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ORIGINAL ARTICLE

Adaptive mixed differential evolution algorithm for bi-objective tooth profile spur gear optimization

Hammoudi Abderazek [1] · **Djeddou Ferhat** [1] · **Atanasovska Ivana** [2]

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Abstract Nowadays, the increasing demand for high-strength, efficient, quiet, and high-precision gear design leads to the use of various optimization methods. In this study, a new evolutionary optimization algorithm, named adaptive mixed differential evolution (AMDE), based on a self-adaptive approach is introduced. The proposed method is applied to solve the problem of the optimal spur gear tooth profile, where the objectives are to equalize the maximum bending stresses and the specific sliding coefficients at extremes of contact path. The mathematical model of the maximum bending stresses is developed using a finite element analysis (FEA) calculation. The effectiveness of the proposed method is demonstrated by solving some well-known practical engineering problems. The optimization results for the test problems show that the AMDE algorithm provides very remarkable results compared to those reported recently in the literature. Moreover, for the spur gear used in this work, a significant improvement in balancing specific sliding coefficients and maximum bending stresses are found.

Keywords Gears · Tooth profile · Bending stresses · Bi-objective optimization · Mixed differential evolution algorithm

✉ Hammoudi Abderazek
a_hammoudi@univ-setif.dz

[1] Applied Precision Mechanics Laboratory, Institute of Optics and Precision Mechanics, Setif -1- University, 19000 Setif, Algeria

[2] Mathematical Institute of Serbian Academy of Sciences and Arts, Kneza Mihaila 36, Belgrade 11000, Serbia

1 Introduction

Gears are the most important components used as a part of mechanical systems to transmit motion and power between rotating shafts. They are widely used in the industry for advantages of compact structure, large power transmission, and high efficiency. However, the gear design is a complicated task involving multiple objectives which often conflict such as strength, pitting resistance, bending stress, scoring wear, etc. Furthermore, the complex shape and geometry of a gear leads to a large number of mixed design parameters and highly non-linear constraints. By considering the precedent factors, the traditional gear design is very difficult hence the computer-aided design is needed [1].

Over the last decades, the optimization of gears using different methods has been a subject of many research reports [2–5]. Gologlu and Zeyveli [6] used a genetic algorithm (GA) to automate a preliminary design of gear drives. The objective was to minimize the volume of gear trains. Mendi et al. [7] employed also a genetic approach to minimize the total volume of a gearbox. Savsani et al. [1] presented two advanced optimization algorithms, particle swarm optimization (PSO) and simulated annealing (SA), to find the optimal combination of design parameters for the minimum weight of a spur gear train. Marjanovic et al. [8] provided a practical approach for gearbox optimization with spur gears based on an optimal selection of materials, position of shaft axes, and gear ratio. Wan and Zhang [9] formulated an optimal design problem of a spur gear drive with a fixed load factor. Three methods were presented to find the globally optimal design scheme on the structure of the spur gear pair.

Recently, Golabi et al. [10] minimized the volume of the gearbox using MATLAB optimization toolbox, where two and three stage gear trains have been considered. Buiga and Tudose [11] optimized the mass of a two-stage helical coaxial speed reducer by using the genetic algorithm. The objective function was described by a set of 17 mixed-design variables. Thoan et al. [12] presented the optimization design of stress-relieving holes at root fillet in spur gear. The authors used the genetic algorithm methodology to find out the optimized locations and sizes of the holes at the root fillet. Salomon et al. [13] formulated an optimization problem of a gearbox for a random variant of torque and speed requirements. Both the gear numbers and their characteristics were optimized to minimize the overall energy consumption and the gearbox cost. The authors employed an active robustness methodology (AR) to solve the problem.

Gear tooth profile has an immense effect on the main operating parameters of gear pairs such as load capacity, working life, efficiency, and vibrations. In recent years, a variety of methods have been introduced to determine the optimal profile in order to satisfy various optimization criteria. Indeed, Divandari et al. [14] presented the effect of different profile modifications and profile error on the dynamic response of gear system in the presence of tooth-localized defect. In their work, a dynamic model including different gear errors and defects was developed. Furthermore, to estimate the optimal values of profile shift coefficients as well as radius of the tooth root and the pressure angle, an optimization method called explicit parametric method (EPM) was developed by Atanasovska et al. [15]. The objective was to balance the maximum bending stresses of spur gears. Bruyère and Velex [16] proposed a simplified multi-objective analysis of optimum profile modifications in spur and helical gears. The gear design criteria such as tooth load, contact pressure, root stress, and friction loss were considered. Diez-Ibarbia et al. [17] studied the influence of profile shift coefficients on the energy efficiency of spur gears using a developed load contact model. Hammoudi et al. [18] optimized the selection values of profile shift coefficients for cylindrical spur and helical gears using differential evolution algorithm (DE). The optimization procedure was developed for exact balancing specific sliding coefficients at extremes of contact path.

In this paper, an efficient adaptive mixed differential evolution (AMDE) algorithm is presented to solve the optimal tooth profile of a specific cylindrical spur gear problem. The optimization procedure is developed based on the principles of equalized maximum bending stresses and the specific sliding coefficients to maximize the service life of the used gear pair. The mathematical model of maximum bending stresses is developed according to Atanasovska et al. [15] using a finite element analysis (FEA) calculations. Minimal transverse contact ratio value, thickness of the tooth tip diameter, and tooth interferences are considered as constraints. Design variables are the profile shift coefficient, the radius of root curvature, and the normal pressure angle. The AMDE algorithm uses a self adaptive approach to adapt the control parameters. The performance of the proposed algorithm is demonstrated by solving three well-studied engineering design problems.

The paper is organized as follows. In Sect. 2, the basic DE algorithm steps are briefly introduced. The proposed AMDE is presented in Sect. 3. The performance of the proposed algorithm is evaluated in Sect. 4. The optimization procedure for gear design profile is described in Sect. 5 and solved in Sect. 6. Finally, the last section is dedicated to the conclusion.

