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ORIGINAL ARTICLE

Rectifying inspection for acceptable quality loss limit based on variable repetitive group sampling plan

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Abstract It is usually assumed that a quality characteristic in an item obeys a normal distribution in the case that the quality of items is evaluated based on the variable property. Then, the concept of Taguchi's quality loss has been accepted as the evaluation measure of quality instead of the traditional attribute property such as the proportion of nonconforming items. From this viewpoint, some variable sampling plans indexed by the quality loss have been investigated before now. As a study earliest among them, the variable single sampling plan based on operating characteristics (OC) indexed by the quality loss was considered. On the other hand, the attribute repetitive group sampling plan on OC was proposed for reducing the sampling number in the inspection. Recently, the variable repetitive group sampling (VRGS) plan on OC indexed by the quality loss has been considered. By the way, the rectifying inspection is known as one of the schemes of acceptance sampling inspection. Then, Dodge-Romig single sampling plans are known as the traditional rectifying inspection based on attribute sampling plans. Dodge-Romig rectifying attribute sampling plans provide the lot tolerance percent defective (LTPD) scheme on each lot and the average outgoing quality limit (AOQL) scheme for many lots. Furthermore, the rectifying variable single sampling (RVSS) plan indexed by the quality loss was investigated. In conformity with the traditional rectifying attribute sampling plans for the LTPD and AOQL schemes, the acceptance quality loss limit (AQLL) and specified permissible average outgoing surplus quality loss limit (PAOSQLL) schemes are respectively proposed in the RVSS plans indexed by the quality loss. In this article, we suppose that the quality characteristic in an item obeys a normal distribution. Under this condition, the rectifying variable repetitive group sampling (RVRGS) plan for AQLL is considered for the purpose of reducing the average total inspection (ATI). Specifically, the design procedure for finding out the required sample size and inspection criteria for satisfying the constraint of the quality assurance is derived. Lastly, it is shown that ATI of the RVRGS plan is reduced in comparison with that of the RVSS plan under the same condition.

Keywords Average total inspection · Acceptance quality loss limit inspection scheme · Patnaik's approximation · Repetitive group sampling · Taguchi's quality loss

1 Introduction

When inspection is for the purpose of acceptance or rejection of a item and/or a lot, based on adherence to a standard, the type of inspection procedure employed is called acceptance sampling. The acceptance sampling plan has an important role in the statistical quality control.

The investigations about acceptance sampling plans have been actively practiced until now. In the detail, for example, see a textbook [1]. Note that the acceptance sampling is not directly productive and profitable. Consequently, reducing the number of samples is requested under the condition

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that the specified requirements for quality are satisfied. For the purpose of reducing the number of samples in the acceptance sampling, a variety of sampling plans have been developed such as double, multiple, and sequential [2].

The repetitive group sampling plan [3] was devised as one of them. The repetitive group sampling plan has two criteria for judging the lot to be accepted, rejected, or pending. When pending, the inspection is repeated by taking a new sample, where each judgment is not influenced by the previous inspection records. The repetitive group sampling plan achieves small number of samples in comparison with the traditional single acceptance sampling plan.

On the other hand, when lots are rejected, acceptance sampling programs frequently require corrective action. As this corrective action, the form of screening through 100 % inspection for rejected lots is taken generally. Then, all discovered nonconforming items are either removed for subsequent rework or returned to the supplier. Such sampling programs are called rectifying inspection programs [4]. In the case of the rectifying inspection programs, it is requested to reduce the total inspection number including the samples of sampling inspection and 100 % inspection. Therefore, the rectifying inspection programs are formulated as the minimizing problems of average total inspection number under the specified requirements for quality.

The quality of lots has been traditionally evaluated based on the attribute property such as the proportion of nonconforming items in a lot. Then, a variety of acceptance samplings have been developed using the proportion of nonconforming items as the quality evaluation. However, the traditional quality evaluation as the proportion of nonconforming items has not distinguished among items that fall within the specification limits. Naturally, the quality between items that fall within the specification limits is not definitely identical. Accordingly, in order to achieve the strict quality assurance, more severe quality evaluations have been required newly. In such a case, it is effective to evaluate the quality of items by the variable property. And then, it is usually assumed that a quality characteristic in an item obeys a normal distribution. Under such a background, a new concept of the quality evaluation has been proposed by Taguchi [5, 6]. Taguchi has proposed the idea of interpreting the departure from the target value as the quality loss.

By introducing the concept of quality loss into the acceptance sampling, Arizono et al. [7] have developed the variable single sampling plan having desired operating characteristics indexed by quality loss. Subsequently, Yen and Chang [8] have considered similar variable acceptance sampling plan. The repetitive group sampling plan by variables has been studied actively, and many research results are reported such as Balamurali et al. [9, 10] and Jun et al. [11]. In particular, Aslam et al. [12] and Tomohiro et al. [13] have proposed the variables repetitive group sampling plan indexed by the quality loss. By introducing the repetitive group sampling plan, the reduction of sample number is realized compared to the variable single sampling plan indexed by quality loss by Arizono et al. [7].

On the other hand, the rectifying inspection programs by quality loss have been considered by Morita et al. [14] and Arizono et al. [15]. Note that their rectifying inspection programs are designed by the variable single sampling plan indexed by quality loss. For the purpose of designing the economic inspection program based on the reduction of the average total inspection (ATI), we consider rectifying variable repetitive group sampling (RVRGS) plan indexed by quality loss. In particular, in conformity with the concept of lot tolerance percent defective (LTPD) protection in the traditional rectifying inspection programs, we design the rectifying inspection programs based on the concept of acceptance quality loss limit (AQLL) protection (RVRGS plan for AQLL).

In this article, we suppose that the quality characteristic of items obeys a normal distribution. Under this condition, the RVRGS plan for AQLL indexed by quality loss for the purpose of designing the economic inspection program based on the reduction of ATI is considered. Under the consideration of the statistical property of the estimator of the quality loss, the design algorithm for the RVRGS plan for AQLL is investigated. Through some numerical simulation, the utility of the proposed RVRGS plan for AQLL is investigated.

The contents of this article are as follows. Section 2 explains the brief of quality loss. In this section, the distribution of the estimator of the quality loss is specified based on the approximation technique proposed by Patnaik [16]. Section 3 presents the RVRGS plan for AQLL. In this section, the design concept of the RVRGS plan for AQLL in consideration of ATI is described. Successively, we show the design procedure of the RVRGS plan for AQLL in Section 4. Then, the design conditions of the RVRGS plan for AQLL in consideration of ATI is considered based on the the statistical property of the estimator. Section 5 formulates the algorithm for deriving the optimal RVRGS plan for AQLL. Through some numerical examples, the reduction of ATI in the proposed plan has been verified in comparison with the RVSS plan indexed by quality loss in Section 6. Finally, we conclude this article in Section 7.

