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ORIGINAL ARTICLE

Manufacturing signature in variational and vector-loop models for tolerance analysis of rigid parts

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Abstract To control and to manage the geometric deviations along the product life cycle, the first step is to consider, during the design stage, the tolerance specification, the tolerance allocation and the tolerance analysis. Many approaches of the literature for tolerance analysis of rigid assemblies exist, and different commercial computer-aided tolerancing (CAT) software packages were developed with those models. However, there is a growing interest in considering working conditions and operating windows in CAT. As a response to these needs, skin model concept was proposed. The aim of this paper is to connect a point cloud-based discrete geometry framework (i.e. a skin model representation) to the manufacturing processes, in order to bring the CAT simulation tools closer to reality. In this work, the effect of a manufacturing process on solving a stack-up function is investigated throughout circular profiles obtained by a turning process. A case study has been defined and solved using two literature models, the variational model and the vector-loop one, by considering the manufacturing signature. The results have been compared to those obtained by the same models without considering the manufacturing signature. Monte Carlo simulations have been carried out by solving the stack-up functions into Matlab® software, and statistical analyses have been carried out by Minitab® software.

Keyword Tolerance analysis · Manufacturing signature · Variational model · Vector-loop model

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Nomenclature

T	Index of data points in each sampled profile
B	Backshift operator
N	Number of equally spaced points each profile
k	Number of undulations per revolution for each profile
a	Means vector of a multivariate normal distribution used in the Armax model
b	Variance-covariance matrix of a multivariate normal distribution used in the Armax model
ε_t	Term modelled as a Gaussian white noise with standard deviation equal to 0.374 μm
L_i	Generic side of the rectangular box (i = 1 to 4)
r_{zi}	Rotation parameter of the generic side of the box (i = 1 to 4)
t_{yi}	Translation parameter of the generic side of the box (i = 1 to 4)
d_i	Geometric tolerance parameter applied to the points i of each circular profile (i = A, B, C, D and E) and used in the variational model
$\{\Delta x\}$	Vector of variations of the dimensional variables
$\{\Delta u\}$	Vector of variations of the assembly variables
$\{\Delta \alpha\}$	Vector of variations of the geometric variables
$[A]$	The first order partial derivate of the dimensional variables in the closed loop
$[B]$	The first order partial derivate of the assembly variables in the closed loop
$[C]$	The first order partial derivate of the dimensional variables in the open loop
$[D]$	The first order partial derivate of the assembly variables in the open loop
$[F]$	The first order partial derivate of the geometric variables in the closed loop
$[G]$	The first order partial derivate of the geometric variables in the open loop

S^d_{ij}	Sensitivity matrix of the dimensional variables
S^α_{ij}	Sensitivity matrix of the geometric variables
m	Number of the dimensional variables
n	Number of the geometric variables
α_j	Geometric tolerance parameter applied to the points j of each circular profile (j = A, B, C, D and E) and used in the vector-loop model
F	Scale factor

1 Introduction

The geometric variation management is a need in design, manufacturing and all other phases of the product life cycle [1]. It is due to the fact that, even though modern manufacturing processes achieve an increasingly high accuracy, geometric deviations have a huge influence on both the function behaviour and on the customers' quality perception of the product. To control and to manage these geometric deviations along the product life cycle, the first step is to consider during the design stage, the tolerance specification, the tolerance allocation and the tolerance analysis [2].

Tolerance analysis is an important task in the design of mechanical assemblies, and it has received considerable attention in literature. Tolerance analysis generally consists in studying one or more tolerance chains, each of which associates a functional requirement on the assembly with a set of tolerances specified on components; the mathematical model of a tolerance chain helps to detect situations where manufacturing errors that are allowed by parts' tolerances are likely to violate the required assembly tolerance. From this basic objective, however, the problem has been defined differently according to the needs of specific applications. Differences can involve properties of parts and assemblies (the geometry of the features and relations), types of tolerance specifications (dimensional or geometric) and assumptions on tolerance stack-up (worst-case or statistical). As consequence, several methods have been proposed for solving tolerance analysis problems under different sets of assumptions.

The most significant models for tolerance analysis of rigid assemblies proposed by the literature are vector loop, matrix, variational, Jacobian, torsor, unified Jacobian-torsor and the T-Map®. The basic idea of the vector-loop model is to represent the variability in the product, due to the tolerances and the assembly conditions, by chains of vectors [3, 4]. The basic idea of the variational model approach is to represent the deviations from nominal due to the tolerances and the assembly conditions through a set of parameters of a mathematical model [5–7]. The matrix model uses a displacement matrix to describe small displacements of a feature within its tolerance zone and clearance between two features [8]. A Jacobian-based model uses pairs of functional elements to represent both the dimensions and the variations in an assembly. The functional element pairs are arranged in chains representing those dimensions that stack together to determine the resultant assembly functional requirement [9–11]. The torsor model uses the idea that tolerances are essentially small linear and angular dispersions of a functional feature with respect to its nominal, and it is based on the concept of a small displacement torsor [12, 13]. The unified Jacobian-torsor model is an innovative tolerance analysis method, which uses the torsor model for tolerance representation and the Jacobian matrix for tolerance propagation [14]. Both deterministic and statistical analysis methods about this model are concise and efficient. The T-Map® model is based on a two-level model: the local model that models a part's variations in order to consider the interactions of the geometric controls applied to a feature of interest, and the global model that inter-relates all control frames on a part or assembly. A tolerance-map is a hypothetical solid of points in n-dimensions, which represent all possible variations in the feature or assembly. Overlaying the coordinates of the T-Map, the stack-up equations to perform the tolerance analysis are obtained [15, 16]. Many commercial CAT software packages were developed on the basis of those models, such as 3-DCS of Dimensional Control Systems®, VisVSA of Siemens®, CETOL® and so on.

