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ORIGINAL ARTICLE

Reliability analysis and condition-based maintenance of systems with dependent degrading components based on thermodynamic physics-of-failure

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Abstract In this paper, we present a new reliability model and a unique condition-based maintenance model for complex systems with dependent components subject to respective degradation processes, and the dependence among components is established through environmental factors. Common environmental factors, such as temperature, can create the dependence in failure times of different degrading components in a complex system. The system under study consists of one dominant/independent component and *n* statistically dependent components that are all subject to degradation. We consider two aspects that link the degradation processes and environmental factors: the degradation of dominant/independent component is not affected by the state of other components, but may influence environmental factors, such as temperature; and the *n* dependent components degrade over time and their degradation rates are impacted by the environmental factors. Based on the thermodynamic study of the relationship between degradation and environmental temperature, we develop a reliability model to mathematically account for the dependence in multiple components for such a system. Considering the unique dependent relationship among components, a novel condition-based maintenance model is developed to minimize the long run expected cost rate. A numerical example is studied to demonstrate our models, and sensitivity analysis is conducted to test the impact of parameters on the models.

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1 Introduction

Complex systems operate under dynamic environmental stresses that impair their functionality and life. Maintenance activities are essential to prevent unexpected sudden failures, and reduce downtime cost and production loss. For maintenance purposes, reliability analysis of such systems should incorporate an accurate description of the degradation evolution under these conditions [1], especially when the degradation processes of different components are not independent under common environmental conditions. In this paper, we analyze the reliability of complex systems with failure-dependent components subject to respective degradation processes, where the dependence among components is established through environmental factors, such as temperature. Using the reliability analysis results, a unique condition-based maintenance scheme is developed for the complex system with an aim to minimize the expected total cost rate.

The degradation of many components can be affected by environmental factors, such as temperature and humidity, which either affect the degradation rate or change the relative frequency of different failure modes of sensitive components. On the other hand, environmental conditions are subject to change due to the degradation of certain components. For instance, the friction of two sliding surfaces in a component can cause wear degradation in the form of material loss in the wear tracks, and the dissipation of frictional energy can increase the local temperature [2]. Consequently, the elevated temperature can accelerate the degradation process of nearby temperature-sensitive components, such as resistors. This causal relationship generates a set of interesting and challenging research problems in reliability analysis and maintenance modeling of systems. In this paper, based on the study of physics-of-failure mechanisms and the relationships between degradation and environmental factors, we analyze the reliability of complex systems with dependent components subject to respective degradation processes.

Extensive studies have been done for reliability analysis of systems experiencing degradation [3–6], and for systems subject to independent or dependent failure processes of degradation and random shocks [7–14]. For systems with multiple dependent components, Schottl [15] studied the dependence caused by random environmental effects concerning all components, such as number of shocks, cracks, or dust particles. Coit and English [16] introduced a system reliability model where components are dependent because of the shared environmental exposure. Zhang and Horigome [17] presented reliability analysis of systems that endure environmental shocks, which can result in the failure of one or more components due to a cumulative shock process. Kotz et al. [18] investigated how the degree of correlation affects the increase in the mean lifetime for parallel redundant systems when the two components are positively quadrant dependent. Burkschat [19] proposed a model for describing the lifetimes of coherent systems, in which the failures of components may have an impact on the lifetimes of the remaining components.

Although various types of dependence among components have been studied in the literature, very little research has been devoted to study the dependence among degrading components when the degradation processes and the failure times of components are dependent due to environmental factors. In this paper, we study a system with multiple components that include one dominant/independent component and n statistically dependent components. The dominant component degrades over time, and its degradation rate or lifetime distribution is not affected by the state of other components. However, the degradation process of the dominant component may cause the change in environmental conditions. For example, as the component wears, its temperature increases, which causes the ambient temperature to increase. In the meanwhile, the dependent components degrade over time and their degradation rates increase as the ambient temperature increases. Therefore, the degradation processes of the dependent components are statistically dependent on the degradation process of the dominant component via the environmental factors. The dependence among different components creates an interesting and challenging problem to analyze the reliability of this type of system, which is lacking in the literature. In this paper, we attempt to fill this void by investigating the dependence between the degradation processes through the analysis of physics-of-failure mechanisms, and developing the reliability and maintenance models for such systems.

Although the maintenance of systems with single components has been extensively studied, the research on maintenance modeling of systems with multiple components is limited. The latter topic is more practical to industry applications, yet much more difficult due to the dependence among components. Typically, three types of dependence are considered for multi-component systems: economic dependence, structural dependence, and failure dependence [20, 21]. Economic dependence considers that there are cost/time-savings to jointly perform maintenance on multiple components, instead of on individual components. Structural dependence implies that the components are structurally connected, and therefore, maintenance actions on a failed component require dismantling other components. Failure dependence refers to the dependence between the failure of one component and that of other components in the system. It also refers to situations when the components suffer from the common-cause failure from external sources.

After the first survey paper on maintenance policies for multi-component systems conducted by Thomas in 1986 [20], this topic has attracted increasing attention. Another three survey papers on this topic are provided by Cho and Parlar [22] for group, block, and opportunistic models; Dekker et al. [23] with a focus on economic dependence; and Wang [21] with an emphasis on single-component systems. More recently, condition-based maintenance of multi-component systems, where economic dependence exists among different components, was investigated [24, 25]. Laggounce et al. [26] proposed a preventive maintenance plan for a multi-component system, where economic dependence is considered to reflect the influence of component operation/maintenance costs on the overall system costs.

Most of the literature on maintenance policies for multi-component systems studies the economic dependence among the components. The failure dependence has rarely been considered in maintenance policies for multi-component systems. In this paper, we develop a unique condition-based maintenance model for a complex system with multi-components that are failure dependent. Each of the components is subject to a respective degradation process, and the dependence among the components is established through environmental factors.

