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ORIGINAL ARTICLE

Numerical simulation of sheet metal forming: a review

Muhammad Ali Ablat[1] · **Ala Qattawi**[1]

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Abstract Numerical simulation is becoming one of the main methods to investigate various engineering problems with sophisticated conditions. A considerable amount of research is conducted on numerical analysis of sheet metal forming process to address different aspects of the problem. Among the numerical simulation methods for sheet metal forming process, finite difference method (FDM) and finite element method (FEM) have been the main methods. In this paper, the progresses in simulation techniques, advantage and disadvantage of numerical methods for simulating sheet metal forming process are discussed based on these numerical methods. Currently, FEM being the main simulation method for sheet metal forming, development in solution strategies and formulation, element selection are further brought into attention. Historical development of anisotropy and yield criteria, which are theoretical foundation for numerical simulation, and their application in simulation software are briefly classified. Formability of sheet metal is presented from the numerical simulation point of view. Numerical investigations on springback are reviewed in terms of simulation techniques and factors influencing springback. Simulation techniques for novel sheet metal forming techniques such as laser forming and incremental sheet forming (ISF) are presented.

Keywords Finite element analysis · Sheet metal · Anisotropy · Springback · Laser forming

1 Introduction

With the development of computers in the last few decades, numerical simulation of engineering problems has achieved significant progress. Along with theoretical and experimental studies, numerical simulation has become one of the major tools for solving engineering problems in research and industrial application due to its efficiency and cost reduction. Numerical simulation has been applied to simulate sheet metal forming since the 1960s. Thus far, various simulation methods have been used for sheet metal forming process. These methods include finite difference method (FDM), finite element method (FEM), meshfree (meshless) method, slip line field method and upper bound method [1, 2]. Some of the methods like FEM are being used extensively while others are seldom in use in research and industrial application. In this paper progress in sheet metal forming simulation using various numerical methods are presented and followed by the discussion of anisotropy and yield criteria on which numerical simulation methods are built. The simulation of sheet metal forming element selection is also presented due to its importance in FEA. In addition, this manuscript highlights the springback effect in simulation as one of the major concerns for precision forming and quality of the final product as well as the simulation of novel forming methods, particularly the laser forming and incremental sheet forming (ISF).

In the past, researchers reported the state-of-the-art for sheet metal forming simulation for various stages. However, the available literature varies largely in terms of the topics covered and depth of discussion. Some numerical simulation methods were not mentioned since most of the past reviews focused on FEM and its related topics such as formulation and solution strategies and element selection. In addition, some reviews extended the discussion to a constitutive model of sheet metal and contact in the sheet metal forming process.

Muhammad Ali Ablat is also known as Abulaiti Maimaitiaili.

✉ Ala Qattawi
aqattawi@ucmerced.edu

[1] Department of Mechanical Engineering, University of California Merced, 5200 North Lake Rd, Merced, CA 95343, USA

This paper is intended to serve as a comprehensive and broad review.

As early as 1980s, Tkekaya and Kaftanoglu [3] briefly summarized the pioneering works in sheet metal forming simulation. Makinouchi [4] discussed application of finite element simulation of sheet metal forming in industry until mid-1990s. Makinouchi developed a summary of existing simulation codes and examples from industry perspective. Makinouchi, Teodosiu, and Nakagawa [5] presented a review in which they overviewed the state-of-the-field focusing on application in automobile industry and sheet metal suppliers in Europe, USA, and Japan. Later, in 2000, Tekkaya [6] presented the progresses in the field with detailed discussion of industrial requirement of sheet metal simulation, existing methodologies, element types, and available software packages at that time. Ability of simulation at that time was illustrated by industry application examples. Tisza [7] overviewed developments in sheet metal forming simulation from industrial and research perspectives by looking at the application of FEM in manufacturing process. Unlike others, Wenner [8] presented the development of sheet metal forming simulation until 2005 by summarizing achievements and progresses presented NUMISHEET conferences held in past years. Ahmed and Sehkon [9] discussed the development of sheet metal simulation from different aspects such as, shell and continuum approach, methods dealing with material nonlinearity, geometrical nonlinearity and frictional contact. Error estimation in simulation, error projection and adaptive mesh refinement were also presented. Lee, Kim, Pavlina, and Barlat [10] presented a detailed discussion related to many aspects of simulation of sheet metal ranging from material models, solution type, element selection, and parameters identification during simulation, contact model in sheet metal forming process and process chain simulation. Banabic [11] and Prasad [12] also presented recent developments until 2012 in the field. However, the review was not comprehensive and discussed few areas in the FEA for sheet metal.

This paper presents a comprehensive survey of historical development in sheet metal forming followed by an overview on other closely related topics each classified based on area of interest and scope of work. The summary of the revised literature are tabulated according to the focus area at the beginning of this manuscript in Table 1. In addition, Fig. 1 illustrates visual representation for the surveyed reported literature in this manuscript; the representation classifies the reported work based on the area of interest and the year of publication.

2 Simulation methods

This section presents various numerical methods used to simulate sheet metal forming process. Focus is given to FEM and topics closely related to it while other numerical simulation methods are also presented.

2.1 Finite difference method

In early stage simulation of sheet metal, forming was limited to 2D symmetric simple problems. First simulation of sheet metal forming started in 1960s for deep drawing process of a 2D cylindrical cup using finite difference method (FDM) [13]. When nonlinear FEM was proven for its accuracy, it took the position of FDM [12]. Attempts were made to implement finite difference method to 3D problems 1990s; however, it was not successful due to the difficulty in applying boundary conditions [6]. However, FDM has been used for simulating thermal effect in sheet metal forming processes [14, 15].