However, there is a growing interest in considering working conditions and operating windows in CAT [17, 18]. These computer models for tolerance simulation and analysis make severe simplifications about observable geometric deviations, since they are reduced to rotational and translational feature defects [19]. This leads to results with large ranges of uncertainty and a discrepancy between the virtual models and the observed reality [20]. Furthermore, the tolerancing tasks in design as well as all other activities of geometric variations management should be incorporated in a complete and coherent tolerancing process [21, 22]. As a response to these needs, skin model concept was proposed [1]. It is a model of the physical workpiece surface in contrast to the nominal model, which is a 'simple' model of the intended workpiece not taking into account inevitable geometric deviations [1].

The aim of this paper is to connect a point cloud-based discrete geometry framework (i.e. a skin model representation) to the manufacturing processes, in order to bring the CAT simulation tools closer to reality. The discrete geometry framework of the skin model has been represented by the pattern left on a surface throughout a turning process that is a widely used manufacturing process. The turning process involves a correlation among the points on the manufactured surface that is called signature; it has been inserted in the framework of the skin model.

The first contribution of this paper is the insertion of a skin model into two models of the literature for tolerance analysis of rigid parts: the variational and the vector-loop ones. The second contribution is that the used skin model is the signature of an actual turning process. To demonstrate the effectiveness of considering the actual turning signature into the models for tolerance analysis, the developed skin models with signature have been applied to a case study made up of three parts: a rigid box and two profiles that fit within it. The case study has been chosen simple in order to be solved manually but representative, since it allows to consider both dimensional and geometrical tolerances applied to the same profile. The obtained results have been compared to those obtained by the use of the two models of the literature. Matlab® and Minitab® software packages have been used to carry out the tolerance analysis and the statistical analysis of the obtained results, respectively.

The paper is organized as follows: in Section 2, the case study and the manufacturing signature of the circular profiles are presented. In Section 3, the variational model with and without manufacturing signature is solved. In Section 4, the vector-loop model with and without manufacturing signature is described in detail and solved. In Section 5, the obtained results are compared and discussed.

2 Case study

The case study is composed by three parts: a hollow rectangular box and two circular profiles that fit within it, as shown in Fig. 1. The aim of this 2D tolerance analysis is the measurement of the variation of the gap g between the second profile and the top side of the box (Δg) as a function of the dimensions and the tolerances applied to the components, as shown in Table 1. The tolerance analysis has been carried out by considering the rectangular box fixed and the circular profiles with and without the manufacturing signature.

In this work, circular profiles machined by turning process have been studied throughout an autoregressive-moving average (ARMAX) model [23]. This harmonic model was combined with a second-order autoregressive model of the noise. Combining this harmonic model and the second-order autoregressive model of the noise, the parametric model of the identified process signature is given by:

$$Y_t = \sqrt{\frac{2}{N}} \sum_{k=2}^{3} \left[b_{2k-1} \cdot \cos\left(\frac{k \cdot t \cdot 2 \cdot \pi}{N}\right) + b_{2k} \cdot \sin\left(\frac{k \cdot t \cdot 2 \cdot \pi}{N}\right) \right] + \frac{1}{1 - a_1 B - a_2 B^2} \cdot \varepsilon_t \tag{1}$$

where $t = 1, 2, \ldots, N$ is the index of data points in the sampled profile, B is the backshift operator, N is the number of equally spaced points measured on that profile. For each index t, Y_t represents the radial distance between the actual point and the least square substitute circle, measured at an angular position $\theta_t = 2\pi t/N$. Thus, the signature model in Eq. (1) is a linear combination of two harmonic terms plus a second-order autoregressive model of the noise. Each term of the first part of Eq. (1) represents the kth harmonic ($k = 2, 3$), characterized by k undulations per revolution. The parameters' vector (a and b) in Eq. (1) forms a stochastic vector, that has a multivariate normal distribution with the mean vector and the variance-covariance matrix. The term ε_t in Eq. (1) was modelled as a Gaussian white noise with standard deviation equal to 0.374 μm.

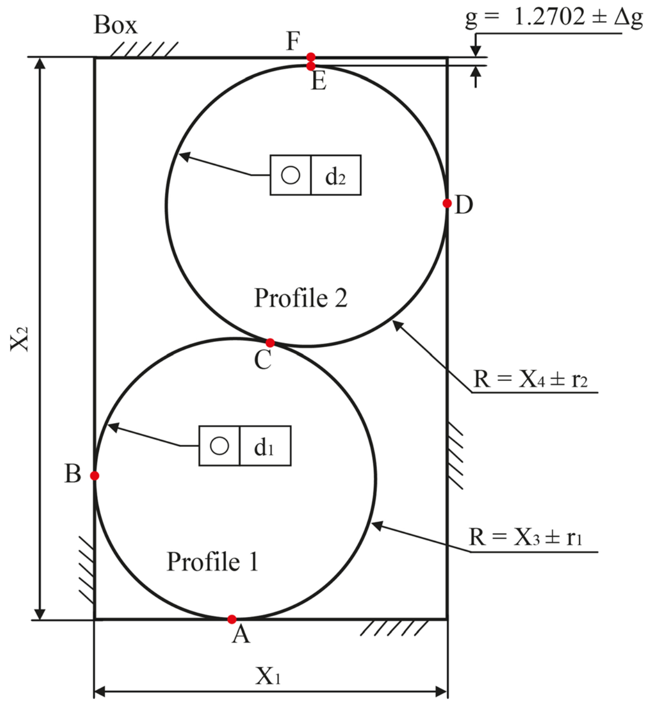

Fig. 1 Case study made up of three parts: a rigid box and two profiles that fit within it

