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ORIGINAL ARTICLE

Experimental study and machining parameter optimization in milling thin-walled plates based on NSGA-II

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Abstract The selection of machining parameters in milling thin-walled plates affects deformation, quality, and productivity of the machined parts. This paper presents an optimization procedure to determine and validate the optimum machining parameters in milling thin-walled plates. The regression models for cutting force and surface roughness are developed as objective functions according to experimental results. Besides, the influences of machining parameters on cutting force and surface roughness are also investigated. The objectives under investigation in this study are cutting force, surface roughness, and material removal rate subjected to constraints conditions. As the effects of milling parameters on optimization objectives are conflicting in nature, the multi-objective optimization problem in thin-walled plates milling is proposed. A non-dominated sorting genetic algorithm (NSGA-II) is then adopted to solve this multi-objective optimization problem. The optimized combinations of machining parameters are achieved by the Pareto optimal solutions, and these solutions are verified by the chatter stability.

Keywords Thin-walled plates · Machining parameters · Optimization · NSGA-II

1 Introduction

The thin-walled plates are widely used in the field of aerospace owing to their physical performances, such as less weight and more structure strength. However, deformation is easy to appear because of their low rigidity in machining process. Ensuring machining quality and improving metal removal rate of the machined thin-walled plates in the practical manufacturing engineering are in the limelight nowadays. As cutting parameters are one of the main factors that affect the milling process, cutting parameters optimization is performed to determine optimal milling parameters to satisfy various industrial demands. The emphasis is to achieve optimal machining parameters for meeting different industrial demands simultaneously in an effective way.

Balancing of machining parameters to achieve optimal combination has been the endeavor of many researchers. Initially, efforts are focused on optimizing parameters for one single objective (e.g., cutting forces, tool wear, and machining efficiency). Oktem et al. [1] resorted to genetic algorithm and neural networks to get the optimal parameters for surface roughness in milling mold faces. Vijayakumar et al. [2] obtained the multipass turning parameters to minimizing the unit production cost by means of ant colony system. Natarajan et al. [3] determined optimal parameters for longer tool life by genetic algorithm and particle swarm method. However, due to the complex nature of the machining processes, often optimization problems have several contradictory objectives to be optimized simultaneously. That optimizing one objective causes the other objectives to be poor and makes the selection of optimal parameters become difficult. Researchers deduced different optimal parameters to satisfy different industrial demands. Multi-objective optimization methodologies have demonstrated their usefulness in finding well-distributed Pareto optimal solutions and have been widely used in various problem-solving tasks. Yang and Natarajan [4] performed simultaneous optimization of tool wear and material removal rate to obtain optimal machining parameters. Mohinder et al. [5] optimized the cutting speed and surface

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roughness simultaneously, which provided a guide for operator. Govindan and Joshi [6] investigated the effects of milling parameters in order to maximize milling efficiency and minimize tool wear rate. Yan and Li [7] optimized milling parameters in terms of cutting energy, cutting quality, and production rate.

Traditional methods used for optimizing machining parameters include calculus method [8], Lagrange method [9], goal programming method [10], sequential quadratic programming method [11], and so on. The main disadvantage of these traditional approaches lies in the fact that the solutions are liable to get trapped into local minima. To solve this problem, various evolutionary optimizations [12] are presented for global solutions, such as genetic algorithm, ant colony optimization, and particle swarm optimization. These evolutionary techniques are good at optimizing machining parameters. Particle swarm optimization is developed by Bharathi and Baskar [13] to find the optimal machining parameters. It is observed that this method is capable of selecting appropriate parameters. Baskar et al. [14] adopted ant colony optimization to optimize surface grinding process. Genetic algorithm [15], which has favorable robustness and global searching ability, is widely applied in multi-objective optimization problems to capture a number of solutions simultaneously. Bouacha et al. [16] took composite desirability function, Gray-Taguchi method, and genetic algorithm as multi-objective optimization method to acquire the optimal solutions of the performance characteristics. They found that genetic algorithm seemed to be the most advantageous approach. Sreeram et al. [17] used genetic algorithm to optimize tool life and production cost in micro end milling, and this method yielded better results than those presented in the previous literature. Regression cutting force model coupled with genetic algorithm was developed by Subramanian et al. [18] to establish optimum end mill process parameter. Adam Khan et al. [19] employed an orthogonal design and analysis of variance to determine effective milling parameters on tool life based on genetic algorithm.

Among the multi-objective optimization approaches enhanced from genetic algorithm, non-dominated sorting genetic algorithm (NSGA-II) is one of the most widely used methods for generating the Pareto frontier based on combination of suitable variables [20]. According to Ref. [4], the non-dominated set obtained from NSGA-II outperformed that obtained from multi-objective differential evolution algorithm in the context of number of solutions and ratio of non-dominated individuals. Chen [21] proposed an improved NSGA-II by extending the non-dominance concept to constraint space in order to deal with multiple constraints in the milling process. It is observed that this approach makes the optimization process more approximate to application, more economical and effective in searching for the Pareto front.

Although many studies have been done in optimization of milling parameters, fewer surveys are performed with consideration of deformation, surface quality, and machining efficiency simultaneously in milling thin-walled plates. Moreover, most of the previous papers use weight factors to solve multi-objective optimization problems. Since no single solution can be considered as the best answer in a multi-objective optimization problem. A successful achievement of optimization objective in one field does not mean that this optimization objective is suitable for all other fields. Thus, it is very important to find a wide range of Pareto-optimal front as possible, from which decision makers can choose the one that suits their demands.

