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ORIGINAL ARTICLE

Phase I monitoring of simple linear profiles in multistage processes with cascade property

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Abstract When a multistage manufacturing process is monitored statistically, the cascade property results in a more complicated condition compared to the case when a single-stage process is controlled. The cascade property usually exists in different stages of a multistage process, where the quality of a stage influences the performance of the next stage. Moreover, sometimes the quality of a product/process is best characterized by a functional relationship. This relationship is referred to as a profile. In this paper, phase I monitoring of simple linear profile is addressed for a multistage process involving the cascade property. To aim this, the capabilities of the methods that may be used to monitor a profile in a multistage process are first assessed. Then, a statistic, named the U statistic, is introduced to provide the opportunity of removing the cascade property. This statistics provides quality engineers a way to reduce the complicated condition of monitoring a multistage process. The new approach also helps quality engineers to diagnose effectively the stage responsible for the out-of-control condition. To evaluate the effectiveness of the proposed approach, different simulated cases are analyzed numerically. In addition, a case study is provided to illustrate the applicability of the proposed method in real-world manufacturing environments.

Keywords Profile monitoring · Multistage process · Cascade property · Test power

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1 Introduction

A regression model can describe the quality of a product or process in many practical applications well. This approach helps quality engineers to monitor the quality of the product or the process by constructing a curve which is referred to as a profile. Profile monitoring attracted the attention of a large group of researchers recently. In general, profile monitoring may be classified into two main branches of (1) profile monitoring of single-stage processes and (2) profile monitoring of multistage processes. Several researchers such as Zou et al. [1], Moguerza et al. [2], Williams et al. [3], Saghaei et al. [4], Noorossana et al. [5], Kazemzadeh et al. [6], Jensen et al. [7], Kazemzadeh et al. [8], Saghaei et al. [9], Zhang et al. [10], Soleimani et al. [11], Zhang and Albin [12], Chicken et al. [13], Shiau et al. [14], Zhu and Lin [15], Noorossana et al. [16, 17], Chang and Yamada [18], Noorossana et al. [19], Amiri et al. [20], Chuang et al. [21], Narvand et al. [22], Soleimani and Noorossana [23], Wang and Tamirat [24], and Atashgar et al. [25] have contributed to the monitoring problem of the profile in a single-stage process. However, many manufacturing processes involve production in several consecutive stages.

The dependence between stages of a multistage process is an important property that must be considered in process monitoring. In other words, the quality of each stage of the process affects directly the performance of the next stage. This property of a multistage process is referred to as the cascade property. The cascade property, in fact, provides a complex condition in comparison to a single process. Literature addresses few works that focused on monitoring the profile in multistage processes. Esmaeeli and Sadegheih [26] developed two methods for monitoring a two-stage process in phase II using the profile approach. Eghbali et al. [27] proposed a phase II monitoring method for a simple linear profile of a process with two stages considering the cascade property. Imani and Amiri [28] addressed the phase II monitoring problem of logistics profiles in a multistage process.

In this paper, phase I profile monitoring of a multistage process involving the cascade property is focused on. The purpose of the statistical control in phase I is to ensure the stability of a process and estimate the parameters of the process. In phase II monitoring, however, one aims to detect the shifts or trends manifested by a special cause in the process. This paper provides a comparative evaluation of three different methods of controlling the parameters of a profile considering the cascade property. While these methods are not capable of removing the impact of the cascade property, a transformation method is proposed in this paper to remove (or minimize) the impact of the cascade property. Through comparative investigations, we will show that monitoring a multistage process based on the proposed transformation is more effective compared to the case where the cascade property is not removed. The proposed approach also allows quality engineers to identify the stage responsible for the out-of-control condition in order to perform an effective root cause analysis.

The rest of the paper is organized as follows. In the next section, a multistage process is formulated by a regression model. The performance of three methods of phase I monitoring is evaluated in Section 3. The proposed transformation method is discussed in Section 4. Section 5 is allocated for evaluating the performance of the proposed approach, numerically. To illustrate the applicability of the proposed model, Section 6 is supported by a real case study. Finally, the last section is devoted to concluding remarks.

2 Multistage process modeling

In a multistage process, the output quality of each stage depends on two main factors. The first factor is the quality of the activities at this stage, and the second is the performance of the previous stage(s). This correlated performance, which is referred to as the cascade property, is of great importance when a multistage process is monitored. A multistage process can be expressed as the following regression equations:

$$y_{ik1} = A_0 + A_1 x_i + \varepsilon_{ik1},\quad \mathrm{i} = 1, 2, \ldots, n; \mathrm{k} = 1, 2, \ldots, m \qquad (1)$$

$$y_{iks} = \varnothing y_{ik(s-1)} + \alpha_{1s} + \alpha_{2s} x_i + \varepsilon_{iks},\qquad \mathrm{s} = 2, 3, \ldots, S \qquad (2)$$

where for the k^{th} sample, (x_i, y_{ik1}) and (x_i, y_{iks}) are the i^{th} observed outputs for the first and the s^{th} stages, respectively. Moreover, A_0 and A_1denote the intercept and the slope of the simple linear profile in Eq. 1, respectively. The special effects of stage s on the intercept and the slope are denoted by α_{1s} and α_{2s},respectively. Furthermore, ε_{ik1} and ε_{iks} are random error terms that follow $N\sim\left(0, \sigma_1^2\right)$ and $N\sim\left(0, \sigma_s^2\right)$, respectively, and $\varnothing$ is the auto-correlation coefficient (stage correlation). For the sake of simplicity, the variance of the error term in all stages is assumed to remain constant, i.e., $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_s^2 = \sigma^2$. Besides, it is possible to rewrite Eq. 2 as Eq. 3.

$$y_{iks} = \varnothing^{s-1} y_{ik1} + \sum_{r=2}^{s} \varnothing^{s-r}\left(\alpha_{1r} + \alpha_{2r} x_i + \varepsilon_{ikr}\right).\qquad \mathrm{s} = 2, 3, \ldots, S \qquad (3)$$

In addition, similar to Mahmoud and Woodall [29], it is assumed that the independent variable x_i takes the values 0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, and 1.8 in both Eqs. 1 and 2.

As phase I monitoring is involved in this paper, it is necessary to estimate the process parameters. Knowing the first stage is independent of the next stages, A_0 and A_1 are estimated using the least squares method for sample k as follows:

$$a_k = \overline{y}_k - b_k \overline{x}_k \qquad (4)$$

$$b_k = \frac{S_{XY(k)}}{S_{XX(k)}}, \qquad (5)$$

where

$$\overline{x}_k = \frac{1}{n}\sum\nolimits_{i=1}^{n} x_{ik} \qquad \overline{y}_k = \frac{1}{n}\sum\nolimits_{i=1}^{n} y_{ik} \qquad (6)$$

$$S_{XX(k)} = \sum\nolimits_{i=1}^{n}\left(x_{ik} - \overline{x}_k\right)^2 \qquad S_{XY(k)} = \sum\nolimits_{i=1}^{n}\left(x_{ik} - \overline{x}_k\right) y_{ik}. \qquad (7)$$

In this phase, $\varnothing$ is estimated as follows:

$$\hat{\varnothing}_{k,s} = \left[Y', Y_{k,s-1}, -, Y'_{k,s-1}, X, \left(X' X\right)^{-1}, X', Y_{k,s-1}\right]^{-1}$$

$$\left[Y'_{k,s} Y_{k,s-1} - Y'_{k,s} X \left(X' X\right)^{-1} X' Y_{k,s-1}\right]$$

$$s = 2, 3, \ldots, S, \qquad (8)$$

that results in

$$\hat{\varnothing} = \frac{\sum\nolimits_{S=2}^{S}\sum\nolimits_{k=1}^{m} \hat{\varnothing}_{k,s}}{(S-1)m}. \qquad (9)$$