The introduction of the manufacturing signature in tolerance analysis is an important step to improve the simulation results due to the solving of the literature models. In fact, the variation of a profile radius is simulated as shown in Fig. 2a into literature models; but actually, we have the configuration shown in Fig. 2b. This real configuration shows a generic signature due to the manufacturing process. However, in this

Table 1 Parameter values of case study

Definition	Parameter	Value [mm]
Width of the box	X_1	50
Height of the box	X_2	80
Radius of the circular profile 1	X_3	20
Radius of the circular profile 2	X_4	20
Dimensional tolerance of the circular profile 1	r_1	0.0145
Dimensional tolerance of the circular profile 2	r_2	0.0145
Form tolerance of the circular profile 1	d_1	0.0145
Form tolerance of the circular profile 2	d_2	0.0145

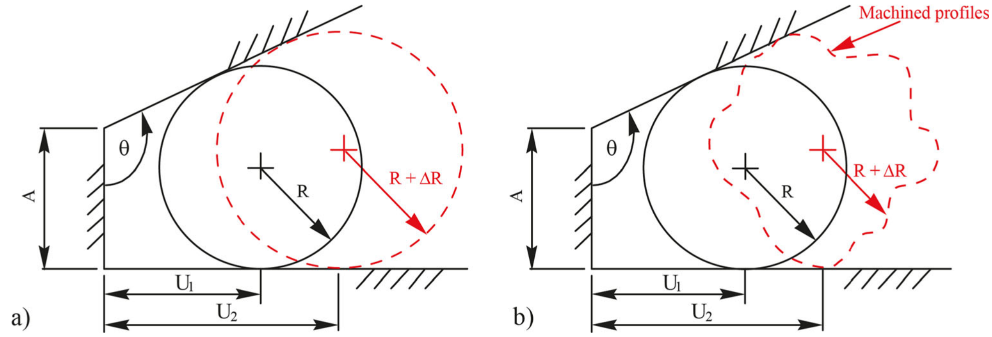

Fig. 2 How to simulate the radial variation of a circular profile

work, the manufacturing process is a turning process, so the signature was mainly affected by bi-lobe and tri-lobe contours. An example of the machined profile due to the ARMAX model is shown in Fig. 3.

3 Variational model with and without manufacturing signature

A mathematical rigorous foundation of the variational model has been proposed by Boyer and Stewart [6] and later by Gupta and Turner [7]. So, the gap g of Fig. 1 has been evaluated by means of the following analytical equation, while all the details are reported in [24]:

$$g = 1.2702 + r_{z3}\Delta X_{13} - \Delta Y_{13} + 5r_{z3} - t_{y3} - r_2 - d_E \quad (2)$$

where

$$\Delta X_{12} = \left[t_{y4} + r_1 + d_B + r_{z4}(40-h)\right] / (1 + r_{z1}r_{z4}) \quad (3)$$

$$\Delta Y_{12} = r_{z1}\Delta X_{12} - 5r_{z1} + t_{y1} + r_1 + d_A \quad (4)$$

$$h = -5r_{z1} + t_{y1} + r_1 + d_A \quad (5)$$

$$\Delta X_{13} = -r_{z2}\Delta Y_{13} - a \quad (6)$$

$$\Delta Y_{13} = b + \sqrt{(b^2 - c)} \quad (7)$$

$$a = 18.73r_{z2} + t_{y2} + r_2 + d_D \quad (8)$$

$$b = [r_{z2}(10 - a - \Delta X_{12}) - 38.73 + \Delta Y_{12}] / \left(1 + {r_{z2}}^2\right) \quad (9)$$

$$c = \left[(10 - a - \Delta X_{12})^2 + (38.73 - \Delta Y_{12})^2 - (40 + r_1 + r_2 + d_C)^2\right] / \left(1 + {r_{z2}}^2\right) \quad (10)$$

In Eqs. (2)–(10), r_{zi} is the rotation parameter of the generic side L_i of the box, t_{yi} is the translation parameter of the generic side L_i of the box, r_1 and r_2 are the model parameters, due to the dimensional tolerances, of the first and the second circular profiles, respectively, d_i is the model parameters due to the form tolerance applied to the points of circular profiles (where i = A, B, C, D and E, as shown in Fig. 1), ΔX_{12}-ΔY_{12} and ΔX_{13}-ΔY_{13} are the assembly parameters of the first and the second profiles on the rectangular box. The parameters r_{zi} and t_{yi} of the sides of the box are equal to zero because the box has fixed planar surfaces. So the set of Eqs. (2)–(10) is drastically simplified in

$$g = 40 - d_E - r_1 - r_2 - \sqrt{(d_C + r_1 + r_2 + 40)^2 - (d_B + d_D + r_1 + r_2 - 10)^2} - d_A \quad (11)$$

where only the model parameters due to the dimensional and form tolerances of two circular profiles are required. The Eq. (11) has been solved throughout of a statistical approach considering the model parameters as statistical variables following Gaussian probability density functions.

