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ORIGINAL ARTICLE

Numerical solution of simultaneous equations based geometric error compensation for CNC machine tools with workpiece model reconstruction

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Abstract This paper presents geometric error compensation of CNC machine tools based on workpiece model modification and the numerical solution of equation set. Firstly, the nominal cutter position is introduced to the geometric error model of machine tool as the compensation goal according to the kinematics of three-axis machine tools. With the polynomials of basic error components, the polynomials of the geometric error model are established, which are the functions about movements of all axes and nominal NC code. The accurate compensation is represented as solution of corresponding simultaneous equations. The high-efficiency numerical calculation can obtain the compensated NC code. Secondly, the polynomial equations about points of workpiece are established based on the simple relationship between workpiece model and NC codes. The compensated points are calculated using numerical solution. STL is chose as the format of reconstructed workpiece model. In order to obtain the precise workpiece model with compensated points, one conversion approach from CAD model to STL model is proposed. The number of points calculated with isoparametric method can control the precision of STL. The reconstructed STL model is inputted to CAM software to generate processing file for machining. The input of the proposed compensation is CAD model of workpiece rather than NC codes and CL data. It makes the compensation convenient and suitable for different three-axis machine tools. Finally, the experiments are carried out on Carver800T CNC machine tool to testify the effectiveness of proposed geometric error compensation.

Keywords Geometric errors · Model reconstruction · Error compensation · Simultaneous equations · CNC machine tools

1 Introduction

The rapid development of manufacturing industry and technology leads to the high demands on machining accuracy and efficiency. The precision of machine tools is becoming more and more important due to their significant role in machining. Error compensation is one economical and efficiency approach to improve the precision. While, the precision of machine tools is impacted by many factors, such as geometric errors, thermal errors, cutting process, workpiece setup, and so on. Among them, geometric errors take a considerable proportion [1]; the geometric error compensation becomes one important way of precision enhancement.

In the past decades, geometric error modeling has been considered by many researchers and many compensation methods also have been developed [2–4]. Lin and Shen proposed the matrix summation approach for error modeling, which divided the geometric errors of machine tools into six components with clear physical meaning [5]. Khan and Chen used recursive method to modify cutter location for generation new tool path to compensate the geometric errors with their proposed geometric error model [6, 7]. Uddin et al. proposed one simulator to predict machining geometric errors to evaluate the accuracy of workpiece based on the geometric error model [8]. Hsu and Wang proposed one decouple approach for geometric error compensation [9]. Tool orientation errors

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were firstly compensated by correcting rotary axes due to tool orientation only relating to rotary axes, and then, tool position errors were compensated by correcting linear axes. Cui et al. proposed the error compensation algorithm through reconstructing NC program according to geometric error model [10]. They discussed the compensation of three main movements, including rapid positioning, linear interpolation, and circular interpolation. Fan et al. used orthogonal polynomials to model error components of machine tools for spatial error model, and they compensated geometric errors based on external coordinate offset of machine tools [11]. Fu et al. used product of exponential (POE) theory to establish integrated geometric error model of machine tools [12, 13]. They used twists and POE models to represent the motion and geometric errors of each axis.

Recursive and iterative technology are always used to guarantee the accuracy of error compensation [2, 6, 14, 15]. Peng et al. applied total differential methods to geometric error compensation, which avoided the inverse kinematics solution problem [16]. Their proposed compensation algorithm used iterative method to seek appropriate increments of movements of each axis for precise compensation. However, the recursive and iterative technology may greatly impact the calculation speed. Compensation with Jacobian matrix is another approach to calculate revision of NC codes. Lei and Hsu established Jacobian matrix according to forward kinematics and homogeneous transformation matrices of machine tools for geometric error compensation [17]. They solved the singular problems of Jacobian through discussing the special cases. Chen et al. proposed geometric error modeling and compensation according to differential transform theory [18]. They established Jacobian matrix with tool pose error vector which was affected by differential movements of each axis. Fu et al. used transforming differential changes between coordinate systems for precision enhancement of five-axis machine tools, including geometric error modeling, error identification, and error compensation [19]. They established 6×6 differential motion matrix of each axis relative to tool to calculate the influence of each axis to integrated errors of tool; then, they constructed Jacobian matrix using differential motion matrix to compensate the integrated errors of machine tools. Jacobian matrix can calculate the compensation of NC codes, but the compensation accuracy is not evaluated. Namely, the geometric errors of compensated NC codes maybe not minimum. Due to the intercoupling between geometric errors of all axes, the compensation needs some technologies to obtain high accuracy such as iteration. If the geometric errors are in form of simultaneous equation set, numerical solution of equations can be applied to obtain the high precise compensation. In addition, numerical solution of equation set is mature enough and has high solution speed.

The relationship between machining of machine tools, geometric error modeling, and compensation is represented in Fig. 1. The machining needs CAM and postprocessing. The tool path, cutter location (CL) data, and NC codes are generated for cutting. The tool paths, CL data, or NC codes are inputted into geometric error models to compute integrated errors of machine tools. The error results are inputted into compensation algorithm to generate corresponding modification. Then, the modification feedbacks to machining for precision enhancement. The state of the art of geometric error compensation can be classified to three main types according to the objects of compensation, including tool trajectories compensation [14, 15, 20–22], CL compensation [6, 8, 23, 24], and NC code compensation [2, 9, 10, 16–19, 25, 26] as shown in Fig. 1. As the name suggest, tool trajectories compensation modifies the tool paths generated by CAM software to remove geometric errors. CL compensation is to revise cutter location to eliminate the geometric errors of nominal CL points. Because the integrated errors of tool are in form of cutter location, CL data can be modified through simple inversing the calculated errors. NC code compensation is most popular because NC codes reflect the movements of all axes and are used for machining directly. Three kinds of approaches are generally suitable for finish machining. The rough and semi finish machining are still generated with

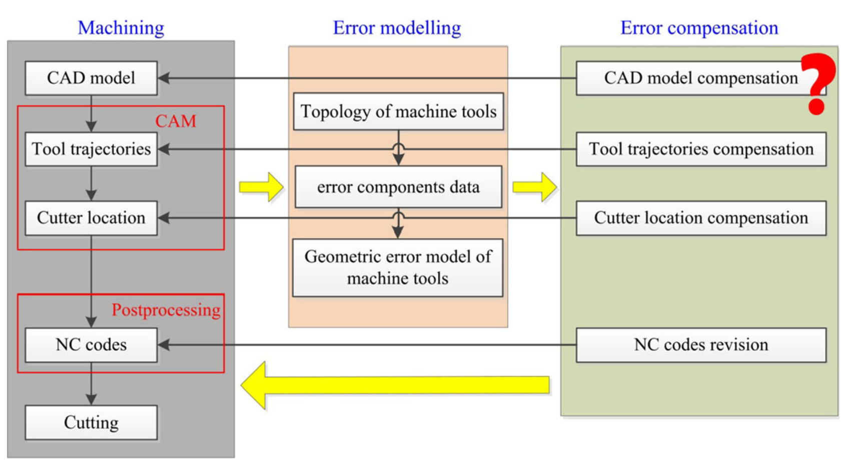

Fig. 1 Relationship between machining of machine tools, geometric error modeling, and compensation

CAM software. In addition, these methods need the input of corresponding data generated by CAM software beforehand, which needs some special knowledge of compensation. According to the flow of machining, CAD model of workpiece is the first step of machining. If the CAD model is reconstructed according to geometric error models, the compensation can be realized with CAM software by generating the corresponding tool paths and NC codes of new workpiece model, including rough and finish machining. Only need of workpiece model makes it convenient. Model reconstruction has been commonly used in the on-machine measurement and compensation [27–29]. On-machine measurement obtains point clouds containing the integrated errors of machining. This method is suitable for mass production. However, in the field of geometric error compensation, model reconstruction has not been spread.