2 Basic differential evolution

The DE algorithm was firstly introduced by Storn and Price [19]. It is a very popular evolutionary algorithm and exhibits remarkable performance in a wide variety of problems from diverse fields [20]. This technique involves three general evolutionary operators, i.e., mutation, crossover, and selection, which are associated with certain control parameters. In short, the procedure of the DE works as follows:

Step 1: *Parameter setting*

The control parameters of DE are the population size NP, the maximum number of generations (termination criterion) G_{max}, the scale factor F, and the crossover rate Cr.

Step 2: *Initialization*

The population NP is initialized by randomly generated individuals within the boundary constraints $\left[x_i^{(L)}, x_i^{(U)}\right]$ $(i=1,2\ldots,D)$, where D is the number of variables and $x_i^{(L)}$ and $x_i^{(U)}$ are the lower and the upper limits of the i^{th} variable problem, respectively.

Step 3: *Mutation*

For each target vector $x_{i,j}^{G}$, a mutant vector $v_{i,j}^{G+1}$ is generated. The most frequently used mutation strategies are [21, 22]

$$\text{“DE/rand/1/bin”}: \quad v_{i,j}{}^{G+1} = x_{i,r1}{}^{G} + F\left(x_{i,r2}{}^{G} - x_{i,r3}{}^{G}\right) \tag{1}$$

$$\text{“DE/best/1/bin”}: \quad v_{i,j}{}^{G+1} = x_{i,\text{best}}{}^{G} + F\left(x_{i,r1}{}^{G} - x_{i,r2}{}^{G}\right) \tag{2}$$

$$\text{“DE/best/2/bin”}: \quad v_{i,j}{}^{G+1} = x_{i,\text{best}}{}^{G} + F_1\left(x_{i,r1}{}^{G} - x_{i,r2}{}^{G}\right) + F_2\left(x_{i,r3}{}^{G} - x_{i,r4}{}^{G}\right) \tag{3}$$

$$\text{“DE/rand/2/bin”}: \quad v_{i,j}{}^{G+1} = x_{i,r1}{}^{G} + F_1\left(x_{i,r2}{}^{G} - x_{i,r3}{}^{G}\right) + F_2\left(x_{i,r4}{}^{G} - x_{i,r5}{}^{G}\right) \tag{4}$$

where j= 1,2,…, NP. r_1, r_2, r_3, r_4, r_5 are the integer indices chosen randomly in the interval [1, 2,…, NP] and

$r_1 \neq r_2 \neq r_3 \neq r_4 \neq r_5 \neq j$. F, F_1 and F_2 are the scaling factors used to control the amplification of the differential variation between two individuals.

If the i^{th} element of $v_{i,j}^{G+1}$ is infeasible (i.e., out of the boundary), it is reset as follows [23]:

$$v_{i,j}^{G+1} = \begin{cases} \min\left\{U_i, 2L_i - v_{i,j}^{G+1}\right\} \text{ if } v_{i,j}^{G+1} < L_i \\ \max\left\{L_i, 2U_i - v_{i,j}^{G+1}\right\} \text{ if } v_{i,j}^{G+1} > U_i \end{cases} \tag{5}$$

Step 4: *Crossover*

The trial vector $u_{i,j}^{G+1}$ is generated using the target and mutated vectors as

$$u_{i,j}^{G+1} = \begin{cases} v_{i,j}^{G+1} \text{ if } rand_{i,j}[0,1] \leq Cr \text{ or } i = i_{\text{rand}} \\ x_{i,j}^{G} \text{ otherwise} \end{cases} \tag{6}$$

where i_{rand} is an integer index randomly chosen in the interval $[1, 2, \ldots, \text{D}]$. The operator ($rand_{i,j}$) creates a random value uniformly distributed in the interval $[0, 1]$.

Step 5: *Selection*

In the selection step, the better one from the target vector $x_{i,j}^{G}$ and the trial vector $u_{i,j}^{G+1}$ will be chosen to enter the next generation according to their fitness value:

$$x_{i,j}^{G+1} = \begin{cases} u_{i,j}^{G+1} \text{ if } f\left(u_{i,j}^{G+1}\right) \leq f\left(x_{i,j}^{G}\right) \\ x_{i,j}^{G} \quad \text{otherwise} \end{cases} \tag{7}$$

Step 6: *Stopping criterion*

If the stopping criterion is satisfied, the best result will be displayed. Otherwise, steps 3, 4, and 5 are repeated.

3 Adaptive mixed differential evolution

The proposed algorithm, AMDE, is developed to solve the engineering design optimization problems of mixed discrete continuous types. The particular variant used throughout this study in order to generate a mutant vector is the DE/rand/2/bin scheme (Eq. (4)). Moreover, the AMDE algorithm uses a self-adaptive approach, proposed by Brest et al. [24] to adapt the control parameters. For each individual, the scaling factors ($F_{1,j}$, $F_{2,j}$) and the crossover factor (Cr_j) are adjusted before the mutation is performed. So, they influence the mutation, the crossover, and the selection operations of the new individual [24]. The initialization process sets are $F_1 = 0.1$, $F_2 = 0.2$, and $Cr_1 = 0.5$. The new control parameters are calculated as follows:

$$F_{1,j,G+1} = \begin{cases} F_l + \text{rand}[0,1] * F_u \text{ if } \beta_1 < \tau_1 \\ F_{1,j,G} \text{ otherwise} \end{cases} \tag{8}$$

$$F_{2,j,G+1} = \begin{cases} F_l + \text{rand}[0,1] * F_u \text{ if } \beta_2 < \tau_2 \\ F_{2,j,G} \text{ otherwise} \end{cases} \tag{9}$$

$$Cr_{j,G+1} = \begin{cases} \text{rand}[0,1] \quad \beta_3 < \tau_3 \\ Cr_{j,G} \text{ otherwise} \end{cases} \tag{10}$$

where $F_l = 0.1$, $F_u = 0.9$, and β_k ($k = 1,\ldots,3$) are uniform random values on [0, 1]; *rand* creates a random value uniformly distributed in the interval $[0, 1]$. τ_1, τ_2, and τ_3 represent the probabilities to adjust the control parameters. In this work, $\tau_1 = 0.5$, $\tau_2 = 0.2$, and $\tau_3 = 0.5$.

