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ORIGINAL ARTICLE

A multi-commodity network flow-based formulation for the multi-period cell formation problem

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Abstract In this paper, we present a new multi-commodity network flow-based formulation for the multi-period cell formation problem. The objective of the model is to minimize the total costs of acquisition, disposal, and relocation of machines, manufacturing, and inter-cell/intra-cell material handling. The main contribution of this paper comes from the fact that we structure the underlying problem as a multi-period multi-commodity network flow problem with general integer variables for machine acquisition, disposal, and relocation connecting one period to the next. This formulation is more efficient than the formulations we have encountered in the literature. Another contribution of this paper is that the flow variables representing the flow of parts through the system is path based; as a result, this approach makes it very easy to model alternate process routings. This paper illustrates the formulation by the use of two examples taken from the literature and presents computational results for other representative problems.

Keywords Cell formation · Mixed integer linear programming · Cellular manufacturing · Group technology · Design of production systems · Multi-commodity network flow · Alternate process routings · Multi-period cell formation problem

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1 Introduction and literature review

The cell formation problem (CFP) is an important problem in facilities planning and design since it is the foundational step in designing cellular manufacturing systems (CMS). While there are many models in the literature to solve the the CFP, we have not come across one where the underlying multi-commodity flow structure is recognized and exploited. While we are specifically interested in the multi-period version of the cell formation problem (MPCFP), the terms MPCFP and CFP are often used interchangeably.

The inputs to the MPCFP problem are multi-period demand, acquisition, relocation, and disposal costs of machines, manufacturing cost, and the cost of inter and intra-cell material handling. The output is a multi-period cell design minimizing the total cost over a multi-period time horizon. During the last three decades, a number of researchers have studied and addressed the cell formation problem in cellular manufacturing design. Several mathematical models and solution algorithms have been developed for the CFP. There are different methods for solving the CFP such as matrix arrangement, similarity coefficients, graph theory, mathematical programming, heuristics, and meta-heuristics such as simulated annealing (SA), genetic algorithm (GA), and TABU search (TS). Papaioannou and Wilson [24] presented a number of solution methods that have been used for CFP focusing on formulations proposed in the last decade.

Matrix arrangement methods deal with arrangement of rows and columns of a part-machine matrix to form the block diagonal matrix. The part families and machine groups can be formed from the blocks, each block representing a manufacturing cell [9, 19]. Similarity coefficient methods classify part families and machine groups based on the similarities between parts or machines. Similarities are based on machines, tools, and fixtures required by parts. Different measures of similarities have been developed to form part families and machine groups [28]. Yin and Yasuda [35] presented a taxonomy and review of similarity coefficient methods published in the literature. In their paper, a three-step procedure attributed to [27] is used in these methods:

1. Form the machine-part incidence matrix.
2. Construct a similarity matrix.
3. Use a clustering algorithm to process the similarity matrix to obtain a dendogram from which groups are obtained.

Another paper by Yin and Yasuda [34] presented a comparative investigation to evaluate the performance of various similarity coefficients methods applied to CFP. The authors classified the similarity coefficients into two categories: more efficient (using three similarity coefficients) and inefficient (using four similarity coefficients) for solving the CFP. They also found that there were three discriminable similarity coefficients and that the Jaccard similarity coefficient is the most stable similarity coefficient. In their definition, discriminability is the number of problems in which a similarity coefficient best performance value for a performance measure (such as number of exceptional elements or grouping efficiency—the paper defines nine such in total). Stability is the number of problems in which a similarity coefficient gives a value of a performance measure that is least the average value or better.

In graph theoretical methods, the machine-part matrix is represented by a graph. The aim of this method is to obtain sub-graphs from the machine-part graph to identify part families and machine groups. For example, Ribeiro [26] computed the dissimilarities between parts and organizes the production system in part-families and group-machines. A graph is generated and a coloring algorithm is used in order to obtain the desired number of cells.

Mathematical programming may also be used to solve the CFP. In these methods, part families and machine groups can be formed simultaneously based on the solution to a MIP. The simplest formulation of the problem is based on clustering, in which cells are formed from the machine-part incidence matrix in order to minimize the number of exceptional elements [8, 13]. Wang [32] proposed two linear assignment models to solve the machine-cell and part-family formation for the design of CMS.

Beyond the standard clustering or assignment approach, designing cells requires making a trade-off between duplicating machines (in one or more cells) and increasing material movement (between cells). The texbook by Askin and Standridge [4] presented one such model. Askin et al. [5] proposed an interactive cell formation method that can be used to design flexible cells. The authors illustrated routing flexibility (i.e., the ability for the cellular system to process parts within multiple cells) and demand flexibility (i.e., the ability of the cellular system to respond quickly to changes in part demand and part mix). A mathematical model for assigning operation types to machine types was presented. Selim et al. [29] introduced a mathematical formulation that includes two additional dimensions of the CFP. The first dimension is grouping workers and the second dimension deals with tooling.

The dynamic or multi-period cell formation problem (MPCFP) was described in [6]. The authors suggested a framework in which the cellular configuration is periodically changed when the cost-benefit analysis favours such a move. In this way, the cellular layout is better suited to the demand in each period and thus more efficient and agile during the planning horizon. Balakrishnan and Cheng [7] conducted a literature review to categorize research that has been done to address cell reconfiguration and uncertainty issues in CMS. They described a deterministic model for CMS reconfiguration due to planned product changes and presented a mathematical programming formulation for multi-period planning with cell reconfiguration. The dynamic CFP and worker assignment problem are considered simultaneously in [2]. Ghotboddini et al. [15] used a decomposition algorithm in order to solve the MPCFP and looked at part family/cell formation (PF/CF) and worker assignment simultaneously over a multiple period planning horizon. Defersha and Chen [11] presented a formulation for the cell formulation. The problem considered in this paper is more detailed than the basic MPCFP and was solved using an MILP formulation. Once cells are formed, the layout needs to be developed, as was done by Altuntas et al. [1] using solution approaches based on fuzzy weighted association rules for the facility layout of cellular manufacturing systems.

