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ORIGINAL ARTICLE

Linear profile monitoring using EWMA structure under ranked set schemes

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Abstract In many manufacturing processes, study variable is not the only quality characteristic, but there may exist some explanatory variable(s) that are associated with the study variable. This association may be linear or nonlinear depending on the nature of variables. The term profiling is used for such relationships among study and explanatory variables. Linear profiles are more common options because of their simplicity and coverage of more common scenarios. A popular proposal for the monitoring of linear profiles is $EWMA-3_{[SRS]}$ chart that detect shifts in the profile parameters including slope, intercept, and error variance. The said chart is designed under simple random sampling. In this study, we have designed and investigated $EWMA-3_{[\tau]}$ chart under the different ranked set sampling strategies (τ) such as ranked set sampling (RSS), median ranked set sampling (MRSS), extreme ranked set sampling (ERSS), double ranked set sampling (DRSS), double median ranked set sampling (DMRSS), and double extreme ranked set sampling (DERSS). We have used average run length (ARL), extra quadratic loss (EQL), relative average run length (RARL), and performance comparison index (PCI) as performance measures for the aforementioned designs of $EWMA-3_{[\tau]}$ chart under different sampling schemes. The computational results of run length properties revealed that the ranked set based $EWMA-3_{[\tau]}$ chart offers better detection ability. More specifically, $EWMA-3_{[ERSS]}$ and $EWMA-3_{[DMRSS]}$ serve the purpose in a more efficient way.

Keywords Error variance · EWMA-3 · Intercept · Linear profiles · Ranked set schemes · Run length properties · Slope

1 Introduction

In the modern era of advanced technology, customers are not only concerned with the product for their need but also look at the product quality and cost efficiency. As a result, many companies are engaged in minimizing the cost with high-quality product. In manufacturing setups, quality is dependent on the state of the process being free from defects and deficiencies. Generally in every process, there exist some variations that affect its performance, categorized as natural variation and assignable cause of variations. The former are the inherent part of the process that cannot be completely eliminated while special cause variations damage the process performance and hence need our attention.

Statistical process control is the collection of tools to monitor the quality of a process. In this toolkit, control chart is the most widely used tool. An exemplary control chart consists of two decision lines namely upper control limit (UCL) and lower control limit (LCL) in addition to a centerline (CL). These lines help us declaring the process to be in-control (IC) or out-of-control (OOC). Control charts are classified into two classes: memory less control charts and memory type control charts. Shewhart [1] proposed Shewhart control chart, which is memory less control chart that utilizes only current information. Cumulative sum (CUSUM) control chart was introduced by Page [2], and exponentially weighted moving average (EWMA) control chart was originated by Roberts [3]. These two charts are the memory type control charts that use

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both the past and current information. Shewhart charts are good to detect large or transient shifts (cf. Riaz et al. [4] and Ali et al. [5]) while CUSUM and EWMA are relatively good at detection of small or persistent shifts (cf. Riaz et al. [6] and Zaman et al. [7]). Weiler [8], Does and Schriever [9], Champ and Woodall [10], Abbas et al. [11], Riaz et al. [12], Riaz and Touqeer [13], Nelson [14], Zaman et al. [15], and Abbasi et al. [16] established the control limit coefficients under different rules.

In many manufacturing processes, variable of interest can be modeled by a relation between a predicted variable and one or more predictor variables. The functional relationship among these variables is referred as profile and is addressed by different authors with fixed and random explanatory variables. For monitoring of simple linear profiles that is related to the control charting of regression, adjusted variables were proposed by Mandel [17], Hawkins [18], Hawkins [19], Wade and Woodall [20] and Hauck et al. [21]. EWMA/R and Hotelings T^2 control charts were proposed by Kang and Albin [22] to monitor different parameters. Mahmoud and Woodall [23] proposed a scheme for multiple linear regression in phase I structure. Noorossana et al. [24] introduced a multivariate cumulative sum (MCUSUM) control chart and R chart. The non-normality in the simple linear profiles is discussed by Noorossana et al. [25], Zou et al. [26], Mahmoud et al. [27] and Yeh and Zerehsaz [28] that used the control charts based on change point model for monitoring the simple linear profiles. The comparison of Shewhart method proposed by Kim et al. [29] and Croarkin and Varner [30] was developed by Gupta et al. [31]. Integrated MCUSUM and χ^2 control charts are introduced by Noorossana and Amiri [32]. Woodall [33] provided a comprehensive review on profile monitoring. For recursive residuals, control chart was proposed by Zou et al. [34] while control chart for mixed model in linear profiles was developed by Jensen et al. [35]. Problem of within autocorrelation in the model of Jensen et al. [35] was eliminated by Soleimani et al. [36]. Moreover, likelihood ratio-based control chart for monitoring the simple linear profile was proposed by Zhang et al. [37]. Saghaei et al. [38] designed an approach based on CUSUM chart while Mahmoud et al. [39] proposed the study for monitoring simple linear profiles when sample size cannot be greater than one or two. A phase II study of linear profiles under random effect model was introduced by Noorossana et al. [40].

All of the above researchers used the idea of linear profiling by using simple random sampling (SRS). The concept of ranked set sampling (RSS) was introduced by McIntyre [41] and more mathematical modifications were developed by Takahasi and Wakimoto [42]. Many of the researchers used RSS in control charts to make them more sensitive against different type of shifts (cf. Al-Saleh and Al-Kadiri [43], Muttlak and Saleh [44], Muttlak and Al-Sabah [45], Abujiya and Muttlak [46], Sinha [47], Al-Omari and Haq [48], Mehmood et al. [49]).

Kim et al. [29] introduced an $EWMA-3$ control chart for simultaneous monitoring of shifts in intercept, slope, and standard deviation of disturbance term. They used SRS in their study for process monitoring. In this study, we intend to use different RSS techniques to enhance the performance of the aforementioned $EWMA-3$ control chart. The organization of the rest of the article is as follows: In Sect. 2, we will describe the brief methodology of linear profiles under different sampling schemes and their monitoring. In addition, we will provide the design structure of control charting constants. In Sect. 3, performance evaluations of the proposed and other competing charts will be given. In Sect. 4, we will discuss the effect of smoothing parameter λ and sample size n, and the case study about electrical phenomena is given in Sect. 5. Finally, in Sect. 6, summary, conclusions, and recommendations are reported.

2 Linear profiles and their monitoring

In this section, we provide the conceptual framework of linear profiles and RSS mechanisms. These are required to develop our proposed structure of profile monitoring using $EWMA-3$ control chart for intercept, slope, and standard deviation of disturbance term under different ranked set schemes.

2.1 Ranked set sampling schemes

McIntyre [41] introduced a sampling strategy named as RSS which is defined as follows: select the n random samples for each of n sets. After sorting (ascending) the samples in each set with respect to concomitant variable, choose the minimum value from the first set then second smallest from the second set and so on. By following this method up to the largest sample from n^{th} set, we get a sample of size n. The cycle may be repeated m times until nm samples have been measured. These nm samples thus form the RSS.

Another type of ranked set scheme is median RSS (MRSS) which is defined as follows (cf. Muttlak [50]): randomly select n samples for each of n sets. By ranking the samples using the concomitant variable, n samples are obtained depending on whether the set size is even or odd. For odd set size, choose the median value of each ranked set (i.e., $\left(\frac{n+1}{2}\right)^{th}$ ranked value). For even set size, select the first half from the smallest rank of $\left(\frac{n}{2}\right)^{th}$ order and the smallest rank of $\left(\frac{n+2}{2}\right)^{th}$ order in the second half. The cycle may be repeated m times until nm samples have been measured. These nm samples thus form the MRSS.

Another sampling strategy is named extreme RSS (ERSS) which is defined by cf. Samawi et al. [51]: select n random sets each of size n and sort each set with respect to concomitant variable. If the set size is odd, select the smallest sample from the first $\frac{n-1}{2}$ sets; the largest sample from the last $\frac{n-1}{2}$ sets; median of the remaining set, for actual measurement. If the set size n is even, select the smallest sample from the first $\frac{n}{2}$ sets and from the other $\frac{n}{2}$ sets the largest sample for actual measurement. The cycle may be repeated m time until nm samples have been measured. These nm samples thus form the ERSS.

Al-Saleh and Al-Kadiri [43] provided the outline of double RSS (DRSS) which is defined as follows: select n^3 samples and further divide them into n sets each of n^2 samples. Apply the RSS on each of the sets and form the new n sets in each of size n then again apply the RSS technique to obtain the second stage samples. The cycle may be repeated m time until nm samples have been measured. These nm samples thus form the DRSS.

The second stage form of MRSS and ERSS named as double median RSS (DMRSS) and double extreme ranked set (DERSS), respectively, may also be obtained by double implementation of their procedures, following the same steps as mentioned above for DRSS. One may see Samawi and Tawalbeh [52] for more details.

It is to be noted that the RSSs are considered as perfect if the correlation (ρ) between study variable and concomitant variable is equal to 1 (i.e., $\rho = 1$) otherwise will be imperfect.

2.2 Linear profiles under ranked set schemes

Linear profile plays a key role in almost every process where the quality characteristic of interest depends on one or more explanatory variables. In this subsection, we describe the monitoring of linear profiles under different sampling strategies (later denoted by (τ) in the study), following Kim et al. [29] who considered SRS for the said purpose. We have covered six choices of (τ) named RSS, MRSS, ERSS, DRSS, DMRSS, and DERSS. One may see, Samawi and Ababneh [53] and Alodat et al. [54] for more details about simple linear profile model under ranked set strategies. The simple linear profile model under ranked set strategies is defined as follows:

$$Y_{[i]k} = \beta_0 + \beta_1 X_{(i)k} + \varepsilon_{[i]k};\ \mathrm{i} = 1, 2, 3, \ldots\ldots.., n;\ \ \mathrm{k} = 1, 2, 3, \ldots\ldots.., m \tag{1}$$

where the terms appearing in model (1) are defined as follows:

$Y_{[i]k}$	Study variable for i^{th} ordered sample in k^{th} cycle.
$X_{(i)k}$	Fixed predictor variable with i^{th} random sample in k^{th} cycle.
$\varepsilon_{[i]k}$	Error term for i^{th} ordered sample in k^{th} cycle.
β_0	Intercept
β_1	Slope
n	Size of samples
k	Number of cycles

It is to be mentioned that in SRS, one may interpret $\varepsilon_{[i]k}$ as i^{th} random error in first cycle (i.e., k=1) and $Y_{[i]k}$ as i^{th} random sample in first cycle. We have also assumed that $\varepsilon_{[i]k} \sim f(\varepsilon_{(\tau)})$. It is to be noted that the probability density function of error term under different rank set samplings $f(\varepsilon_{(\tau)})$ is given in the Appendix. $f(\varepsilon_{(r)})$ is the probability density function of error term under RSS while $f\left(\varepsilon^*_{(r)}\right)$, $f\left(\varepsilon_{(m)}\right)$, $f\left(\varepsilon^*_{(m)}\right)$, $f\left(\varepsilon_{(e1,en)}\right)$, and $f\left(\varepsilon^*_{(e1,en)}\right)$ are the probability density functions of error term under DRSS, MRSS, DMRSS, ERSS, and DERSS, respectively. The least square estimates of the parameters are given by the following expressions:

$$\hat{\beta}_{1[\tau]} = \frac{\sum_{i=1}^{n}\sum_{k=1}^{m}\left(X_{(i)k} - \overline{X}_{[\tau]}\right)Y_{[i]k}}{\sum_{i=1}^{n}\sum_{k=1}^{m}\left(X_{(i)k} - \overline{X}_{[\tau]}\right)^2} = \frac{S_{xy_{[\tau]}}}{S_{xx_{[\tau]}}}$$

$$\hat{\beta}_{0[\tau]} = \overline{Y}_{(\tau)} - \hat{\beta}_{1[\tau]}\overline{X}_{[\tau]}$$

where $\overline{Y}_{[\tau]} = \frac{\sum_{i=1}^{n}\sum_{k=1}^{m} Y_{[i]k}}{nm}$, $\overline{X}_{[\tau]} = \frac{\sum_{i=1}^{n}\sum_{k=1}^{m} X_{[i]k}}{nm}$ and the conditional mean, variance, and covariance of $\hat{\beta}_{0[\tau]}, \hat{\beta}_{1[\tau]}$ are defined as

$$E\left[\hat{\beta}_{0[\tau]}|X\right] = \beta_0, E\left[\hat{\beta}_{1[\tau]}|X\right] = \beta_1,$$

$$var\left[\hat{\beta}_{0[\tau]}|X\right] = \sigma^2_{e[\tau]}\left[\frac{1}{nm} + \frac{\overline{X}^2_{[\tau]}}{S_{xx_{[\tau]}}}\right],$$

$$var\left[\hat{\beta}_{1[\tau]}|X\right] = \frac{\sigma^2_{e[\tau]}}{S_{xx_{[\tau]}}}$$

$$cov\left[\hat{\beta}_{0[\tau]}, \hat{\beta}_{1[\tau]}|X\right] = -\frac{\sigma^2_{e[\tau]}\,\overline{X}_{[\tau]}}{\mathrm{S}_{\mathrm{xx}_{[\tau]}}}.$$

Mean square error is an unbiased estimator of the variance of error term σ^2, which is defined as

$$MSE_{[i]k} = \frac{\sum_{i=1}^{n}\sum_{k=1}^{m} \mathrm{e}^2_{[i]k}}{nm-2}$$

where $e_{[i]k} = y_{[i]k} - \hat{y}_{[i]k}$ is the i^{th} residual in the k^{th} cycle and $\hat{y}_{ik}$ is the i^{th} fitted regression line in the k^{th} cycle.

It is to be mentioned that model (1) may be transformed using the transformation $X_{(i)k}{}^* = X_{(i)k} - \overline{X}_{[\tau]}$ given in Appendix. After transforming $X_{(i)k}$, we obtain a modified form of the aforementioned model (1) named as transformed model which is defined as follows:

$$Y_{[i]k} = \left(A_0{}^*\right) + \left(A_1{}^*\right)X_{(i)k}{}^* + \varepsilon_{[i]k} \tag{2}$$

where $A_0{}^* = \beta_0 + \beta_1\overline{X}_{[\tau]} + (\beta\sigma)\overline{X}_{[\tau]}$ and $A_1{}^* = (\beta_1 + \beta\sigma)X_{(i)k}{}^*$. Here, shift is defined in σ units with reference to slope of model (1) (i.e., $\beta\sigma$). One may define the expressions of means, variances, and covariance of of A_0^* and A_1^*. It is to be noted that the covariance of A_0^* and A_1^* will be zero as the average of $X_{(i)k}^*$ is zero. It is to be mentioned that the estimated intercept and slope of transformed model will be denoted by $\hat{A}_{0[i]}^*$ and $\hat{A}_{1[i]}^*$, respectively.

2.3 $EWMA-3_{[\tau]}$ charting structure

Kim et al. [29] introduced $EWMA-3$ control chart under SRS ($EWMA-3_{[SRS]}$) for simultaneous monitoring of shifts in linear profile parameters including intercept, slope, and standard deviation of disturbance term. We introduce here that the $EWMA-3$ charting structure under different ranked set strategies (τ) on the base of transformed model is defined as

$$\mathrm{EWMA}_{I[i][\tau]} = \lambda\left(\hat{A}_{0[i]}^*\right) + (1-\lambda)\mathrm{EWMA}_{I[i-1][\tau]}$$
$$\mathrm{EWMA}_{S[i][\tau]} = \lambda\left(\hat{A}_{1[i]}^*\right) + (1-\lambda)\mathrm{EWMA}_{S[i-1][\tau]}$$
$$\mathrm{EWMA}_{E[i][\tau]} = \max\left\{\lambda ln\left(MSE_{[i]k}\right) + (1-\lambda)\mathrm{EWMA}_{E[i-1][\tau]}, \ln\left(\sigma^2{}_0\right)\right\}$$

where $EWMA_{I[i][\tau]}$ is the ith EWMA statistic for intercept under different samplings (τ); $EWMA_{S[i][\tau]}$ and $EWMA_{E[i][\tau]}$ are the ith EWMA statistics for slope and error variance, respectively, under different strategies (τ); λ is the smoothing parameter that ranges between zero and one (i.e., $0<\lambda\leq 1$). The popular choices of λ fall in the interval $0.05\leq\lambda\leq 0.25$ (cf. Lucas and Saccucci [55]).

The mean and variance of each of the three $EWMA_{[\tau]}$ statistic are given as follows:

$$E\left(\mathrm{EWMA}_{I[i][\tau]}\right) = A_0{}^*, E\left(\mathrm{EWMA}_{S[i][\tau]}\right) = A_1{}^*, E\left(\mathrm{EWMA}_{E[i][\tau]}\right) = \ln\left(\sigma^2{}_0\right),$$
$$Var\left(\mathrm{EWMA}_{I[i][\tau]}\right) = \frac{\lambda}{2-\lambda}\sigma^2_{e[\tau]}\left[\frac{1}{nm} + \frac{\overline{X}^2_{[\tau]}}{S_{xx_{[\tau]}}}\right], Var\left(\mathrm{EWMA}_{S[i][\tau]}\right) = \frac{\lambda}{2-\lambda}\frac{\sigma^2_{e[\tau]}}{S_{xx_{[\tau]}}},$$

$$Var\left(\mathrm{EWMA}_{E[i]}\quad[\tau]\right) = Var\left(ln\left(MSE_{[i]k}\right)\right) \cong \frac{2}{n-2} + \frac{2}{(n-2)^2} + \frac{4}{3(n-2)^3} - \frac{16}{15(n-2)^5}.$$ (cf. Crowder and Hamilton [56]).