Further, the penalty function method is applied to handle the design constraints. The basic idea of this method is to transform a constrained optimization problem into an unconstrained one, by adding the penalty terms into the objective function to penalize the constraint violations. The pseudocode of the AMDE algorithm is given in Fig. 1.

3.1 Discrete and integer variables handling

The main idea to handle a discrete variable is to use the variable position index in the array of normalized values [25–27]. For example, when the created index is 2, this is the second normalized value that will be taken. So, if there are n normalized values in the set, there are n index positions from 1 to n. The integer variable can be treated in a similar way as the discrete.

More specifically, instead of optimizing the values of the discrete and the integer variables directly, we optimize the values of their indexes which are created as real values, and then they are transformed to the nearest integer by truncation. During the evaluation of the objective function and the constraints, the discrete values are used instead of their indexes. In general, the handling of the discrete and the integer variables are performed in two procedures: initialization and mutation. The indexes are created as follows:

$$\begin{cases} x_{i,j}^{G=0} \; = \; INT\left(x_i^{(L)} + rand_{i,j}[0,1] * \left(x_i^{(U)} - x_i^{(L)} + 1\right)\right) \\ v_{i,j}^{G+1} = INT\left(x_{i,r1}^{G} + F_1\left(x_{i,r2}^{G} - x_{i,r3}^{G}\right) + F_2\left(x_{i,r4}^{G} - x_{i,r5}^{G}\right)\right) \end{cases} \tag{11}$$

where *INT* is a function that transforms the real value into the nearest integer value of $x_{i,j}^{G=0}$ and $v_{i,j}^{G+1}$ by truncation.

Fig. 1 Pseudocode of the AMDE

Input: NP, G_{max}, the probabilities to adjust the control parameters: $\tau_1 = 0.5$, $\tau_2 = 0.2$ and $\tau_3 = 0.5$
The initial values of control parameters: $F_1 = 0.1$, $F_2 = 0.2$, $Cr_1 = 0.5$
$G = 0$;
Generate an initial population P_0:
For discrete and integer variables: $x_{i,j}^{G=0} = INT\left(x_i^{(L)} + rand_{i,j}[0,1] * (x_i^{(U)} - x_i^{(L)} + 1)\right)$
For continuous variables: $x_{i,j}^{G=0} = x_i^{(L)} + rand_{i,j}[0,1] * (x_i^{(U)} - x_i^{(L)})$
Evaluate the initial population using penalty function to handle constraints as in Eq. (13)
Perform the elitism to choose f_{best} and x_{best} of $G = 0$
For $G = 1$ to G_{max} do
For $j = 1$ to NP do
Select randomly five integer indices $r_1 \neq r_2 \neq r_3 \neq r_4 \neq r_5 \neq j$
Adjust the parameters $F_{1,j}$, $F_{2,j}$ and Cr_j according to Eqs. (8), (9) and (10)
Generate the mutant vector $v_{i,j}^{G+1}$ using DE/rand/2 mutation strategy
Handle boundary constraint violations according Eq. (5)
Generate the trial vector $u_{i,j}^{G+1}$ using binomial crossover
Randomly generate an integer i_{rand} in the range $[1, D]$
For $i = 1$ to D do
If $rand_{i,j}[0,1] \leq Cr_j$ or $i = i_{rand}$
$u_{i,j}^{G+1} = v_{i,j}^{G+1}$
Else
$u_{i,j}^{G+1} = x_{i,j}^{G}$
End If
End For
Evaluate the trial vector $u_{i,j}^{G+1}$ using penalty function to handle constraints
Selection
If $f\left(u_{i,j}^{G+1}\right) \leq f\left(x_{i,j}^{G}\right)$, for minimization case
$x_{i,j}^{G+1} = u_{i,j}^{G+1}$
Else
$x_{i,j}^{G+1} = x_{i,j}^{G}$
End If
Perform the elitism to choose f_{best} and x_{best} of G
End For
End For
Output: the best individual with the smallest objective function value in the population

3.2 Constraints functions treatment

The exterior penalty method has been widely used to deal with constraints, due to its simple principle and easy implementation [28–31]. However, the drawback of this technique is the large number of the penalty parameters that must be set especially for the optimization problems with high constraints [32].

In order to overcome the above limitation, we propose in this study to transform the constraint violation sum to an inequality constraint (Eq. (12)). Then, the penalty term will be introduced into the objective function to treat this constraint (Eq. (13)). The main advantage of this method is that only one penalty factor will be set to solve the problem with constraints:

$$\phi(x) = \sum_{i=1}^{m} \max(0, g_i(x)) - \upsilon \leq 0 \tag{12}$$

$$F(x) = f(x) + \lambda\left(\max(0, \phi(x))^2\right) \tag{13}$$

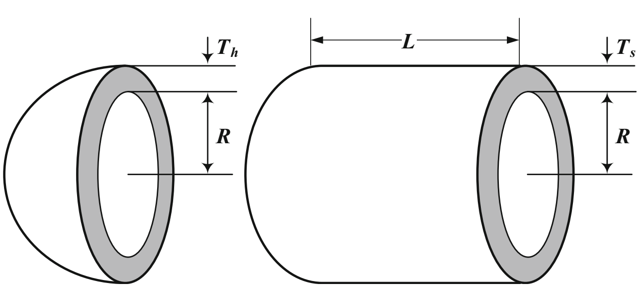

Fig. 2 Cylindrical pressure vessel design problem

where $\varnothing$ is the sum of constraint violation, g_i are the inequality constraints, m is the number of inequality constraints, υ is a very small positive number chosen in the range $\left[10^{-6}, 10^{-20}\right]$ depending on the number and the constraint function complexity of the problem, F is the expanded objective function to be optimized, and λ is the penalty factor.

