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ORIGINAL ARTICLE

Theoretical and experimental investigation on modeling of surface topography influenced by the tool-workpiece vibration in the cutting direction and feeding direction in single-point diamond turning

Fujing Tian[1,2] **· Ziqiang Yin**[1,2] **· Shengyi Li**[1,2]

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Abstract The fabrication of high-quality freeform surfaces is based on ultra-precision diamond turning with fast tool servo (FTS) technology which allows direct machining of the freeform surfaces with sub-micrometric form accuracy and nanometric surface roughness. Surface roughness is an important factor in evaluating the performance of the optical freeform surfaces. This paper presents a theoretical and experimental analysis of surface generation in ultra-precision single-point diamond turning. In this model, we take into consideration the basic machining parameters as well as the relative vibration between the workpiece and the tool in both the cutting and feeding directions. Theoretical model is built to predicting the surface roughness of machined flat surface as well as freeform surfaces. A series of experiments have conducted and the results show good correlation between the theoretical model and the fabricated surfaces.

Keywords Surface topography · Modeling · Single-point diamond turning · Optical freeform surfaces

1 Introduction

Ultra-precision single-point diamond turning (UPSPDT) is an ultra-precision machining process to remove surface material at micro-scale for fabricating non-rotational freeform surfaces with sub-micrometric form accuracy and nanometric surface roughness, without the need for any subsequent polishing. Surface topography plays an extremely important role in estimating the surface quality of machined surfaces.

In UPSPDT, surface roughness is influenced by many factors, including cutting conditions, material properties, relative vibration between tool and workpiece, tool vibration, and spindle vibration. Cheung [1] found that the relative vibration between tool and workpiece and tool trajectories were the most important factors influencing the surface topography. The material properties influencing the surface topography was based on the crystal orientation and grain size of the workpiece material. In UPSPDT, a few researchers have studied the influences of tool-workpiece vibration in computer numerical control turning and diamond turning on surface topography [2–10]. SAbouelatta and Madl [2] employed a mathematical tool to analyze the vibration and surface roughness data obtained in a series of cutting experiments. However, the proposed correlation model did not take into consideration the mechanism of metal-cutting process; therefore, no physical model was proposed to explain how vibration affects the surface roughness. Cheung and Lee [3] proposed a surface roughness simulation model, which takes into account the effect of process parameters, tool geometry, and relative vibration between the workpiece and the tool. The authors took on the assumption that the relative vibration between the tool and the workpiece is a steady simple harmonic motion with small amplitude and a low frequency. However, only the relative vibration in the cutting direction is considered which is adopted from the assumptions made by Dong-Sik Kim et al. [8] and H. Wang et al. [9]; the influences of the relative vibration in the feeding direction is neglected. Current models focus on the relative tool-workpiece vibration in the cutting direction. However, based on our experimental measurements, the contribution of relative tool-workpiece

✉ Shengyi Li
18674440178@163.com

[1] School of Mechatronics Engineering and Automation, National University of Defense Technology, Changsha, China

[2] Hu'nan Key Laboratory of Ultra-precision Machining Technology, Changsha, China

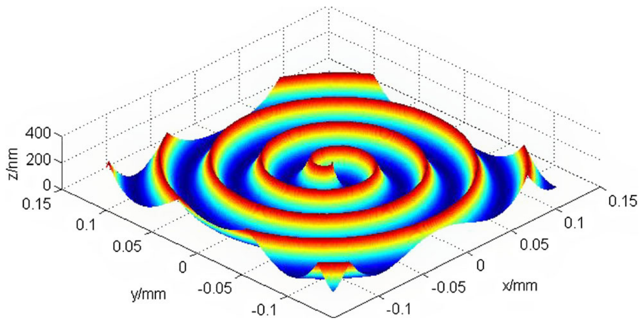


Fig. 1 Simulated three-dimensional surface topography of a diamond-turned flat surface

vibration in the feeding direction is significant and cannot be ignored in the model. As compared with Chih-yu Huang et al. [10], this proposed model can predict the surface roughness of machined freeform surface accurately.