2.2 Finite element method

FEM is a main method for simulation of sheet metal forming. The solution accuracy achieved in FEM led to shifting from FDM to FEM. In the late 1970s, Wifi [16] presented a finite element analysis of elastoplastic, axisymmetric circular blank sheet for stretch forming and deep drawing process. Gotoh and Ishise [17] presented a general formulation of finite element analysis for the flange in deep drawing process based on rigid-plastic material law, where the analysis was carried out using the quadratic yield function and fourth-degree yield function. Wang and Budiansky [18] presented a general finite element procedure for sheet metal stamping, they assumed small thickness of the sheet material to be able to apply membrane theory in the analysis. In addition, the material was assumed to obey elastic-plastic material law and rate-independent, J2-type flow rule. By comparison, they showed that both elastic-plastic material law and rigid-plastic material law yielded the same strain distribution to the point of material unloading.

In 1980s, Tang, Chu, and Samanta [19] expanded application of finite element simulation from 2D to 3D by simulating deformed automobile body panel. Toh and Kobayashi [20] also presented a 3D simulation of sheet metal in general shape. In these simulations, either static implicit method or static explicit method were used [8, 9] and elastoplasticity was used as the material model [11]. Later, Benson and Hallquist [21] introduced deformation mechanics to simulation software DYNA3D. Dynamic explicit method was applied in simulation based on work of Belytschko and Mullen [22]. The concept of using an artificial force for replacing drawbead was introduced by Massoni [23] while the viscous effects are taken into consideration by Wang and Wenner [24].

Since late 1990s onward, accurately predicting springback has become the focus of many researchers, which in turn directed the development of sheet metal simulation into

Table 1 A summary of the literature revised in numerical sheet metal forming

Affiliation	Focus area
Kaftanoglu and Tekkaya, [3]	Review
Makinouchi, [4]	Review
A. Makinouchi, Teodosiu, and Nakagawa, [5]	Review
Tekkaya, [6]	Review
Tisza, [7]	Review
Wenner, [8]	Review
Ahmed and Sekhon, [9]	Review
Lee, Kim, Pavlina, and Barlat, [13]	Review
Banabic, [11]	Review
Reddy, Reddy, and Prasad, [12]	Review
Woo, [14]	FMD
Doege and Ropers, [15]	FMD
TSENG, [16]	FMD
Wifi, [17]	FEM
Gotoh and Ishisé, [18]	FEM
N.-M. Wang and Budiansky, [19]	FEM
Tang, Chu, and Samanta, [20]	FEM
Toh and Kobayashi, [21]	FEM
Benson and Hallquist, [22]	FEM
Belytschko and Mullen, [23]	FEM
Massoni, Bellet, Chenot, Detraux, and De Baynast, [24]	FEM
N-M Wang and Wenner, [25]	FEM
Ofiate, Rojek and Carino, [26]	Formulation and solution type
Yang, Jung, Song, Yoo, and Lee, [27]	Formulation and solution
Nakamachi, [28]	Formulation and solution type
Mamalis, Manolakos, and Baldoukas, 1997a, 1997b	Formulation and solution
Jung, [29]	Formulation and solution type
Carleer and Hu, [30]	Formulation and solution type
Finn et al., [31]	Formulation and solution type
Micari, Forcellese, Fratini, Gabrielli, and Alberti, [32]	Formulation and solution
Tang, Li, and Lu, [33]	Formulation and solution type
Azizi, [34]	Formulation and solution type
Kim, Pavlina, and Barlat, [10]	Formulation and solution type
Wang and Budiansky, [19]	Element selection
Chung, Kim, Lee, Ryu, and Joun, [35]	Element selection
Chung et al., [35];	Element selection
Xu, Liu, Zhang, and Du, [36]	Element selection
Papeleux and Ponthot, [37]	Element selection
Cardoso, and Alves de Sousa, [38]	Element selection
Alves de Sousa, Yoon, Cardoso, Fontes Valente, and Grácio, [39]	Element selection
Lee, Chung, Jang, and Joun, [40]	Element selection
Menezes and Teodosiu, [41]	Element selection
Chung et al., [35]	Element selection
Cueto and Chinesta, 2013	Meshfree method
Yoon, Wu, Wang, and Chen, [42]	Meshfree method
Yoon and Chen, [43]	Meshfree method
Botkin, Guo, and Wu, [44]	Meshfree method
H. S. Liu, Xing, and Yang, [45]	Meshfree method
Sidibe and Li, [46]	Meshfree method

Table 1 (continued)

