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ORIGINAL ARTICLE

# A novel tolerance analysis for mechanical assemblies based on Convex Method and non-probabilistic set theory

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**Abstract** Tolerance directly affects the performance and cost of the mechanical product. Tolerance analysis is a very useful approach for evaluating the accumulation of uncertainties caused by individual part tolerances. Worst Case (W-C) method and statistical methods are commonly used tolerance analysis methods. However, the result of W-C method is overly pessimistic, and the statistical methods adopt idealized distribution assumptions. In this paper, a novel C-NPS method combing Convex Method and non-probabilistic set theory (NPS) is put forward to address the above tolerance analysis problem. In this method, uncertainties of both part tolerances and assembly variations are modeled using NPS, then these part uncertainties are accumulated together to calculate the assembly function using Convex Method. Thus, the variation caused by each feature in the mechanical assembly can be estimated. C-NPS method is more suitable for tolerance analysis of different tolerances when the tolerance probability distributions are unavailable. The application of the method is illustrated through a one-way clutch mechanism assembly problem, and the advantages of this method are presented. The proposed method can be regarded as an attractive supplement to the tolerance analysis field.

**Keywords** Tolerance analysis · Convex method · Non-probabilistic set · Mechanical assembly

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## 1 Introduction

When engineering parts are manufactured, the design parameters may deviate from their intended design values. These deviations are usually called the manufacturing tolerances. During the assembly process, tolerances of individual part may ultimately affect the performance of final product. Dimensions with tolerances inherently generate an uncertain circumstance in a mechanical assembly. Though the deviations can be relatively diminutive, their impacts can be significant. Tolerance analysis for stacked parts of an assembly is vital to ensure the designed quality requirements. Two commonly used methods of tolerance analysis are worst case (W-C) method and the statistical tolerance method. The W-C method [1–4] is established based on the complete interchangeability of the parts. In W-C analysis, the assembly deviation is determined by summing the part tolerances linearly. Each part dimension is assumed to be at its maximum or minimum value, resulting in the worst possible assembly circumstance. Statistical tolerance methods are mainly referred as the root sum squares (RSS) method and Monte Carlo method, which are based on the theory of probability and statistics for tolerance analysis. In these methods, each part tolerance is regarded as a random variable. Khodaygan et al. [5] proposed a new method based on RSS to estimate the accumulative tolerances in a mechanical assembly. Gavankar et al. [6] used RSS method to allocate the appropriate tolerances to ensure that components fit right and function satisfactorily. Etienne et al. [7] conducted genetic algorithm to quantify functional tolerances. To evaluate the whole assembly quality, Li et al. [8] constructed the quality-tolerance assembly functions for accumulation and propagation. Zhang et al. [9] presented an analytical methodology to control the total tolerance accumulation in a disk cam-translating follower system. Franciosa et al. [10] proposed a novel general approach to automatically calculate the variational parameters. Whitney et al. [11] proposed a closed-form algorithm for modeling variations caused by geometric deviations. Zuo et al. [12] and Shen et al. [13] used Jacobian-Torsor model to provide mathematical models of tolerance analysis. Armillotta [14] puts forward that assembly requirements which have been used in a rule-based geometric reasoning procedure select datum reference frames for each part and assign tolerance types to part features. Monte Carlo method is also widely used in tolerance analysis. Beaucaire et al. [15] adopted Monte Carlo method to evaluate a predicted quality level at the design stage, while Qureshi et al. [16] used the same method for over-constraint mechanisms. Dantan et al. [17] simulated the influences of geometrical deviations on the geometrical behavior of the mechanism. Governi et al. [18] conducted a Monte Carol method for automatic tolerance allocation. Yang et al. [19] proposed a novel variation propagation control method, and comparisons were made against Monte Carlo simulations for the purposes of validation. Huang et al. [20] presented a process plan to predict machining tolerances via Monte Carlo simulation.