Based on the above mentioned properties of the $EWMA_{[\tau]}$ statistics, the asymptotic limits for each $EWMA_{[\tau]}$ plotting statistic are given as follows:

$$\text{or } \mathrm{EWMA}_{I[i][\tau]}: \left\{\begin{array}{l} UCL_I = A_0{}^* + L_{I[\tau]}\sqrt{\frac{\lambda}{2-\lambda}\sigma^2_{e[\tau]}\left[\frac{1}{nm}+\frac{\overline{X}^2_{[\tau]}}{S_{xx_{[\tau]}}}\right]} \\ LCL_I = A_0{}^* - L_{I[\tau]}\sqrt{\frac{\lambda}{2-\lambda}\sigma^2_{e[\tau]}\left[\frac{1}{nm}+\frac{\overline{X}^2_{[\tau]}}{S_{xx_{[\tau]}}}\right]} \end{array}\right\}$$

$$\text{for } \mathrm{EWMA}_{S[i][\tau]}: \left\{\begin{array}{l} UCL_S = A_1{}^* + L_{S[\tau]}\sqrt{\frac{\lambda}{2-\lambda}\frac{\sigma^2_{e[\tau]}}{S_{xx_{[\tau]}}}} \\ LCL_S = A_1{}^* - L_{S[\tau]}\sqrt{\frac{\lambda}{2-\lambda}\frac{\sigma^2_{e[\tau]}}{S_{xx_{[\tau]}}}} \end{array}\right\}$$

$$\text{for } \mathrm{EWMA}_{E[i][\tau]}: \left\{UCL_E = ln\left(\sigma^2{}_0\right) + L_{E[\tau]}\sqrt{\frac{\lambda}{2-\lambda}Var\left(ln\left(MSE_{[i]k}\right)\right)}\right\}$$

where $L_{I[\tau]}$, $L_{S[\tau]}$, and $L_{E[\tau]}$ are the control limit coefficients for intercept, slope, and standard deviation of error term, respectively, under different sampling strategies (SRS, RSS, MRSS, ERSS, DRSS, DMRSS, and DERSS); $\sigma^2_{e[r]}$ is the error variance of RSS and $\sigma^2_{e[dr]}, \sigma^2_{e[m]}, \sigma^2_{e[dm]}, \sigma^2_{e[e1,en]}$, and $\sigma^2_{e[de1,den]}$ are the error variances of DRSS, MRSS, DMRSS, ERSS, and DERSS, respectively. The error variances for different RSSs $\left(\sigma^2_{e[\tau]}\right)$ are given in the Appendix.

The above mentioned three $EWMA_{[\tau]}$ statistics are combined in such a way to evaluate simultaneous monitoring of the three parameters of interest namely intercept, slope, and error variance. The said combined structure is designed under different ranked set methodologies for $EWMA-3$ setup named as $EWMA-3_{[\tau]}$ chart in the later part of this study.

3 Performance evaluations and comparisons

In this section, we provide performance evaluations and comparative analysis of the proposed charts relative to some existing charts. First, we give a brief description of different measures we have used in our study, followed by their computations and discussions of the results.

Average run length (ARL) is a popular measure that is defined as the number of samples until a signal occurs. It has two types (i) IC average run length (ARL_0) and (ii) OOC average length (ARL_1). There are certain other measures for chart's performance such as extra quadratic loss (EQL), relative average run length (RARL), and performance comparison index (PCI) which are defined as follows (for the more details, see Wu et al. [57], Ou et al. [58] and Ahmad et al. [59]):

$$EQL = \frac{1}{\nabla_{max}-\nabla_{min}}\int_{\nabla_{min}}^{\nabla_{max}} \nabla^2 ARL(\nabla)d\nabla$$
$$\mathrm{RARL} = \frac{1}{\nabla_{max}-\nabla_{min}}\int_{\nabla_{min}}^{\nabla_{max}} \frac{ARL(\nabla)}{ARL_{bmk}(\nabla)}d\nabla$$
$$PCI = \frac{EQL}{EQL_{\text{best chart}}}$$

where $ARL(\nabla)$ is the ARL of the particular chart at shift ∇ and $ARL_{bmk}(\nabla)$ is the ARL of the benchmark chart (we consider $EWMA_{[SRS]}$ as a benchmark) at shift ∇.

3.1 Control chart constants for $EWMA-3_{[\tau]}$ structure

For our study purposes, we have used IC linear profile model (fixed effect) which is mentioned by Kang and Albin [22] and defined as follows: $Y_{[i]k} = 3 + 2X_{(i)k} + \varepsilon_{[i]k}$. For our study purposes, we have fixed sample size ($n=4$), and values of the explanatory variable are fixed at $X_{(i)k}$ = 2, 4, 6, 8 with average equals to 5, following Kang and Albin [22]. The transformed model given in Eq. (2) is given as follows:

Table 1 Control charting constants for $EWMA-3_{[\tau]}$ chart at fixed $ARL_0 = 200$

Strategies	**SRS**	**RSS**				**MRSS**				**ERSS**			
rho	**0.00**	**0.25**	**0.50**	**0.75**	**1.00**	**0.25**	**0.50**	**0.75**	**1.00**	**0.25**	**0.50**	**0.75**	**1.00**
$L_{I[\tau]}$	3.016	2.967	2.793	2.479	1.960	2.940	2.773	2.420	1.810	2.969	2.810	2.550	2.110
$L_{S[\tau]}$	3.011	3.870	4.848	5.750	6.550	3.140	3.350	3.420	3.300	4.100	5.360	6.500	7.520
$L_{E[\tau]}$	1.372	1.310	1.150	0.860	0.480	1.315	1.154	0.850	0.465	1.360	1.310	1.290	1.190
Strategies		**DRSS**				**DMRSS**				**DERSS**			
rho		**0.25**	**0.50**	**0.75**	**1.00**	**0.25**	**0.50**	**0.75**	**1.00**	**0.25**	**0.50**	**0.75**	**1.00**
$L_{I[\tau]}$		2.960	2.840	2.498	1.950	2.930	2.670	2.100	1.160	2.950	2.720	2.440	1.850
$L_{S[\tau]}$		4.000	4.900	5.750	6.550	3.040	3.000	2.750	2.150	4.600	6.380	7.900	9.400
$L_{E[\tau]}$		1.310	1.150	0.870	0.480	1.300	1.050	0.810	0.029	1.400	1.415	1.462	1.465

$Y_{[i]k} = (A_0^*) + (A_1^*)X_{(i)k}^* + \varepsilon_{[i]k}$ where $A_0^* = 13 + 5(\beta\sigma)$, $A_1^* = (2+\beta\sigma)X_{(i)k}^*$, and $X_{(i)k}^* = -3, -1, 1, 3$ with average equals to 0. For the said parameter settings, $EWMA-3_{[\tau]}$ chart is designed under different sampling strategies.

Table 2 ARL comparison of $EWMA-3_{[\tau]}$ chart for slope shifts $\left(A_1^* \text{ to } A_1^* + \beta\sigma\right)$ on fixed intercept ($\delta = 0.15$)

Strategies	**SRS**	**RSS**				**MRSS**				**ERSS**			
β	**ρ = 0**	**ρ = 0.25**	**ρ = 0.5**	**ρ = 0.75**	**ρ = 1**	**ρ = 0.25**	**ρ = 0.5**	**ρ = 0.75**	**ρ = 1**	**ρ = 0.25**	**ρ = 0.5**	**ρ = 0.75**	**ρ = 1**
0.000	89.16	90.89	82.43	68.29	46.79	85.67	83.55	66.30	40.84	86.58	79.95	72.08	53.45
0.025	85.18	75.95	70.35	58.37	43.28	73.45	70.65	56.53	36.19	74.37	70.71	63.80	48.06
0.050	70.94	59.08	54.28	47.73	36.14	57.80	54.67	44.08	29.11	58.96	55.70	49.90	38.82
0.075	53.31	43.44	40.81	36.06	28.24	43.09	39.58	31.81	21.55	42.59	41.28	37.58	29.85
0.100	38.24	31.60	30.02	26.82	22.18	31.12	28.45	23.02	16.06	31.40	30.48	28.02	22.70
0.125	28.05	23.48	22.43	20.58	17.42	22.94	20.79	17.05	12.25	23.71	23.12	21.30	18.14
0.150	20.95	17.93	17.57	16.18	14.25	17.42	16.05	13.47	9.93	17.96	18.08	17.00	14.85
0.175	15.88	14.49	14.35	13.54	12.03	13.45	12.72	10.83	8.30	14.52	14.90	14.04	12.63
0.200	12.46	11.87	12.09	11.49	10.52	11.07	10.51	8.95	7.07	12.09	12.45	12.01	11.08
0.225	10.37	9.99	10.25	10.03	9.30	9.28	8.78	7.75	6.21	10.22	10.71	10.56	9.83
0.250	8.73	8.57	8.98	8.81	8.40	7.96	7.58	6.79	5.54	8.76	9.34	9.37	8.87
EQL	0.3605	0.3218	0.3178	0.297	0.2613	0.3068	0.2865	0.2418	0.1817	0.3241	0.3277	0.3109	0.2744
RARL	1.0000	0.8888	0.8619	0.7871	0.6671	0.8515	0.7988	0.6643	0.4805	0.8884	0.8828	0.8268	0.7061
PCI	1.0000	0.8926	0.8815	0.8238	0.7249	0.8509	0.7948	0.6708	0.5039	0.8991	0.9090	0.8625	0.7613
Strategies		**DRSS**				**DMRSS**				**DERSS**			
β		**ρ = 0.25**	**ρ = 0.5**	**ρ = 0.75**	**ρ = 1**	**ρ = 0.25**	**ρ = 0.5**	**ρ = 0.75**	**ρ = 1**	**ρ = 0.25**	**ρ = 0.5**	**ρ = 0.75**	**ρ = 1**
0.000		90.17	88.63	78.93	71.71	87.14	74.78	50.04	17.37	87.03	75.70	66.42	43.81
0.025		77.49	71.70	58.28	51.06	75.96	63.64	42.78	16.22	73.27	63.88	58.12	40.14
0.050		59.65	51.58	40.97	34.11	60.07	48.78	32.61	13.51	57.42	52.97	44.23	32.85
0.075		43.59	36.31	28.88	23.55	43.91	35.67	23.71	10.55	42.07	40.04	33.07	25.82
0.100		32.32	26.11	21.01	17.64	31.91	25.64	17.27	8.27	31.36	30.36	25.39	20.54
0.125		24.25	20.13	16.46	14.00	23.27	18.76	12.91	6.74	23.63	23.17	19.68	16.66
0.150		18.31	15.75	13.53	11.78	17.81	14.53	10.27	5.68	18.28	18.48	16.23	14.14
0.175		14.61	12.94	11.37	10.17	13.83	11.37	8.44	4.88	14.78	15.18	13.81	12.20
0.200		12.04	10.95	9.98	8.99	11.04	9.35	7.10	4.28	12.21	12.89	11.91	10.95
0.225		10.03	9.42	8.72	8.08	9.18	8.03	6.20	3.85	10.42	11.13	10.55	9.84
0.250		8.77	8.35	7.87	7.39	7.86	6.92	5.43	3.48	9.08	9.85	9.49	9.09
EQL		0.3268	0.2880	0.2508	0.2221	0.3104	0.2584	0.1873	0.1043	0.3272	0.3331	0.2989	0.2630
RARL		0.9013	0.7980	0.6837	0.6011	0.8657	0.7193	0.5100	0.2625	0.8917	0.8802	0.7825	0.6587
PCI		0.9064	0.7989	0.6956	0.6161	0.8609	0.7167	0.5196	0.2894	0.9077	0.9239	0.8293	0.7296

Table 3 ARL comparison of $EWMA-3_{[\tau]}$ chart for intercept shifts $\left(A_0^* \text{ to } A_0^* + \delta\sigma\right)$ on fixed intercept ($\beta = 0.075$).

	SRS	RSS				MRSS				ERSS			
δ	ρ = 0	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1
0.05	75.38	54.44	51.12	46.78	40.02	54.27	50.07	41.71	28.81	53.80	53.51	48.32	39.75
0.10	67.33	51.87	47.86	42.08	35.53	49.17	45.80	37.94	26.25	48.78	48.31	43.57	35.68
0.15	53.31	43.44	40.81	36.06	28.24	43.09	39.58	31.81	21.55	42.59	41.28	37.58	29.85
0.20	41.67	36.35	33.24	28.15	21.37	35.28	31.84	25.85	16.95	35.64	34.00	29.55	23.28
0.25	32.22	28.65	26.35	22.22	15.71	28.11	25.44	20.34	12.96	28.32	26.48	23.32	17.61
0.30	24.77	22.40	20.57	17.36	11.95	21.82	19.98	15.90	10.10	22.54	21.22	18.14	13.38
EQL	1.1427	0.9808	0.9045	0.7742	0.5775	0.9578	0.8727	0.7034	0.4603	0.9651	0.9193	0.8096	0.6270
RARL	1.0000	0.8144	0.7551	0.6597	0.5189	0.7970	0.7305	0.5958	0.4007	0.7979	0.7714	0.6871	0.5458
PCI	1.0000	0.8583	0.7915	0.6775	0.5054	0.8382	0.7637	0.6155	0.4028	0.8445	0.8044	0.7085	0.5486
		DRSS				DMRSS				DERSS			
δ		ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1
0.05		55.51	42.05	32.53	25.92	57.36	44.59	30.86	13.48	53.53	52.20	41.92	35.45
0.10		51.26	40.24	31.36	25.11	51.21	41.21	27.80	12.53	49.48	47.11	38.64	31.63
0.15		43.59	36.31	28.88	23.55	43.91	35.67	23.71	10.55	42.07	40.04	33.07	25.82
0.20		36.70	31.38	25.17	20.11	36.06	28.52	19.13	8.35	35.40	32.22	26.98	19.62
0.25		29.01	25.54	21. 00	16.01	27.92	23.11	15.21	6.51	28.15	25.74	21.49	14.53
0.30		22.86	20.65	17.09	12.30	22.04	18.49	12.12	5.19	22.28	19.81	16.93	10.90
EQL		0.9903	0.8509	0.6888	0.5341	0.9704	0.7906	0.5250	0.2285	0.9589	0.8819	0.7366	0.5282
RARL		0.8217	0.6812	0.5452	0.4306	0.8177	0.6586	0.4439	0.1985	0.7944	0.7446	0.6161	0.4703
PCI		0.8666	0.7446	0.6028	0.4673	0.8492	0.6918	0.4594	0.1999	0.8391	0.7718	0.6446	0.4622

In order to fix the overall ARL_0 at a prefixed level, we need to set the control limit coefficients including $L_{I[\tau]}$, $L_{S[\tau]}$, and $L_{E[\tau]}$ for different combinations of the design parameters such as n, λ, and ρ with the above mentioned settings of the profile parameters. One may obtain results for different combinations of the design parameters at different values of ARL_0. We have evaluated the results for some selective choices of these design parameters, and the results are reported in Table 1 to achieve an overall $ARL_0 = 200$. For computations, we used Monte Carlo simulation study with 10,000 iterations.