4 Evaluation of AMDE approach

The performance of the proposed algorithm is evaluated based on three well-known constrained mixed engineering

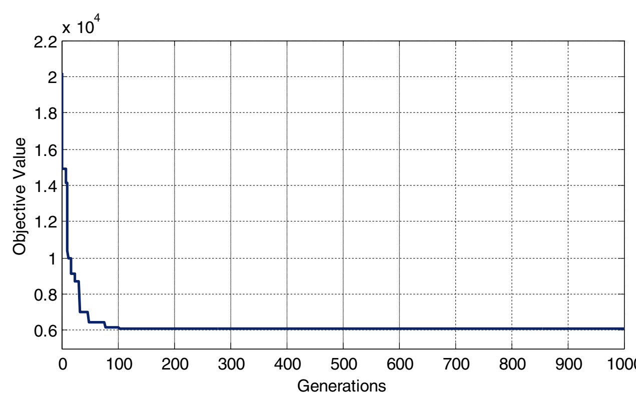

Fig. 3 Convergence of AMDE algorithm to the best fitness for pressure vessel

Table 1 Pressure vessel problem: comparison of AMDE results with literature

Design variables	*PSO-DE*	*ABC*	*CS*	*MOCoDE*	*AMDE*
Ts	0.8125	0.8125	0.8125	0.812500	0.8125
T_h	0.4375	0.4375	0.4375	0.437500	0.4375
R	42.098445596	42.098446	42.0984456	42.09844559585	42.098445595854919
L	176.636595842	176.636596	176.6365958	176.63659584337	176.63659584243942
f_{min}	6059.714335048	6059 .714339	6059.7143348	6059.7143	6059.7143350484

problems: pressure vessel, speed reducer, and multiple disc clutch brake design.

The AMDE parameters for the engineering design problems are set as follows: NP = 100, initialization process sets are $F_1=0.1$, $F_2=0.2$, $Cr_1=0.5$, $\tau_1=0.5$, $\tau_2=0.2$, and $\tau_3=0.5$. The maximum number of generations varied from 200 to 1200 depending on the problem. For each problem, we independently run AMDE 100 times to measure the quality of the results and the robustness of the proposed algorithm. The AMDE was implemented in MATLAB and the optimization runs were executed on a PC with a 2.2 GHz Intel Dual Core processor and 2 GB of RAM memory.

4.1 Pressure vessel

This optimization problem was originally formulated by Sandgren [33]. The cylindrical vessel capped at both ends by hemispherical heads, as shown in Fig. 2, must be designed for the minimum total fabrication cost, including the material cost, forming, and welding. The problem involves four mixed-design variables: the thickness of the cylindrical shell (T_s), the thickness of the spherical head (T_h), the inner radius (R), and the length of the cylindrical segment of the vessel (L). T_s and T_h are integer multiples of 0.0625 in. The problem can be expressed as follows:

$$\begin{aligned} \text{Minimize}: f(T_s, T_h, R, L) = 0.6224\ T_s R\ L \\ + 1.7781\ T_h R^2 \\ + 3.1611\ {T_s}^2 L \\ + 19.84\ {T_s}^2 R \end{aligned} \tag{14}$$

Subject to

$$\begin{aligned} g_1 &= -T_s + 0.0193\ R \leq 0 \\ g_2 &= -T_h + 0.00954\ R \leq 0 \\ g_3 &= -\pi R^2 L - \frac{(4\pi R^3)}{3} + 1296000 \leq 0 \\ g_4 &= L - 240 \leq 0 \end{aligned} \tag{15}$$

where $0.0625 \leq T_s \leq 6.1875$, $0.0625 \leq T_h \leq 6.1875$, $10 \leq R \leq 200$, and $10 \leq L \leq 200$.

The above problem was recently studied by many researchers using different optimization methods like Hybrid PSO-DE [34], artificial bee colony (ABC) [35], cuckoo search (CS) [36], and composite differential evolution with modified oracle penalty (MOCoDE) [37]. The objective function convergence over the generations is plotted on Fig. 3. A comparison of results is listed in Table 1 and the statistical results are shown in Table 2. The constraint values, for the best solution obtained by AMDE, are [0, −3.588083E−2; 0, −6.336340E+1].

As it can be seen, the best feasible solution obtained for the pressure vessel design example using the above methods is 6059.71433, which is also provided by AMDE. In addition, it is observed from the statistical results that the proposed algorithm is more robust in solving this problem with *8.2267E−12* standard deviation. So, it can be said that AMDE outperforms the four compared approaches in terms of robustness.

SD standard deviation, *NA* not available

4.2 Speed reducer

The objective is to optimize the total weight of the speed reducer [38]. The problem (Fig. 4) is subjected to constraints on bending stress of the gear teeth, surfaces stress, transverse deflections of the shafts, and stresses in the shafts. The variables are the face width (b), module of teeth (m), number of

Table 2 Pressure vessel problem: comparison of statistical results of AMDE with literature

Statistical	*PSO-DE*	*ABC*	*CS*	*MOCoDE*	*AMDE*
Best	6059.714335	6059.714736	6059.714	6059.7143	6059.7143350484
Mean	6059.714335	6245.308144	6447.736	6059.7143	6059.7143350484
Worst	6059.714335	NA	6495.347	6059.7143	6059.7143350484
SD	1.0E−10	2.05E+02	502.693	1.9444E−8	*8.2267 E−12*

