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ORIGINAL ARTICLE

Variable sampling plan for resubmitted lots based on process capability index and Bayesian approach

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Abstract Acceptance sampling plans are applied for quality inspection of products. Among the design approaches of sampling plan, the most important one is to use process capability indices in order to improve the quality of manufacturing processes and the quality inspection of products. But, selection of estimators of process capability index and their sampling distribution is very important. Bayesian statistical technique can be used to obtain the sampling distribution. In this paper, a variable sampling plan is developed for resubmitted lots based on process capability index and Bayesian approach. In the proposed sampling plan, lots are inspected several times depending on the quality level of the process. In addition, this paper presents an optimization model for determining the decision parameters of developed sampling plan with regards to the constraints related to the risk of consumer and producer. Two comparison studied have been done including: First, the methods of double sapling plan (DSP), multiple dependent state (MDS) sampling plan, and repetitive group sampling (RGS) plan are elaborated, and also in order to comparing developed sampling plans, an expected number of products as average sample number (ASN) is used for different developed plans; second, a comparison study between Bayesian approach and exact probability distribution is carried out and their results are analyzed. It is observed that the ASN values of MDS sampling plan is less than ASN values of other methods, and also the ASN values of different variable sampling plans based on Bayesian approach is less than ASN values obtained using exact approach.

Keywords Process capability index · Resubmitted lot · Average sample number (ASN) · Bayesian approach · Exact approach

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1 Introduction

Due to the influence of many factors in manufacturing processes, it is not possible to produce products without any fault or to control these factors completely. Therefore, the quality of products should be controlled. To produce a product with proper quality, some indexes should be measured for quality control in all stages of production. One type of these indexes is referred as process capability index. Since the application of process capability index has increased, thus the selection of estimators and distribution of these estimators will be very important. Bayesian statistical technique can be used to obtain the distribution of these estimators. This technique specifies a priori distribution function for the given parameters and then forms a posterior distribution function for these parameters using collected data. One application of process capability index is to use them in the context of lot acceptance sampling plan to make decision about received lot from supplier or finished goods in production environments so that the risk of producer and consumer falls within standards [1].

Therefore, the decision makers must be familiar with appropriate sampling plans and then consider the best methods to make a decision about lot. One common application of sampling plan is when a supplier sends a lot to a company. Usually, after receiving this lot, a sample is selected and desired quality characteristic is inspected.

Based on the information obtained from this sample, a decision is made to reject or accept the lot. Accepted lots are used in production process and rejected lots are returned to the supplier or inspected by other methods [2].

Among sampling procedures for lot sentencing problem, variable sampling plan for resubmitted lot is very important. Using this plan is more difficult but it has better performance in comparison with classical sampling plan [3]. Meanwhile, variable sampling plan for resubmitted lots was first developed by Govindaraju and Ganesalingam [4]. In this sampling method, lot inspections may be repeated, in the several stages. A repetitive group sampling plan was designed by Sherman [5]. Repetitive mixed sampling plan based on the process capability index is proposed by Aslam et al. [3, 6, 7]. Their plan is applicable for the inspection of the products whose lifetime follows the normal distribution. Balamurali and Jun [8, 9] proposed repetitive group sampling procedure for variables inspection and they also designed variables repetitive group sampling plan indexed by point of control. Aslam et al. [7, 10] proposed a mixed repetitive sampling plan based on process capability index and using process capability index of multiple quality characteristics, respectively.

Soundararajan and Vijayaraghavan [11] proposed a method for designing multiple dependent (deferred) state sampling plans. Vaerst [12] introduced a procedure to construct multiple deferred state sampling plans. A new MDS sampling plan based on process capability index has been proposed by Aslam et al. [3, 6, 7]. Also, a new method of MDS sampling plan for lot acceptance problem is introduced by Balamurali and Jun [13]. An optimal double-sampling plan based on process capability index is proposed by Fallah Nezhad and seifi [14] in order to reduce the average sample number, time, and cost of sampling. Also in previous decades, variable sampling plans have been commonly developed by Moskowitz and Tang [15], Tagaras [16], Arizono et al. [17], Fallah Nezhad and Hosseini Nasab [18], Wu [19], Wu et al. [20], Miao et al. [21], Pearn and Wu [22], Yen and Chang [23], and Negrin et al. [24].