Table 4 ARL comparison of $EWMA-3_{[SRS]}$ chart for joint shifts $\left(A_0^* \text{ to } A_0^* + \delta\sigma\right)$ and $\left(A_1^* \text{ to } A_1^* + \beta\sigma\right)$

β	δ = 0.05	δ = 0.10	δ = 0.15	δ = 0.20	δ = 0.25	δ = 0.30
0.000	175.73	131.91	89.16	59.39	40.24	28.73
0.025	154.64	122.76	85.18	57.12	39.34	27.15
0.050	112.42	91.87	70.94	51.83	36.20	26.79
0.075	75.38	67.33	53.31	41.67	32.22	24.77
0.100	47.75	44.69	38.24	33.19	27.09	21.94
0.125	31.86	30.85	28.05	25.22	21.81	18.71
0.150	22.43	21.75	20.95	19.69	17.82	15.94
0.175	16.84	16.42	15.88	15.38	14.40	13.18
0.200	13.09	13.10	12.46	12.23	11.94	11.15
0.225	10.51	10.53	10.37	10.05	9.97	9.47
0.250	8.84	8.84	8.73	8.67	8.53	8.28
EQL	1.8873	1.8968	1.9135	1.9283	1.9438	1.9597
RARL	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
PCI	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

3.2 Performance evaluations for $EWMA-3_{[\tau]}$ structure

In order to evaluate the performance of the $EWMA-3_{[\tau]}$ chart under different sampling strategies, we have considered shifts of different amounts in the profile parameters. The description of these shifts is given as follows:

(i) Shifts for the intercept ($\delta = 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8,$ and 2.0) in transformed model,
(ii) Shifts for the slope ($\beta = 0.025, 0.050, 0.075, 0.100, 0.125, 0.150, 0.175, 0.200, 0.225,$ and 0.250) in the original model,
(iii) Shifts for error variance ($\gamma = 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 2.8,$ and 3.0) in original model,
(iv) Negative shifts for the slope ($\beta = -1.0, -0.9, -0.8, -0.7, -0.6, -0.5, -0.4, -0.3,$ and -0.2) in transformed model,

Table 5 ARL comparison of $EWMA-3_{[RSS]}$ chart for joint shifts $\left(A_0^* \text{ to } A_0^* + \delta\sigma\right)$ and $\left(A_1^* \text{ to } A_1^* + \beta\sigma\right)$

	δ = 0.05				δ = 0.10				δ = 0.15			
β	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1
0.000	178.75	175.59	166.33	157.63	131.92	126.95	116.19	86.92	90.89	82.43	68.29	46.79
0.025	129.64	124.43	117.59	109.58	104.00	99.44	90.06	73.02	75.95	70.35	58.37	43.28
0.050	84.34	80.01	73.71	65.90	73.29	69.23	63.26	51.61	59.08	54.28	47.73	36.14
0.075	54.44	51.12	46.78	40.02	51.87	47.86	42.08	35.53	43.44	40.81	36.06	28.24
0.100	36.18	34.17	31.04	26.81	34.21	32.59	29.43	25.02	31.60	30.02	26.82	22.18
0.125	25.73	24.21	22.42	19.53	25.28	23.87	21.64	18.78	23.48	22.43	20.58	17.42
0.150	19.08	18.48	17.51	15.44	18.72	18.30	16.94	14.99	17.93	17.57	16.18	14.25
0.175	14.93	14.87	14.14	12.63	14.72	14.56	13.90	12.55	14.49	14.35	13.54	12.03
0.200	12.20	12.16	11.71	10.81	11.95	12.03	11.49	10.75	11.87	12.09	11.49	10.52
0.225	10.09	10.37	10.13	9.50	10.11	10.18	10.07	9.44	9.99	10.25	10.03	9.30
0.250	8.71	8.95	8.90	8.50	8.65	9.02	8.94	8.47	8.57	8.98	8.81	8.40
EQL	0.3519	0.3447	0.3268	0.2945	0.3409	0.3339	0.3139	0.2812	0.3218	0.3178	0.2970	0.2613
RARL	1.7319	1.6759	1.5774	1.4627	0.8157	0.7790	0.7095	0.5765	0.4949	0.4635	0.4037	0.3142
PCI	0.1865	0.1827	0.1732	0.1561	0.1797	0.1760	0.1655	0.1483	0.1682	0.1661	0.1552	0.1366

	δ = 0.20				δ = 0.25				δ = 0.30			
β	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1
0.000	57.67	53.46	42.30	27.27	39.33	35.45	28.18	17.45	27.14	25.09	19.33	12.43
0.025	52.51	47.76	39.69	26.09	37.43	33.27	27.13	17.39	26.92	23.60	19.12	12.55
0.050	45.39	41.59	34.16	24.19	32.75	30.68	24.73	16.69	25.29	22.52	18.79	12.21
0.075	36.35	33.24	28.15	21.37	28.65	26.35	22.22	15.71	22.40	20.57	17.36	11.95
0.100	27.66	26.89	22.85	18.02	23.98	22.10	19.05	14.37	19.60	18.28	15.55	11.32
0.125	21.66	20.92	18.53	15.32	19.38	18.60	16.19	12.76	16.97	15.90	14.28	10.69
0.150	17.43	16.62	15.19	13.13	15.72	15.24	14.02	11.53	14.68	13.79	12.55	10.04
0.175	13.96	13.83	13.08	11.39	13.12	13.01	12.05	10.42	12.40	12.22	11.15	9.27
0.200	11.48	11.63	11.14	10.07	11.04	11.14	10.65	9.45	10.75	10.58	10.07	8.63
0.225	9.88	9.96	9.81	9.02	9.56	9.65	9.52	8.59	9.26	9.39	9.05	7.99
0.250	8.41	8.84	8.66	8.18	8.24	8.67	8.47	7.90	8.25	8.38	8.23	7.44
EQL	0.3007	0.296	0.2742	0.2376	0.2748	0.2704	0.2505	0.2122	0.2534	0.2464	0.2278	0.1877
RARL	0.3411	0.3201	0.2736	0.2061	0.2523	0.2362	0.2024	0.1497	0.1971	0.1830	0.1582	0.1163
PCI	0.156	0.1535	0.1422	0.1232	0.1414	0.1391	0.1288	0.1092	0.1293	0.1257	0.1162	0.0958

Table 6 ARL comparison of $EWMA-3_{[MRSS]}$ chart for joint shifts $\left(A_0^* \text{ to } A_0^*+\delta\sigma\right)$ and $\left(A_1^* \text{ to } A_1^*+\beta\sigma\right)$

	δ = 0.05				δ = 0.10				δ = 0.15			
β	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1
0.000	173.31	184.28	171.43	150.37	128.76	128.15	111.29	79.42	85.67	83.55	66.30	40.84
0.025	129.94	127.82	114.99	95.68	100.95	99.94	84.50	62.27	73.45	70.65	56.53	36.19
0.050	85.02	80.34	68.55	50.61	72.88	69.27	58.20	40.71	57.80	54.67	44.08	29.11
0.075	54.27	50.07	41.71	28.81	49.17	45.80	37.94	26.25	43.09	39.58	31.81	21.55
0.100	36.43	32.24	26.46	18.68	34.33	31.18	25.31	17.53	31.12	28.45	23.02	16.06
0.125	24.92	22.60	18.59	13.34	24.00	22.12	18.17	13.02	22.94	20.79	17.05	12.25
0.150	18.29	16.92	14.04	10.37	17.96	16.42	13.73	10.19	17.42	16.05	13.47	9.93
0.175	14.19	13.00	11.13	8.49	13.91	12.97	11.05	8.42	13.45	12.72	10.83	8.30
0.200	11.26	10.59	9.18	7.19	11.17	10.58	9.17	7.18	11.07	10.51	8.95	7.07
0.225	9.38	8.91	7.74	6.30	9.23	8.84	7.78	6.29	9.28	8.78	7.75	6.21
0.250	8.00	7.65	6.82	5.58	7.93	7.60	6.79	5.56	7.96	7.58	6.79	5.54
EQL	0.3374	0.3130	0.2662	0.2018	0.3232	0.3026	0.2569	0.1934	0.3068	0.2865	0.2418	0.1817
RARL	1.7077	1.7136	1.5429	1.2816	0.7931	0.7676	0.6502	0.4685	0.4785	0.4527	0.3663	0.2456
PCI	0.1788	0.1659	0.1410	0.1069	0.1704	0.1595	0.1354	0.1019	0.1603	0.1497	0.1264	0.0949

	δ = 0.20				δ = 0.25				δ = 0.30			
β	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1
0.000	56.40	52.09	40.93	23.09	38.52	34.72	26.80	15.15	27.22	24.69	18.58	10.86
0.025	51.82	47.81	38.12	22.25	35.73	32.74	25.29	14.64	25.62	23.60	18.44	10.72
0.050	44.14	40.10	32.14	19.75	32.82	30.06	23.37	14.08	24.59	22.33	17.31	10.51
0.075	35.28	31.84	25.85	16.95	28.11	25.44	20.34	12.96	21.82	19.98	15.90	10.10
0.100	26.83	25.23	20.37	13.83	22.73	20.93	17.21	11.54	19.11	17.74	14.26	9.40
0.125	21.01	19.20	16.06	11.26	18.66	17.20	14.04	9.90	16.37	15.00	12.30	8.61
0.150	16.32	14.94	12.65	9.36	15.22	14.13	11.87	8.64	13.88	12.70	10.67	7.77
0.175	13.07	12.27	10.49	8.02	12.49	11.53	9.99	7.55	11.69	10.84	9.39	6.98
0.200	10.75	10.19	8.81	6.95	10.48	9.83	8.52	6.68	9.94	9.43	8.21	6.33
0.225	9.17	8.64	7.64	6.14	8.75	8.47	7.42	5.96	8.62	8.15	7.24	5.74
0.250	7.85	7.53	6.65	5.50	7.68	7.37	6.60	5.41	7.51	7.29	6.47	5.24
EQL	0.2845	0.2654	0.2257	0.1690	0.2608	0.244	0.2076	0.1551	0.2386	0.223	0.1904	0.1414
RARL	0.3305	0.3043	0.2476	0.1609	0.2439	0.2242	0.1808	0.1186	0.1894	0.1745	0.1412	0.0938
PCI	0.1475	0.1376	0.1170	0.0876	0.1342	0.1255	0.1068	0.0798	0.1218	0.1138	0.0971	0.0722

Table 7 ARL comparison of $EWMA-3_{[ERSS]}$ chart for joint shifts $\left(A_0^* \text{ to } A_0^*+\delta\sigma\right)$ and $\left(A_1^* \text{ to } A_1^*+\beta\sigma\right)$

	δ = 0.05				δ = 0.10				δ = 0.15			
β	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1
0.000	177.36	166.62	172.86	156.38	130.18	122.63	118.43	95.25	86.58	79.95	72.08	53.45
0.025	129.08	120.33	120.97	106.92	102.05	96.24	91.30	74.58	74.37	70.71	63.80	48.06
0.050	83.47	80.83	76.08	65.56	72.60	70.07	64.92	52.22	58.96	55.70	49.90	38.82
0.075	53.80	53.51	48.32	39.75	48.78	48.31	43.57	35.68	42.59	41.28	37.58	29.85
0.100	36.95	35.45	32.21	26.90	34.92	33.42	30.69	25.33	31.40	30.48	28.02	22.70
0.125	26.11	25.56	22.98	20.11	24.80	24.73	22.40	19.21	23.71	23.12	21.30	18.14
0.150	19.22	19.18	18.12	15.81	18.75	18.96	17.74	15.60	17.96	18.08	17.00	14.85
0.175	15.06	15.36	14.63	13.18	14.87	15.00	14.50	13.04	14.52	14.90	14.04	12.63
0.200	12.21	12.76	12.31	11.36	12.25	12.64	12.30	11.25	12.09	12.45	12.01	11.08
0.225	10.36	10.75	10.71	10.05	10.26	10.89	10.60	9.94	10.22	10.71	10.56	9.83
0.250	8.86	9.47	9.45	9.00	8.83	9.47	9.41	8.98	8.76	9.34	9.37	8.87
EQL	0.3553	0.3585	0.3404	0.3041	0.3419	0.3466	0.3287	0.2916	0.3241	0.3277	0.3109	0.2744
RARL	1.7229	1.6326	1.6321	1.4479	0.8033	0.7716	0.7270	0.5976	0.4876	0.4683	0.4272	0.3391
PCI	0.1883	0.1899	0.1804	0.1611	0.1803	0.1828	0.1733	0.1537	0.1694	0.1712	0.1625	0.1434

	δ = 0.20				δ = 0.25				δ = 0.30			
β	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1
0.000	57.24	51.75	45.62	31.40	40.01	35.55	29.58	20.35	27.55	25.27	20.93	14.31
0.025	53.12	47.79	41.59	30.30	36.58	33.66	28.85	20.01	26.79	24.10	20.01	14.41
0.050	44.36	41.41	36.44	27.24	33.16	30.72	26.36	18.94	24.94	22.79	19.31	13.92
0.075	35.64	34.00	29.55	23.28	28.32	26.48	23.32	17.61	22.54	21.22	18.14	13.38
0.100	27.41	26.39	24.00	19.36	23.58	22.51	20.31	15.75	19.86	18.53	16.34	12.64
0.125	21.43	21.24	19.44	16.17	19.62	18.84	17.11	13.99	17.00	16.57	14.75	11.71
0.150	17.07	17.27	15.95	13.98	15.81	15.87	14.65	12.28	14.60	14.33	13.19	10.89
0.175	14.07	14.38	13.57	12.12	13.39	13.43	12.65	11.15	12.49	12.61	11.70	9.98
0.200	11.72	12.10	11.64	10.60	11.32	11.65	11.20	10.11	10.73	11.07	10.60	9.23
0.225	9.96	10.42	10.30	9.57	9.74	10.13	9.99	9.13	9.39	9.81	9.51	8.60
0.250	8.69	9.18	9.17	8.65	8.54	9.05	9.00	8.42	8.27	8.77	8.70	7.94
EQL	00.3013	00.3048	00.2875	00.2529	00.2786	00.2796	00.2638	00.2283	00.2544	00.2558	00.2389	00.2032
RARL	00.3390	00.3219	00.2886	00.2272	00.2527	00.2401	00.2139	00.1659	00.1974	00.1879	00.1656	00.1290
PCI	00.1563	00.1581	00.1491	00.1311	00.1433	00.1439	00.1357	00.1174	00.1298	00.1305	00.1219	00.1037

Table 8 ARL comparison of $EWMA-3_{[DRSS]}$ chart for joint shifts $\left(A_0^* \text{ to } A_0^*+\delta\sigma\right)$ and $\left(A_1^* \text{ to } A_1^*+\beta\sigma\right)$.

	δ = 0.05				δ = 0.10				δ = 0.15			
β	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1
0.000	186.82	181.77	170.21	194.19	135.74	133.6	127.12	135.11	90.17	88.63	78.93	71.71
0.025	130.97	111.24	89.21	81.53	106.15	93.60	78.56	70.57	77.49	71.70	58.28	51.06
0.050	87.08	68.03	51.12	41.63	77.04	61.31	47.73	39.04	59.65	51.58	40.97	34.11
0.075	55.51	42.05	32.53	25.92	51.26	40.24	31.36	25.11	43.59	36.31	28.88	23.55
0.100	36.97	28.77	22.74	18.55	34.96	28.22	22.14	18.42	32.32	26.11	21.01	17.64
0.125	26.36	20.88	17.29	14.43	25.40	20.53	17.22	14.31	24.25	20.13	16.46	14.00
0.150	19.43	16.37	13.87	11.89	18.65	16.05	13.77	11.88	18.31	15.75	13.53	11.78
0.175	14.90	13.21	11.65	10.23	15.06	13.10	11.57	10.22	14.61	12.94	11.37	10.17
0.200	12.36	11.10	10.02	9.04	12.22	11.10	10.01	9.02	12.04	10.95	9.98	8.99
0.225	10.24	9.59	8.82	8.11	10.11	9.47	8.81	8.11	10.03	9.42	8.72	8.08
0.250	8.79	8.39	7.92	7.40	8.78	8.38	7.87	7.39	8.77	8.35	7.87	7.39
EQL	0.3574	0.3055	0.2628	0.2296	0.3450	0.2989	0.2592	0.2273	0.3268	0.2880	0.2508	0.2221
RARL	1.7816	1.5838	1.3663	1.3841	0.8347	0.7318	0.6267	0.5784	0.5007	0.4476	0.3721	0.3229
PCI	0.1894	0.1619	0.1392	0.1217	0.1819	0.1576	0.1366	0.1199	0.1708	0.1505	0.1311	0.1161

	δ = 0.20				δ = 0.25				δ = 0.30			
β	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1
0.000	59.05	58.15	49.43	36.80	39.43	38.13	31.47	20.91	28.04	26.74	21.30	13.87
0.025	53.62	50.47	41.20	31.71	36.87	34.44	28.37	20.19	27.40	25.48	20.40	13.77
0.050	45.95	40.56	32.29	25.60	34.00	30.74	24.96	18.03	25.38	23.39	18.80	13.07
0.075	36.70	31.38	25.17	20.11	29.01	25.54	21.00	16.01	22.86	20.65	17.09	12.30
0.100	28.35	23.90	19.47	16.09	23.75	21.06	17.32	13.90	19.98	17.87	14.91	11.37
0.125	21.72	18.66	15.80	13.34	19.41	17.25	14.63	12.15	16.98	15.45	12.95	10.51
0.150	17.16	15.18	13.08	11.40	16.07	14.28	12.35	10.59	14.45	13.23	11.46	9.61
0.175	14.09	12.57	11.17	9.92	13.38	12.16	10.77	9.50	12.45	11.47	10.19	8.83
0.200	11.64	10.75	9.71	8.85	11.26	10.47	9.50	8.57	10.73	10.03	9.16	8.13
0.225	9.91	9.33	8.71	8.03	9.72	9.13	8.51	7.80	9.26	8.87	8.30	7.55
0.250	8.67	8.31	7.81	7.31	8.44	8.17	7.71	7.21	8.29	8.03	7.54	6.97
EQL	0.3034	0.2729	0.2399	0.2116	0.2790	0.2552	0.2260	0.1969	0.2538	0.2361	0.2099	0.1800
RARL	0.3459	0.3125	0.2598	0.2102	0.2546	0.2322	0.1952	0.1520	0.1991	0.1836	0.1540	0.1176
PCI	0.1573	0.1415	0.1244	0.1097	0.1435	0.1313	0.1162	0.1013	0.1295	0.1205	0.1071	0.0919

Table 9 ARL comparison of $EWMA-3_{[DMRSS]}$ chart for joint shifts $\left(A_0^* \text{ to } A_0^*+\delta\sigma\right)$ and $\left(A_1^* \text{ to } A_1^*+\beta\sigma\right)$.