The same mathematical model has been used by considering the manufacturing signature. This means that the values of the form tolerance of the two circular profiles have been substituted by the value of the form deviations computed by means of the manufacturing signature (i.e. by means of Eq. (1)). Therefore, the roundness of the two circles has been simulated by the local values of Y_t evaluated for the points on the circular profile. The obtained values have been used to define the values of the parameters d_i of the variational model, where i = A, B, C, D and

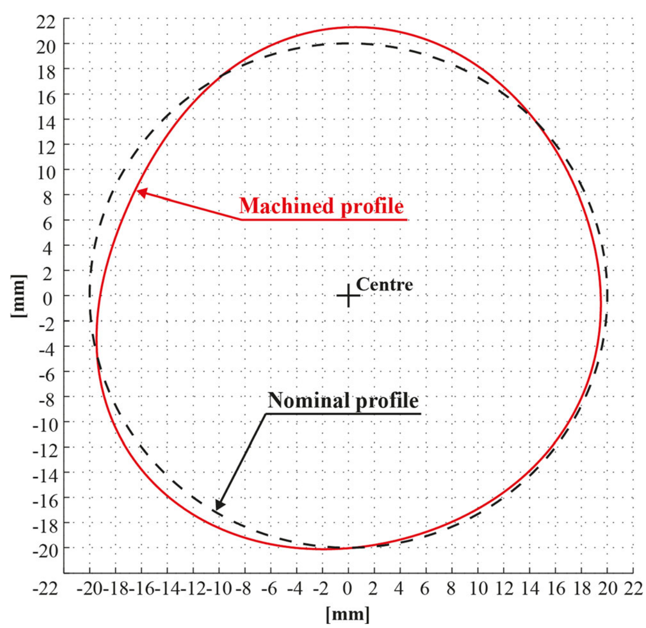

Fig. 3 Machined profile generated by the Armax model

E with A, B, C and D the contact points between the profiles and the box. The values of r_1 and r_2 remain equal to ±0.0145 mm, since the manufacturing signature affects only the values of the d_i parameters.

4 Vector-loop model with and without manufacturing signature

The vector-loop-based tolerance analysis method has been proposed by Chase et al. [3]. The assembly equations expressed by the vector-loop-based assembly models which use vectors to represent either component dimensions or assembly dimensions take three main sources of variation into account in a mechanical assembly: dimensional, kinematic and geometric variations.

The methodology of the vector-loop model may be summarized in three main steps. First of all, one should create the assembly vector loops. The whole assembly is modelled with a graph representation, in which each edge corresponds to a joining feature, while each vertex is a part being assembled. Then, equations are written for each independent loop. Assembly constraints for each vector loop may be expressed as a concatenation of homogeneous rigid body transformation matrices. These equations are linearized using Taylor's series expansion (direct linearization method—DLM). Finally, loop equations are solved.

Taylor's first-order series expansion of assembly constraint equations for a closed loop can be written as

$$\{\Delta H_C\} = [A]\cdot\{\Delta x\} + [F]\cdot\{\Delta\alpha\} + [B]\cdot\{\Delta u\} = \{0\} \quad (12)$$

and for an open loop, it is:

$$\{\Delta H_o\} = [C]\cdot\{\Delta x\} + [G]\cdot\{\Delta\alpha\} + [D]\cdot\{\Delta u\} \quad (13)$$

where $\{\Delta H_C\}$ is the vector of clearance variations in a closed loop, $\{\Delta H_o\}$ is the vector of assembly variations in an open loop; $\{\Delta x\}$ is the vector of variations of dimensional variables; $\{\Delta u\}$ is the vector of variations of assembly variables; $\{\Delta\alpha\}$ is the vector of variations of geometric feature variables; $[A]$ and $[C]$ are the first-order partial derivatives of the dimensional variables in the closed loop and open loop, respectively; $[B]$ and $[D]$ are the first-order partial derivatives of the assembly variables in the closed loop and open loop, respectively; $[F]$ and $[G]$ are the first-order partial derivatives of the geometric feature variables in the closed loop and open loop, respectively.

Among Eqs. (12)–(13), $\{\Delta u\}$ is obtained by solving these two equations. For the closed loop, $\{\Delta u\}$ is given in Eq. (14) if $[B]$ is a full-ranked matrix.

$$\{\Delta u\} = -[B]^{-1}\cdot[A]\cdot\{\Delta x\} - [B]^{-1}\cdot[F]\cdot\{\Delta\alpha\} \quad (14)$$

From Eqs. (13)–(14), it can obtain the $\{\Delta u\}$ in the open loop as

$$\{\Delta u\} = \left([C]-[D]\cdot[B]^{-1}\cdot[A]\right)\cdot\{\Delta x\} + \left([G]-[D]\cdot[B]^{-1}\cdot[F]\right)\cdot\{\Delta\alpha\} \quad (15)$$

Therefore, the gap g of Fig. 1 has been evaluated by means of the three main steps and the analytical equation previously mentioned. The assembly graph of Fig. 4 shows two joints of 'cylinder-slider' kind between box and profile 1 at points A and B, respectively, one joint of 'parallel-cylinder' kind between profile 1 and profile 2 at point C, one joint of 'cylinder-slider' kind between profile 2 and box at point D, and the measure to perform g. The vector loops have been created and placed on the assembly, as shown in Fig. 5. The first (closed) loop joins the box and the profile 1 by the links passing from points A and B. The second (closed) loop joins the subassembly box—profile 1 and the profile 2 by the links passing through points D and C. The third (open) loop defines the gap g. Once the vector loops have been defined, the relative equations have been generated and solved. So, the variation of gap g (Δg) can be estimated with a worst-case way and a statistical way (with the root sum square), as shown in Eqs. (16)–(17):

$$\Delta g_{wc} = \pm\left(\sum_{j=1}^{m} |S^d|\cdot\Delta x_j + \sum_{j=1}^{n} |S^\alpha|\cdot\Delta\alpha_j\right) \quad (16)$$

$$\Delta g_{Stat} = \pm\sqrt{\left(\sum_{j=1}^{m} |S^d|\cdot\Delta x_j\right)^2 + \left(\sum_{j=1}^{n} |S^\alpha|\cdot\Delta\alpha_j\right)^2} \quad (17)$$

where the sensitivity matrices of dimensional variables S_{ij}^d and geometric variables S_{ij}^α are known, in accordance with [25], which are the coefficients of the $\{\Delta x\}$ and $\{\Delta\alpha\}$ in Eqs. (14)–(15); m and n are the number of dimensional and geometric variables, respectively.