This paper focuses on workpiece model reconstruction for compensation of three-axis machine tools based on numerical calculation of simultaneous equations. Firstly, the novel geometric error model will be proposed by introducing the goal of compensation to evaluate the effect of compensation. Secondly, with polynomial models of all basic error components, geometric error model will be established as polynomials of movements of axes. The compensation can be seen as numerical solution of equation set. Thirdly, compensated points of workpiece model will be obtained with the relationship between workpiece model and movements of all axes. Then, the workpiece model will be reconstructed in the form of STL with compensated point cloud. The precision and rule of conversion from CAD to STL model will be established in order to ensure the precision of model reconstruction. The reconstructed model can be inputted to CAM software for machining.

The remains of the paper are arranged as follows: Section 2 establishes geometric error models of three-axis machine tools in the form of equations in order for numerical calculation. The forward kinematics is established, which is the basis of modeling and compensation. Then, compensation goal is introduced to geometric error model to establish equations. In section 3, workpiece model reconstruction is represented. First, compensated points are calculated based on established geometric error model. Second, conversion from CAD to STL model is proposed for high precision. In section 4, experiments are carried out on Carver800T machine center to testify the effectiveness of the proposed geometric error compensation.

2 Geometric error modeling for numerical calculation

This section establishes the polynomials of geometric error models of three-axis machine tools, more specifically, Carver800T machine center in our lab as shown in Fig. 2a. According to kinematics of machine tools, the models take the compensation goal into account. Then, geometric error compensation is represented as numerical solution of equation set.

2.1 Kinematics of machine tools

The kinematics of machine tools represents the kinematic relationship between workpiece and tool. In other words, it represents the relationship between cutter location and movements of all axes. Carver800T machine center belongs to XFYZ type of three-axis machine tool. Figure 2b represents the topological structure of this machine tool.

Every machine tool has one reference point O_P, which is defined as the machine zero point. At this point, the absolute movements of all linear axes are zero. The origin of machine coordinate frame (MCF) is set at this point. The reference coordinate frame (RCF) is also located on this point as shown in Fig. 2b. Namely, MFC coincides with RCF. Working table coordinate frame (WCF) is set at the origin of the frame of workpiece. Namely, WCF represents the workpiece. Then, there exists one vector $\mathbf{V_w}$ (w_x, w_y, w_z) between WCF and MCF at initial position of machine tools, which represents the position of workpiece on the working zone of the machine tool. This vector can be measured through tool setting before machining. It can be seen as the position of the program zero point of cutting process in MCF. For this type of machine tool,

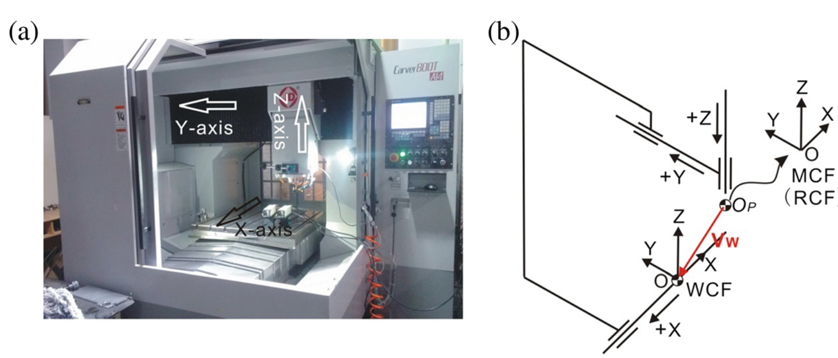

Fig. 2 The structure and the topological structure of Carver800T machine center

X-axis belongs to working table chain and the coordinate frame is in line with RCF at its zero position. While, the positive direction of its motion is opposite to x direction of RCF; so, the movement of X-axis is opposite in the reference coordinate frame.

As shown in Fig. 2b, when all axes are at their zero positions, there exists displace vector $\mathbf{V_w}$ between tool tip and workpiece. Vector $\mathbf{V_W}$ makes the chain of the machine tool closed. The forward kinematics of this machine tool is represented as follows:

$$T_t^w = (T_X)^{-1} \cdot T_Y \cdot T_Z \cdot T_W = \begin{bmatrix} 1 & 0 & 0 & -x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -w_x \\ 0 & 1 & 0 & -w_y \\ 0 & 0 & 1 & -w_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & x-w_x \\ 0 & 1 & 0 & y-w_y \\ 0 & 0 & 1 & z-w_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where T_W represents the homogenous transformation matrix of tool tip relative to working table at zero position; x, y, and z represent the movements of X-axis, Y-axis, and Z-axis relative to their zero positions, respectively. Tool position relative to working table is expressed as follows:

$$[\mathrm{P_t}; 1] = \left[x_p, y_p, z_p, 1\right]^T = T_t^w \cdot [0, 0, 0, 1]^T \quad (1)$$

where $\mathbf{P}_t$ also represents the cutter location. Due to lack of rotary axes for three-axis machine tools, the inverse kinematics is easily obtained. Because $\mathbf{V_w}$ is known, the NC program can be calculated easily with cutter location data according to Eq. (1). When cutting workpiece, the tool tip works on workpiece; so, the tool position represents the point on workpiece model essentially. Equation (1) represents relationship between movements of all axes and workpiece points. It lays the foundation for compensation of model reconstruction.

2.2 Geometric error modeling

2.2.1 Novel geometric error model with compensation goal

There exist 21 error components for three-axis machine tools, including six basic error components of each axis and four squareness errors between three axes. Errors of different axes are intercoupling. The geometric error model should consider these intercoupling. The general geometric error model of Carver800T machine center can be established using POE formulas according to our previous work in [12] and [13] with the topological structure and the kinematics. The general geometric error model with symbols of error components is represented as follows:

$$\begin{cases} p_{ex} = -zS_{xz} - \delta_{xx} + \delta_{xy} + \delta_{xz} + y\varepsilon_{zx} + z(-\varepsilon_{yx} + \varepsilon_{yy}) - w_z(-\varepsilon_{yx} + \varepsilon_{yy} + \varepsilon_{yz}) + w_y(-\varepsilon_{zx} + \varepsilon_{zy} + \varepsilon_{zz}) \\ p_{ey} = -xS_{xy} - zS_{yz} - \delta_{yx} + \delta_{yy} + \delta_{yz} - x\varepsilon_{zx} + z(\varepsilon_{xx} - \varepsilon_{xy}) + w_z(-\varepsilon_{xx} + \varepsilon_{xy} + \varepsilon_{xz}) - w_x(-\varepsilon_{zx} + \varepsilon_{zy} + \varepsilon_{zz}) \\ p_{ez} = -xS_{xy} - zS_{yz} - \delta_{yx} + \delta_{yy} + \delta_{yz} - x\varepsilon_{zx} + z(\varepsilon_{xx} - \varepsilon_{xy}) + w_z(-\varepsilon_{xx} + \varepsilon_{xy} + \varepsilon_{xz}) - w_x(-\varepsilon_{zx} + \varepsilon_{zy} + \varepsilon_{zz}) \end{cases} \quad (2)$$

where p_{ex}, p_{ey}, and p_{ez} represent tool position errors in x, y, and z directions, respectively; δ_{xj}, δ_{yj}, and δ_{zj} represent the linear errors of axis j in x, y, and z direction, respectively; ε_{xj}, ε_{yj}, and ε_{zj} represent the angular errors of axis j in x, y, and z direction, respectively; S_{jk} represents squareness errors between axis j and axis k. This model can calculate integrated errors of certain NC code (x, y, z). Compensation is to reduce or even eliminate the geometric errors of machine tools for each NC code. It is not convenient for this type of model due to lack of compensation goal. It is hard to evaluate the effect of compensated NC code. Integrated errors of each compensated NC code relative to corresponding nominal NC code should be calculated.