	δ = 0.05				δ = 0.10				δ = 0.15			
β	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1
0.000	180.95	166.57	160.32	111.99	128.59	119.69	92.47	39.17	87.14	74.78	50.04	17.37
0.025	137.63	116.54	93.91	54.16	107.10	91.35	67.31	30.31	75.96	63.64	42.78	16.22
0.050	90.05	72.66	52.45	23.64	76.52	62.67	43.47	19.10	60.07	48.78	32.61	13.51
0.075	57.36	44.59	30.86	13.48	51.21	41.21	27.80	12.53	43.91	35.67	23.71	10.55
0.100	37.16	29.21	19.76	9.33	35.14	27.83	18.76	9.05	31.91	25.64	17.27	8.27
0.125	25.54	20.79	13.99	7.12	24.77	19.89	13.81	7.07	23.27	18.76	12.91	6.74
0.150	18.47	15.19	10.81	5.85	17.91	14.86	10.66	5.78	17.81	14.53	10.27	5.68
0.175	14.23	11.91	8.76	4.95	13.94	11.80	8.57	4.95	13.83	11.37	8.44	4.88
0.200	11.30	9.66	7.30	4.33	11.27	9.55	7.16	4.30	11.04	9.35	7.10	4.28
0.225	9.28	8.11	6.18	3.85	9.37	8.00	6.18	3.86	9.18	8.03	6.20	3.85
0.250	7.97	7.00	5.43	3.49	7.89	6.9	5.49	3.50	7.86	6.92	5.43	3.48
EQL	0.3427	0.2844	0.2066	0.1134	0.3286	0.2733	0.1984	0.1097	0.3104	0.2584	0.1873	0.1043
RARL	1.7914	1.5531	1.3252	0.8189	0.8208	0.7015	0.5112	0.2326	0.4909	0.4067	0.2765	0.1194
PCI	0.1816	0.1507	0.1095	0.0601	0.1733	0.1441	0.1046	0.0578	0.1622	0.1350	0.0979	0.0545

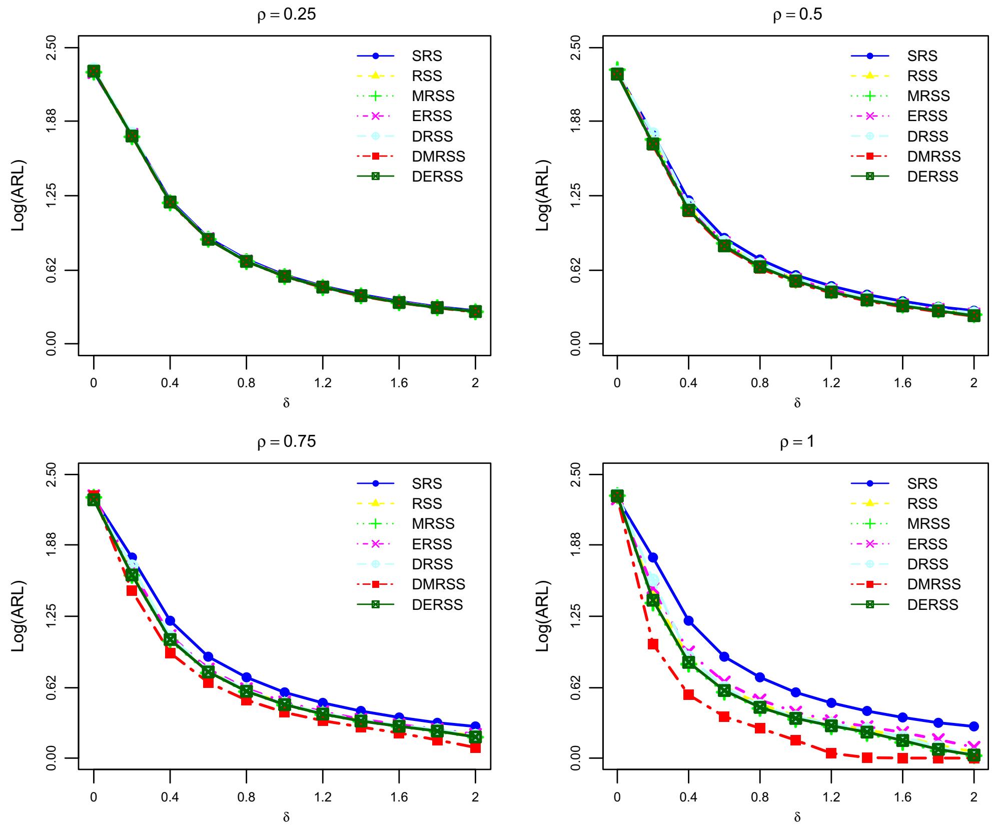
	δ = 0.20				δ = 0.25				δ = 0.30			
β	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1
0.000	56.10	47.85	29.73	10.07	38.93	31.27	19.64	6.92	27.07	22.46	13.97	5.33
0.025	51.99	43.23	27.58	10.06	36.37	30.09	18.88	6.96	26.62	21.69	13.80	5.32
0.050	43.54	36.50	23.75	9.30	32.99	26.87	17.01	6.85	24.82	20.26	13.03	5.25
0.075	36.06	28.52	19.13	8.35	27.92	23.11	15.21	6.51	22.04	18.49	12.12	5.19
0.100	27.79	22.35	15.17	7.26	23.53	19.09	12.82	6.09	19.12	15.99	10.83	5.07
0.125	21.44	17.33	12.13	6.24	18.88	15.57	10.84	5.51	16.62	13.83	9.62	4.81
0.150	16.52	13.82	9.80	5.42	15.07	12.66	9.08	4.99	13.63	11.82	8.37	4.47
0.175	13.29	11.12	8.20	4.76	12.65	10.65	7.80	4.52	11.73	10.01	7.38	4.19
0.200	10.81	9.26	6.98	4.24	10.44	8.90	6.80	4.10	10.02	8.53	6.53	3.87
0.225	9.13	7.89	6.12	3.81	8.77	7.76	5.98	3.74	8.62	7.49	5.83	3.58
0.250	7.81	6.83	5.36	3.47	7.65	6.71	5.33	3.43	7.44	6.58	5.23	3.35
EQL	0.2874	0.2408	0.1754	0.0985	0.2620	0.2220	0.1622	0.0916	0.2389	0.2043	0.1502	0.0841
RARL	0.3328	0.2756	0.1845	0.0817	0.2461	0.2035	0.1364	0.0628	0.1911	0.1598	0.1085	0.0512
PCI	0.1491	0.1249	0.0909	0.0511	0.1348	0.1142	0.0834	0.0471	0.1219	0.1042	0.0767	0.0429

Table 10 ARL comparison of $EWMA-3_{[DERSS]}$ chart for joint shifts $\left(A_0^* \text{ to } A_0^*+\delta\sigma\right)$ and $\left(A_1^* \text{ to } A_1^*+\beta\sigma\right)$.

	δ = 0.05				δ = 0.10				δ = 0.15			
β	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1
0.000	176.12	164.23	161.93	157.41	130.44	114.76	108.42	82.79	87.03	75.70	66.42	43.81
0.025	126.22	122.25	106.88	100.53	100.17	92.91	81.79	67.16	73.27	63.88	58.12	40.14
0.050	82.55	79.13	65.32	57.81	70.97	67.31	56.73	46.72	57.42	52.97	44.23	32.85
0.075	53.53	52.20	41.92	35.45	49.48	47.11	38.64	31.63	42.07	40.04	33.07	25.82
0.100	35.97	35.50	28.66	24.32	33.99	33.39	27.50	22.87	31.36	30.36	25.39	20.54
0.125	25.71	26.02	21.35	18.51	24.90	24.84	20.78	17.86	23.63	23.17	19.68	16.66
0.150	19.40	19.92	17.01	15.15	18.94	19.38	16.85	14.73	18.28	18.48	16.23	14.14
0.175	15.27	15.86	14.23	12.89	14.98	15.64	14.00	12.67	14.78	15.18	13.81	12.20
0.200	12.52	13.24	12.19	11.36	12.44	13.11	12.16	11.19	12.21	12.89	11.91	10.95
0.225	10.60	11.37	10.74	10.16	10.54	11.35	10.69	10.09	10.42	11.13	10.55	9.84
0.250	9.21	10.03	9.68	9.30	9.12	9.89	9.58	9.24	9.08	9.85	9.49	9.09
EQL	0.3577	0.3682	0.3238	0.2934	0.3449	0.3537	0.3144	0.2811	0.3272	0.3331	0.2990	0.2630
RARL	1.7021	1.6286	1.4818	1.3933	0.7967	0.7469	0.6576	0.5379	0.4840	0.4485	0.3910	0.2962
PCI	0.1895	0.1951	0.1716	0.1555	0.1818	0.1865	0.1658	0.1482	0.1710	0.1741	0.1562	0.1374

	δ = 0.20				δ = 0.25				δ = 0.30			
β	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1	ρ = 0.25	ρ = 0.5	ρ = 0.75	ρ = 1
0.000	56.88	48.57	41.84	24.81	38.75	32.27	27.21	15.96	27.07	23.13	19.13	11.31
0.025	51.50	44.55	38.35	23.83	36.68	30.81	25.88	16.00	26.03	22.48	18.70	11.42
0.050	43.63	38.46	33.30	22.22	32.59	28.40	24.53	15.08	24.44	21.26	18.05	11.28
0.075	35.40	32.22	26.98	19.62	28.15	25.74	21.49	14.53	22.28	19.81	16.93	10.90
0.100	27.60	26.04	22.17	17.15	23.63	21.90	18.83	13.44	19.80	18.07	15.33	10.60
0.125	21.55	21.15	18.20	14.74	19.36	18.65	16.29	12.52	16.92	15.93	13.97	10.14
0.150	17.44	17.38	15.27	12.89	16.26	15.98	14.19	11.30	14.72	14.16	12.69	9.64
0.175	14.17	14.54	13.23	11.47	13.47	13.67	12.34	10.41	12.73	12.64	11.40	9.15
0.200	11.96	12.46	11.56	10.44	11.53	11.91	11.08	9.61	10.99	11.23	10.38	8.67
0.225	10.24	10.95	10.35	9.52	10.06	10.64	10.01	8.93	9.61	10.10	9.51	8.20
0.250	8.98	9.69	9.36	8.79	8.84	9.53	9.11	8.35	8.55	9.16	8.78	7.75
EQL	0.3057	0.3088	0.2793	0.2398	0.2827	0.2836	0.2573	0.2130	0.2579	0.2561	0.2329	0.1862
RARL	0.3368	0.3098	0.2692	0.1957	0.2521	0.2313	0.2008	0.1428	0.1963	0.1807	0.1571	0.1105
PCI	0.1585	0.1602	0.1448	0.1244	0.1454	0.1459	0.1324	0.1096	0.1316	0.1307	0.1189	0.0950

Fig. 1 ARL comparison of $EWMA-3_{[\tau]}$ chart for intercept shifts (A_0^* to $A_0^* + \delta\sigma$)



(v) Shifts for both intercept and slope of transformed model,
(vi) Shifts in the slope of transformed model by changing the $X_{(i)k} = 1, 2, 3, 4$.

For these shifts, we have carried an extensive simulation study to evaluate run length properties in the form of ARL, EQL, RARL, and PCI of $EWMA-3_{[\tau]}$ for different settings of design parameters. In addition, we have also done the same for the existing $EWMA-3$ chart. These results are reported in Tables 2, 3, 4, 5, 6, 7, 8, 9, and 10. We have also produced some useful figures based on these tabular results that are provided in Figs. 1, 2, 3, 4, and 5 for some selective cases. The results of other cases may also be obtained on the same lines.

3.3 Comparative analysis

The ARL, EQL, RARL, and PCI measures of different competing structures, as provided above in Tables 2, 3, 4, 5, 6, 7, 8, 9 and 10 and Figs. 1, 2, 3, 4, and 5, reveal useful findings. These results are discussed here in detail for different profile parameters including intercept, slope, and error variance.

Shift in slope parameter The shifts for slope (at $\delta = 0.15$) are reported in Table 2.

- The results depict that when slope is IC, the shift in intercept may cause the 55.42 percentage loss in the ARL of $EWMA-3_{[SRS]}$ while under $EWMA-3_{[RSS]}$ 54.56 % on ($\rho = 0.25$), 58.79 % on ($\rho = 0.5$), 65.85 % on ($\rho = 0.75$), and 76.60 % on ($\rho = 1$) loss are reported. On fixed $\rho = 0.25$, 57.17, 56.71, 54.92, 56.43, and 56.48 % loss in ARL's of $EWMA-3_{[\tau]}$ are reported with respect to MRSS, ERSS, DRSS, DMRSS, and DERSS schemes
- When $\rho = 0.5$, then 58.23, 60.03, 55.69, 62.61, and 62.15 % loss are reported in ARL's of $EWMA-3_{[\tau]}$ under MRSS, ERSS, DRSS, DMRSS, and DERSS, respectively.
- When the ρ is fixed 0.75, then 6.85%, 63.96%, 60.54%, 74.98%, and 66.79 % and on $\rho = 1$, 79.58%, 73.27%, 64.14%, 91.31%, and 78.09 % loss in ARL's are reported for $EWMA-3_{[\tau]}$ under MRSS, ERSS, DRSS, DMRSS, and DERSS schemes.
- Finally, increase in the slope on fixed intercept resulted decrease in the ARL of all $EWMA-3_{[\tau]}$ schemes and least values of EQL, RARL, and PCI are the evidence that $EWMA-3_{[DMRSS]}$ scheme performs comparatively better than others.

Shift in intercept parameter Shifts in intercept at fixed slope ($\beta = 0.075$) are reported in Table 3.

- The results depict that the shift in intercept ($\delta = 0.05$) may cause 62.31 percentage loss in the ARL of $EWMA-3_{[SRS]}$ while in $EWMA-3_{[RSS]}$, 72.78 % on ($\rho = 0.25$), 74.44 % on ($\rho = 0.5$), 76.61 % on ($\rho = 0.75$), and 79.99 % on ($\rho = 1$)

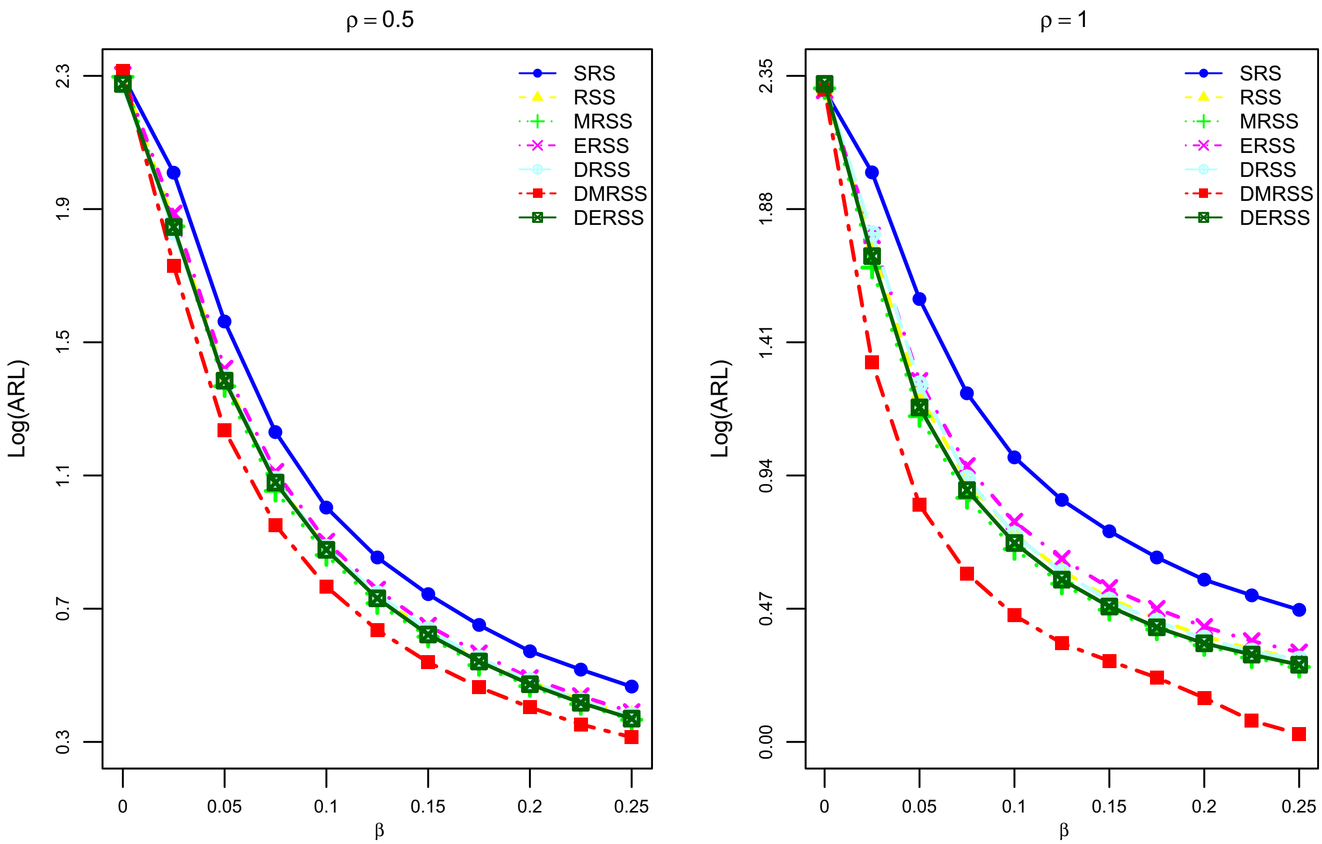


Fig. 2 ARL comparison of $EWMA-3_{[\tau]}$ chart for slope shifts (β_1 to $\beta_1+\beta\sigma$)

loss are reported in the ARL's and on $\rho=0.25$, 72.87 %, 73.10 %, 72.24 %, 71.32 %, and 73.24% loss are reported in ARL's of $EWMA-3_{[\tau]}$ with respect to MRSS, ERSS, DRSS, DMRSS, and DERSS schemes.