Best results are shown in italics form

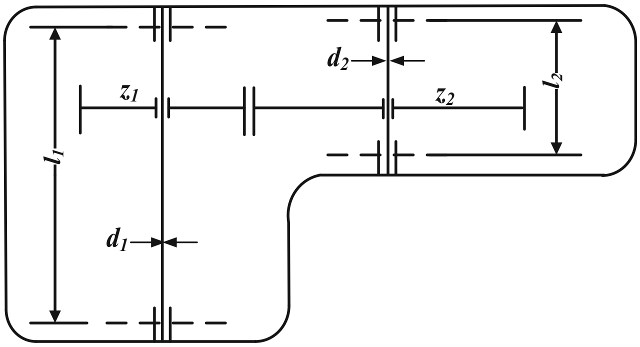

Fig. 4 The speed reducer design problem

teeth on pinion (z), length of first shaft between bearings (l_1), length of second shaft between bearings (l_2), diameter of shaft 1 (d_1), and diameter of shaft 2 (d_2). The third variable is integer and the rest of them are continuous. The mathematical formulation of this problem can be described as follows:

$$\text{Minimize}: f(b,m,z,l_1,l_2,d_1,d_2) = 0.7854\, b\, m^2 \left(3.3333\, z^2 + 14.9334\, z - 43.0934\right) - 1.508\, b\left(d_1^2 + d_2^2\right) + 7.4777\left(d_1^3 + d_2^3\right) + 0.7854\left(l_1 d_1^2 + l_2 d_2^2\right) \quad (16)$$

Subject to

$$\begin{aligned}
g_1 &= \frac{27}{b\, m^2 z} - 1 \leq 0\\
g_2 &= \frac{397.5}{b m^2 z^2} - 1 \leq 0\\
g_3 &= \frac{1.93 l_1^3}{m z d_1^4} - 1 \leq 0\\
g_4 &= \frac{1.93 l_2^3}{m z d_2^4} - 1 \leq 0\\
g_5 &= \frac{\sqrt{\left(\frac{745 l_1}{mz}\right)^2 + 16.9 \times 10^6}}{\left(110 d_1^3\right)} - 1 \leq 0\\
g_6 &= \frac{\sqrt{\left(\frac{745 l_2}{mz}\right)^2 + 157.5 \times 10^6}}{\left(85 d_2^3\right)} - 1 \leq 0\\
g_7 &= \frac{mz}{40} - 1 \leq 0\\
g_8 &= \frac{5m}{b} - 1 \leq 0\\
g_9 &= \frac{b}{12m} - 1 \leq 0\\
g_{10} &= \frac{1.5 d_1 + 1.9}{l_1} - 1 \leq 0\\
g_{11} &= \frac{1.1 d_2 + 1.9}{l_2} - 1 \leq 0
\end{aligned} \quad (17)$$

where $2.6 \leq b \leq 3.6$, $0.7 \leq m \leq 0.8$, $17 \leq z \leq 28$, $7.3 \leq l_1 \leq 8.3$, $7.3 \leq l_2 \leq 8.3$, $2.9 \leq d_1 \leq 3.9$, and $5 \leq d_2 \leq 5.5$.

The convergence of AMDE to the best fitness is given on Fig. 5. The best results obtained by AMDE and the other algorithms are given in Table 3. The statistical results are shown in Table 4. The best feasible solution found by AMDE is 2994.47106 and the constraints are [−7.39152E−2, −1.97998E−1, −4.99172E−1, −9.04643E−1, −7.00E−16, 0, −7.0250E−1, −2.00E−16, −5.83333E−1, −5.13257E−2, −9.00E−16].

It can be seen from Table 3 that the feasible solution found by AMDE is the best among those of all the compared approaches. Furthermore, it is observed from the statistical results that AMDE presents the smaller standard deviation *1.3711E−12*. So, our method is more efficient than are the compared approaches in terms of quality solutions and robustness.

4.3 Multiple disc clutch brake

The multiple disc clutch brake [39] (Fig. 6) must be designed for the minimum weight using five discrete variables: inner radius (r_i)∈(60, 61, 62,…, 80), outer radius (r_0)∈(90, 91,…,110), thickness of discs (t)∈(1, 1.5, 2, 2.5, 3), actuating force (F)∈(600, 610, 620,…, 1000), and number of friction surfaces (Z)∈(2, 3, 4, 5, 6, 7, 8, 9). The problem can be stated as

$$\text{Minimize}: f(r_i, r_0, Z, t) = \pi\, t\rho\left(r_0^2 - r_i^2\right)(Z+1) \quad (18)$$

Subject to

$$\begin{aligned}
g_1 &= r_0 - r_i - \Delta r \geq 0\\
g_2 &= l_{\max} - (Z+1)\left(\left(t + \delta\right) \geq 0\right.\\
g_3 &= p_{\max} - p_{rz} \geq 0\\
g_4 &= p_{\max} v_{sr\max} + p_{rz} v_{sr} \geq 0\\
g_5 &= v_{sr\max} - v_{sr} \geq 0\\
g_6 &= T_{\max} - T \geq 0\\
g_7 &= M_h - s\, M_s \geq 0\\
g_8 &= T \geq 0
\end{aligned} \quad (19)$$

$$\text{where } M_h = \frac{2}{3}\mu F Z \frac{r_0^3 - r_i^3}{r_0^2 - r_i^2}, p_{rz} = \frac{F}{\pi\left(r_0^2 - r_i^2\right)}, v_{sr} = \frac{2\pi n\left(r_0^3 - r_i^3\right)}{90\left(r_0^2 - r_i^2\right)}, T = \frac{I_z \pi n}{30\left(M_h + M_f\right)} \quad (20)$$

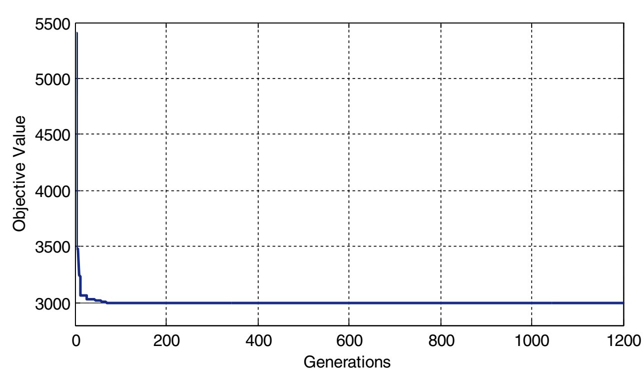

Fig. 5 Convergence of AMDE algorithm to the best fitness for the speed reducer design