The novel geometric error model is established by introducing compensation goal. The compensation goal is that the machine tool can move to nominal position after compensation. According to kinematics of three-axis machine tools in Eq. (1), one NC code can easily obtain its nominal position with the known vector $\mathbf{V_W}$. If the actual position of one code is equal or very close to nominal NC code, it is the compensated NC code. The position deviation of one NC code can be calculated with above-mentioned error model. The actual position of this NC code can be obtained by adding its nominal tool position. The novel geometric error model is established as follows:

$$\begin{cases} \Delta_x = (p_{ex} + x - w_x) - (x_o - w_x) = (p_{ex} + x) - x_o \\ \Delta_y = \left(p_{ey} + y - w_y\right) - \left(y_o - w_y\right) = \left(p_{ey} + y\right) - y_o \\ \Delta_z = (p_{ez} + z - w_z) - (z_o - w_z) = (p_{ez} + z) - z_o \end{cases} \quad (3)$$

where x_o, y_o, and z_o represent one nominal NC code; Δ_x, Δ_y, and Δ_z represent deviation vector between actual position of one code and nominal position. When one certain code (x, y, z) makes deviation vector relative to one nominal code reduce to zero, the compensation goal is achieved and the code is the compensated code. In addition, compensated code should not be greatly different from nominal NC code.

2.2.2 Geometric error model for numerical calculation

The 21 basic error components of three-axis machine tools can be measured or identified by laser interferometer. Six basic error components of each axis change along with the displacement of axis. They can be represented as polynomials of movement of corresponding axis. The basic error components are zero when the axis is at its zero position; so, the constant term of polynomials should be set as zero. All errors of Carver800T machine center are identified with nine-line method with laser interferometer. The optimal polynomials are established with the proposed method in our previous work [30]. The degree of polynomials is determined with F value. The polynomial models of basic errors of Y-axis are represented as follows:

$$\begin{cases} \delta_{xy} = 7.4204 \times 10^{-5} y^2 - 0.009y \\ \delta_{yy} = -0.0194y \\ \delta_{zy} = -1.1532 \times 10^{-4} y^2 + 0.0054y \\ \varepsilon_{xy} = -0.2089y \\ \varepsilon_{yy} = 0.2921y \\ \varepsilon_{zy} = 2.1927 \times 10^{-4} y^2 + 0.0361y \end{cases} \tag{4}$$

The unit of y is mm, and the units of linear errors and angular errors are μm and μrad, respectively. The polynomial curves and identified data of geometric errors of Y-axis are shown in Fig. 3. The polynomials of the other basic error components of this machine tool are represented in Appendix A. Table 1 shows the squareness errors of Carver800T machine center.

Through taking polynomials of all basic error components and the value of squareness errors into Eq. (3), the polynomials of geometric error models are established. Before that, the units of all symbols in the geometric error model should be agreed. They should be represented with international units. While, in order to accord with the units of NC code, the unit of movements of axes is set as mm, which is also in line with polynomials of basic errors. Then, geometric error model of Eq. (3) is expressed as follows:

$$\begin{cases} \Delta_x = \delta_{xy} - \delta_{xx} + \delta_{xz} + \left(-zS_{xz} + y\varepsilon_{zx} + z\left(-\varepsilon_{yx} + \varepsilon_{yy}\right) - w_z\left(-\varepsilon_{yx} + \varepsilon_{yy} + \varepsilon_{yz}\right) + w_y\left(-\varepsilon_{zx} + \varepsilon_{zy} + \varepsilon_{zz}\right)\right) \times 10^{-3} + (x - x_o) \times 10^3 \\ \Delta_y = \delta_{yy} - \delta_{yx} + \delta_{yz} + \left(xS_{xy} - zS_{yz} - x\varepsilon_{zx} + z\left(\varepsilon_{xx} - \varepsilon_{xy}\right) + w_z\left(\varepsilon_{xy} - \varepsilon_{xx} + \varepsilon_{xz}\right) - w_x\left(\varepsilon_{zy} - \varepsilon_{zx} + \varepsilon_{zz}\right)\right) \times 10^{-3} + (y - y_o) \times 10^3 \\ \Delta_z = \delta_{zy} - \delta_{zx} + \delta_{zz} + \left(-y\varepsilon_{xx} + x\varepsilon_{yx} - w_y\left(-\varepsilon_{xx} + \varepsilon_{xy} + \varepsilon_{xz}\right) + w_x\left(-\varepsilon_{yx} + \varepsilon_{yy} + \varepsilon_{yz}\right)\right) \times 10^{-3} + (z - z_o) \times 10^3 \end{cases} \tag{5}$$

where the unit of Δ_x, Δ_y, and Δ_z is μm; the unit of x, y, and z is mm. With the polynomials of basic error components, the polynomials of geometric error model are obtained. The polynomial of Δ_x is shown as follows:

$$\begin{aligned} \Delta_x = & -1000x_o + \left(1000.1093 - 2.057 \times 10^{-4} w_y + 1.161 \times 10^{-4} w_z\right)x - \left(0.009 - 3.61 \times 10^{-5} w_y - 2.921 \times 10^{-4} w_z\right)y \\ & + \left(7.4204 \times 10^{-5} + 2.1927 \times 10^{-7} w_y\right)y^2 + \left(0.0211 + 8.54 \times 10^{-5} w_y - 1.325 \times 10^{-4} w_z\right)z \\ & + 2.0570 \times 10^{-4} xy - 1.161 \times 10^{-4} xz + 2.921 \times 10^{-4} yz + \left(9.8497 \times 10^{-6} - 4.4335 \times 10^{-7} w_z\right)z^2 \end{aligned} \tag{6}$$

The total polynomials of geometric error models are represented in Appendix B. The geometric error models can be represented in the form of functions as follows:

$$\begin{cases} \Delta_x = f_x(x, y, z, x_o, w_y, w_z) \\ \Delta_y = f_y(x, y, z, y_o, w_x, w_z) \\ \Delta_z = f_z(x, y, z, z_o, w_x, w_y) \end{cases} \tag{7}$$

Vector $\mathbf{V_W}$ can be obtained with tool setting, so for a certain nominal NC code (x_o, y_o, z_o), the variables of geometric error models are x, y, and z.

The compensation is to seek NC code to make deviation vector as zero. The compensation can be represented as the solution of simultaneous equation set:

$$\begin{cases} f_x(x, y, z, x_o, w_y, w_z) \equiv 0 \\ f_y(x, y, z, y_o, w_x, w_z) \equiv 0 \\ f_z(x, y, z, z_o, w_x, w_y) \equiv 0 \end{cases} \tag{8}$$

For one nominal NC code, the unknown variable is NC code. The simultaneous equations contain three unknowns. Numerical solution of equation set can be used for calculation.