Affiliation	Focus area
H. Liu, Xing, Sun, and Bao, [47]	Meshfree method
Hill, 1948	Anisotropic yield criteria
Hill, [48]	Anisotropic yield criteria
Hill, [49]	Anisotropic yield criteria
Vial, Hosford, and Caddell, [50]	Anisotropic yield criteria
F. Barlat and Richmond, [51]	Anisotropic yield criteria
Frédéric Barlat, Lege, and Brem, [52]	Anisotropic yield criteria
F. Barlat et al., [53, 54]	Anisotropic yield criteria
F Barlat et al, [53, 54]	Anisotropic yield criteria
Karafillis and Boyce, [55]	Anisotropic yield criteria
Gotoh, [56]	Anisotropic yield criteria
Budiansky, 2013	Anisotropic yield criteria
F. Barlat et al., [57]	Anisotropic yield criteria
Banabic, Balan, and Comsa, [58]	Anisotropic yield criteria
Cazacu and Barlat, 1994	Anisotropic yield criteria
Carleer, Meinders, and Vegter, [59]	Anisotropic yield criteria
Vegter and Van Den Boogaard, [60]	Anisotropic yield criteria
Hu, [61]	Anisotropic yield criteria
D. S. Comsa and Banabic, [62]	Anisotropic yield criteria
Soare, [63]	Anisotropic yield criteria
Banabic, Aretz, Comsa, and Paraianu, [64]	Anisotropic yield criteria
D. Comsa and Banabic, [65]	Anisotropic yield criteria
Version, 1997	Anisotropic yield criteria
Emmens, [66]	Formability
Goodwin, [67]	Formability
Keeler, [68]	Formability
Havranek, [69]	Formability
Arrieux, [70]	Formability
Manoj Simha, Gholipour, Bardelcik, and Worswick, [71]	Formability
Janssens, Lambert, Vanrostenberghe, and Vermeulen, [72]	Formability
Blek, Deng, Papamantellos, and Gusek, [73]	Formability
Hu, Ma, Liu, and Zhu, [74]	Formability
Zimniak, [75]	Formability
Berstad, Lademo, Pedersen, and Hopperstad, [76]	Formability
Samuel, [77]	Formability
Duan, Jain, and Wilkinson, [78]	Formability
Takuda, Ozawa, Hama, Yoshida, and Nitta, [79]	Formability
Hajian and Assempour, [80]	Formability
Hamid Baseri, Rahmani, and Bakhshi-Jooybari, [81]	Springback
Tekiner, [82]	Springback
Moon, Kang, Cho, and Kim, [83]	Springback
Gomes, Onipede, and Lovell, [84]	Springback
Li, Carden, and Wagoner, [85]	Springback
Nakamachi et al., [86]	Springback
H. Baseri, Rahmani, and Bakhshi-Jooybari, [87]	Springback
Hamid Baseri et al., [81]	Springback
Jamli, Ariffin, and Wahab, [88]	Springback
Ank and Barauskas, [89];	Springback
Banabic, [11]	Springback

Table 1 (continued)

Affiliation	Focus area
Xu, Ma, Li, and Feng, [90]	Springback
Geiger and Vollertsen, [91]	Laser forming
Shen and Vollertsen, [92]	Laser forming
Shichun and Jinsong, [93]	Laser forming
Ji and Wu, [94]	Laser forming
Shichun and Zhong, [95]	Laser forming
Hoseinpour Gollo, Mahdavian, and Moslemi Naeini, [96]	Laser forming
Kheloufi and Amara, [97]	Laser forming
Shichun and Jinsong, [93]	Laser forming
Ji and Wu, [94]	Laser forming
Shichun and Zhong, [95]	Laser forming
Venkadeshwaran, Das, and Misra, [98]	Laser forming
Y. J. Shi, Shen, Yao, and Hu, [99, 100]	Laser forming
Yu, Masubuchi, Maekawa, and Patrikalakis, [101]	Laser forming
Zohoor and Zahrani, [102]	Laser forming
Gollo, Naeini, and Arab, [103]	Laser forming
Pitz, Otto, and Schmidt, [104]	Laser forming
Che Jamil, Sheikh, and Li, [105]	Laser forming
Lambiase, [106]	Laser forming
Paramasivan, Das, and Misra, [107]	Laser forming
Guan, Sun, Zhao, and Luan, [108]	Laser forming
J, S, and L, 2011	Laser forming
Y. Shi, Yao, Shen, and Hu, [99, 100]	Laser forming
Leszak, [109]	ISF
Jeswiet et al., [110]	ISF
Emmens, Sebastiani, and van den Boogaard, [111]	ISF
Kumar and Kumar, [112]	ISF
Iseki, [113]	ISF
Shim and Park, [114]	ISF
Hirtl and Germany, n.d.	ISF
Sebastiani, Brosius, Tekkaya, Homberg, and Kleiner, [115]	ISF
Yamashita, Gotoh, and Atsumi, [116]	ISF
Lequesne, Henrard, and Bouffioux, [117]	ISF
Hadoush and van den Boogaard, [118]	ISF
Ben Ayed, Robert, Delamézière, Nouari, and Batoz, [119]	ISF
Zhang, Lu, Chen, Long, and Ou, [120]	ISF

designing and developing of robust, efficient and accurate solution methods and algorithms.

2.2.1 Formulation and solution strategies

Literatures [4–6, 8, 11, 12, 25] discussed different solution methods in simulation of sheet metal forming. Makinouchi [5] classified formulation into three main categories, which are dynamic explicit, static explicit, and static implicit formulation. Further, solution strategies were categorized into three, which are incremental method, large step method and one-step method. Thus, in FEM codes the various FEM formulations are combined with different solution strategies. In summary, one can classify solution methods into five different categories. They are static explicit method, dynamic explicit method, static implicit incremental method, static implicit large step method, and static implicit one-step method. Characteristics of these methods were studied in comparison to one another and reported in [26, 27].

Unlike others, Banabic [11] classified simulation approaches into static implicit (solid) approach, static explicit approach, flow approach, rigid-plastic approach, and dynamic explicit approach based on description of motion and constitutive relations.