Heuristic methods use rules that guide the search process. Under this classification, there are heuristics and meta-heuristics such as SA, GA, and TS. Heuristics represent decision procedures and rules of thumb that expert users use to solve a problem. Heragu and Gupta [16] developed a heuristic for forming part families and machine groups. Kochikar and Narendran [20] developed a heuristic for solving the flexible manufacturing system (FMS) CFP. They introduced a heuristic which uses a grouping criterion that reflects the multi-faceted nature of flexibility: a composite of routing, machine, and part transfer flexibility. The evaluation shows that the heuristic has a tendency to create a large number of small cells. Liu et al. [21] proposed a mathematical model to deal with the CFP which incorporates production factors such as production volume, batch size, alternative process routing, cell size, unit cost of inter and intra-cell movements, and path coefficient of material flow. A three-stage heuristic has been developed to solve the NP-hard problem.

Among the papers in the literature that use metaheuristics to solve the CFP is the one by Filho and Tiberti [14]. This paper presented a group GA for the cell layout design problem with several new features such as chromosome codification scheme, correction mechanism and the crossover and mutation operators that work directly with the group of machines as opposed to individual machines. Moghaddam et al. [23] presented traditional meta-heuristic methods to solve the MPCFP. In this paper, a nonlinear integer model of the MPCFP was provided and then solved by GA, SA, and TS. Hu and Yasuda [17] presented a formulation for the MPCFP and used a specialized GA algorithm to solve small, medium, and large scale instances. Mahdavi et al. [22] proposed a mathematical model for the CFP based on the cell utilization concept. The aim of the model is to simultaneously minimize the number of voids and exceptional elements in cells, to achieve higher cell utilization. The authors presented an algorithm based on GA to solve the mathematical model. Tunnukij and Hicks [31] presented an enhanced grouping GA to solve the CFP without predetermining the number of manufacturing cells or the number of machines and parts within each cell. The enhanced grouping GA employs a rank-based roulette-elitist strategy as a new mechanism for creating successive generations. Deljoo et al. [12] extended previous MPCFP models presented in the literature and uses GA as the solution methodology. Very recently, Deep and Singh [10] benchmarked classic MPCFP problem solutions. The authors also presented their own MILP formulation for the problem, though this was not solved other than by using GA.

One of the examples in this paper used to illustrate our approach is taken from [33], who considered the dynamics of the production environment by incorporating a multi-perod forecast of product mix and demand. The authors presented a mixed-integer nonlinear program with quadratic and cubic terms for the design of CMS under fluctuations in the demand for products and product mix. The objective function in their formulation is to minimize the total cost of material handling and machine relocation over a forecast period. One extreme solution for the cellular manufacturing design problem is to purchase as few machines as possible, resulting in a high cost of material handling. The other extreme solution is to duplicate machines indiscriminately to reduce inter-cell traffic. However, this strategy results in higher acquisition costs. Finding an optimal solution to the cell design problem using the formulation in [33] is difficult and GA is proposed as the preferred solution mechanism for the problem. The model suffers from the following limitations:

- The model is difficult to solve because it has nonlinear and integer variables.
- The model assumes that each part has only one machine-type sequence. This is very restrictive. With the choice of technologies in modern manufacturing, it may be possible to use a 5-axis CNC milling centre to machine a part as one possible sequence. On the other hand, the part may also be machined using a routing through conventional machines such as lathes, drilling machines, and milling machines. It is important to be able to model the inherent trade-offs between the cost purchasing high-technology equipment resulting in simpler routings versus lower technology with lower costs, which result in longer routings and higher material handling costs.
- The model implicitly assumes growth in demand. With negative scenarios involving reduced demand, it should be possible to discard machines.

Another model used for detailed comparison is taken from [18], who presented a case study of a valve manufacturer. The objective of the model is to minimize the sum of machine purchase costs, the operating cost, inter-cell, and intra-cell material handling costs for the given periods. Although the [18] model tries to balance these costs, the original objective function is a nonlinear integer equation with absolute value terms (that take on binary values 0 or 1) in the third and fourth terms of the objective function to represent whether cell transfer has occurred between two successive operations. The authors converted the nonlinear absolute value function into a linear function by transforming the absolute terms into their corresponding linear terms (with the introduction of non-negative difference variables) to solve the problem. This type of formulation is computationally inefficient because it creates many branches in the branch-and-bound search tree if strong cuts are not used.

This paper makes two contributions to the literature. First, we structure the underlying problem as a multi-period multi-commodity network flow problem [30] with side constraints involving general integer variables for machine acquisition, disposal, and relocation connecting the periods. Second, because the flow variables represent start to end arc-paths, alternative machine sequences may be modeled very generally. The remainder of the problem is structured as follows: In Section 2, we present a new multi-commodity network flow-based formulation for the multi-period cell design problem. The solution toolkit is outlined briefly in Section 3, the examples and computation results are presented in Section 4, and conclusions and directions for future research are presented in Section 4.

2 A multi-commodity network flow based formulation for the MPCFP

The following notation is used to develop the mathematical model using the well-known multi-commodity network flow (MCNF) problem in operational research. In doing so, the formulation is structured using the arc-path formulation instead of the node-arc formulation [30]:

2.1 Indices and sets

$i =$ Parts
$j =$ Machine types
$k =$ Cells
$t =$ Time periods
$p =$ Start-to-finish sequence for part i.
$\{S_i\} =$ Set of all possible sequences for part i.
$\{P_i\} =$ Set of start-to-finish sequences for product i based on $\{S_i\}$.