- On $\rho=0.5$, loss in the ARL's are reported as 74.96 %, 73.24 %, 78.98 %, 77.70 %, and 73.90% for $EWMA-3_{[\tau]}$ under MRSS, ERSS, DRSS, DMRSS, and DERSS schemes, respectively.
- Further, when ρ is fixed 0.75, then 79.15 %, 75.84 %, 83.74 %, 84.57 %, and 79.04% loss in ARL's are reported for $EWMA-3_{[\tau]}$ with respect to MRSS, ERSS, DRSS, DMRSS, and DERSS schemes.
- On $\rho=1$, 85.59%, 80.13%, 87.04%, 93.26%, and 82.28 % loss in ARL of $EWMA-3_{[\tau]}$ for MRSS, ERSS, DRSS, DMRSS, and DERSS schemes are reported.
- Moreover, increases in the intercept δ on fixed slope ($\beta=0.075$) may result decrease in the ARL of $EWMA-3_{[\tau]}$ schemes. The smallest values of the EQL, RARL and PCI reveal that $EWMA-3_{[DMRSS]}$ scheme outperforms all others.

Joint shifts in slope and intercept parameters

- Shifts in both linear profile parameters (intercept and slope) of $EWMA-3_{[SRS]}$ are described in Table 4. Where the result reveals that when slope is IC ($\beta=0$), then 12.14, 34.05, 55.42, 70.31, 79.88, and 85.64 percentage loss in the ARL's are reported with respect to shifts in intercept ($\delta=0.05$ upto 0.30). While on fixed intercept $\delta=0.05$, the loss in the ARL's due to shifts in slope ($\beta=0$ upto 0.25) are reported as follows: 12.14 %, 22.68 %, 43.79 %, 62.31 %, 76.12 %, 84.07 %, 88.78 %, 91.58 %, 93.46 %, 94.75 %, and 95.58 %.
- Results for $EWMA-3_{[RSS]}$ are reported in Table 5. Where the findings depict that when slope is IC, then shift in intercept ($\delta=0.05$) may cause 10.63, 12.21, 16.84, and 21.19 percentage loss in the ARL with respect to $\rho=0.25$, 0.50, 0.75, and 1. While on fixed $\delta=0.05$ and $\rho=0.25$, 10.63%, 35.18%, 57.83%, 72.78%, 81.91%, 87.14%, 90.46%, 92.53%, 93.90%, 94.96%, and 95.65 % loss in the ARL's are reported with respect to shift in slope ($\beta=0$ upto 0.25).
- Furthermore, performance of $EWMA-3_{[MRSS]}$ is described in Table 6. The results reveal that when slope is IC, then shift in intercept ($\delta=0.10$) may cause 35.62, 35.93, 44.36, and 60.29 percentage loss in the ARL's with respect to ρ = 0.25, 0.50, 0.75, and 1. While on fixed $\delta=0.10$ and $\rho=0.5$, 35.93%, 50.03%, 65.37%, 77.10%, 84.41%, 88.94%, 91.79%, 93.52%, 94.71%, 95.58%, and 96.20 % loss in the ARL's are reported with respect to shifts in slope ($\beta=0$ upto 0.25).

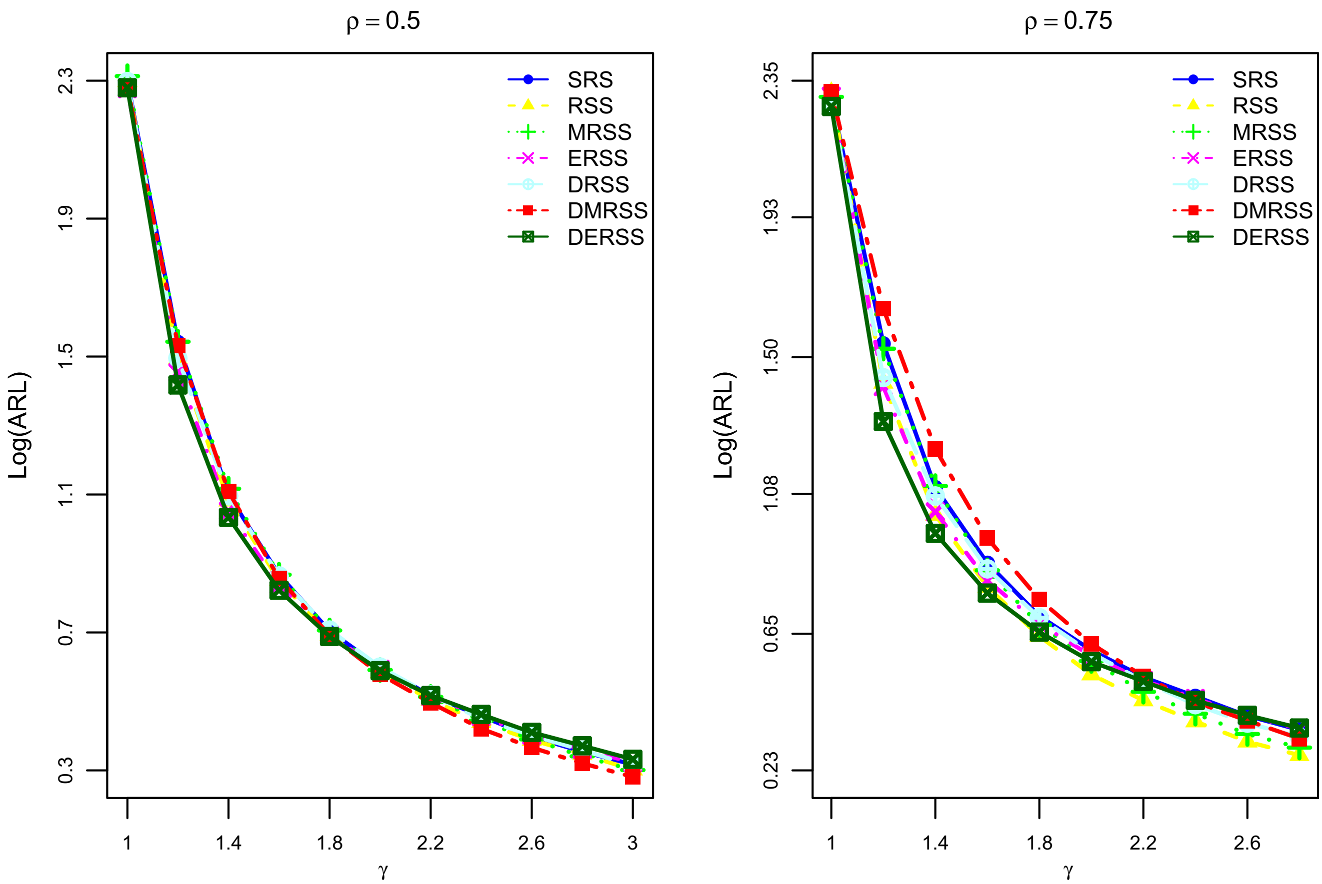


Fig. 3 ARL comparison of $EWMA-3_{[\tau]}$ chart for standard deviation shifts (σ to $\gamma\sigma$)

- $EWMA-3_{[ERSS]}$ scheme is reported in Table 7. Where the results depict that when slope is IC ($\beta=0$), then shift in intercept ($\delta=0.15$) may cause 56.71, 60.03, 63.96, and 73.27 percentage loss in the ARL's with respect to ρ = 0.25, 0.50, 0.75, and 1. While on fixed $\delta=0.15$ and $\rho=0.75$, 63.96%, 68.10%, 75.05%, 81.21%, 85.99%, 89.35%, 91.50%, 92.98%, 93.99%, 94.72%, and 95.32 % loss in the ARL's are reported with respect to shifts in slope from $\beta=0$ to $\beta=0.25$.
- Findings of $EWMA-3_{[DRSS]}$ are reported in Table 8, which depict that when slope is IC, then shift in intercept ($\delta=0.20$) may result in 70.47, 70.93, 75.28, and 81.60 percentage loss in ARL's with respect to ρ = 0.25, 0.50, 0.75, and 1. While on fixed $\delta=0.20$ and $\rho=1$, loss in the ARL's with respect to shifts in slope from $\beta=0$ to $\beta=0.25$ be reported as follows: 81.60%, 84.14%, 87.20%, 89.95%, 91.96%, 93.33%, 94.30%, 95.04%, 95.57%, 95.99% and 96.34 %.
- Shifts in both linear profile parameters (intercept and slope) of $EWMA-3_{[DMRSS]}$ are described in Table 9. Results depict that when slope is IC, then shift in intercept ($\delta=0.25$) may cause 80.53, 84.37, 90.18, and 96.54 percentage loss in the ARL's with respect to ρ = 0.25, 0.50, 0.75, and 1. While on fixed δ = 0.25 and ρ = 0.50, 84.37, 84.95, 86.57, 88.45, 90.45, 92.21, 93.67, 94.67, 95.55, 96.12, and 96.64 % loss in the ARL's are reported with respect to shifts in slope ($\beta=0$ upto 0.25).
- Results of $EWMA-3_{[DERSS]}$ are reported in Table 10, which reveals that when slope is in control, then shift in intercept ($\delta=0.30$) may cause 86.47, 88.43, 90.44, and 94.34 percentage loss in the ARL's with respect to $\rho=0.25, 0.50, 0.75$, and 1. While on fixed δ = 0.30 and ρ = 0.75, loss in the ARL's with respect to shifts in slope from $\beta=0$ to $\beta=0.25$ be reported as follows**:** 90.44, 90.65, 90.98, 91.53, 92.33, 93.02, 93.66, 94.30, 94.81, 95.25, and 95.61 %.
- Finally, it is concluded that in joint shifts, least values of the EQL, RARL, and PCI reveal that $EWMA-3_{[DMRSS]}$ scheme outperform all others.

Shift in error variance parameter

- The detection ability of shifts in the error variance depicts that all the $EWMA-3_{[\tau]}$ schemes perform almost the same. However, at larger choices of ρ (say $\rho=0.75$), $EWMA-3_{[DERSS]}$ scheme performs slightly better than the others (up to 1.8σ shift), while $EWMA-3_{[DMRSS]}$ offers relatively poor performance in this case (cf. Fig. 3).

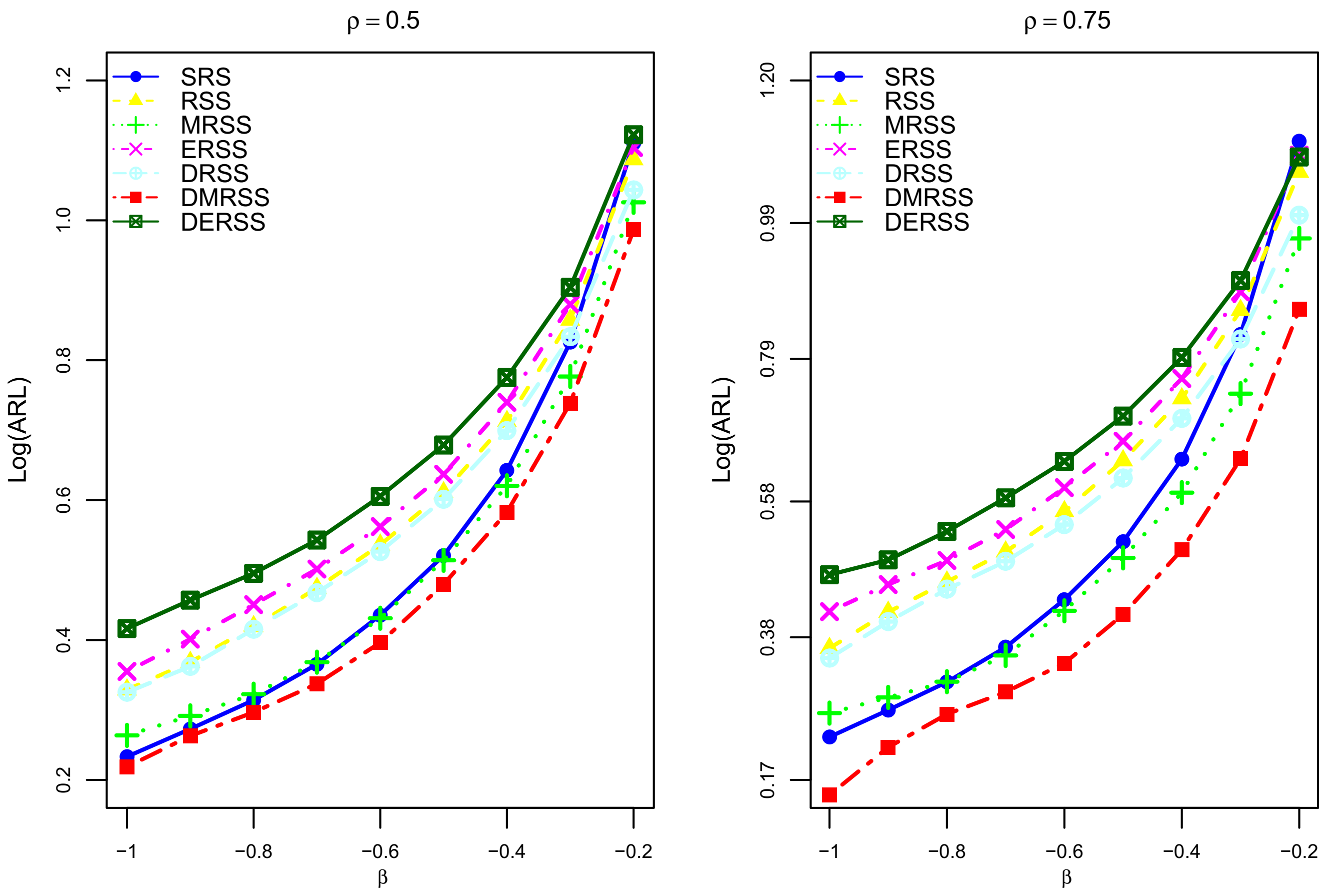


Fig. 4 ARL comparison of $EWMA-3_{[\tau]}$ chart for slope shifts (A_1^* to $A_1^* + \beta\sigma$)

An overall view

- The logarithmic ARL's for different shifts in intercept are portrayed in Fig. 1. The results depict that when RSS is imperfect ($\rho = 0.25$), no real change is experienced in the performance $EWMA-3_{[\tau]}$ schemes. With the increase of ρ, we have observed significant changes in the performance of $EWMA-3_{[\tau]}$ schemes. Overall, $EWMA-3_{[DMRSS]}$ scheme outperforms all other schemes under consideration. The same may also be noticed from Fig. 2, where shifts in term of σ unit are considered for the slope in original model. Figure 3 (showing the detection of shifts in error variance) reveals that $EWMA-3_{[DERSS]}$ is marginally better than others while $EWMA-3_{[DMRSS]}$ is a poor performer, especially at larger values of shifts and ρ. Figure 4 (negative shifts in the slope of transformed model) depicts that $EWMA-3_{[DMRSS]}$ scheme surpass all other schemes under consideration. Figure 5 (referring to different values of independent variable) exhibits that $EWMA-3_{[DMRSS]}$ scheme beats all the other schemes under consideration.

