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ORIGINAL ARTICLE

A new computer-aided tolerance analysis and optimization framework for assembling processes using DP-SDT theory

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Abstract Tolerance analysis is frequently used in predicting the product quality and balancing the design tolerances in mechanical assemblies. Generally, the tolerance analysis procedure is rather complex and cumbersome, and the existing computer-aided tolerance analysis methods are insufficient in dealing with some assembly information and user-defined quality requirements. This paper presents a new comprehensive tolerance analysis and optimization framework using deviation propagation and small displacement torsor (DP-SDT) theory. In this framework, four modules are designed to model the tolerances, analyze the assembly processes, predict the product quality, and optimize the tolerances respectively. Comparing with the existing methods, this framework can better support the complex assembly information like 3D dimensional tolerances, geometric dimensioning and tolerancing (GD&Ts), different tolerance zones, geometric information, assembly sequence, and various kinds of locating modes. And more practical quality requirements besides the distance precisions can be analyzed. The framework is a helpful supplement in tolerance analysis field. An application prototype using the framework has been developed for SolidWorks, and a tolerance optimization example of lathe saddle is provided to verify the performance of the framework.

Keywords Tolerance analysis · Quality prediction · Tolerance optimization · Deviation propagation

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1 Introduction

In product design, tolerance analysis is a powerful tool to predict the product quality and balance the quality and cost. The main work in the tolerance analysis process is to construct and analyze the quality tolerance (Q–T) function, which describes the relationship between the target quality requirements and design tolerances. Researches have shown that the product quality is comprehensively affected by the part geometric information, tolerances and tolerance types, assembly sequence, locating modes, fixture designs, etc. [1–4]. And in a mechanical assembly, there are many parts, various kinds of tolerances, tolerance zones and locating modes. Consequently, the formulation expression of the Q–T function is very difficult to acquire and analyze [5]. In this situation, many tolerance analysis methods have been proposed.

In order to get a reliable prediction of the product quality, the whole assembly design process should be considered and different kinds of tolerances and locating modes should be supported in the tolerance analysis method. Traditional tolerance analysis methods (such as linearized method, root sum squares, System Moments method, Taguthi Test, etc.) can get a relatively simple Q–T function, but they are not completely reliable for some of the following reasons [6–9]: (1) the parts should be completely constrained to construct the Q–T functions; (2) the models are insufficient in dealing with the exceptional kinds of tolerances and quality requirements; (3) the tolerances should be independent; and (4) the methods adopt different degrees of simplification by ignoring some of the locating information, sequence information. For example, we can only analyze 1D tolerance analysis in Pro/ENGINEER Tolerance Analysis, and the supported tolerances are limited to dimensional tolerances, position, and profile. In this case, methods like stream of variation (SoV) and modal interval and small degrees of freedom (MI-SDOF) are developed to have a better performance in describing the assemblies [2, 10]. Yet the definition of the tolerance model and assembly process of these methods are very cumbersome, and the constructed Q–T functions are rather complex, which make it difficult to optimize the tolerances in the following steps.

DP-SDT theory provides a novel method to synthesize the deviation information in the assembly system [11]. Based on this theory, we can intuitively describe different kinds of tolerances, locating modes, assembly sequence, and fixtures in the tolerance analysis procedure, but it still needs to be consummated: (1) there are too many kinds of tolerances and tolerance zones supported in this method, and we have designed a special solving algorithm for each situation, which make the solving procedure of DP-SDT method rather complex and cumbersome, while there is not an effective and integrated analysis system to process the solving procedure automatically; (2) the model information (such as the locations, dimensions, and directions of the features and points) is measured by manual work, and a software program for CAD systems with interaction module is the best way to get the information; (3) the method can only deal with 1D dimensional tolerances, geometric dimensioning and tolerancing (GD&Ts), and the “3-2-1” location mode; and (4) the solving procedure is not consummate in multi-stage assemblies.

This paper presents a comprehensive framework for tolerance analysis and optimization based on DP-SDT theory. In this framework, we design four modules to predict the product quality and optimize the tolerances: tolerance modeling, deviation propagation, tolerance analysis, and tolerance optimization. Tolerance modeling module can deal with almost all kinds of tolerances, such as 3D dimensional tolerances and standard GD&Ts (ASME Y14.5M-1994). Deviation propagation module can calculate the feature deviations and part deviations depending on the assembly sequence and locating modes. Tolerance analysis module can evaluate the required quality using fraction defective, influence chart, frequency chart and contribution chart. Tolerance optimization module can help the designers to balance the tolerances and improve the product quality. Comparing with the existing researches and computer-aided tolerance (CAT) analysis systems, this framework does better in the following: (1) supporting various kinds of location patterns, constraint patterns, and fit patterns; (2) supporting 3D dimensional tolerances, GD&Ts, different kinds of tolerance zones, and user-defined quality requirements; (3) correlated tolerances are analyzed in a more reasonable way; and (4) assembly information like location mode, assembly sequence, and fixtures are integrated in the framework. Meanwhile, a tolerance analysis application for a CAD system has been developed, which has greatly simplified the analysis procedure and parameter inputs comparing with the original DP-SDT theory. In a word, the proposed framework provides a helpful solution in tolerance analysis and optimization field.

The paper is organized as follows. A literature review about typical tolerance analysis methods is presented in Section 2. The structure and components of this CAT framework is described in Section 3. The detailed module design of the framework is provided in Section 4. A CAT application prototype is developed in Section 5. Section 6 gives out an example about lathe saddle tolerance analysis to verify the performance of this framework, and Section 7 presents a conclusion of the method.

2 Related works

Tolerance analysis aims at predicting the product quality and improving the tolerance design. However, we have to deal with various kinds of assembly information and complex Q–T function in the analysis procedure. In order to solve the above problem, most of the researches process in the following four steps: (1) construct a unified tolerance model to synthesize the dimensional tolerances, GD&Ts, and geometric information; (2) construct an assembly model to integrate the process information such as assembly sequence, locating modes, etc.; (3) predict the product quality using mathematical tools; and (4) evaluate and optimize the tolerances using statistical charts, algorithms, etc.