2 Brief of Taguchi's quality loss

In this section, the quality loss in the Taguchi method as the quality evaluation of items based on the variable property is briefed. When the mean and variance of the quality characteristics in individual items are given as μ and σ^2, the expected loss per item can be evaluated as

$$k\tau^2 = E[k(x-\mu_T)^2] = k\{(\mu-\mu_T)^2+\sigma^2\}, \tag{1}$$

where k denotes the proportional coefficient based on the functional limit of quality characteristic and gives the monetary loss brought by the item which cannot fulfill its fundamental function, and μ_T indicates the mean in the ideal quality characteristic distribution for items. Then, without loss of generality, k can be specified as 1, because k is a constant. Consequently, τ^2 can be redefined as the quality loss. In this article, we treat τ^2 as the new evaluation measure of quality instead of traditional attribute property such as the proportion of nonconforming items.

In the viewpoint of the quality loss, even if the proportion of nonconforming items was identical, the quality loss may not be identical. That is, from Eq. 1, it is found that there are innumerable combinations of (μ, σ^2) yielding same τ^2.

Suppose that the quality characteristic in an item obeys a normal distribution $N(\mu, \sigma^2)$ where μ and σ^2 are unknown parameters, respectively. Then, let $x_i,\ i = 1, 2, \cdots, n$ be observations obtained from random samples from the normal distribution $N(\mu, \sigma^2)$. In this situation, we have the estimator $\hat{\tau}^2$ of the quality loss τ^2 as follows:

$$\hat{\tau}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu_T)^2 = (\bar{x}-\mu_T)^2+s^2, \tag{2}$$

where $\bar{x}$ and s^2 denote the maximum likelihood estimators of μ and σ^2 calculated as follows:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n}x_i,\ \ s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2. \tag{3}$$

Further, the statistic $n\hat{\tau}^2/\sigma^2$ obeys the non-central chi-square distribution with n degrees of freedom and non-centrality parameter $n\xi$, where ξ is defined as

$$\xi = \frac{(\mu-\mu_T)^2}{\sigma^2}. \tag{4}$$

Note that the minimum variance is supposed to be given as feasible performance depending on manufacturing costs and environments. Then, the feasible minimum variance under the ideal manufacturing environment is presented as σ_T^2.

Since the non-central chi-square distribution is complex and difficult in the stochastic analysis, Arizono et al. [7, 14, 15] have employed an approximation for the distribution of $\hat{\tau}^2$ based on the approximation technique proposed by Patnaik [16]. Then, Arizono et al. have considered the following statistic ρ:

$$\rho = \frac{1+\xi}{1+2\xi}\frac{n\hat{\tau}^2}{\sigma^2}. \tag{5}$$

Based on the non-central chi-square distribution with n degrees of freedom and non-centrality parameter $n\xi$, the mean and variance of the statistic ρ are given by

$$E[\rho] = \frac{1+\xi}{1+2\xi}E\left[\frac{n\hat{\tau}^2}{\sigma^2}\right] = \frac{n(1+\xi)^2}{1+2\xi}, \tag{6}$$

$$V[\rho] = \left(\frac{1+\xi}{1+2\xi}\right)^2 V\left[\frac{n\hat{\tau}^2}{\sigma^2}\right] = \frac{2n(1+\xi)^2}{1+2\xi}. \tag{7}$$

It is found that the mean and variance of the statistic ρ coincide with those of the central chi-square distribution with ϕ degrees of freedom where

$$\phi = \frac{n(1+\xi)^2}{1+2\xi}. \tag{8}$$

Accordingly, the central chi-square distribution with ϕ degrees of freedom in Eq. 8 can be employed as the approximate distribution of ρ. Further, it is easy to derive that the function ϕ is the monotonous increasing function in ξ. Then, it can be presented that $\phi \geq n$. It can be easily known that the minimum value $\phi_{\min} = n$ is given by the condition of $\xi = 0$.

From Eqs. 2–8, the statistic ρ can be rewritten as $\rho = \phi\hat{\tau}^2/\tau^2$. Hereby, the distribution of the estimator $\hat{\tau}^2$ is specified approximately as follows:

$$\hat{\tau}^2 \sim \frac{\tau^2}{\phi}\chi_\phi^2, \tag{9}$$

where χ_ϕ^2 means the central chi-square distribution with ϕ degrees of freedom. Note that ϕ is a function consisting of μ and σ^2. Hence, the distribution of $\hat{\tau}^2$ is not unique even if the value of τ^2 is identical.

3 Proposal of RVRGS plan for AQLL in consideration of ATI

The feasible minimum variance under the ideal manufacturing environment can be supposed as σ_T^2. From this fact, the state described by the combination (μ_T, σ_T^2) can be defined as the ideal state and the quality loss yielded by this combination is expressed as $\tau_T^2 (= \sigma_T^2)$.

Arizono et al. [15] have presented the RVSS plan for the AQLL. In the RVSS plan for AQLL, the quality loss τ_1^2 which should be rejected and the probability of consumer's risk β are specified. And, it is necessary to determine the inspection plan satisfying the probability of consumer's risk β about the arbitrary combination (μ, σ^2) yielding τ_1^2. Then, Arizono et al. [15] have considered the RVSS plan for minimizing the average total inspection (ATI) in the ideal state (μ_T, σ_T^2). Remark that the incentive to improving the quality of items by optimising the RVSS plan for AQLL based on ATI in the ideal state (μ_T, σ_T^2) is formed.

In this section, we formulate the RVRGS plan for AQLL in order to reduce the cost of sampling inspection in comparison with the RVSS plan for AQLL. Let c_0 and c_1 be the acceptance and rejection criteria, respectively. And then, the acceptance rule in the RVRGS plan for AQLL is constructed as

$$\begin{cases} \text{if} \quad \hat{\tau}^2 \le c_0, \quad \text{then accept the lot,} \\ \text{if} \quad c_0 < \hat{\tau}^2 \le c_1, \quad \text{then continue the inspection,} \\ \text{otherwise, reject the lot.} \end{cases} \tag{10}$$

Then, the rejected lot is totally inspected. Note that, if the inspection is continued, the judgment of successive inspection stage is not affected by the judgment of previous inspection stage. This is a unique feature of the repetitive group sampling plan introduced by Sherman [3].

In the RVRGS plan for AQLL, the quality loss τ_1^2 which should be rejected and the probability of consumer's risk β are specified. And, it is necessary to determine the sampling plan (n, c_0, c_1) satisfying the probability of consumer's risk β about the arbitrary combination (μ, σ^2) yielding τ_1^2.