The remaining sections are arranged as follows. Section 2 describes the thermodynamic study in analyzing the relationships between degradation and temperature. Section 3 presents the system reliability model. The proposed condition-based maintenance model for multi-component systems is introduced in Section 4. Section 5 gives a numerical example to demonstrate our models with sensitivity analyses. Concluding remarks are summarized in Section 6.

2 Thermodynamic study for physics-of-failure

For a system consisting of one dominant/independent component and n statistically dependent components that are all subject to degradation, we consider two aspects that link the degradation processes and environmental factors [27]:

- The dominant/independent component degrades over time, and its degradation rate or lifetime distribution is not affected by the state of other components. However, the degradation process of the dominant component may influent environmental factors, such as temperature. For example, the wear degradation of a microengine increases ambient temperature.
- The n dependent components degrade over time, and their degradation rates are impacted by the environmental factors. For instance, the elevated temperature accelerates the degradation of resistors.

To demonstrate the thermodynamic analysis and reliability modeling, we use an example application. The dominant component in an example system is a microengine that experiences wear degradation over time, and the wear-out process increases the ambient temperature. In the system, there are n temperature-sensitive thin film resistors whose resistances increase over time, and the degradation rates increase as the temperature elevates due to the wear-out process of the microengine. Considering that the subsystem of dependent components (e.g., thin film resistors) is typically sealed in a small package, the temperature rise among dependent components is not significantly different. There may be other examples where the temperature differentials are not small and our model is not applicable for those examples. In order to analyze reliability performance of this system, we need to understand physics-of-failure mechanisms for these degradation processes, specifically through the study of thermodynamics.

The relationship between wear degradation and temperature has been of great interest to many researchers in thermodynamics. Bryant et al. [28] developed a thermodynamic characterization of degradation dynamics, which employs entropy, a measure of thermodynamic disorder, as the fundamental measure of degradation. Ramalho and Miranda [29] conducted experimental studies on the relationship between wear and dissipated energy in sliding systems using the energetic approach, and the results show that the dissipated energy is linearly related to wear volume. The experimental work on the relationship between wear and thermal response in sliding systems from Amiri et al. [30] shows that the temperature rise is linearly correlated with the material loss, and the slope of the linear relationship is a measure of the wear coefficient. On the other hand, the impact of elevated temperature on component degradation is usually modeled by the Arrhenius relationship in the literature. Tencer et al. [31] presented a method of assessing the effective temperature essential for predicting the temperature acceleration of the wear-out mechanism using the Arrhenius equation. Kuehl [32] developed a method for prediction of resistive value changes due to aging for any relevant condition in the temperature-time expanse, and the method is based on and derived from the Arrhenius equation.

2.1 Wear degradation and thermal response

The degradation due to wear over time can follow various degradation path models, such as a linear degradation path with random coefficients or a randomized logistic degradation path [10]. For the dominant component (e.g., a microengine), its wear degradation $X(t)$ follows a linear degradation path, $X(t)=\varphi+\beta t$ [33]. The initial value φ and the degradation rate β are both random variables following normal distributions, $\varphi\sim N(\mu_\varphi,\sigma_\varphi^2)$ and $\beta\sim N(\mu_\beta,\sigma_\beta^2)$, respectively, characterizing the unit-to-unit variability. σ_φ and σ_β are assumed to be substantially smaller than μ_φ and μ_β, respectively, and the probability that φ or β takes negative values is negligible. The microengine is considered to be failed when the wear degradation is greater than a failure threshold value H.

The degradation of the dominant component leads to the rise of ambient temperature. According to Amiri et al. [30], the temperature rise ΔT at the interface during steady state operation has a linear relationship with the wear degradation rate:

$$\Delta T = \frac{\Psi}{\xi}\beta, \tag{1}$$

where ξ is a constant, $\xi = \frac{K}{\eta\mu_{\mathrm{ave}}h}$, K is the wear coefficient, η is the heat partitioning factor, μ_{ave} is the friction coefficient, h is the material hardness, and Ψ is a constant. Because β follows a normal distribution, $\beta\sim N(\mu_\beta,\sigma_\beta^2)$, the temperature rise ΔT at the steady state is also a normal random variable with mean of $\mu_\beta\Psi/\xi$ and variance of $\sigma_\beta^2\Psi^2/\xi^2$.

2.2 Arrhenius relationship

Similar to the degradation process modeling of the dominant component, we want to incorporate the unit-to-unit variability in the degradation process modeling of the dependent components as well. For the n dependent components, such as thin film resistors, the resistance $r_i(t)$ increases linearly over time, $r_i(t)=r_{0i}+\rho_i t$. The initial resistance of component i, r_{0i}, is a random variable following a normal distribution, $r_{0i}\sim N(\mu_{ri},\sigma_{ri}^2)$, for component i, $i=1, 2, \ldots, n$. The degradation rate of component i, ρ_i, is affected by the temperature via the Arrhenius relationship [31, 32]:

$$\rho_i = A_i \exp\left(-\frac{E_a}{kT}\right), \tag{2}$$

where E_a is the activation energy in eV, k is the Boltzmann constant, T is the temperature in Kelvin, and A_i is an experimental constant. Because the ambient temperature rise ΔT is a function of β given in Eq. (1), the degradation rate ρ_i is a random variable depending on β, characterizing the unit-to-unit variability among the n dependent components. Therefore, the resistance is expressed as

$$r_i(t) = r_{0i} + \rho_i t = r_{0i} + A_i t \exp\left(-\frac{E_a}{kT}\right). \tag{3}$$

A thin film resistor is considered to be failed when the resistance is beyond the failure threshold value, L_i, $i=1, 2, \ldots, n$. Figure 1 shows 30 pairs of simulated degradation processes of a dominant component and a dependent component. The parameters and their values used for this simulation are listed in Table 1. We can notice that the lifetime of the dependent component has a much larger variance than that of the dominant component, because the degradation rate of the dominant component significantly affects the degradation rate of the dependent component. For a series system with dependent components, we develop its reliability function and a unique condition-based maintenance policy in the following sections.