Table 1 T^2 statistics of Kang and Albin [31], Stover and Brille [30], and Williams [3]

T^2 of Kang & Albin	T^2 of Stover & Brille	T^2 of Williams
$T_k^2 = \frac{m(Z_k - \bar{Z})^T S^{-1}(Z_k - \bar{Z})}{m-1}$ $k = 1,2,\ldots,m$ where $Z_k = (a_k \quad b_k)^T$ $\quad \bar{Z} = (\bar{a} \quad \bar{b})^T$ $\bar{a} = \frac{1}{m}\sum_{k=1}^{m} a_k$ $\bar{b} = \frac{1}{m}\sum_{k=1}^{m} b_k$ $S = MSE\begin{pmatrix} \frac{1}{n} + \frac{\bar{X}^2}{S_{xx}} & \frac{-\bar{X}}{S_{xx}} \\ \frac{-\bar{X}}{S_{xx}} & \frac{1}{S_{xx}} \end{pmatrix}$ $MSE = \frac{1}{m}\sum_{k=1}^{m} MSE_k \quad \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ $S_{xx} = \sum_{i=1}^{n}(X_i - \bar{X})^2$ $MSE_k = \frac{1}{(n-2)}\sum_{i=1}^{n} e_{ik}^2$ $e_{ik} = y_{ik} - a_k - b_k X_{ik}$ $i = 1,2,\ldots,n \quad k = 1,2,\ldots,m$	$T_k^2 = (Z_k - \bar{Z})^T S^{-1}(Z_k - \bar{Z})$ $k = 1,2,\ldots,m$ where $Z_k = (a_k \quad b_k)^T$ $\quad \bar{Z} = (\bar{a} \quad \bar{b})^T$ $\bar{a} = \frac{1}{m}\sum_{k=1}^{m} a_k \quad \bar{b} = \frac{1}{m}\sum_{k=1}^{m} b_k$ $S = \begin{pmatrix} \frac{\sum_{k=1}^{m}(a_k - \bar{a})^2}{m-1} & \frac{\sum_{k=1}^{m}(a_k - \bar{a})(b_k - \bar{b})}{m-1} \\ \frac{\sum_{k=1}^{m}(a_k - \bar{a})(b_k - \bar{b})}{m-1} & \frac{\sum_{k=1}^{m}(b_k - \bar{b})^2}{m-1} \end{pmatrix}$	$T_k^2 = \left(\widehat{\beta_k} - \bar{\hat{\beta}}\right)^T S^{-1}(\widehat{\beta_k} - \bar{\hat{\beta}})$ $k = 1,2,\ldots,m$ where $\hat{\beta}_k = (X\acute{X})^{-1}\acute{X}Y$ $\bar{\hat{\beta}} = \frac{1}{m}\sum_{k=1}^{m}\widehat{\beta_k}$ $\hat{v}_{k+1} = \hat{\beta}_{k+1} - \hat{\beta}_k$ $k = 1,2,\ldots.m-1$ $\hat{V} = \begin{bmatrix} \hat{v}_2^T \\ \vdots \\ \hat{v}_m^T \end{bmatrix}$ $S = \frac{\hat{V}^T \times \hat{V}}{2(m-1)}$
Upper control limit$=2F_{2,m(n-2),\alpha}$	$Upper\ control\ limit = \frac{(m-1)^2 B_{1,{(m-3)}/{2},\alpha}}{m}$	$Upper\ control\ limit = \frac{(f-1)^2}{f} Beta_{1-\alpha,\frac{p}{2},\frac{f-p-1}{2}}$ *where* $f = \frac{2(m-1)^2}{3m-4}$ *p= Number of parameters*

3 Performance evaluation of profile monitoring methods

Literature introduces several methods for phase I monitoring of a simple linear profile in a single-stage process. In this section, the performances of three well-known methods including T^2 of Stover and Brille [30], T^2 of Kang and Albin [31], and T^2 of Williams et al. [3] in profile monitoring of a multistage process in phase I are evaluated.

Table 2 Test powers of the three methods per change of the intercept in A_0 to $A_0+\lambda\frac{\sigma}{\sqrt{n}}$

	λ									
T^2 Kang & Albin	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Ø = 0	0.1729	0.1738	0.1747	0.1765	0.1773	0.1790	0.1810	0.1815	0.1856	0.1890
Ø = 0.1	0.1792	0.1815	0.1775	0.1895	0.2000	0.2212	0.2284	0.2501	0.2684	0.3096
Ø = 0.3	0.1863	0.2107	0.2533	0.3659	0.4924	0.6553	0.8009	0.9094	0.9681	0.9911
Ø = 0.5	0.1965	0.2656	0.4434	0.6901	0.8925	0.9769	0.9975	1	1	1
Ø = 0.7	0.2115	0.3639	0.6528	0.9014	0.9889	0.9998	1	1	1	1
Ø = 0.9	0.2157	0.4431	0.7960	0.9767	0.9989	1	1	1	1	1
T^2 Stover & Brill	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Ø = 0	0.1805	0.1809	0.1830	0.1850	0.1852	0.1856	0.1861	0.1949	0.1971	0.2008
Ø = 0.1	0.1806	0.1807	0.1809	0.1817	0.1856	0.1929	0.2012	0.2045	0.2093	0.2262
Ø = 0.3	0.1921	0.1993	0.2062	0.2298	0.2832	0.3405	0.4193	0.4934	0.5756	0.6485
Ø = 0.5	0.1988	0.2104	0.2657	0.3562	0.4829	0.5975	0.7132	0.7964	0.8654	0.9261
Ø = 0.7	0.1955	0.2380	0.3446	0.4877	0.6490	0.7650	0.8662	0.9336	0.9676	0.9871
Ø = 0.9	0.2057	0.2722	0.4159	0.5921	0.7508	0.8639	0.9392	0.9766	0.9926	0.9980
T^2 Williams	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Ø = 0	0.2512	0.2593	0.2659	0.2663	0.2665	0.2669	0.2688	0.2693	0.2800	0.2900
Ø = 0.1	0.2612	0.2642	0.2651	0.2838	0.2936	0.3101	0.3275	0.3492	0.3662	0.4015
Ø =0.3	0.2690	0.3099	0.3669	0.4625	0.5837	0.7170	0.8243	0.8973	0.9493	0.9771
Ø = 0.5	0.2954	0.3938	0.5911	0.7810	0.9152	0.9769	0.9966	0.9992	0.9997	1
Ø = 0.7	0.3244	0.5486	0.8191	0.9611	0.9955	0.9997	1	1	1	1
Ø = 0.9	0.3684	0.7052	0.9437	0.9968	1	1	1	1	1	1

Table 3 Test powers of the three methods per change of the slope in A_1 to $A_1+\beta\frac{\sigma}{\sqrt{S_{xx}}}$