2 Modeling of surface topography

Under ideal conditions, the three-dimensional surface profile of the diamond-turned surface is shown in Fig. 1. The cutting tool will follow the real tool path to reproduce the tool profile on the machined surface in the form of feed marks.

However, during the actual cutting process, there are several factors which influence the surface roughness such as the vibration between the tool and the workpiece, air nozzle direction, thermal variation, and cutting fluids. Thermal variation, air nozzle direction, and cutting fluids can be recognized as environmental factors and are not the main focus of this paper. Above all, the vibration between the tool and the workpiece is one of the most important factors influencing the machined surface roughness. Some of developed models are based on the relative vibration between the tool and the workpiece in the cutting direction [3, 4]. However, due to air bearings in our developed experimental setup, there is also significant relative tool-workpiece vibration in the feeding direction. Although this vibration is not as evident in modeling flat surfaces, it is equally important to the vibration in the cutting direction when considering curved surfaces. Subsequently, we developed a more accurate model based on these two vibrations and other cutting parameters to predict the surface topography of the flat surface as well as complex surfaces of the workpiece. Figure 2 shows diagrammatic drawing of the cutting direction and the feeding direction in diamond turning process.

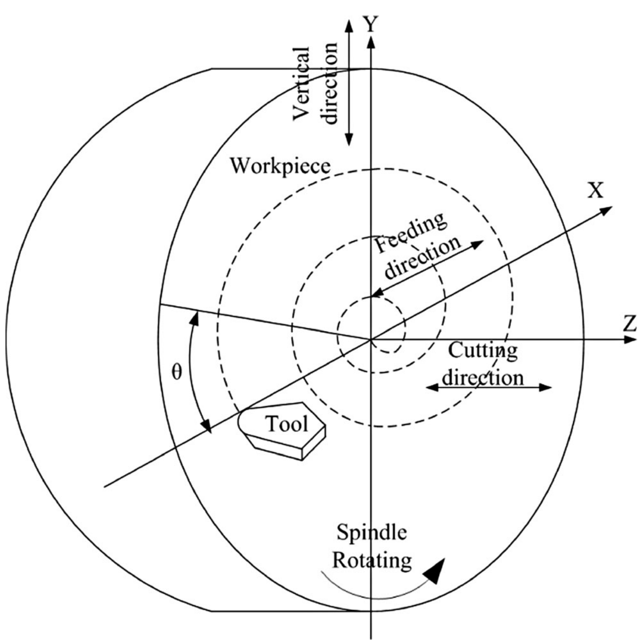


Fig. 2 Diagrammatic drawing of cutting direction and feeding direction in diamond turning process