This paper focuses on optimizing machining parameters in milling thin-walled plates based on NSGA-II. Simultaneous optimization of cutting force, surface roughness, and material removal rate in thin-walled parts machining process are carried out practically in this study. Milling tests are conducted on a vertical machining center. According to the experimental results, the regression equations of cutting force and surface roughness are established with parameters of spindle speed, feed per tooth, and milling depth. Then, different optimization objectives are proposed, and a group of optimum combinations of milling parameters that meet these optimization objectives are selected by NSGA-II. Finally, milling stability is used to verify whether the optimized milling parameters can be directly applied to actual manufacturing.

2 Experimental procedure

In this work, a series of cutting tests of milling thin-walled plates have been conducted to develop statistical models of cutting force and surface roughness by means of regression analysis. These models are applied in the procedure of searching for optimal machining parameters. Considering that this paper focuses on optimizing the machining parameters, only the experimental conditions and results are listed in this paper.

The parts are machined on ME650 three-dimensional vertical machining center. The cutter used in experiments is a four-flute flat-end milling cutter (diameter 16 mm and helix angle 30°), which is made of solid carbide and coated with TiSiN. A rectangular thin-walled plate with size of 120 × 100 × 6 mm is selected as the workpiece. The workpiece selected for the experiments is die steel NAK80, the hardness of which is HRC 43-46. The physical properties are shown in Table 1. A Kistler 9123C dynamometer is used to measure the cutting force, as shown in Fig. 1. The cutting forces and surface roughness are measured at each milling test. Surface roughness of the machined part is achieved by a portable profilometer MarSurf PS1.

The milling process involves various milling parameters such as spindle speed, feed per tooth, and axial depth of cut. In the process of milling thin-walled part, the radial depth of

Table 1 Physical properties of NAK80

Tensile (MPa)	Elongation (%)	Contraction (%)	Yield (MPa)	Modulus of elasticity (GPa)
1319	14.6	51.2	1186	199

cut a_e is fixed to 1 mm in this study. The tested parameters including spindle speed n, feed per tooth f, and axial depth of cut a_p are employed to build experimental tables. The Taguchi method is used to design the cutting tests considering that the Taguchi method can realize the optimal and robust operation under different environment with high efficiency. Table 2 shows the milling parameters and their levels employed in the milling experiment. The three parameters with every three levels constitute the L 27 orthogonal array, as shown in Table 3. It also lists the experimental results of the cutting force Fa, surface roughness Ra, and material removal rate M. In this table, the cutting force Fa refers to the mean value in each milling process.

3 Statistical model for optimization

Regression analysis is a method for investigating functional relationships between input and output parameters. It focuses on exploratory data analysis rather than statistical theory, so it is suitable for manufacturing output parameters estimation. In this study, input parameters include spindle speed, feed per tooth, and axial depth of cut. The experimental results of cutting force and surface roughness are applied to establish statistical model by regression analysis method. The statistical models of cutting force and surface roughness should include all factors and their interactions. Also, these models are reduced by eliminating terms with no significant effect on the responses. Thus, the regression equations between cutting force, surface roughness, and input parameters, i.e., spindle speed, feed per tooth, and axial depth of cut, can be modeled based on second-order polynomial equation as given in the following:

$$Y = \alpha 0 + \sum_{i=1}^{k} \alpha i X i + \sum_{i \le j}^{k} \alpha i \mathrm{jXiXj} \tag{1}$$

where α_i and α_{ij} represent the coefficients, X_i is the input parameter. All the unknown coefficients are determined using regression with the aid of Matlab software.

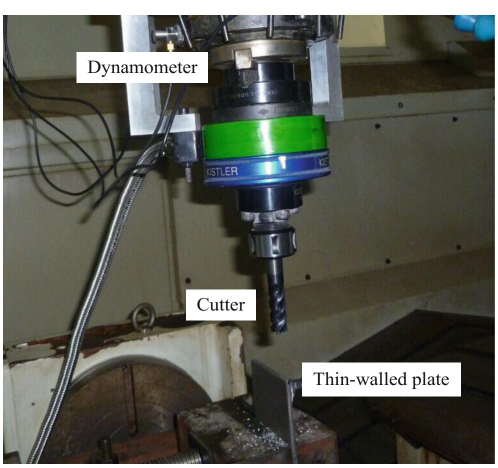


Fig. 1 Milling experiment setup

3.1 Cutting force model

Cutting force has a direct effect on cutting heat and can cause vibration. Moreover, the cutting force not only affects the machining deformation and the machined surface quality but also is a vital parameter in optimizing the milling process. The cutting force model based on regression method is developed according to the measured results in Table 3 and the model is given as follows:

$$\begin{aligned} \mathrm{Fa} = & -52.07 - 0.006n + 381.01f + 80.35ap + 0.0243n \cdot f \\ & -13.333f \cdot ap + 0.0116n \cdot ap - 784.44f^2 + 9.8765ap^2 \end{aligned} \tag{2}$$

The acquired cutting force model cannot be directly applied in the process of analysis and estimation. It is necessary to further confirm the statistical regularity of the model. The regression equation needs to be verified by the normality assumption [22]. F calibration is adopted to test the reliability of the regression equation. The closer determination coefficient R^2 is to 1, the higher the reference value of the regression equation is.

The R^2 value of the second-order polynomial model for cutting force is 98.64 %, which is close to 1. It shows that the regression equation is highly significant at a 95 % confidence value. Figure 2 displays the normal probability plot of residuals for cutting force. Obviously, the residuals are located on a straight line, which means the errors are well distributed. Hence, the cutting force model can provide an excellent explanation between the input parameters and cutting force values.