In this paper, a variable sampling program for resubmitted lots is developed based on process capability index using Bayesian approach. Also, the optimal parameters of developed sampling are determined based on the constraints related to the risk of consumer and procedure. Also, a comparison study is carried out between average sample numbers of different sampling plan and the results are analyzed. The main contributions of this research are as following:

(i) Developing a sampling plan for resubmitted lots based on process capability index and Bayesian approach.

(ii) Different variable sampling plans (which includes single sampling plan (SSP), DSP, RGS plan and MDS sampling plan) are developed based on Bayesian approach.

(iii) The developed sampling plans are compared and the optimal plan is determined.

(iv) The proposed sampling plans are compared based on two approaches (which includes Bayesian approach and Exact approach) and the suitable approach is analyzed.

2 Proposed variables sampling plan

2.1 Procedure of variable sampling plan for resubmitted lots

As explained, the variable sampling plan for resubmitted lots has many applications. The parameters used in this sampling plan are as follows:

m Number of resubmissions
n Sample size
k_a Minimum acceptable value of process capability index

And its procedure is as follows:

Step 1. collect a sample with n observation from lot and compute $\hat{C}_{pk}$.

Step 2. if $\hat{C}_{pk} \geq k_a$ then accept the lot otherwise If the lot was not accepted after repeating the step 2 for m times then reject the lot else resubmit the lot and go to step 1.

Suppose that the quality variable X follows a normal distribution with a mean of μ and variance of σ^2. Process capability index is defined as follows:

$$C_{pk} = \min\left\{\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right\} = \frac{d-|\mu-M|}{3\sigma} \qquad (1)$$

Where LSL and USL are lower and upper specification limit, $d=(USL-LSL)/2$ is half distance between USL and LSL, and $M=(USL+LSL)/2$ is the midpoint of specification limits. Also, since the mean value of process is unknown, thus, the average of the observations can be used to estimate the mean value $(\overline{X})$.

2.2 Parameter estimation based on the multiple samples

In cases where m subsamples have been gathered and the studied quality characteristics of the process is normally distributed, so that the sample size of i_{th} subsample is equal to n_i, for each i and j, $i = 1,2,....,m$ and $j = 1,2,....,n_j$, we consider x_{ij} as the random observation from normal distribution with the mean μ and variance σ^2. We consider that the process is controlled by $\overline{X}$ and S control charts thus, assuming that during sampling, the process is under statistical control, for each subsample, then $\overline{X}_i$ and S_i^2 will be defined as the mean and variance of the i$_{\text{th}}$ sample are obtained as follows:

$$\hat{\mu} = \bar{\bar{x}}_i = \frac{1}{N}\sum_{j=1}^{n_j} n_i \bar{x}_i \quad , \quad S_i^2 = \frac{1}{n_i - 1}\sum_{j=1}^{n_j}\left(x_{ij} - \bar{x}_i\right)^2 \quad , \quad and\ N = \sum_{i=1}^{m} n_i \tag{2}$$

Assuming $N_1 = \sum_{i=1}^{m}(n_i - 1)$, we use the average of samples mean and accumulation of samples variance as an unbiased estimators of μ and σ^2, then the estimation of process capability index based on the observations of multiple samples is as follows:

$$\hat{\mu} = \bar{\bar{x}}_i = \frac{1}{N}\sum_{j=1}^{n_j} n_j \bar{x}_j \quad , \quad \hat{\sigma}^2 = S_p{}^2 = \frac{1}{N_1}\sum_{j=1}^{n_j}(n_i - 1)\, S_i{}^2 \tag{3}$$

$$\hat{C}^*_{pk} = \min\left\{\frac{USL - \bar{\bar{x}}}{3s_p}, \frac{\bar{\bar{x}} - LSL}{3s_p}\right\} = \frac{d - \left|\bar{\bar{x}} - M\right|}{3s_p} \tag{4}$$

Therefore, the estimator $\hat{C}^*_{pk}$ can be used in quality evaluation of produced lots.