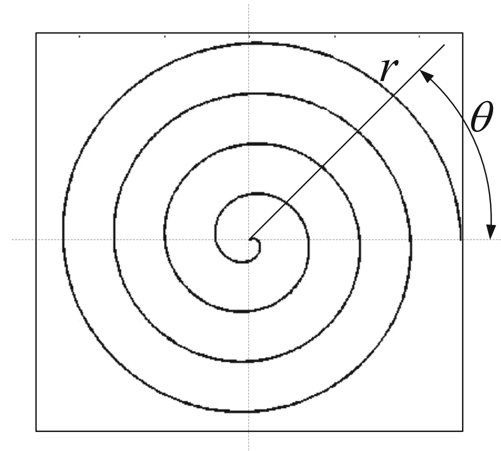


Fig. 3 Schematic diagram of tool path

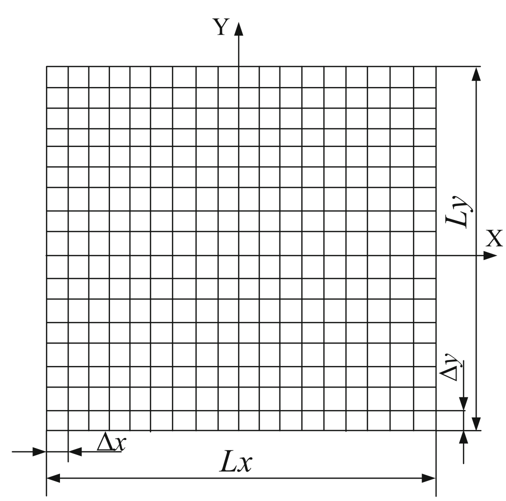


Fig. 4 Schematic diagram of meshed surface

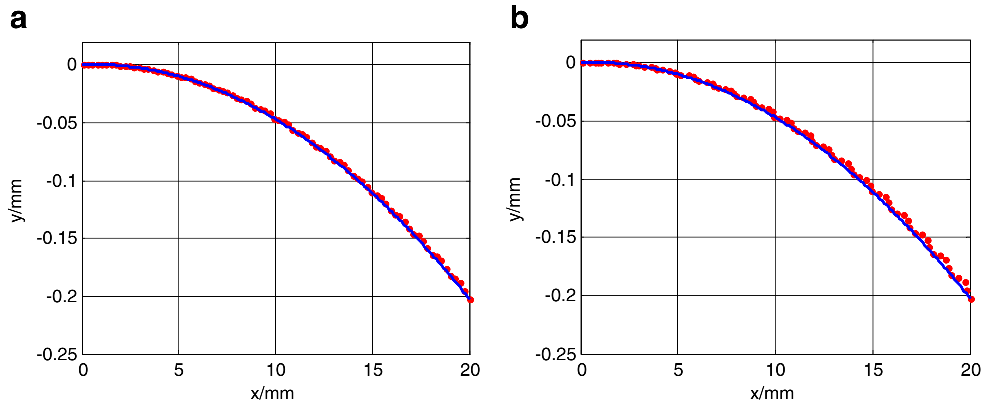


Fig. 5 Tool tip position on a freeform surface with relative tool-workpiece vibration in **a** cutting direction only and **b** both cutting and feeding directions

To simulate the three-dimensional surface topography, the surface is sampled into a finite number of equally spaced radial sections. The number of sections Ns can be expressed as

$$N_s = 2\pi \big/ \Delta\theta \tag{1}$$

where $\Delta\theta$ is the angular resolution, and the number of spindle rotation rounds during machining Nr can be calculated as

$$N_r = R_w \big/ f \tag{2}$$

where R_w is the radius of the workpiece and f is the federate (mm/rev). Therefore, the total number of simulated points is

$$N_t = N_s N_r + 1 \tag{3}$$

In this model, we apply the meshing method. Through calculating the contour height value of all mesh points in simulated area, we get the resulted surface topography based on Z-map simulation algorithm. Figure 3 shows the tool path and Fig. 4 shows the meshing of the simulated surface.

In Fig. 3, the polar accordinated position of the tool tip is shown as

$$\begin{cases} r = R_w - f \cdot f_r \cdot t \\ \theta = 2\pi f_r \cdot t \end{cases} \tag{4}$$

where n is the spindle speed (rpm), $f_r = n/60$ is the spindle rotational frequency (Hz), t is the time (s), s, and $\theta \in [0, 2N_r\pi)$. To convert the tool path into a rectangular coordinate system, there are

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \tag{5}$$

We assume these two vibrations to be simple harmonic motions. This gives us a relatively simple model to build, but can provide the most useful information from the diamond turning process. The simple harmonic motion in the x and z directions can be described as shown in Eq. (6), respectively.

$$\begin{cases} X_v(t) = A_x(1 - \cos(2\pi f_x t + \varphi_x)) \\ Z_v(t) = A_z\left(1 - \cos\left(2\pi f_z t + \varphi_z\right)\right) \end{cases} \tag{6}$$

where A_x and A_z are the amplitudes of vibration in each direction and f_x and f_z are the frequencies of the vibration in each direction. The phase φ_x and φ_z are the initial phase shifts for the vibration in each direction. For simplicity, we can assume $\varphi_x = \varphi_z = 0$.

The machined surface topography under vibration is directly determined by the relation between the vibration frequencies f_x and f_z and the spindle rotational frequency f_r. Their ratios are

$$\begin{cases} f_x/f_r = I_{fx} + D_{fx} \\ f_z/f_r = I_{fz} + D_{fz} \end{cases} \tag{7}$$

where I_{f_x} and I_{f_z} are the nonnegative integer, D_{fx} and D_{fz} are the fractional part, and $\in (-0.5, 0.5]$.