	β									
T^2 Kang & Albin	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Ø = 0	0.1717	0.1726	0.1741	0.1750	0.1751	0.1753	0.1804	0.1841	0.1890	0.2000
Ø = 0.1	0.1798	0.1818	0.1891	0.1913	0.2053	0.2261	0.2395	0.2642	0.2851	0.3329
Ø = 0.3	0.1868	0.2161	0.2725	0.4038	0.5489	0.7141	0.8597	0.9464	0.9849	0.9963
Ø = 0.5	0.1978	0.2906	0.4894	0.7580	0.9274	0.9916	0.9992	1	1	1
Ø = 0.7	0.2182	0.3914	0.7179	0.9395	0.9969	0.9998	1	1	1	1
Ø = 0.9	0.2250	0.4925	0.8538	0.9899	0.9999	1	1	1	1	1
T^2 Stover & Brill	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Ø = 0	0.1833	0.1848	0.1853	0.1857	0.1936	0.1940	0.1947	0.1950	0.1956	0.1962
Ø = 0.1	0.1957	0.1959	0.1976	0.1980	0.1982	0.1986	0.2019	0.2080	0.2124	0.2246
Ø = 0.3	0.1981	0.1970	0.2113	0.2463	0.3092	0.371	0.4505	0.5375	0.6218	0.6904
Ø = 0.5	0.1982	0.2130	0.2783	0.3918	0.5116	0.6441	0.7493	0.8386	0.8973	0.9465
Ø = 0.7	0.1955	0.2550	0.3720	0.5271	0.6899	0.8104	0.8981	0.9537	0.9823	0.9945
Ø = 0.9	0.2025	0.2829	0.4465	0.6421	0.7982	0.9007	0.9590	0.9885	0.9960	0.9997
T^2 Williams	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Ø = 0	0.2553	0.2558	0.2560	0.2562	0.2565	0.2596	0.2626	0.2626	0.2635	0.2650
Ø = 0.1	0.2459	0.2570	0.2607	0.2751	0.2791	0.2969	0.3230	0.3550	0.3805	0.4089
Ø = 0.3	0.1947	0.2270	0.3069	0.4115	0.5383	0.6831	0.8023	0.8966	0.9431	0.9750
Ø = 0.5	0.1218	0.2112	0.4124	0.6541	0.8383	0.9429	0.9867	0.9982	0.9998	1
Ø = 0.7	0.0648	0.2235	0.5341	0.8076	0.9485	0.9901	0.9992	0.9998	1	1
Ø = 0.9	0.0395	0.2312	0.5970	0.8631	0.9679	0.9952	0.9996	1	1	1

Suppose the following regression models define the first and the second stages of a two-stage process with the cascade property, respectively.

$$Y_{ik1} = 3 + 2x_i + \varepsilon_{ik1} \quad (10)$$

$$y_{ik2} = (3\varnothing + \alpha_1) + (2\varnothing + \alpha_2)x_i + \varnothing\varepsilon_{ik1} + \varepsilon_{ik2}, \quad (11)$$

where $(3\varnothing + \alpha_1) = 2$ and $(2\varnothing + \alpha_2) = 1$. Let ε_{ik1} and ε_{ik2} follow the standard normal distribution (mean zero and variance 1). Here, we evaluate the performances of the three methods where the process is modeled by Eq. 11 and experiences different shifts. Table 1 shows the defined statistics of each method along with their upper control limits.

Tables 2 and 3 show the detection probability of out-of-control conditions for intercept and slope, respectively. These tables show a comparative report of the capabilities of the three studied methods. These data are produced by simulation. The simulation is iterated 10,000 times for each step-shift size. In other words, the experiments shown in Tables 2 and 3 lead into the changes that may be experienced by the slope and the intercept of a multistage process. In this evaluation, the intercept A_0 and the slope A_1 are shifted as $A_0 + \lambda\frac{\sigma}{\sqrt{n}}$ and $A_1 + \beta\frac{\sigma}{\sqrt{S_{xx}}}$, respectively, assuming $\alpha = 0.01$. As shown in Tables 2 and 3, the analysis is reported for λ = 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, and 5.0 and β = 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, and 5.0.

Figures 1, 2, and 3 depict the capability of the three studied methods in terms of the test power. In this analysis, a scheme signals in the second stage of a two-stage process when the intercept of the process is shifted in the first stage. The capabilities of the methods are shown graphically in Figs. 4, 5, and 6, when the slope is shifted in the first stage. In both cases, it is assumed $\alpha = 0.01$.

The curves shown in the above six figures address clearly that the test power term has an increasing trend. In other words, as the shift size increases, the power of detecting an out-of-control condition for all levels of ∅ increases as well. Meanwhile, when the value of ∅ (the cascade property) is increased, the test power, i.e., the detection probability of out-of-control signal increases as expected.

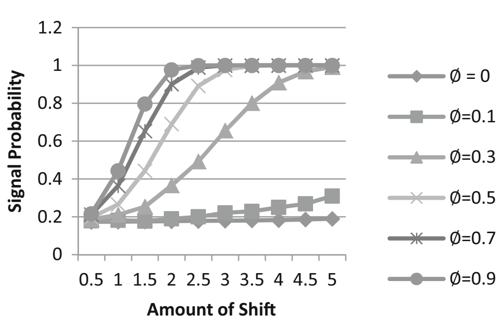

Fig. 1 Capability of the T^2 of Kang and Albin [31] method with intercept change

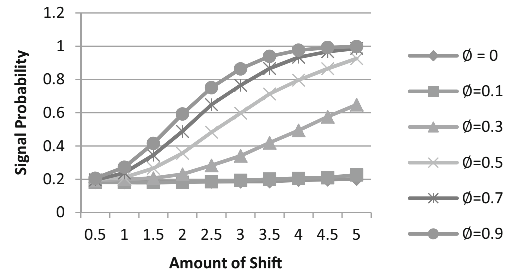

Fig. 2 Capability of the T^2 of Stover and Brille [30] method with intercept change

The above analysis indicates that the T^2 of Kang and Albin [31] is superior compared to the other two methods in terms of the test power. The figures also indicate that more increase in the shift size leads into more increase in the value of the test power for the superior method. This change occurs faster for bigger ∅ values. In other words, for the case that the values of the shift size and ∅ are high, the method signals an out-of-control faster compared to other cases.

In the above analysis, the cascade property has not been removed by the methods under investigation. In other words, these methods are not capable of removing the cascade property involved in a multistage process. In addition, the above methods are not capable of performing diagnostic analysis either. In the next section, a novel approach is proposed to not only remove the cascade property but also to perform a diagnostic analysis in order to determine the stage responsible for an out-of-control signal.

4 The proposed method

As discussed above, the cascade property of a multistage process is an important factor that affects the test power of a scheme. If a method addresses the capability of removing the impact of the cascade property, it may increase the signaling power. Furthermore, this remove of the cascade property provides the opportunity of conducting diagnostic analysis that leads into the identification of the stage(s) responsible for the process change. However, the methods investigated in Section 3 are only capable of signaling an out-of-control condition, and not able to diagnose the fault. In other words,

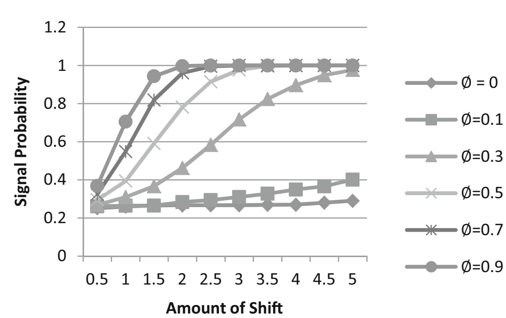

Fig. 3 Capability of the T^2 of Williams [3] method with intercept change

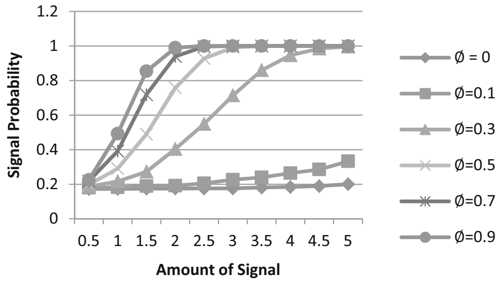

Fig. 4 Capability of the T2 of Kang and Albin [31] method with slope change

eliminating the cascade property at any stage of a multistage process provides the opportunity to test each stage independently. In this case, the test power term shows the change of a stage without having the impact of the previous stage(s). This opportunity provides the capability of identifying the fault at a stage where the fault manifested itself to the multistage process. In this paper, the U-transformation (firstly introduced by Hauck et al. [32]) is tailor made for phase I profile monitoring of a multistage process as follows.