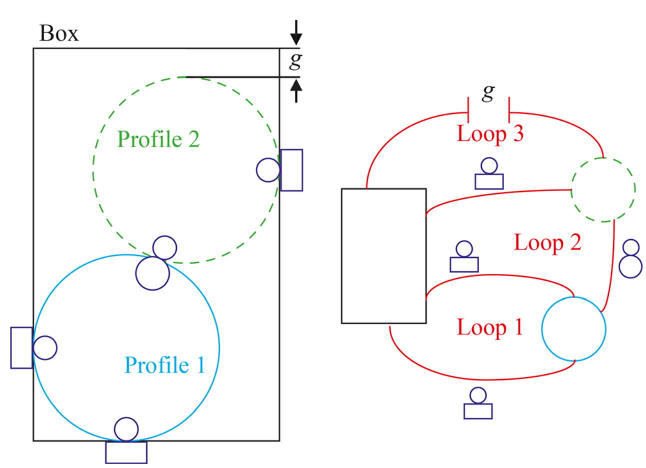

Fig. 4 Assembly graph of the case study

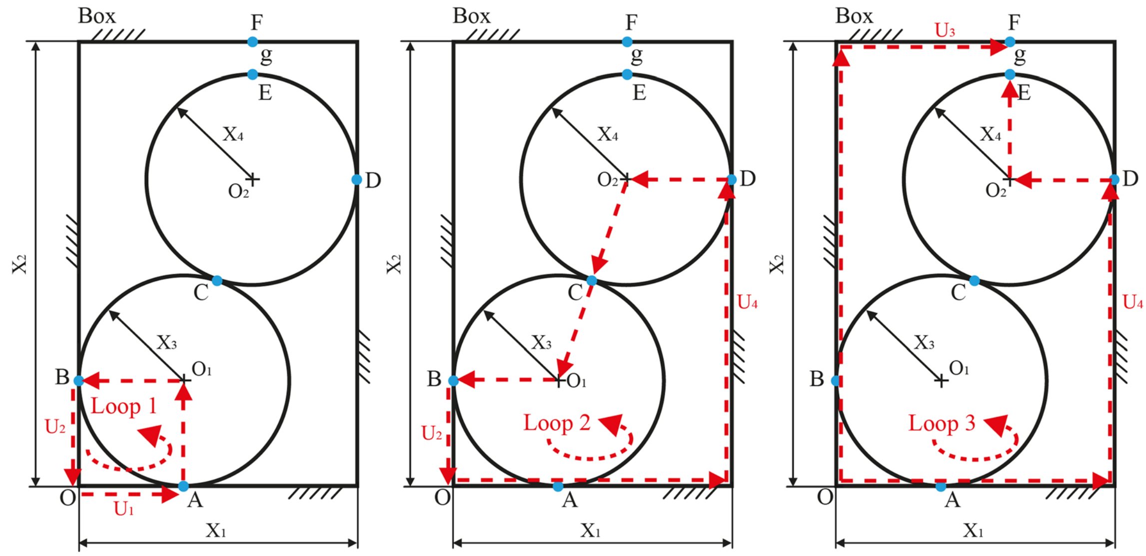

Fig. 5 Assembly loops of the case study

Also, the variation of gap g (Δg) can be estimated with a Monte Carlo simulation. In this case, it is just necessary to solve the assembly problem, to write the final stack-up function and to vary the model parameters randomly. Therefore, the problem can be drastically simplified in Eq. (18):

$$g = X_2 - X_3 - X_4 - \alpha_A - \alpha_E - (\alpha_{C_1} + \alpha_{C_2} + X_3 + X_4) \cdot \sin\left(\arccos\left(\frac{X_1 - X_3 - X_4 - \alpha_B - \alpha_D}{\alpha_{C_1} + \alpha_{C_2} + X_3 + X_4}\right)\right) \quad (18)$$

where X_1 and X_2, the width and the height of the box, respectively, are constant; X_3 and X_4, the radii of two profiles, have a Gaussian density function with a mean value equal to radius and standard deviation equal to a third of the dimensional tolerance (r_1 and r_2, respectively); α_A, α_B, α_{C_1}, α_{C_2}, α_D and α_E, the geometrical parameters of two profiles, have a Gaussian density function with mean value equal to zero and standard deviation equal to a sixth of the form tolerance (d_1 and d_2).

The same mathematical model has been used by considering the manufacturing signature. This means that the values of the form tolerance of the two circular profiles have been substituted by the value of the form deviations computed by means of the manufacturing signature. Therefore, the roundness of the two circles has been simulated by the local values of Y_t of Eq. (1) evaluated for the points on the circular profile. The obtained values have been used to define the values of the parameters $\Delta\alpha_j$ of the vector-loop model, where j = A, B, C, D and E with A, B, C and D are the contact points between the profiles and the box. The value of Δx_j remains equal to ±0.0145 mm, since the manufacturing signature affects only the values of the $\Delta\alpha_j$ parameter.