Table 1 Surface parameter for the freeform surface

X^5Y^0	2.0880e−012	X^3Y^0	−1.5960e−007	X^1Y^4	2.1470e−012	X^0Y^5	−1.2870e−023
X^4Y^1	−1.3210e−023	X^2Y^3	−1.5970e−024	X^1Y^3	7.2520e−022	X^0Y^4	1.9100e−009
X^4Y^0	1.7350e−009	X^2Y^2	3.6450e−009	X^1Y^2	−1.5960e−007	X^0Y^3	4.3670e−020
X^3Y^2	4.2430e−012	X^2Y^1	5.3830e−020	X^1Y^1	4.3660e−019	X^0Y^2	−5.3840e−004
X^3Y^1	−5.6410e−022	X^2Y^0	−5.3840e−004	X^1Y^0	7.0160e−004	X^0Y^1	−3.3880e−018

In Fig. 4, the length and width of simulated area are L_x and L_y, and the resolution are Δ_x and Δ_y, respectively. The simulated center is the center of workpiece, and the coordinate of point (i,j) is

$$\begin{cases} X_{i,j} = i\Delta x - L_x/2 \\ Y_{i,j} = j\Delta y - L_y/2 \end{cases} \tag{8}$$

The corresponding polar coordinate is

$$\begin{cases} r(i,j) = \sqrt{X_{i,j}^2 + Y_{i,j}^2} \\ \theta(i,j) = \arctan\left(Y_{i,j}/X_{i,j}\right) \end{cases} \tag{9}$$

The all possible radius values of the tool tip position in the θ radial section are

$$r_k = R_w - S_f\left((k-1) + \theta(i,j)/2\pi\right) + A_x\left\{1-\cos\left[I_{fx}\cdot\theta(i,j) + D_{fx}\cdot(2(k-1)\pi + \theta(i,j))\right]\right\} \tag{10}$$

where $k = 1, 2, \cdots, N_t$. The height value of the tool edge profiles on this point is

$$h_k = r_t\left[1-\sqrt{1-\left(\frac{r(i,j)-r_k}{r_t}\right)^2}\right] + A_z\left\{1-\cos\left[I_{fz}\cdot\theta(i,j) + D_{fz}\cdot(2(k-1)\pi + \theta(i,j))\right]\right\} \tag{11}$$

Then, comparing all the tool edge profiles with the depth of cut d_c, it is easy to get the residual contour through achieving the minimum.

$$Z_{i,j} = \min(h_k) \tag{12}$$

Then, comparing all the tool edge profiles with the depth of cut d_c, it is easy to get the residual contour through achieving the minimum.

During the actual diamond turning process, the actual cutting point on the diamond tool changes constantly based on the local shape of the workpiece it cuts, and the diamond tool always cuts the local surface of the workpiece perpendicularly. As a result, the contributions of the two vibrations in the cutting and feeding directions are different along a curved surface. We apply the coordinate rotation matrix to solve this problem, as shown in Eq. (13).

$$\begin{bmatrix} X'_v(t) \\ Z'_v(t) \end{bmatrix} = \begin{bmatrix} \cos\beta - \sin\beta \\ \sin\beta \cos\beta \end{bmatrix}\begin{bmatrix} X_v(t) \\ Z_v(t) \end{bmatrix} \tag{13}$$