The result of the W-C method is too strict. Therefore, the machining precision of parts should be improved, and the manufacturing cost increases correspondingly. Statistical tolerance methods regard part variations of the assembly as randomness. However, the probability distribution is assumed, which may result in irrationality in many practical engineering. Uncertain parameter may not be a random variable, but a fuzzy or unknown-but-bounded variable [21]. Generally, it is difficult to verify whether the actual tolerances of assembly satisfy a particular probability distribution or not. And, the Monte Carlo method requires repeating experiments and large data to define the design parameters. It will be time-consuming when the data are extremely large.

In this paper, we proposed a novel tolerance analysis method, Convex Method and non-probabilistic set theory (C-NPS), to address the above problems. In this method, uncertainties of both part tolerances and assembly variations are modeled using NPS, and then these uncertainties are accumulated together to calculate the assembly variations using Convex Method. In NPS theory, the uncertainties are described as the bounds of the parameters, without knowing the probabilistic distributions. Convex Method [22–26] provides the ability to explicitly express the uncertain variables. C-NPS is an optional tolerance analysis method when scarce data are available. The aim of our work is to acquire accumulated variations in mechanical assemblies. To the best of our knowledge, non-probabilistic for tolerance analysis is a relatively rare research topic. Our work makes a supplement for tolerance analysis field.

The rest of this paper is organized as follows. In Sect. 2, C-NPS method is presented to estimate the accumulation of assembly, and the basic concepts of the proposed method are briefly introduced. A case study is presented in Sect. 3. Finally, the conclusions of the method are presented in Sect. 4.

## 2 C-NPS method

In this section, we propose the C-NPS method for tolerance analysis. Firstly, the Convex Method and NPS theory are briefly introduced in Sect. 2.1 and 2.2. Then, the C-NPS method is constructed in Sect. 2.3. Finally, the tolerance analysis procedures of C-NPS method are illustrated in Sect. 2.4.

### 2.1 Convex Method

Convex Method is a new kind of uncertainty analysis methodology. There exist two main models, the interval one and the ellipsoid one. In the former model, the uncertainty domain is simply a multidimensional box, so theoretically, it can only deal with the problems involving independent variables. By contrast, in the latter model, the parametric uncertainty is assumed to lie within a multidimensional ellipsoid, which can be easily obtained based on a small number of samples or just the engineering experience [27]. It has been proved that the ellipsoid model can deal with not only the independent but also the correlated problems. The degree of uncertainty and correlation of the variables are described by the size and shape of the ellipsoid.

Tolerance is the allowable variation of design parameter. During the manufacturing process, the actual dimensions of design parameter will fluctuate within the tolerance region if the process capability is satisfied. It is much more easily to identify the upper and lower limit of the fluctuation than to precisely estimate the probability distribution. According to Convex Method, we assume that all these $m$ uncertain-but-bounded parameters are correlated and fluctuate within a $m$-dimensional ellipsoid region $E(\delta,\theta)$, as shown in Eq. 1.

$$E(\delta, \theta) = \left\{\delta :\ \delta^T \omega \delta \leq \theta^2\right\} \tag{1}$$

In Eq. 1, $\delta=(\delta_i)_m=(\delta_1,\delta_2,\ldots,\delta_m)^T$ is the $m$-dimensional variation of nominal value which varies inside this convex region; $\omega$ is an $m\times$m symmetric positive definite matrix (SPDM), which determines the orientation of the ellipsoid; $\theta$ ($0\leq\theta\leq1$) denotes the radius of ellipsoid [28] and is used to define the size of the ellipsoid. Function $E(\delta,\theta)$ is an ellipsoid of $m$-dimensional space, containing all the uncertain parameters. The shape and size of the ellipsoid are determined by $\omega$ and $\theta$, which are chosen to represent available information concerning the variability of the uncertain parameters.

Convex Method has many advantages over traditional tolerance analysis methods: (1) Convex Method does not need the distributions of the parameters and greatly reduces the demand of original data, (2) Convex Method can get a relatively reliable variation interval of the results depend on a small amount of data, and (3) the result of W-C method is overly pessimistic, the distribution assumptions of statistical methods are excessively ideal, while the result of Convex Method is closer to the engineering practice [29]. In this method, it is assumed that uncertainty of the parameters belongs to a convex region; thus, the uncertainty boundary can be obtained based on a small number of samples instead of an exact probability distribution. The convex method is highly suitable for uncertainty analysis of many complex tolerance analysis problems.