3 System reliability modeling

Consider a series system with one dominant component and n dependent components, e.g., a microengine and n thin film resistors connected in series. System reliability at time t is the probability that it survives by time t, that is, the degradation level of each component should be less than the corresponding failure threshold level [27]:

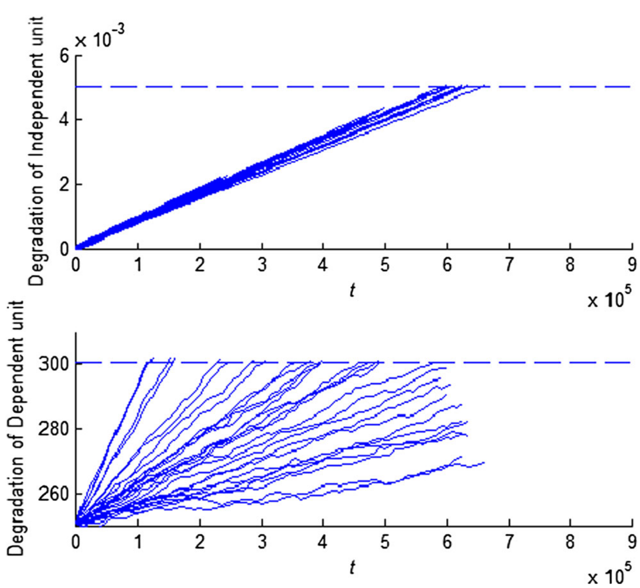

Fig. 1 Simulation of the stochastic degradation processes for dominant/independent and dependent components

Table 1 Parameters and values

Parameters	Values	Sources
k	8.6171×10^{-5} eV/K	
E_a	1.29 eV (for TaN)	[34]
A_1, A_2	2.911×10^{10} (for TaN)	[34]
T_0	293 K	
h	11.5 Gpa	[35]
K	1×10^{-4}	[35]
μ_{ave}	0.7	[35]
η	0.5	[30]
ξ	2.484×10^{-14}/pa	Calculation
Ψ	4.55×10^{14} K/W	Assumption
H	$0.005\,\mu m^3$	Assumption
β	$\sim N(\mu_\beta,\sigma_\beta^2)$ $\mu_\beta=8.4823\times10^{-9}$ μm^3/time unit $\sigma_\beta=6.0016\times10^{-10}$ μm^3/time unit	[10, 36]
φ	$\sim N(\mu_\varphi,\sigma_\varphi^2)$ $\mu_\varphi=0\,\mu m^3$ $\sigma_\varphi=5.0000\times10^{-5}\,\mu m^3$	Assumption
r_{01}	$\sim N(\mu_{r1},\sigma_{r1}^2)$ $\mu_{r1}=250.48$ Ω, $\sigma_{r1}=0.5$ Ω	Assumption
r_{02}	$\sim N(\mu_{r2},\sigma_{r2}^2)$ $\mu_{r2}=250.48$ Ω, $\sigma_{r2}=0.5$ Ω	Assumption
L_1, L_2	300.58 Ω	[34]

$$R(t) = P(X(t) < H, r_1(t) < L_1, \ldots, r_n(t) < L_n). \tag{4}$$

Because the degradation processes of these components are dependent through temperature change, we need to compute it by finding the conditional probability given ΔT. Based on the law of total probability, we then integrate this conditional probability multiplied by the probability density function (pdf) of ΔT to derive the system reliability, as shown in Eq. (5):

$$\begin{aligned} R(t) &= P(X(t) < H, r_1(t) < L_1, \ldots, r_n(t) < L_n) \\ &= \int_{-\infty}^{+\infty} P(X(t) < H, r_1(t) < L_1, \ldots, r_n(t) < L_n | \Delta T = s) f_{\Delta T}(s) ds \\ &= \int_{-\infty}^{+\infty} P(X(t) < H | \Delta T = s) \prod_{i=1}^{n} P(r_i(t) < L_i | \Delta T = s) f_{\Delta T}(s) ds, \end{aligned} \tag{5}$$

where the conditional probabilities of $X(t)$ and $r_i(t)$ given ΔT are derived, respectively:

$$\begin{aligned} P(X(t) < H | \Delta T = s) &= P(\varphi + ts\xi/\psi < H) \\ &= P\left(\varphi < H - \frac{\xi}{\psi} st\right) \\ &= \Phi\left(\frac{1}{\sigma_\varphi}\left(H - \frac{\xi}{\psi} st - \mu_\varphi\right)\right), \end{aligned} \tag{6}$$

$$
\begin{aligned}
&P(r_i(t) < L_i|\Delta T = s)\\
&= P\left(r_{0i} + A_i t\exp\left(-\frac{E_a}{k(T_0+s)}\right) < L_i\right)\\
&= P\left(r_{0i} < L_i - A_i t\exp\left(-\frac{E_a}{k(T_0+s)}\right)\right)\\
&= \Phi\left(\frac{1}{\sigma_{r_i}}\left(L_i - \mu_{ri} - A_i t\exp\left(-\frac{E_a}{k(T_0+s)}\right)\right)\right). \qquad (7)
\end{aligned}
$$

The temperature rise ΔT is a normal random variable with mean of $\mu_\beta \Psi/\xi$ and variance of $\sigma_\beta{}^2\Psi^2/\xi^2$, and its pdf can be expressed as

$$
f_{\Delta T}(s) = \frac{\xi}{\sigma_\beta \psi}\phi\left(\frac{s - \mu_\beta \psi/\xi}{\sigma_\beta \psi/\xi}\right), \qquad (8)
$$

where $\phi(\cdot)$ denotes the pdf of a standard normal random variable.