2.2 Parameters

$c_j =$ Cost of purchasing one unit of machine type j.
$c'_j =$ Cost of discarding one unit of machine type j.
$R_j =$ Cost of relocating machine type j between cells.
$f_j =$ Fixed cost of one unit of machine type j per period.
$D_{it} =$ Demand of part i in period t.
$h_p^1 =$ Number of inter-cell transfers in sequence p.
$h_p^2 =$ Number of intra-cell transfers in sequence p.
$H_i^1 =$ Cost of inter-cell material handling for one unit of part i.
$H_i^2 =$ Cost of intra-cell material handling for one unit of part i.
$m_{ipj} =$ Unit manufacturing cost for part i on machine type j and sequence p.
$q_{ipjk} =$ Unit processing time part i on machine type j in cell k and sequence p.
$C_j =$ Time availability of one unit of machine type j per time period.
$LM =$ Minimum number of machines per cell (Lower limit).
$UM =$ Maximum number of machines per cell (Upper limit).

2.3 Decision Variables

$x_{ipt} =$ No. of parts of type i routed through sequence p in period t.
$n_{jkt} =$ No. of machines of type j available in cell k in period t.
$u_{jkt} =$ No. of machines of type j moved into cell k at the start of period t.
$v_{jkt} =$ No. of machines of type j moved out of cell k at the start of period t.
$a_{jkt} =$ No. of machines of type j purchased in cell k at the start of period t.
$b_{jkt} =$ No. of machines of type j discarded from cell k at the start of period t.

2.4 Illustration of multi-period multi-commodity network flow model

The objective in the MPCFP is to minimize the total costs of machine acquisition, relocation and disposal, production, and inter and intra-cell handling. It is important to underline that the inter- and intra-cell material handling costs for as well as production costs are path (routing) dependent. They can be estimated in a pre-processing step where all combinations of alternate routings and cell visits are enumerated. The only questions for the model then are to decide on how much to produce in a sequence and how many machines of each type to maintain going from period to period by purchasing, discarding, or relocating machines.

It is useful to think of the MPCFP problem using the time layered multi-commodity flow networks shown in Fig. 1, where the commodities are the parts. In each time period, the nodes (a combination of machine type j, cell type k, and time-period t) represent the machine types in each cell. The arcs represent the flow between machines (based on routing sequence). Node capacities depend on the (n_{jkt}) decision variable. The number of machines n_{jkt} may be varied by installing new machines, discarding existing machines, or relocating existing machines between cells. It is assumed that machines are purchased (and installed), relocated or retired at the time instant that occurs at the end of period t and the beginning of period $t + 1$. The flows and the number of machines available, on the other hand, are during a period.

Figure 1 shows a product with two alternative machine-type routing sequences (1,2) and (2,1). There are two cells: cell one is represented in blue and cell two in yellow. Let

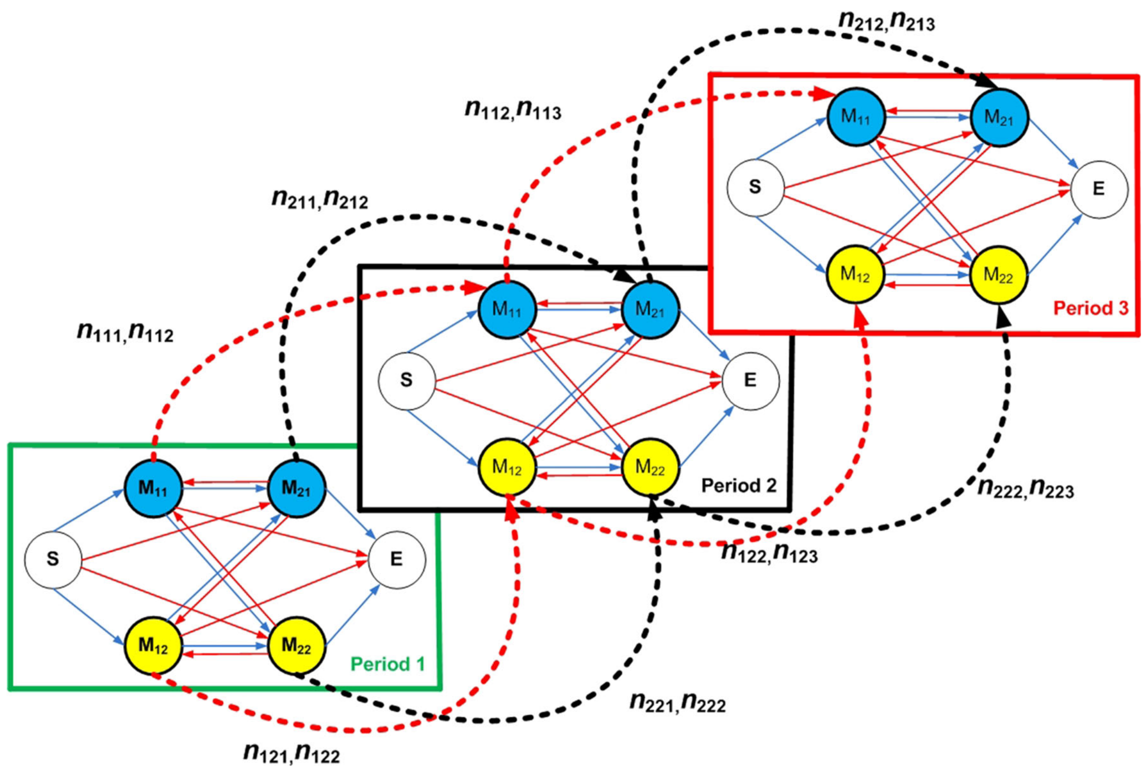

Fig. 1 Multi-period multi-commodity network flow model

M_{jk} denote machine type j in cell k. It is easy to see that are eight different processing sequences (paths) for the product. In any time period, a valid start-end (S, E) path for the product could be any one of the following:

$S \rightarrow M_{11} \rightarrow M_{21} \rightarrow E$
$S \rightarrow M_{11} \rightarrow M_{22} \rightarrow E$
$S \rightarrow M_{12} \rightarrow M_{22} \rightarrow E$
$S \rightarrow M_{12} \rightarrow M_{21} \rightarrow E$
$S \rightarrow M_{21} \rightarrow M_{11} \rightarrow E$
$S \rightarrow M_{21} \rightarrow M_{12} \rightarrow E$
$S \rightarrow M_{22} \rightarrow M_{11} \rightarrow E$
$S \rightarrow M_{22} \rightarrow M_{12} \rightarrow E$

The dashed red lines in Fig. 1 are for machine transitions, not *part* flows. They relate the number of machines of type M1 in period t (in both cell 1 and cell 2) to period t+1. Similarly, the dashed black lines relate the number of machines of type M2 from period to period.