### 2.2 Non-probabilistic set theory

Probabilistic reliability can be very sensitive to small inaccuracy of the disturbance. Once those probability distribution assumptions of tolerances do not comply with the real distribution, then the rationality of the probability statistical tolerance analysis loses significance. This can result in an unrealistic state in many practical engineering. The non-probabilistic concept is useful when sufficient information is unavailable for substituting a probabilistic model.

For a parameter $a$ with known limits, it can be described as an NPS model as follows:

$$a = \left[\underline{a}, \overline{a}\right] = \left\{v : \underline{a} \le v \le \overline{a}\right\} \tag{2}$$

From the design perspective, the product deviation originating from the dimension tolerances will finally affect the product quality. Generally, the relationship between the input parameters and the accumulated variation $\varphi$ in a mechanical assembly can be expressed as

$$\varphi = \varnothing(x) = \varnothing(x_1, x_2, \cdots, x_m) \tag{3}$$

$\varphi = \varnothing(x)$ is the accumulated variation, which is usually called the assembly function. It can be presented by the NPS model:

$$\varphi = \left[\underline{\varphi}, \overline{\varphi}\right] \tag{4}$$

$x_1, x_2, \ldots, x_m$ are the input parameters in the form of NPS model. $x_i$ is the dimension with tolerance can be expressed as

$$x_i = \left[\underline{x_i}, \overline{x}_i\right] = \left\{v : \underline{x_i} \le v \le \overline{x_i}\right\},\ i = 1, 2 \ldots, m \tag{5}$$

where $\overline{x_i}$ and $\underline{x_i}$ are the upper limit and lower limit value of the $i$th dimension respectively.

According to the GD&T standards (ASME Y14.5M-1994), the dimensional and geometrical tolerance zones can be divided into the following six kinds. Table 1 illustrates the tolerance zones described by NPS model. 2D and 3D tolerance zones can be decomposed to the deviations in X-axis, Y-axis, and Z-axis according to Table 1. $\underline{x}$, $\underline{y}$, and $\underline{z}$ are the lower limit variations of each axis, $\overline{x}$, $\overline{y}$, and $\overline{z}$ are the upper limit variations of each axis.

All those tolerances described by NPS model can be solved using Convex Method. Therefore, we come up with a novel method named C-NPS by combining the NPS theory and Convex Method together. The details of C-NPS method will be discussed extensively in the next section.

### 2.3 Assembly function based on C-NPS model

The assembly tolerances are the allowances of initial design requirements. The assembly function is the most essential equation for tolerance analysis and allocation that describes relations between the assembly and manufacturing tolerances. And, the result of assembly function is the most significant factor that could estimate the performance.

Assuming that $\boldsymbol{x}^0 = (x_i^0)_m = (x_1^0, x_2^0, \cdots, x_m^0)^T$ is the median value of the dimension vector $\boldsymbol{x} = (x_i)_m = (x_1, x_2, \cdots, x_m)^T$ with tolerances, $x$ can be represented as

$$x = x^0 + \delta \tag{6}$$

or

$$x_i = x_i^0 + \delta_i,\ i = 1, 2, \ldots m \tag{7}$$

where

$$\delta_i \le \left(\overline{x_i} - \underline{x_i}\right)/2,\ i = 1, 2, \ldots m \tag{8}$$

Equation 8 expresses the level of the input dimensional parameters uncertainty. Thus, the input uncertain parameters $x$ will be converted to uncertainties of parameter $\boldsymbol{\delta} = (\delta_i)_m = (\delta_1, \delta_2, \ldots, \delta_m)^T$. Equation 3 is expressed by taylor expansion and preserved the first order items:

$$\begin{aligned} \varnothing x &= \varnothing\left(x^0 + \delta\right) = \varnothing\left(x^0\right) + \sum_{i=1}^{m} \frac{\partial \varnothing(x^0)}{\partial x_i} \delta_i \\ &= \varphi_{0+} g^T \delta \end{aligned} \tag{9}$$