A practical tolerance model is the premise of tolerance analysis method. Most of the researchers are absorbed in designing a tolerance model that is more close to reality, applicable, and easier to analyze. Franciosa et al. [12] presented a general approach to automatically calculate the variation parameters for planar or cylindrical features using the design tolerance parameters. Dantan et al. [13] proposed an uncertainty formulation method based on the uncertainty classification. Chen et al. [14] used a unified Jacobian-Torsor model for tolerance representation. And Schleich et al. [15] proposed a method to construct a model to describe the geometric tolerances based on points and boundary definitions.

Assembly models mainly describe the connection relationships between the parts, analyze the deviations of the parts, and finally construct the Q–T functions. A commonly used tolerance analysis model is the full contact theory [16]. In this model, six points are chosen in a completely located part, and the tolerance deviations propagate through the deviations of these points. Based on this conception, linearized methods and “dimensional hierarchization matrix” are applied to construct the Q–T functions [17–19]. Meanwhile, to make the assembly model more applicable, many researchers constructed their models for over-constraint assemblies [20, 21]. Liu et al. [22] adopted a generic state space approach to model the variation propagation in general MAPs. And, Asante et al. [23] presented a constraint-based approach for tolerance analysis in a multi-operation single setup and multi-operation multi-setup part–fixture assembly.

The commonly used quality prediction methods include the linearized methods, statistical methods, and test methods. Linearized methods hypothesize that the Q–T function is linear, and usually analyze the function in two situations: worst case (WC) and root sum squares (RSS) [24–26]. Statistical methods analyze the Q–T function using probability and statistics methods. Researchers have proposed many kinds of statistical methods, such as system reliability method [27] and system moment method [28]. Monte Carlo method is the most frequently used test method, and many researches applied this method to tolerance analysis [29].

Tolerance optimization can help the designers to improve the product quality, balance the tolerances, and control the manufacturing cost. As the Q–T functions usually have many variables (design tolerances) and constraints, most of the researchers adopt optimization algorithms to adjust the design tolerances. Wu et al. [30] adopted genetic algorithm to make a discrete optimization of the quality and cost. Piepel et al. [31] optimized the manufacturing cost using quality loss function. Chiang et al. [32] proposed an integrated optimization algorithm based on tolerance cost and quality cost to balance the design tolerances.

3 Tolerance analysis framework by DP-SDT theory

DP-SDT is a novel deviation synthesis theory based on the propagation and accumulation analysis of feature deviations [11]. In this theory, three kinds of deviations are designed to describe the variation information of the assembly process, include tolerance deviation, feature deviation, and quality point deviation. Tolerance deviation describes the direct influence of an individual tolerance to the corresponding feature or part. Feature deviation represents the rotation and translation of the feature contrast to the ideal position. Quality point deviation is the deviation of the quality point designed to calculate the quality requirements.

Tolerance deviation and feature deviation are expressed as a 6D vector $\tau_{\mathrm{SDT}}=(\alpha,\beta,\gamma,u,v,w)$ using small displacement torsor (SDT) model, and quality point deviation is expressed as a 3D vector $\Delta p_{\mathrm{SDT}}=(u,v,w)$, where α, β, γ, u, v, *and* w represent the rotation values and translation values along the x/y/z-axes, respectively [33]. Then the tolerance deviations accumulate and propagate through the assembly process to calculate the predicted quality requirements using deviation propagation (DP) theory.

Assume there are m key features, and the feature deviations are $\tau_1^F,\ \tau_2^F,\cdots,\tau_m^F$; there are n tolerances, and the values are t_1, $t_2,\cdots,t_n$; there are r tolerance deviations for the features, and we note them as $\tau_t^{*1},\tau_t^{*2},\cdots,\tau_t^{*r}$; there are e parts, named π_1, $\pi_2,\cdots,\pi_e$, and the tolerance deviations of locating tolerances to the parts are $\tau_{\pi1}^L,\tau_{\pi2}^L,\cdots,\tau_{\pi e}^L$; there are s quality points, and the quality point deviations are $\Delta p_1,\Delta p_2,\cdots,\Delta p_s$; and the parameter of the final quality requirement is q. Let $t=(t_1,\cdots,t_n)^{\mathrm{T}}$, $\tau_t=(\tau_t^{*1};\cdots;\tau_t^{*r})$, $\tau^L=(\tau_{\pi1}^L;\cdots;\tau_{\pi e}^L)$, $\tau^F=(\tau_1^F;\cdots;\tau_m^F)$, and $\Delta p_{\mathrm{list}}=(\Delta p_1;\cdots;\Delta p_s)$. According to DP-SDT theory, the Q–T function can be formulated using the following V/F/G functions.

$$\begin{cases}\left(\boldsymbol{\tau}_t,\boldsymbol{\tau}^L\right) = \mathrm{V}\ (\boldsymbol{t})\\ \boldsymbol{\tau}^F = \mathrm{F}\ \left(\boldsymbol{\tau}_t,\boldsymbol{\tau}^L\right)\\ (q,\Delta\boldsymbol{p}_{\mathrm{list}}) = \mathrm{G}\left(\boldsymbol{\tau}^F,\boldsymbol{t}\right)\end{cases} \tag{1}$$

The tolerance analysis procedure of DP-SDT theory is designed in three steps.

Step 1. In each part, calculate the tolerance deviations for the related features and locating tolerance deviation.
Step 2. Calculate the part deviations and feature deviations according to the assembly sequence.
Step 3. Calculate the deviations of selected quality points, and solve the quality results.

Figure 1 shows the scheme of the tolerance analysis and optimization framework proposed in this paper. In the framework, we have designed four modules to realize the above analysis procedure: tolerance modeling, deviation propagation, tolerance analysis, and tolerance optimization. By means of these modules, the product quality can be predicted and optimized using the following assembly information: design tolerances, assembly sequence, location modes, and geometric information of the product. The detailed module design is presented in the next section.

4 Module design of the framework

4.1 Tolerance modeling module

This module is designed to calculate the feature deviations and part deviations which are directly caused by the tolerances, and expresses them using SDT model. All of the calculated tolerance deviations use the temporary feature coordinate system (FCS) for each tolerance.