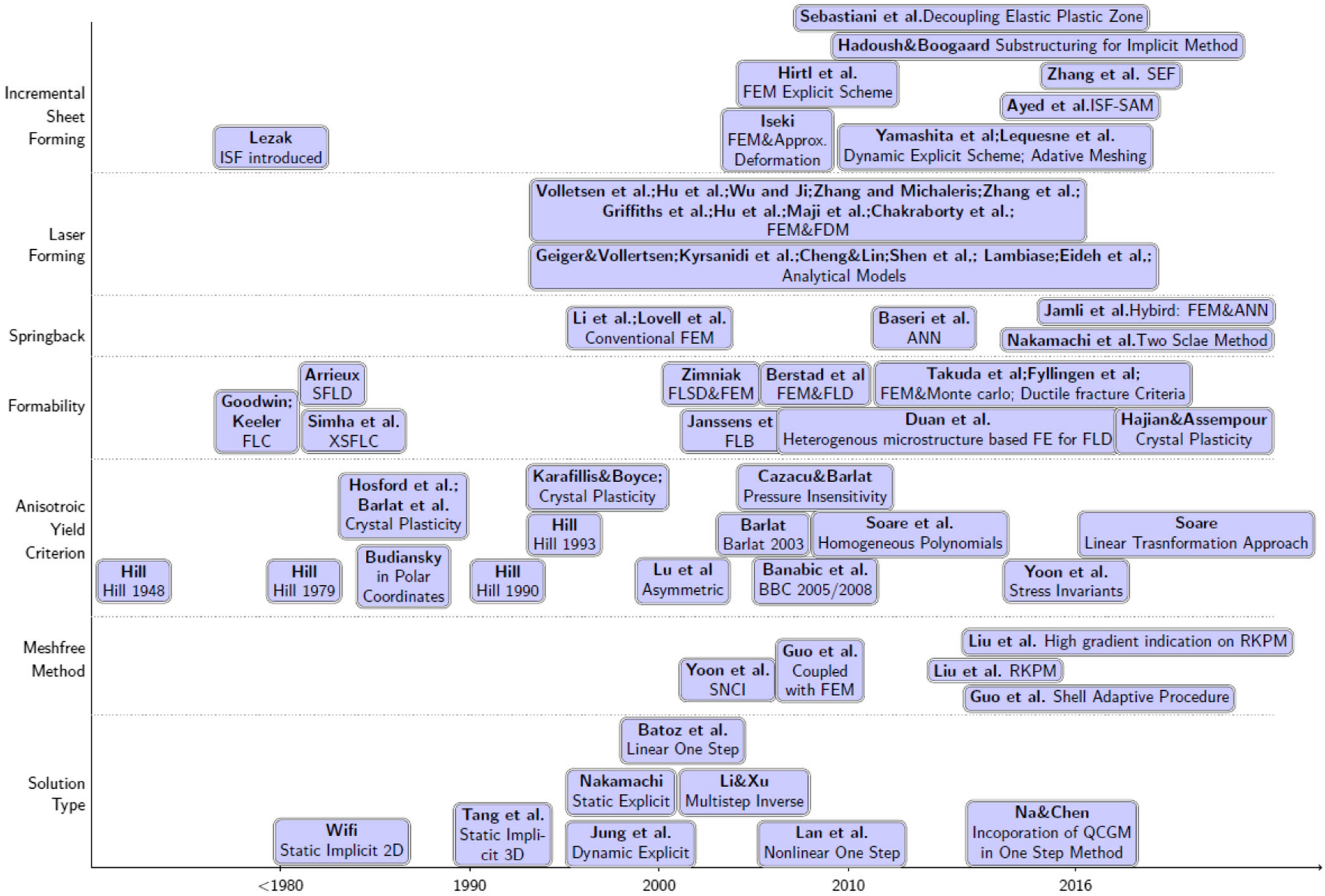

Fig. 1 A visual representation for the surveyed literature reported in this manuscript classified into categories and year of publication

2.2.2 Implicit method

The implicit method uses an iterative procedure in solving linear systems. It yields accurate result for the simulation and is unconditionally stable. Consequently, large time steps can be set in the simulation process. Due to the iterative procedure in solving process, it requires large memory and long computation time. Convergence is difficult to achieve when a large number of elements are involved in the deformation.

2.2.3 Explicit method

Explicit method, on the other hand, is quick in computation and less memory demanding. This method can easily parallelize and convergence is easy to achieve. The disadvantage is, however, that it is stable only under certain condition.

2.2.4 Static implicit method

In static implicit method, also known as solid approach, repeated solutions of large linear systems is required to guarantee the equilibrium at each incremental step. Simulations in early stages of sheet metal forming were solved using this method [6, 16–17, 20, 28], since static implicit method is quite accurate and efficient for 2D problems. When expanding simulations from 2D to 3D, static implicit method brought the disadvantages mentioned earlier. The disadvantages of this method were overcome by static explicit method where a system of equations is integrated using forward Euler scheme [4, 6, 12, 29, 30].

2.2.5 Dynamic explicit method

Dynamic explicit method is an inertia-based process where static equilibrium is not required. However, the dynamic explicit method has conditional stability and mass or time scaling procedure need to be employed [31, 32]. Thus, it assumes high velocity and acceleration, which is rather unrealistic since in most sheet metal forming process inertia can be neglected [33]. Finn et al. [34] and Micari et al. [35] purposed coupled method to exploit the advantages of both explicit and implicit methods. In this coupled approach, the explicit method is used to simulate forming process while the implicit method is used for springback simulation.

