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ORIGINAL ARTICLE

Experimental and modeling study on cutting forces of feed direction ultrasonic vibration-assisted milling

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Abstract An ultrasonic vibration-assisted milling experiment was made for studying the cutting forces. The experimental results showed that small fluctuations caused by vibration of milling machine and cutting fluid system were restrained after exerting ultrasonic vibration. Horizontal force was used for analyzing the effect of exerted ultrasonic vibration on milling forces. And for ultrasonic vibration with different amplitudes, the horizontal cutting force firstly increases and then decreases. An ultrasonic vibration-assisted milling cutting mechanism model was made in this paper to compute the cutting forces of tangential, radial, and axial directions. A cutting force model was made and it showed that all the three direction forces are influenced by the feed speed, vibration amplitude, and the ratio between vibration frequency and the rotation speed. Air cutting takes place every vibration period when certain relationship was reached. Tangential force is the vector addition of *X* and *Y* direction forces, which are the combinations of a linear part and trigonometric parts affected by rotation speed and vibration frequency, respectively. *Z* direction force changes proportionally to the tangential force. The results indicate that the cutting forces are decreased when feed direction ultrasonic vibration is exerted especially when the feed per tooth is 12 μm/tooth and the ultrasonic vibration with amplitude 12 μm is exerted.

Keywords Cutting forces · Modeling · Ultrasonic vibration-assisted milling · Experiment · Cutting mechanism

Nomenclature

F_t	Cutting force along tangential direction
F_r	Cutting force along radial direction
F_v	Horizontal cutting force
F_a	Z direction cutting force
b	Width of cut
h	Chip thickness
v_f	Feed speed
r	Tool radius
ω	Rotational velocity of spindle
A	Ultrasonic vibration amplitude
f	Ultrasonic vibration frequency
λ_0	Starting phase angle
N	Cutter teeth number
n	Spindle speed (r/min)
β	Tool helix angle
θ_s	Angle that the cutter starts cutting
θ_e	Angle that the cutter stops cutting

1 Introduction

The original and classical analytical cutting force model of traditional end milling processing without considering tip radius was made by Tlusty and MacNeil [1]. This model gives support to general and micro cutting force models of many researchers such as Bao [2] and Kang [3]. Omar [4] made an improved model to predict the cutting forces of traditional side milling processing, in consideration of several machining errors. Azmi [5] studied end milling machining forces of GFRP by experimental. Wang [6] predicts the milling force by a

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mechanistic model and the cutting force coefficients are obtained by the least square regression method. Aydin [7] studied side milling force based on force distributions by a mechanical model and the coefficients based on a practical experiment. Some other researches on five-axis milling forces have also been made by modeling or experimental methods [8–10].

Ultrasonic vibration-assisted milling (UVAM) is an advanced machining technology by introducing ultrasonic vibration into traditional milling process. UVAM helps in improving the whole milling processing and enlarging the applicable area, especially for difficult-to-machine materials. Some studies have been made on the UVAM force by modeling and experiment. Shen [11] and Ding [12] analyzed the changing rule of UVAM force and main influence factors by modeling based on chip thickness. Xiao [13] made a rotary direction ultrasonic vibration-assisted milling cutting force model and confirmed it by experiment.

In this paper, an analytical model of cutting force for feed direction UVAM was made based on the time. Cutting thickness is not essential to be calculated firstly because the component force is only related with the component cutting depth by the mathematical physical model. Air cutting happens every vibration period when certain relationship between the feed speed, vibration conditions, and spindle rotating speed is reached. An UVAM force experiment was conducted and the experimental results proved the model to be reasonable.

2 Experimental details

Feed direction ultrasonic vibration-assisted end milling was done with a three-axis CNC end milling machine, XK7120XD. The range of speed is from 1000 to 12,000 rpm. The positioning accuracy is ±5 μm and the repositioning accuracy is±3 μm. KISTLER 9129A three-component dynamic force sensor was used for force measurement, cooperated with KISTLER 6129A charge-amplifier. The sampling frequency was 10 KHz. The vibrator system is constituted with ultrasonic transducer and amplitude pole. The ultrasonic power supply is GCH intelligent ultrasonic generator. The vibration was applied along feed direction with a resonant frequency of 20,045 Hz and the vibration error was ±200 Hz. LDT1-028K piezoelectricity thin-film sensor was glued to the amplitude transformer connected to Agilent InfiniiVision DSO-X 2024A oscilloscope. The minimum impedance of the sensor is 1 MΩ and the analog bandwidth of the oscilloscope is 200 MHz. Figure 1a, b shows the illustration and actual experiment setups, respectively.