In a practical assembly system, the tolerances can directly affect the shape of the parts and the assembly gaps in locating process. For convenience, the two effects to the product should be analyzed separately. In this module, "tolerance deviations for features" is designed to describe the effect of an individual tolerance to a feature (in Section 4.1.1), and "locating tolerance deviations for parts" is designed to describe the effect of the locating tolerances to the installing part (in Section 4.1.2).

4.1.1 Tolerance deviations for features

Tolerance deviations for the features are decided by the tolerance value, tolerance type, tolerance zone type, and geometric

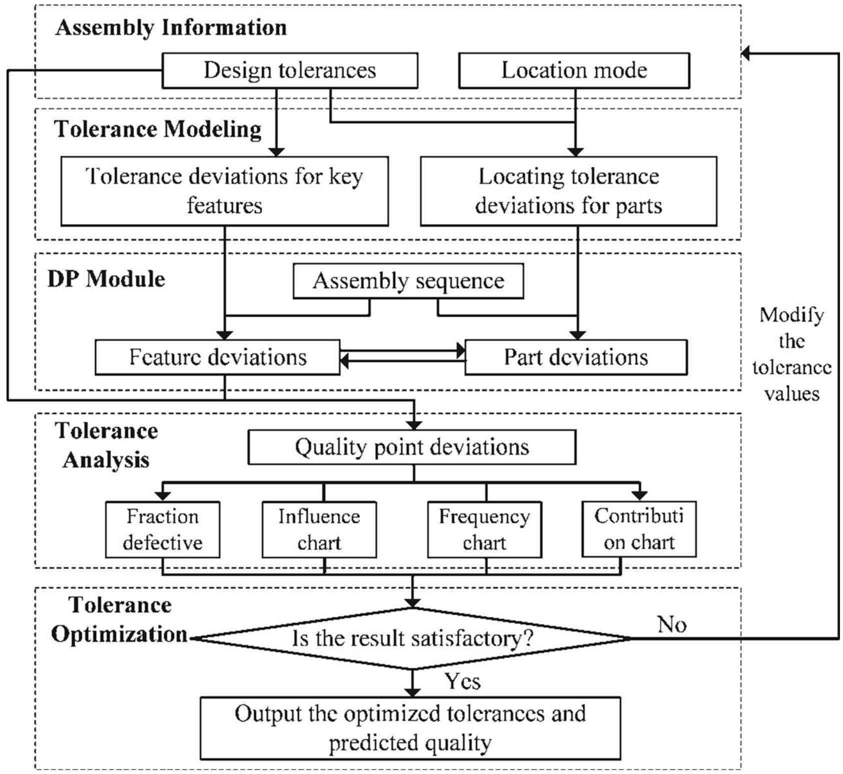


Fig. 1 Tolerance analysis and optimization framework

design of the parts at the same time. In this framework, we can analyze 3D dimensional tolerances and 14 kinds of GD&Ts (ASME Y14.5 M-1994) with four kinds of tolerance zones. Particularly, the GD&Ts are defined in the state of RFS, and then the corresponding deviations are independent.

For 3D dimensional tolerances analysis, we should firstly construct the FCS. Assume the tolerance is T, tolerance value is t, and there are three key tolerance action points (P_1, P_2, and P_3). The points are non-collinear, and defined for each dimensional tolerance. If tolerance T affects P_1, we choose P_1 as the origin, $\overline{P_2P_3}$ as the y-axis, and normal vector (n) of the surface $P_1P_2P_3$ as the z-axis, and construct the required coordinate system FCS. The tolerance deviation is $\tau^{\mathrm{FCS}}=(\alpha, \beta, \gamma, u, v, w)$ can be formulated as follows.

$$\begin{cases} \alpha = c1 \times k \times t \times |\cos\theta|/L \\ w = c1 \times t \times |\cos\theta| \\ \beta = 0, \quad \gamma = 0, \quad u = 0, \quad v = 0 \end{cases} \tag{2}$$

Where $c1$ represent the action coefficient of P_1, k represent the rotation coefficient of P_1, θ represent the angle of n and the dimension direction, and L represents the distance of P_1 to P_2P_3. If P_1 is defined as the increasing link, $c1=1$, otherwise $c1=-1$. If the x-component of P_1 is positive, $k=1$, otherwise $k=-1$.

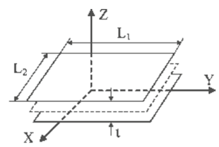
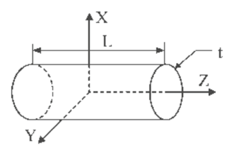
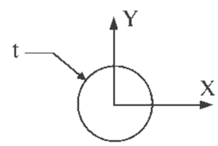
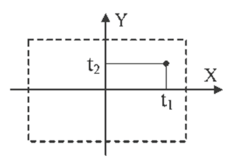
For GD&T analysis, assume the tolerance deviation is $\tau^{\mathrm{FCS}}=(\alpha, \beta, \gamma, u, v, w)$. In this module, four kinds of tolerance zones are modeled, and Table 1 shows the FCS and deviation for each tolerance zone type by parametric equations, where t, t_1, and t_2 represent the tolerance values, L, L_1, and L_2 represent the dimensions of the features.

4.1.2 Locating tolerance deviations for parts

The locating tolerance deviations depend on several correlated tolerances and the locating mode. In a practical assembly, these tolerances can be synthesized as the assembly gaps. In addition, the role of a locating feature in the locating process is extremely important, and we distinguish the locating features as the main locating feature and assistant locating feature. In this framework, up to ten kinds of location modes are considered. Meanwhile, we can select all kinds of possible constraint patterns (under-constraint, completely constraint, and over-constraint) and fit patterns (loose fit, transition fit, and tight fit). Thus the analysis results can be more reliable.