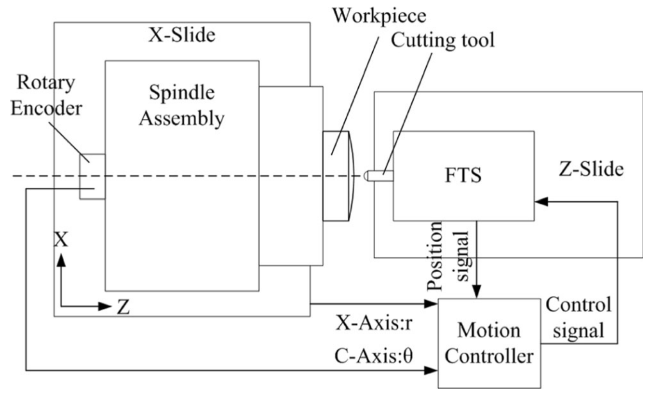


Fig. 6 FTS mounted on diamond turning machine with cutting

where β is the angle of rotation from the X-axis. Figure 5 gives an example of the tool tip position on a freeform surface (rear-view mirror), and its surface parameters are shown in Table 1 [11], $A_x = 100\mu\text{m}$, $A_z = 10\text{nm}$, $f_x = 20\text{Hz}$ and $f_z = 200\text{Hz}$. The amplitude and frequency of both vibrations are purposely exaggerated to have a clearer demonstration. In Fig. 5a, we only consider the vibration in the cutting direction, and the influence of this vibration continuously decreases from center to the outer edge of the radial section surface ($y = 0$). This is because when the tool is at or near the center, the vibration in the cutting direction plays nearly perpendicularly on the surface. On the other hand, when we consider the vibrations in both cutting and feeding directions as in Fig. 5b, the influences of these two vibrations act consistently throughout the whole surface. As a result, it is sufficient to model freeform surfaces when we take into consideration both the relative tool-workpiece vibrations in the cutting and feeding directions.

Once we have acquired the surface topography, it is straightforward to calculate the maximum height (peak to valley) of the surface profile (Rt), the arithmetic average surface roughness (Ra), and root mean squared surface roughness

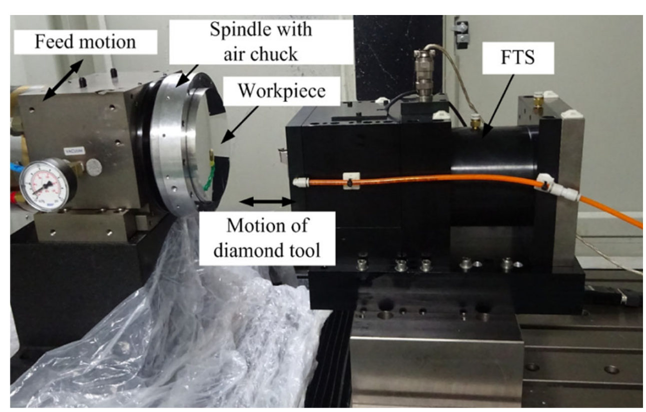


Fig. 7 FTS mounted on diamond turning machine with cutting

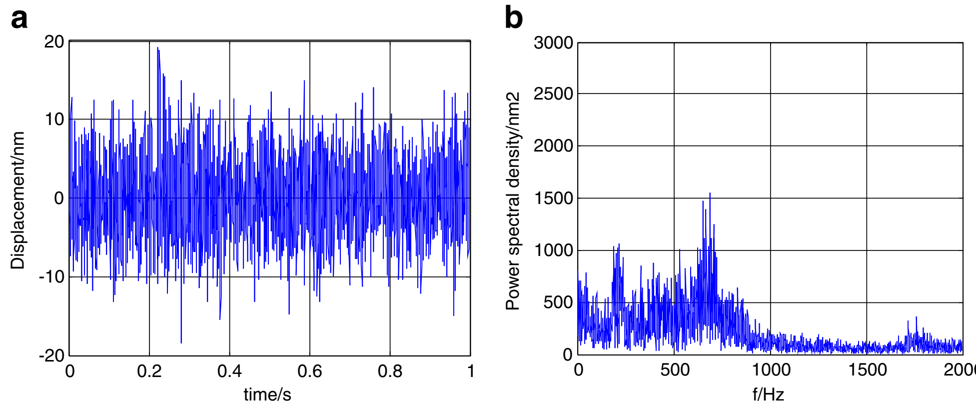


Fig. 8 The relative tool-workpiece vibration in **a** cutting direction and **b** its spectral plot

(Rq). The formula for each parameter described above is shown in the following equations.

$$R_t = \max(Z_{i,j}) - \min(Z_{i,j}) \quad (14)$$

$$R_a = \frac{1}{MN}\sum_{j}^{N}\sum_{i}^{M}\left(\left|Z_{i,j}\right|\right) \quad (15)$$

$$R_q = \sqrt{\frac{1}{MN}\sum_{j}^{N}\sum_{i}^{M}\left(Z_{i,j}^2\right)} \quad (16)$$