Threaded work-pieces made by Ti-6Al-4V fix on the transformer by thread. The 20 mm × 18 mm × 2 mm work-piece was obtained by wire-electrode cutting and the thread was by turning. Before and after milling processing, 10-min ultrasonic cleaning with acetone was carried out to clean up the dust and cutting chips. The upper surface was adjusted to a certain position and parallel to the workbench with an error of ±50 μm.

Plane milling with a two-flute 5-mm diameter end mill was conducted before the slot milling to make sure the same cutting depth and the cutting depth was 0.1 mm. Fifteen-millimeter length slot milling with a two-flute 3-mm diameter end mill was carried out with and without vibration. PRO-CUT^{TM} CCF-10 μ-emulsified cutting fluid applied during processing the ratio between the cutting fluid and the water was 1:3.

Detailed experimental conditions are presented as Table 1. Experimental parameters settings are presented as Table 2.

3 Results and discussions

3.1 Results

Figure 2 shows the randomly selected samples in 50 ms and the respective feed speed from left to right is 4, 8, and 12 $\mu m/$tooth. Figure 2a shows three direction forces of traditional end milling without vibration. Figure 2b, c shows measured three direction forces by UVAM and the amplitudes are 6 and 12 μm, respectively. Figure 3 shows the horizontal force computed with the measured data shown in Fig. 2.

From Fig. 2a, c, gradually increasing forces of all the three directions can be observed with the feed per tooth rising. Force of horizontal directions changed obviously, especially the feed direction force. And the changing rate is gradually improving. Small fluctuations caused by vibration of milling machine and cutting fluid system accompany the cutting force trend and the fluctuations are irregular.

As can be seen in Fig. 2b, c, machine and cutting fluid system yawp is restrained after exerting ultrasonic vibration, particularly when the amplitude is 6 μm. Macro wave trend is accompanied with regular small fluctuations. However, as the sampling frequency was less than the ultrasonic frequency, the actual changing trend cannot be observed by this experiment. The macro changing was caused by spindle rotation, whose speed was 5000 r/min. Then, for two-flute milling, the changing period shall be computed as follows.