In order to calculate the locating tolerance deviations, we should firstly acquire the assembly gaps and locating parameters, and construct the locating coordinate system. The coordinate system can be also regarded as the coordinate system of the installing part or installing feature. Generally, the assembly gaps are key design parameters of the parts, and the related tolerances are designed for them. Therefore, we can define the assembly gaps directly. In this way, the solving procedure can be simplified, and the model is more closely to reality. Particularly, the locating tolerance deviations do not include

Table 1 Deviation of GD&T

Tolerance zone	FCS	Deviations (the other components of $\boldsymbol{\tau}^{FCS}$ are 0)
Parallel planes		$\begin{cases} w = k_1 t/2 \\ \alpha = k_2(1-\lvert k_1\rvert)t/L_2 \\ \beta = k_3(1-\lvert k_1\rvert)(1-\lvert k_2\rvert)t/L_1 \end{cases}$ $k_1, k_2 \in [-1,1],\ k_3 \in \{1,-1\}$
Cylinder		$\begin{cases} u = k_1 t\cos\varphi/2 \\ v = k_2 t\sin\varphi/2 \\ \alpha = (1-k_1)\,t\cos\varphi/L \\ \beta = (1-k_2)t\sin\varphi/L \end{cases}$ $k_1, k_2 \in [-1,1],\ \varphi \in [0,2\pi]$
Circle		$u = t\cos\varphi/2$ $v = t\sin\varphi/2$ $\varphi \in [0,2\pi]$
Rectangle		$u = t_1$ $v = t_2$

the deviations of the installing features and locating features, for they are feature deviations and processed in the deviation propagation module.

Table 2 shows the optional locating modes in this framework. For situation 1–9, there are three kinds of fit patterns, and different strategies have been adopted by the tolerance modeling module to deal with the locating tolerances.

Situations 1–3 represent the “3-2-1” or “two-pin” location mode. This location mode is frequently adopted by designers to joint two different parts. Generally the location mode has two pins and holes (or slot) to ensure the completely constraint of the part. While sometimes only 1 pin and hole can meet our requirements (under-constraint). And sometimes more than two pins and holes are needed to ensure the stiffness of the product (over-constraint), and in this case, we should select tight fit although the fit tolerances are loose.

Situation 4–7 represent the “cylinder” location mode. This location mode is usually adopted in spindle assembling,

Table 2 Optional locating modes

Situation	Locating features		Fit patterns
	Main feature	Assistant features	
1	Plane	Hole and hole /pin and pin	Loose fit, transition fit, tight fit
2	Plane	Hole and slot/pin and pin	
3	Plane	Hole/pin	
4	Cylinder	Key and plane/slot and Plane	
5	Cylinder	Key/slot	
6	Cylinder	Plane/plane	
7	Cylinder	None	
8	Groove (free)	Plane (cross-section)	
9	Groove	Plane (cross-section)	
10	Weld, over-constraint, and the other tight fit situations		

bearing assembling, gear assembling, etc. Particularly, if the spindle is located by several centered cylinder surfaces at the same time (for example, we install the spindle with two bearings at the both ends), these surfaces can be regarded as one cylinder surface, and the GD&Ts are retained.

Situation 8 and 9 represent the groove location mode. This locating mode is mainly adopted in the assembling process of guide rail. In order to construct the FCS, we define the sub-surface of the guide rail as the main feature, and select the cross-section as the assistant feature.

Situation 10 includes the other locating modes that have not been modeled in the framework, such as weld, over-constraint, and undefined tight fit situations. In these situations, the locating tolerance deviation is 0, for the parts to be assembled are completely fixed to the locating features. In this case, the parts only have the accumulated deviations from the locating features and assistant features, and do not have the locating deviation.

4.2 Deviation propagation module

Feature deviations and part deviations are calculated using the tolerance deviations, assembly sequence, locating modes and geometric information in deviation propagation module. All of the deviations use the unified assembly coordinate system (ACS).

Deviation propagation module is a series of algorithms. The main algorithm take steps according to the assembly process, and the tolerance deviation for features and locating tolerance deviations for parts from tolerance modeling module are processed according to the assembly relationships (see Fig. 1).

First of all, we present the algorithms to calculate the feature deviations and part deviation in an individual process. There are five kinds of key features in this framework: main locating features, assistant locating features, main installing features, assistant installing features, and quality point features. Main (assistant) locating features are the locating surfaces for the other parts. Main (assistant) installing features are the installing surfaces of the part. Quality point features are used to select the quality points for tolerance analysis module. Figure 2 shows the deviation accumulation and propagation in an individual process.

The main work in this process is to calculate the feature deviations (each τ_F^{ACS} in Fig. 2), and the solving procedure is conducted along the directions of the arrows in Fig. 2. First of all, we assume the related deviations in Table 3. Particularly, we have defined two assistant locating feature deviations for the part is usually located by a main locating feature and 0/1/2 assistant locating features.

As the main & assistant locating feature deviations are correlated deviations according to DP-SDT theory [11], we should firstly use a special designed algorithm to get the independent accumulated deviation $\tau_{\mathrm{Acc}}^{\mathrm{ACS}}$ as follows:

$$\tau_{\mathrm{Acc}}^{\mathrm{ACS}} = H\left(\tau_{\mathrm{MLF}}^{\mathrm{ACS}}, \tau_{\mathrm{ALF1}}^{\mathrm{ACS}}, \tau_{\mathrm{ALF2}}^{\mathrm{ACS}}\right) \tag{3}$$

The accumulated deviations in Fig. 2 represent the variation accumulation and propagation of the assembly, and affect the whole deviations of the following parts. Then we will calculate the main installing feature deviation and part deviation according to Fig. 2. As the tolerance deviations are described in the feature coordinate systems, respectively, the module firstly transform the tolerance deviations to the corresponding deviations in the unified assembly coordinate systems, e.g., $\tau_t^{\mathrm{FCS}\rightarrow\mathrm{ACS}}$. Meanwhile, as the tolerances are in the state of RFS, and the accumulated deviation is also independent, we can regard the relationship between these deviations as linear [11]. Then the main installing feature deviation and part deviation are as follows:

$$\tau_{\mathrm{MIF}}^{\mathrm{ACS}} = \tau_{\mathrm{Acc}}^{\mathrm{ACS}} + \tau_{\mathrm{LTD}}^{\mathrm{FCS}\rightarrow\mathrm{ACS}} - \sum\nolimits_{j=1}^{n} \tau_{AIF,\,tj}^{FCS\rightarrow ACS} \tag{4}$$

$$\tau_{\pi\mathrm{m}}^{\mathrm{ACS}} = \tau_{\mathrm{MIF}}^{\mathrm{ACS}} - \sum\nolimits_{j=1}^{k} \tau_{MIF,\,tj}^{FCS\rightarrow ACS} \tag{5}$$