2.3 Posterior probability distribution of process capability index

A Bayesian method was proposed by Pearn and Wu [22] to determine the probability distribution function of process capability index C_{pk} based on multiple samples, also they developed a Bayesian method to obtain a confidence interval for C_{pk} based on multiple samples. Therefore, the probability $p = \Pr(\text{the process is capable}|x)$ can be obtained using Bayesian approach and posterior probability distribution function of C_{pk}.

Then, the probability of having a capable process using posterior distribution of C_{pk} based on the specified threshold of $w > 0$, will be as follows [19]:

$$p = \Pr\{\text{the process is capable}|X\} = \Pr\{C_{pk} > w|X\} = \int_0^\infty \frac{1}{\Gamma(\alpha) y^{\alpha+1}} \exp\left(-\frac{1}{y}\right) \times \{\phi[b_1(y)] + \phi[b_2(y)] - 1\} dy \tag{5}$$

where $\phi(u) = \int_{-\infty}^{u} (2\pi)^{-1/2} \exp\left(-t^2/2\right) dt$ is the cumulative distribution function of standard normal distribution and other parameters will be as follows [19]:

$$\alpha = (N - 1)/2 \quad , \quad \delta = \frac{\left|\bar{\bar{x}} - M\right|}{s_p} \tag{6}$$

$$\gamma = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n_j}\left(x_{ij} - \bar{x}_i\right)^2/2}{\sum_{i=1}^{m}\sum_{j=1}^{n_j}\left(x_{ij} - \bar{\bar{x}}\right)^2/2} = \frac{N_1 s_p^2}{N_1 s_p^2 + \sum_{i=1}^{m} n_i\left(\bar{x}_i - \bar{\bar{x}}\right)^2} \tag{7}$$

$$b_1(y) = 3\sqrt{N}\left(\hat{C}^*_{pk} \times \sqrt{\frac{2\gamma}{N_1 y}} - w\right) \tag{8}$$

$$b_2(y) = 3\sqrt{N}\left(\left(\hat{C}^*_{pk} + \frac{2}{3}\delta\right) \times \sqrt{\frac{2\gamma}{N_1 y}} - w\right) \tag{9}$$

According to proposed sampling plan, the OC function of the variables sampling plan for resubmitted lot at the give quality level (p) will be as follows [4]:

$$P_A(p) = 1 - (1 - P_a)^m \tag{10}$$

Where P_a is the acceptance probability in a single stage that is determined as the following:

$$P_a = P\left\{\hat{C}_{pk} \geq k_a\right\} = \int_0^\infty \frac{1}{\Gamma(\alpha) y^{\alpha+1}} \exp\left(-\frac{1}{y}\right) \times \left\{\begin{array}{l} \phi\left[3\sqrt{N}\left(C_{AQL} \times \sqrt{\frac{2\gamma}{N_1 y}} - k_a\right)\right] + \\ \phi\left[3\sqrt{N}\left(\left(C_{AQL} + \frac{2}{3}\delta\right) \times \sqrt{\frac{2\gamma}{N_1 y}} - k_a\right)\right] \end{array}\right\} dy \tag{11}$$

It is essential to note that when $m = 1$, then developed sampling plan would be similar to a single sampling plan. Therefore, sampling plan for resubmitted lot can be considered as a more general form of single-stage sampling plan. The average sample number of developed sampling plan is as follows [4]:

$$\text{ASN}(p) = \frac{n\left(1 - (1 - P_a)^m\right)}{P_a} \tag{12}$$

Now, to achieve optimal parameters of developed sampling plan and to minimize the average sample number, we try to solve an optimization problem by considering the constraints of type I error probability and type II error probability.

Producer needs that the probability of accepting the lot at the quality level of AQL would be more than $1-\alpha$, and the consumer wants that the probability of accepting the lot at the quality level of LQL would be less than β. Therefore, the optimization problem by considering the values of p_1,p_2 is as follows:

$$\text{Minimize ASN}(p) = \frac{n\left(1-(1-P_a)^m\right)}{P_a}$$
$$\text{subject to :}$$
$$P_A(p_1)\geq 1-\alpha \quad \text{and} \quad P_A(p_2)\leq\beta \tag{13}$$

where p_1 is equal to AQL and p_2 is equal to LQL. In this paper, the parameters of developed sampling plan can be obtained by minimizing the average sample number at the quality levels of C_{AQL} and C_{LQL}. Therefore, considering posterior distribution function of process capability index, optimization problem for specified values of C_{AQL} and C_{LQL} can be written as the following:

$$\text{Minimize ASN} = \frac{n\left(1-(1-P_a)^m\right)}{P_a}$$
$$\text{subject to :}$$
$$C = C_{AQL}\Rightarrow P_A\left(C_{AQL}\right)\geq 1-\alpha \quad \text{and} \quad C = C_{LQL}\Rightarrow P_A\left(C_{LQL}\right)\leq\beta \tag{14}$$