The set P_i is used in the model to represent all feasible start-to-finish paths for product i, such as the ones enumerated above. P_i may be generated beforehand depending on processing sequences. The size of this set is $T \times \sum_r C^{n_r}$, where n_r is the number of steps in each alternative routing r, C is the number of cells, and T is the number of periods in the problem. The enumeration is exponential and could limit model implementation if the number is very large (e.g., if the order of magnitude of path variables is 10^6). However, partial enumeration is also possible if the problem size gets very big or the designer wishes to include constraints such as:

– Production for a product can occur only in one cell.
– Production for a product can occur in up to two cells.
– Production for a product can occur in up to two cells, but no more than one process can occur in the second cell.

This construct is quite general. If a new machine type is expected to be available at a later time period, it can still be represented from time period 1, but with capacity equal to zero. Similarly, if a new part is expected to be produced at a later period, or an existing part discontinued, the demands for these can be adjusted accordingly.

The mathematical formulations including objective function and system constraints are now written as follows:

Minimize:

$$Z = \sum_j \sum_k \sum_t \left(c_j\, a_{jkt} + c'_j\, b_{jkt} + R_j\, u_{jkt} + f_j\, n_{jkt} \right) + \sum_i \sum_{p \in P_i} \sum_j \sum_t \left(H_i^1\, h_p^1 + H_i^2\, h_p^2 + m_{ipj} \right) x_{ipt} \quad (1)$$

Subject to:

$$n_{jkt} = a_{jkt} \quad \forall j, \forall k, t = 1 \quad (2)$$

$$n_{jkt} = n_{jk(t-1)} + a_{jkt} - b_{jkt} + u_{jkt} - v_{jkt} \quad \forall j, \forall k, \forall t > 1 \quad (3)$$

$$\sum_k u_{jkt} = \sum_k v_{jkt} \quad \forall j, \forall t > 1 \quad (4)$$

$$LM \leq \sum_j n_{jkt} \leq UM \quad \forall k, \forall t \quad (5)$$

$$\sum_i \sum_{p \in P_i} q_{ipjk}\, x_{ipt} \leq C_j\, n_{jkt} \quad \forall j, \forall k, \forall t \quad (6)$$

$$\sum_{p \in P_i} x_{ipt} = D_{it} \quad \forall i, \forall t \quad (7)$$

$$x_{ipt} >= 0 \quad \forall i, \forall t \quad (8)$$

$$n_{jkt}, u_{jkt}, v_{jkt}, a_{jkt}, b_{jkt} \in \mathbb{Z} \quad \forall j, \forall k, \forall t \quad (9)$$

The overall objective of the MPCFP problem is to minimize the total system cost. The total system cost in the objective function consists of two terms:

1. The first sum in the objective function (1) minimizes the sum of purchasing, disposal, relocation, and fixed costs of machines in of each type across cells and time periods. The purchase (c_j) and disposal (c'_j) costs are per machine unit. The fixed costs f_j are per machine unit per time period. These fixed costs are in addition to purchase costs and may represent the cost of space, maintenance, tied-up capital, etc., not related to machine operation. To account for the relocation cost, it is assumed that every machine time is moved into a cell, a unit cost of R_j is incurred. By definition, a move into a cell involves another move out of a cell; in order to avoid double counting, we multiply the u_{jkt} variables by the unit cost of relocation, R_j. Therefore, the total cost of purchasing, discarding, and relocating machines is $\sum_j \sum_k \sum_t \left(c_j\, a_{jkt} + c'_j\, b_{jkt} + R_j\, u_{jkt} \right)$.
2. The second sum in the objective function (1) minimizes the sum of total inter and intra-cell material handling and manufacturing costs. These are summed overall parts (i) and part routings (p) in each multi-commodity flow network for each time period. H_i^1 is the unit handling cost per inter-cell transfer of part i, while H_i^2 is the unit handling cost per intra-cell transfer of part i. The total number of inter and intra-cell transfers in path $p \in P_i$ may be pre-computed based on the sequence of cells visited by the part in the path. Similarly, the manufacturing cost per unit m_{ipj} on path p may be calculated by summing the production costs on each of the machine types visited by the routings. Since x_{ipt} is the flow of part i using path p in time period t, the total cost of inter and intra-cell material handling and manufacturing is $\sum_i \sum_{p \in P_i} \sum_j \sum_t \left(H_i^1\, h_p^1 + H_i^2\, h_p^2 + m_{ipj} \right) x_{ipt}$.

