 0^{\circ}$. $\mathbf{b} \alpha = 15^{\circ}$, $\beta = 5^{\circ}$. $\mathbf{c} \alpha = 30^{\circ}$, $\beta = 10^{\circ}$. **d** $\alpha = 45^{\circ}, \beta = 0^{\circ}$

Table 1 Dynamic cutting force coefficients in different tool postures

Lead angle	Tilt angle	$K_{\rm tc}$	$K_{\rm rc}$	$K_{\rm ac}$	K_{te}	K_{re}	K_{ae}
$(^\circ)$	(°)	(N/mm ²)	(N/mm ²)	(N/mm ²)	(N/mm)	(N/mm)	(N/mm)
30	θ	-68.22	-875.25	727.41	-52.54	-43.94	-34.72
45	θ	-12.73	323.46	-600.18	-46.24	-65.65	-99.99
60	θ	-15.41	-117.57	29.15	-40.61	-37.97	-50.21

workpiece engagement region varies with the change of tool posture, which determines dynamic cutting force.

Considering the stability lobe diagram, the cutting parameters such as spindle speed 2000 rmp, depth of cut 0.2 mm, and cutting width 0.2 mm are selected in milling. The acceleration sensor (Dytran 3055B, Sensitivity 52.5 mV/g, frequency range $1 \sim 10,000$ Hz, linearity $\pm 2\%$ FS, weight 10 g, temperature range −60~250 °C) is placed on the pressure surface. Numerical simulation predicted and measured vibration acceleration signal are analyzed for obtaining the effects of tool posture on the chatter stability. The results are illustrated in Fig. [12.](#page-10-0) It can be seen that the lead angles have the same effects on the vibration acceleration in different tilt angles and the amplitude of the vibration acceleration increases with the growth of the lead angles, which are shown in Fig. [12a](#page-10-0), b. While in large tilt angles, the amplitude of the vibration acceleration is large, and it decreases with the lead angle increasing firstly then, goes up, which is shown in Fig. [12c](#page-10-0). Consequently, the experimental results show that reasonable tool posture can suppress the vibration effectively.

In order to further verify the effectiveness and feasibility of the proposed method, the chatter stability problem should be deeply investigated by the method of energy spectrum. In milling, the change of vibration frequency can be used to

Fig. 9 Comparison of dynamic cutting forces with the measured and predicted cutting force in lead angle 45° and tilt angle 0°. a Cutting force component in X direction. **b** Cutting force component in Y direction. **c** Cutting force component in Z direction

Fig. 10 Comparison of the effect of tool posture on the maximum cutting force in different feed speed. $a f = 0.04$ mm/z. $b f = 0.05$ mm/z. $c f = 0.06$ mm/z

depict the milling state. Therefore, to effectively investigate the chatter stability, according to Parseval's theorem, the vibration signal energy E in frequency domain can be represented as $E = \int_{-\infty}^{+\infty} |s(f)|^2 df$ using energy spectrum density $s(f)$. However, for the discrete signal in milling, the vibration energy $E(i)$ in the *i*th spindle frequency integer times frequency band can be expressed as $E(i) = \sum_{i=0}^{i \times f + 5}$ $\sum_{n=i\times f-4}^{\infty} |A(n)|^2$ using the amplitude of energy spectral density $A(n)$ in frequency n (f is the spindle rotation frequency). Then, the signal frequency band energy ratio η can be defined as $\eta = \frac{\sum E(i)}{E}$ (0 < η < 1). It can be seen from the ratio η that the amplitude of vibration energy signals is large in stable milling. Nevertheless, in chatter, the energy signal shifts from signal basic frequency to chatter frequency, and the ratio η is small in this state. Consequently, in order to analyze the effectiveness of this

Fig. 11 Stability lobe diagrams under different combinations of lead and tilt angles

Fig. 12 Comparison of the vibration acceleration response with the changes of lead angle. a Tilt angle $\beta = 0^\circ$. b Tilt angle $\beta = 30^\circ$. c Tilt angle $\beta = 60$

method, one case, tool posture ($\alpha = 10^{\circ}$, $\beta = 5^{\circ}$), is used to investigate the vibration properties in different tool postures; the analysis results are shown in Fig. [13](#page-10-0).

In Fig. [13](#page-10-0), it is noted that when the vibration energy signals are around the spindle frequency and its integer times frequency, the ratio η is larger than 0.5 shown in 6– 15 s. While in chatter, dynamic cutting force signals change obviously and the energy ratio curve near 0 in 15–17 s. In addition, in 0–5 s, the energy ratio is not equal to 0; the reason for this is the machine tool vibration when it is operating. Therefore, signal frequency band energy ratio curve can effectively analyze the chatter stability in complex machining environment.

6 Conclusions

Due to the flexibilities of the thin-walled workpiece, obvious chatter is generated in five-axis ball-end milling process. In this work, a chatter stability model considering the tool posture is developed in milling. The changes of the tool posture will affect the cutter-workpiece engagement region in multiaxis milling thin-walled free surface parts. In addition, the dynamic cutting force, which is associated with chatter stability, depended on the cutter-workpiece engagement region to some extent. The proposed method is validated by experiments and numerical simulations. The main conclusions of this work are drawn as follows:

- 1. A new method of extracting the cutter-workpiece engagement region is proposed based on VERICUT secondary development. This method can rationally consider the actual cutter geometry parameters in machining compared with the other simulation approaches. In addition, the calculation efficiency in VERICUT is very high.
- 2. A dynamic cutting force model considering the tool posture (lead and tilt angle) is developed. The changes of tool posture have effects on dynamic cutting force. Therefore, according to the dynamic properties of the workpiece, the tool posture is adjusted to obtain small cutting force to suppress chatter.
- 3. A chatter stability model including the tool posture is proposed and analyzed. The model demonstrates that the feasibility of adjusting tool posture can effectively suppress machining chatter. Due to the varying engagement region and dynamic cutting force coefficients along the tool axis, it can be seen that the effects of tool posture such as lead and tilt angles on the stability limit can be significant. The lead and tilt angles can greatly change the chatter behavior and stability limit, and improve the surface quality and productivity. Finally, several experiments are conducted to verify the effectiveness of the proposed method.

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