$$\mathrm{T}=\frac{60}{5000}/2\times 1000=6\mathrm{ms} \tag{1}$$

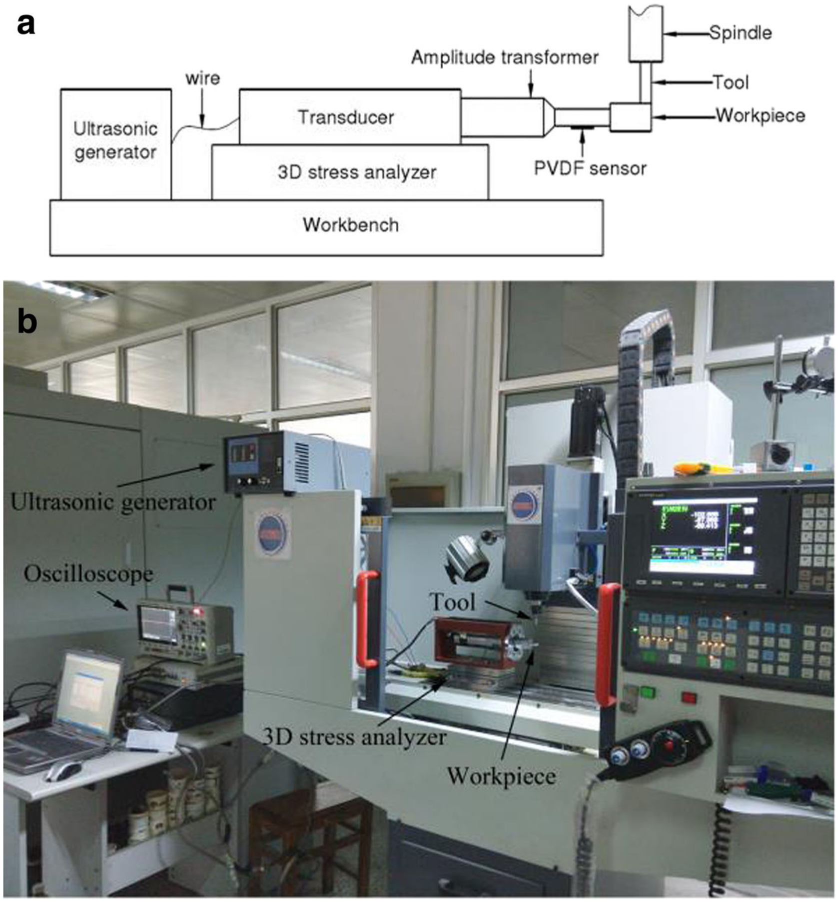

Fig. 1 Experimental setups. **a** Illustration. **b** Devices

Figure 2b, c also indicates that ultrasonic vibration with low amplitude will increase component forces of all the three directions because of the additional part caused by ultrasonic vibration. But when the vibration amplitude is 12 μm and the vibration causes air cutting every period of vibration. Then, the measured cutting forces reduce to some extent.

As the rotation angle shall not be measured under the current experimental conditions around the world and the measured forces in feed and nominal directions cannot be converted to tangential force, the resultant force in horizontal direction is used to analyze the improving effect of the exerted ultrasonic vibration on the milling processing. And the horizontal force is obtained by Eq.(2).

$$F_v = \sqrt{F_x^2 + F_y^2} \tag{2}$$

As shown in Fig. 3, the ultrasonic vibration has little effect on the cutting force with feed per tooth 4 μm/tooth. When feed speed is 8 μm/tooth in Fig. 3b, the horizontal cutting force firstly increases and then decreases. And the obvious effect is mainly caused by feed direction impact as shown in Fig. 2. At the same time, the horizontal cutting force rises after exerting ultrasonic vibration. Figure 3c shows the effect for the feed per tooth 12 μm/tooth. The horizontal cutting force also firstly increases and then decreases. However, when ultrasonic vibration amplitude is 12 μm, the horizontal cutting force is almost of the same value as milling processing without vibration and even smaller. And the force values return to zero obviously for UVAM with large amplitude.

Table 1 Experimental conditions

Items	Item parameters
Workpiece	Ti-6Al-4 V, 20 mm × 18 mm × 2 mm
Tool	Two-flute end mills, VOLKSTIGERVTMF2 030 08 04 50, with a helix angle of 45°
Cutting depth	0.15 mm

Table 2 Experimental parameters settings

Samples	RS(r/min)	FPT(μm/tooth)	Amplitude
1	5000	4	0
2	5000	8	0
3	5000	12	0
4	5000	4	6μm
5	5000	8	6μm
6	5000	12	6μm
7	5000	4	12μm
8	5000	8	12μm
9	5000	12	12μm

RS rotation speed, *FPT* feed per tooth

3.2 Discussions

The cutting force was studied ignoring the tool tip radius as it is much smaller than the cutting depth. And the model in this paper is based on the following assumptions.

- The cutting force along tangential direction changes proportionally to the product of the cutting width and thickness of cutting.

$$F_t = K_m bh \tag{3}$$

- Cutting force along radial direction changes proportionally to the tangential force because of the helix angle.

$$F_r = pF_t \tag{4}$$

Figure 4 shows that the modeling coordinate system and feed direction ultrasonic vibration is exerted. The X and Y direction components of chip thickness can be computed by the difference between the $k^{\#}$ flute tip position A at time t_0 and $(k+1)^{\#}$ flute tip position B at time t_1. Two-flute milling tools are used for model analysis, and there is no matter assuming k is zero.