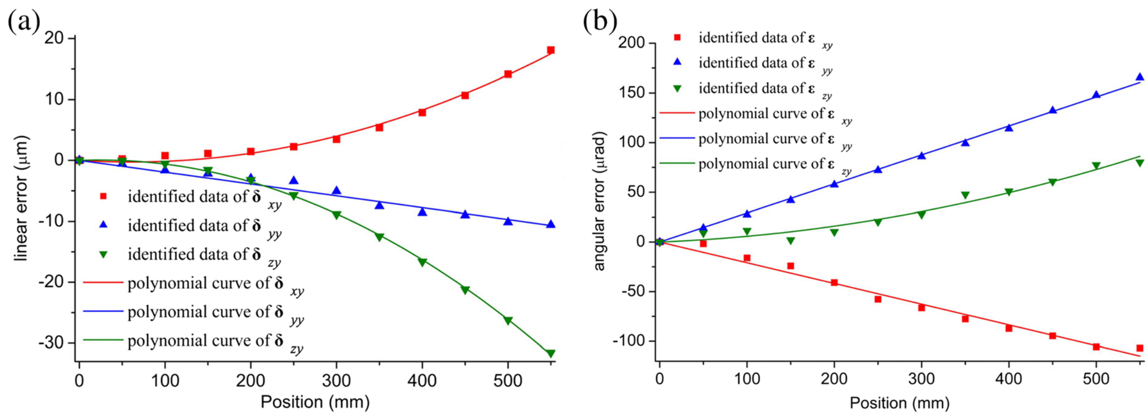

Fig. 3 Polynomial curves and identified data of basic geometric errors of Y-axis

The solution around nominal NC code (x_o, y_o, z_o) is the compensated code. The numerical solution can guarantee the accuracy of compensation.

3 Model reconstruction for compensation

The proposed expression set for NC code compensation needs nominal NC code, which is the reference of numerical calculation. The NC code file is the prerequisite. While, the formats of NC code may be different for different numerical control (NC) systems. It is difficult for this NC code compensation to be suitable for different machine tools due to difficulty of reading different formats of NC code. In addition, the whole NC code of finish machining of one workpiece contains different auxiliary information besides cutting, such as tool lift and off, oil on-off, and so on. It is bad for reading of movements of axes and calculation of equation set. In order for precise cutting, some NC codes are air moving to link the different tool trajectories. The compensation of these codes has no effect on precision improvement. The compensation of these codes also increases the amount of calculation. These matters make the NC code compensation not convenient. What is worse, some NC systems are not open enough that NC codes of workpiece are not allowed to read. This Carver800T machine center has its matched CAM system which is not open enough. The NC codes are not shown for workers, and cannot be exported. As the result, the NC code compensation is not vary suitable for this machine tool. According to relationship between workpiece model and NC codes of three-axis machine tools, this section proposes workpiece model reconstruction for compensation based on numerical solution of section 2. The reconstructed model is inputted into CAM to generate corresponding cutting process.

3.1 Compensated points of workpiece model

For three-axis machine tools, the inverse kinematics is obtained according to forward kinematics of Eq. (1) as follows:

$$\begin{cases} x = x_p + w_x \\ y = y_p + w_y \\ z = z_p + w_z \end{cases} \tag{9}$$

The nominal NC code of cutter location is calculated according to Eq. (9). When the tool is cutting workpiece, point $\mathbf{P}_t=[x_p, y_p, z_p]^T$ represents one point on workpiece model. Without rotary axes of three-axis machine tools, the movements of all axes are obtained with the coordinates of points of workpiece model by simple addition. From the view of transformation of coordinate systems, Eq. (9) represents that the workpiece model translates from the origin to the point $[w_x, w_y, w_z]^T$ in the machine coordinate frame. The relationship between workpiece model and NC codes also can be explained in this way. However, due to geometric errors of all axes, the zone of machine coordinate frame is deformed. The translation introduces the deformation of workpiece model. The model can be reconstructed before translation in order to eliminate deformation. The points on workpiece model are recalculated to remove the errors of machine coordinate frame.

The compensated points can be calculated using numerical solution of simultaneous equation set like section 2. The nominal NC code and the variable code in Eq. (7) can be replaced by coordinates of workpiece model according to Eq. (9) as follows:

$$\begin{cases} x = x_{pc} + w_x \\ y = y_{pc} + w_y \\ z = z_{pc} + w_z \end{cases} \quad \begin{cases} x_o = x_{po} + w_x \\ y_o = y_{po} + w_y \\ z_o = z_{po} + w_z \end{cases} \tag{10}$$

where x_{po}, y_{po}, and z_{po} represent the nominal point of workpiece model; x_{pc}, y_{pc}, and z_{pc} represent the

Table 1 Squareness errors of Carver800T machine center

S_{xy} (μrad)	S_{yz} (μrad)	S_{xz} (μrad)
−57.781	−29.2832	48.9912

corresponding compensated point. Then, the new simultaneous equations about point of workpiece model can be obtained by bringing Eq. (10) into Eq. (7). The nominal coordinates of points can be obtained with CAD model of workpiece. The numerical solution of equation set computes the compensated points, which should be near by the corresponding nominal points. Theoretically speaking, all points of workpiece model can be compensated in this way, and the whole model is reconstructed. The geometric errors are compensated with the reconstructed model.

3.2 Model reconstruction

The number of points of model is infinite, and the number calculation only can obtain the finite points. The model reconstruction method in on-machine measurement and compensation can be used here. The big difference between geometric error compensation and on-machine compensation is that the point clouds are calculated rather than measured with workpiece. Workpiece model can be reconstructed by reverse engineering. While, the NURBS surface fitting with point clouds is a grand challenge. The parametric formula of NURBS surface is hard to define, including the order of surface and number of knots. The precision of fitting is also hard to evaluate. In addition, the fitted surface may not pass through these compensated points, which may reduce the accuracy of compensation. As the result, STL format is chose to remodel the point cloud. The vertexes of STL are the compensated points. The generation of STL is simple with points. And the precision of STL can be controlled by the number of compensated points.

The STL model can be directly generated with some CAD modelers based on point cloud. The rule of generating facets has influences on the precision of STL model. Many useless and chaotic facets may be generated, which make the model deformed and great different from nominal model. The compensated points are different from original points; so, the location and geometrical relationship of points are changed. For example, some points are in one plane or in one straight line in the original model; however, the relationship may be different after compensation. The corresponding points may not in one plane or in one straight line. It also increases difficulty of the generation of STL model. There is another method for STL model reconstruction with the help of CAD software. First, the STL model of nominal workpiece model can be generated with CAD software. Then, the vertexes of STL model are recalculated with proposed method in section 3.1. While, the CAD software automatically minimizes the number of facets; so, the obtained STL model is not uniformly meshed. The vertexes are not uniformly distributed. It affects the accuracy of compensation. In this method, the STL model needs further processed before numerical calculation.