2.2.6 *One-step (inverse) method*

Afterward, one step approach was developed where a single time step was used and initial blank sheet metal was obtained from the final deformed shape of the sheet. It is based on assumptions like linear strain path, neglecting history of contact, ignoring friction, etc. This approach requires short computation time [6, 36–38]. Lan et al. [39] extended this approach to nonlinear problems. Based on the one-step approach Tang, Kim et al. [40] developed multistep inverse method. Li and Xu [41] proposed a multistep inverse method for simulation of the stamping process. One-step analysis is carried out by comparing the node position, initial blank thickness and strain distribution of initial configuration to that of final configuration. Multistep analysis is basically an extension of the one-step approach repeatedly between two subsequent steps. In each step, current state is set to final configuration and former step is set to initial configuration. This process is repeated until reaching final desired configuration. Different implementation of one-step approach in sheet metal forming was investigated by Azizi [42] in terms of solution type, equations solving time and speed of convergence. Na and Chen [43] incorporated quasi-conjugate-gradient method in the one-step method.

As simulation techniques have been increasingly gaining attention in industrial application, commercial software were developed correspondingly. LS-DYNA, ABAQUS/Explicit are the most widely used general purpose FE software in sheet metal forming simulation while specialized sheet metal forming simulation software such as AutoForm, PAMSTAMP, OPTRIS, INDEED, MTLFROM, and STAMPACK were also widely used [11]. Despite significant improvement in computing power of current computers, the simulation results still do not satisfy the industrial requirement. Demand for high accuracy in industrial applications is directing further investigations to improve static implicit method in the long run [10].

2.2.7 *Element selection*

Element type and formulation is also one of the main factors influencing accuracy of simulation results in FEM. Thus, different element type was used in simulation in various problems. Membrane [18], shell (thin/thick) and continuum (solid) elements were used in FEM. Membrane elements are limited in cases where the bending radius of sheet is larger than 20 times of sheet thickness. Therefore, deep drawing processes are simulated with shell elements instead of the membrane element [6]. But shell elements are not good choice for simulations in which predictions in through thickness plastic deformation are required [44]. Continuum element is capable of describing deformation in thickness directions and thus they are good for simulation of hydroforming and blanking [30]. For this reason, 3D solid and solid-shell elements were developed for simulation of sheet metal in cases where through thickness information is critical [44, 45]. Menezes and Teodosiu [46] developed 3D isoparametric elements with selective reduced integration to simulate the deep drawing process and it was proven that the result was in good agreement with experiments. Ponthot and Papeleux [47] compared enhanced assumed strain elements with other elements like selective reduced integration and selective uniform integration in springback simulation. Another example of using 3D solid or solid-shell elements were presented in the literature [48, 49]. In addition, eight-node hexahedron elements and four-node tetrahedral elements were used to simulate the sheet metal forming [46, 50]. Recently feasibility of applying tetrahedron-MINI elements in simulation of single and multi-layer sheet metal forming were presented by Chung et al. [44].

2.3 Meshfree method

Along with finite element method, since 2000 meshfree (meshless) method was also used to simulate sheet metal forming process [51, 52]. Chen, Yoon, Wu, and Wang [51] presented a meshfree formulation for metal forming simulation by introducing stabilized conforming nodal integration (SCNI) method, which overcame the disadvantages of Galerkin-based meshfree method that it has high CPU cost. They showed examples of flanging operation and springback effect in the operation.

Chen and Yoon [53] extended SCNI to history dependent problems and implemented their results in cylindrical punch and springback in flinging. To maintain the advantages of both FEM and meshfree method, Coupled FEM/Meshfree simulation were proposed by Guo, Wu, and Borkin [54]. Liu [55] presented deep drawing and hemisphere drawing processes using meshfree method to reproduce the kernel particle method (RKPM) to show the effectiveness of meshfree method. Sidibe and Li [56] presented draw bending of sheet metal using the reproducing kernel particle method (RKPM). Liu et al. [57] studied springback using adaptive multiple scale meshless simulation, where RKPM was adopted to analyze springback in two scales. High and low components of effective strain were obtained by integrating the decomposed high and low scales of RKPM shape function into a nonlinear elasto-plastic formulation. Once high-strain areas identified, the proper node refinement scheme was implemented to accurately calculate stress and thus predict springback. They also made a comparison between experiment, FEM (ABAQUS), meshless method, and adaptive meshless method and showed that results from adaptive meshless method was the closest to experiment results. Liu and Fu [58] showed deep drawing process simulation implementing high gradient indicator on RKPM based adaptive multi-scale meshless algorithm to achieve higher accuracy. Guo, Wu, and Park [59] developed meshless shell adaptive procedure for sheet metal forming simulations.

3 Anisotropy and yield criteria

The plastic deformation is the main concern in the sheet metal forming process. Due to anisotropy, which is caused by crystallographic structure and the characteristics of the rolling process [11]. In sheet metal forming simulation, plastic constitutive models and yield criteria, which depict anisotropic material behavior, are the starting point in studies.

There has been extensive effort on the development of anisotropic yield criteria. Numerous yield criteria have been proposed to address various aspects of the problem. Banabic [11] classified the yield criteria into several families, which are classical yield criteria, advanced anisotropic yield criteria, Banabic-Balan-Cosma (BBC) yield criteria, BBC 2005 yield criteria, and BBC 2008 yield criteria. These families are further divided into sub-families as listed in Table 2.

3.1 Classical yield criterion

This family includes Hill's family yield criteria, yield functions based on crystal plasticity (Hershey's family), yield criteria expressed in polar coordinates. Hill's family has four yield criteria, which are Hill 1948 [60], Hill 1979 [61], Hill 1990 [62] and Hill 1993 [63]. Crystal Plasticity based yield criterion are Hosford yield criterion [64], Barlat 1989 yield criteria [65], Barlat 1991 [66], Barlat 1994 and 1996 [67, 68], Karafillis–Boyce yield criterion [69]. Yield criteria in polar coordinates are Budiansky yield criteria [70]. Other than these, Gotoh [71] also proposed a yield criterion in classical yield criterion category.