Where C_{AQL} is the minimum value of process capability index which is acceptable for the producer and C_{LQL} is the maximum value of process capability index that is not desirable for consumer. Finally, the above optimization problem can be solved using numerical simulation procedures and parameters of developed sampling plan can be obtained for specific values of C_{AQL} and C_{LQL}.

3 Double-sampling plan

Double-sampling plans are widely applied in production environments. At first, a primary sample is taken from lot and a judgment is done on it. In the case of not selecting any decision about the lot, a second sample is taken under certain conditions. Then, decisions are made on the lot. In the modified double-sampling model based on process capability index, the parameters are defined as follows:

- n_1 Sample size of the first sample
- k_1 The lower threshold of Process capability index for rejecting the lot based on the first sample
- n_2 Sample size of the second sample
- k_2 The upper threshold of Process capability index for accepting the lot based on the first sample
- k_3 The threshold of Process capability index for accepting the lot based on the second sample

Decision making is summarized as follows:

Step 1. Take the first sample with n_1 observations and compute $\hat{C}_{pk}$.
If $\hat{C}_{pk}\geq k_2$, Accept the lot and reject the lot if $\hat{C}_{pk}\leq k_1$ where $k_2>k_1$. If $k_1<\hat{C}_{pk}<k_2$, then take the second sample with n_2 observations.

Step 2. compute $\hat{C}_{pk}$ for the second sample. Accept the lot if $\hat{C}_{pk}\geq k_3$; otherwise, reject the lot.
Now, we try to formulate the optimization problem of acceptance sampling plan by considering the constraints of type I and type II errors in order to obtain the optimal value of required sample size by minimizing ASN. Therefore, the general equation for obtaining the ASN, concerning the posterior probability distribution function of C_{pk} will be as the following:

$$\min ASN = n_1 + n_2\left[\Pr\left(C_{pk} > k_1\right)-\left(C_{pk} > k_2\right)\right] \tag{15}$$

The above objective function is minimized by considering the constraints of C_{AQL} and C_{LTPD} as following,

$$\mathrm{C} = \mathrm{C_{AQL}}\Rightarrow \mathrm{P}\left(\hat{\mathrm{C}}_{\mathrm{pk}}\geq \mathrm{k_2}|\mathrm{n}=\mathrm{n_1}\right) + \mathrm{P}\left(\hat{\mathrm{C}}_{\mathrm{pk}}\geq \mathrm{k_3}|\mathrm{n}=\mathrm{n_2}\right) . \ \mathrm{P}\left(\mathrm{k_1}\leq\hat{\mathrm{C}}_{\mathrm{pk}}\leq \mathrm{k_2}|\mathrm{n}=\mathrm{n_1}\right)\geq 1-\alpha \tag{16}$$

$$\mathrm{C} = \mathrm{C_{LTPD}}\Rightarrow \mathrm{P}\left(\hat{\mathrm{C}}_{\mathrm{pk}}\geq \mathrm{k_2}|\mathrm{n}=\mathrm{n_1}\right) + \mathrm{P}\left(\hat{\mathrm{C}}_{\mathrm{pk}}\geq \mathrm{k_3}|\mathrm{n}=\mathrm{n_2}\right) . \ \mathrm{P}\left(\mathrm{k_1}\leq\hat{\mathrm{C}}_{\mathrm{pk}}\leq \mathrm{k_2}|\mathrm{n}=\mathrm{n_1}\right)\leq\beta \tag{17}$$

4 Proposed variables RGS model

Variables repetitive group sampling plan is one of the most effective sampling plans and its parameters are as following:

- n Sample size
- k_1 The lower threshold of Process capability index for rejecting the lot based on the single sample
- k_2 The upper threshold of Process capability index for accepting the lot based on the single sample

And its procedure, based on the process capability index is summarized as following:

Step 1. Select a sample with n observations from the lot.