The nominal workpiece model is stored in a standard file format of IGES. Arbitrary points of workpiece model can be calculated with its parametric equation. In order to obtain uniformly distributed vertexes, one conversion method from IGES to STL is developed. The points are calculated with isoparametric method for uniform distribution in parametric space. The precision of STL model can be controlled by the number of points. The size of calculated points is set as $n_u \times n_v$. n_u and n_v mean number of points along u direction and v direction, respectively. The parametric equation of nominal model is known as $S(u, v)$, $u \in [0,1]$, and $v \in [0,1]$. The coordinates of one point are calculated as follows:

$$P(i,j) = S\left(\frac{i-1}{n_u-1}, \frac{j-1}{n_v-1}\right) \quad i = 1,2,\cdots,n_u; j = 1,2,\cdots,n_v \tag{11}$$

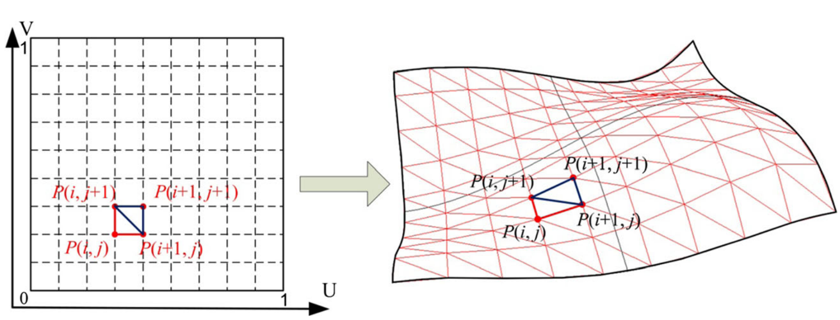

Fig. 4 The generation of triangular facets with isoparametric points

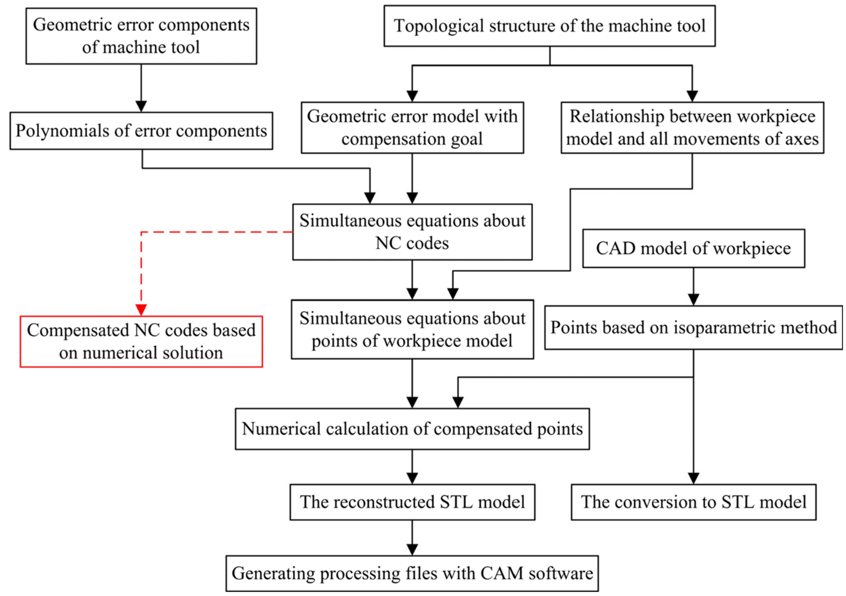

Fig. 5 The flow chart of geometric compensation based on the numerical calculation of simultaneous equations and model reconstruction

The data structure of the triangular facet is constructed as follows:

$$\text{struct } Triangle \{ \\ \quad \text{Point} \quad A, B, C \\ \quad \text{Vector} \quad normal \\ \};$$

In this structure, A, B, and C are the vertexes of triangular facet, and *normal* is the normal vector of the facet. The normal vector can be calculated with three vertexes as follows:

$$normal = \text{normalize}[(B-A) \times (C-B)] \tag{12}$$

where normalize [] means normalizing the vector. Once three vertexes are set, the triangular facet is obtained. In addition, the vertexes of all facets should be in the same sequence, such as clockwise. The generation of triangular facets is represented in Fig. 4. Two facets are generated with four points. The two facets T1 and T2 are constructed as follows:

$$\begin{cases} \text{T1}.A = P(i,j); \quad \text{T1}.B = P(i,j+1); \quad \text{T1}.C = P(i+1,j); \\ \text{T2}.A = P(i+1,j); \quad \text{T2}.B = P(i,j+1); \quad \text{T2}.C = P(i+1,j+1); \end{cases} \quad i = 1,2,\cdots n_u-1; j = 1,2,\cdots n_v-1 \tag{13}$$

All triangular facets are generated with Eqs. (11), (12), and (13) based on the calculated points. The number of facets is $2(n_u-1)(n_v-1)$. The corresponding compensated points are calculated with the proposed method in section 3.1. The reconstructed STL model is generated based on compensated points using the proposed conversion. The reconstructed

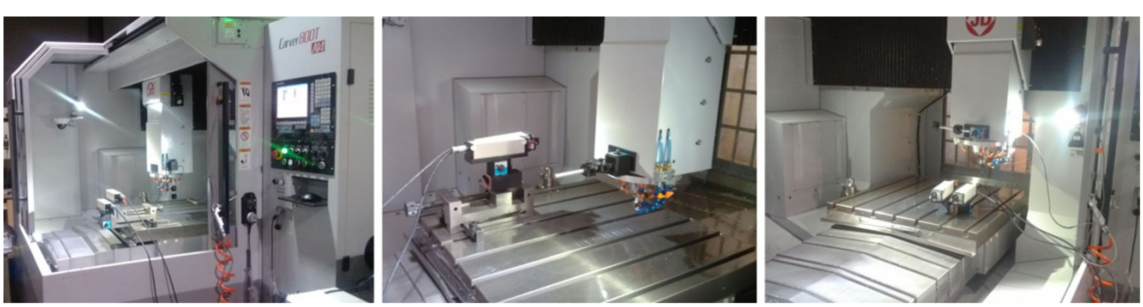

Fig. 6 The geometric error measurement with LDDM

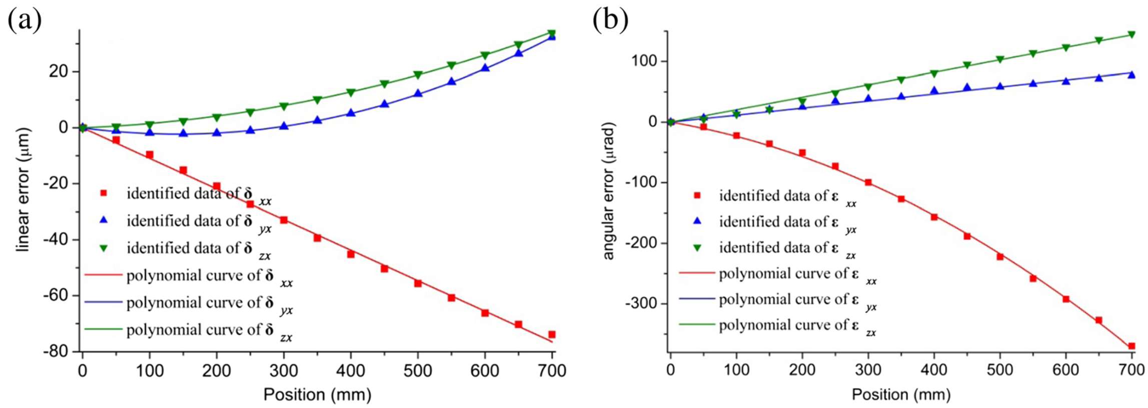

Fig. 7 The polynomial curves and identified data of basic geometric errors of X-axis

model is inputted into CAM software to generate the corresponding machining file. The rough and finish machining are generated together with the new model, which simples the operations of workers. This compensation does not need special knowledge of compensation for workers. It is convenient and available for different numerical control systems.