Without loss of generality, consider a two-stage process. In the case that the specifications of the process are characterized effectively by linear profiles, the U-transformation can be defined for the first and the second stages as shown in Eqs. 12 and 13, respectively.

$$U_{j1} = \boldsymbol{A}_1 \tag{12}$$

$$U_{js} = \boldsymbol{A}_s - \Sigma_{s(s-1)} \Sigma^{-1}_{(s-1)(s-1)} \boldsymbol{A}_{s-1}, \tag{13}$$

where $\boldsymbol{A}_{s-1}$ is a vector estimator of the intercept and the slope in stage $S-1$, $\boldsymbol{A}_s$ is a vector estimator of the intercept and the slope of the output of the stage S, $\sum_{S(S-1)}$ is a covariance matrix of the estimator of the intercept and the slope between stage S and stage $S-1$, and finally, $\sum_{(S-1)(S-1)}$ is covariance matrix of the slope and the intercept estimators at stage $S-1$.

The mean vector and the covariance matrix for the U-transformation in the first stage are obtained by the following equations, respectively.

$$\boldsymbol{\mu}_{U_{j1}} = \boldsymbol{\mu}_{A_1} \tag{14}$$

$$\Sigma_{U_{j1}} = \Sigma_{A_1} = \Sigma_{11}. \tag{15}$$

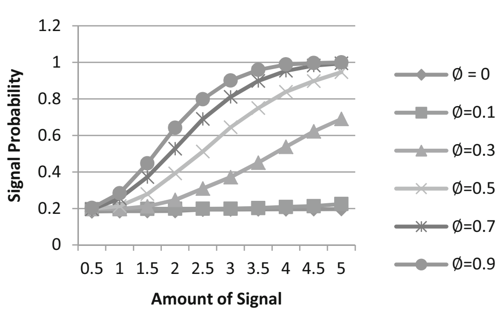

Fig. 5 Capability of the T2 of Stover and Brille [30] method with slope change

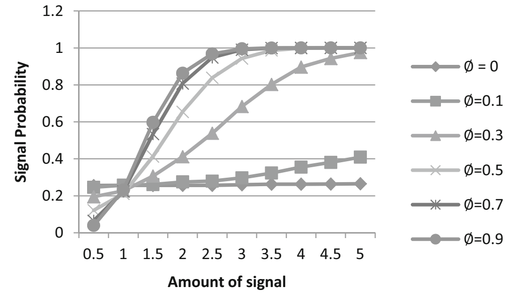

Fig. 6 Capability of the T2 of Williams [3] method with slope change

For the stage S, the mean vector and the covariance matrix are obtained, respectively, as follows.

$$\boldsymbol{\mu}_{U_{js}} = \boldsymbol{\mu}_{A_s} - \Sigma_{s(s-1)} \Sigma^{-1}_{(s-1)(s-1)} \boldsymbol{\mu}_{A_{s-1}} \tag{16}$$

$$\Sigma_{U_{js}} = \Sigma_{ss} - \Sigma_{s(s-1)} \Sigma^{-1}_{(s-1)(s-1)}. \tag{17}$$

As discussed in Section 3, the comparative analysis of the three studied methods indicates that the T2 of Kang and Albin [31] is the superior approach compared to the other two methods. In order to employ the U-transformation method in phase I monitoring of simple linear profiles in a multistage process with the cascade property, the transformation is employed based on the T2 of Kang and Albin [31]. In other word, for the j^{th} sample observed from the process in each stage, the following U statistics is used.

$$T^2_{U_{js}} = \left(U_{js} - \mu_{U_s}\right) \Sigma^{-1}_{U_s} \left(U_{js} - \mu_{U_s}\right)^T. \tag{18}$$

Then, the stage is diagnosed out-of-control if this statistics falls above the upper control limit of the chart that is $\chi^2_{\alpha,2}$.

5 Performance evaluation of the proposed method

In this section, we evaluate the performance of the proposed approach in phase I monitoring of linear profiles used in multistage processes. The proposed method is evaluated in terms of the test power when the process under investigation experiences several shifts as follows:

a) Different shift sizes for the intercept of the profile in the first stage of the process.
b) Different shift sizes for the slope of the profile in the first stage of the process.
c) Different combination shifts for the intercepts corresponding to the first and the second stages of the process, simultaneously.
d) Different combination shifts for the slopes corresponding to the first and the second stages of the process, simultaneously.

Table 4 Test powers of the proposed method under different shifts in the intercept and slope of the profile

	λ	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Shifts in intercept	$\emptyset = 0$	0.1370	0.1397	0.1360	0.1385	0.1357	0.1471	0.1387	0.1399	0.1449	0.1428
	$\emptyset = 0.1$	0.1432	0.1380	0.1381	0.1373	0.1402	0.1441	0.1372	0.1465	0.1445	0.1393
	$\emptyset = 0.3$	0.1401	0.1380	0.1408	0.1458	0.1412	0.1365	0.1369	0.1428	0.1458	0.1411
	$\emptyset = 0.5$	0.1370	0.1366	0.1302	0.1357	0.1373	0.1381	0.1446	0.1362	0.1379	0.1400
	$\emptyset = 0.7$	0.1410	0.1375	0.1365	0.1460	0.1425	0.1360	0.1394	0.1431	0.1423	0.1441
	$\emptyset = 0.9$	0.1425	0.1411	0.1417	0.1364	0.1323	0.1422	0.1452	0.1412	0.1453	0.1420
	β	0.5	1	1.5	2	2.5	3	3.5	4	4.5	**5**
Shifts in slope	$\emptyset = 0$	0.1370	0.1487	0.1360	0.1385	0.1357	0.1471	0.1387	0.1399	0.1449	0.1428
	$\emptyset = 0.1$	0.1441	0.1453	0.1392	0.1400	0.1401	0.1402	0.1424	0.1412	0.1354	0.1409
	$\emptyset = 0.3$	0.1385	0.1451	0.1448	0.1364	0.1323	0.1392	0.1349	0.1413	0.1408	0.1380
	$\emptyset = 0.5$	0.1443	0.1389	0.1412	0.1422	0.1393	0.1458	0.1459	0.1336	0.1433	0.1340
	$\emptyset = 0.7$	0.1395	0.1426	0.1413	0.1380	0.1379	0.1323	0.1369	0.1380	0.1432	0.1373
	$\emptyset = 0.9$	0.1383	0.1406	0.1451	0.1424	0.1394	0.1417	0.1353	0.1383	0.1362	0.1386