The constraints in the model are as follows:

- Constraint sets (2–4) together are machine balance constraints. Constraint (2) ensures that the number of machines in a cell during the first time period t=1 is equal to the number of machines purchased (a_{jkt}) and installed in cell k at the beginning of period t.
- Constraint (3) ensures that the number of machines in a cell during any time period $t > 1$ is equal to the number of machines in time period $t - 1$ plus the number of machines purchased (a_{jkt}) minus the number of machines discarded (b_{jkt}) plus the number of machines relocated (u_{jkt}) into the cell minus the number of machines relocated out of the cell (v_{jkt}) in that time period.
- Constraint set (4) ensures that the total number of machines of type j moved into all cells during time period $t > 1$ must be equal to the number of machines of types j moved out of all cells in period t. This is a flow conservation equation that makes sure that all machines (in a time period) moved in to cells have to come from other cells. An aggregate constraint across all cells is sufficient because the objective function penalizes cell movement (with positive cost coefficients R_j for both an "out" move and an "in" move). Therefore, it would never be optimal to move a machine to another cell and moved it right back because this would increase the objective function cost. In other words, for a particular (j, k, t) combination, if u_{jkt} is positive, v_{jkt} will be zero in the optimal solution and vice versa. This is similar to how the absolute value function is linearized in linear programming. It may also be noted that if there are two machine moves, say from cell 1 to cell 2 and another from cell 3 to 4, the net movement in cells 1 and cell 3 is -1 while in cells 2 and 4 it is +1. Another possibility with the same net movement is to move the machine from cell 1 to cell 4 and another from cell 3 to cell 2. The model does not differentiate between these two moves, but it does capture the relocation costs correctly.
- Constraint set (5) limits the number of machines in each cell k during each time period t based on lower and upper bounds.
- Constraint set (6) is the capacity constraint in the model. Here, capacity is written in terms of processing time. However, it may be extended to include the availability of tools, labor, and other inputs such as machine setup time. The total processing time on machine type j in cell k during time period t over all part routings $p \in P_i$ is $\sum_i \sum_{p \in P_i} q_{ipjk}\, x_{ipt}$. This has to be less than or equal to the time availability of machine type j in cell k during time period t, i.e., $C_j\, n_{jkt}$. It is to be noted that the the

arc-path formulation of the multi-commodity network flow problem [30] is the basis for modeling capacity in our formulation. The flow on an arc-path or processing sequence (x_{ipt}) cannot be such that the capacity of any node, i.e., machine (n_{jkt}) is exceeded.

- Constraint set (7) ensures that the the sum of production $\sum_{p \in P_i} x_{ipt}$ of part i routed through path p during period t should be equal to the demand for that part (D_{it}).
- Constraint set (8) defines the domain of x_{ipt} as continuous and positive.
- Constraint (9) states that the variables $n_{jkt}, u_{jkt}, v_{jkt}, a_{jkt}$, and b_{jkt} are non-negative integers.

Naturally, there is an interaction between the constraints. The number of machines n_{jkt} is governed through constraint sets (2) and (3) for purchasing and installing, relocation, and disposal through the a_{jkt}, b_{jkt}, u_{jkt}, and v_{jkt} variables. Constraint set ensures valid relocations (each movement in is due to a movement out from somewhere else). These machines are make available the capacity requirements, depending on the flows x_{ipt}, in constraint set (6).

The second summation in the objective function and constraints 6 (for capacity) and 7 (for demand) represent a node-arc formulation of the time-phased multicommodity flow problem [30], with no flows from one period to another (as shown in Fig. 1). The nodes of the time-phased multicommodity network in each period are the machines in the various cells (j, k combinations), the commodities are the products (i). The x_{ipt} variables (feasible routing sequences) may be thought of as the start-to-finish arc-paths for each time period t. The cost of an arc-path x_{ipt} in the second sum of the objective function is the sum of the costs of inter-cell and intra-cell material handling and the manufacturing cost at each machine in the arc-path. The MPCFP problem is thus a time phased multi-commodity flow network problem with side constraints (2-4) for machine balance and constraint set (5) to limit cell size.

The MPCFP can be used to model real-life cell formation problems in the group technology or cellular manufacturing context in discrete manufacturing with medium part variety and volume. Examples can be seen in an example from the valve manufacturing industry [18], which we will look at in more detail later on, and the gear manufacturing industry [25].

This model does not explicitly consider worker assignment to cells or machines and implicitly assumes that a worker can either move between cells from period to period. Sometimes, a worker may be trained for a particular set of machines and when only one of the machines is moved, the worker assignment may need to change or retraining costs may be involved. Askin [3] presented a very general formulation for manufacturing cell design that includes tool sharing and setup compatibility, floor space requirements, subcontracting, alternate process plans, assignment of workers to cells and specific tasks, potential training of workers for new skills. Thus, the assignment of workers to machines and/or cells can be included in the model, but the trade-off would be that an optimal solution may take longer to obtain.

A toolkit for developing optimization-based analytical decision support applications was developed. The MPCFP formulation was coded using IBM CPLEX Optimization Studio (version 12.5). Python 2.7 was used to generate $\{P_i\}$ using complete enumeration. It is the set of start-to-finish paths for product i based on the alternative process plans in $\{S_i\}$. As it generates the sequences, the Python code also pre-computes the inter- and intra-cell material handling and total production costs associated with the paths. The Python output ($\{P_i\}$) is then entered in CPLEX Optimization Studio.

Table 1 The [33] solution to the example problem

Period	Cell number	Machine type in the cell (number of machines)	Part families
	1	A(1), C(1), D(1), F(1), I(1), J(1) K(1)	Parts (1, 8, 9, 10, 13, 16, 20, 23)
1	2	A(1), F(1), G(1), J(1)	Parts (14, 17, 18)
	3	B(1), E(1), G(1), H(1), I(1)	Parts (2, 5, 6, 12, 21)
	1	A(1), C(1), D(1), E(1), F(2), I(1), J(1) K(1)	Parts (1, 4, 6, 7, 8, 9, 10, 13, 16 17, 18, 19, 20, 23)
2	2	A(1), B(1), F(1), G(1), J(1), K(1)	Parts (3, 14, 22)
	3	B(1), E(1), G(1), H(1), I(1)	Parts (2, 5, 12, 21)
	1	A(1), C(1), D(1), E(1), F(2), I(1), J(1) K(1)	Parts (1, 4, 7, 8, 9, 10, 11, 13, 15, 16, 17, 18, 19, 20, 22, 23)
3	2	A(1), B(1), F(1), G(1), J(1), K(1)	Parts (3, 24, 25)
	3	B(1), E(1), G(1), H(1), I(1)	Parts (2, 5, 12, 21)