Table 2 Cutting parameters and their levels

Levels	Milling parameters		
	Spindle speed (n, r/min)	Feed per tooth (f, mm)	Axial depth of cut (a_p, mm)
1	1600	0.15	0.4
2	2100	0.20	0.55
3	2600	0.25	0.7

Table 3 Experimental conditions and results

No.	Experimental conditions			Experimental results		
	Spindle speed (n, r/min)	Feed per tooth (f, mm)	Axial depth of cut (a_p, mm)	Cutting force (Fa, N)	Roughness (Ra, μm)	Removal rate (M, mm^3/min)
1	1600	0.15	0.4	27.3	0.532	384
2	1600	0.15	0.55	42.7	0.853	503
3	1600	0.15	0.7	58.5	1.215	672
4	1600	0.2	0.4	33.2	0.587	512
5	1600	0.2	0.55	52.8	0.912	704
6	1600	0.2	0.7	70.2	1.316	896
7	1600	0.25	0.4	39.2	0.671	640
8	1600	0.25	0.55	54.6	1.030	880
9	1600	0.25	0.7	69.8	1.487	1120
10	2100	0.15	0.4	35.3	0.498	504
11	2100	0.15	0.55	49.6	0.834	693
12	2100	0.15	0.7	68.5	1.042	882
13	2100	0.2	0.4	39.7	0.566	672
14	2100	0.2	0.55	56.5	0.878	924
15	2100	0.2	0.7	73.4	1.195	1176
16	2100	0.25	0.4	43.9	0.627	840
17	2100	0.25	0.55	58.3	0.902	1155
18	2100	0.25	0.7	73.6	1.268	1470
19	2600	0.15	0.4	38.5	0.468	624
20	2600	0.15	0.55	52.7	0.815	858
21	2600	0.15	0.7	72.3	0.926	1092
22	2600	0.2	0.4	43.6	0.487	832
23	2600	0.2	0.55	61.3	0.773	1144
24	2600	0.2	0.7	82.3	1.026	1456
25	2600	0.25	0.4	48.9	0.568	1040
26	2600	0.25	0.55	71.4	0.869	1430
27	2600	0.25	0.7	85.6	1.056	1820

3.2 Surface roughness model

Surface roughness is an important criterion for machining quality evaluation. It is the description of surface geometry and texture. In addition, it directly affects the wear resistance and fatigue strength of the machined part. The formation mechanism of surface roughness is quite complicated and mainly depends on milling parameters. In this paper, the surface roughness model and the cutting force model have the same regression equation form. Thus, the regression model for surface roughness is described as:

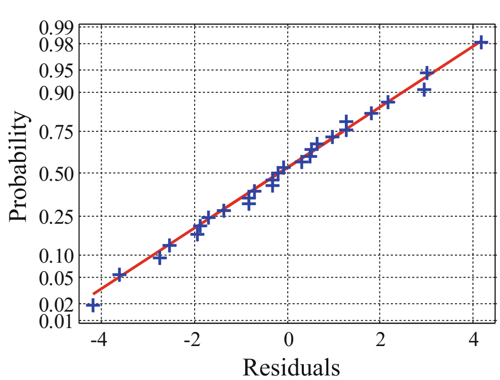


Fig. 2 Normal probability plot of residuals for cutting force, N

$$\begin{aligned}\mathrm{Ra} = &-1.2445 + 0.0005n + 0.3691f + 3.738ap - 0.001n{\cdot}f \\ &+2.8889f{\cdot}ap - 0.0008n{\cdot}ap + 4.0222f^2 - 0.4864ap^2\end{aligned} \tag{3}$$

Similarly, the regression equation also needs to be verified by F calibration. The fitting coefficient R^2 of the regression equation is 98.82 %, verifying the high reliability of the regression equation. Figure 3 displays the normal probability plot of residuals for surface roughness. It is observed that the residuals are located on a straight line, which means the errors are well distributed. On this account, we conclude that the

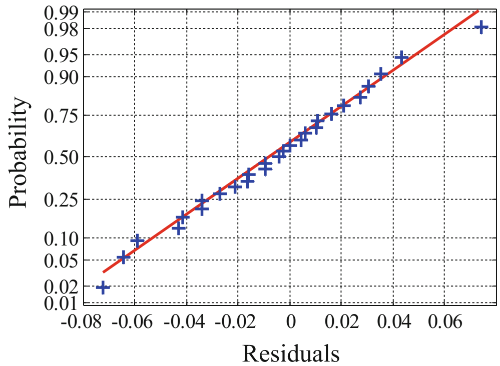


Fig. 3 Normal probability plot of residuals for surface roughness, μm

surface roughness model reflects the relationship between the milling parameters and surface roughness values fairly well.

In order to validate the abovementioned regression models, a series of random milling experiments are conducted, shown in Table 4. The comparison of the predicted and measured values of cutting force and surface roughness is presented in Fig. 4. It can be seen from Fig. 4 that the predicted values are in well agreement with the experimental data, which further verifies the high reliability of these regression models. Therefore, the two models can be used for optimizing the machining parameters.

3.3 Material removal rate

Material removal rate needs to be considered when determining the machining parameters because the higher efficiency is of great significance in such operations. Material removal rate is defined as the volume of material that is removed from the part per unit time, and it is a function of the milling parameters, which is expressed as:

$$M = n{\cdot}f{\cdot}Z{\cdot}ap{\cdot}ae \quad (4)$$

where Z is the tooth number. The material removal rates under different milling conditions are shown in Table 3.