3.2 Advanced anisotropic yield criterion

This family includes Barlat yield criterion [72], Banabic–Balan–Comsa (BBC) yield Criteria [73], Cazacu–Barlat yield Criteria [74], Vegter yield criteria [75, 76], Hu yield criteria [77], Cosma yield criteria [78], Soare yield criteria [79], and Wang yield criteria [80].

In addition, there are BBC 2005 yield criterion [81], BBC 2008 yield criterion [82], which was developed to enhance the flexibility of BBC 2005.

Recent development on anisotropic yield criterion are by Soare and Barlat [83], Desmorat and Marull [84], Taherizadeh et al. [85], Gawad et al. [86].

Among all these yield criteria, the widely used anisotropic yield criterion is the one proposed by Hill [60] due to its mathematical simplicity; only four mathematical coefficients are needed in identification. However, it fails to describe

Table 2 Classification of anisotropic yield criteria

Families		Yield criteria
Classical yield criteria	Hill's family yield criteria	Hill 1948
		Hill 1979
		Hill 1990
		Hill 1993
	Hershey's family (crystal plasticity) yield criteria	Hosford
		Barlat 1989
		Barlat1991
		Barlat 1994
		Barlat 1996
		Karafillis–Boyce
	Yield criteria in polar coordinates	Budiansky
	Others	Gotoh yield criteria
Advanced anisotropic yield criteria		Barlat yield criterion
		Banabic–Balan–Comsa (BBC)
		Cazacu–Barlat
		Vegter
		Hu
		Cosma
		Soare
		Wang
		BBC 2005
		BBC 2008

uniaxial yield stress and uniaxial coefficient of plastic isotropy. Hill 1990 [62], Barlat 1989 [65] are also used by many.

These three yield criteria were implemented in many commercial sheet metal forming simulation software like Abuqus, AutoForm, LS_DYNA, OPTRIS, PAMSTAMP and STAMPACK. The mathematical formulation of these respective yield criteria, comparison, advantage, and disadvantage is discussed in detail by Banabic [11] and Nielsen [87].

3.3 Asymmetric anisotropic yield criterion

Another class of yield criterion that are being actively developed now is the asymmetric yield criterion.

Unlike symmetric yield criterion in which the yield surface is initially symmetric to origin of stress space, asymmetric yield criterion considers the situation where the yield surface is not symmetric with respect to origin of stress space initially or due to plastic deformation. There are two factors, pressure dependence and intrinsic asymmetry, that contribute to asymmetry. Works for incorporating asymmetry were done by Liu et al. [88], Cazacu and Barlat [89], Soare, Yoon, and Cazacu [90], Plunkett et al. [91], Yoon et al. [92], and Soare [93].

4 Formability and simulation

Formability is the ability of sheet metal to withstand deformation before fracture occurs [94]. Formability of sheet metal can be described in forming limit diagram/curve (FLD/FLC), which is determined by measuring two principle strains ε_1 and ε_2 at failure [11]. Normal anisotropy, which is referred to as the ratio between width-strain and thickness-strain, and work-hardening coefficient are the two factors influencing formability [94].

This concept, FLC, was originated by Goodwin [95] and Keeler [96] in 1960s. Later wrinkling limit diagram [97] and the stress forming limit diagram [98] (SFLD) were proposed based on the concept of FLC. Then stress limit curve were extended to 3D stress state by extended stress-based limit curve (XSFLC) [99]. There are conditions for FLCs to be valid, which include no bending, straight strain path, plain strain and no shear stress in the test. In addition, FLC is specific to material type and temper or batch. Janssens [100] proposed a more general approach, i.e., the forming limit band (FLB), to overcome the deficiency of FLC in describing forming. Formability is discussed in detail by Wilko [94].

Bleck et al. [101] presented a comparative study of different forming limit diagrams. At an early stage, FLC was served as an instrument to study industrial metal forming operations; the idea was to compare the actual strain state of a product to the FLC of the particular material. Later mismatches were found in the comparison. As a result, in the 1990s, FLC was studied more in the direction of developing and verifying material models rather than its role in 1960s [94]. Currently, the formability analysis has become integral parts of design and manufacturing in industrial practice [102].

Numerical simulation approaches have been proposed for predicting FLDs and FLSDs, since numerical approaches saves costly experimental trial and error investigations and shortens the time-consuming design process. Zimniak [103] showed implementation of FLSD into FE package Mark using perturbation theory. Berstad et al. [104] presented FEM using a FLD calculator using LS-DYNA. The FEM based calculation was found to be in good agreement with the analytical and experimental results.

Samuel [105] predicted FLD, FLSD and thickness distribution of a deep drawing quality steel (DDQS) using FEA software MARCK7.1-3D. In this study, the numerical model was based on rigid-plastic method, taking into account planar anisotropic value, the material properties, blank shape, and coefficient of friction. Duan, Jian, and Wilkinson [106] showed a heterogeneous microstructurally based FE model for the prediction of FLD. This approach was developed to treat the effect of local microstructure like texture, grain size, and particle distributions on the macroscale response of the structure on the FLD. To achieve this, a two-scale model was used where particle size and position were measured by image analysis software, particle field was generated by preprocessing of FE software, stress-strain curve was obtained by micromechanical analysis and then the curve was assigned to elements in structural models. Elements of structural model that are close to predicted intense shear band chosen for extracting strain limit curve. Different strain path was considered by changing either specimen geometry or loading ratio. The simulation was implemented using general FEA package MSC.MARC based on von Mises yield criteria.