For developing the design procedure, we define the following probabilities $P_a(\tau^2)$ and $P_r(\tau^2)$:

$$P_a(\tau^2) = \Pr\{\hat{\tau}^2 \le c_0 \mid \tau^2\}, \tag{11}$$

$$P_r(\tau^2) = \Pr\{\hat{\tau}^2 > c_1 \mid \tau^2\}, \tag{12}$$

where $P_a(\tau^2)$ and $P_r(\tau^2)$ are defined as the acceptance probability and rejection probability of the lot with the quality loss τ^2 at each inspection stage with sampling plan (n, c_0, c_1), respectively. Based on these two probability functions $P_a(\tau^2)$ and $P_r(\tau^2)$, the probability $P_A(\tau^2)$ that the lot with the quality loss τ^2 is finally accepted is derived as

$$\begin{aligned} P_A(\tau^2) &= \sum_{k=1}^{\infty} P_a(\tau^2) \left\{1 - P_a(\tau^2) - P_r(\tau^2)\right\}^{k-1} \\ &= \frac{P_a(\tau^2)}{P_a(\tau^2) + P_r(\tau^2)}. \end{aligned} \tag{13}$$

In the same manner, we have the probability $P_R(\tau^2)$ that the lot with the quality loss τ^2 is finally rejected as

$$\begin{aligned} P_R(\tau^2) &= \sum_{k=1}^{\infty} P_r(\tau^2) \left\{1 - P_a(\tau^2) - P_r(\tau^2)\right\}^{k-1} \\ &= \frac{P_r(\tau^2)}{P_a(\tau^2) + P_r(\tau^2)}. \end{aligned} \tag{14}$$

Furthermore, when ATI with the quality loss τ^2 in the RVRGS plan is presented as $ATI(\tau^2)$, $ATI(\tau^2)$ is given as

$$\begin{aligned} ATI(\tau^2) &= n \sum_{k=1}^{\infty} k P_a(\tau^2) \left\{1 - P_a(\tau^2) - P_r(\tau^2)\right\}^{k-1} \\ &\quad + N \sum_{k=1}^{\infty} P_r(\tau^2) \left\{1 - P_a(\tau^2) - P_r(\tau^2)\right\}^{k-1} \\ &= P_A(\tau^2) ASN(\tau^2) + P_R(\tau^2) N, \end{aligned} \tag{15}$$

where N and $ASN(\tau^2)$ describe the lot size and average sample number in the repetitive group sampling.

Then, $ASN(\tau^2)$ is derived as

$$ASN(\tau^2) = \frac{n}{P_a(\tau^2) + P_r(\tau^2)}. \tag{16}$$

Notice that there are innumerable combinations of (μ, σ^2) yielding same τ^2 due to the relation of $\tau^2 = (\mu - \mu_T)^2 + \sigma^2$. Therefore, the required conditional expression for satisfying the requirement of AQLL inspection scheme is

$$\max_{(\mu, \sigma^2) \in \Omega(\tau_1^2)} P_A(\tau_1^2) \le \beta, \tag{17}$$

where $\Omega(\tau^2)$ represents the set consisting of the combination in (μ, σ^2) satisfying the quality loss τ^2. Consequently, the feasible sampling plan (n, c_0, c_1) should satisfy Eq. 17. Then, there are many sampling plans (n, c_0, c_1) satisfying the requirement of AQLL inspection scheme prescribed by Eq. 17. In this article, we define the RVRGS plan for AQLL in order to minimize ATI based on Eq. 15 in the case of the ideal state (μ_T, σ_T^2). That is, the design problem is to look for the sampling plan (n, c_0, c_1) which satisfies Eq. 17 and minimizes $ATI(\tau_T^2)$ based on Eq. 15 under the acceptance rule of Eq. 10.

Although we can obtain the RVRGS plan for AQLL optimised on ATI under any other assigned state (μ, σ^2) except the ideal state (μ_T, σ_T^2), we define the RVRGS plan for AQLL in order to minimize ATI under the ideal state (μ_T, σ_T^2). This is because the purpose of the quality inspection is to promote quality improvement in addition to guaranteeing quality. It is obvious that the RVRGS plan for AQLL optimised on $ATI(\tau_T^2)$ brings the incentive to improve the quality of items.

4 Design procedure for RVRGS plan for AQLL in consideration of ATI

In this section, we develop the design procedure for the RVRGS plan for AQLL defined in the previous section. Then, since $P_A(\tau_1^2)$ is a function composed of $P_a(\tau_1^2)$ and $P_r(\tau_1^2)$, it is difficult to show the condition satisfying Eq. 17 analytically under the arbitrary combination of

(μ, σ^2) yielding τ_1^2. So, about Eq. 17, the following relation is derived:

$$\begin{aligned}\max_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} P_A(\tau_1^2) &= \max_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} \frac{P_a(\tau_1^2)}{P_a(\tau_1^2)+P_r(\tau_1^2)} \\ &= \max_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} \frac{1}{\dfrac{P_r(\tau_1^2)}{P_a(\tau_1^2)}+1} \\ &\le \frac{1}{\dfrac{\min_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} P_r(\tau_1^2)}{\max_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} P_a(\tau_1^2)}+1}. \end{aligned} \tag{18}$$

From this relation, the required condition of Eq. 17 is always satisfied if the following inequality is established:

$$\frac{1}{\dfrac{\min_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} P_r(\tau_1^2)}{\max_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} P_a(\tau_1^2)}+1} \le \beta. \tag{19}$$

Therefore, we consider the combination (μ, σ^2) maximizing $P_a(\tau_1^2)$ and the combination (μ, σ^2) minimizing $P_r(\tau_1^2)$ individually. Note that it is not necessary that the combination (μ, σ^2) maximizing $P_a(\tau_1^2)$ and the combination (μ, σ^2) minimizing $P_r(\tau_1^2)$ are the same combination (μ, σ^2) in order to guarantee the establishment of Eq. 17.

At first, we consider the maximization of $P_a(\tau_1^2)$. We define

$$\max_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} P_a(\tau_1^2) \equiv \beta^\dagger \ (0 < \beta^\dagger \le \beta), \tag{20}$$

and represent (μ_1^*, σ_1^{2*}) as (μ, σ^2) yielding the situation of $\max_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} P_a(\tau_1^2)$. Note that the relation of $P_a(\tau_1^2) \le P_A(\tau_1^2)$ is satisfied from Eq. 13. Furthermore, the relation of $\max_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} P_a(\tau_1^2) = \beta^\dagger$ is changed to

$$\begin{aligned}\beta^\dagger &= \max_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} P_a(\tau_1^2) \\ &= \max_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} \Pr\{\hat{\tau}^2 \le c_0 \mid \tau_1^2\} \\ &= \Pr\left\{\hat{\tau}^2 \le \min_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} \frac{\chi^2_{\phi_1}(1-\beta^\dagger)}{\phi_1}\tau_1^2\right\} \\ &= \Pr\left\{\hat{\tau}^2 \le \frac{\chi^2_{\phi_1^*}(1-\beta^\dagger)}{\phi_1^*}\tau_1^2 \;\middle|\; (\mu_1^*, \sigma_1^{2*})\right\}, \end{aligned} \tag{21}$$

where

$$\phi_1 = \frac{n(1+\xi_1)^2}{1+2\xi_1}, \tag{22}$$

$$\xi_1 = \frac{(\mu_1-\mu_T)^2}{\sigma_1^2}, \tag{23}$$

and ϕ_1^* and ξ_1^* are in conformity to Eqs. 22 and 23. Hereby, based on Eq. 21, the acceptance criterion c_0 is derived as

$$c_0 \equiv \min_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} \frac{\chi^2_{\phi_1}(1-\beta^\dagger)}{\phi_1}\tau_1^2. \tag{24}$$