Finally, the reliability function in Eq. (5) is expressed as

$$
R(t) = \int_{-\infty}^{+\infty} \Phi\left(\frac{1}{\sigma_\varphi}\left(H - \frac{\xi}{\psi}st - \mu_\varphi\right)\right)\prod_{i=1}^{n}\Phi\left(\frac{1}{\sigma_{r_i}}\left(L_i - \mu_{ri} - A_i t\exp\left(-\frac{E_a}{k(T_0+s)}\right)\right)\right) \times \frac{\xi}{\sigma_\beta \psi}\phi\left(\frac{s - \mu_\beta \psi/\xi}{\sigma_\beta \psi/\xi}\right)ds, \qquad (9)
$$

where $\Phi(\cdot)$ denotes the cumulative distribution function (cdf) of a standard normal random variable.

4 Condition-based maintenance modeling

Due to the unique relationship between the dominant and dependent components and their characteristics, we propose a new maintenance model for this type of system. Since the dominant component plays a key role in this system and it is typically expensive, we consider the case when the replacement cost of the dominant component is much higher than the replacement cost of all the dependent components combined (or the subsystem). We assume that the system is non-repairable or not worth repairing rather than replacing. The replacement time for the whole system and the subsystem of all dependent components is negligible. With more attention on the expensive dominant component, the proposed maintenance strategy is designed as follows and illustrated in Fig. 2.

- Periodic inspection of length τ is carried out to observe or measure the degradation level $X(t)$ of the dominant component. If the degradation level is less than a warning limit, D, no action is taken; and if the degradation level is between the warning limit D and the failure threshold H, preventive replacement takes place.
- If the dominant component fails (the degradation level is beyond the failure threshold H) between two inspection actions, it is self-announcing and corrective replacement is implemented.
- Every time the dominant component is replaced preventively or correctively, the whole subsystem of dependent components is replaced preventively for the purpose of saving time/labor, shown as 'PM' in Fig. 2.
- The conditions of dependent components are not checked during the periodic inspection actions. However, the failure of any dependent component is self-announcing. If one of the dependent components in the subsystem fails, we replace the whole subsystem correctively for the purpose of saving time/labor.

To determine the inspection interval τ and the warning limit D, we need to derive and optimize the expected total maintenance cost per unit of time:

$$
\text{Expected cost rate} = \frac{\text{Expected cost per cycle}}{\text{Expected cycle length}} = \frac{E(\text{Total Cost})}{E(\text{Cycle Length})}. \qquad (10)
$$

As illustrated in Fig. 2, a renewal cycle of the dominant component can be terminated due to a preventive replacement (the degradation level is between D and H) or a corrective replacement (the degradation level is beyond H). To find the expected cost per renewal cycle and the expected renewal cycle length, we need to consider these two cases.

4.1 Renewal cycle terminated due to preventive replacement

We start with the case that a renewal cycle is terminated when the degradation level exceeds the warning limit, and therefore, preventive replacement is performed for the dominant component. Let N_{PM} denote the inspection count at which a preventive maintenance/replacement is

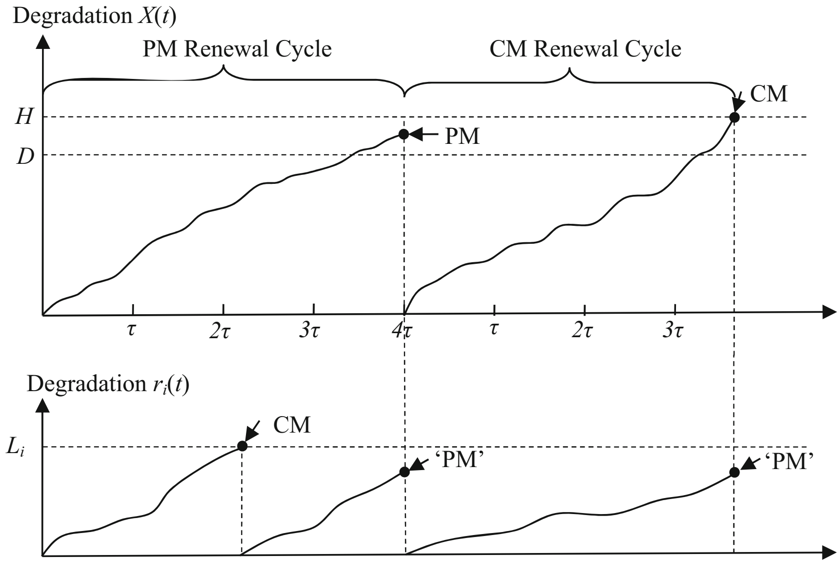

Fig. 2 Proposed condition-based maintenance model

implemented. The probability of performing preventive replacement is derived as follows.