Table 3 Speed reducer problem: comparison of AMDE results with literature

Design variables	*ABC*	*PSO-DE*	*CS*	*AMDE*
b	3 .499999	3.5000000	3.5015	3.5000000001
m	0.7	0.7000000	0.7000	0.7000000000
z	17	17.000000	17.0000	17
l_1	7.3	7.300000000013	7.6050	7.3000000012
l_2	7.8	7 .800000000005	7.8181	7.715319911478252
d_1	3.350215	3.3502146 66,097	3.3520	3.350214666096448
d_2	5.287800	5.286683229758	5.2875	5.286654464980222
f_{min}	2997.058412	2996.3481649	3000.9810	*2994.471066146*

Best results are shown in italics form

Δr = 20 mm, t_{max} = 3 mm, t_{min} = 1.5 mm, l_{max} = 30 mm, Z_{max} = 10, v_{max} = 10 m/s, μ = 0.5, δ = 0.5 mm, M_s = 40 Nm, = 3 Nm, input speed n = 250 rpm, p_{max} = 1 MPa, I_Z = 55 kg.m^2, T_{max} = 15 s, F_{max} = 1000 N, r_{imin} = 55 mm, r_{omax} = 110 mm, and ρ = 7800 kg/m^3.

Multiple disc clutch brake problem has been solved using non-dominated sorting genetic algorithm (NSGA-II) [40] and teaching learning-based optimization (TLBO) [39]. Figure 7 demonstrates the convergence function values with respect to the number of generation for the multiple disc clutch brake design problem.

The objective function value is 0.313656611 and the constraints are [0, 24, 9.19427E−1, 9.83037E+3, 7.89469E+3, 7.02013E−1, 3.77062E+4, 14.29798]. According to Table 5, the best feasible solution found by AMDE is better than obtained by NSGA-II and is similar for that provided by TLBO. However, it is observed from the statistical results (Table 6) that AMDE outperforms TLBO for the best, the mean, and the worst solutions. Hence, AMDE is effective to solve the multiple disc clutch brake design problem.

5 Optimization procedure for gear design profile

The flowchart in Fig. 8 displays a brief description of the developed optimization procedure for selecting the optimal tooth profile parameters of the spur gear.

Table 4 Speed reducer: comparison statistical results of AMDE with literature

Statistical	ABC	PSO-DE	CS	AMDE
Best	2997.058412	2996.348165	3000.9810	*2994.471066146*
Mean	2997.058412	2996.348165	3007.1997	*2994.471066146*
Worst	NA	2996.348165	3.0090	*2994.471066146*
SD	0	1.0E-07	4.9634	*1.3711E−12*

Best results are shown in italics form

In Atanasovska et al. [15], an explicit parametric method (EPM) was developed to estimate the optimal spur gear tooth profile. The objective was to balance the maximum bending stresses in gear pair. The FEA was used for all necessary stress and strain calculations. The EPM was applied to optimize the profile tooth of a real spur gear in a large transport machine. The main characteristics of this gear are as follows: number of teeth z_1= 20, z_2= 96; face width b= 175 mm; normal module m_n= 24 mm; normal pressure angle α_n = 20°; sum of profile shift coefficients $x_1 + x_2$ = 0.5; rotational wheel speed n_2= 4.1596 rpm and wheel torque T_2= 1263 kN m. The authors found that the exponential functions, as shown in Eqs. (20) and (21), describe how the maximum root stresses depend on the profile shift coefficient.

$$\sigma_{F1}(x_1) = a_1 e^{b_1 x_1} \tag{21}$$

$$\sigma_{F2}(x_1) = a_2 e^{b_2 x_1} \tag{22}$$

where a_1, a_2, b_1, and b_2 are coefficients depending on the variables α_n and ρ_f; e is the natural base logarithm; and σ_{F1} and σ_{F2} are the maximum bending tooth root stress of the pinion and the wheel, respectively.

In this work, the coefficients a_1, a_2, b_1, and b_2 are expressed according to variables α_n and ρ_f based on the results of numerical experiments found by Atanasovska et al. [15]. The toolbox Surface Fitting Tool of MATLAB is used to create 2D polynomials. So, the final mathematical

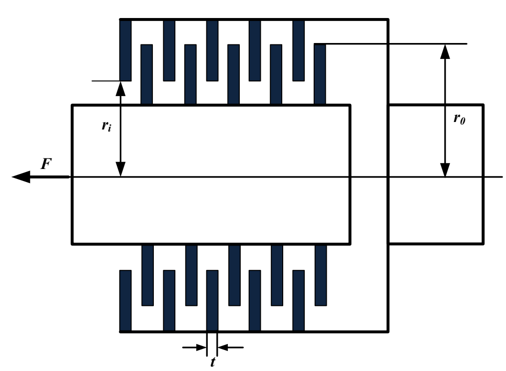

Fig. 6 Multiple disc clutch brake design problem

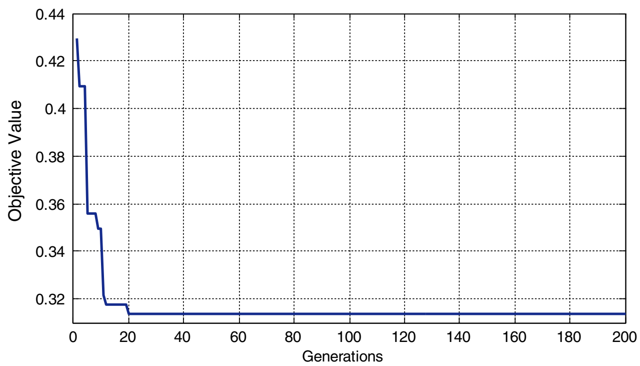

Fig. 7 Convergence of AMDE algorithm to the best fitness for the multiple disc clutch brake design

models of the maximum bending stresses depending on the three parameters are presented as follows:

$$\sigma_{F1}\left(x_1,\alpha_n,\rho_f\right) = \left(2065-19.53\alpha_n-96.6\rho_f\right)e^{\left(-1.437-0.00403\alpha_n+0.09107\rho_f\right)x_1} \tag{23}$$

$$\sigma_{F2}\left(x_1,\alpha_n,\rho_f\right) = \left(1237-13.39\alpha_n-43.48\rho_f\right)e^{\left(0.3755-0.02243\alpha_n+0.06075\rho_f\right)x_1} \tag{24}$$