5 Discussion of results

Monte Carlo simulations have been carried out by implementing 50,000 runs; this value has been chosen after performing a sensitivity analysis. In particular, the sensitivity analysis has been carried out all models by varying the number of Monte Carlo simulations and considering a scale factor $F = 1$. The results about the standard deviations due to the sensitivity analysis are shown in Fig. 6. It is evident that results are very stable if 50,000 runs of Monte Carlo simulation are carried out.

Three scale factors (1, 10 and 50) of the applied tolerance ranges have been considered to simulate three different

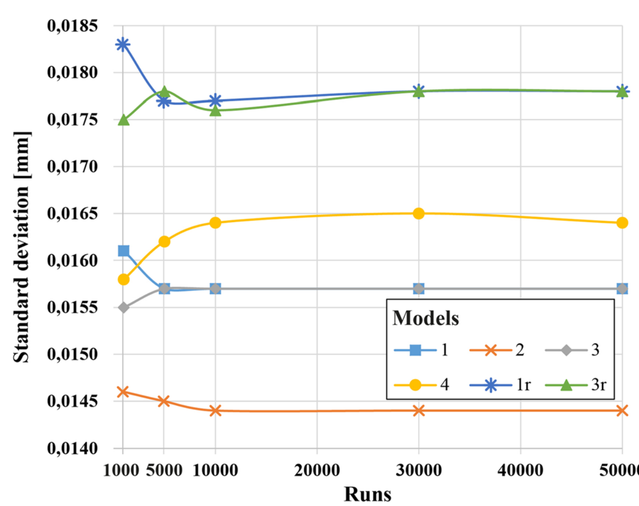

Fig. 6 Results about standard deviation of the sensitivity analysis ($F = 1$)

Table 2 Variational model results considering 50,000 cycles of assembling

Model	Scale factor	Mean [mm]	3 · StDev [mm]	A^2	*P* value	Skewness	Kurtosis
1	1	1.2562	0.047	0.150	0.966	−0.005	−0.012
	10	1.1307	0.467	0.230	0.796	0.017	−0.002
	50	0.5763	2.351	0.260	0.705	0.010	−0.033
2	1	1.2701	0.043	0.470	0.247	0.004	−0.039
	10	1.2693	0.434	0.350	0.467	0.015	0.023
	50	1.2713	2.175	0.420	0.320	0.009	0.014

Table 3 Vector-loop model results considering 50,000 cycles of assembling

Model	Scale factor	Mean [mm]	3 · StDev [mm]	A^2	*P* value	Skewness	Kurtosis
3	1	1.2561	0.047	0.250	0.744	0.000	0.000
	10	1.1319	0.472	0.430	0.313	0.004	−0.037
	50	0.5849	2.344	0.550	0.159	0.020	0.002
4	1	1.2702	0.049	0.200	0.891	−0.005	−0.028
	10	1.2705	0.494	0.660	0.083	−0.013	−0.006
	50	1.2673	2.445	0.300	0.584	0.016	0.004

dimensional and geometrical conditions of the circular profiles. The measurement of the *g* gap has been evaluated at every assembly cycle, and a statistical analysis has been carried out by Minitab® software. The normality of the obtained distributions of the *g* gap has been evaluated by means of Anderson-Darling test. All results are shown in Tables 2, 3 and 4 with the mean, the standard deviation, the skewness and kurtosis values (model 1 is the variational model with manufacturing signature, model 2 is the variational model without manufacturing signature, model 3 is the vector-loop model with manufacturing signature and model 4 is the vector-loop model without manufacturing signature).

The mean value of the gap *g* due to the variational and vector-loop models without the manufacturing signature is very near to the nominal value of 1.2702 mm even varying the scale factor. When the scale factor increases, the standard deviations of the gap *g* increase in each model and there are no significant differences among models.

The variational and vector-loop models with the manufacturing signature involve a decrease of the gap mean value with the increase of the scale factor. It is due to the fact that when the ARMAX model generates the profiles, they are always generated in very similar and nearby positions as shown in Fig. 7, and the next effect is the shift of the second profile towards the upper side of the box, until exiting.

To solve this problem, other simulations have been carried out in which the profiles have been generated by the ARMAX model, at first, and then they have been casually rotated; only after the rotation, the parameter values for two models, in the contact points, have been extracted by profiles. In this way, an operating condition of the assembly has been introduced.

The results of these last simulations are shown in Table 4 (model 1r is the variational model with manufacturing signature and model 3r is the vector-loop model with manufacturing signature, both with profiles casually rotated). The variational and vector-loop models with manufacturing signature and operating condition involve a smaller decrease of the gap mean value with the

Table 4 Results of variational and vector-loop models with manufacturing signature and operating conditions

Model	Scale factor	Mean [mm]	3 · StDev [mm]	A^2	*P* value	Skewness	Kurtosis
1r	1	1.2701	0.053	0.430	0.307	0.011	−0.038
	10	1.2690	0.535	0.830	0.033	−0.014	−0.026
	50	1.2400	2.676	0.640	0.098	−0.001	−0.024
3r	1	1.2702	0.053	0.310	0.549	0.008	−0.029
	10	1.2691	0.534	0.380	0.404	0.023	−0.017
	50	1.2407	2.685	0.300	0.592	0.007	−0.066