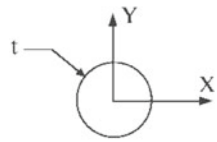
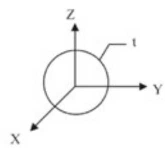
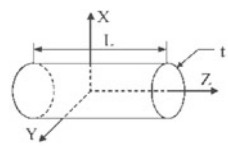
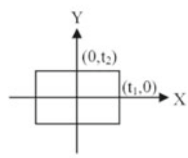
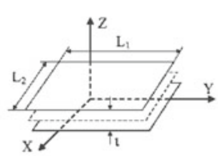
where

$$\varphi_0 = \varnothing\left(x^0\right) \tag{10}$$

$$g = \left(\frac{\partial \varnothing x^0}{\partial x_1}, \frac{\partial \varnothing(x^0)}{\partial x_2}, \cdots, \frac{\partial \varnothing(x^0)}{\partial x_m}\right)^T \tag{11}$$

According to Convex Method, the uncertain parameter $\boldsymbol{\delta}$ varies inside boundary of Eq. 1 in Sect. 2.1. We can determine the limits of the assembly function $\varnothing x$ through the optimization method so that the objective function is Eq. 9 and the constraint condition is Eq. 1.

**Table 1** Tolerance zones and their descriptions using NPS

| Tolerance zones | | Description |
|---|---|---|
| Linear |  | $x = [t_1, t_2]$ |
| Circular |  | $x = [\underline{x}, \overline{x}]$ $y = [\underline{y}, \overline{y}]$ $x^2 + y^2 = t^2$ |
| Spherical |  | $x = [\underline{x}, \overline{x}]$ $y = [\underline{y}, \overline{y}]$ $z = [\underline{z}, \overline{z}]$ $x^2 + y^2 + z^2 = t^2$ |
| Cylindrical |  | $x = [\underline{x}, \overline{x}],\ y = [\underline{y}, \overline{y}]$ $z = [-\frac{L}{2}, \frac{L}{2}]$ $x^2 + y^2 = t^2$ |
| Rectangle |  | $x = [-t_1, t_1]$ $y = [-t_2, t_2]$ |
| Two parallel plans |  | $x = [-\frac{L_2}{2}, \frac{L_2}{2}]$ $y = [-\frac{L_1}{2}, \frac{L_1}{2}]$ $z = [-\frac{t}{2}, \frac{t}{2}]$ |

Equation 4 can be rewritten as

$$\overline{\varphi} = \varphi_{max} = \max_{\delta \epsilon E(\delta,\theta)} \left\{ \varphi_{0+} g^T \delta \right\} \tag{12}$$

$$\underline{\varphi} = \varphi_{min} = \min_{\delta \epsilon E(\delta,\theta)} \left\{ \varphi_{0+} g^T \delta \right\} \tag{13}$$

As Eq. 1 is a convex domain, the extreme value of $\varphi$ will arrive at the ellipsoid boundary. Therefore, Eqs. 12 and 13 can be transformed into an extreme value problem under the constraint of Eq. 1. Using the C-NPS method, the extreme value problem can be rewritten as

maximize

$$\varphi = \varphi_{0+} g^T \delta c \tag{14}$$

$$\text{subject to} \begin{cases} \delta^T \omega \delta = \theta^2 \\ 0 \leq \theta \leq 1 \end{cases}$$