After modeling the maximum bending stresses, the model formulation of the studied spur gear is developed for bi-objective optimization. Finally, the AMDE algorithm is used to solve the problem.

6 Optimization of gear design profile using AMDE

The studied case is a real gear pair in a large transport machine [15]. The optimal tooth profile problem is formulated as a bi-objective optimization problem, among the objectives are equalized the maximum bending stresses and the specific sliding coefficients.

Table 5 Multiple disc clutch brake problem: comparison of AMDE results with literature

Design variables	*NSGA-II*	*TLBO*	*AMDE*
r_i	70	70	70
r_o	90	90	90
Z	3	3	3
t	1.5	1	1
F	1000	810	810
f_{min}	0.4704	0.313656611	0.313656611

Table 6 Multiple disc clutch brake problem: comparison statistical results of AMDE with literature

Statistical	*NSGA-II*	*TLBO*	*AMDE*
Best	NA	0.313657	*0.313656610*
Mean	NA	0.3271662	*0.313656610*
Worst	NA	0.392071	*0.313656610*
SD	NA	NA	*2E−016*

Best results are shown in italics form

The parameters used by the AMDE for optimization search process are NP = 100, G_{max} = 1000, the scaling factors and the crossover factor kept the same as in Sect. 4.

6.1 Model formulation

6.1.1 Objective functions

To provide an equally strong teeth on the pinion and the wheel, their maximum bending stresses should be balanced as

$$\text{Minimize}: f_1\left(x_1,\alpha_n,\rho_f\right) = |\sigma_{F1}-\sigma_{F2}| \tag{25}$$

To maximize the wear resistance of the gear pair, the maximum specific sliding coefficients must be equal at extremes of contact path [18] (points A and E, Fig. 9):

$$\text{Minimize}: f_2(x_1,\alpha_n) = |\gamma_{2\max}-\gamma_{1\max}| \tag{26}$$

The maximal specific sliding coefficients $\gamma_{1\max}$ and $\gamma_{2\max}$ are given by

$$\gamma_{1\max} = \frac{\tan\alpha_{at1}-\tan\alpha_t'}{(1+u)\tan\alpha_t'-\tan\alpha_{at1}}(u+1) \tag{27}$$

$$\gamma_{2\max} = \frac{\tan\alpha_{at2}-\tan\alpha_t'}{\left(1+\frac{1}{u}\right)\tan\alpha_t'-\tan\alpha_{at2}}\left(\frac{u+1}{u}\right) \tag{28}$$

where u is the transmission ratio, α_t' is pressure angle at the pitch cylinder, and α_{at1} and α_{at2} are the tip transverse pressure angles of the pinion and the wheel respectively.

For solving this bi-objective problem, the ε-constraint method [41] is used. The objective of the specific sliding coefficients is converted into an inequality constraint and that of the maximum bending stresses is minimized.

$$f_2(x_1,\alpha_n)\leq 10^{-5} \tag{29}$$

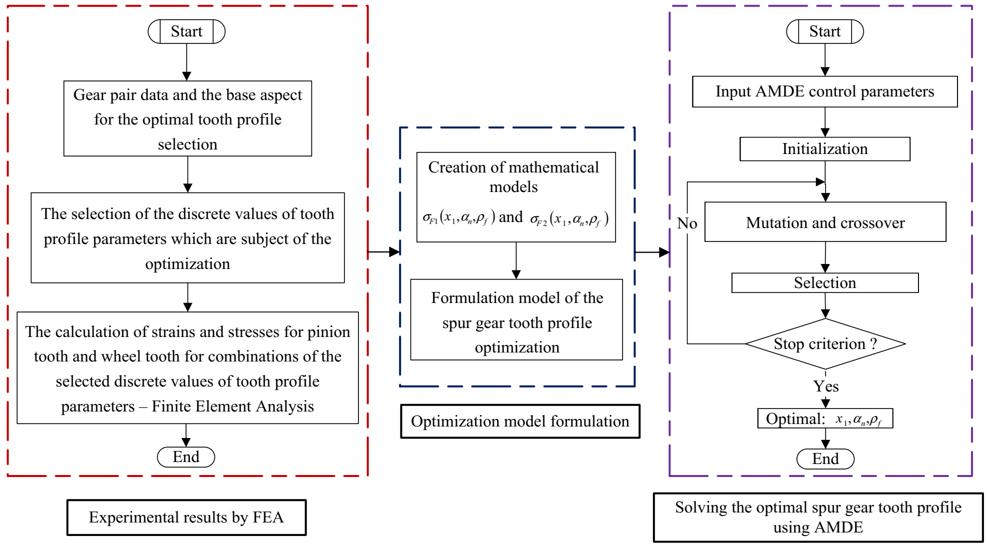

Fig. 8 Flowchart of optimization procedure for gear design profile

6.1.2 Design variables

The final optimization model of the gear design profile is developed for three mixed-design variables. The profile shift x_1 and the radius of root curvature ρ_f are continuous, and the pressure angle α_n is discrete. The upper and the lower variable limits are $-0.5 \leq x_1 \leq 0.8$, $6 \leq \rho_f \leq 8.3$ mm, and $\alpha_n(°) \in (18, 19, 20, 21, 22)$.