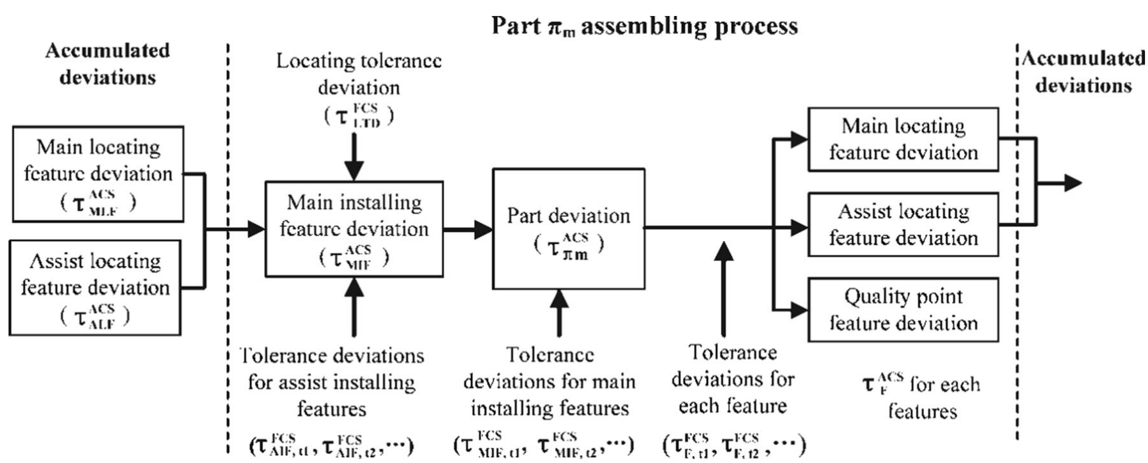


Fig. 2 Deviation accumulation and propagation in an individual process

Table 3 Deviations in an individual process

Deviations	Symbols
Main locating feature deviation	$\tau_{\mathrm{MLF}}^{\mathrm{ACS}}$
Assistant locating feature deviation	$\tau_{\mathrm{ALF1}}^{\mathrm{ACS}}, \tau_{\mathrm{ALF2}}^{\mathrm{ACS}}$
Accumulated deviations	$\tau_{\mathrm{Acc}}^{\mathrm{ACS}}$
Main installing feature deviation	$\tau_{\mathrm{MIF}}^{\mathrm{ACS}}$
Locating tolerance deviation	$\tau_{\mathrm{LTD}}^{\mathrm{FCS}}$
Tolerance deviation for main installing feature	$\tau_{\mathrm{MIF},\,t1}^{\mathrm{FCS}}, \tau_{\mathrm{MIF},t2}^{\mathrm{FCS}}, \ldots \tau_{\mathrm{MIF},\,tk}^{\mathrm{FCS}}$
Tolerance deviation for assistant installing feature	$\tau_{\mathrm{AIF},\,t1}^{\mathrm{FCS}}, \tau_{\mathrm{AIF},\,t2}^{\mathrm{FCS}}, \ldots \tau_{\mathrm{AIF},\,tn}^{\mathrm{FCS}}$
Part deviation	$\tau_{\pi\mathrm{m}}^{\mathrm{ACS}}$
The other feature deviations (example)	$\tau_{\mathrm{F}}^{\mathrm{ACS}}$
Tolerance deviation for the other features	$\tau_{\mathrm{F},\,t1}^{\mathrm{FCS}}, \tau_{\mathrm{F},\,t2}^{\mathrm{FCS}}, \ldots \tau_{\mathrm{F},\,ts}^{\mathrm{FCS}}$

The deviations of the assistant installing features, main (assistant) locating features for the other parts, and quality point features are calculated using the part deviation and tolerance deviations for these features. Then the feature deviation of the other key features (τ_F^{ACS}) can be formulated as:

$$\tau_F^{\mathrm{ACS}} = \tau_{\pi\mathrm{m}}^{\mathrm{ACS}} + \sum\nolimits_{j=1}^{S} \tau_{\mathrm{MIF,\,tj}}^{\mathrm{FCS}\rightarrow\mathrm{ACS}} \tag{6}$$

The above of this section has provided the deviation propagation module design in an individual process. Assume there are totally p processes in the assembly, the main procedure of this module is as follows:

Set 0 *to* accumulated deviations of fixed parts
For process *from* 1 *to p*
Calculate the accumulated deviation
Calculate the installing feature deviation
Calculate the part deviation
Calculate the other feature deviations
End

4.3 Tolerance analysis module

This module is designed to calculate the product quality using the quality point feature deviations (the deviations are calculated in the deviation propagation module according to Eq. 6), and present a statistical result of the quality requirement, including the fraction defective, frequency chart, and contribution chart.

First of all, the deviations of several quality points must be calculated according to Eq. 1 using the quality point feature deviations and form tolerances. Assume P is a quality point, $\overline{p}^{\mathrm{ACS}} = (\overline{px}, \overline{py}, \overline{pz})$ is the original location of P in the unified assembly coordinate system, and Δp^{ACS} is the deviation of P, then:

$$\Delta p^{\mathrm{ACS}} = \left(\tau_F^{\mathrm{ACS}} + \tau_{\mathrm{F,tm}}^{\mathrm{FCS}\rightarrow\mathrm{ACS}}\right) \times R\left(\overline{p}^{\mathrm{ACS}}\right) \tag{7}$$

Where τ_F^{ACS} is the deviation of the quality point feature, *tm* is the form tolerance of the feature (flatness, straightness, circularity, and cylindricity), $\tau_{\mathrm{F,tm}}^{\mathrm{FCS}\rightarrow\mathrm{ACS}}$ is the form tolerance deviation for the feature that has been transformed into the assembly coordinate systems, and $R(p)$ is the rotation and translation matrix:

$$R\left(\overline{p}^{\mathrm{ACS}}\right) = \begin{bmatrix} 0 & \overline{pz} & -\overline{py} & 1 & 0 & 0 \\ -\overline{pz} & 0 & \overline{px} & 0 & 1 & 0 \\ \overline{py} & -\overline{px} & 0 & 0 & 0 & 1 \end{bmatrix}^T \tag{8}$$

Then the designers should define the quality requirements with a quality definition function (QDF) using the quality point deviations according to the technical requirements. In a practical assembly, there are many kinds of technical requirements, such as distance precision, location precision, vibration and noise, motion path precision etc. In this module, we have designed four kinds of quality requirements to try to describe the above technical requirements: (1) distance precision, (2) location precision of a point, (3) deviation of a point, and (4) user-defined hybrid requirement. Meanwhile, we have designed the corresponding algorithms and input rules of QDF for each quality requirements. For convenience, the quality requirements use a user-defined quality coordinate system (QCS). The definition of the QCS had better refer to the inspection of product quality, which can greatly simplify the quality requirement function.