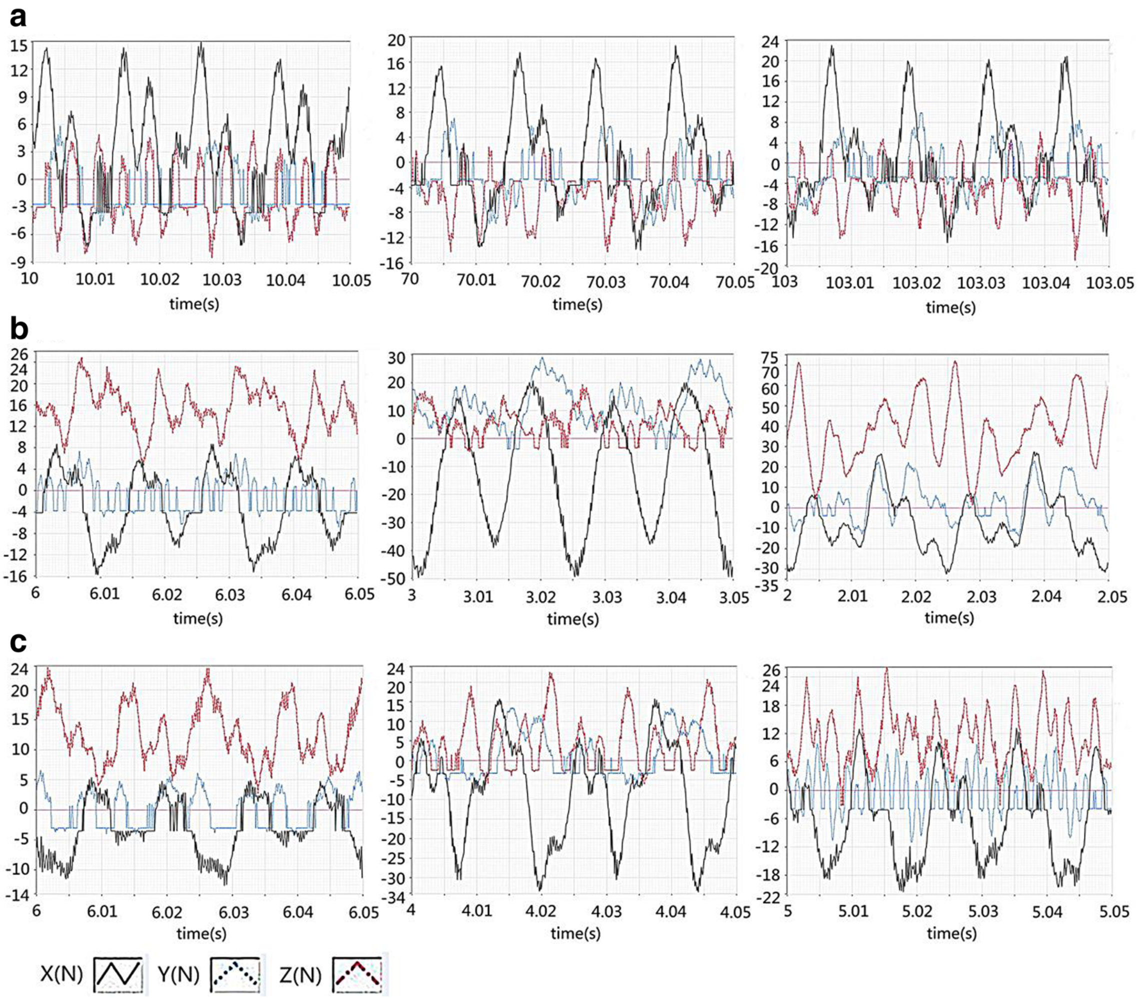

Fig. 2 Random selected force measurement results. **a**–**c** Vibration amplitude is 0 , 6 , and 12 μm