1) The preventive replacement is performed at the 1st inspection, or $N_{PM}=1$:

$$\begin{aligned}P(N_{PM}=1)&=P(D<X(\tau)<H)\\&=P(D<\varphi+\beta\tau<H)\\&=\Phi\left(\frac{H-\mu_\varphi-\mu_\beta\tau}{\sqrt{\sigma_\varphi^2+\sigma_\beta^2\tau^2}}\right)-\Phi\left(\frac{D-\mu_\varphi-\mu_\beta\tau}{\sqrt{\sigma_\varphi^2+\sigma_\beta^2\tau^2}}\right).\end{aligned}\tag{11}$$

2) The preventive replacement is performed at the i^{th} inspection, or $N_{PM}=i>1$:

$$\begin{aligned}P(N_{PM}=i>1)&=P\Big(D<X(i\tau)<H,X\Big((i-1)\Big)\tau\Big)<D\Big)\\&=\int_{-\infty}^{+\infty}P(D<X(i\tau)<H,X((i-1)\tau)<D|\beta=b)f_\beta(b)db\\&=\int_{-\infty}^{+\infty}P(D<\varphi+bi\tau<H,\varphi+b(i-1)\tau<D)f_\beta(b)db\\&=\int_{-\infty}^{+\infty}P(D-bi\tau<\varphi<\min(H-bi\tau,D-b(i-1)\tau))f_\beta(b)db\\&=\int_{-\infty}^{\frac{H-D}{\tau}}\left(\Phi\left(\frac{D-b(i-1)\tau-\mu_\varphi}{\sigma_\varphi}\right)-\Phi\left(\frac{D-bi\tau-\mu_\varphi}{\sigma_\varphi}\right)\right)f_\beta(b)db\\&\quad+\int_{\frac{H-D}{\tau}}^{+\infty}\left(\Phi\left(\frac{H-bi\tau-\mu_\varphi}{\sigma_\varphi}\right)-\Phi\left(\frac{D-bi\tau-\mu_\varphi}{\sigma_\varphi}\right)\right)f_\beta(b)db.\end{aligned}\tag{12}$$

4.2 Renewal cycle terminated due to corrective replacement

When the degradation level of the dominant component exceeds the failure threshold H, a renewal cycle is terminated and corrective replacement is performed. To find the probability of performing corrective replacement upon failure, we need to derive the failure time distribution of the dominant component. The degradation process $X(t)$ follows a normal distribution with mean $\mu_{X(t)}=\mu_\varphi+\mu_\beta t$, and variance $\sigma_{X(t)}^2=\sigma_\beta^2 t^2+\sigma_\varphi^2$. If we denote T_x as the time of the degradation path reaching a threshold x, then the cdf of T_D is

$$\begin{aligned}P(T_D<t)&=P(X(t)>D)=P(\varphi+\beta t>D)\\&=1-\Phi\left(\frac{D-\mu_\varphi-\mu_\beta t}{\sqrt{\sigma_\varphi^2+\sigma_\beta^2\tau^2}}\right).\end{aligned}\tag{13}$$

Its pdf can be calculated by taking the first derivative of the cdf with respect to t, which is

$$\begin{aligned}f_{T_D}(t)&=\frac{dP(T_D<t)}{dt}\\&=\phi\left(\frac{D-\mu_\varphi-\mu_\beta t}{\sqrt{\sigma_\varphi^2+\sigma_\beta^2\tau^2}}\right)\frac{\mu_\beta\sigma_\varphi^2+\Big(D-\mu_\varphi\Big)\sigma_\beta^2 t}{\Big(\sigma_\varphi^2+\sigma_\beta^2 t^2\Big)^{3/2}}.\end{aligned}\tag{14}$$

In the case of a failure occurring between inspections, it indicates that at the previous inspection the degradation level of the dominant component does not reach the warning limit D yet. We need to include this condition in our derivation of the failure distribution for the dominant component. The cdf of the failure time T_H conditioning on T_D is

$$
\begin{aligned}
P(T_H < t | T_D = t_0) &= P(X(t) > H | X(t_0) = D) \\
&= P(X(t) - X(t_0) > H - D) \\
&= P(\beta(t - t_0) > H - D) \\
&= 1 - \Phi\left(\frac{H - D - \mu_\beta(t - t_0)}{\sigma_\beta(t - t_0)}\right).
\end{aligned}
\tag{15}
$$

Similarly, the pdf of the failure time T_H conditioning on T_D can be derived as

$$
\begin{aligned}
f_{T_H|T_D}(t|t_0) &= \frac{dP(T_H < t | T_D = t_0)}{dt} \\
&= \phi\left(\frac{H - D - \mu_\beta(t - t_0)}{\sigma_\beta(t - t_0)}\right)\frac{H - D}{\sigma_\beta(t - t_0)^2}.
\end{aligned}
\tag{16}
$$

4.3 Optimization model

The dominant component is either preventively replaced at inspection or correctively replaced upon failure between inspections. Based on Eqs. (11)–(16), the expected renewal cycle length can be found as:

$$
E(\text{Cycle Length}) = \sum_{i=1}^{\infty} i\tau P(N_{PM} = i) + \sum_{i=1}^{\infty} \int_{(i-1)\tau}^{i\tau} \int_{t_0}^{i\tau} t \cdot f_{T_H|T_D}(t|t_0) dt \cdot f_{T_D}(t_0) dt_0. \tag{17}
$$

The overall maintenance cost includes preventive and corrective replacement costs for the dominant component, C_{PI} and C_{CI}; preventive and corrective replacement costs for the subsystem of all dependent components, C_{PD} and C_{CD}; and the inspection cost C_I. The system downtime cost is not considered, as we assume that the time for maintenance actions, such as inspection and replacement, is negligible.