Takuda et al. [107] demonstrated possibility of predicting FLD using the ductile fracture criterion proposed by Cockcroft and Latham in dynamic explicit FE software LS_DYNA ver.970. Fyllingen et al. [108] showed a new approach for predicting FLD using FEM and Monte Carlo simulation. This approach, known as stochastic finite element-based approach, FEA was used to simulate a square patch of material subjected to a set of proportional strain paths. Inhomogeneities were related to variation of special thickness of material assuming weaker zones represent thinner thickness and stronger zones represent thicker thickness. Thickness variation was realized using the Matérn Covariance function in which change of a parameter would lead to different field realizations. For each realizations, there exists difference forming limits. In next step Monte Carlo simulation could generate stochastic forming limit diagram. Hajian and Assempour [109, 110] presented crystal plasticity approach for predicting FLD of 1010 steel sheet. Material behavior was obtained using a user material (UMAT) subroutine in which rate dependent crystal plasticity model was incorporated with power law hardening. For determination of FLD, second-order derivative of sheet thickness variation with respect to time was set to necking criteria.

5 Springback

Springback refers to the phenomenon of elastic recovery of sheet metal upon unloading during a forming operation. This is a critical issue since it is directly involved with the precision of forming process [111]. Comprehensive reviews of springback were presented in Literature [112–114]. Springback has been studied using numerical simulation along with experiments. Tekiner [115] presented an experimental study of springback in which the amount of springback of several sheet metals with different bending angles was obtained on a modular V bending die. Moon, Kang, Cho, and Kim [116] experimentally validated that springback could be reduced by 20 % through using hot die and cold punch. Lovell et al. [117] presented numerical studies based on various yield criteria and compared the results with experiments to show that springback varies with orientation of anisotropic sheet in the U-die bending process. Springback in the bend test over various variables were simulated for three typical sheet alloys [118]. Nakamachi et al. [119] developed a two-scale finite-element procedure for springback evaluation based on the crystallographic homogenization method, where an elastic plastic continuum procedure could predict anisotropic plastic deformation of sheet metal in macroscale while a microscopic polycrystal structure could predict the crystal texture and hardening evolutions in the microscale. Though these numerical, experimental studies provided important information, they involved numerous simplification and assumptions, which is critical to the success of these approaches.

Other methods are developed to overcome the drawbacks of experimental and numerical ones using artificial neural network (ANN) [111, 120]. Though ANN can reduce the number of experiments and replace finite element analysis, size of input data is critical in ANN to achieve accurate prediction. Jamli et al. [121] proposed hybrid approach for more accurately predicting springback, where artificial neutral network (ANN) and finite element are combined.

Parameters studied to investigate the influences of springback are nodal transient softening of hardening curve, reduction of Young's modulus, damping values, number of integration points, element size, punch velocity, blankholder force (BHF), friction coefficient, die gap, time step, material model, and drawbead model [11, 122, 123].

Finite element formulation has significant impact on simulation results; there are two types of elements that are used for simulation. One is bending enhanced membrane (BEM) and elastic-plastic shell (EPS). To achieve accurate simulation results, element size, which is equal to 1/2–1/3 of relative bending radios, are required. Further refinement in element size may increase computation time, the cost of contact search algorithm and matrix solution without substantially improving the simulation result. Gauss integration, Gauss-Simpson combined integration and Lobatto integration are the recommended integration method for simulation of springback.

Though smaller time step, generally, increases the simulation accuracy, a time step less than 2.5 s (provided tool velocity is 1 mm/s) concentrates only small band area and achieves no further improvements in simulation [11].

6 Simulation of non-conventional forming methods

Progress in manufacturing technology brought novel forming approaches like laser forming and incremental forming into exist. In this section, simulation of this non-conventional sheet metal forming processes is discussed.

6.1 Laser forming simulation

Laser forming, as one of the special approach of sheet metal forming, started in 1980s. Laser forming is to utilize thermal stresses irradiated by laser to achieve the structural shape in sheet metals [124]. Therefore, it is a thermo-mechanical process. There is no springback effect in laser forming, which is one of the advantages over stamping. Research on laser forming was started in mid-1980s. Three mechanisms were proposed by Geiger and Vollertsten [124] to explain laser bending process. They are temperature gradient mechanism (TGM), bulking mechanism (BM) and upsetting mechanism (UM) [125]. Other analytical models for laser forming process are presented in [126, 127–130].

Experimental and numerical approaches always went hand in hand in investigating parameters that influence the laser bending of sheet metals. Numerical analysis can be classified into two main groups, which are analysis using FEM/FEM, analysis using soft computing techniques. FEM/FDM simulation attempts are by Volletsen et al. [131], Hu et al. [132], Wu and Ji [133], Zhang and Michaleris [134], Zhang et al. [135], Griffiths et al. [136], Hu et al. [137], Maji et al. [138], and Chakraborty et al. [139].

Soft computing techniques are also used in analyzing the laser-forming process. Among them are back propagation neural network technique [140–142], neural network related techniques [143–145], fuzzy logic [146–149], response surface method [150], synthesis method [151], minimization technique [152].