Table 4 Experimental conditions of validation tests

No.	Spindle speed (n, r/min)	Feed per tooth (f, mm)	Axial depth of cut (a_p, mm)
1	1500	0.15	0.6
2	1700	0.18	0.7
3	2000	0.2	0.8
4	4200	0.3	0.4
5	3000	0.25	0.5

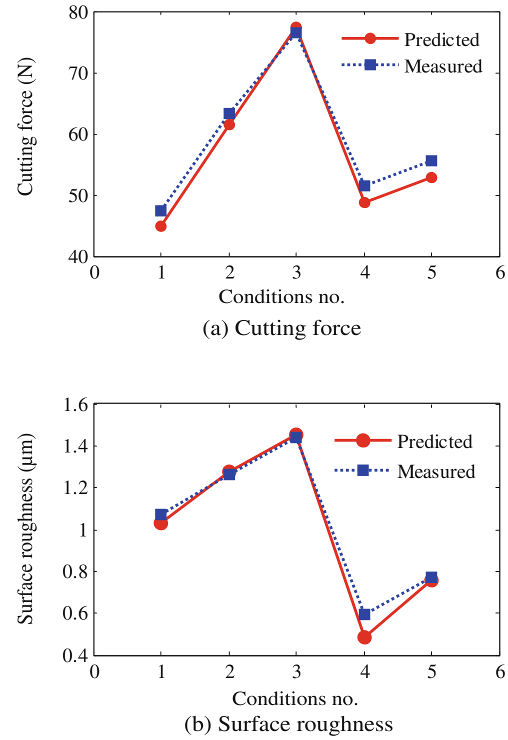


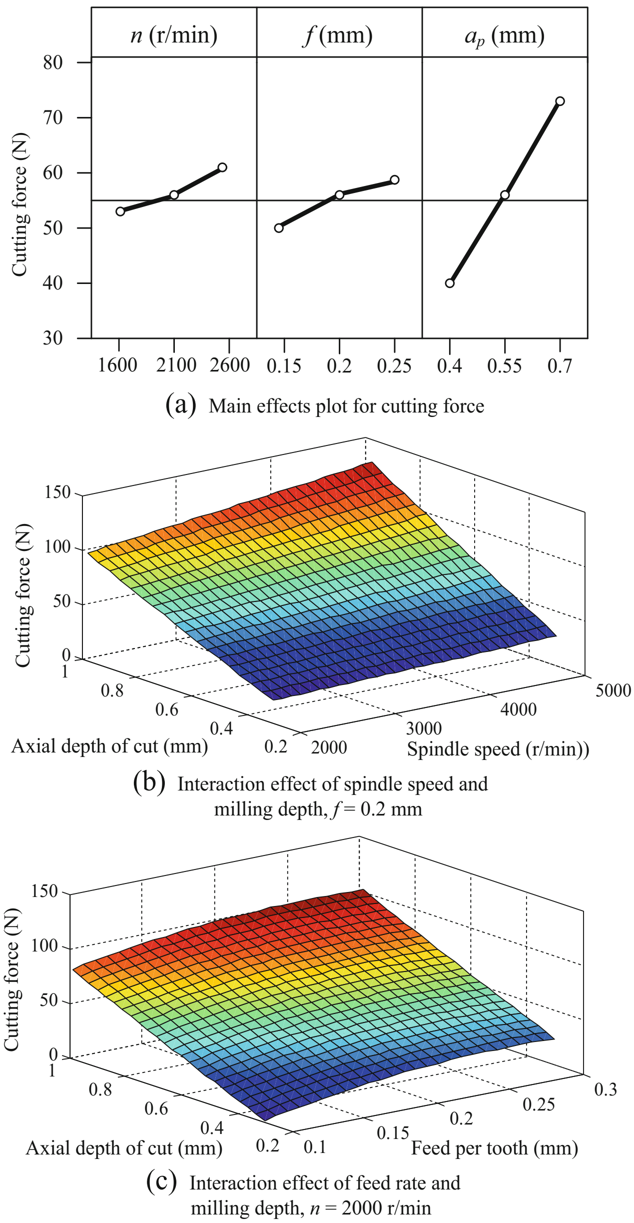
Fig. 4 Comparison between the predicted and measured values

3.4 Influence of milling parameters on cutting force and surface roughness

Based on these regression models, a further explanation is made for the influence of milling parameters.

Figure 5 illustrates the main effects for cutting force. Obviously, it is observed from Fig. 5a that there is an increase of cutting force with increase of milling parameters. However, the influence of each machining parameters on cutting force is different. That is to say, the axial depth of cut has a larger effect on the cutting force than the other two factors. Figure 5b shows the interaction effect of spindle speed and axial depth of cut on the cutting force while Fig. 5c depicts the interaction effect of spindle speed and feed per tooth on the cutting force. It can be seen that smaller cutting force can be obtained with the combination of higher spindle speed and smaller axial depth of cut or the combination of higher feed per tooth and smaller axial depth of cut.