When a renewal cycle is terminated at the i^{th} inspection due to preventive replacement, the incurred cost includes the preventive replacement cost of the dominant component C_{PI}, the preventive replacement cost of the subsystem C_{PD}, the cost for i inspection actions, and the cost to correctively replace the subsystem before $i\tau$. The subsystem can be correctively replaced multiple times whenever one of the dependent components fails before $i\tau$. The number of corrective replacements (or the number of failures) of the subsystem prior to $i\tau$ can be calculated by the renewal function, $M(t)$, which is derived in the next section. When a renewal cycle is terminated between $(i-1)\tau$ and $i\tau$ due to failure, the incurred cost includes the corrective replacement cost of the dominant component C_{CI}, the preventive replacement cost of the subsystem C_{PD}, the cost for $i-1$ inspection actions before failure, and the cost to correctively replace the subsystem before failure. Then, the expected total maintenance cost is derived to be:

$$
\begin{aligned}
E(\text{Total Cost}) &= \sum_{i=1}^{\infty} P(N_{PM} = i) \cdot (C_{PI} + C_{PD} + iC_I + M(i\tau)C_{CD}) \\
&\quad + \sum_{i=1}^{\infty} \int_{(i-1)\tau}^{i\tau} \int_{t_0}^{i\tau} (C_{CI} + C_{PD} + (i-1)C_I + M(t)C_{CD}) \cdot f_{T_H|T_D}(t|t_0) dt \cdot f_{T_D}(t_0) dt_0.
\end{aligned}
\tag{18}
$$

Based on Eqs. (17) and (18), we propose the following constrained nonlinear optimization problem for the maintenance optimization:

$$
\begin{aligned}
&\text{Min} \quad c(\tau, D) = \frac{E(\text{Total Cost})}{E(\text{Cycle Length})} \\
&\text{Subject to}: \quad 0 < D < H \\
&\qquad\qquad\qquad 0 < \tau < t_{\max}
\end{aligned}
\tag{19}
$$

where $t_{\max}$ is the allowed upper bound of the inspection interval. The Sequential Quadratic Programming algorithm (Matlab optimization toolbox) is used to solve this constrained nonlinear optimization problem.

4.4 Renewal function of the subsystem

To calculate the expected total cost per cycle, we need to have the number of corrective replacements (or the number of failures) of the subsystem in a renewal cycle, namely, the renewal function, which requires the reliability function of the subsystem of dependent components, $R_{\text{Sub}}(t)$:

$$
\begin{aligned}
R_{\text{Sub}}(t) &= P(r_1(t) < L_1, \ldots, r_n(t) < L_n) \\
&= \int_{-\infty}^{+\infty} P(r_1(t) < L_1, \ldots, r_n(t) < |L_n \Delta T = s) f_{\Delta T}(s) ds \\
&= \int_{-\infty}^{+\infty} \prod_{i=1}^{n} P(r_i(t) < L_i | \Delta T = s) f_{\Delta T}(s) ds \\
&= \int_{-\infty}^{+\infty} \prod_{i=1}^{n} \Phi\left(\frac{1}{\sigma_{r_i}}\left(L_i - \mu_{ri} - A_i t \exp\left(-\frac{E_a}{k(T_0 + s)}\right)\right)\right) \\
&\quad \cdot \frac{\xi}{\sigma_\beta \psi} \phi\left(\frac{s - \mu_\beta \psi / \xi}{\sigma_\beta \psi / \xi}\right) ds
\end{aligned}
\tag{20}
$$

The renewal function is calculated to be $M(t)=F_{\mathrm{Sub}}(t)+\int_0^t M(t-u)f_{\mathrm{Sub}}(u)du$, where $F_{\mathrm{Sub}}(t)$ and $f_{\mathrm{Sub}}(t)$ are the cdf and pdf of the subsystem, respectively. It is difficult to derive the closed form of the renewal function given the complicated subsystem reliability function. Estimation of the renewal function is typically applied [37]:

$$M(t) = F_{\mathrm{Sub}}(t) + \int_0^t \frac{F^2_{\mathrm{Sub}}(u)}{\int_0^u R_{\mathrm{Sub}}(v)dv} du.$$

Even using the estimate of the renewal function, the complex nonlinear optimization model is still difficult to solve mathematically. One approach demonstrated in the numerical example is to simplify the subsystem reliability function by fitting it to a simple regression model that could lead to a closed-form renewal function. For example, when the failure time follows an exponential distribution with arrival rate λ, its renewal function is simply $M(t)=\lambda t$.

5 Numerical example

In this numerical example, a system consisting of one microengine (the dominant component) and two identical resistors (dependent components) is studied. The three components are dependent because the degradation of the microengine causes the temperature rise in the surrounding environment, while the temperature rise accelerates the degradation of both resistors. For this type of system, we are interested in determining reliability over time and the optimal maintenance strategies using the reliability and maintenance models that we developed. The parameters and their values used in our models are listed in Table 1. Figure 3 plots the reliability of the system over time.

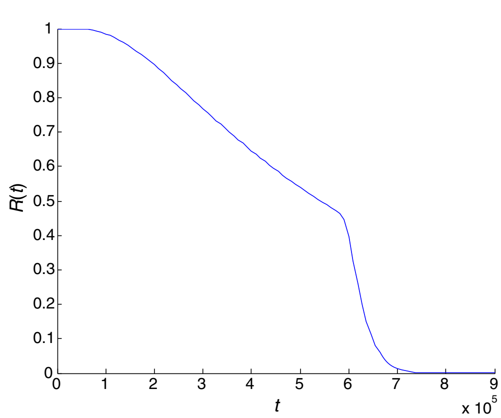

Fig. 3 System reliability over time

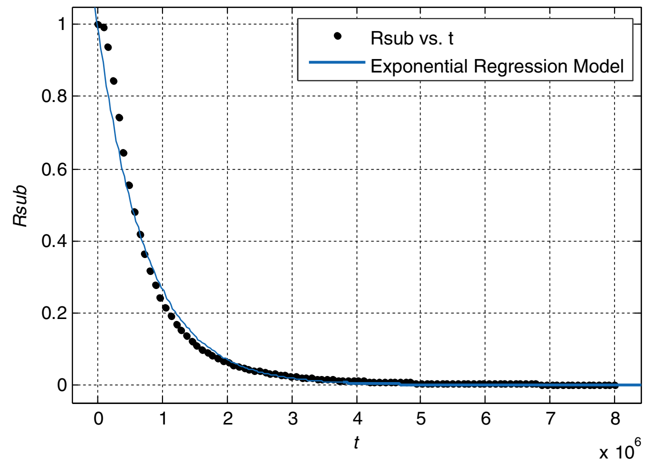

Fig. 4 The subsystem reliability and exponential regression model

For the condition-based maintenance model, we assume that the preventive and corrective replacement costs for the subsystem of two dependent components are 40 and 50, respectively. Because the replacement cost of the dominant component is far more expensive than that of the dependent components, the preventive and corrective replacement costs for the dominant component are 400 and 500, respectively. The inspection cost is 10 per inspection for the dominant component.