3 Experimental verification

3.1 Experimental setup

The basic principle of the FTS system used in machining the freeform surfaces is shown in Fig. 6. In this paper, the developed FTS system consists of an air-bearing stage driven by a unique voice coil motor (Fig. 7). The FTS system has a total stroke of 30 mm and an acceleration of 920 m/s^2. Position feedback sensor is provided by a Heidenhain linear encoder, which has a theoretical resolution of 0.5 nm. The glass scale is attached to the air slide of the FTS system. The developed FTS system mounted on the diamond turning machine is shown in Fig. 8. This machine utilizes two hydrostatic linear axes. The straightness on all slides is less than 200 nm over 100 mm of each axis. The main spindle is a bi-directional air-bearing spindle with vacuum and air feed through the shaft. The radial and axial error motion of the spindle is less than 50 nm [11].

3.2 Experimental results and discussions

To verify this proposed model, we carry out a series of experiments by cutting aluminum flat surfaces as well as freeform surfaces. The experiments can be divided into two groups. The machining parameters are shown in Table 2. Group 1 analyzes the relationship between spindle speed and surface roughness by changing the spindle speed from 300 to 600 rpm while keeping other cutting parameters constant. Group 2 acts freeform surface cutting test to verify the surface topography obtained in the previous section. The surface topography and the surface roughness of the machined surface are measured with a ZYGO NewView Scanning White Light Interferometer (SWLI).

The relative tool-workpiece vibrations in both the cutting and feeding directions are measured by the program Power PMAC IDE provided with the FTS motion controller and the laser displacement sensor, respectively, and the measurement results are shown in Figs. 8 and 9. The relative tool-workpiece vibration in the cutting direction is shown in Fig. 8a and Fig. 8b shows its spectral plot. Similarly, Fig. 9a shows the relative tool-workpiece vibration in the feeding direction and

Table 2 Machining parameters for the four groups

Group	Surface	Spindle speed/rpm	Feedrate /mm/min	Depth of cut/µm	Tool nose radius/mm	Workpiece material
1	Flat surface	300	2	2	1	Aluminum
		500	2	2	1	
		600	2	2	1	
2	Freeform surface	150	0.75	2	0.5	

Fig. 9 The relative tool-workpiece vibration in **a** feeding direction and **b** its spectral plot

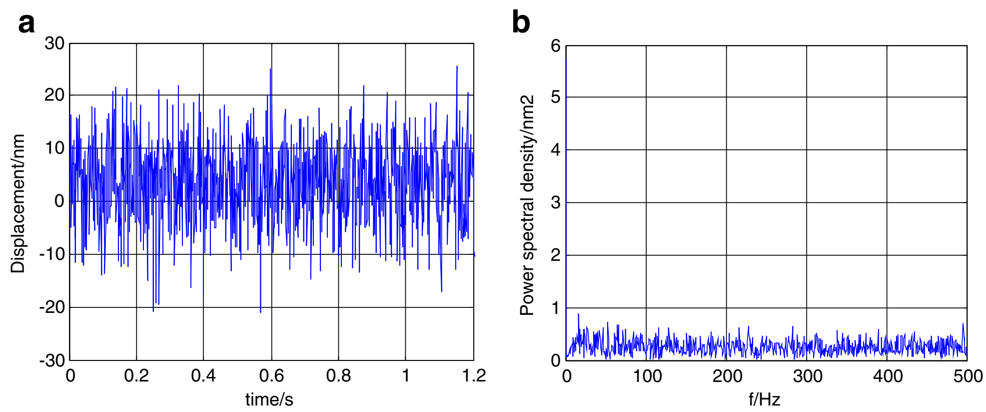
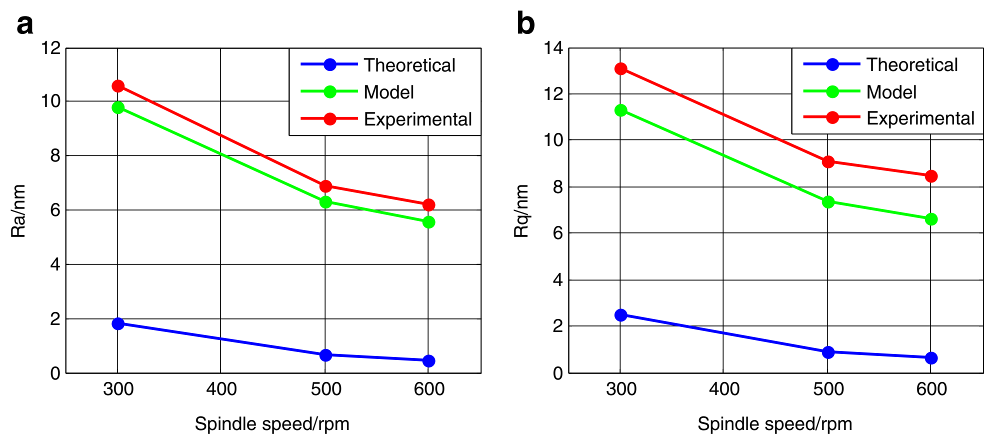