In this case, by applying the Wilson-Hilferty approximation [17], the behavior of $\chi^2_{\phi_1}(1-\beta^\dagger)/\phi_1$ in τ_1^2 against ϕ_1 can be specified approximately as follows:

$$\frac{\chi^2_{\phi_1}(1-\beta^\dagger)}{\phi_1} = \left(1 - \frac{2}{9\phi_1} + u_{1-\beta^\dagger}\sqrt{\frac{2}{9\phi_1}}\right)^3, \tag{25}$$

where $u_{1-\beta^\dagger}$ denotes the upper $100(1-\beta^\dagger)$ percentile of the standard normal distribution. Further, the differential coefficient (primary derivative) for ϕ_1 can be derived as

$$\begin{aligned}\frac{d}{d\phi_1}\left(\frac{\chi^2_{\phi_1}(1-\beta^\dagger)}{\phi_1}\right) &= \sqrt{\frac{1}{2\phi_1^3}} \\ &\times \left(1 - \frac{2}{9\phi_1} + u_{1-\beta^\dagger}\sqrt{\frac{2}{9\phi_1}}\right)^2 \\ &\times \left(\sqrt{\frac{8}{9\phi_1}} - u_{1-\beta^\dagger}\right). \end{aligned} \tag{26}$$

Because $\beta^\dagger$ should be less than β, and the small value such as 0.10 is assigned to β, $1-\beta^\dagger$ is close enough to 1. Therefore, $u_{1-\beta^\dagger} < 0$ and Eq. 26 are positive. Hereby, Eq. 25 is a monotonous increasing function in ϕ_1 and minimized when $\phi_1 = \phi_{1\min}$, where $\phi_{1\min}$ is the minimum of ϕ_1.

Furthermore, ϕ_1 is specified as a function related to ξ_1 and $\xi_1 \ge 0$. By differentiating ϕ_1 from ξ_1, the following relation is obtained:

$$\frac{d\phi_1}{d\xi_1} = \frac{2n\xi_1(1+\xi_1)}{(1+2\xi_1)^2} \ge 0. \tag{27}$$

By Eq. 27, it is obvious that ϕ_1 is a monotonous increasing function in ξ_1 and minimized when ξ_1 is minimized. So, ϕ_1 is minimized when $\xi_1 = 0$ because $\xi_1 \ge 0$. Based on this logic, it is seen that Eq. 25 is minimized and $P_a(\tau_1^2)$ is maximized under the condition of $(\mu_1^*, \sigma_1^{2*}) = (\mu_T, \tau_1^2)$ yielding $\xi_1 = 0$.

Next, we consider the minimization of $P_r(\tau_1^2)$ under the condition of $\max_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} P_a(\tau_1^2) = \beta^\dagger$. From Eq. 19, the following relation is established:

$$\begin{aligned}\frac{1}{\dfrac{\min_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} P_r(\tau_1^2)}{\max_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} P_a(\tau_1^2)}+1} &= \frac{1}{\dfrac{\min_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} P_r(\tau_1^2)}{\beta^\dagger}+1} \\ &\le \beta. \end{aligned} \tag{28}$$

By changing Eq. 28, we obtain the following relation:

$$\min_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} P_r(\tau_1^2) \geq \frac{1-\beta}{\beta}\beta^{\dagger}. \tag{29}$$

Consequently, we consider the following inequality:

$$\max_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} \{1-P_r(\tau_1^2)\} \leq 1-\frac{1-\beta}{\beta}\beta^{\dagger}, \tag{30}$$

where we define $1-(1-\beta)\beta^{\dagger}/\beta \equiv \beta^{\ddagger}\,(\beta \leq \beta^{\ddagger} < 1)$ and represent $(\mu_1^{**}, \sigma_1^{2**})$ as (μ, σ^2) yielding the situation of $\max_{(\mu,\sigma^2)\in\Omega(\tau_1^2)}\{1-P_r(\tau_1^2)\}$. Successively, the relation of $\max_{(\mu,\sigma^2)\in\Omega(\tau_1^2)}\{1-P_r(\tau_1^2)\} = \beta^{\ddagger}$ is presented as

$$\begin{aligned}\beta^{\ddagger} &= \max_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} \{1-P_r(\tau_1^2)\}\\ &= \max_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} \Pr\{\hat{\tau}^2 \leq c_1 \mid \tau_1^2\}\\ &= \Pr\left\{\hat{\tau}^2 \leq \min_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} \frac{\chi^2_{\phi_1}(1-\beta^{\ddagger})}{\phi_1}\tau_1^2\right\}\\ &= \Pr\left\{\hat{\tau}^2 \leq \frac{\chi^2_{\phi_1^{**}}(1-\beta^{\ddagger})}{\phi_1^{**}}\tau_1^2 \,\middle|\, (\mu_1^{**}, \sigma_1^{2**})\right\},\end{aligned} \tag{31}$$

where ϕ_1^{**} is in conformity to Eq. 22. Hereby, based on Eq. 31, the rejection criterion c_1 is derived as

$$c_1 = \min_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} \frac{\chi^2_{\phi_1}(1-\beta^{\ddagger})}{\phi_1}\tau_1^2. \tag{32}$$

By applying the Wilson-Hilferty approximation, the behavior of $\chi^2_{\phi_1}(1-\beta^{\ddagger})/\phi_1$ in τ_1^2 against ϕ_1 can be specified approximately as follows:

$$\frac{\chi^2_{\phi_1}(1-\beta^{\ddagger})}{\phi_1} = \left(1-\frac{2}{9\phi_1}+u_{1-\beta^{\ddagger}}\sqrt{\frac{2}{9\phi_1}}\right)^3. \tag{33}$$

Further, the differential coefficient (primary derivative) for ϕ_1 can be derived as

$$\begin{aligned}\frac{d}{d\phi_1}\left(\frac{\chi^2_{\phi_1}(1-\beta^{\ddagger})}{\phi_1}\right) &= \sqrt{\frac{1}{2\phi_1^3}}\\ &\quad\times\left(1-\frac{2}{9\phi_1}+u_{1-\beta^{\ddagger}}\sqrt{\frac{2}{9\phi_1}}\right)^2\\ &\quad\times\left(\sqrt{\frac{8}{9\phi_1}}-u_{1-\beta^{\ddagger}}\right).\end{aligned} \tag{34}$$

If $1-\beta^{\ddagger} > 0.5$, namely $\beta^{\ddagger} < 0.5$, Eq. 33 is a monotonous increasing function in ϕ_1 and minimized when ϕ_1 is minimized. So, $P_r(\tau_1^2)$ is minimized under the condition of $(\mu_1^{**}, \sigma_1^{2**}) = (\mu_T, \tau_1^2)$ yielding the maximization of $P_a(\tau_1^2)$.