Step 2. Accept the lot if $\hat{C}_{pk} \geq k_2$ and reject it if $\hat{C}_{pk} \leq k_1$ where $k_2 > k_1$. If $k_1 < \hat{C}_{pk} < k_2$, then repeat step 1 and step 2.

The OC function of the variables RGS plan is as follows:

$$P_a(C_{pk}) = \frac{P_a}{P_r + P_a} \tag{18}$$

Where P_a and P_r are the probabilities of accepting and rejecting the lot based on single sample.

The above equation can be rewritten as following:

$$P_a(C_{pk}) = \frac{P_a}{P_r + P_a} = \frac{\Pr\left(\hat{C}_{pk} \geq k_2\right)}{1 - \Pr\left(\hat{C}_{pk} \geq k_1\right) + \Pr\left(\hat{C}_{pk} \geq k_2\right)} \tag{19}$$

Therefore, based on the model of Balamurali and Jun [8] and quality levels of C_{AQL}, C_{LQL} and specified values of the producer risk, α and consumer risk β, the following constraints are obtained:

$$C = C_{AQL} \Rightarrow \frac{\Pr\left(\hat{C}_{pk} \geq k_2\right)}{1 - \Pr\left(\hat{C}_{pk} \geq k_1\right) + \Pr\left(\hat{C}_{pk} \geq k_2\right)} \geq 1 - \alpha \tag{20}$$

And

$$C = C_{LQL} \Rightarrow \frac{\Pr\left(\hat{C}_{pk} \geq k_2\right)}{1 - \Pr\left(\hat{C}_{pk} \geq k_1\right) + \Pr\left(\hat{C}_{pk} \geq k_2\right)} \leq \beta \tag{21}$$

Where in the first constraint, $\Pr\left(\hat{C}_{pk} \geq k_2\right)$, $\Pr\left(\hat{C}_{pk} \leq k_1\right)$, are the probabilities of accepting and rejecting the lot based on single sample at AQL point, respectively. Also in the second constraint, $\Pr\left(\hat{C}_{pk} \geq k_2\right)$ and $\Pr\left(\hat{C}_{pk} \leq k_1\right)$ are the probabilities of accepting and rejecting the lot based on single sample at LQL point, respectively.

The objective function of problem is to minimize the required average sample number and it is clear that the number of sampling stages is equal to $\left(\frac{1}{\Pr\left(\hat{C}_{pk} \leq k_1\right) + \Pr\left(\hat{C}_{pk} \geq k_2\right)}\right)$. In fact, the number of sampling stages is equal to the mean value of a geometric distribution which its success probability is equal to $\Pr\left(\hat{C}_{pk} \leq k_1\right) + \Pr\left(\hat{C}_{pk} \geq k_2\right)$. In each sampling stage, the sample size is equal to n. Thus, the average sample number, that is the objective function of problem, is determined as follows:

$$Min\,T = \frac{n}{\Pr\left(\hat{C}_{pk} \leq k_1\right) + \Pr\left(\hat{C}_{pk} \geq k_2\right)} \tag{22}$$

5 Proposed variable MDS sampling plan

MDS sampling plan is a type of conditioned sampling plans which considers sampling results of past or future lots. The parameters of MDS plan are defined as follows:

- n Sample size
- m Number of preceding lots
- k_1 The upper threshold of Process capability index for accepting the lot
- k_2 The lower threshold of Process capability index for rejecting the lot

And its procedures are as follows:

Step 1. Take a random sample of size n and compute process capability index, $\hat{C}_{pk}$.

Step 2. If $\hat{C}_{pk} \geq k_1$, then accept the lot and if $\hat{C}_{pk} \leq k_2$, then reject it. If $k_2 \leq \hat{C}_{pk} \leq k_1$, then if the value of process capability index in each of the samples taken from the m preceding lots is more than k_1 then accept the lot; otherwise reject the lot.

It is noted that if $k_1 = k_2$, then proposed sampling plan converts to the SSP.