6.1.3 Constraints formulation

The objective functions listed above are subjected to the following constraints:

- To reduce the vibrations, the contact ratio ε_α should be greater than 1.5 [42]:

$$g_1 = 1.5 - \varepsilon_\alpha \leq 0 \tag{30}$$

- To avoid the narrow top lands of the teeth, the thickness of the tooth tip diameter should be greater than or equal to 0.4 m_t [43]:

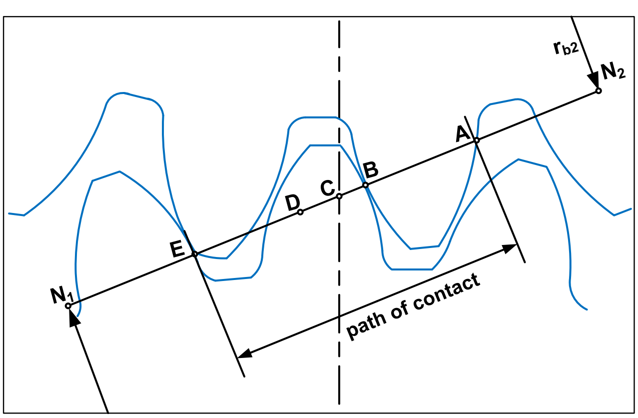

Fig. 9 Characteristic points on the path of contact

$$g_2 = 0.4m_t - s_{at1} \leq 0 \tag{31}$$

$$g_3 = 0.4m_t - s_{at2} \leq 0 \tag{32}$$

where m_t is the transverse module, s_{at1} and s_{at2} are the transversal arc thickness of the tooth at the tip diameter for the pinion and the wheel respectively.

- To avoid the operating interference on the two mating gears, the following relations should be checked:

$$g_4 = \frac{m_t(z_2 + 2 + 2x_2)}{2} - \sqrt{r_{b2}{}^2 + \left(a' \sin\alpha_t'\right)^2} < 0 \tag{33}$$

$$g_5 = \frac{m_t(z_1 + 2 + 2x_1)}{2} - \sqrt{r_{b1}{}^2 + \left(a' \sin\alpha_t'\right)^2} < 0 \tag{34}$$

where a' is the working center distance and r_{b1} and r_{b2} are the base radii of the pinion and the wheel, respectively.

6.1.4 Results and discussion

The found specific sliding distribution along the contact path is as plotted in Fig. 10. The best solution reported by the proposed algorithm is (0.4018724121, 6.2888763721 mm, 21°), corresponding to the objective functions values $f_1 = 3.3537 * 10^{-11}$ *MPa* and $f_2 = 5.5444 * 10^{-6}$. The constraint violation are [−1.58464E−2, −2.69271, −8.90131, −42.39185, −6.050492421E+2].

From the obtained results, it can be clearly seen that the design variable values really lead to a perfect balancing in both parameters the tooth root stresses as well as the maximum specific sliding coefficients for the studied gear pair ($\sigma_{F1} = \sigma_{F2}$ = 765.642771 MPa and $\gamma_{1\max} = \gamma_{2\max}$ = 1.0093).

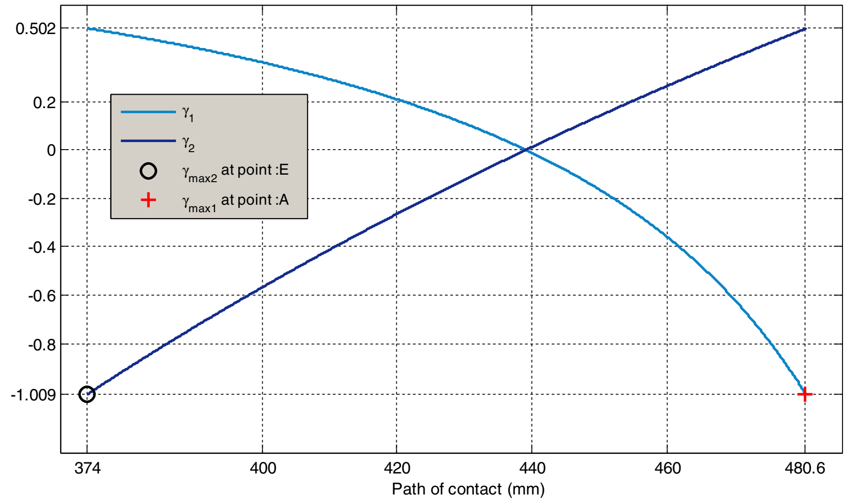

Fig. 10 Specific sliding distribution along the path of contact

Consequently, it may be assumed that the mating gear teeth flanks make sure a regulated resistance to bending stresses and to wear, leading to an extended gear service life.

7 Conclusion

Gear pair design is a highly complex task including different non-linear constraints and requires the use of mixed-design variables. In this paper, a new evolutionary optimization algorithm named AMDE has been presented based on a self-adaptive approach. The method was applied for solving the optimal tooth profile of a specific cylindrical spur gear. The optimization purposes were to compromise between the maximum bending stresses and the specific sliding. The mathematical model of the maximum bending stresses was developed using FEA calculations. For the specific spur gear studied in this work, a significant improvement in balancing specific sliding coefficients and maximum bending stresses were found.

The proposed algorithm was implemented to solve three well-studied engineering design examples, pressure vessel, speed reducer, and multiple disc clutch brake design. Simulation results showed that AMDE algorithm provides very remarkable results compared to those reported recently in the literature using different optimization methods. So, it can be concluded that the AMDE is a very effective algorithm for solving the engineering optimization problems with high-quality solutions and robustness.

As part of our future work, the verified optimization algorithm gives a base for developing a multi-objective optimization algorithm by taking into account all the other involved aspects for solving the optimal tooth profile of cylindrical spur and helical gears.

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Compliance with ethical standards

Conflicts of interest The authors declare that they have no conflicts of interest.

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