Table 2 Summary of the Solution obtained by MPCFP Model

Period	Cell number	Machine type in the cell (number of machines)	Part families
	1	E(1), F(1)	Part (17)
		K(1)	None
1	2	F(1), J(2), K(1)	Part (9)
	3	A(1), B(1), C(1), D(1), E(1), F(1), G(1), H(1), I(1), J(1), K(1)	Parts (1, 2, 5, 6, 8, 10, 12, 13, 14, 16, 18, 20, 21, 23)
	1	E(1), F(1)	Part (17)
		K(1)	None
2	2	F(1), J(2), K(1)	Part (9)
	3	A(1), B(1), C(1), D(1), E(1), F(1), G(1), H(1), I(1), J(1), K(1)	Parts (1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 13, 14, 16, 18, 19, 20, 21, 22, 23)
	1	E(1), F(1)	Part (17)
		K(1)	None
3	2	F(1), J(2), K(1)	Part (9)
	3	A(1), B(1), C(1), D(1), E(1), F(1), G(1), H(1), I(1), J(1), K(1)	Parts (1, 2, 3, 4, 5, 7, 8, 10, 11, 12, 13, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25)

3 Comparative experiments

Two problems are selected from the literature to present detailed results from the proposed model for the MPCFP. These examples are representative of what is available in the literature and provide a general perspective on the applicability of the proposed model for solving the MPCFP. The first illustrative example is adopted from the frequently cited paper by Wicks and Reasor [33], while the second case study is adopted from [18]. While the former paper is older, it has often been cited in the literature. The latter is a relatively new paper that roughly summarizes the state of the art in MPCFP formulation and solution.

It may be noted that the design decisions made by the MPCFP model include part routings and quantities; cell configurations; and machine acquisition, disposal, and relocation decisions. The costs of inter-cell/intra-cell material handling along with production and machine acquisition, disposal, and relocation are a function of the design decisions.

3.1 First illustrative example

The illustrative example in [33] consists of three time periods (index t in the model), 11 machine types (index j in the model), and 25 parts for production (index i in the model. In this example, the intra-cell material handling cost is zero and the fixed cost of machines is also zero. The disposal cost for all machines is assumed to be high since disposal is not considered in the example (a value of $100,000 was used). The operation sequences, processing times, machine capacities, and acquisition and relocation costs are assumed to be the same over the three time periods. The following cell capacity constraints were placed on the design of the CMS to be consistent with the example in the paper:

- Three machine cells to be formed.
- The machine lower limit is three per cell, while the upper limit of machines in each cell is open-ended.
- Each part has only one operation sequence, which implies that the manufacturing sequences is pre-defined (this is a restrictive assumption in [33] illustrative example).

The authors in [33] use GA to solve the problem and their solution may be seen in Table 1.

Table 3 Comparison between [33] and MPCFP solution

	Wicks & Reasor ($)	Our solution ($)
Machine acquisition cost		
Period 1	94,000	90,000
Period 2	18,000	0
Period 3	0	0
Sub-total	112,000	90,000
Material handling cost		
Period 1	8000	0
Period 2	3500	0
Period 3	1000	0
Sub-total	12,500	0
Machine relocation cost		
All periods	0	0
Total system cost	124,500	90,000

Table 4 [18] case solution

Period	Cell number	Machine type in the cell (number of machines)	Part families
	1	M3(2), M6(1), M7(1), M8(1)	P1, P7, P10
1	2	M1(2), M2(1), M4(1), M7(1)	P2, P4, P12
	3	M2(1), M4(2), M5(1), M6(1)	P5, P8, P9
	1	M3(1), M7(1), M8(1)	P1, P10
2	2	M1(2), M4(1)	P4, P12
	3	M2(1), M4(1), M5(1), M6(1)	P3, P6, P9
	1	M2(1), M3(1), M6(1), M7(1)	P5, P10
3	2	M4(2), M7(1)	P4
	3	M2(1), M4(1), M5(1), M6(1), M7(1)	P2, P6, P7, P8, P11

The data presented in the illustrative example was used to test the MPCFP model. As mentioned earlier, each part in [33] illustrative example has only one operation sequence, implying that the manufacturing sequences are pre-defined. The path generation code in Python was used to generate all possible sequences for each part routing through machines and cells. This resulted in hundreds of sequences for which the inter- and intra-cell material handling costs were determined.

Finally, the sequences were entered as input to the optimization model in IBM CPLEX Studio 12.5 with the [33] data, to run on a 64-bit machine with an Intel i5 chipset at 3.20 Ghz clock speed. The optimal solution was reached in 5.76 s. The output of the MPCFP model is shown in Table 2.

Even though part demands are different in each period, the machine composition of the three cells is the same across periods. Also, cell 1 should has a redundant machine of type K in each period. This is a result of the machine limit constraint in [33] illustrative example which states that the machine lower limit in each cell must be three machines. The MPCFP model adjusts to this constraint by acquisition and use of the cheapest machine—the resulting solution is still cheaper than the one in [33]. When LM is set to zero, this machine is dropped in the solution to the MPCFP and there are only two machines in each of the three periods in cell 1.

Table 3 compares the two solutions. The material handling cost in [33] for the three periods are $8000 for period one, $3500 for period two, and $1000 for period three, with a total cost of $12,500 for all periods. The cost of machines purchasing machines in [33] in period one is $94,000; in period two, two machines (E and F) are added to cell 1 with an extra purchasing cost of $8000 and two machines ($B$ and K) are added to cell 2 with an extra purchasing cost of $10,000. Therefore, the total purchasing cost in [33] is $ 112,000. In contrast, there is no cost associated with material handling in the MPCFP solution since each part is manufactured within one cell in every time period. The machine acquisition cost in the MPCFP solution is $ 90,000; all machines are purchased in period one and are retained for the subsequent periods, since the disposal cost is $100,000. Even when the disposal cost was reduced to $100,000, there was no change in the solution. This is because the fixed cost of machines per time period is zero and there is no incentive to discard machines.