The OC function of MDS sampling plan is as follows [13]:

$$P_a(C_{pk}) = \Pr\left\{\hat{C}_{pk} \geq k_1 | X\right\} + \Pr\left\{k_2 \leq \hat{C}_{pk} \leq k_1 | X\right\} \cdot \left[\Pr\left\{\hat{C}_{pk} \geq k_1 | X\right\}\right]^m \tag{23}$$

Where $\Pr\{\hat{C}_{pk} \geq k_1 | X\}$ is the probability of accepting the lot based on single sample. In addition, the term $\Pr\{k_2 \leq \hat{C}_{pk} \leq k_1 | X\}.[\Pr\{\hat{C}_{pk} \geq k_1 | X\}]^m$ shows the probability of accepting the lot based on the quality level of m preceding lots.

An optimization model is presented to minimize ASN with considering the constraints of first and second type error. Also the optimal value of m can be obtained by simulation studies. The mentioned model is as follows:

$$\begin{aligned} &\text{Minimize}\, n \\ &\text{subject to}: \\ &C = C_{AQL} \Rightarrow P_A(C_{AQL}) \geq 1 - \alpha \\ &\text{and} \\ &C = C_{LTPD} \Rightarrow P_A(C_{LTPD}) \leq \beta \end{aligned} \tag{24}$$

6 Designing proposed sampling plans based on exact probability distribution

The exact probability distribution function of process capability index can be determined using statistical techniques. So the acceptance sampling plan can be designed based on exact probability distribution function of process capability index. Considering the natural estimator of C_{pk} that is $\hat{C}_{pk}$, for a normally distributed process, a simple and exact form of the cumulative distribution function of estimated parameter, $\hat{C}_{pk}$ can be determined. The cumulative distribution function is as follows [25].

$$F_{\hat{C}_{pk}}(Y) = 1 - \int_{0}^{b\sqrt{n}} G\left(\frac{(n-1)(b\sqrt{n}-t)^2}{9.n.y^2}\right) \left[\phi\left(t + \xi\sqrt{n}\right) + \phi\left(t - \xi\sqrt{n}\right)\right] dt \quad (25)$$

Where $b = \frac{d}{\sigma}$ and b values can be rewritten as $b = 3C_{pk} + |\xi|$ and $\xi = \frac{\mu - M}{\sigma}$. In addition, $G(.)$ shows the CDF of the chi-square distribution, χ^2_{n-1} with n-1 degrees of freedom and $\phi(.)$ shows the PDF of the standard normal distribution $N(0,1)$.

With regard to OC function of the mentioned sampling plans (DSP, MDS sampling plan, RGS plan, and sampling plan for resubmitted lot) and exact probability distribution function, the required sample size of mentioned plans can be obtained using designed optimization models.

Now in order to obtain the optimal parameters of sampling plans, we present methodology of how mentioned plan parameters can be obtained. In the case of sampling for resubmitted lot with using a grid search, we can obtain the minimum ASN plans searching in the multi-dimensional grid formed setting $m = 1,2,3,4$, $n = 3(1)\ 100$, $k_a = 0.8(0.001)\ 2.0$. The optimal solution of the proposed plans is determined by solving the related nonlinear optimization model using MATLAB R2015a software and applying a grid search procedure. Since all feasible solutions are analyzed in grid search methods, thus we can be sure that optimal solution is obtained. Also the results coincided approximately with the findings of Aslam et al. [3, 6, 7]. Also only one continues decision variable existed in the model, and we discretized this variable with an accuracy of one-thousandth for a grid search procedure.