It becomes difficult and inefficient to directly solve the optimization problem, because of the complex form of the subsystem reliability function and the resulting renewal function. An alternative way is to simplify the subsystem reliability function by using a regression model to approximate it. Using the set of parameter values provided in Table 1, we find that the exponential regression model fits the subsystem reliability well, as

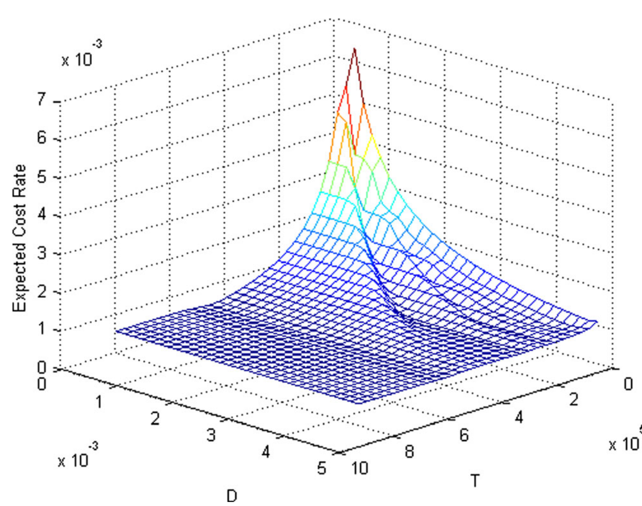

Fig. 5 3D plot of the expected cost rate vs τ and D

Table 2 Sensitivity analysis result on H

$H(\mu m^3)$	τ^*	D^*	Min expected cost rate
0.0030	3.34E+05	1.51E-03	1.46E-03
0.0035	3.90E+05	1.76E-03	1.26E-03
0.0040	4.45E+05	2.01E-03	1.11E-03
0.0045	5.01E+05	2.25E-03	9.91E-04
0.0050	5.57E+05	2.50E-03	8.98E-04
0.0055	6.13E+05	2.75E-03	8.22E-04
0.0060	6.68E+05	3.00E-03	7.59E-04
0.0065	7.24E+05	3.25E-03	7.06E-04
0.0070	7.80E+05	3.50E-03	6.60E-04

Table 4 Sensitivity analysis result on C_{PI}/C_{PD}

C_{PI}/C_{PD}	C_{PI}	C_{CI}	τ^*	D^*	Min expected cost rate
4	160	200	5.65E+05	2.50E-03	4.51E-04
5	200	250	5.62E+05	2.50E-03	5.26E-04
6	240	300	5.60E+05	2.50E-03	6.00E-04
7	280	350	5.59E+05	2.50E-03	6.75E-04
8	320	400	5.58E+05	2.50E-03	7.49E-04
9	360	450	5.57E+05	2.50E-03	8.24E-04
10	400	500	5.57E+05	2.50E-03	8.98E-04
11	440	550	5.56E+05	2.50E-03	9.72E-04
12	480	600	5.56E+05	2.50E-03	1.05E-03
13	520	650	5.56E+05	2.50E-03	1.12E-03
14	560	700	5.56E+05	2.50E-03	1.20E-03
15	600	750	5.55E+05	2.50E-03	1.27E-03

shown in Fig. 4. The fitted exponential model is $\hat{R} = e^{-1.332\times10^{-6}t}$, and the corresponding R^2 value is 0.9869.

After fitting the subsystem reliability to an exponential regression model, the renewal function has a simple form, which is $M(t)=1.332\times10^{-6}t$. Then, we use the Sequential Quadratic Programming algorithm (in Matlab R2013a) to solve this constrained nonlinear optimization problem in Eq. (19) and obtain the minimum expected cost rate, 8.98×10^{-4}, when $\tau^*=5.57\times10^5$ and $D^*=0.0025$. The expected cost rates at different τ and D levels are plotted in Fig. 5.

5.1 Sensitivity analysis

Sensitivity analysis is conducted to see the sensitivity of the optimal results to the change of parameter values. The parameters of interest are the degradation failure threshold H, the ratio of resistor failure threshold to its initial value L/r_0, the ratio of preventive replacement cost for the dominant component to the preventive replacement cost for the subsystem of all dependent components C_{PI}/C_{PD}, and the ratio of inspection cost to preventive replacement cost for the subsystem of all dependent components C_I/C_{PD}. The sensitivity analysis results are listed in Tables 2, 3, 4, and 5 and plotted in Figs. 6, 7, and 8.

Table 2 shows the values of the optimal inspection interval τ^* and warning limit D^*, the minimum expected cost rate at different H values from 0.003 to 0.007. When H increases from 0.003 to 0.007, τ^* and D^* linearly increases (also shown in Fig. 6), while the minimum expected cost rate decreases. This is reasonable, since a higher failure threshold means the system can survive longer, requiring less frequent inspections and a higher warning limit, leading to a reduced cost.