Finally, the product quality is predicted by calculating the user-defined QDF. Since the Q–T function of this framework

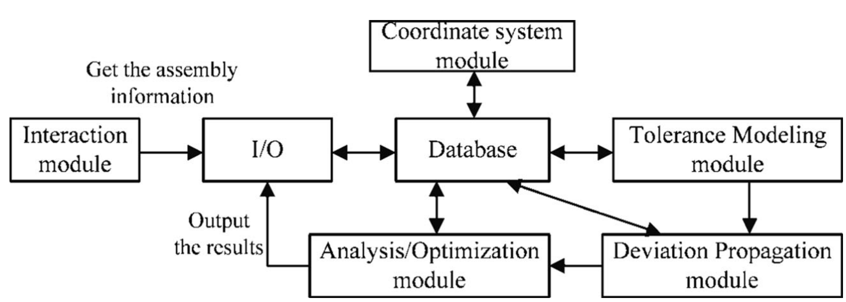
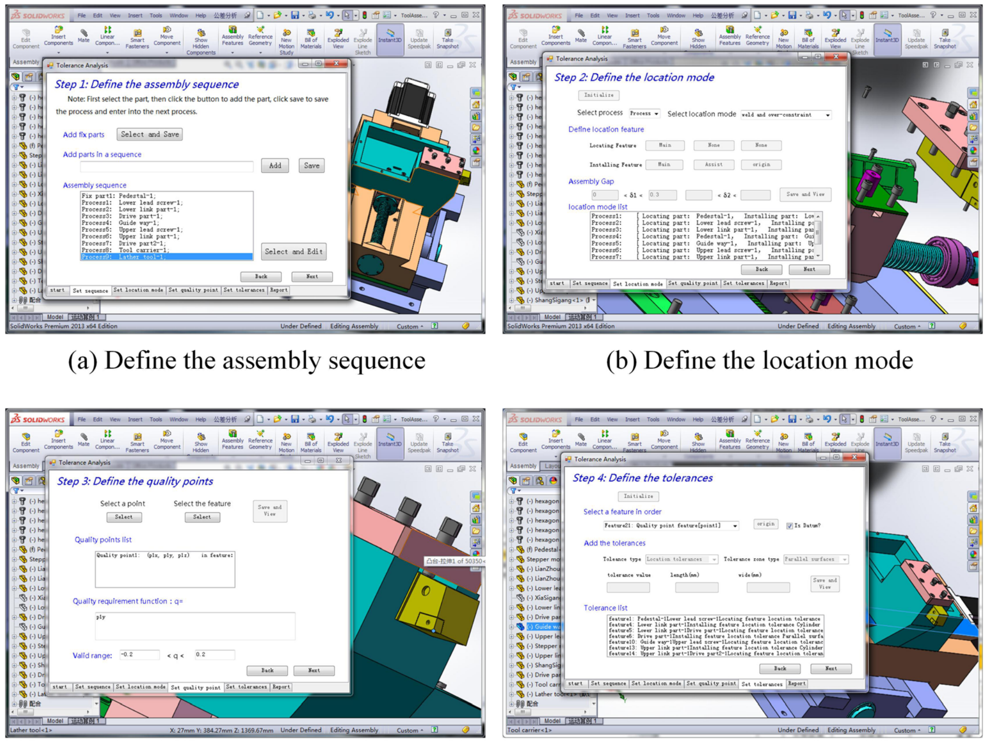


Fig. 3 Relationships between the main modules of the application

Fig. 4 I/O design of the application



(a) Define the assembly sequence (b) Define the location mode

(c) Define the quality requirement (d) Define the tolerances

is implicit, the tolerance analysis module could not adopt linearized methods and formulary statistical methods to describe the result of the quality requirements. In this module, Monte Carlo method is adopted to simulate the whole procedure of the framework. By means of statistical analysis, the framework can present four kinds of results to describe the product quality: fraction defective, influence chart, frequency chart, and contribution chart.

4.4 Tolerance optimization module

This module is designed to optimize the tolerances and recalculate the product quality to reduce the fraction defective and balance the influences of the tolerances. In this module, all of the tolerances and assembly gaps are indexed, and the designers can select the bad designed items and adjust their parameters according to the predicted product quality.

On the one hand, the influence chart and contribution chart provide a visualized guidance for selecting the bad designed tolerances. The tolerances or parts which make more contribution to the product deviation should be firstly considered to adjust according to the two charts. On the other hand, the fraction defective and frequency chart provide a visualized evaluation of the product quality. The designers can acquire the adjustment range for the tolerances according to them. Therefore, the designers can distinguish the bottleneck of the product quality, and assign more reasonable parameters to specific tolerances or parts to improve the product quality.

5 Application prototype

Using the above framework, a tolerance analysis and optimization add-in application for SolidWorks 2013 x64 Edition has been developed with C# and SolidWorks API. The application consists of several modules and sub-modules. Figure 3 shows the relationships between the main modules of the

Table 4 Database design of the application

Objects	Attributes
Location mode	Process number, type, assembly gap, related feature numbers, FCS number, etc.
Dimension tolerance	Number, values, tolerance deviation, feature number, FCS number, etc.
GD&T	Number, values, type, tolerance zone type, tolerance deviation, feature number, FCS number, etc.
Point	Number, type, location, deviation, feature number, etc.
Feature	Number, name, type, location, direction, deviation, part number, tolerance lists, etc.
Part	Number, name, deviation, feature list, etc.
Coordinate system	FCS for each tolerance, ACS, QCS.
Index	Dimensional tolerance and feature list, GD&T and feature list, locating information list, etc.