Fig. 10 Measurement results for group 1. **a** Ra and **b** Rq



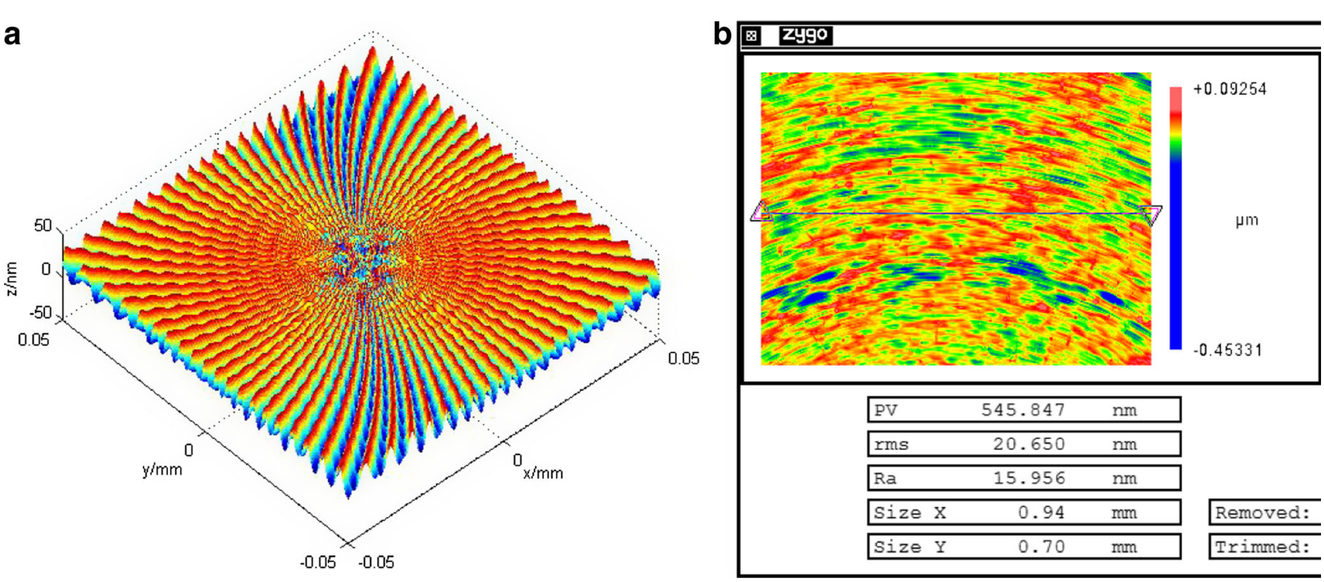


Fig. 11 Surface topography of the freeform surface for group 2. **a** Simulated results. **b** Measurement results

Table 3 Comparison of surface roughness

	Theoretical	Model	Experimental result
Ra/nm	2.08	15.53	15.96
Rq/nm	2.8	18.46	20.65

Fig. 9b shows its spectral plot. We can find that a dominant mode of vibration with an average magnitude of about 15 nm and frequency of 192 Hz occurred in the cutting direction, and a dominant mode of vibration with an average magnitude of about 25 nm and frequency of 16.88 Hz occurred in the feeding direction. However, there are also a few weaker modes of vibration in the results, we ignore those vibration modes and only consider the dominant mode in our model.