4 Effect of λ and n on the performance of $EWMA-3_{[\tau]}$

The Greek letter λ is the smoothing parameter of $EWMA-3_{[\tau]}$ chart, which is used to assign the weight to current information and previous information. λ always lie between 0 and 1 (i.e., (0,1]). As the value of λ increase, more weight is assigned to the current information and less to the previous information and vice versa. The effects of different λ such as 0.20, 0.35, and 0.75 on $EWMA-3_{[\tau]}$ chart with fixed sample size ($n = 4$) and correlation ($\rho = 0.75$) are portrayed in Fig. 6. Figure 6a is about the $EWMA-3_{[DMRSS]}$ scheme where the intercept is fixed (i.e., $\delta = 0.10$) and the shifts in slope parameter are portrayed with respect to λ. The results reveal an inverse relation between λ and the performance of $EWMA-3_{[DMRSS]}$ chart. Change in intercept parameter on the fixed slope parameter (i.e., $\beta = 0.10$) is plotted in Fig. 6b. The results also reveal inverse relation between λ and the performance of $EWMA-3_{[DMRSS]}$. Kang and Albin [22] reported that an increase in the standard deviation may cause increase in the variance of the estimators (intercept and slope). That's why EWMA structure is not reliable in this case. Same results are also captured in Fig. 6c that on the small value of λ, $EWMA-3_{[DERSS]}$ chart fails to show good performance but as the λ increase

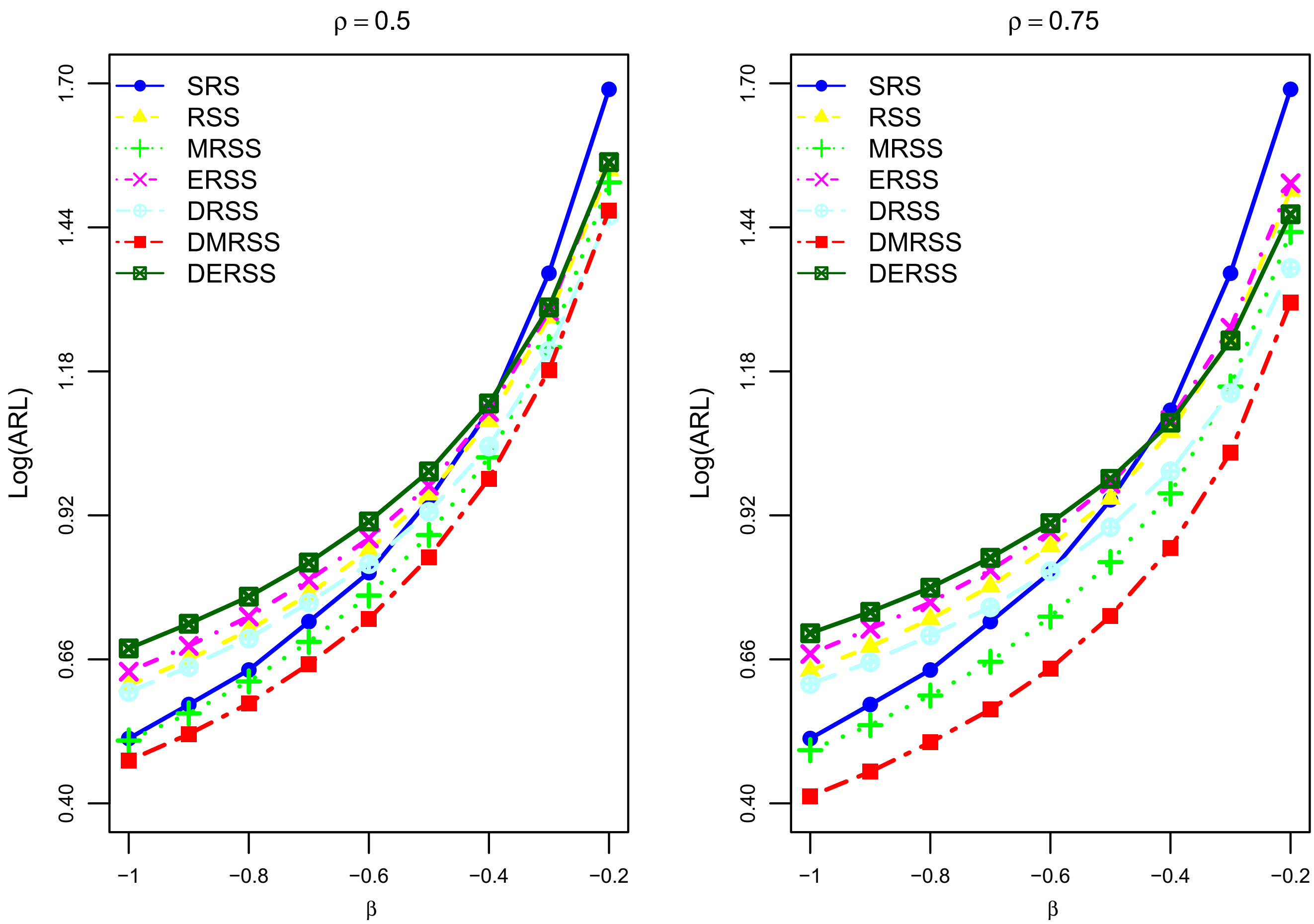


Fig. 5 ARL comparison of $EWMA-3_{[\tau]}$ chart for slope shifts (A_1^* to $A_1^* + \beta\sigma$) with the change in value of X.

$EWMA-3_{[DERSS]}$ chart performs better. Negative Shifts for slope parameter with the change in value of predictor variable $X_{(i)k}$ are reported in Fig. 6d, which depicts that on large value of λ, $EWMA-3_{[DMRSS]}$ scheme performed well as compared to small values of λ.

The size of sample n plays an important role during the experimental studies as well as in the precision of the studies. Effect of sample size in $EWMA-3_{[\tau]}$ chart on fixed smoothing parameter $\lambda = 0.20$ and $\rho = 0.5$ are discussed in Fig. 7. Shifts in slope parameter on the fixed intercept parameter (i.e., $\delta = 0.20$) are plotted in Fig. 7a. The results reveal direct relation between n and the performance of $EWMA-3_{[DMRSS]}$ chart. Same findings are also highlighted in Fig. 7b, where the shifts in intercept parameter on fixed slope parameter (i.e., $\beta = 0.125$) are portrayed. Figure 7c is about the $EWMA-3_{[DERSS]}$ chart where the shifts in standard deviation of error are portrayed with respect to sample size. The findings reveal direct relation between n and the performance of $EWMA-3_{[DERSS]}$ chart. While negative shifts in the slope parameter with the change in value of predictor variable $X_{(i)k}$ are plotted in the Fig. 7d, which also depicts that as the sample size increase, performance of $EWMA-3_{[DMRSS]}$ scheme also increased.

5 A real application

In this section, illustrations with the real-life example of linear profiles between voltage and capacitance of Z-source inverter system is discussed.

5.1 Data description about Z-source inverter system

In conventional voltage source inverter (VSI), a DC voltage source is connected in parallel to a capacitor which is connected to three phase (3-φ) bridge whereas in conventional current source inverter (CSI), a DC current source is connected to 3-φ bridge inverter through inductor. The VSI is a buck inverter, i.e., the output voltage is always less than the dc input voltage while the CSI is a boost inverter, i.e., the output voltage is always greater than the input dc voltage. In a grid connected PV system, the Z-source inverter has been used by many researchers instead of conventional VSI or CSI to improve the overall system efficiency. The conventional inverters can only buck or boost the output voltage; in addition, the electromagnetic interferences (EMI) and other restrictions lower their efficiency. However, the Z-source inverter is buck-boost inverter which eliminates the need of buck-boost converter and also overcomes various

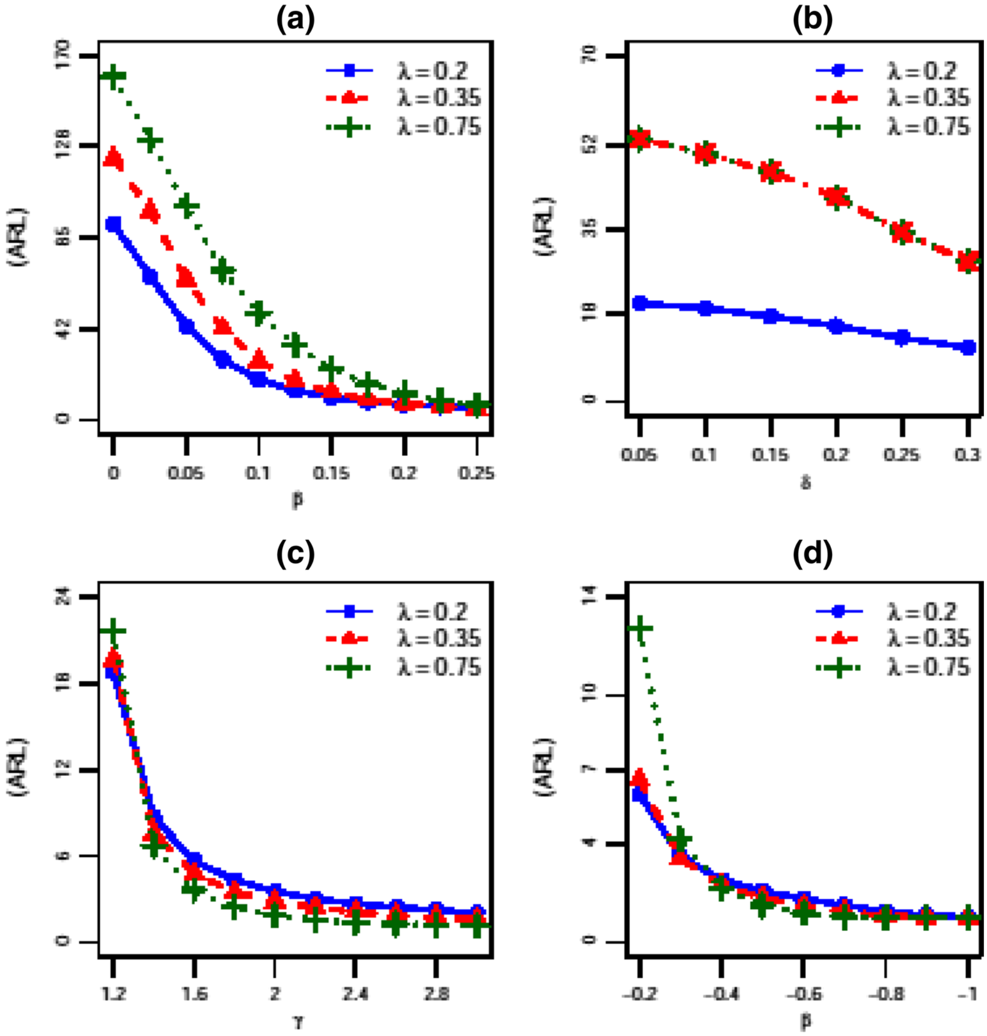


Fig. 6 ARL comparison of $EWMA-3_{[\tau]}$ chart with respect to smoothing parameter (λ).

problems associated with conventional inverters. The Z-source inverter consists of two inductors, two capacitors, and 3-φ bridge inverter connected. The switches used in bridge circuit can have either series or anti-parallel diodes as shown in Fig. 8.

In grid-connected PV system, the output of PV arrays is connected to DC link capacitor through maximum power point tracker (MPPT) to get the maximum available power from PV system. The MPPT control is connected to DC-DC boost converter which maintains the constant voltage at DC link by adjusting the duty cycle of boost converter. This DC voltage is then converted to AC through inverter and finally connected to local grid. A diagram of grid-connected PV system is shown in Fig. 9.

Usually, parallel plate capacitors are used as a DC link which consists of two conductive plates separated by a dielectric material. In a parallel plate capacitor, capacitance is directly proportional to the surface area of the conductive plates and inversely proportional to the distance between them. If the charge on the plates are $+q$ and $-q$, and V is the potential difference between the plates, then the capacitance C is given by

$$\mathrm{C} = \frac{\mathrm{Q}}{\mathrm{V}}$$

5.2 Implementation of $EWMA-3_{[\tau]}$ chart

For the illustrative example, we get 75,456 sample values of voltage (V) against each level of capacitance (C) given in Mukhtar [60]. There exist 7 different capacitance levels such as $50\mu F, 100\mu F, 150\mu F, 200\mu F, 250\mu F, 300\mu F$, and 350 μF. In the stated study, we consider voltage (V) as a dependent variable and capacitance (C) as an independent variable. Further, we proceed with the following steps:

Step 1: For the IC regression model, we run 75,456 sample values of V against fixed values of C and get a following model

$$V = 398.9197 - 0.005015086\ \mathrm{C}$$

Step 2: We applied DMRSS and DERSS techniques on the observations of V at each level of C and get 1533

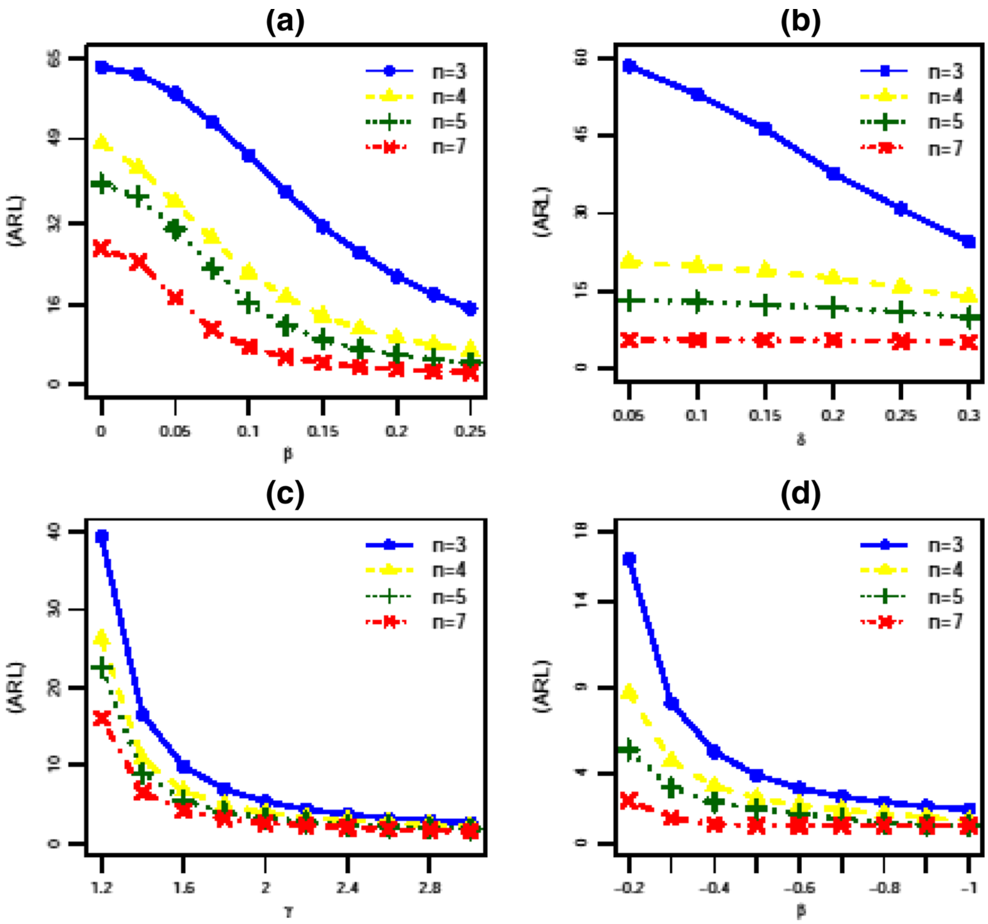


Fig. 7 ARL comparison of $EWMA-3_{[\tau]}$ chart with respect to sample size (n).

ranked set observations of V at each level of C. Further, 1533 groups of V values are created which are parallel to different levels of C. Finally, 1533 profiles under DMRSS and DERSS are generated by the use of 1533 groups of respected ranked set V against transformed C (i.e., $C^{*}=-150,-100,-50,0,50,100,150$).

Step 3: For the analysis purpose, we calculate the standard deviation of V which is $\sigma=11.44713$ and we fixed $\lambda=0.20$, $ARL_0=200$, $L_{I[DMRSS]}=942.7$, $L_{I[DERSS]}=943.54$, $L_{S[DMRSS]}=14.1$, $L_{S[DERSS]}=14.84$, $L_{E[DMRSS]}=0.1543$, $L_{E[DERSS]}=0.155$, $\sigma^2_{MSE[DMRSS]}=2355.329$ and $\sigma^2_{MSE[DERSS]}=2324.417$.