For arbitrary three-axis machine tool, the polynomials of geometric error components are established in the form of Eq. (4) with the measured error data. Based on the topological structure of the three-axis machine tool, geometric error model with compensation goal is established with Eq. (3). The equation set of compensation is expressed as Eq. (8). Then, if the nominal NC codes are known, the NC code compensation can be realized with numerical solution of equations. With kinematics of the machine tool, the new polynomial equation set is obtained by replacing NC code by points of workpiece model. The numerical calculation can calculate compensated points of workpiece model. With the standard file format of workpiece model, the original points are generated with isoparametric method. The corresponding compensated points are computed for reconstructing STL model using the proposed conversation method in Eq. (13). The machining of the reconstructed model can compensate the geometric errors.

The error compensation based on numerical solution of simultaneous equations and model reconstruction is available for different three-axis machine tools. Figure 5 represents the flow chart of the proposed geometric error compensation for any type of three-axis machine tools.

4 Experiments and results

In order to verify the effectiveness of proposed geometric error compensation based on numerical solution and model reconstruction, experiments are carried out on this Carver800T machine center. At first, the geometric errors are measured by the MCV-500 Optodune laser interferometer system (LDDM). Nine-line method is used to identify the data of all 21 geometric errors. The stroke of this machine tool is 800 × 800 × 420 mm. Due to the limitation of setup of LDDM, the measurement working zone is set as 700 × 550 × 360 mm. This zone can satisfy the machining of major of workpieces. Figure 6 shows the scenes of measurement with LDDM. The basic errors of each axis are changed along with movements of the corresponding axis. The polynomials of all basic errors are established in section 2.2.2. Figures 7 and 8

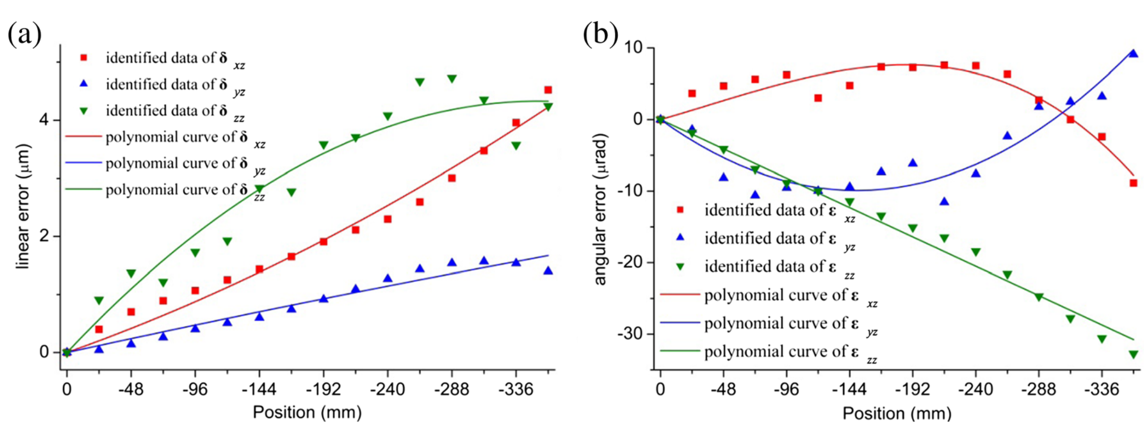

Fig. 8 The polynomial curves and identified data of basic geometric errors of Z-axis

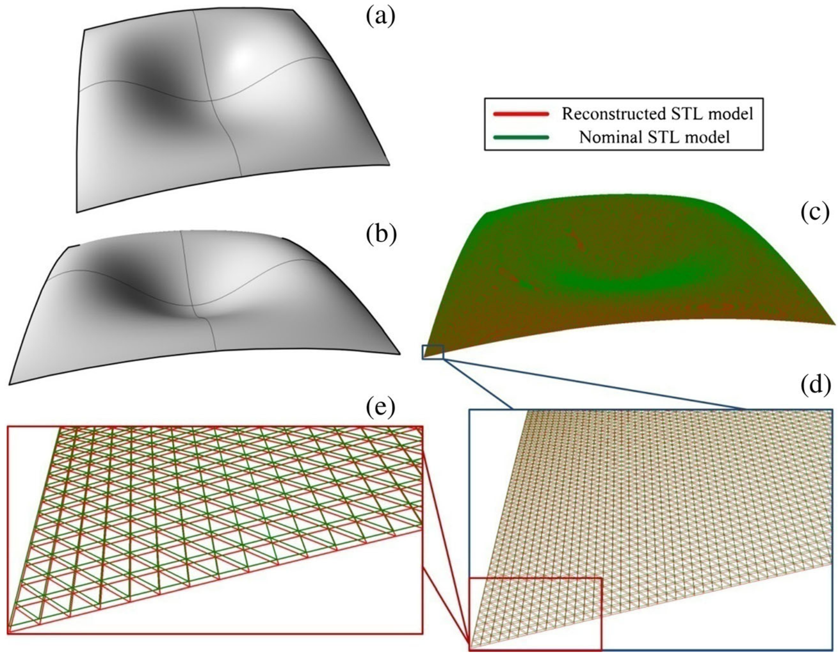

Fig. 9 Workpiece models of IGES model, nominal STL model, and reconstructed STL model

show the polynomial curves and identified data of basic geometric errors of X-axis and Z-axis, respectively.

The proposed geometric error compensation is implemented with MATLAB software, including geometric error modeling, IGES file reading and points generation with isoparametric method, numerical solution of equation set, and STL model generation. MATLAB has the strong ability of numerical calculation. The module of geometric error modeling is to establish the polynomial equations about points of workpiece model. The value of vector $\mathbf{V_w}$ is measured by tool setting process to finish the geometric error modeling. $\mathbf{V_w}$ represents the position of workpiece relative to MCS. As long as the tool and the workpiece are installed, $\mathbf{V_w}$ can be obtained. In our experiment, $\mathbf{V_w}$ is [178.844, 232.105,-278.022] mm. With the CAD model of workpiece, the compensated points can be obtained using numerical solution of simultaneous equations.