Table 4 contains the probability of receiving an out-of-control signal for the two cases of shift sizes. The first case corresponds to different shifts of the intercept of the first stage profile when the value of A_0 changes to $A_0 + \lambda \frac{\sigma}{\sqrt{n}}$. The second case focuses on different shifts of the slope of the first stage profile when the value A_1 changes to

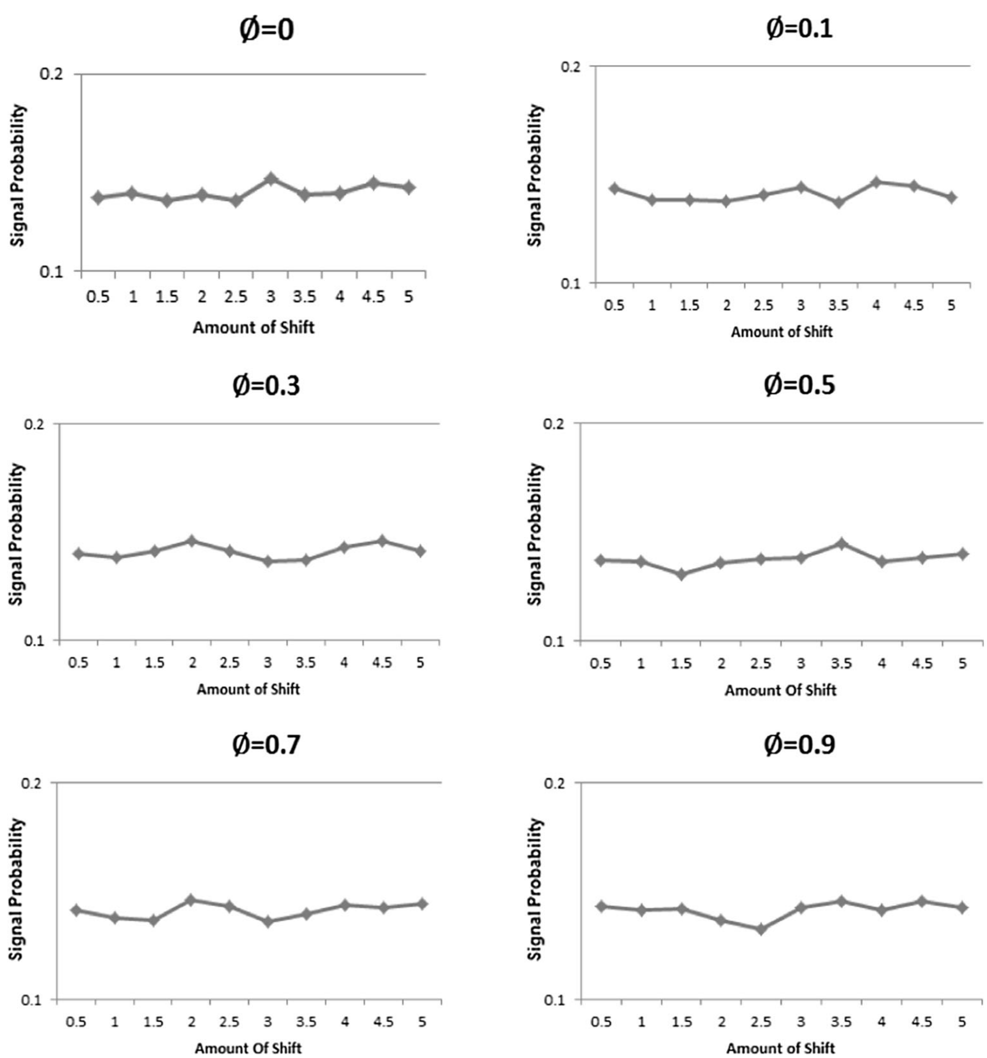

Fig. 7 Test powers of the proposed method under different shifts in the intercept

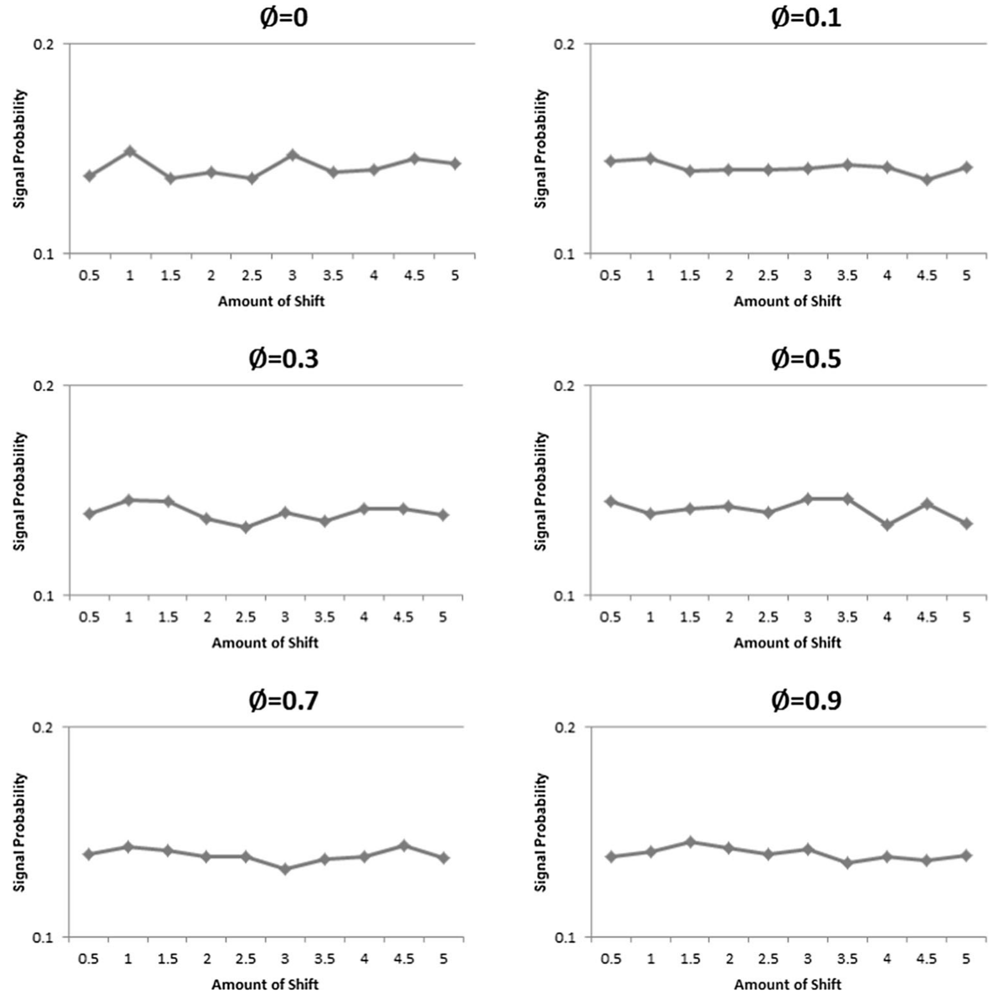

Fig. 8 Test powers of the proposed method under different shifts in the slope

$A_1+\beta\frac{\sigma}{\sqrt{S_{xx}}}$. The average test power shown in Table 4 are obtained using 10,000 replications for λ = 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, and 5.0 and assuming $\alpha=0.01$. In addition, Figs. 7 and 8 are the plots of the test power in the second stage based on the data shown in Table 4. In other words, the two figures show the monitoring capability of the second stage profile using the proposed U-statistic when different shifts are manifested into the parameters of the first stage; however, the second stage works normally.