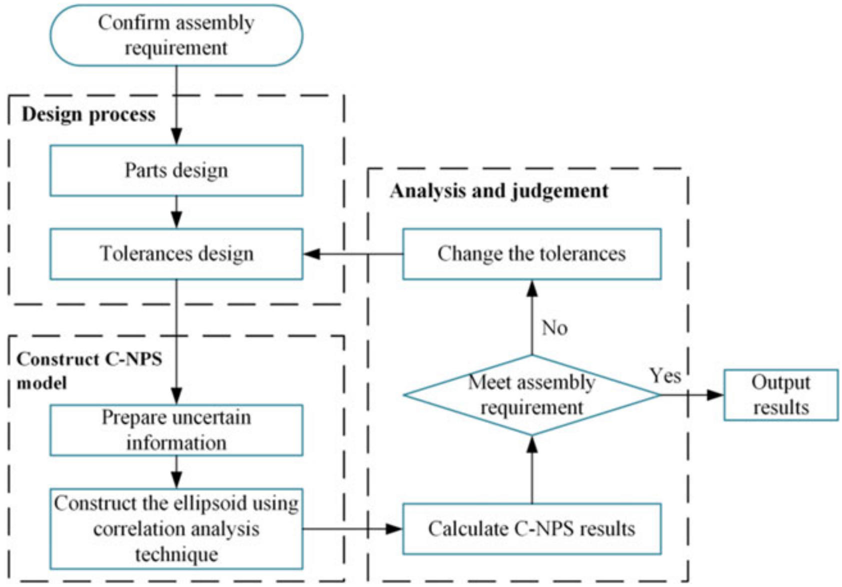



**Fig. 1** Analysis procedures of C-NPS method

and

$$\begin{array}{ll} \text{Manimize} & \varphi = \varphi_{0+} g^T \delta \\ \text{Subject to} & \begin{cases} \delta^T \omega \delta = \theta^2 \\ 0 \le \theta \le 1 \end{cases} \end{array} \tag{15}$$

Lagrangian equation is established based on Eqs. 14 and 15 as follows:

$$L = \varphi_{0+} g^T \delta + \mu \left( \delta^T \omega \delta - \theta^2 \right) \tag{16}$$

where $\mu$ is a Lagrangian multiplier. The necessary conditions for the extreme value is

$$\frac{\partial L}{\partial \delta} = g + 2\mu\omega\delta = 0 \tag{17}$$

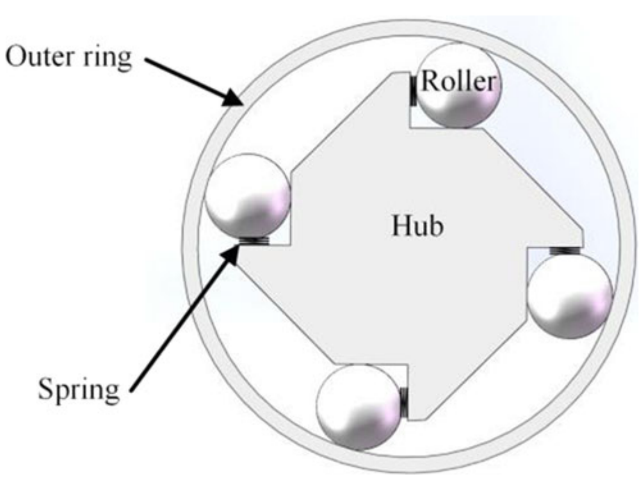



**Fig. 2** One-way clutch assembly

then

$$\delta = -\frac{1}{2\mu} \omega^{-1} g \tag{18}$$

Substituting Eq. 16 to constraint condition in Eqs. 14 and 15, then

$$\mu^2 = \frac{1}{4\theta^2} g^T \omega^{-1} g \tag{19}$$

then

$$\mu = \pm \frac{1}{2\theta} \sqrt{g^T \omega^{-1} g} \tag{20}$$

$$\overline{\varphi} = \varphi_{max} = \varphi_0 + \theta \sqrt{g^T \omega^{-1} g} \tag{21}$$

$$\underline{\varphi} = \varphi_{min} = \varphi_0 - \theta \sqrt{g^T \omega^{-1} g} \tag{22}$$

The SPDM of the ellipsoid in Eq. 1 is often taken as the following dimensional diagonal matrix form:

$$\omega = diag\left( {}^{1}/_{e_i^2} \right) \tag{23}$$

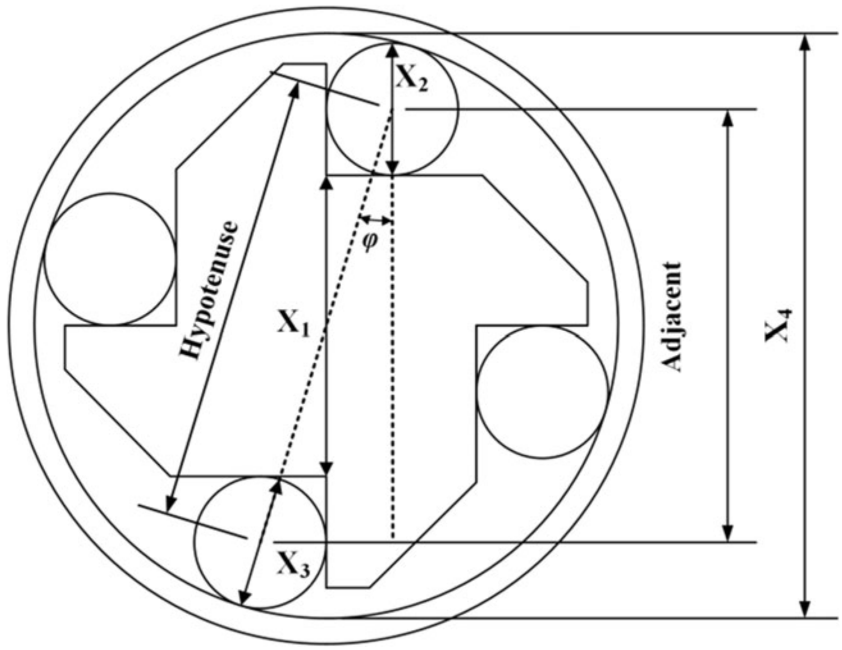
where $e_i > 0$, $i = 1, 2, \cdots m$.

Then, the ellipsoid of Eq. 1 can be converted to

$$\begin{aligned} E(\delta, \theta) &= \left\{ \delta : \ \delta^T \omega \delta \le \theta^2 \right\} \\ &= \left\{ \delta : \ \sum_{i=1}^{m} \frac{\left( \delta_i - \delta_i^0 \right)^2}{e_i^2} \le \theta^2 \right\} \end{aligned} \tag{24}$$

The size parameter $\theta$ and the semi-axis vector $\boldsymbol{e} = (e_1, e_2, \cdots, e_m)^T$ of the ellipsoid are obtained by means of the

**Fig. 3** One-way clutch assembly dimensions




constraint condition of Eq. 5. When $\theta\neq0$, Eq. 22 can be converted to

$$E(\delta,\theta)=\left\{\delta:\ \sum_{i=1}^{m}\frac{\left(\delta_i-\delta_i^0\right)^2}{\left(\theta e_i\right)^2}\leq 1\right\} \tag{25}$$

In the Convex Method, the shell of this ellipsoid should contain the minimum volume of ellipsoid. And, this minimum volume ellipsoid can be quantified as

$$\sum_{i=1}^{m}\frac{\delta_i^2}{\left(\frac{m}{2}\Delta x_i\right)^2}\leq 1 \tag{26a}$$

or

$$\frac{\delta_1^2}{\left(\frac{m}{2}\Delta x_1\right)^2}+\frac{\delta_2^2}{\left(\frac{m}{2}\Delta x_2\right)^2}+\cdots+\frac{\delta_m^2}{\left(\frac{m}{2}\Delta x_m\right)^2}\leq 1 \tag{26b}$$

where

$$\Delta x_i=\frac{T_i}{2} \tag{27}$$

$T_i$ is the $i$th designed tolerance. From equations above, we can see that the uncertain level of feature variations increases with the addition of dimensions and tolerances. It is necessary to lower the extent of input uncertainties for exacting assembly accumulation. Thus, the upper limit and lower limit of assembly function can be written as

$$\begin{cases}\overline{\varphi}=\varphi_{max}=\varphi_0+\theta\sqrt{\sum_{i=1}^{m}\left(\frac{m}{2}\Delta x_i\frac{\partial\varnothing(x^0)}{\partial x_i}\right)^2}\\ \underline{\varphi}=\varphi_{min}=\varphi_0-\theta\sqrt{\sum_{i=1}^{m}\left(\frac{m}{2}\Delta x_i\frac{\partial\varnothing(x^0)}{\partial x_i}\right)^2}\end{cases} \tag{28}$$

Equation 26b is the assembly function based on C-NPS. The equation gives out the limits of target assembly requirement.