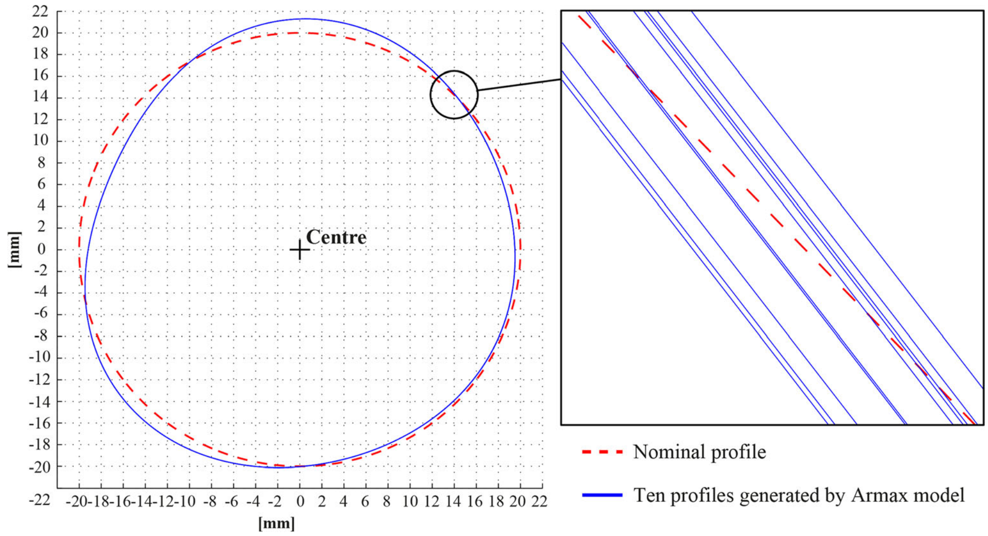

Fig. 7 Ten profiles generated by Armax model

increase of the scale factor, but these gap mean values are very near to the nominal value of 1.2702 mm and to the mean values evaluated previously. The standard deviations change if compared to that previously obtained and with this changing, a magnitude of the manufacturing signature effect on the tolerance analysis has been evaluated, as shown in Fig. 8.

All Anderson-Darling tests show that the obtained distributions of the *g* gap are Gaussian, symmetric and normally concentrated around the mean value.

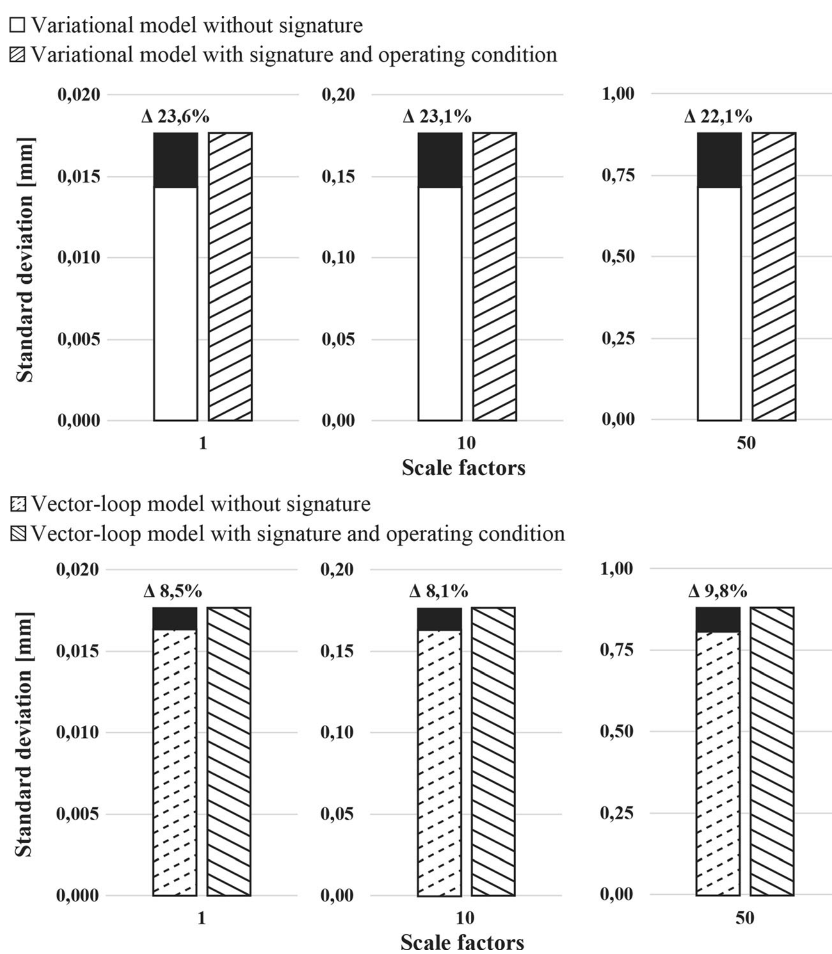

Fig. 8 Percentage variation of three·standard deviation due to the manufacturing signature

6 Conclusions

This paper presents two skin models for tolerance analysis of rigid parts that take into account the pattern left by the turning process on the obtained surfaces, it has been called signature. The developed two models have been applied to a case study in order to compare them with the models of the literature that consider the surfaces as a single feature. The adopted case study involves three parts. The manufacturing signature has been simulated by means of bi-lobe or tri-lobe profiles, which are the contours typically given by a turning process. The first Monte Carlo simulations have shown that the mean values of the gap *g*, due to the variational and vector-loop models without manufacturing signature, are very near to the mean value of 1.2702 mm even by varying the scale factor, but they decrease when the manufacturing signature is taken into account. Moreover, the standard deviations of the gap *g* increase in each model and there are not significant differences with and without considering the manufacturing signature.