The accuracy of workpiece can reflect the precision of the machine tool. If the precision of machine tool is improved, the accuracy of workpiece is also improved. The cutting test of one surface is carried out to testify the effect of proposed compensation. The surface is shown in Fig. 9a, b. It is a NURBS surface like a bowl. It is 80×80 mm^2 in size and 13.842 mm in height. The appropriate value of n_u and n_v is

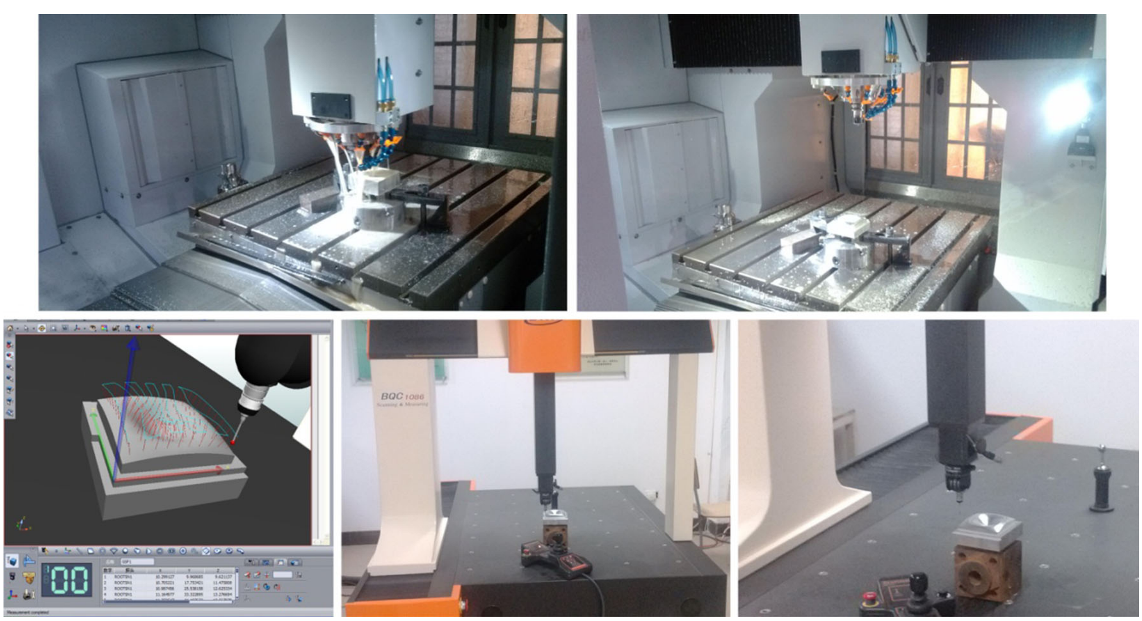

Fig. 10 The machining on the machine tool and the measurement with CMM

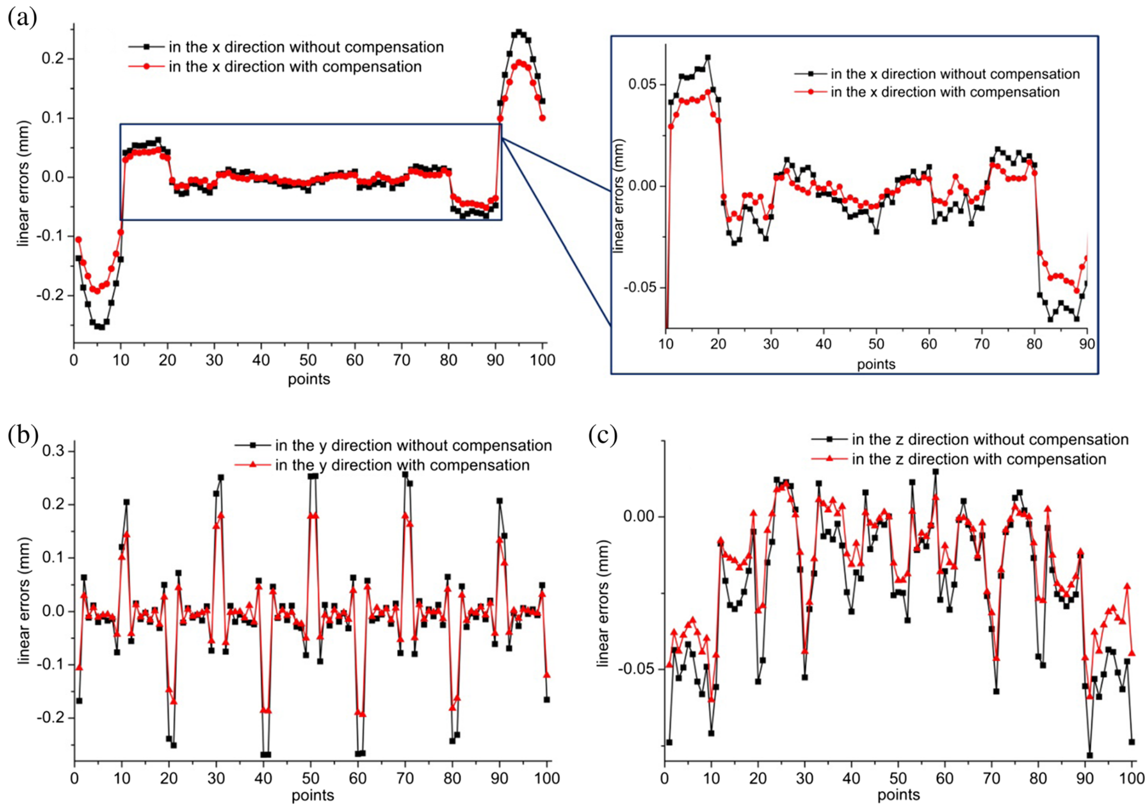

Fig. 11 The linear errors of 100 points in three directions with/without compensation

important for high precision of conversion to STL model. When the cutter location is generated using CAM software with the chord height of 0.001 mm based on IGES file, the number of cutter location points is about 80×540. In addition, when conversing to STL model using CAD software with the chord height of 0.0005 mm, the number of triangular facets is 345,920. So, the value of n_u and n_v is both set as 550, which can guarantee the precision of STL model. The original points are calculated with isoparametric method. Then, the numerical solution of equation set calculates the compensated points. The nominal and compensated STL models are generated with the proposed conversion method. Figure 9c shows the two STL models. The reconstructed model removes the geometric errors of the machine tool. Figure 9d, e represents the difference between nominal and reconstructed STL model in wireframe mode.

The nominal CAD model and reconstructed STL model are used to generate processing documents with the CAM software of this machine tool, respectively. Then, two workpieces are machined with the two processing documents. Next, the errors of the two workpieces are measured using the coordinate measuring machine (CMM). The 100 points on the surface are selected to represent the whole workpiece. Figure 10 shows the scene of the machining on the machine tool and the

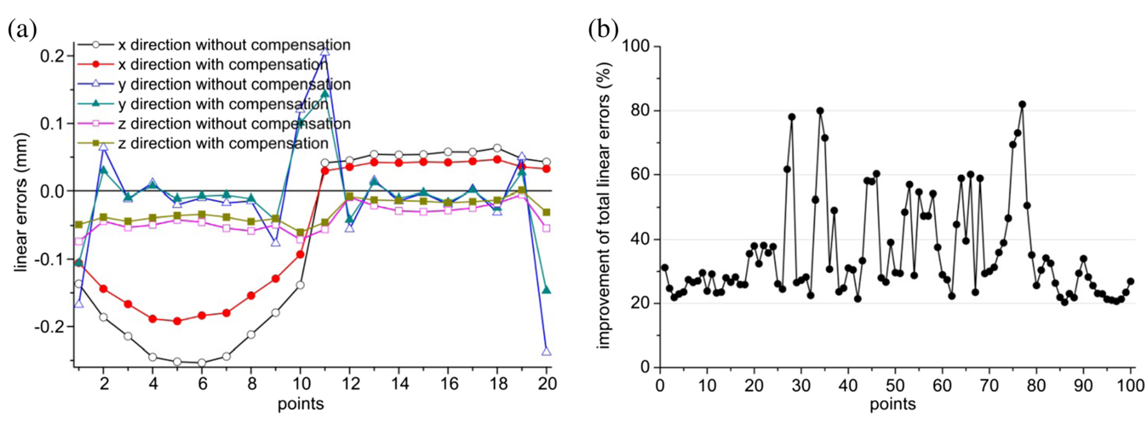

Fig. 12 **a** The linear errors in the three directions of 20 points with/without compensation. **b** The improvement of workpiece with the geometric error compensation