Figure 6a indicates that an increase of feed per tooth or axial depth of cut leads to the increase of surface roughness while the increase of spindle speed results in a slightly decrease of surface roughness. The same conclusion can also be drawn from Fig. 6b, c. Besides, the axial depth of cut affects surface roughness more than feed per tooth. Figure 6b shows the surface roughness in relation to feed per tooth and axial depth of cut when spindle speed is 2000 r/min. And from Fig. 6c, we can deduce that the higher

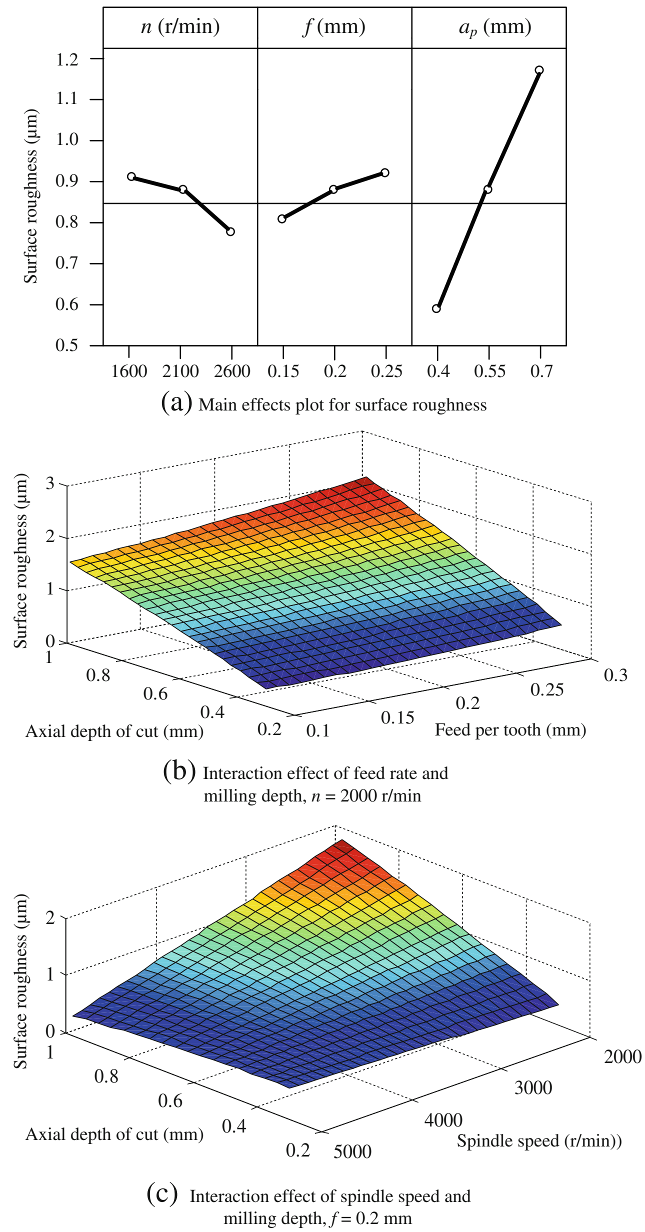


(a) Main effects plot for cutting force

(b) Interaction effect of spindle speed and milling depth, $f = 0.2$ mm

(c) Interaction effect of feed rate and milling depth, $n = 2000$ r/min

Fig. 5 The influences of milling parameters on cutting force



(a) Main effects plot for surface roughness

(b) Interaction effect of feed rate and milling depth, $n = 2000$ r/min

(c) Interaction effect of spindle speed and milling depth, $f = 0.2$ mm

Fig. 6 The influences of milling parameters on surface roughness

the spindle speed is and the smaller the milling depth is, the lower the surface roughness will be.

4 Results of multi-objective optimization

Genetic algorithm is a powerful tool for optimization of complicated problems in terms of biological heredity and evolutionary mechanism. Enhanced from single objective optimization genetic algorithm, many multi-objective optimization approaches are proposed. Among these approaches, NSGA-II is most widely used to optimize the machining parameters. The algorithm adopts non-dominated sorting approach, crowded distance estimation procedure, and simple crowded comparison operator to find a set of evenly distributed solutions to a simultaneous optimization problem [23]. In this study, the simultaneous optimization of cutting force, surface roughness, and material removal rate is performed using NSGA-II. A set of optimal solutions called Pareto optimal fronts would be obtained that in general, any of these solutions has no predominance to another [23]. Figure 7 shows the flow chart of NSGA-II algorithm. Generally, the main steps of the NSGA-II are given as follows.

Step 1: Randomly initialize the parent population based on the problem range and constraint.

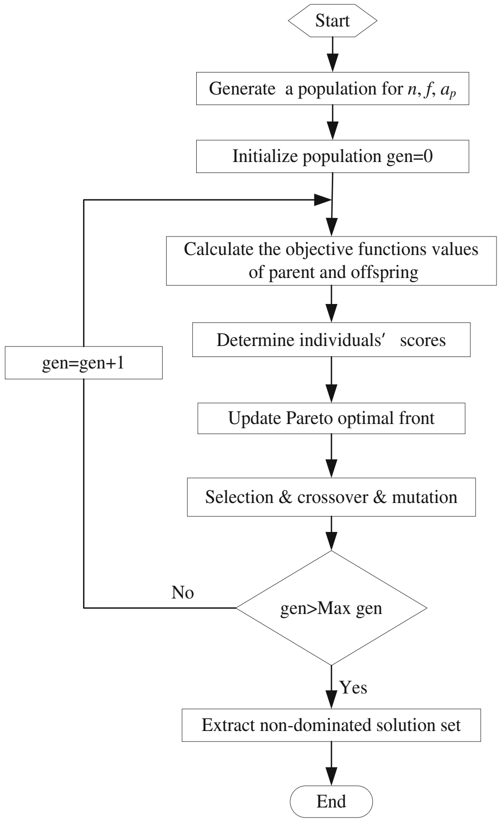


Fig. 7 Flow chart of machining parameters optimization algorithm with NSGA-II

Step 2: Rank the population using non-domination criteria. The individuals in population are selected based on rank and crowding distance.