On the other hand, when $1-\beta^{\ddagger} \leq 0.5$, we consider based on the comparison of $\beta^{\ddagger}$ and γ yielding $u_\gamma = \sqrt{8/9n}$, where $\sqrt{8/9n}$ is the maximum of $\sqrt{8/9\phi_1}$.

Then, if $0 \leq 1-\beta^{\ddagger} \leq \gamma$, namely $1-\gamma \leq \beta^{\ddagger} < 1.0$, $u_{1-\beta^{\ddagger}}$ is more than $\sqrt{8/9n}$ $\left(\geq \sqrt{8/9\phi_1}\right)$. In this case, Eq. 34 is negative regardless of the value of ϕ_1. Hereby, Eq. 33 is a monotonous decreasing function in ϕ_1 and minimized when $\phi_1 = \phi_{1\,\max}$, where $\phi_{1\,\max}$ is the maximum of ϕ_1. Because ϕ_1 is a monotonous increasing function in ξ_1, Eq. 33 is minimized when ξ_1 is maximized. Further, from Eq. 23, because ξ_1 is the monotonous decreasing function in σ_1^2, ξ_1 is maximized when $\sigma_1^2 = \sigma_T^2$, where σ_T^2 is the minimum of σ_1^2. As the result, Eq. 33 is minimized under the condition of $(\mu_1^{**}, \sigma_1^{2**}) = \left(\mu_T \pm \sqrt{\tau_1^2-\sigma_T^2}, \sigma_T^2\right)$.

Moreover, if $\gamma < 1-\beta^{\ddagger} \leq 0.5$, namely $0.5 \leq \beta^{\ddagger} < 1-\gamma$, Eq. 33 is minimized when ϕ_1 is maximized or minimized, because $\chi^2_{\phi_1}(1-\beta^{\ddagger})/\phi_1$ is concave in ϕ_1. In this case, if $\sqrt{8/9\phi_{1\,\max}} - u_{1-\beta^{\ddagger}} \geq 0$, Eq. 33 is minimized when ϕ_1 is $\phi_{1\,\min}$. On the other hand, if $\sqrt{8/9\phi_{1\,\min}} - u_{1-\beta^{\ddagger}} \leq 0$, Eq. 33 is minimized when ϕ_1 is $\phi_{1\,\max}$. Further, if both relations of $\sqrt{8/9\phi_{1\,\min}} - u_{1-\beta^{\ddagger}} > 0$ and $\sqrt{8/9\phi_{1\,\max}} - u_{1-\beta^{\ddagger}} < 0$ are satisfied simultaneously, Eq. 33 is minimized when ϕ_1 is $\phi_{1\,\max}$ or $\phi_{1\,\min}$. Hereby, we have the condition of $(\mu_1^{**}, \sigma_1^{2**}) = (\mu_T, \tau_1^2)$ when Eq. 33 is minimized at $\phi_{1\,\min}$. On the other hand, the condition of $(\mu_1^{**}, \sigma_1^{2**}) = \left(\mu_T \pm \sqrt{\tau_1^2-\sigma_T^2}, \sigma_T^2\right)$ is derived in the case that Eq. 33 is minimized at $\phi_{1\,\max}$.

Table 1 Sampling plans in AQLL inspection scheme under $\beta^{\dagger} = 0.001$ in the case of $\tau_1^2 = 2.25$, $\beta = 0.10$, $N = 500$, and $\tau_T^2 = 1.00$

n	c_0	c_1	$ATI(\tau_T^2)$
48	1.089	3.252	69.74
49	1.098	3.241	69.51
50	1.107	3.230	69.36
51	1.116	3.219	69.26
52	1.125	3.209	69.23
53	1.134	3.199	69.25
54	1.142	3.189	69.32
55	1.150	3.180	69.44
56	1.158	3.171	69.60
57	1.166	3.162	69.81

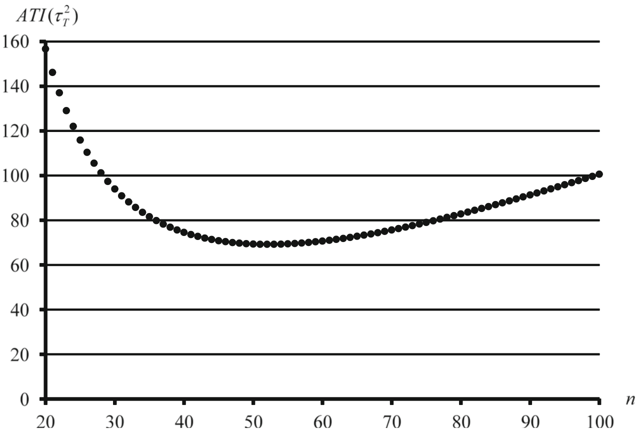

Fig. 1 Fluctuation of $ATI(\tau_T^2)$ for each of the sample size under $\beta^\dagger = 0.001$ in the case of $\tau_1^2 = 2.25$, $\beta = 0.10$, $N = 500$, and $\tau_T^2 = 1.00$ in AQLL inspection scheme

By the argument so far, the acceptance criterion under the prescribed sample size n can be defined as

$$\begin{aligned} c_0 &= \min_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} \frac{\chi^2_{\phi_1}(1-\beta^\dagger)}{\phi_1}\tau_1^2 \\ &= \frac{\chi^2_{\phi_{\min}}(1-\beta^\dagger)}{\phi_{\min}}\tau_1^2 \\ &= \frac{\chi^2_{n}(1-\beta^\dagger)}{n}\tau_1^2. \end{aligned} \tag{35}$$

And the rejection criterion under the prescribed sample size n can be defined as

$$\begin{aligned} c_1 &= \min_{(\mu,\sigma^2)\in\Omega(\tau_1^2)} \frac{\chi^2_{\phi_1}(1-\beta^\ddagger)}{\phi_1}\tau_1^2 \\ &\equiv \frac{\chi^2_{\phi_1^{**}}(1-\beta^\ddagger)}{\phi_1^{**}}\tau_1^2, \end{aligned} \tag{36}$$