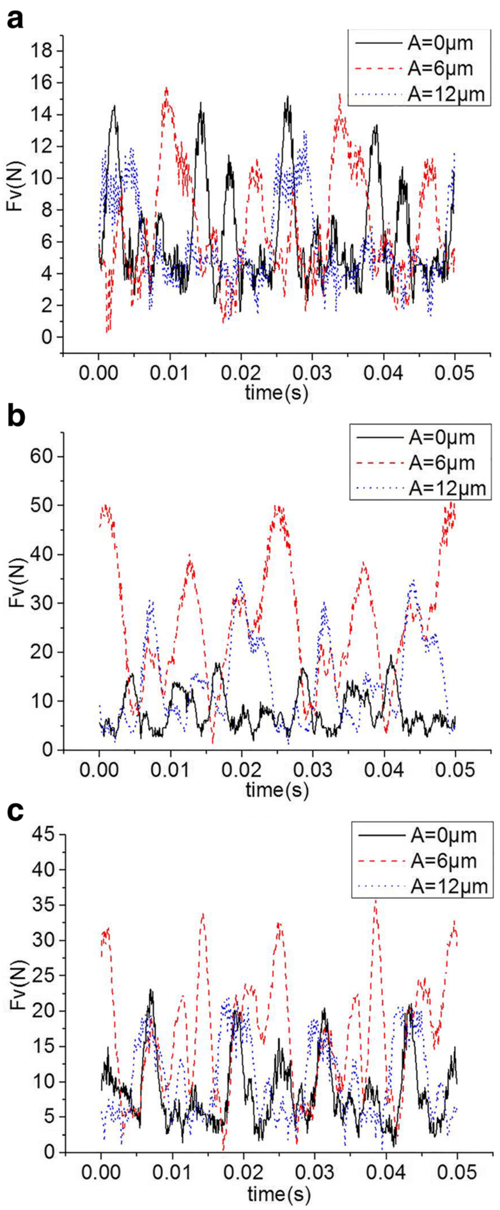

Fig. 3 Horizontal force. **a**–**c** Feed per tooth is 4 , 8 , and 12 μm/tooth

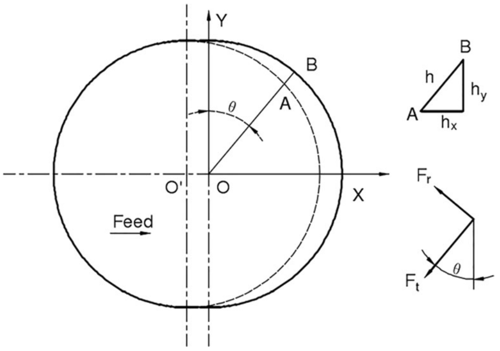

Fig. 4 End milling processing

$$\begin{cases} h_x = \dfrac{Nnf_z}{60}\Delta t + 2AM \\ \quad -2r\sin(\theta_1)\cos\dfrac{\omega\Delta t}{2} \\ h_y = \\ \quad -2r\cos\left(\omega t_1 - \dfrac{\omega\Delta t}{2} + \lambda\right)\cos\dfrac{\omega\Delta t}{2} \end{cases} \tag{5}$$

The time difference can be computed since that the points of O, A, B are collinear.

$$r\sin(\omega\Delta t) + \left[\frac{Nnf_z}{60}\Delta t + 2\mathrm{AM}\right]\cos(\omega t_1 + \lambda) \\ -2\mathrm{AH}\cos\left(\omega t_1 - \frac{\omega\Delta t}{2} + \lambda\right) = 0 \tag{6}$$

Where,

$$\begin{cases} \Delta t = t_1 - t_0 \\ M = \\ \quad \cos(2\pi f t_1 - \pi f\Delta t + \varphi)\sin\pi f\Delta t \\ H = \\ \quad \cos(\omega\Delta t/2)\sin(2\pi f t_1 + \varphi) \end{cases} \tag{7}$$

When the point A is between O and B, the milling is normal with the generation of chip. At the same time, no material shall be removed when the point B is between O and A. For a certain time t_1, The smallest Δt is a constant.

λ is the additional angle in consideration of the cutting start angle, cutter runout and tilt, and the additional angle $\delta\theta(Z_k)$ because of the helix angle at height Z_k.

$$\lambda = \lambda_0 - \delta\theta(Z_k) \tag{8}$$

$$\delta\theta(Z_k) = \frac{Z_k\tan\beta}{r} \tag{9}$$

The width of cut b depends on the helix angle, the circumferential direction feed.

$$\mathrm{db} = \frac{r\omega}{\tan\beta}\mathrm{dt} \tag{10}$$

As the difficulty for a direct calculation of the total cutting force as the chip thickness is a complex function of the rotation angle especially for UVAM processing, component forces on the feed and normal directions are used for studying the force system.

$$\begin{cases} d\mathrm{F_x} = -d\mathrm{F_t}\cos\theta - d\mathrm{F_r}\sin\theta \\ d\mathrm{F_y} = d\mathrm{F_t}\sin\theta - d\mathrm{F_r}\cos\theta \end{cases} \tag{11}$$

When expression (5) is substituted for expression (11), component forces on the feed and normal directions can be represented with the projection of cutting thickness on the *X* and *Y* direction.

$$\begin{cases} d\mathrm{F}_x = -\dfrac{k_m h_y r\omega}{\tan\beta}\mathrm{dt} - \dfrac{k_m p h_x r\omega}{\tan\beta}dt \\ d\mathrm{F}_y = \dfrac{k_m h_x r\omega}{\tan\beta}\mathrm{dt} - \dfrac{k_m p h_y r\omega}{\tan\beta}dt \end{cases} \tag{12}$$

Taking integration, the expression (12) can be rewritten as follows.