It is to be noted that the x_{ipt} variables in our model are declared continuous for two reasons:

1. The time periods are fairly long and fractional production on paths can be rounded up or down without much practical significance in the MPCFP. In fact, flows in MCNF problems are often modelled as linear variables for this very reason.
2. Even without integer restrictions on the x_{ipt} variables in the [33] case, all flow variables turned out to be integer. With the integer restriction on x_{ipt}, we get the same optimal solution (value and decision variables). in 6.74 s, as opposed to 5.76 s. Therefore, using continuous variables is very practical. Note that the other variables ($n_{jkt}, a_{jkt}, b_{jkt}, u_{jkt}$, and$v_{jkt}$) must always be integer.

Table 5 MPCFP Solution to [18]

Period	Cell number	Machine type in the cell (number of machines)
	1	M6(1), M7(1), M8(1)
1	2	M2(1), M3(1), M4(1), M5(1), M6(1)
	3	M1(1), M4(1), M6(1), M7(1)
	1	M7(1), M8(1)
2	2	M2(1), M3(1), M4(1), M5(1), M6(1)
	3	M1(1), M4(1), M6(1), M7(1)
	1	M7(1)
3	2	M2(1), M3(1), M4(1), M5(1), M6(1)
	3	M1(1), M4(1), M6(1), M7(1)

Table 6 Comparison between [18] and MPCFP solutions

	[18] solution ($)	MPCFP solution ($)
Machine acquisition and fixed cost	56,400	49,700
Machine disposal cost	0	700
Machine relocation cost	4525	0
Material handling cost	20,114.29	26,434.54
Production cost	118,356	118,877.04.4
Total system cost	199,395.29	195,711.58

3.2 Second illustrative example

The [18] case study has eight different type of machines to manufacture 12 part types. The operating cost, machine capacities, and acquisition and relocation cost are the same over the three time periods. The inter- and intra- cell movement costs they use are $40 and $6, respectively. The case study has the following characteristics:

- The machines are grouped into three cells.
- The upper limit on the number of machines in a cell is five machines. However, there is no lower limit of machines. The authors state that smaller cells are preferable, but provide no lower bound.
- This example has alternate processing sequences. Each part type has three operations. Each operation can be performed on one of two alternative machine types. It is to be noted that the MPCFP model is much more general and can accept sequences of varying lengths or allow for several different alternatives at each step.
- The paper is ambiguous on what the disposal costs are, but seems to suggest that the disposal costs are half of what the relocation costs are.

Jayakumar and Raju [18] report the best integer feasible solution found after 8 h of solution time using the Branch and Bound method in the Lingo 11.0 optimal software package. The cell configuration for this case is shown in Table 4. The MPCFP solution for this case is summarized in Table 5. Table 6 compares the solutions obtained by [18] and the MPCFP.

It may be noted that:

- The machine acquisition and fixed costs in [18] are $56,400. There are relocation costs of $4525 but no disposal costs. The material handling costs (including inter-cell and intra-cell) for the three periods are $20,114.29. The production cost in [18] is reported as $118,356. However, there are some inconsistencies in the solution. For example, the routings in period 1 of parts 7, 8, 9, and 10 are inconsistent with the routing data. For example, the part routing data for part 7 is (1,4), (1,6), (2,6). Which is to say that the first operation is on either machine 1 or 4, the second on machine 1 or 6, and the third is on machine 2 or 6. Now, part 7 is assigned to cell 1 in period 1 and it is clear that since both machines 1 and 4 are absent in cell 1, the first operation of this part cannot be performed. This type of inconsistency is observed in the results of period 2 for parts 3, 9, and 10 and in period 3 for part 10.
- The machine acquisition cost in the MPCFP solution is $49,700. There are no relocation costs in this model, but there is a $700 disposal cost. The material handling cost, including inter-cell and intra-cell costs, is $26,434.94, which is significantly higher than in the Jayakumar and Raju [18] solution. The solution does have an inter-cell cost of $2181.06, which is about 8.25% of the total material handling cost.
- The production cost in the solution is $118,877.04.
- The MPCFP solution is optimal and superior to the solution in the best obtained solution in [18].

In order to illustrate that the MPCFP model can relocate machines depending on parameters, the [18] case was solved with the following modifications: all f_j variable valuess were set to zero, all H_i^2's were multiplied by 100, and all R_j's were set to zero. This is not to claim that the parameters are realistic, but rather to show that cell relocations do take place when the relocation costs are lowered and inter-cell handling costs increased. The resulting solution may

Table 7 MPCFP Solution to modified [18] case showing machine relocations

Period	Cell number	Machine type in the cell (number of machines)
	1	M6(1), M7(1), M8(1)
1	2	M2(1), M4(1), M5(1), M6(1)
	3	M1(1), M4(2), M6(1), M7(1)
	1	M1(1), M4(1), M6(1), M7(1)
2	2	M2(1), M3(1), M4(1), M5(1), M6(1)
	3	M4(1), M6(1), M7(1), M8(1)
	1	M4(1), M6(1), M7(1)
3	2	M1(1), M4(1), M6(1), M7(1), M8(1)
	3	M2(1), M3(1), M4(1), M5(1), M6(1)

Table 8 List of problems solved

Problem number	Source	Parts	Machine types	Cells	Maximum route length	Periods	Path enumeration time(s)
1	[33]	25	11	3	3	3	0.045
2	[18]	12	8	3	3	3	0.012
3	[11]	25	10	3	9	2	0.077
4	[17] (Medium 1)	20	12	3	6	1	0.17
5	[17] (Medium 2)	20	14	3	6	1	0.271
6	[17] (Medium 3)	30	18	3	5	1	0.109

be seen in Table 7. In this solution, machines 1, 4, and 8 are swapped between cells 1 and 3 from period 1 to period 2. Similarly, machines 1, 2, 3, 5, 7, and 8 are exchanged between cells 1, 2, and 3 from period 2 to period 3. The relocations in the solution (u_{jkt} and v_{jkt} variables) were consistent with the cell configuration variables (n_{jkt} variables). All machine acquisitions in this solution are in period 1 and there are no disposals since f_j's are zero.