Table 1 Optimal values of parameters for different values of α, β

$\delta = 0.5$, $\gamma = 0.8$, $C_{AQL} = 1.8$, $C_{LQL} = 0.9$
n, k_a, m, ASN

α	β	n	k_a	m	ASN	α	β	n	k_a	m	ASN	α	β	n	k_a	m	ASN
0.01	0.01	10	1.675	2	19.86	0.01	0.01	11	1.629	3	28.32	0.01	0.01	10	0.985	4	38.52
	0.03	12	1.698	2	21.02		0.03	13	1.514	3	22.13		0.03	15	1.517	4	24.14
	0.05	31	0.949	2	61.21		0.05	28	1.735	3	76.33		0.05	28	1.786	4	76.65
	0.07	7	1.537	2	13.74		0.07	6	1.712	3	17.70		0.07	7	0.898	4	19.28
	0.09	31	0.925	2	60.49		0.09	29	1.735	3	68.32		0.09	26	1.757	4	78.23
0.03	0.01	14	0.934	2	18.17	0.03	0.01	8	1.978	3	20.64	0.03	0.01	8	1.965	4	26.63
	0.03	9	0.945	2	12.61		0.03	5	1.834	3	14.79		0.03	6	1.817	4	18.78
	0.05	10	1.547	2	19.86		0.05	33	0.915	3	97.32		0.05	32	0.906	4	112.18
	0.07	11	1.551	2	21.61		0.07	33	0.964	3	96.64		0.07	32	0.913	4	102.95
	0.09	7	1.489	2	11.65		0.09	4	1.814	3	13.62		0.09	5	1.761	4	21.65
0.05	0.01	13	1.552	2	25.95	0.05	0.01	12	1.596	3	28.31	0.05	0.01	12	1.523	4	37.54
	0.03	12	1.545	2	23.79		0.03	15	1.547	3	29.65		0.03	15	1.616	4	41.44
	0.05	7	1.727	2	13.85		0.05	8	1.715	3	21.12		0.05	9	1.768	4	29.19
	0.07	10	1.546	2	16.94		0.07	10	1.544	3	18.63		0.07	11	1.434	4	19.67
	0.09	11	1.537	2	11.49		0.09	7	1.794	3	14.20		0.09	5	1.705	4	26.98
0.07	0.01	9	1.779	2	17.92	0.07	0.01	9	1.773	3	27.72	0.07	0.01	10	1.714	4	37.19
	0.03	11	1.548	2	22.60		0.03	13	1.566	3	31.60		0.03	13	1.329	4	43.27
	0.05	12	1.525	2	23.70		0.05	12	1.428	3	29.70		0.05	13	0.916	4	37.67
	0.07	30	0.932	2	61.41		0.07	31	0.977	3	83.42		0.07	32	0.975	4	127.16
	0.09	30	0.940	2	58.47		0.09	32	0.884	3	102.86		0.09	33	0.893	4	120.82
0.09	0.01	9	1.795	2	17.94	0.09	0.01	8	0.919	3	19.87	0.09	0.01	9	0.916	4	111.73
	0.03	5	1.786	2	9.91		0.03	11	0.916	3	23.17		0.03	8	0.957	4	124.13
	0.05	8	1.771	2	18.22		0.05	34	1.66	3	100.29		0.05	33	0.899	4	109.46
	0.07	6	1.737	2	13.46		0.07	34	1.574	3	99.62		0.07	34	0.895	4	132.76
	0.09	4	1.769	2	12.22		0.09	10	0.877	3	26.97		0.09	11	0.882	4	33.81

Table 2 Results of comparison study ($\delta = 0.5$, $\gamma = 0.8$, $C_{AQL} = 1.8$, $C_{LQL} = 0.9$)

	Average sample number					
α	β	SSP	DSP	Variables MDS sampling plan	Variables (RGS) plan	Sampling plan for resubmitted lots ($m = 2$)
0.01	0.03	53	39.46	11	14.62	21.02
0.01	0.07	42	27.31	5	7.40	13.74
0.01	0.09	67	62.87	17	28.66	60.49
0.05	0.01	73	57.62	8	10.89	25.95
0.07	0.03	56	43.19	18	22.24	22.60
0.07	0.09	64	59.27	19	21.88	58.47
0.09	0.05	43	39.54	10	14.44	18.22
0.09	0.09	38	31.28	4	6.33	12.22

7 Simulation studies

In Table 1, the optimal parameters of variable sampling plan for resubmitted lots, including n, k_a for the specified values of parameters $C_{AQL} = 1.8$, $C_{LQL} = 0.9$, $\delta = 0.5$, $\gamma = 0.8$ and different values of α, β are denoted. For example, if, $C_{AQL} = 1.8$, $C_{LQL} = 0.9$, $\delta = 0.5$, $\gamma = 0.8$, $m = 2$, and $\alpha = 0.05$, $\beta = 0.03$, then Table 1 gives $n = 12$, $k_a = 1.545$ as the optimal solution; thus, the decision making procedure will be as follows:

Step 1. Collect a sample of $n = 12$ observations from lot and compute $\hat{C}_{pk}$.