$$\begin{aligned} F_x = {} & \frac{2F_u r}{\omega}\cos\frac{\omega\Delta t}{2}(\sin\theta_{t_e} - \sin\theta_{t_s}) - pF_u v_f \Delta t(t_e - t_s) \\ & - \frac{2pF_u r}{\omega}\cos\frac{\omega\Delta t}{2}(\cos\theta_{t_e} - \cos\theta_{t_s}) - \frac{pF_u A}{\pi f}\sin(\pi f\Delta t)\left(\sin\theta'_{t_e} - \sin\theta'_{t_s}\right) \end{aligned} \tag{13}$$

$$\begin{aligned} F_y = {} & F_u v_f \Delta t(t_e - t_s) + \frac{2F_u r}{\omega}\cos\frac{\omega\Delta t}{2}(\cos\theta_{t_e} - \cos\theta_{t_s}) \\ & + \frac{F_u A}{\pi f}\sin\pi f\Delta t\left(sic\,\theta'_{t_e}\sin\theta'_{t_s}\right) + \frac{2pF_u r}{\omega}\cos\frac{\omega\Delta t}{2}(\sin\theta_{t_e} - \sin\theta_{t_s}) \end{aligned} \tag{14}$$

Where,

$$\begin{aligned} F_u &= \frac{k_m r\omega}{\tan\beta} \\ \theta_{t_e} &= \omega t_e - \frac{\omega\Delta t}{2} + \lambda \\ \theta_{t_s} &= \omega t_s - \frac{\omega\Delta t}{2} + \lambda \\ \theta'_{t_e} &= 2\pi f t_e - \pi f\Delta t + \varphi \\ \theta'_{t_s} &= 2\pi f t_s - \pi f\Delta t + \varphi \end{aligned}$$

t_s and t_e are the cutting start time and cutting end time, respectively.

Cutting forces of tangential, radial, and axial directions can be computed by Eq.(4) and Eq.(11).

$$\begin{cases} F_t = F_x\cos\theta - F_y\sin\theta \\ F_r = F_x\sin\theta + F_y\cos\theta \\ F_a = F_t\cos(\pi/2 - \beta) \end{cases} \tag{15}$$

Case 1

$$f_z - 2A\left|\sin\left(\frac{30\pi f}{n}\right)\right| > 0 \tag{16}$$

No additional intersection of tip trajectories shall be caused by ultrasonic vibration in this case. And the cutting force can be included in three cases as shown in the study by Bao and Tansel [2]. The cutting forces shall increase since that an additional part caused by ultrasonic vibration is involved in Eq.(13) and Eq.(14).

Case 2

$$f_z - 2A\left|\sin\left(\frac{30\pi f}{n}\right)\right| < 0 \tag{17}$$

Additional tip trajectory intersections are present in every vibration period and no material will be cut in a time range of each vibration period. For the nearest several vibration periods, sinusoidal variations with a certain duty factor caused by ultrasonic vibration are present with the trend of sinusoidal variations caused by spindle rotation as a whole. *X* and *Y* direction forces are the combinations of a linear part and trigonometric parts affected by rotation speed and vibration frequency, respectively. *Z* direction force is proportional to the horizontal resultant force.

From the model, the regular fluctuations in Fig. 2 are caused by ultrasonic vibration from Eq.(13) and Eq.(14). However, as the sampling frequency was smaller than ultrasonic frequency, the changing period of measured regular fluctuations was multiple of ultrasonic period as in Eq.(18).

$$T = \frac{f_{\text{ultrasonic}}\, T_{\text{sampling}}}{f_{\text{sampling}}\quad N} \approx 0.01\mathrm{s} \tag{18}$$

4 Conclusions

In this study, UVAM cutting force was investigated by modeling and experiment. It has an evident effect on the three direction cutting forces by exerting large amplitude ultrasonic vibration to the milling process. The major conclusions of this paper are as follows:

1. When the expression (17) was reached, air cutting happened every vibration period and force was back to zero at this time.
2. There were significant changes happened in all the three direction component forces, and small fluctuations caused by vibration of milling machine and cutting fluid system were restrained after exerting ultrasonic vibration.
3. The assisted ultrasonic vibration has little effect when the feed speed is small. As the increasing of the feed speed, the horizontal cutting force firstly increases and then decreases. And for ultrasonic vibration with different amplitudes, the horizontal cutting force firstly increases and then decreases.

The results indicate that the cutting forces are improved when feed direction ultrasonic vibration is exerted especially when the feed speed is large and the ultrasonic vibration with large amplitude is exerted. When the feed speed is large, the force becomes almost the same and even smaller after exerting a vibration with a large amplitude.

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