Table 2 Sampling plans in AQLL inspection scheme for each $\beta^\dagger$ in the case of $\tau_1^2 = 2.25$, $\beta = 0.10$, $N = 500$, and $\tau_T^2 = 1.00$

$\beta^\dagger$	n	c_0	c_1	$ATI(\tau_T^2)$
0.048	17	1.137	2.294	25.82
0.049	17	1.142	2.276	25.76
0.050	17	1.147	2.259	25.73
0.051	17	1.152	2.241	25.71
0.052	17	1.157	2.224	25.70
0.053	17	1.161	2.207	25.71
0.054	17	1.166	2.189	25.74
0.055	18	1.196	2.177	25.76
0.056	18	1.200	2.160	25.79
0.057	18	1.205	2.144	25.84

where $\beta^\ddagger$ is defined as

$$\beta^\ddagger = 1 - \frac{1-\beta}{\beta}\beta^\dagger, \tag{37}$$

and ϕ_1^{**} is represented as follows:

$$\phi_1^{**} = \begin{cases} \phi_{1\,\min} = n, & (0 \le \beta^\ddagger < 0.5) \\ \phi_{1\,\max} = \dfrac{n\left(\dfrac{\tau_1^2}{\sigma_T^2}\right)^2}{2\dfrac{\tau_1^2}{\sigma_T^2} - 1}, & (1-\gamma \le \beta^\ddagger < 1.0) \\ \phi_{1\,\min} \text{ or } \phi_{1\,\max}, & (0.5 \le \beta^\ddagger < 1-\gamma) \end{cases} \tag{38}$$

where ϕ_1^{**} making Eq. 36 smaller is adopted in the case of $0.5 \le \beta^\ddagger < 1-\gamma$.

5 Algorithm for designing optimal RVRGS plan for AQLL

From the results mentioned above, the following algorithm for the purpose of specifying the RVRGS plan (n, c_0, c_1) in the AQLL inspection scheme is obtained:

(i) Set the initial value $\beta^\dagger = 0.001$.
(ii) Set the initial value $n = 2$.
(iii) Derive the acceptance criterion c_0 based on Eq. 35 using the value of $\beta^\dagger$, n, and given τ_1^2.
(iv) Derive $\beta^\ddagger$ based on Eq. 37 and calculate the rejection criterion c_1 based on Eq. 36 using the value of $\beta^\ddagger$, n, and given τ_1^2.
(v) Evaluate the value of $ATI(\tau_T^2)$ in Eq. 15 for the sampling plan (n, c_0, c_1).
(vi) Reset n to $n+1$. If $n < N$, then, go to (iii). Otherwise, go to (vii).

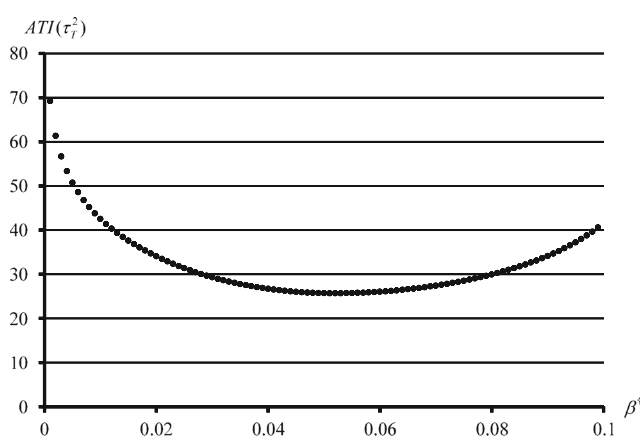

Fig. 2 The relation of $\beta^\dagger$ and $ATI(\tau_T^2)$ in the case of $\tau_1^2 = 2.25$, $\beta = 0.10$, $N = 500$, and $\tau_T^2 = 1.00$ in AQLL inspection scheme

Table 3 Sampling plans in AQLL inspection scheme for each τ_1^2 in the case of $\beta = 0.10$, $N = 500$ and $\tau_T^2 = 1.00$

τ_1^2	n	c_0	c_1	$ATI(\tau_T^2)$
1.25	97	0.994	1.191	216.68
1.50	50	1.054	1.484	87.25
1.75	31	1.098	1.738	49.85
2.00	22	1.129	1.988	34.06
2.25	17	1.157	2.224	25.70

Table 5 Sampling plans in AQLL inspection scheme for each τ_1^2 in the case of $\beta = 0.05$, $N = 500$, and $\tau_T^2 = 1.00$

τ_1^2	n	c_0	c_1	$ATI(\tau_T^2)$
1.25	126	0.988	1.136	270.43
1.50	66	1.054	1.415	109.46
1.75	40	1.091	1.654	62.01
2.00	28	1.121	1.861	42.24
2.25	22	1.156	2.067	31.85

(vii) Reset $\beta^\dagger$ to $\beta^\dagger + 0.001$, if $\beta^\dagger < \beta$. Then, go to (ii). Otherwise, go to (viii).

(viii) Specify (n, c_0, c_1) minimizing the value of $ATI(\tau_T^2)$ in the plans obtained by (i)-(vii) as the optimal RVRGS plan for AQLL.

6 Numerical examples

In this section, we design the RVRGS plan for AQLL specifically and verify the efficacy of the RVRGS plan for AQLL proposed in this article through some numerical examples. Let $\beta = 0.10$, $\tau_1^2 = 2.25$, $N = 500$ and $\tau_T^2 = 1.00$.

At first, set the initial value $n = 2$ and $\beta^\dagger = 0.001$ and calculate the value of c_0 and c_1. Table 1 shows a part of calculated results of (n, c_0, c_1) and $ATI(\tau_T^2)$ under $\beta^\dagger = 0.001$. Then, Fig. 1 illustrates the relation of sample size n and $ATI(\tau_T^2)$ under $\beta^\dagger = 0.001$. In this case, the sampling plan $(n, c_0, c_1) = (52, 1.125, 3.209)$ is obtained.

Then, Table 2 shows a part of calculated results of (n, c_0, c_1) and minimum value of $ATI(\tau_T^2)$ for each $\beta^\dagger (\leq \beta)$, and Fig. 2 illustrates a part of calculated results of (n, c_0, c_1) and value of $ATI(\tau_T^2)$. Consequently, $(n, c_0, c_1) = (17, 1.157, 2.224)$ and $ATI(\tau_T^2) = 25.70$ under $\beta^\dagger = 0.052$ is obtained as the optimal sampling plan and minimum value of $ATI(\tau_T^2)$ through the above numerical results.