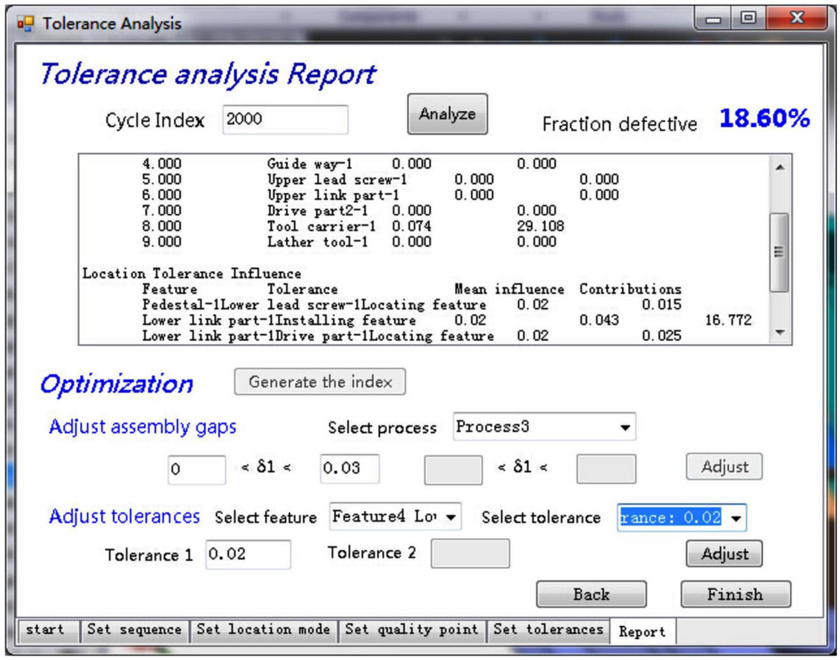


Fig. 5 Analysis/optimization module

application system, and the rest of this section explains the function of each module in detail.

I/O module This module provides a wizard data definition and a visualized result display. The data definition procedure follows these steps: (1) define the assembly sequence (Fig. 4a); (2) for each assembly process, select the locating pattern and define the parameters including the locating features, installing features, and assembly gaps (Fig. 4b); (3) define the quality coordinate system, select the quality points and the homologous features, and then define the quality description function and the quality requirement (Fig. 4c); (4) for each feature, define the tolerance types, tolerance zone types, tolerance values, and related dimensions if necessary (Fig. 4d); and (5) set the simulation times of the Monte Carlo method, the program will calculate the predicted quality automatically and present a statistical chart of the result, and we can also select the tolerances and assembly gaps to be optimized and input the adjusted values if necessary.

Interaction module This module provides a convenient way for the designer to select the points, lines, surfaces, and entities in the SolidWorks assembling environment. Information like the name, locations and directions of the selected objects are available by background processing.

Coordinate system module This module can construct a user-defined coordinate system, and transform the location and deviation of an object from one coordinate system to another automatically. There are 3 kinds of coordinate systems (FCS, ACS, and QCS) in this framework. In order to get the correct coordinate system in different situations, the

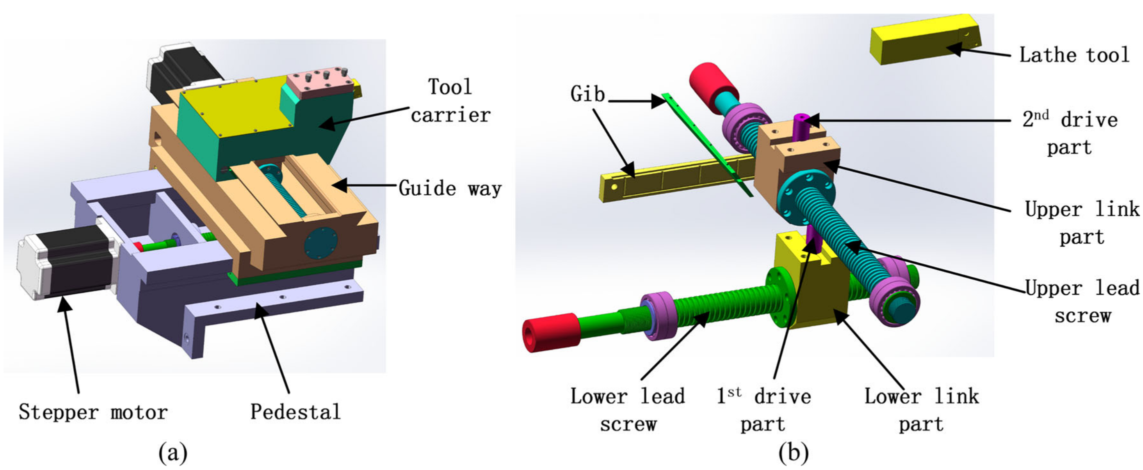


Fig. 6 **a** assembled lathe saddle, **b** drive mechanism of the lathe saddle

Table 5 Assembly sequence and location modes of the parts

Order	Installing part	Location mode
1	Pedestal	Fixed
2	Lower lead screw	Cylinder and plane
3	Lower link part	Over-constraint
4	1st drive part	Groove (free)
5	Guide way	Groove
6	Upper lead screw	Cylinder and plane
7	Upper link part	Over-constraint
8	2nd drive part	Groove (free)
9	Tool carrier	Groove
10	Lather tool	Over-constraint

coordinate system can be constructed by an origin and an axis, or an origin and two axes.

Database module The database of the application is designed in Table 4. This database includes all of the user-defined information and calculated information. Almost all kinds of objects are related between each other, so indexes are used to describe the relationships between the objects.

Tolerance modeling, deviation propagation, and analysis/optimization module These modules provide a series of background algorithms to analyze/optimize the quality from the defined information. The modules can deal with the tolerances, assembly sequence, locating mode, geometric information, and quality requirements comprehensively and finally predict the product quality. Figure 5 shows the output fraction defective, influence chart, and contribution chart. The designers can also select the assembly gaps and tolerances to optimize the tolerances by adjusting the design values and recalculating.