Many researchers studied parameters in laser forming process. Wu and Zheng [153] investigated energy parameters, material parameters and geometry parameters for laser bending of sheet metal. Wu and Zhong [154] presented finite element simulation of temperature field during the laser forming process of sheet metal. Gollo et al. [155] presented a study in which effect of parameters such as material, laser power, beam diameter, scan velocity, sheet thickness, pass number and pulse duration on bending angle were investigated using simulation techniques and results are compared with experiments [153, 155, 156]. The topics researched included, but not limited to, temperature filed [154], deformation field [133, 157], temperature gradient mechanism for high accuracy bending [158], finite element model for thermo-mechanical forming of sheet metal [159], influence of scanning strategies on bending angle [160], relationship between temperature distribution and bending angle [161], moving mesh strategies on laser path to reduce computation time [162], impact of beam geometry on bending of sheet metal under buckling mechanism [163], analytical model for bending angle [129], and effect of cutouts on laser forming of plates [164]. Many parameters have impact on laser forming process, including laser power density, wavelength, temporal energy variation, speed motion relative to the work piece, absorption coefficient and thermal conductivity etc. [125, 155]. Large bending angle can be produced with the material with lower Young's modulus as well as from a material with lower specific heat and density. It is also proportional to thermal expansion coefficient. Increase in the heat conductivity decreases bending angle [165]. From the component geometry point of view, wider beams produce larger bending radius and narrower beams generate smaller bending radius. Square beam produce highest bending angle compared with rectangular and triangular beams [166]. Based on buckling critical condition, bending direction of the plate can be exactly determined [167].

7 Incremental sheet forming

Incremental sheet forming (ISF) was firstly proposed by Lezak [168] in 1967. But it was not implemented until a decade ago when Jeswiet [169] presented state-of-the-art incremental sheet forming fundamental concepts in ISF. The development history of incremental sheet metal forming are discussed in detail [170]. ISF can be classified into conventional incremental sheet forming (CISF) and hybrid incremental sheet metal forming (HISF). CISF is further divided into single-point incremental forming (SPIF) and two-point incremental forming (TPIF). HISF is classified into single-point incremental hydroforming, TPIF with partial die, TPIF with full die [171], see Table 3.

From simulation point of view, Iseki [172], Shim and Park [173] presented FEA for simple tool path for incremental sheet forming. Since contact area changes as the tool moves along the path, the computation time for simulation is long and FE model requires fine mesh. Hirtl et al. [174] presented FEA using explicit scheme for investigation of limitation on the maximum achievable wall angle and the occurrence of geometric deviations. In their work, they proposed multi-stage forming instead of single stage and correction algorithm to increase accuracy. Sebastiani et al. [175] presented a decoupled simulation method for ISF in which the decoupling algorithm separated the domain into an elastic deformation zone and an elastoplastic deformation zone in order to reduce long computation time for simulation. Yamashita et al. [176] showed numerical simulation of deformation behavior in incremental sheet forming process using dynamic explicit method and different tool paths were tested to see effect of tool path on deformation behavior of sheet. In order to reduce long computation time in implicit method, Lequesne et al. [177] showed implementation of adaptive re-meshing where elements near to tool were divided into small elements to have fine mesh at the location where high deformation occurs. Hadoush and van den Boogaard [178] presented substructuring method for implicit scheme in order to reduce computation time. Substructuring method divides mesh into plastic-nonlinear-structures and elastic-pseudo-linear-substructures assuming that plastic deformation is localized. The plastic part, which is in contact with tool, is iteratively updated while elastic part models the elastic deformation. Recently, Ayed et al. [179] presented a novel numerical approach called ISF-Simplified Analysis Modeling (ISF-SAM) for ISF. In ISF-SAM, a simplified contact procedure was proposed to predict nodes, which are in contact with tool, and to estimate their displacement. A Kirchhof triangle shell element called DKT12 was used considering membrane and bending effects. Elastoplastic material model was exploited for material law and nonlinear equilibrium equation was solved using static scheme. Result showed that CUP time was reduced more than 60 %. Zhang et al. [180] presented selective element fission (SEF) approach based

Table 3 Classification of incremental sheet forming

Main types	Sub types
Conventional incremental sheet forming (CISF)	Single-point incremental forming (SPIF) Two-point incremental forming (TPIF)
Hybrid incremental sheet metal forming (HISF)	Single-point incremental hydroforming (SPIHF) TPIF with partial die TPIF with full die

on LS-DYNA where a background mesh was introduced for simulation data storage and separate simulation mesh with varied mesh density for simulation to reduce unnecessary calculation.

Nomenclature

ANN	Artificial Neural Network
BM	Bulking Mechanism
BBC	Banabic-Balan-Cosma
BHF	Blank Holder Force
BEM	Bending Enhanced Membrane
CISF	Conventional Incremental Sheet Forming
DDQS	Deep Drawing Quality Steel
EM	Finite Element Method
EPS	Elastic–plastic Shell
FED	Finite Difference Method
FLC	Forming Limit Curve
FLD	Forming Limit Diagram
FLB	Forming Limit Band
FLSD	Forming Limit Stress Diagram
HISF	Hybrid Incremental Sheet Metal Forming
ISF	Incremental Sheet Forming
ISF-SAM	ISF-Simplified Analysis Modelling
RKPM	Reproducing Kernel Particle Method
SCNI	Stabilized Conforming Nodal Integration
SFLD	Stress Forming Limit Diagram
SPIF	Single-Point Incremental Forming
SEF	Selective Element Fission
TGM	Temperature Gradient Mechanism
TPIF	Two-Point Incremental Forming
UMAT	User Material
UM	Upsetting Mechanism
XSFLC	Extended Stress-Based Limit Curve

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