Both Figs. 7 and 8 indicate that the test power is distributed randomly for different ∅ values. It means that the monitoring routine is not influenced by the values of ∅ which is referred to as the cascade property. As shown in Figs. 7 and 8, the results are distributed around the line of zero value for ∅. The analysis denotes that the performance of the proposed approach for different ∅ values is the same as the case where there is no cascade property, i.e., ∅=0. This case refers to monitoring a multistage process using several single-stage monitoring schemes. However, when the multistage process is monitored by the proposed approach, both figures address the shift size insensitivity of the model. In addition, Figs. 7 and 8 show that the values of the test power are between 0.13 and 0.15 when different shift sizes are induced to the process. This somehow indicates the robust property of the proposed approach for monitoring the profile of a multistage process.

Now, the performance of the proposed U-statistics is analyzed in the case where the parameters of the profiles in both stages are influenced by different shift sizes, simultaneously. In this analysis, it is assumed that the output of the first stage is produced under an out-of-control condition. It means that the first stage provides input of the second stage when a change takes place in the first stage. Furthermore, the evaluation is allowed to consider an influenced change of the second stage of the process. In this case, both stages work under out-of-control conditions simultaneously.

Table 5 shows the probabilities of identifying an out-of-control condition when the intercept A_0 changes to $A_0+\lambda_1\frac{\sigma}{\sqrt{n}}$ per λ_1 = 0.5,1, 1.5, 2, 2.5, 3, 3.5, and 4 for the first stage and

Table 5 Test powers of the proposed method under shifts in the intercept of the first stage from A_0 to $A_0 + \lambda_1 \frac{\sigma}{\sqrt{n}}$ and the second stage from A_0 to $A_0 + \lambda_2 \frac{\sigma}{\sqrt{n}}$

	λ_2	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
$\lambda_1 = 0.5$	∅= *0.1*	0.8258	0.8249	0.8221	0.82	0.8282	0.8255	0.8313	0.8228	0.8213	0.8195
	∅ = *0.3*	0.8215	0.8198	0.8293	0.8166	0.8231	0.8234	0.8222	0.8171	0.8221	0.8169
	∅ = *0.5*	0.8247	0.8210	0.8268	0.8205	0.8247	0.822	0.8213	0.8252	0.8247	0.8235
	∅= *0.7*	0.8235	0.8238	0.8259	0.8206	0.8193	0.8224	0.8281	0.8194	0.8105	0.8218
	∅= *0.9*	0.8183	0.8173	0.8264	0.8212	0.8207	0.8171	0.8307	0.8253	0.8350	0.8303
$\lambda_1 = 1$	∅= *0.1*	0.8294	0.8258	0.8235	0.8186	0.8291	0.8262	0.8246	0.8222	0.8207	0.8214
	∅= *0.3*	0.8224	0.8191	0.8205	0.8232	0.8240	0.8225	0.8203	0.8227	0.8210	0.8249
	∅= *0.5*	0.8259	0.8233	0.8190	0.8201	0.8247	0.8160	0.8235	0.8222	0.8237	0.8103
	∅= *0.7*	0.8243	0.8129	0.8210	0.8239	0.8257	0.8185	0.8152	0.8243	0.8200	0.8229
	∅= *0.9*	0.8255	0.8287	0.8269	0.8264	0.8215	0.8308	0.8211	0.8315	0.8188	0.8229
$\lambda_1 = 1.5$	∅= *0.1*	0.8218	0.8172	0.8267	0.8270	0.8173	0.8177	0.8243	0.8170	0.8238	0.8264
	∅= *0.3*	0.8221	0.8229	0.8142	0.8247	0.8238	0.8237	0.8202	0.8291	0.8246	0.8237
	∅= *0.5*	0.8270	0.8228	0.8210	0.8197	0.8298	0.8233	0.8223	0.8307	0.8209	0.8203
	∅= *0.7*	0.8228	0.8193	0.8288	0.8231	0.8240	0.8235	0.8188	0.8226	0.8281	0.8202
	∅= *0.9*	0.8190	0.8228	0.8241	0.8248	0.8274	0.8263	0.8155	0.8209	0.8205	0.8242
$\lambda_1 = 2$	∅= *0.1*	0.8170	0.8170	0.8193	0.8171	0.8229	0.8225	0.8201	0.8151	0.8192	0.8199
	∅= *0.3*	0.8161	0.8230	0.8270	0.8219	0.8207	0.8171	0.8212	0.8205	0.8294	0.8263
	∅= *0.5*	0.8220	0.8236	0.8235	0.8282	0.8259	0.8247	0.8246	0.8242	0.8208	0.8293
	∅= *0.7*	0.8222	0.8241	0.8261	0.8243	0.8224	0.8137	0.8197	0.8209	0.8262	0.8296
	∅= *0.9*	0.8256	0.8217	0.8206	0.8253	0.8219	0.8278	0.8202	0.8234	0.8236	0.8266
$\lambda_1 = 2.5$	∅= *0.1*	0.8168	0.8239	0.8232	0.8220	0.8196	0.8163	0.8240	0.8281	0.8274	0.8234
	∅= *0.3*	0.8170	0.8260	0.8270	0.8288	0.8247	0.8223	0.8200	0.8188	0.8267	0.8286
	∅= *0.5*	0.8213	0.8263	0.8224	0.8268	0.8263	0.8165	0.8274	0.8193	0.8203	0.8221
	∅= *0.7*	0.8280	0.8211	0.8259	0.8184	0.8205	0.8227	0.8221	0.8237	0.8201	0.8275
	∅= *0.9*	0.8251	0.8189	0.8254	0.8228	0.8243	0.8300	0.8264	0.8219	0.8214	0.8224
$\lambda_1 = 3$	∅= *0.1*	0.8249	0.8253	0.8248	0.8203	0.8200	0.8128	0.8150	0.8231	0.8255	0.8257
	∅= *0.3*	0.8159	0.826	0.8198	0.8217	0.8175	0.8220	0.8262	0.8211	0.8168	0.8219
	∅= *0.5*	0.8242	0.8208	0.8227	0.8214	0.8244	0.8184	0.8295	0.8187	0.8226	0.8143
	∅= *0.7*	0.8228	0.8227	0.8240	0.8278	0.8229	0.8147	0.8228	0.8232	0.8225	0.8257
	∅= *0.9*	0.8232	0.8255	0.8146	0.8249	0.8222	0.8251	0.8154	0.8225	0.8191	0.8260
$\boldsymbol{\lambda}_1 = 3.5$	∅= *0.1*	0.8232	0.8241	0.8255	0.8287	0.8133	0.8204	0.8179	0.8131	0.8179	0.8262
	∅= *0.3*	0.8235	0.8208	0.8246	0.8221	0.8275	0.8207	0.8247	0.8187	0.8238	0.8189
	∅= *0.5*	0.8176	0.8221	0.8199	0.8195	0.8244	0.8276	0.8178	0.8195	0.8170	0.8226
	∅= *0.7*	0.8230	0.8221	0.8220	0.8204	0.8223	0.8230	0.8187	0.8183	0.8260	0.8131
	∅= *0.9*	0.8284	0.8201	0.8240	0.8218	0.8220	0.8231	0.8170	0.8216	0.8269	0.8230
$\lambda_1 = 4$	∅= *0.1*	0.8272	0.8254	0.8215	0.8232	0.8225	0.8209	0.8188	0.8244	0.8269	0.8245
	∅= *0.3*	0.8188	0.8242	0.8169	0.8225	0.8242	0.8274	0.8168	0.8192	0.8242	0.8244
	∅= *0.5*	0.8202	0.8229	0.8184	0.8205	0.8221	0.8179	0.8139	0.8237	0.8246	0.8243
	∅= *0.7*	0.8239	0.8209	0.8184	0.8257	0.8228	0.8185	0.8271	0.8231	0.8252	0.8239
	∅= *0.9*	0.8246	0.8196	0.8196	0.8268	0.8281	0.8203	0.8216	0.8271	0.8205	0.8248

to $A_0 + \lambda_2 \frac{\sigma}{\sqrt{n}}$ per λ_2 = 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, and 5 for the second stage, assuming $\alpha = 0.01$. The test powers have been obtained using 10,000 simulation replications. Moreover, Fig. 9 is a plot of the results in Table 5 for the case where the shift value of the first stage is equal to 0.5. The other plots corresponding to the other values, i.e., 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, and 4.0, indicate the same pattern.