Step 2. If $\hat{C}_{pk} \geq 1.545$ then accept the lot; otherwise if the lot was not accepted after repeating the step 2 for two times then reject the lot else resubmit the lot and go to step 1.

It is noted that the results in Table 1 describe the advantages of proposed sampling plan in the case of $m = 2$ in comparison with the cases $m = 3$ and $m = 4$. Because for any given parameters α and β, the value of ASN in the case $m = 2$ is smaller than ASN values in the cases $m = 3$ and $m = 4$. For example, where $\alpha = 0.05$, $\beta = 0.09$, ASN of proposed sampling plan in the case of $m = 2$ is equal to 11.49, and for the cases of $m = 3$ and $m = 4$, ASN is equal to 14.20 and 26.98, respectively. In the other words, a sampling plan that has smaller ASN would be more economical and desirable in practical situations. Because of less inspection cost, so we have used the case ($m = 2$) in the comparison study with the other methods.

8 Comparison study

8.1 Comparison of different sampling plans

To compare the performance of the proposed variable sampling plan ($m = 2$), with classical single sampling plan [26], DSP, MDS sampling plan, and variables RGS plan, the ASN values of plans are denoted in Table 2.

Table 3 Results of comparison study

		DSP		MDS sampling plan		RGS plan		Sampling plan for resubmitted lots	
α	β	(Exact approach)	(Bayesian approach)	(Exact approach)	(Bayesian approach)	(Exact approach)	(Bayesian approach)	(Exact approach)	(Bayesian approach)
0.01	0.03	41.37	39.46	13	11	17.91	14.62	23.64	21.02
0.01	0.07	29.64	27.31	7	5	9.37	7.40	15.39	13.74
0.01	0.09	64.46	62.87	21	17	31.48	28.66	66.49	60.49
0.05	0.01	58.97	57.62	12	8	15.34	10.89	27.43	25.95
0.07	0.03	46.35	43.19	23	18	23.54	22.24	25.71	22.60
0.07	0.09	63.93	59.27	22	19	24.67	21.88	63.42	58.47
0.09	0.05	42.28	39.54	13	10	17.38	14.44	22.79	18.22
0.09	0.09	32.58	31.28	8	4	9.64	6.33	17.62	12.22

In Table 2, it is clear that the ASN values of sampling plan for resubmitted lots in the specified values of C_{AQL}, C_{LQL}, δ, and γ is less than ASN values of SSP and DSP. In addition, it is observed that the ASN values of RGS plan and MDS sampling plan are less than the ASN values of sampling plan for resubmitted lots. In general, it is observed that MDS sampling plan is more economical than the other sampling plans.

8.2 Analysis of proposed sampling plans based on Bayesian approach and exact probability distribution

In this section, we analyze the ASN of mentioned sampling plan (DSP, MDS sampling plan, RGS plan and sampling plan for resubmitted lot) based on Bayesian approach and exact probability distribution (exact approach). With regard to the given values of $\delta = 0.5$, $\gamma = 0.8$ for Bayesian approach and given value of $\xi = 1$ for exact approach, it is observed that the ASN of different variable sampling plans based on Bayesian approach is less than ASN values obtained using based on exact approach. The results are denoted in Table 3.

It is also observed that MDS sampling plan has the least values of ASN in the case of exact probability distribution.

Thus, when some historical data of the process is available, then applying the Bayesian inference approach is preferred, but when we do not have access to such data, then we must apply the exact probability distribution.

9 Conclusion

In this paper, we developed a variable sampling plan for resubmitted lots based on process capability index and Bayesian approach. Also, we obtained the optimal parameters of developed sampling plan at the specified values of acceptable quality level and limiting quality level and different values of consumer and producer risk. In addition, two comparison studies are performed. First, we developed DSP, RGS plan, and MDS sampling plan and compared all methods with each other based on Bayesian approach and it is observed that the ASN values of sampling plan for resubmitted lots are smaller than ASN values of SSP and DSP but ASN values of MDS sampling plan is less than ASN values of other methods. Second, we developed mentioned sampling plans based on exact probability distribution and concluded that the ASN values of different variable sampling plans based on Bayesian approach is less than ASN values obtained using exact approach.

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