Step 4: For the diagnosis purpose, we calculate 1533 plotting statistics for $EWMA-3_{[DMRSS]}$ and $EWMA-3_{[DERSS]}$ by using 1533 profiles of under DMRSS and DERSS, respectively, and then plotted against the control limits such as the following: $UCL_{I[DMRSS]}=799.7551$, $LCL_{I[DMRSS]}$

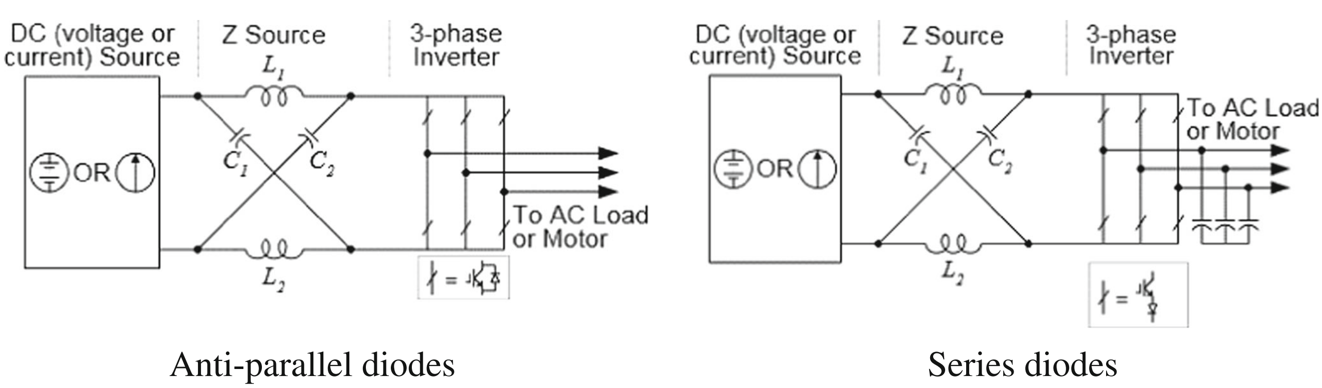


Fig. 8 Z-source inverter with different switches

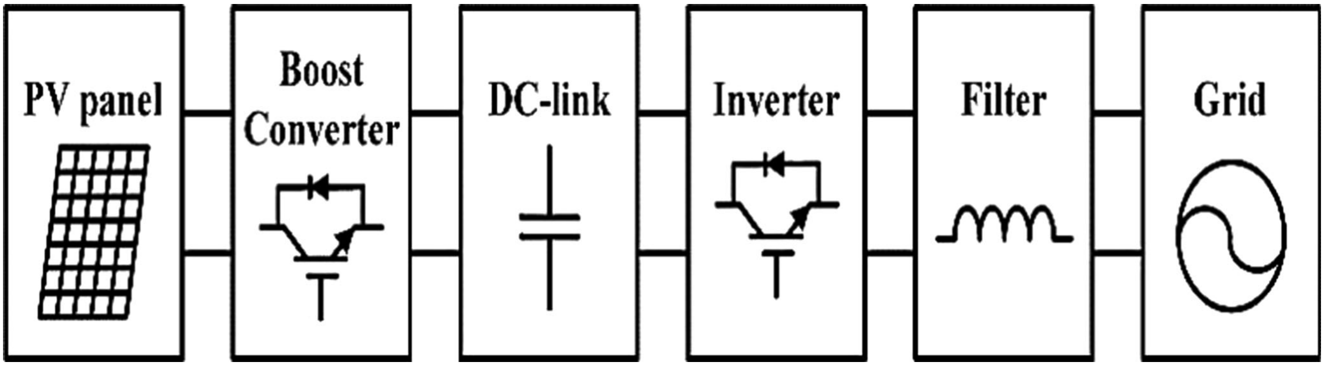


Fig. 9 Grid-connected PV system

$= -3.921784$, $UCL_{I[DERSS]} = 800.1132$, $LCL_{I[DERSS]} = -4.279846$, $UCL_{S[DMRSS]} = 0.05508804$, $LCL_{S[DMRSS]} = -0.06511822$, $UCL_{S[DERSS]} = 0.05824239$, $LCL_{S[DERSS]} = -0.06827256$, $UCL_{E[DMRSS]} = 4.933888$, and $UCL_{E[DERSS]} = 4.928703$.

Step 5: In the first stage of diagnosis analysis: in case of $EWMA-3_{[DMRSS]}$, 122 profiles are declared OOC and excluded while in case of $EWMA-3_{[DERSS]}$, 83 profiles are declared OOC and excluded. In the second stage: in case of $EWMA-3_{[DMRSS]}$, 7 profiles are declared OOC and excluded while in case of $EWMA-3_{[DERSS]}$, only 1 profile is declared OOC and excluded.

Step 6: Once we get, 1404 IC profiles for $EWMA-3_{[DMRSS]}$ and 1449 IC profiles for $EWMA-3_{[DERSS]}$, we used only first hundred profiles as an IC profiles shaded pink in Figs. 1 and 2. Further, following phases for several shifts are made

(i) For the detection of shifts in the intercept, we used $C^* = -550, -500, -450, -400, -350, -300, -250$, against the profiles indexed 101 to 125.

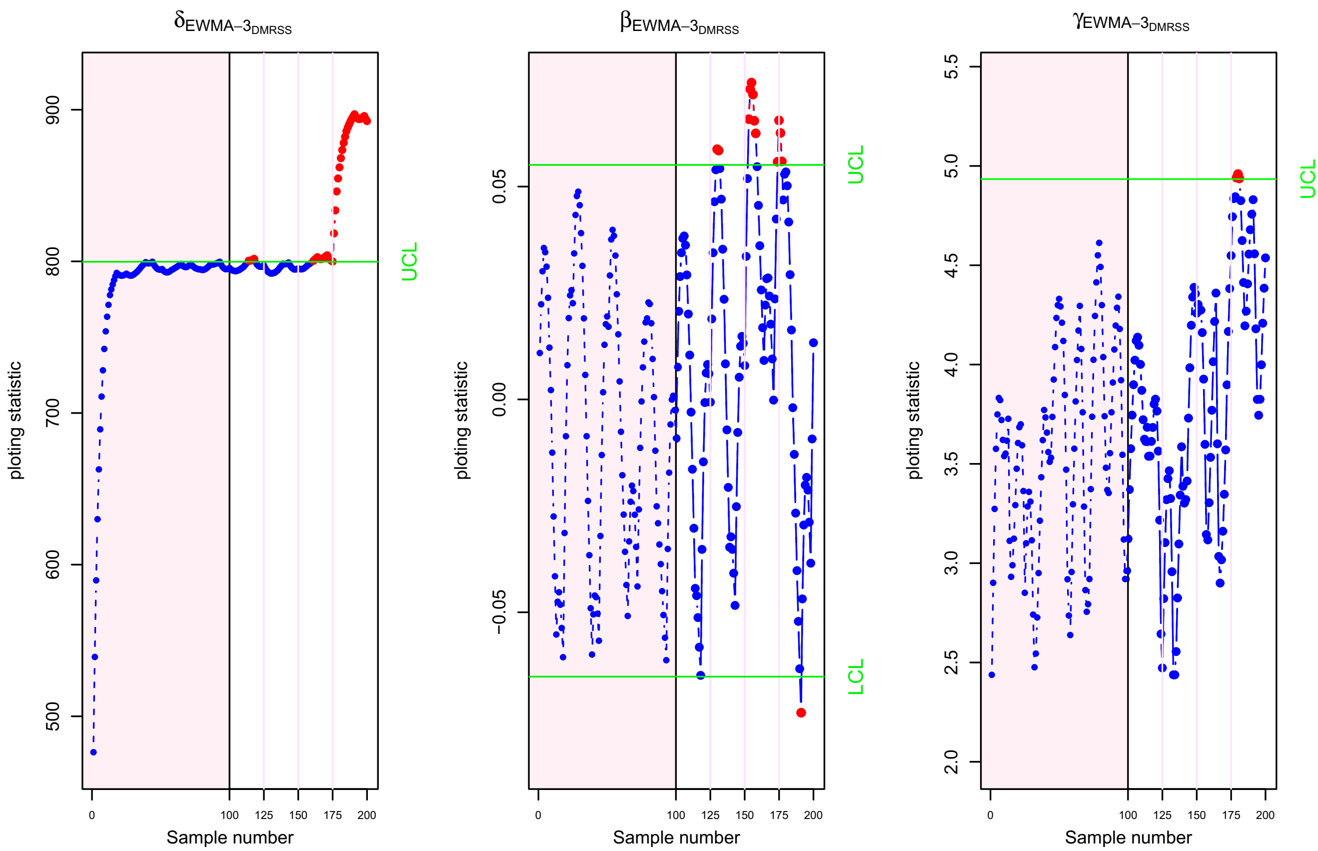


Fig. 10 $EWMA-3_{[DMRSS]}$ chart for different phases of illustrative example

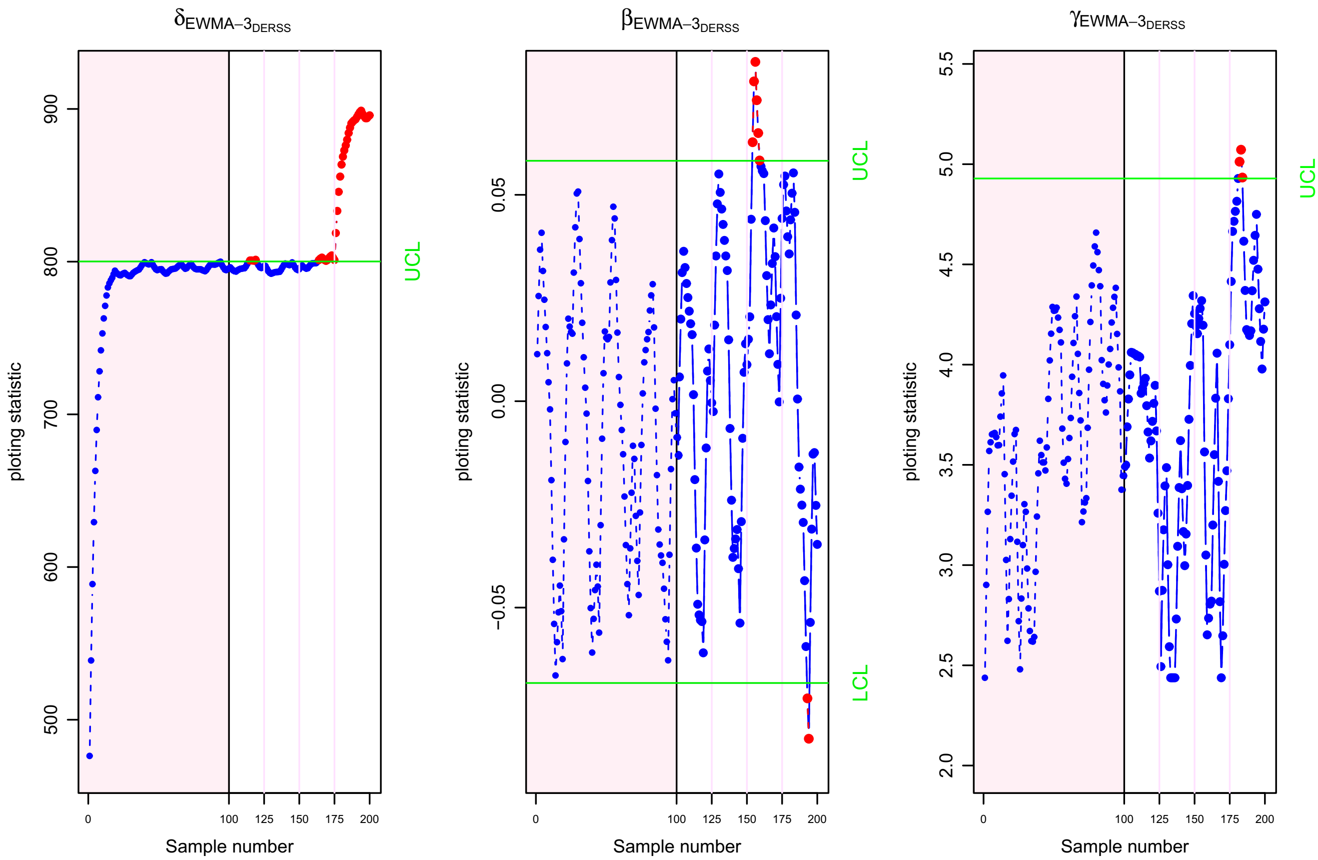


Fig. 11 $EWMA-3_{[DERSS]}$ chart for different phases of illustrative example

(ii) For the detection of shift in the slope, we used $C^{*}=75, 50, 25, 0, -25, -50, -75$, against the profiles indexed 126 to 150.

(iii) For the detection of shift in the both intercept and slope, we used $C^{*}=500, 0, -500, -1000, -1500, -2000, -2500$, against the profiles indexed 151 to 175.

(iv) For the detection of shift in the error variance, we multiply profiles indexed 176 to 200 with 1.5 and then run against $C^{*}=-150, -100, -50, 0, 50, 100, 150$.

The findings of each phase are portrayed in Figs. 10 and 11. The indexed about the OOC profiles are given in Table 11 which depicts that our proposed $EWMA-3_{[DMRSS]}$ surpass the other chart for intercept, slope, and joint shifts while $EWMA-3_{[DERSS]}$ offers slightly better ability for shifts in error variance.

6 Summary, conclusions, and recommendations

In many manufacturing processes, study variable is not only single quality characteristic but there exist some explanatory variable(s) that is/are linearly associated with study variable. Monitoring the process when study variable has linear functionality with other independent variable(s) is termed as linear profiling. A popular scheme $EWMA-3_{[SRS]}$ was proposed for detecting simultaneous shifts in linear profile parameters including

Table 11 Indexed of OOC profiles at every stage of illustrative example

Interval of Points	Parameters	Intercept (A_0^*)		Slope (A_1^*)		Error Variance (σ^2)	
	$EWMA-3_{[\tau]}$	DMRSS	DERSS	DMRSS	DERSS	DMRSS	DERSS
1–100		NA	NA	NA	NA	NA	NA
101–125		114–118	115–119	NA	NA	NA	NA
126–150		NA	NA	130–131	NA	NA	NA
151–175		161–175	164–175	153–158, 174–175	154–159	NA	NA
176–200		176–200	176–200	176–177,191	193,194	179–181	182–184

*NA = No OOC points

intercept, slope, and error variance. In this study, we have designed $EWMA-3_{[\tau]}$ chart under different sampling strategies such as RSS, MRSS, ERSS, DRSS, DMRSS, and DERSS.

In this study, we consider different shifts for the monitoring of linear profile parameters including intercept, slope, and error variance. In case of several shifts in intercept term, it is concluded that $EWMA-3_{[DMRSS]}$ scheme outperforms all other schemes under consideration. The same may also be noticed where shifts in terms of σ unit are considered for the slope in original model. The detection of shifts in error variance reveals that $EWMA-3_{[DERSS]}$ is marginally better than others while $EWMA-3_{[DMRSS]}$ is a poor performer, especially at larger values of shifts and ρ. Further, negative shifts in the slope of transformed model depicts that $EWMA-3_{[DMRSS]}$ scheme surpass all other schemes under consideration and referring to different values of independent variable exhibits that $EWMA-3_{[DMRSS]}$ scheme beats all the other schemes under consideration.

Overall, when RSS is imperfect ($\rho=0.25$), no real change is experienced in the performance $EWMA-3_{[\tau]}$ schemes and with the increase of ρ; we have observed significant improvement in the performance of $EWMA-3_{[\tau]}$ schemes. Further, as the smoothing parameter λ decreases, performance of $EWMA-3_{[\tau]}$ also increased, but in some cases, reverse results are observed, and as the sample size n increases, performance of $EWMA-3_{[\tau]}$ also increased. The scope of the study may be extended in other directions such as implementation of run rules, addition of fast initial response (FIR) feature, and coverage of multiple and non-linear profiles.

7 Nomenclature

τ; Sampling strategies

UCL_E; Upper control limit for error term

SRS; Simple random sampling

L_I; Charting constant of EWMA for intercept term

RSS; Ranked set sampling

L_S; Charting constant of EWMA for slope term

ERSS; Extreme ranked set sampling

L_E; Charting constant of EWMA for error term

MRSS; Median ranked set sampling

$\sigma^2_{e[r]}$; Variance of error under RSS

DRSS; Double ranked set sampling

$\sigma^2_{e[dr]}$; Variance of error under DRSS

DMRSS; Double median ranked set sampling

$\sigma^2_{e[m]}$; Variance of error under MRSS

DERSS; Double extreme ranked set sampling

$\sigma^2_{e[dm]}$; Variance of error under DMRSS

$EWMA-3_{[\tau]}$; EWMA-3 chart under specified sampling

$\sigma^2_{e[e1]}$; Variance of smallest error under ERSS

EQL; Extra quadratic loss

$\sigma^2_{e[de1]}$; Variance of smallest error under DERSS

RARL; Relative average run length

$\sigma^2_{e[en]}$; Variance of largest error under ERSS

PCI; Performance comparison index

$\sigma^2_{e[den]}$; Variance of largest error under DERSS

LCL; Lower control limit

ARL_0; In-control Average rum length

UCL; upper control limit

ARL_1; Out-of-control Average rum length

CL; Center line

δ; Shift in Intercept

IC; In-control

β; Shift in slope

OOC; Out-of-control

γ; Shift in error term

EWMA; Exponentially weighted moving average

VSI; Voltage Source Inverter

CUSUM; Cumulative sum

CSI; Current Source Inverter

MCUSUM; Multivariate cumulative sum

PV; Photo voltaic

n; Size of samples

DC; Direct Current

k; Number of cycles

AC; Alternating Current

λ; Smoothing parameter of EWMA statistics

V; Voltage

ρ; correlation coefficient

C; Capacitence

$Y_{[i]k}$; Study variable (In SRS, i^{th} random sample in first cycle (i.e. $k=1$) otherwise i^{th} ordered sample in k^{th} sample)

E[.]; Expected value of [.]

$X_{(i)k}$; Predictor variable (fixed)

Var[.]; Variance of [.]