Figure 9 indicates that the proposed approach is capable of removing (or minimizing) the impact of the cascade property successfully. Note that Fig. 7 shows the test power values for the case that a fault affects the first

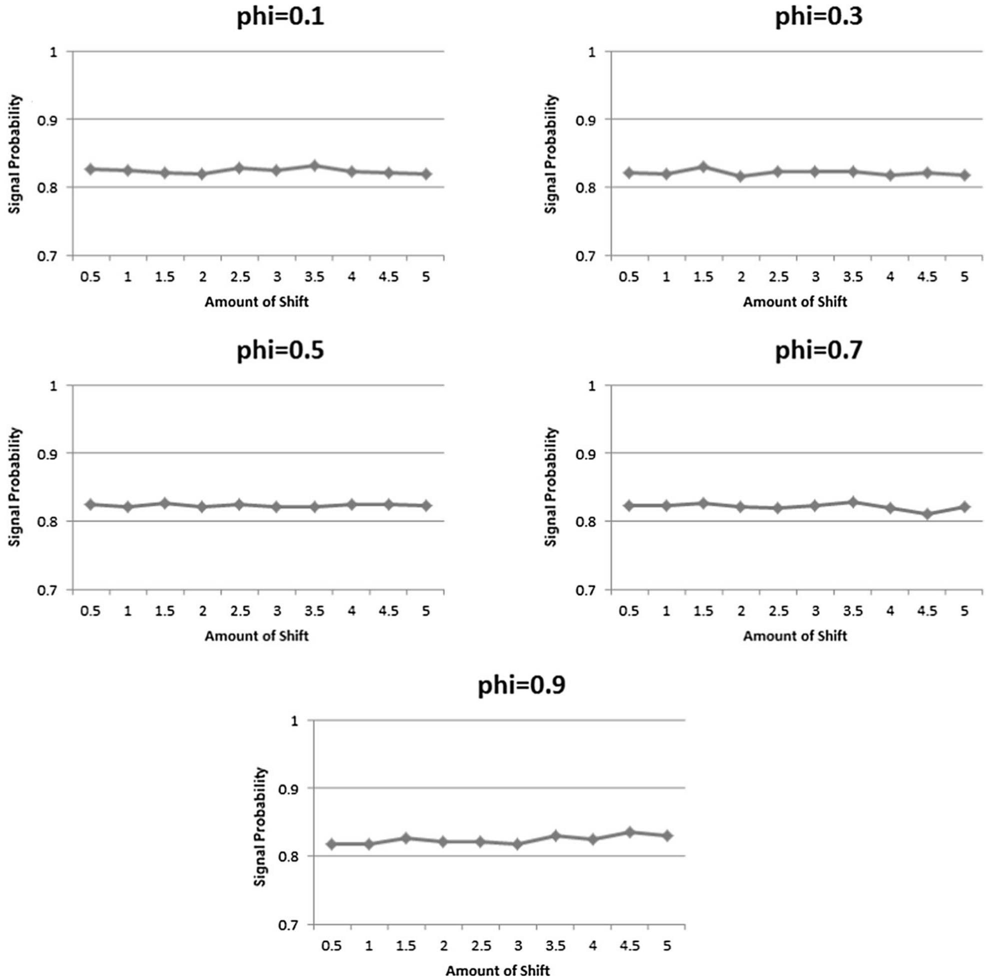

Fig. 9 Test powers of the proposed method under shifts in the intercept of stage 1 from A_0 to $A_0+\lambda_1\frac{\sigma}{\sqrt{n}}$ and stage 2 from A_0 to $A_0+\lambda_2\frac{\sigma}{\sqrt{n}}$

stage; however, the process is monitored in the second stage. A comparison of the patterns in Figs. 7 and 9 addresses an increase in the value of the test power. In addition, the comparative analysis shows that the proposed method is capable of reducing the impact of the first stage performance on the performance of the profile in the second stage.

Table 6 contains the test powers of the proposed approach when the slopes of the profiles in both stages are shifted (assuming $\alpha=0.01$). The values shown in this table are the probabilities of identifying an out-of-control condition in the multistage process where the slope A_1 changes to $A_1+\beta_1\frac{\sigma}{\sqrt{S_{xx}}}$ per β_1 = 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, and 4.0 for the first stage, and to $A_1+\beta_2\frac{\sigma}{\sqrt{S_{xx}}}$ per β_2 = 0.25, 0.5, 0.75, 1, 2, 3, 4, and 5 for the second stage. Again, the results are obtained using 10,000 independent replications. Besides, Fig. 10 is a plot of the values in Table 6 for the case the shift value of the first stage is equal to 0.5. The other plots corresponding to the other values of the shift indicate the same pattern.

Once again, Fig. 10 and Table 6 both indicate that the proposed approach is capable of removing (or minimizing) the impact of the cascade property, effectively. As discussed above, Fig. 8 shows the test power of the proposed approach in the case that a special cause affects the first stage; however, the profile of the process is monitored in the second stage. A comparison of the patterns in Figs. 8 and 10 indicate that the value of the test power increases in the case both stages are shifted. In other words, the proposed method is capable of reducing the impact of the cascade property on the performance of the profile monitoring method in the second stage, significantly.

6 A real case study

In this section, a piston manufacturing line, firstly considered by Fong and Lawless [33], is used to illustrate the capability of the proposed methodology in real-world manufacturing environments. In this practical case, a piston is produced in a four-stage machining process, where in each stage, the diameters of a piston are inspected in a micron precision at heights 4, 10, 36.7, and 58.7 mm from the bottom of the part. In this case, the functional relationship between the diameter and the height of each piston allows

Table 6 Test powers of the proposed method under shifts in the slope of stage 1 from A_1 to $A_1+\beta_1\frac{\sigma}{\sqrt{s_{xx}}}$ and stage 2 from A_1 to $A_1+\beta_2\frac{\sigma}{\sqrt{s_{xx}}}$