Figure 10 shows the measurement results for group 1, which indicates the relationship between the spindle speed, the arithmetic average surface roughness (Ra), and root mean squared surface roughness (Rq). Suppose both the amplitude of the relative tool-workpiece vibrations in the cutting and feeding directions are zero. The theoretical value of the Ra and RMS is calculated by Eqs. (15) and (16).

In this figure, we can see that the proposed model gives a better evaluation of the surface roughness than the theoretical prediction. The theoretical surface roughness is much lower than the experimental results. This is because in the theoretical case, we assume the cutting conditions to be ideal and ignore all other influenced factors, including dynamic cutting force, spindle chatter, tool wear, cutting heat, material properties, system errors of machine tool, environmental vibration, and so on. However, we take into consideration the tool-workpiece vibration. As a result, the prediction of the model is more accurate than the theoretical value. Due to the environmental factors such as thermal variation, air nozzle direction, and cutting fluids, there is still a small discrepancy between the simulated results and the experimental results. The simulated surface topography and the measurement results are shown in Fig. 11, and Table 3 shows the comparison of the surface roughness for group 2. We can see that the model predicts well as compared to the theoretical value.

4 Conclusions

We propose a model to simulate the surface topography generated by a single-point diamond turning with FTS technology. In this model, we take into consideration the relative tool-workpiece vibration in both the cutting and feeding directions. By introducing a proper coordinate rotation matrix, the proposed model can describe the flat surface as well as freeform surfaces. We give an example by showing the tool tip position on a radial section of freeform surface with and without considering the vibration in the feeding direction, and conclude that it is necessary to include both the vibrations in the cutting and feeding directions in order to make the model accurate while calculating freeform surfaces. A series of experiments to cut flat surfaces and freeform surface are carried out to verify the proposed model. There is a good correlation between the simulated results and the experimental results. In the future, we will give a more general simulation algorithm to predict the surface topography of freeform surface in FTS turning.

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References

1. Cheung CF, Lee WB (2000) A multi-spectrum analysis of surface roughness formation in ultra-precision machining. Precis Eng 24 (1):77–87
2. Abouelatta OB, Madl J (2001) Surface roughness prediction based on cutting parameters and tool vibrations in turning operations. J Mater Process Technol 118:269–277
3. Cheung CF, Lee WB (2000) A theoretical and experimental investigation of surface roughness formation in ultra-precision diamond turning. Int J Mach Tools Manuf 40:979–1002
4. Cheung CF, Lee WB (2003) Surface generation in ultra-precision diamond turning: modeling and practices. Prof Eng
5. Wang H, To S, Chan CY, Cheung CF, Lee WB (2010) A theoretical and experimental investigation of the tool-tip vibration and its influence upon surface generation in single-point diamond turning. Int J Mach Tools Manuf 50:241–252
6. Risbood KA, Dixit US, Sahasrabudhe AD (2003) Prediction of surface roughness and dimensional deviation by measuring cutting forces and vibrations in turning process. J Mater Process Technol 132:203–214
7. Chen T, Li S, Han B, Liu G (2014) Study on cutting force and surface micro-topography of hard turning of GCr15 steel. Int J Adv Manuf Technol 72:1639–1645
8. Kim D-S, Chang I-C, Kim S-W (2002) Microscopic topographical analysis of tool vibration effects on diamond turned optical surfaces. J Int Soc Precis Eng Nanotechnol 26:168–174
9. Wang H, Ton S, Chan CY, Cheung CF, Lee WB (2010) A theoretical and experimental investigation of the tool-tip vibration and its influence upon surface generation in single-point diamond turning. Int J Mach Tools Manuf 50:241–252
10. Huang C-Y, Liang R (2015) Modeling of surface topography in single-point diamond turning machine. Appl Opt 54(23):6969–6985
11. F Tian, Z Yin, S Li (2015) Fast tool servo diamond turning of optical freeform surfaces for rear-view mirrors. Fast tool servo diamond turning of optical freeform surfaces for rear-view mirrors. Int J Adv Manuf Technol