3.3 Computational performance

In order to benchmark the computational performance of our model, other representative examples are taken from the literature. The problem sources with sizes are shown in Table 8. This table also shows the CPU time for route generation in Python 2.7. It is to be noted that all problems were run on a 64-bit machine running the Windows 7 ™ operating system with 4 GB RAM using an Intel i5 ™ chipset running at 3.20 Ghz. The IBM CPLEX Studio 12.5 ™ optimization environment was used throughout.

For each of these problems, the number of variables and constraints in the MPCFP model is shown in Table 9. It is worth mentioning that the problem in [11]* is an expanded version of the MPCFP with many other side constraints arising from tooling, workload balancing, machine adjacency, etc. It can be seen that the number of constraints in our model is relatively small because once the paths are enumerated, the constraints are only on machine balance, machine capacity, and job demand. Moreover, though this case has disposal costs (c'_j), it does not have any fixed costs (f_j). Hence, there is no incentive to dispose of machines once purchased.

Finally, Table 10 provides the computational time for each of the problems. With our model, these representative problems (among the largest solved in the literature with the exception of the large problem in [17]) are solved within 1000 s of CPU time, which is better than the time it took for the GAs in the original papers in all cases, with the exception of [18] in [10]. Please note that the time reported in [18] was the best integer solution found after the time limit of 8 h of CPU time had elapsed. It must be noted that in the [11] and the [17] cases, we do not solve the actual problem presented in the case. Rather, we solve an MPCFP problem of representative size using the data provided. In the [11] case, this is because the stated problem is an expanded version of the MPCFP will several other features. In the [17] cases, this is because purchasing, relocation, disposal, and fixed costs of machines are not provided. The only trade-off in the paper is between inter-cell and intra-cell handling. Since our interest was mainly to test CPU time, we randomly chose c_j, c'_j, and R to be uniformly distributed within the intervals (5000, 10,000), (500, 1000), and (1000, 5000), respectively. The value of f_j was set to zero.

Table 9 Number of variables and constraints in MPCFP

Problem number	Source	Variables (Original paper)	Constraints (Original paper)	Variables (Our model)	Constraints (Our model)
1	[33]	–	–	2185	304
2	[18]	6725	2151	8163	279
3	[11]	16, 930*	24, 030*	7983	268
4	[17] (Medium 1)	–	–	4881	133
5	[17] (Medium 2)	–	–	10,791	151
6	[17] (Medium 3)	–	–	5376	197

Table 10 Computational Time

Problem number	Source	CPU time in seconds (optimal in original paper)	CPU time in seconds (GA, original Paper)	CPU time in seconds (GA, [10])	CPU time in seconds (optimal using our model)
1	[33]	–	–	222.295	5.76
2	[18]	$28,800^+$	–	557.326	786.35
3	[11]	–	–	409.468	13.68
4	[17] (Medium 1)	–	218.825	–	65.8
5	[17] (Medium 2)	–	266.123	–	75.99
6	[17] (Medium 3)	–	363.312	–	23.21

4 Summary, conclusions, and further study

In this paper, a mathematical model based on the arc-path formulation of multi-commodity network problem was developed to solve the multi-period cell formation problem (MPCFP). Since both the inter- and intra-cell flows are known from the output of the model in this paper, spatial designs for the cells could be subsequently designed using a layout optimization model. Since the scale of the problems would be large, a metaheuristic approach might have to be used in practical instances.

The two main contributions of this paper are as follows:

1. With this formulation, there is no need to use GA for problems of the size typically solved in the literature with the exception of the large scale problem in [17]. While, GA or other metaheuristics are still useful tools for larger problems, the solutions were obtained with a standard commercial and academic MIP software package.
2. The model can handle alternate machine routings because the flow variables representing the flow of parts through system path based (start to end arc-paths).

Some areas for future research are:

- As discussed, some rules were discussed to reduce the number of path variables. A column generation approach can be developed to enumerate routings and understand the loss in optimality incurred by limiting the number of paths.
- A computational study could be undertaken to compare this model with other models in the literature. Also, the MPCFP is likely to be run repeatedly for different demand scenarios, for example, in a stochastic programming setting. For such problems, the basic and alternative routings can be enumerated once and for all. The repeated runs then are only for different demands, the rest of the problem data remains the same.
- Since the multicommodity flow part of the problem can be solved using state-of-the-art LP solvers, a GA or any other metaheuristic approach can be easily designed to search for solutions over the n_{jkt} space, with the multicommodity network flow LP as the fitness evaluation function. In other words, the n_{jkt} variables can be fixed by appropriately choosing a_{jkt}, b_{jkt}, u_{jkt}, andv_{jkt} through a structured neighbourhood search procedure. Thus, the fitness function for a given time period would decompose into a MCNF problem.
- It would be interesting to develop a robust version of the MPCFP model in this paper to look at the optimal design under different demand scenarios.
- The model also has a lot of utility for further research in the domain. For example, it can be used to design different types of cellular manufacturing systems. It can be used to tailor cell formation in order for production researchers to study the trade-offs between flexibility and efficiency in cellular manufacturing systems. We are currently pursuing this line of research.
- This paper looked at the MPCFP which is a strategic cell formation problem. There are some industries where machines may be relocated without incurring very high costs. In such cases, since the planning periods are shorter and the tactical cell formation problem with inventory and subcontracting variables could be formulated to avoid costly future purchases of machines.
- Finally, a machine could be moved out of one cell at the beginning of period t, stored for one or more periods, and then relocated to another cell. This would allow for a practical alternative that would avoid the need to buy a costly, new machine. To model this problem, we believe that our formulation can be revised so that the linking constraints between periods is made more general than from one period to the next only.

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