6 Application case of the framework

The application system developed in this paper has been applied to the tolerance optimization project for a lathe saddle. The lathe saddle is an important component of the special purpose lathe for train wheel machining. Figure 6 shows the assembled product and drive mechanism of the lathe saddle. To ensure the machining quality of the wheels, the positioning precision of the lathe tool is strictly specified. As the mainly used GD&Ts (such as perpendicularity and concentricity) are not supported in Pro/ENGINEERING Tolerance Analysis module, and the quality requirement is not supported in SolidWorks TolAnalyst module, a new way to analyze the tolerances is needed. This section will optimize the tolerances by the requirement using the application prototype in Section 5.

Step 1. Define the assembly information. According to Fig. 6, 10 parts have been defined in the assembling process, and the locating modes are assigned in Table 5. The other parts (such as the holding-down clip) can also be defined, but obviously they scarcely make any contributions to the lathe tool deviation.

Step 2. Define the quality requirement. The quality point is selected on the cutting edge of the lathe tool (Fig. 6a). Firstly, we set the direction of feed as y-axis and motion direction of the tool carrier as x-axis to construct the QCS. Then the quality requirement is designed as the y-axis component ($dp1y$) of the quality point deviation. In this project, we care most about the synthetic deviation of the lathe tool, besides, the fraction defective can present an intuitional description about the deviation, so we set a variation range to the deviation: $-0.2mm < dp1y < 0.2mm$.

Step 3. Define the tolerances for each feature, including the features of the tolerances, tolerance types, tolerance values, tolerance zone type, and additional dimensions.

Table 6 Result of the predicted quality

order	Tolerance		Part	Influence	Contribution (%)
1	Gap	0.30	Tool carrier	0.074	24.141
2	Perpendicularity	0.02	1st drive part	0.063	20.181
3	Concentricity	0.02	Lower link part	0.058	18.765
4	Concentricity	0.02	Pedestal	0.029	9.512
5	Perpendicularity	0.02	Lower link part	0.027	8.807
6	Gap	0.05	1st drive part	0.025	8.077
7	Perpendicularity	0.02	2nd drive part	0.015	4.922
8	Concentricity	0.02	Upper link part	0.007	2.304
9	Perpendicularity	0.02	Upper link part	0.006	1.932
10	Concentricity	0.02	Guide way	0.004	1.325

Synthetic deviation: 0.310 mm, fraction defective: 24.15 %

Table 7 Results of the optimized quality

order	Tolerance		Part	Influence	Contribution (%)
1	Gap	0.15	Tool carrier	0.038	20.695
2	Perpendicularity	0.01	1st drive part	0.032	17.596
3	Concentricity	0.02	Pedestal	0.029	16.119
4	Concentricity	0.01	Lower link part	0.029	15.939
5	Gap	0.03	1st drive part	0.015	8.134
6	Perpendicularity	0.01	Lower link part	0.014	7.465
7	Perpendicularity	0.01	2nd drive part	0.008	4.280
8	Concentricity	0.02	Upper link part	0.007	3.955
9	Perpendicularity	0.02	Upper link part	0.006	3.443
10	Concentricity	0.02	Guide way	0.004	2.316

Synthetic deviation: 0.183 mm, fraction defective: 3.65 %

Step 4. Calculate the product quality. Set the cycle index with 2000. The predicted quality is presented in Table 6. We have analyzed 24 related tolerances, and this table shows the top ten tolerances according to the contributions.

According to the results in Table 6, the synthetic deviation is 0.310 mm. Furthermore, we can calculate that the contribution of the drive parts (the product uses two drive parts) is 33.180 %, the contribution of the lower link part is 27.572 %, and the contribution of the tool carrier (assembly gap) is 24.141 %. In order to improve the product quality, the tolerances of the three parts should be optimized first of all.

Step 5. Optimize the tolerances. According to the result previously mentioned, two suggestions are made and are as folows: (1) improve the manufacturing precision of the drive part and lower link part; (2) adjust the gib of the tool carrier locating process to reduce the assembly gap. The optimized result is shown in Table 7.

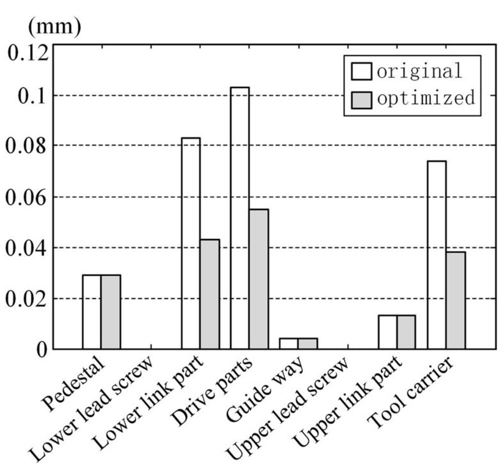


Fig. 7 Comparison of the part influence before and after optimization

Comparing to the original results, we can see that: (1) the synthetic deviation is reduced from 0.310 to 0.183 mm, so the product quality has been greatly improved; (2) the tolerances is more balanced according to Fig. 7; (3) the analysis procedure in this application only takes about 5 s (the cycle index is 2000), so the time cost is controlled; (4) we only make a small adjustment to the tolerances of three small parts to get a better product quality, so the manufacturing cost is controlled.

7 Conclusions

A comprehensive tolerance analysis and optimization framework for mechanical assemblies has been proposed in this paper. The framework integrates various kinds of assembly information with the tolerance module, deviation propagation module, tolerance analysis module, and tolerance optimization module based on DP-SDT theory, and realizes to predict the product quality and optimize the tolerances. The framework does better in supporting complex assembly information and quality requirements, such as 3D dimensional tolerances, GD&Ts, assembly sequence, geometric information, various kinds of location modes, and quality requirements like distance precision, location precision and hybrid requirements. An application prototype using the framework has been developed for SolidWorks, and a tolerance optimization example of lathe saddle is presented to verify the performance of the application.

The paper provides a helpful tool in computer-aided tolerance analysis and optimization field. More work about tolerance analysis in kinematic precision analysis situations and GD&T modeling in maximum material condition and least material condition should be done in the future.

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