$\varepsilon_{[i]k}$; Error term (In SRS, i^{th} random sample in first cycle (i.e. $k=1$) otherwise i^{th} ordered sample in k^{th} sample).

Cov[.]; Co-variance of [.]

β_0; Intercept

F [.]; Cumulative distribution function of [.]

β_1; Slope

f [.]; Probability density function of [.]

$\sigma^2_{e[\tau]}$; Variance of error under Sampling strategies

w; Standardized form of error term

$e_{[i]k}$; Residual term (In SRS, i^{th} random sample in first cycle (i.e. $k=1$) otherwise i^{th} ordered sample in k^{th} sample).

$\varepsilon_{(r)}$; Error term of RSS

$MSE_{[i]k}$; Mean square error (In SRS, i^{th} random sample in first cycle (i.e. $k=1$) otherwise i^{th} ordered sample in k^{th} sample).

$\varepsilon^*_{(r)}$; Error term of DRSS

$X^*_{(i)k}$; Fixed predictor variable (Transformed)

w ; Standardized error of RSS

A^*_0; Intercept (Transformed)

ε ; Error term of MRSS

A^*_1; Slope (Transformed)

$\varepsilon^*_{(m)}$; Error term of DMRSS

$EWMA-3_{I[i][\tau]}$; EWMA-3 statistic for intercept under specified sampling

$EWMA-3_{S[i][\tau]}$; EWMA-3 statistic for slope under specified sampling

$EWMA-3_{E[i][\tau]}$; EWMA-3 statistic for error term under specified sampling

LCL_I; Lower control limit for intercept term

UCL_I; Upper control limit for intercept term

LCL_S; Lower control limit for slope term

UCL_S; Upper control limit for slope term

$w_{(m)}$; Standardized error of MRSS

$\varepsilon_{(1)}$; Smallest error term of ERSS

$\varepsilon_{(n)}$; Largest error term of ERSS

$\varepsilon^*_{(1)}$; Smallest error term of DERSS

$\varepsilon^*_{(n)}$; Largest error term of DERSS

$w_{(1)}$; Standardized smallest error of ERSS

$w_{(n)}$; Standardized largest error of ERSS

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Appendix

(i) Transformed linear model under different ranked set samplings

The simple linear profile model under ranked set strategies is defined as follows:

$$Y_{[i]k} = \beta_0 + \beta_1 X_{(i)k} + \varepsilon_{[i]k};\ \mathrm{i} = 1, 2, 3, \ldots\ldots.., r;\ \ \mathrm{k} = 1, 2, 3, \ldots\ldots.., m$$

writing the shifted β_1 such as $\beta_1 = \beta_1 + \beta\sigma$ in the linear regression model under different ranked set strategies (τ) given in above model, we obtain

$$Y_{[i]k} = \beta_0 + (\beta_1 + \beta\sigma)X_{(i)k} + \varepsilon_{[i]k}$$

where β is the shift for slope and by adding or subtracting with $(\beta_1 + \beta\sigma)\overline{X}_{[\tau]}$, we get

$$Y_{[i]k} = \beta_0 + (\beta_1 + \beta\sigma)X_{(i)k} + \varepsilon_{[i]k} + (\beta_1 + \beta\sigma)\overline{X}_{[\tau]} - (\beta_1 + \beta\sigma)\overline{X}_{[\tau]}$$
$$Y_{[i]k} = \left[\beta_0 + (\beta_1 + \beta\sigma)\overline{X}_{[\tau]}\right] + \left[(\beta_1 + \beta\sigma)X_{(i)k} - (\beta_1 + \beta\sigma)\overline{X}_{[\tau]}\right] + \varepsilon_{[i]k}$$
$$Y_{[i]k} = \left[\left(\beta_0 + \beta_1\overline{X}_{[\tau]}\right) + (\beta\sigma)\overline{X}_{[\tau]}\right] + \left[(\beta_1 + \beta\sigma)\left(X_{(i)k} - \overline{X}_{[\tau]}\right)\right] + \varepsilon_{[i]k}$$

Since average of $X_{(i)k}{}^*$ is zero, so the covariance will be also zero and assumed $A_0 = \beta_0 + \beta_1\overline{X}_{[\tau]}$, $A_1 = \beta_1$ and $X_{(i)k}{}^* = X_{(i)k} - \overline{X}_{[\tau]}$ then the above equation is written as

$$Y_{[i]k} = \left(A_0 + (\beta\sigma)\overline{X}_{[\tau]}\right) + (A_1 + \beta\sigma)X_{(i)k}{}^* + \varepsilon_{ik}$$
$$Y_{[i]k} = \left(A_0{}^*\right) + \left(A_1{}^*\right) X_{(i)k}{}^* + \varepsilon_{[i]k}$$

(ii) Properties of error term in different ranked set samplings

In the simple regression, we assumed that the error term is normally distributed having the mean zero and constant variance (i.e., $\varepsilon \sim \mathrm{N}(0, \sigma^2)$). So, the standardized form of error is defined as

$$w = \frac{\varepsilon - 0}{\sigma} = \frac{\varepsilon}{\sigma}$$

In this study, we are focusing on different strategies (τ) named RSS, MRSS, ERSS, DRSS, DMRSS, and DERSS so the probability density function f(.), mean E(.) , and variance Var(.) for error term under RSS ($\varepsilon_{(r)}$)and DRSS $\left(\varepsilon^*_{(r)}\right)$ are defined as

$$f\left(\varepsilon_{(r)};\sigma^2\right) = n!\prod_{r=1}^{n}\left[\frac{n!}{(r-1)!(n-r)!}\left\{F\left(\frac{\varepsilon}{\sigma}\right)\right\}^{r-1}\left\{1-F\left(\frac{\varepsilon}{\sigma}\right)\right\}^{n-r} f\left(\frac{\varepsilon}{\sigma}\right)\frac{1}{\sigma}\right]$$
$$E\left(\varepsilon_{(r)}\right) = \sigma\int_{-\infty}^{+\infty} w\,n!\prod_{r=1}^{n}\left[\frac{n!}{(r-1)!(n-r)!}\{F(w)\}^{r-1}\{1-F(w)\}^{n-r} f(w)dw\right]$$
$$E\left(\varepsilon_{(r)}\right) = \sigma D_{r1}$$
$$Var\left(\varepsilon_{(r)}\right) = \sigma^2\int_{-\infty}^{+\infty} w^2 \mathrm{n}!\prod_{r=1}^{n}\left[\frac{n!}{(r-1)!(n-r)!}\{F(w)\}^{r-1}\{1-F(w)\}^{n-r} f(w)dw\right] - \left(E\left(\varepsilon_{(r)}\right)\right)^2$$
$$\sigma^2_{e[r]} = Var\left(\varepsilon_{(r)}\right) = \sigma^2 D_{r2} - (\sigma D_{r1})^2 = \sigma^2\left(D_{r2} - D^2_{r1}\right)$$
$$f\left(\varepsilon^*_{(r)};\sigma^2\right) = n!\prod_{r=1}^{n}\left[\frac{n!}{(r-1)!(n-r)!}\left\{F\left(\frac{\varepsilon_{(r)}}{\sigma}\right)\right\}^{r-1}\left\{1-F\left(\frac{\varepsilon_{(r)}}{\sigma}\right)\right\}^{n-r} f\left(\frac{\varepsilon_{(r)}}{\sigma}\right)\frac{1}{\sigma}\right]$$
$$E\left(\varepsilon^*_{(r)}\right) = \sigma\int_{-\infty}^{+\infty} w_{(r)}\mathrm{n}!\prod_{r=1}^{n}\left[\frac{n!}{(r-1)!(n-r)!}\left\{F\left(w_{(r)}\right)\right\}^{r-1}\left\{1-F\left(w_{(r)}\right)\right\}^{n-r} f\left(w_{(r)}\right)dw_{(r)}\right]$$
$$E\left(\varepsilon_{(r)}\right) = \sigma D^*_{r1}$$
$$Var\left(\varepsilon^*_{(r)}\right) = \sigma^2\int_{-\infty}^{+\infty} w^2_{(r)}\mathrm{n}!\prod_{r=1}^{n}\left[\frac{n!}{(r-1)!(n-r)!}\left\{F\left(w_{(r)}\right)\right\}^{r-1}\left\{1-F\left(w_{(r)}\right)\right\}^{n-r} f\left(w_{(r)}\right)dw_{(r)}\right] - \left(E\left(\varepsilon^*_{(r)}\right)\right)^2$$
$$\sigma^2_{e[dr]} = Var\left(\varepsilon^*_{(r)}\right) = \sigma^2 D^*_{r2} - \left(\sigma D^*_{r1}\right)^2 = \sigma^2\left(D^*_{r2} - D^{*2}_{r1}\right)$$

For odd set ($n = 2m-1$), the probability density function f(.), mean E(.) , and variance Var(.) for error term under MRSS ($\varepsilon_{(m)}$) and DMRSS $\left(\varepsilon^*_{(m)}\right)$ are defined

$$f\left(\varepsilon_{(m)};\sigma^2\right) = \frac{(2m-1)!}{(m-1)!(m-1)!}\left\{F\left(\frac{\varepsilon}{\sigma}\right)\right\}^{m-1}\left\{1-F\left(\frac{\varepsilon}{\sigma}\right)\right\}^{m-1} f\left(\frac{\varepsilon}{\sigma}\right)\frac{1}{\sigma}$$
$$E\left(\varepsilon_{(m)}\right) = \sigma\int_{-\infty}^{+\infty} w\ \frac{(2m-1)!}{(m-1)!(m-1)!}\{F(w)\}^{m-1}\{1-F(w)\}^{m-1} f(w)\,dw$$
$$E\left(\varepsilon_{(m)}\right) = \sigma D_{m1}$$
$$Var\left(\varepsilon_{(m)}\right) = \sigma^2\int_{-\infty}^{+\infty} w^2\ \frac{(2m-1)!}{(m-1)!(m-1)!}\{F(w)\}^{m-1}\{1-F(w)\}^{m-1} f(w)\,dw - \left(E\left(\varepsilon_{(m)}\right)\right)^2$$
$$\sigma^2_{e[m]} = Var\left(\varepsilon_{(m)}\right) = \sigma^2 D_{m2} - (\sigma D_{m1})^2 = \sigma^2\left(D_{m2} - D^2_{m1}\right)$$
$$f\left(\varepsilon^*_{(m)};\sigma^2\right) = \frac{(2m-1)!}{(m-1)!(m-1)!}\left\{F\left(\frac{\varepsilon_{(m)}}{\sigma}\right)\right\}^{m-1}\left\{1-F\left(\frac{\varepsilon_{(m)}}{\sigma}\right)\right\}^{m-1} f\left(\frac{\varepsilon_{(m)}}{\sigma}\right)\frac{1}{\sigma}$$
$$E\left(\varepsilon^*_{(m)}\right) = \sigma\int_{-\infty}^{+\infty} w_{(m)}\ \frac{(2m-1)!}{(m-1)!(m-1)!}\left\{F\left(w_{(m)}\right)\right\}^{m-1}\left\{1-F\left(w_{(m)}\right)\right\}^{m-1} f\left(w_{(m)}\right)dw_{(m)}$$
$$E\left(\varepsilon^*_{(m)}\right) = \sigma D^*_{m1}$$
$$Var\left(\varepsilon^*_{(m)}\right) = \sigma^2\int_{-\infty}^{+\infty} w^2_{(m)}\ \frac{(2m-1)!}{(m-1)!(m-1)!}\left\{F\left(w_{(m)}\right)\right\}^{m-1}\left\{1-F\left(w_{(m)}\right)\right\}^{m-1} f\left(w_{(m)}\right)dw_{(m)} - \left(E\left(\varepsilon_{(m)}\right)\right)^2$$
$$\sigma^2_{e[dm]} = Var\left(\varepsilon^*_{(m)}\right) = \sigma^2 D^*_{m2} - \left(\sigma D^*_{m1}\right)^2 = \sigma^2\left(D^*_{m2} - D^{*2}_{m1}\right)$$

Let ε_1 is the error of smallest sample and ε_n is the error of largest sample then for odd sets, the probability density function f(.), mean E(.),and variance Var(.) for error term under ERSS ($\varepsilon_{(1)}, \varepsilon_{(n)}$) and DERSS ($\varepsilon^{*}_{(1)}, \varepsilon^{*}_{(n)}$) are defined

$$f\left(\varepsilon_{(1)};\sigma^2\right) = n\left\{1-F\left(\frac{\varepsilon}{\sigma}\right)\right\}^{n-1} f\left(\frac{\varepsilon}{\sigma}\right)\frac{1}{\sigma}$$

$$f\left(\varepsilon_{(n)};\sigma^2\right) = n\left\{F\left(\frac{\varepsilon}{\sigma}\right)\right\}^{n-1} f\left(\frac{\varepsilon}{\sigma}\right)\frac{1}{\sigma}$$

$$E\left(\varepsilon_{(1)}\right) = \sigma\int_{-\infty}^{+\infty} w\ n\ \{1-F(w)\}^{n-1} f\ (w)\ dw$$

$$E\left(\varepsilon_{(1)}\right) = \sigma\ D_{11}$$

$$E\left(\varepsilon_{(n)}\right) = \sigma\int_{-\infty}^{+\infty} w\ n\ \{F(w)\}^{n-1} f\ (w)\ dw$$

$$E\left(\varepsilon_{(n)}\right) = \sigma\ D_{n1}$$

$$Var\left(\varepsilon_{(1)}\right) = \sigma^2\int_{-\infty}^{+\infty} w^2\ \mathrm{n}\ \{1-F(w)\}^{n-1} f\ (w)\ dw - \left(E\left(\varepsilon_{(1)}\right)\right)^2$$

$$\sigma^2_{e[e1]} = Var\left(\varepsilon_{(1)}\right) = \sigma^2\ D_{12} - (\sigma D_{11})^2 = \sigma^2\ \left(D_{12} - D^2_{11}\right)$$

$$Var\left(\varepsilon_{(n)}\right) = \sigma^2\int_{-\infty}^{+\infty} w^2\ \mathrm{n}\ \{F(w)\}^{n-1} f\ (w)\ dw - \left(E\left(\varepsilon_{(n)}\right)\right)^2$$

$$\sigma^2_{e[en]} = Var\left(\varepsilon_{(n)}\right) = \sigma^2\ D_{n2} - (\sigma D_{n1})^2 = \sigma^2\ \left(D_{n2} - D^2_{n1}\right)$$

$$f\left(\varepsilon^{*}_{(1)};\sigma^2\right) = n\left\{1-F\left(\frac{\varepsilon_{(1)}}{\sigma}\right)\right\}^{n-1} f\left(\frac{\varepsilon_{(1)}}{\sigma}\right)\frac{1}{\sigma}$$

$$f\left(\varepsilon^{*}_{(n)};\sigma^2\right) = n\left\{F\left(\frac{\varepsilon_{(n)}}{\sigma}\right)\right\}^{n-1} f\left(\frac{\varepsilon_{(n)}}{\sigma}\right)\frac{1}{\sigma}$$

$$E\left(\varepsilon^{*}_{(1)}\right) = \sigma\int_{-\infty}^{+\infty} w_{(1)}\ \mathrm{n}\left\{1-F\left(w_{(1)}\right)\right\}^{n-1} f\ \left(w_{(1)}\right)\ dw_{(1)}$$

$$E\left(\varepsilon_{(1)}\right) = \sigma\ D^{*}_{11}$$

$$E\left(\varepsilon^{*}_{(n)}\right) = \sigma\int_{-\infty}^{+\infty} w_{(n)}\ \mathrm{n}\ \left\{F\left(w_{(n)}\right)\right\}^{n-1} f\ \left(w_{(n)}\right)\ dw_{(n)}$$

$$E\left(\varepsilon^{*}_{(n)}\right) = \sigma\ D^{*}_{n1}$$

$$Var\left(\varepsilon^{*}_{(1)}\right) = \sigma^2\int_{-\infty}^{+\infty} w^2_{(1)}\ \mathrm{n}\ \left\{1-F\left(w_{(1)}\right)\right\}^{n-1} f\left(w_{(1)}\right)dw - \left(E\left(\varepsilon^{*}_{(1)}\right)\right)^2$$

$$\sigma^2_{e[de1]} = Var\left(\varepsilon^{*}_{(1)}\right) = \sigma^2 D^{*}_{12} - \left(\sigma D^{*}_{11}\right)^2 = \sigma^2\left(D^{*}_{12} - D^{*2}_{11}\right)$$

$$Var\left(\varepsilon^{*}_{(n)}\right) = \sigma^2\int_{-\infty}^{+\infty} w^2_{(n)}\ \mathrm{n}\ \left\{F\left(w_{(n)}\right)\right\}^{n-1} f\left(w_{(n)}\right)dw - \left(E\left(\varepsilon^{*}_{(n)}\right)\right)^2$$

$$\sigma^2_{e[den]} = Var\left(\varepsilon^{*}_{(n)}\right) = \sigma^2 D^{*}_{n2} - \left(\sigma D^{*}_{n1}\right)^2 = \sigma^2\left(D^{*}_{n2} - D^{*2}_{n2}\right)$$

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