To overcome these limitations, due to the Armax model, into the generations of profiles with signature, other simulations have been carried out introducing an operating condition of assembly, the casual rotation of profiles. With this trick, the mean values of the gap *g* due to the variational and vector-loop models with and without manufacturing signature are very near to the mean value of 1.2702 mm even by varying the scale factor with a maximum percentage variation of about 2.5 % when a scale factor equal to 50 is considered. The standard deviations change if compared to that obtained with the first simulations, in particular there are mean variations equal to 23.2 and 8.7 % between the variational model with and without signature and the vector-loop model with and without signature, respectively.

Future works aim to verify the effectiveness to consider manufacturing signature for solving 3D tolerance analysis problems that involve further signatures of manufacturing processes.

References

1. Schleich B, Anwer N, Mathieu L, Wartzack S (2014) Skin Model Shapes: a new paradigm shift for geometric variations modelling in mechanical engineering. Comput Aid Des 50:1–15
2. Armillotta A, Semeraro Q (2011) Geometric tolerances-impact on product design, quality inspection and statistical process monitoring. In: Colosimo BM, Senin N (eds) Geometric tolerance specification, 1st revision. Springer, New York
3. Chase KW, Gao J, Magleby SP (1995) General 2-D tolerance analysis of mechanical assemblies with small kinematic adjustments. J Des Manuf 5(4):263–274
4. Gao J, Chase KW, Magleby SP (1998) General 3-D tolerance analysis of mechanical assemblies with small kinematic adjustments. IIE Trans 30:367–377
5. Martino PM, Gabriele GA (1989) Application of variational geometry to the analysis of mechanical tolerance. In: 8th Biennal Conference on Failure Prevention and Reliability. Quebec, Canada
6. Boyer M, Stewart NF (1991) Modeling spaces for toleranced objects. Int Journal Rob Res 10:470–582
7. Gupta S, Turner JU (1993) Variational solid modelling for tolerance analysis. IEEE Comput Graph 13:64–74
8. Desrochers A, Rivière A (1997) A matrix approach to the representation of tolerance zones and clearances. Int J Adv Manuf Technol 13:630–636
9. Lafond P, Laperrière L (1999) Jacobian-based modeling of dispersions affecting pre-defined functional requirements of mechanical assemblies. In: Proceedings of IEEE International Symposium on Assembly and Task Planning, Porto, Portugal
10. Laperrière L, Lafond P (1999) Modeling tolerances and dispersions of mechanical assemblies using virtual joints. In: Proceedings of ASME Design Engineering Technical Conferences, Sept. 12–15, Las Vegas, NV
11. Clément A, Riviére A, Serré P, Valade C (1998) The TTRSs: 13 constraints for dimensioning and tolerancing. In: ElMaraghy HA (ed) Geometric design tolerancing: theories, standards and applications, 1st edn. Chapman and Hall, London, pp 123–131
12. Rivest L, Fortini C, Morel C (1994) Tolerancing a solid model with a kinematic formulation. Comput Aid Des 26:465–476
13. Bourdet P, Mathieu L, Lartigue C, Ballu A (1996) The concept of the small displacement torsor in metrology. Advanced Mathematical Tools in Metrology II, World Scientific, Singapore, pp 110–122
14. Desrochers A, Ghie W, Laperrière L (2003) Application of a unified Jacobian-Torsor model for tolerance analysis. J Comput Inf Sci Eng 3:1–13
15. Davidson JK, Mujezinovic A, Shah JJ (2002) A new mathematical model for geometric tolerances as applied to round faces. J Mech Des-T ASME 124:609–622
16. Ameta A, Davidson JK, Shah JJ (2007) Influence of form on frequency distribution for 1-D clearance which is generated from tolerance-maps. In: Proceedings of the Tenth CIRP Conference on Computer Aided Tolerancing, Mar. 21–23, Erlangen, Germany
17. Anselmetti B, Chavanne R, Yang J-X, Anwer N (2010) Quick GPS: a new CAT system for single-part tolerancing. Comput Aided Des 42(9):768–780
18. Armillotta A, Semeraro Q (2012) Critical operating conditions for assemblies with parameter-dependent dimensions. In: Proceedings of the 12th CIRP Conference on Computer Aided Tolerancing
19. Polini W (2012) Taxonomy of models for tolerance analysis in assembling. Int J Prod Res 50(7):2014–2029
20. Ameta G, Serge S, Giordano M (2011) Comparison of spatial math models for tolerance analysis: tolerance-maps, deviation domain, and TTRS. J Comput Inf Sci Eng 11(2):1–10
21. Charpentier F, Ballu A, Pailhes J (2012) A scientific point of view of a simple industrial tolerancing process. In: Proceedings of the 12th CIRP Conference on computer aided tolerancing
22. Dantan JK, Anwer N, Mathieu L (2003) Integrated tolerancing process for conceptual design. CIRP Ann Manuf Technology 52(1):135–138
23. Moroni G, Pacella M (2008) An approach based on process signature modeling for roundness evaluation of manufactured items. J Comput Inf Sci Eng 8:1–10
24. Marziale M, Polini W (2010) A new model based on variational solid modelling. In: Proceedings of the ASME 2010 10th Biennial Conference on Engineering Systems Design and Analysis, Instanbul, Turkey, July 12–14
25. Marziale M, Polini W (2009) A review of two models for tolerance analysis of an assembly: vector loop and matrix. Int J Adv Manuf Tech 43(11-12):1106–1123