error measurement with CMM. The errors of the two workpieces of all 100 points are compared as shown in Fig. 11, including the linear errors in three directions. In x direction, the maximum value of errors is 0.2458 mm and the minimum is −0.2535 mm before compensation. After compensation, the maximum and minimum value is reduced to 0.194 and −0.1922 mm, respectively. The errors of the first 10 points and last 10 points in x direction are great larger than the other 80 points. Even though the geometric error compensation eliminates a part of errors, the errors of these 20 errors are still large. It may be caused by other factors, such as the vibration, thermal errors, and so on. In y direction, the maximum error is reduced from 0.2569 to 0.1794 mm with error compensation and the minimum value is decreased from −0.2683 to −0.1936 mm. According to the measurement sequence of 100 points as shown in Fig. 10, the points whose point number is times of 10 are close to the boundary of workpiece. As shown in Fig. 11b, the errors of boundary of workpiece are larger than other parts and geometric errors may be only the small part of the integrated errors of these points. The measurement of CMM may not precise enough for the boundary of workpieces. The mean errors in z direction are −0.0234 mm without compensation and −0.0163 mm with compensation. The maximum value is reduced from 0.0149 mm to 0.0108 with the geometric error compensation. The errors in z direction are smaller than the other two directions. Figure 12a represents the errors of first 20 of 100 points in three directions in details. The errors in all directions are reduced with the proposed compensation method. The improvement of total linear errors with compensation is shown in Fig. 12b. The precision is improved by about 35 % on average with the compensation. So, the machining accuracy of the machine tool is enhanced with the proposed geometric error compensation. In a word, the geometric error compensation based on numerical solution and model reconstruction is effective to improve the accuracy of three-axis machine tools.

5 Conclusions

This paper applies the numerical solution of equation set and model reconstruction to the geometric error compensation of three-axis machine tools for precision improvement. First, the novel geometric error model is established to evaluate the effect of compensation by introducing the compensation goal. The compensation goal, the nominal cutter position, is represented by nominal NC code based on kinematics of machine tools. The novel model is established as the actual position of one code minus the original position of nominal NC code. Second, the polynomials of geometric error model are established as the functions of NC code with the polynomials of all basic error components. The numerical calculation of the corresponding equation set can obtain the compensated NC code. Third, with the simple relationship between workpiece points and NC codes, the polynomials of geometric error model are converted to the functions of the coordinates of points of workpiece model. The compensated points of workpiece model are computed with numerical solution. What is more, one conversion method from IGES to STL is proposed. The reconstructed STL model is obtained based on the conversion with the compensation points. The reconstructed model is inputted into CAM software to generate the corresponding processing file to realize the compensation. The input of the proposed method is CAD model of workpiece and the output is the reconstructed model. It is not necessary for workers to learn and know well the special knowledge of compensation. Finally, experiments are carried on the Carver800T machine center; results show that the proposed compensation is effective. The model reconstruction-based compensation is convenient and available for different three-axis machine tools.

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Appendix A

The units of linear errors and angular errors are μm and μrad, respectively. The units of movements of three axes are mm. The polynomials of basic error components of Carver800T machine center are shown as follows:

$$\begin{cases} \delta_{xx} = -0.1093x \\ \delta_{yx} = 1.112 \times 10^{-4}x^2 - 0.0316x \\ \delta_{zx} = 5.5563 \times 10^{-4}x^2 + 0.01x \\ \varepsilon_{xx} = -4.9712 \times 10^{-4}x^2 - 0.1863x \\ \varepsilon_{yx} = 0.1161x \\ \varepsilon_{zx} = 0.2057x \end{cases} \tag{A.1}$$

$$\begin{cases} \delta_{xy} = 7.4204 \times 10^{-5}y^2 - 0.009y \\ \delta_{yy} = -0.0194y \\ \delta_{zy} = -1.1532 \times 10^{-4}y^2 + 0.0054y \\ \varepsilon_{xy} = -0.2089y \\ \varepsilon_{yy} = 0.2921y \\ \varepsilon_{zy} = 2.1927 \times 10^{-4}y^2 + 0.0361y \end{cases} \tag{A.2}$$

$$\begin{cases} \delta_{xz} = 9.8497 \times 10^{-6}z^2 - 0.0082z \\ \delta_{yz} = -1.1313 \times 10^{-6}z^2 - 0.005z \\ \delta_{zz} = -3.4788 \times 10^{-5}z^2 - 0.0245z \\ \varepsilon_{xz} = 8.2506 \times 10^{-7}z^3 + 8.9761 \times 10^{-5}z^2 - 0.053z \\ \varepsilon_{yz} = 4.4335 \times 10^{-4}z^2 + 0.1325z \\ \varepsilon_{zz} = 0.0854z \end{cases} \tag{A.3}$$

Appendix B

The polynomials of geometric error models of Carver800T machine center are shown as:

$$\begin{cases} \Delta_x = -1000x_o + \left(1000.1093-2.057\times10^{-4}w_y + 1.161\times10^{-4}w_z\right)x - \left(0.009-3.61\times10^{-5}w_y-2.921\times10^{-4}w_z\right)y \\ \quad + \left(7.4204\times10^{-5}+2.1927\times10^{-7}w_y\right)y^2 + \left(0.0211+8.54\times10^{-5}w_y-1.325\times10^{-4}w_z\right)z \\ \quad + 2.0570\times10^{-4}xy-1.161\times10^{-4}xz+2.921\times10^{-4}yz+\left(9.8497\times10^{-6}-4.4335\times10^{-7}w_z\right)z^2 \\ \Delta_y = -1000y_o + \left(999.9806-3.61\times10^{-5}w_x-2.089\times10^{-4}w_z\right)y + \left(0.0834+2.057\times10^{-4}w_x+1.863\times10^{-4}w_z\right)x \\ \quad -\left(3.169\times10^{-4}-4.9712\times10^{-7}w_z\right)x^2-2.1927\times10^{-7}w_xy^2-\left(0.054+8.54\times10^{-5}w_x+5.3\times10^{-5}w_z\right)z \\ \quad -1.863\times10^{-4}xz-4.9712\times10^{-7}x^2z+2.089\times10^{-4}yz-\left(1.1313\times10^{-6}-8.9761\times10^{-8}w_z\right)z^2+8.2506\times10^{-10}w_zz^3 \\ \Delta_z = -1000z_o + \left(999.9755+1.325\times10^{-4}w_x+5.3\times10^{-5}w_y\right)z-\left(0.01+1.161\times10^{-4}w_x+1.863\times10^{-4}w_y\right)x \\ \quad -\left(4.3953\times10^{-4}+4.9712\times10^{-7}w_y\right)x^2+\left(0.0054+2.921\times10^{-4}w_x+2.089\times10^{-4}w_y\right)y+1.863\times10^{-4}xy \\ \quad +4.9712\times10^{-7}x^2y-1.1532\times10^{-4}y^2-\left(3.4788\times10^{-5}-4.4335\times10^{-7}w_x+8.9761\times10^{-8}w_y\right)z^2-8.2506\times10^{-10}w_yz^3 \end{cases}$$

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