Next, we consider the proportion of the reduction of ATI in the RVRGS plan for AQLL from $\mathrm{ATI_S}$ in the RVSS plan proposed by Arizono et al. [15] under the same condition. Table 3 shows some calculated results of (n, c_0, c_1) and $ATI(\tau_T^2)$ for each τ_1^2 against the variation of quality loss τ_1^2 under $\beta = 0.10$, $\tau_T^2 = 1.00$, and $N = 500$. Further, Table 4 shows the relative reduction rates in ATI under the proposed inspection in Table 3 and RVSS plan under the same conditions. Then, n_S and $ATI_S(\tau_T^2)$ indicate the sample size and ATI in the RVSS plan under the same conditions. In addition, the values in reduction rate are given as $\frac{ATI_S(\tau_T^2)-ATI(\tau_T^2)}{ATI_S(\tau_T^2)}$. From the results in Table 4, the efficacy of the RVRGS plan to the reduction of ATI is confirmed.

In the same manner, Table 5 shows some calculated results of (n, c_0, c_1) and $ATI(\tau_T^2)$ for each τ_1^2 against the variation of quality loss τ_1^2 under $\beta = 0.05$, $\tau_T^2 = 1.00$, and $N = 500$. Then, Table 6 shows the reduction rates in ATI against the variation of quality loss τ_1^2. From comparison of Tables 4 and 6, we find that the smaller the probability of consumer's risk β is, the larger the sample size and ATI are. Then, the rates of reduction in ATI are decreased in the respective τ_1^2.

Additionally, for the purpose of verifying the efficiency of the proposed RVRGS plan, the reduction rates of the designed RVRGS plans have been investigated in the cases of various combinations of (μ, σ^2). For reference, Tables 7, 8, 9, and 10 are shown, where $P_{RS}(\tau^2)$ describes the

Table 4 The reduction rate in ATI for each τ_1^2 in the case of $\beta = 0.10$, $N = 500$, and $\tau_T^2 = 1.00$

τ_1^2	n_S	$ATI_S(\tau_T^2)$	Reduction rate (%)
1.25	154	243.29	10.94
1.50	91	120.83	27.79
1.75	60	75.37	33.86
2.00	44	53.79	36.68
2.25	35	41.61	38.24

Table 6 The reduction rate in ATI for each τ_1^2 in the case of $\beta = 0.05$, $N = 500$, and $\tau_T^2 = 1.00$

τ_1^2	n_S	$ATI_S(\tau_T^2)$	Reduction rate (%)
1.25	186	287.41	5.91
1.50	111	145.75	24.90
1.75	74	91.11	31.94
2.00	54	65.08	35.09
2.25	43	50.40	36.81

Table 7 The reduction rate in ATI for each (μ, σ^2) in the case of $\beta = 0.10$, $N = 500$, $(\mu_T, \sigma_T^2) = (0.0, 1.0)$, and $\tau_1^2 = 1.50$

τ^2	(μ, σ^2)	$P_R(\tau^2)$	$ATI(\mu, \sigma^2)$	$P_{RS}(\tau^2)$	$ATI_S(\mu, \sigma^2)$	Reduction rate (%)
1.10	(0.00, 1.10)	0.103	142.34	0.218	180.33	21.06
1.10	(0.10, 1.09)	0.102	142.34	0.218	180.33	21.06
1.10	(0.32, 1.00)	0.101	142.12	0.218	179.99	21.04
1.30	(0.00, 1.30)	0.571	339.04	0.640	352.68	3.87
1.30	(0.30, 1.21)	0.572	339.36	0.640	352.85	3.82
1.30	(0.55, 1.00)	0.576	342.79	0.645	354.61	3.33
1.50	(0.00, 1.50)	0.900	459.09	0.900	459.10	0.00
1.50	(0.30, 1.41)	0.902	459.33	0.901	459.28	−0.01
1.50	(0.71, 1.00)	0.918	466.69	0.913	464.78	−0.41
1.70	(0.00, 1.70)	0.978	490.63	0.980	491.83	0.24
1.70	(0.40, 1.54)	0.979	490.87	0.981	492.01	0.23
1.70	(0.84, 1.00)	0.988	494.82	0.988	495.17	0.07
1.90	(0.00, 1.90)	0.995	497.70	0.997	498.64	0.19
1.90	(0.50, 1.65)	0.995	497.86	0.997	498.74	0.18
1.90	(0.95, 1.00)	0.998	499.29	0.999	499.60	0.06

rejected probability by the RVSS plan. Then, we know that under the situation of the poor quality such as $\tau^2 \geq \tau_1^2$ there is not a difference between the power of the RVRGS plan and the RVSS plan. Further, although some cases that the value of $ATI_S(\mu, \sigma^2)$ is smaller than the value of $ATI(\mu, \sigma^2)$ are founded in Tables 7–10 under the situation that the value of τ^2 is relatively big, these differences are not so big. On the other hand, it is seen that the reduction rate in ATI grows big as τ^2 becomes small and approaches τ_T^2. This fact means that the sampling cost for guaranteeing the quality of items decreases by realizing good quality. By this feature, it is confirmed that the RVSS plan for AQLL optimised on ATI in the ideal state (μ_T, σ_T^2) brings the incentive to improve the quality of items.

Table 8 The reduction rate in ATI for each (μ, σ^2) in the case of $\beta = 0.10$, $N = 500$, $(\mu_T, \sigma_T^2) = (0.0, 1.0)$, and $\tau_1^2 = 2.00$

τ^2	(μ, σ^2)	$P_R(\tau^2)$	$ATI(\mu, \sigma^2)$	$P_{RS}(\tau^2)$	$ATI_S(\mu, \sigma^2)$	Reduction rate (%)
1.10	(0.00, 1.10)	0.020	46.70	0.064	73.12	36.14
1.10	(0.10, 1.09)	0.020	46.69	0.064	73.11	36.14
1.10	(0.32, 1.00)	0.019	46.51	0.063	72.79	36.10
1.30	(0.00, 1.30)	0.129	110.61	0.248	156.86	29.48
1.30	(0.30, 1.21)	0.128	110.43	0.247	156.67	29.51
1.30	(0.55, 1.00)	0.122	108.59	0.243	154.70	29.80
1.50	(0.00, 1.50)	0.398	236.36	0.501	272.28	13.19
1.50	(0.30, 1.41)	0.398	236.45	0.501	272.32	13.17
1.50	(0.71, 1.00)	0.397	239.41	0.504	273.82	12.57
1.70	(0.00, 1.70)	0.685	360.27	0.717	370.89	2.86
1.70	(0.40, 1.54)	0.686	361.10	0.718	371.38	2.77
1.70	(0.84, 1.00)	0.721	378.06	0.740	381.33	0.86
1.90	(0.00, 1.90)	0.853	433.50	0.856	434.31	0.18
1.90	(0.50, 1.65)	0.857	435.11	0.857	435.39	0.06
1.90	(0.95, 1.00)	0.901	455.52	0.890	449.64	−1.31
2.10	(0.00, 2.10)	0.931	468.22	0.932	468.74	0.11
2.10	(0.50, 1.85)	0.933	469.15	0.933	469.44	0.62
2.10	(1.05, 1.00)	0.968	485.38	0.962	482.44	−0.61
2.30	(0.00, 2.30)	0.966	484.09	0.969	485.68	0.33
2.30	(0.50, 2.05)	0.967	484.59	0.969	486.06	0.30
2.30	(1.14, 1.00)	0.990	495.36	0.989	494.88	−0.10