In this numerical example, we consider two dependent resistors with identical failure threshold L and initial resistance r_0. The change of L/r_0 affects the

Table 3 Sensitivity analysis result on L/r_0

L/r_0	L	λ	τ^*	D^*	Min expected cost rate
1.10	275.53	2.6810E-06	5.57E+05	2.50E-03	9.65E-04
1.15	288.05	1.7720E-06	5.57E+05	2.50E-03	9.20E-04
1.20	300.58	1.3320E-06	5.57E+05	2.50E-03	8.98E-04
1.25	313.10	1.0650E-06	5.57E+05	2.50E-03	8.85E-04
1.30	325.62	8.8640E-07	5.57E+05	2.50E-03	8.76E-04
1.35	338.15	7.5940E-07	5.57E+05	2.50E-03	8.69E-04
1.40	350.67	6.6440E-07	5.57E+05	2.50E-03	8.65E-04

Table 5 Sensitivity analysis result on C_I/C_{PD}

C_I/C_{PD}	C_I	τ^*	D^*	Min expected cost rate
0.1	4	5.55E+05	2.50E-03	8.88E-04
0.2	8	5.56E+05	2.50E-03	8.95E-04
0.3	12	5.57E+05	2.50E-03	9.01E-04
0.4	16	5.59E+05	2.50E-03	9.07E-04
0.5	20	5.60E+05	2.50E-03	9.13E-04
0.6	24	5.61E+05	2.50E-03	9.19E-04
0.7	28	5.63E+05	2.50E-03	9.25E-04
0.8	32	5.65E+05	2.50E-03	9.31E-04
0.9	36	5.67E+05	2.50E-03	9.37E-04
1.0	40	5.69E+05	2.50E-03	9.42E-04

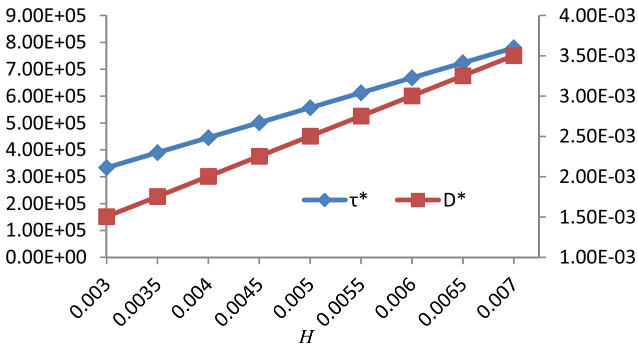

Fig. 6 Sensitivity analysis of τ* and D* on H

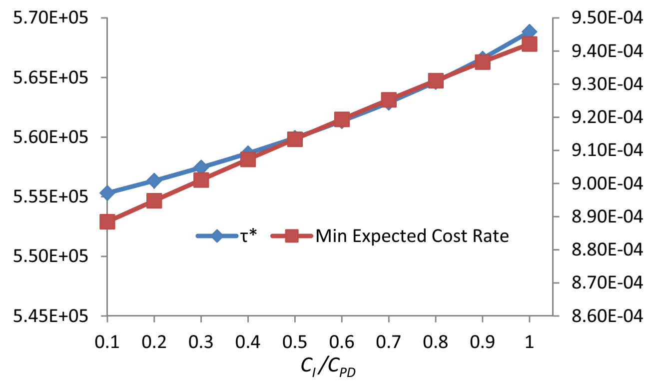

Fig. 8 Sensitivity analysis of τ* and D* on C_I/C_{PD}

subsystem reliability and further the fitted regression model parameter λ, shown in Table 3. From the sensitivity analysis result, we can see that the increase of the ratio of L to r_0 does not affect τ* and D*. This result implies that the L/r_0 of dependent components has no impact on determining τ* and D* on the dominant component.

In the sensitivity analysis on C_{PI}/C_{PD}, C_{PD} is fixed and the value of C_{PI} is changed, while maintaining the ratio of C_{PI} to C_{CI} at 4/5 (when C_{PI} increases, C_{CI} increases accordingly). In Table 4, when the ratio of C_{PI} to C_{PD} increases, τ* decreases and the minimum expected cost rate increases (shown in Fig. 7), while D* stays at a constant value of 0.0025. With the increasing costs of preventive and corrective replacement, inspections should be performed more frequently to prevent failure and reduce cost. However, the increasing cost does not affect the optimal warning limit D* notably.

The sensitivity analysis result in Table 5 shows that when the ratio of C_I to C_{PD} increases, τ* and the minimum expected cost rate increase (also shown in Fig. 8), which indicates that the inspection cost has great impact on the optimal maintenance strategies. However, the inspection cost change has no impact on the optimal warning limit D*.

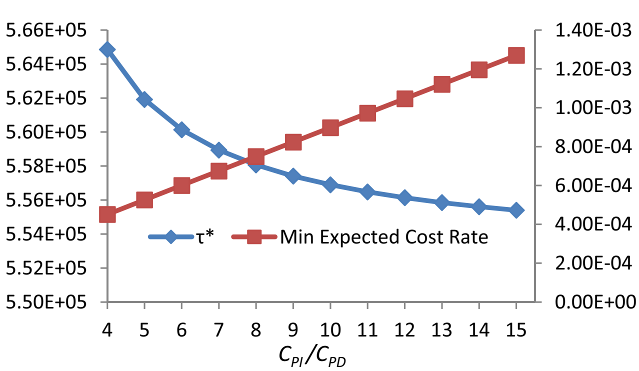

Fig. 7 Sensitivity analysis of τ* and D* on C_{PI}/C_{PD}

6 Conclusions

In this paper, we study a complex system with dependent components subject to respective degradation processes, and the dependency among components is established via environmental factors. We develop a new reliability model for this type of system and use temperature as an example application to demonstrate our model. Relationships between degradation and environmental temperature are studied, and then, the reliability function is derived for such a system. Based on the unique dependent relationship among components within the system and the reliability analysis, a novel condition-based maintenance model is developed to assist system maintenance and minimize cost. To illustrate our reliability and maintenance models, a numerical example is used and sensitivity analysis is also conducted to test the model sensitivity to parameter changes.

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