Step 3: Apply the genetic operators of selection, crossover, and mutation to generate offspring population and validate the offspring population.

Step 4: Offspring population and current generation population are combined and the individuals of the next generation are set by selection.

Step 5: The termination principle is that the current evolutionary generation exceeds the maximum evolutionary generation. Record the candidate Pareto optimal set if it meets the termination principle. Otherwise, return to Step 2.

4.1 Mathematical equations for the optimization

Machining the thin-walled plates has attracted great attention due to their poor machinability. During the milling process, we need to control the deformation, guarantee the surface roughness, and improve the machining efficiency. Thus, the minimal cutting force, optimal surface quality, and best milling efficiency are the desired objectives of this work.

4.1.1 Minimal cutting force

In thin-walled plates milling, the cutting force is directly related to the deformation. That is to say, smaller deformation can be achieved by reducing the cutting force. In order to minimize the deformation in the production process, the cutting force should be as small as possible. The mathematical equation of cutting force is given by Eq. 2.

4.1.2 Optimal surface quality

Surface quality has a great influence on the part functionality. Surface roughness affects the properties of fatigue strength, friction coefficient, and wear rate. As the purpose of this paper is to acquire better surface roughness of the thin-walled plates, the optimal surface quality is set as the minimal surface roughness. Its mathematical equation is expressed in Eq. 3.

4.1.3 Best milling efficiency

The milling efficiency is directly related to material removal rate. It can achieve the best milling efficiency by getting the maximal material removal rate. The mathematical relation between material removal rate and machining variables is given in Eq. 4.

4.1.4 Constraint conditions of optimization

The machining parameters should be designed on the basis of operation constraints. These constraints need to combine various considerations, such as the scope of spindle speed, the availability of feed rate. In this paper, the constraint condition of spindle speed n is from 1000 to 3000 r/min. The range of feed per tooth f is from 0.1 to 0.3 mm. The scope of axial depth of cut a_p is between 0.3 and 1 mm.

4.2 Optimization objectives

In production process, the major goal is to fabricate high quality thin-walled parts with high efficiency. That is, the main problem is to obtain the minimum deformation, best milling efficiency, and optimal surface quality concerning the industrial demands. Thus, the minimal cutting force, the minimal surface roughness, and the maximal material removal rate are the optimization objectives. The objective functions are given as follows:

$$\begin{cases} \text{Objective}(1) = \min[\text{Fa}] \\ \text{Objective}(2) = \min[\text{Ra}] \\ \text{Objective}(3) = \max[M] \end{cases} \tag{5}$$

Objective (1) and Objective (2) are the minimum while Objective (3) is the maximum. Regarding the fact that it is convenient to calculate the minimal values based on NSGA-II, the objective functions could be rewritten in the following form:

$$\begin{cases} \text{Objective} = \min[\text{Fa}, \text{Ra}, 1/M] \\ \text{s.t.} = \begin{cases} n = 1000\text{~}3000\text{r/min} \\ f = 0.1\text{~}0.3\text{mm} \\ ap = 0.3\text{~}1\text{mm} \end{cases} \end{cases} \tag{6}$$

In this work, these optimization objectives should satisfy the following criteria:

$$\text{Fa} \leq 50\text{N}; \text{Ra} \leq 1.3\mu\text{m}; M \geq 1000\text{mm}^3/\text{min} \tag{7}$$

4.3 Results and discussion

In the optimization procedure, the population size is taken as 200 and the evolutionary generation is adopted as 500. Crossover and mutation probabilities are set to 0.9 and 0.1. After optimization, the Pareto-optimal front obtained by NSGA-II is shown in Fig. 8.

All the points in Fig. 8 denote an optimal combination of machining parameters that has no predominance to each other. Any one of them is an acceptable solution, and none of the solutions in the non-dominated set can be said to be absolutely better than any other. The choice of optimum combination of milling parameters over the other depends on the desired machining criterion of the thin-walled plate.

As it is depicted in Fig. 8, it is clear that surface roughness Ra value decreases with the increase of cutting force Fa value, and material removal rate M increases with surface roughness Ra and cutting force Fa increasing. The Pareto optimal solutions in region A are good to reduce cutting force, in the meanwhile, not increase surface roughness. In addition, it can be seen from Fig. 8b that the solutions in region B can get optimal cutting force and material removal rate. Similarly, the Pareto optimal solutions in region C are relevant to reduce surface roughness and increase material removal rate. For the purpose of acquiring large material removal rate as well as high quality parts, the intersection sets of the regions A and B are selected in the Pareto optimal solutions. Besides, these solutions are selected after comprehensive consideration of machining quality and milling efficiency. For instance, some optimum combinations of machining parameters for milling thin-walled plate are presented in Table 5. These solutions can well ensure the machining quality and efficiency in thin-walled plates milling.