	β_2	0.25	0.5	0.75	1	2	3	4
$\beta_1=0.5$	∅= *0.1*	0.3566	0.8812	0.9994	1	1	1	1
	∅= *0.3*	0.3527	0.8718	0.9988	1	1	1	1
	∅= *0.5*	0.355	0.8726	0.9992	1	1	1	1
	∅= *0.7*	0.3673	0.8786	0.9993	1	1	1	1
	∅= *0.9*	0.3607	0.874	0.9997	1	1	1	1
$\beta_1=1$	∅= *0.1*	0.3506	0.8751	0.9990	1	1	1	1
	∅= *0.3*	0.3597	0.8744	0.9985	1	1	1	1
	∅= *0.5*	0.3629	0.8705	0.9993	1	1	1	1
	∅= *0.7*	0.3447	0.8725	0.9992	1	1	1	1
	∅= *0.9*	0.3559	0.8709	0.9991	1	1	1	1
$\beta_1=1.5$	∅= *0.1*	0.3518	0.8735	0.9990	1	1	1	1
	∅= *0.3*	0.3616	0.8757	0.9993	1	1	1	1
	∅= *0.5*	0.3606	0.8741	0.9994	1	1	1	1
	∅= *0.7*	0.3478	0.877	0.9994	1	1	1	1
	∅= *0.9*	0.3476	0.8744	0.9992	1	1	1	1
$\beta_1=2$	∅= *0.1*	0.3527	0.8701	0.9994	1	1	1	1
	∅= *0.3*	0.3577	0.8701	0.9984	1	1	1	1
	∅= *0.5*	0.3503	0.8774	0.9991	1	1	1	1
	∅= *0.7*	0.3506	0.8700	0.9991	1	1	1	1
	∅= *0.9*	0.3533	0.8750	0.9990	1	1	1	1
$\beta_1=2.5$	∅= *0.1*	0.3566	0.8812	0.9994	1	1	1	1
	∅= *0.3*	0.3527	0.8718	0.9988	1	1	1	1
	∅= *0.5*	0.3550	0.8726	0.9992	1	1	1	1
	∅= *0.7*	0.3673	0.8786	0.9993	1	1	1	1
	∅= *0.9*	0.3607	0.874	0.9997	1	1	1	1
$\beta_1=3$	∅= *0.1*	0.3483	0.8715	0.9992	1	1	1	1
	∅= *0.3*	0.3508	0.8764	0.9991	1	1	1	1
	∅= *0.5*	0.3568	0.8762	0.9995	1	1	1	1
	∅= *0.7*	0.3608	0.8730	0.9989	1	1	1	1
	∅= *0.9*	0.3572	0.8783	0.9989	1	1	1	1
$\beta_1=3.5$	∅= *0.1*	0.3486	0.8739	0.9993	1	1	1	1
	∅= *0.3*	0.3506	0.8730	0.9994	1	1	1	1
	∅= *0.5*	0.3573	0.8762	0.9991	1	1	1	1
	∅= *0.7*	0.3601	0.8673	0.9993	1	1	1	1
	∅= *0.9*	0.3565	0.8699	0.9995	1	1	1	1
$\beta_1=4$	∅= *0.1*	0.3593	0.8701	0.9991	1	1	1	1
	∅= *0.3*	0.3592	0.8737	0.9996	1	1	1	1
	∅= *0.5*	0.3547	0.8701	0.9993	1	1	1	1
	∅= *0.7*	0.358	0.8718	0.9993	1	1	1	1
	∅= *0.9*	0.3424	0.8820	0.999	1	1	1	1

one to provide a profile model to monitor each stage of the process. In this study, 25 profiles with 4 replications in each of the four heights are considered. Without loss of generality and for the sake of simplicity, let investigate the first two stages of this process.

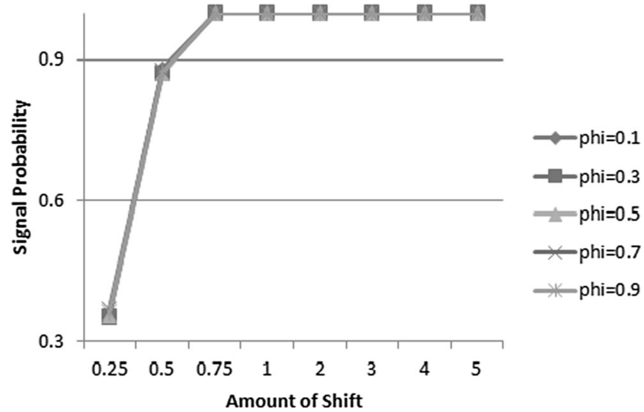

Fig. 10 Test powers of the proposed method under shifts in the slope of stage 1 from A_1 to $A_1+\beta_1\frac{\sigma}{\sqrt{s_{xx}}}$ and stage 2 from A_1 to $A_1+\beta_2\frac{\sigma}{\sqrt{s_{xx}}}$

Table 7 Estimated profile parameters and the UCL of the two-stage piston manufacturing process

Parameter	Value
Stage 1:	
A_0	89.11867
A_1	–0.01322
$UCL_{T^2,1}$	10.11
Stage 2:	
A_0	89.12032
A_1	–0.01328
$UCL_{T^2,2}$	9.21

The estimated parameters of the simple regression profiles in each stage along with the upper control limits of the monitoring schemes derived based on the proposed methodology are shown in Table 7 when $\alpha=0.01$. In this table, the statistic T_U^2 is calculated for each profile of each stage.

The results of employing the proposed approach are plotted on corresponding control charts for stage 1 and stage 2, as shown in Figs. 11 and 12, respectively.

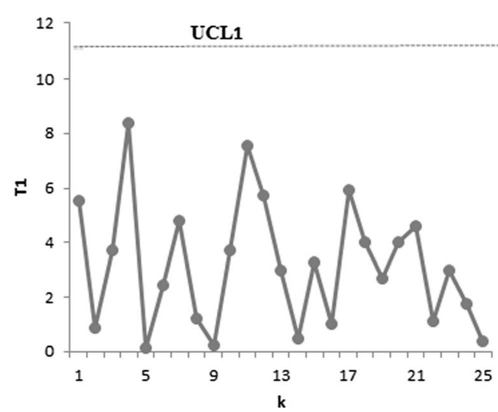

Fig. 11 The proposed T^2 control chart employed in the first stage of the piston manufacturing process

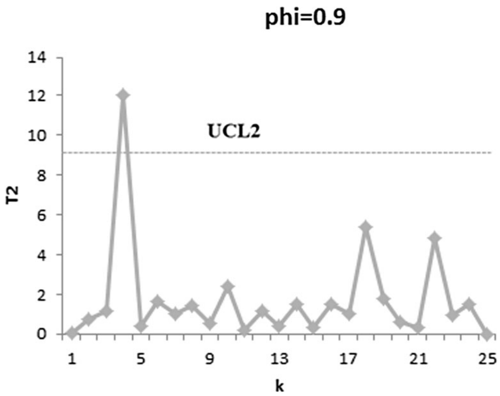

Fig. 12 The proposed T^2 control chart employed in the second stage of the piston manufacturing process

The results indicate that although all T_U^2 values for the first stage are within the control limit, the T_U^2 statistic for the fourth profile in the second stage exceeds the control limit. Consequently, it can be stated that the first stage of the multistage machining process works statistically in-control condition, whereas an assignable cause can be observed in the second stage of the process to diagnose an out-of-control condition.

7 Conclusions

The cascade property of multistage processes leads into a more complex monitoring procedure compared to the case of a single-stage process. In this paper, an approach using the U-transformation was proposed for phase I profile monitoring of a multistage process that decisively reduce the complexity involved based on the cascade property. An extensive numerical simulation based on several cases indicated that the proposed method is capable of detecting an out-of-control condition effectively by removing (or minimizing) the cascade property. An analysis of the profile monitoring indicated that the proposed method not only is capable of identifying an out-of-control condition when a fault manifests itself to each stage of the process but also the method simultaneously is capable of diagnosing the stage(s) contributing to the out-of-control condition.

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