Table 9 The reduction rate in ATI for each (μ, σ^2) in the case of $\beta = 0.05$, $N = 500$, $(\mu_T, \sigma_T^2) = (0.0, 1.0)$, and $\tau_1^2 = 1.50$

τ^2	(μ, σ^2)	$P_R(\tau^2)$	$ATI(\mu, \sigma^2)$	$P_{RS}(\tau^2)$	$ATI_S(\mu, \sigma^2)$	Reduction rate (%)
1.10	(0.00, 1.10)	0.121	179.73	0.273	216.99	17.17
1.10	(0.10, 1.09)	0.121	179.73	0.273	216.99	17.17
1.10	(0.32, 1.00)	0.119	179.61	0.272	216.71	17.12
1.30	(0.00, 1.30)	0.683	390.66	0.737	397.87	1.81
1.30	(0.30, 1.21)	0.684	391.03	0.738	398.09	1.77
1.30	(0.55, 1.00)	0.692	394.97	0.744	400.40	1.36
1.50	(0.00, 1.50)	0.950	480.16	0.950	480.55	0.08
1.50	(0.30, 1.41)	0.950	480.31	0.950	480.68	0.08
1.50	(0.71, 1.00)	0.962	484.93	0.960	484.41	−0.11
1.70	(0.00, 1.70)	0.992	496.69	0.994	497.51	0.17
1.70	(0.40, 1.54)	0.992	496.80	0.994	497.59	0.16
1.70	(0.84, 1.00)	0.996	498.44	0.997	498.80	0.07
1.90	(0.00, 1.90)	0.999	499.41	0.999	499.74	0.07
1.90	(0.50, 1.65)	0.999	499.46	0.999	499.77	0.06
1.90	(0.95, 1.00)	1.000	499.87	1.000	499.95	0.02

Table 10 The reduction rate in ATI for each (μ, σ^2) in the case of $\beta = 0.05$, $N = 500$, $(\mu_T, \sigma_T^2) = (0.0, 1.0)$, and $\tau_1^2 = 2.00$

τ^2	(μ, σ^2)	$P_R(\tau^2)$	$ATI(\mu, \sigma^2)$	$P_{RS}(\tau^2)$	$ATI_S(\mu, \sigma^2)$	Reduction rate (%)
1.10	(0.00, 1.10)	0.022	58.42	0.078	88.97	34.34
1.10	(0.10, 1.09)	0.022	58.41	0.078	88.97	34.34
1.10	(0.32, 1.00)	0.021	58.23	0.078	88.62	34.29
1.30	(0.00, 1.30)	0.166	142.73	0.309	191.87	25.61
1.30	(0.30, 1.21)	0.165	142.60	0.309	191.73	25.62
1.30	(0.55, 1.00)	0.159	141.32	0.306	190.25	25.72
1.50	(0.00, 1.50)	0.508	293.71	0.598	320.68	8.41
1.50	(0.30, 1.41)	0.509	297.53	0.598	322.79	7.83
1.50	(0.71, 1.00)	0.517	300.83	0.607	324.57	7.31
1.70	(0.00, 1.70)	0.794	410.93	0.809	414.57	0.88
1.70	(0.40, 1.54)	0.796	411.77	0.810	415.11	0.80
1.70	(0.84, 1.00)	0.833	428.44	0.834	425.85	−0.61
1.90	(0.00, 1.90)	0.921	464.50	0.920	464.35	−0.03
1.90	(0.50, 1.65)	0.923	465.69	0.922	465.23	−0.10
1.90	(0.95, 1.00)	0.955	479.65	0.947	476.16	−0.73
2.10	(0.00, 2.10)	0.968	485.30	0.969	486.25	0.19
2.10	(0.50, 1.85)	0.969	485.87	0.970	486.69	0.17
2.10	(1.05, 1.00)	0.988	494.64	0.987	494.02	−0.12
2.30	(0.00, 2.30)	0.986	493.51	0.989	494.91	0.28
2.30	(0.50, 2.05)	0.987	493.77	0.989	495.10	0.27
2.30	(1.14, 1.00)	0.997	498.64	0.997	498.80	0.03

7 Concluding remarks

In this article, we have considered the RVRGS plan indexed by quality loss for the purpose of designing the economic inspection program based on the reduction of ATI. At first, the design concept of the RVRGS plan for AQLL in consideration of ATI has been described. Successively, under the consideration of the statistical property of the estimator $\hat{\tau}^2$ of the quality loss τ^2, we have developed the design algorithm for the RVRGS plan for AQLL. Through some numerical evaluation, the reduction of ATI in the proposed plan has been verified in comparison with the RVSS plan indexed by quality loss. As the result, the economic program of the rectifying inspection for the repetitive group sampling plan indexed by quality loss has been established.

Michlin and Pistiner [18] have investigated the volumetric tightness testing of underground storage tanks. In this article, the probability of a leak has been evaluated under the standard normal distribution. Then, it seems that the RVRGS plan for AQLL can contribute the volumetric tightness testing of underground storage tanks. Separately from this, the variable sampling plan based on the process loss index L_e has been considered by Yen and Chang [8] and Aslam et al. [12]. In addition, Pearn and Wu [19] consider the variable sampling plan indexed by the process capability index C_{pm}. Then, the process loss index is described as

$$L_e = \frac{(\mu - \mu_T)^2 + \sigma^2}{d^2},$$

where $d = (USL - LSL)/2$ is the half specification width, USL and LSL are the upper and lower specification limits. Then, d is a fixed value. Further, the process capability index C_{pm} is defined as

$$C_{pm} = \frac{d}{3\sqrt{(\mu - \mu_T)^2 + \sigma^2}}.$$

Then, we can know easily that the process loss index L_e and the process capability index C_{pm} are reduced to the quality loss τ^2. Consequently, in the situations that the variable sampling plan based on the process loss index L_e and the variable sampling plan indexed by the process capability index C_{pm} are adopted, the RVRGS plan for AQLL considered in this article can be applied. Then, the adoption of the RVRGS plan for AQLL in the real case would like to be a future problem.

Furthermore, as mentioned previously, Arizono et al. [15] have considered the rectifying inspection programs by quality loss for the AQLL inspection scheme and PAOSQLL inspection scheme. In this article, the rectifying inspection program for AQLL based on the repetitive group sampling plan is investigated. We would like to consider the rectifying inspection program for PAOSQLL based on the repetitive group sampling plan as a near future subject.

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