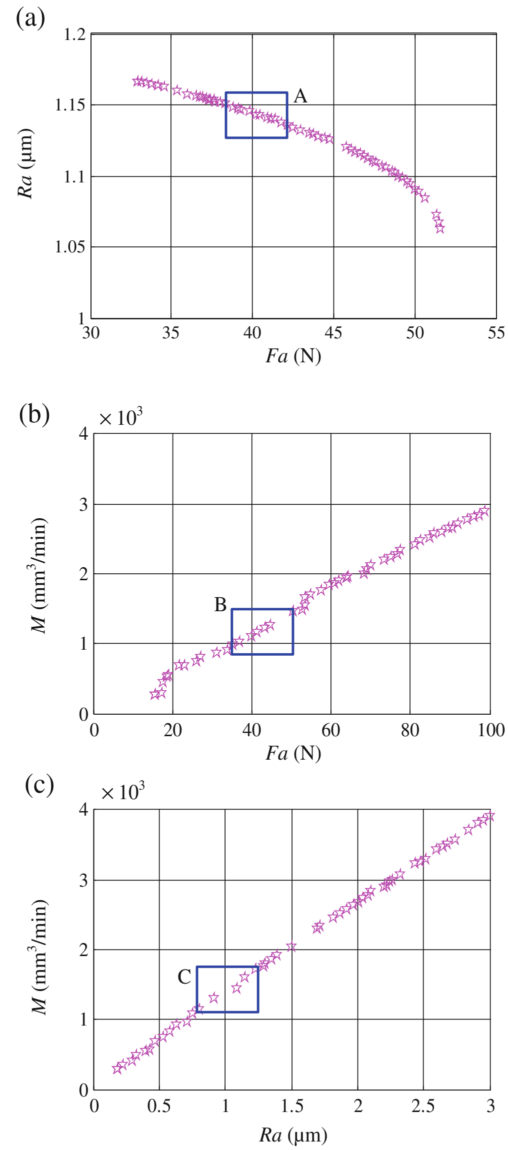


Fig. 8 Pareto optimal front for optimization objectives: **a** the relationship between surface roughness Ra and cutting force Fa. **b** The relationship between material removal rate M and cutting force Fa. **c** the relationship between material removal rate M surface roughness Ra

As mentioned in Section 4.1.4, the constraint condition of spindle speed is from 1000 to 3000 r/min, the range of feed per tooth is from 0.1 to 0.3 mm, the scope of axial depth of cut is between 0.3 and 1 mm. According to Table 5, the optimized spindle speeds are about 2800 r/min, close to upper limit 3000 r/min, optimized feeds per tooth are in the region of 0.24 mm, close to upper limit 0.3 mm, optimized milling depth are about 0.4 mm, close to lower limit 0.3 mm. Hence, it can be asserted that the acquired optimal solutions concentrate upon higher spindle speed, higher feed per tooth, and lower milling depth, which is in accord with the analysis in Section 3.4.

Table 5 Pareto front of optimum combinations

No.	Spindle speed (n, r/min)	Feed per tooth (f, mm)	Axial depth of cut (a_p, mm)	Cutting force (Fa, N)	Roughness (Ra, μm)	Removal rate (M, mm^3/min)
1	2816	0.2474	0.4363	43.971	1.169	1215
2	2835	0.2442	0.3982	39.365	1.059	1103
3	2811	0.2475	0.4098	40.814	1.085	1140
4	2832	0.2427	0.4282	42.811	1.154	1175
5	2819	0.2461	0.3917	38.639	1.032	1086

5 Optimization results verification considering chatter stability

During the optimization, constraint conditions do not include chatter stability for the reason that it will be very complex to perform optimization if the chatter stability is directly taken as the constraint. However, chatter may occur if the optimized parameters are directly applied to machining operations. Hence, it is necessary to further verify the solutions in the light of machining stability.

The dynamic model of the thin-walled plates can be reduced to a single-degree freedom system [24], shown in Fig. 9. The dynamic equation of the cutter-workpiece system is described as:

$$my\ddot{y}(t) + cyy(t) + kyy(t) = Fy(t) \tag{8}$$

where m_y, c_y, and k_y represent the modal mass, modal damping, and modal stiffness of the dynamic system, respectively. $y(t)$ is vibrational displacement in the Y direction. $F_y(t)$ is the cutting force in the Y direction.

According to Altintas's regenerative chatter theory [25], the cutting force in the Y direction can be expressed as

$$F\mathrm{y}(t) = \frac{Z}{4\pi}\mathrm{apktc}\alpha yy\Delta y \tag{9}$$

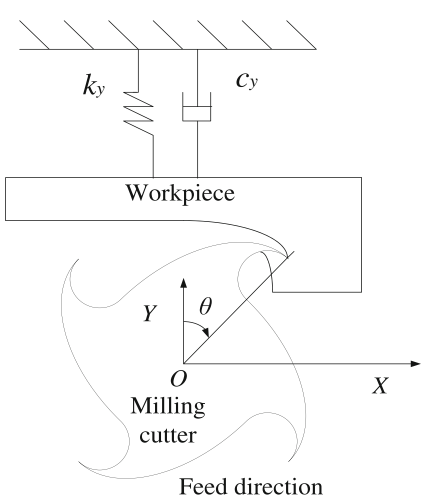


Fig. 9 Dynamic model of the system

where Z is the tooth number, a_p is axial depth of cut, k_{tc} is tangential force coefficient, α_{yy} is the directional dynamic milling coefficient in the Y direction, and Δy is the vibration displacement in the Y direction.

Eq. 8 can be rewritten as:

$$\{Fy(t)\}e^{i\omega ct} = \frac{1}{2}\mathrm{apktc}\left(1-e^{-i\omega cT}\right)\alpha yy[Gyy(i\omega c)]\{Fy(t)\}e^{i\omega ct} \tag{10}$$

where ω_c is chatter frequency, T is tooth passing period, $G_{yy}(i\omega_c)$ is frequency response function of the dynamic system in the Y direction.

By solving Eq. 10, the eigenvalues $\Lambda = \Lambda_R + i\Lambda_I$ are determined. And then the critical depth of cut $a_{p\lim}$ and corresponding spindle speed n for the stability lobes are obtained as:

$$\begin{cases} \mathrm{aplim} = -\dfrac{2\pi\Lambda R\left(1+\kappa^{-2}\right)}{\mathrm{Zktc}} \\ n = \dfrac{60\omega c}{((1+2k)-2\arctan\kappa)} \end{cases} \tag{11}$$

where κ is the ratio of imaginary and real parts of the eigenvalues Λ, k is the number of stability lobes.

The stiffness and damping of the wall will be dropping as more material is removed. The dynamic behavior of the wall depends on the position of the tool. The variation in the dynamics of the wall due to material removal can not be neglected, which leads us to a three-dimensional lobe diagram construction. First of all, we need to identify the modal parameters in the first tool position and acquire the two-dimensional stability lobe. The wall is divided into several zones at the same height, and impact tests are conducted in each zone to obtain corresponding dynamics. It is assumed that the variation of dynamic parameters resulting from material removal is neglected in each zone. Hence, the more zones the wall is divided into, the more accurate the acquired modal parameters are. In this study, the thin-walled plate is divided into four zones, shown in Fig. 10. Then, repeat the previous step and plot two-dimensional stability limits in all tool positions. Finally, these acquired two-dimensional stability limits are analyzed and then the three-dimensional stability lobe diagram is determined, shown in Fig. 11.

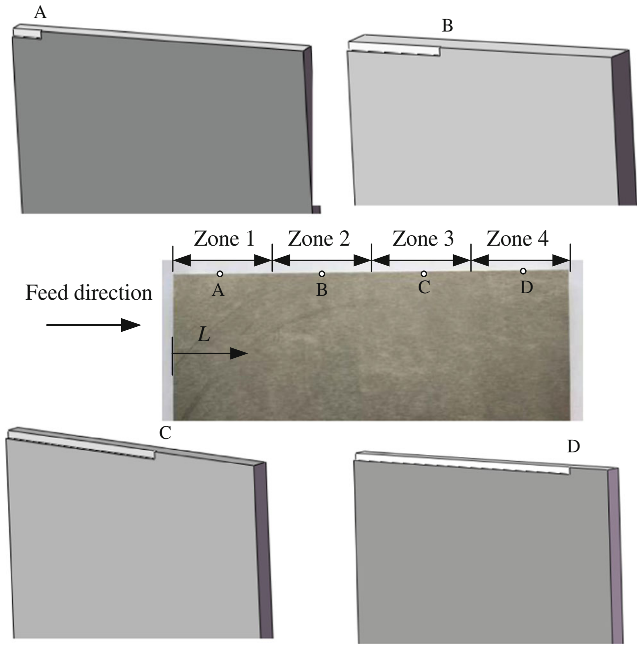


Fig. 10 Divided zones used for measuring modal parameters

The curved surface in Fig. 11 is the boundary between chatter-free operations and unstable process. It is observed from Fig. 11 that the critical milling depths at the same spindle speed gradually decreases with the increment of tool position. In Table 5, we have concluded that the optimum spindle speeds, feeds per tooth, and milling depths are about 2800 r/min, 0.24 mm, and 0.4 mm, respectively. According to the three-dimensional stability lobes, when the spindle speed is 2800 r/min, the corresponding critical milling depth in zone 4 is minimal. And its value is 0.6 mm, larger than the optimized milling depth 0.4 mm. It is well known that chatter stability is not dependent on feed rate [25]. Hence, the acquired optimum combinations of machining parameters can be applied to machining operations, which can not cause chatter. After considering the milling stability, the optimized milling parameters

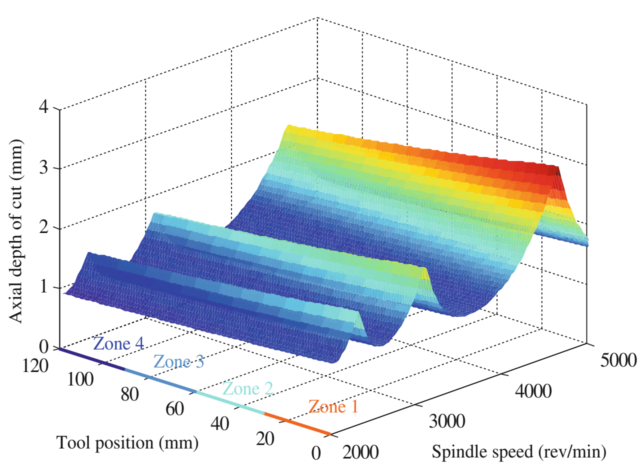


Fig. 11 Three-dimensional stability lobes

can be used to manufacture thin-walled plates for optimal milling quality and efficiency simultaneously.

5.1 Conclusions

1. In this paper, an attempt has been made to solve multi-objective optimization problem in milling thin-walled plates by using NSGA-II. Three conflicting objectives, namely cutting force, surface roughness, and material removal rate, have been considered for optimization simultaneously. As a consequence, the optimum combinations of machining parameters and output objectives are acquired. Besides, the optimized milling parameters have been verified by the milling stability.
2. Cutting force and surface roughness behaviors are investigated in milling thin-walled plates. The second-order polynomial models for cutting force and surface roughness are developed for optimization.
3. The influences of machining parameters on cutting force and surface roughness are analyzed. It is found that both cutting force and surface roughness increase with the increasing of feed per tooth and milling depth. However, the increase of spindle speed has opposing effects on these two output objectives. That is, as spindle speed increases, cutting force increases while surface roughness decreases.
4. The proposed method provides an applicable range wide range of solutions for decision maker, which helps to guarantee the high quality of the machined thin-walled plates as well as improve milling efficiency.

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