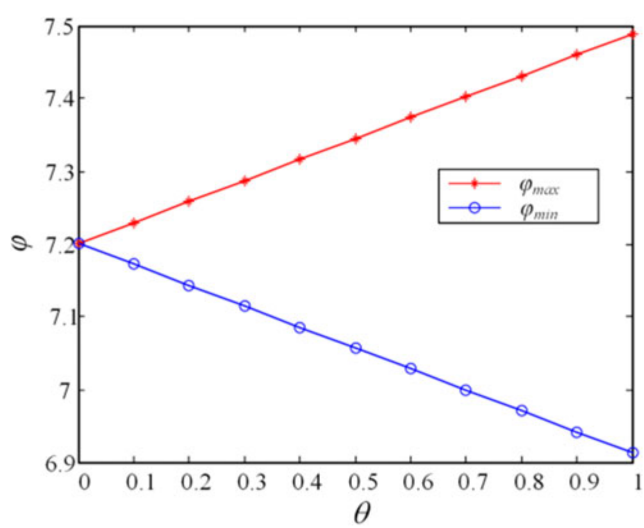



**Fig. 4** Results of accumulation $\varphi$ based on C-NPS

**Table 2** Specifications of manufactured variables of the one-way clutch assembly

| Variables | Nominal | LL | UL |
|---|---|---|---|
| $X_1$ | 55.29 | −0.090 | 0.010 |
| $X_2$ | 22.86 | −0.001 | 0.005 |
| $X_3$ | 22.86 | −0.001 | 0.005 |
| $X_4$ | 101.60 | −0.030 | 0.020 |

### 2.4 Tolerance analysis procedures of C-NPS method

The analysis procedures of C-NPS method is illustrated in Fig. 1, and the detailed procedures can be summarized as follows

Step 1 Confirm assembly requirement. Assembly requirement is determined by functional requirements and is usually described as the distances of features or angels.

Step 2 Design process. For each part, requirement drives a design procedure which includes two main steps: an assignment of dimensions to all parts, the design of tolerances of each feature according to empirical priority criteria.

Step 3 Construct C-NPS model. From step 2, the uncertainties of each initial parameter are acquired. The distribution of the design factors or the performance variables is unknown, or the designed tolerance is non-probabilistic. Thus, the C-NPS model can be constructed using correlation analysis from Sect. 2.3

Step 4 Analysis and judgment. The calculated results are obtained from C-NPS model and then compared with assembly requirement. Repeat step 2 and step 3 until the result reaches the stopping criteria and outputs the optimal results

## 3 Case study

### 3.1 Case description

In this section, the example of one-way clutch mechanism assembly is presented in detail to exhibit the application of the proposed tolerance analysis method. The example is adopted from a real-life industrial application and properly simplified to be easily presented and discussed in this context. One-way clutch is an important component for power transmission between the internal drive shaft and driven shaft. The dimensions and tolerances are critical engineering design information for defining force transmission requirements of manufactured parts.

Figure 2 shows the simplified one-way clutch mechanism assembly. This assembly consists of the following components: an outer ring, a hub, four rollers, and four springs. When the hub rotates counterclockwise, the rollers wedge between the hub and the ring, then the clutch is engaged. When the hub rotates clockwise, the rollers back into the wide grooves, the outer ring is stationary, and then the clutch is separated. The size of pressure angle, the limits of pressure angle, and the variation range of the pressure angle are the key characteristics to the assembly. These data are vital to the designers. It will be of great help to the designer if these key characteristics are simulated in the design process.

As shown in Fig. 3, the pressure angle $\varphi$ should be limited to a certain range to guarantee the force transmission efficiency of the whole mechanism. Four manufactured dimensions, $X_1$,$X_2$,$X_3$, and $X_4$ control the pressure angle $\varphi$, where $X_1$ is the height of the hub, $X_2$ and $X_3$ are the diameter of the rollers, respectively, and $X_4$ is the diameter of the outer ring. The normal values, the upper deviations, and the lower deviations of $X_1$,$X_2$,$X_3$, and $X_4$ are shown in Table 1.

In this clutch assembly, the value and the variation range of the pressure angle $\varphi$ are the assembly requirement.

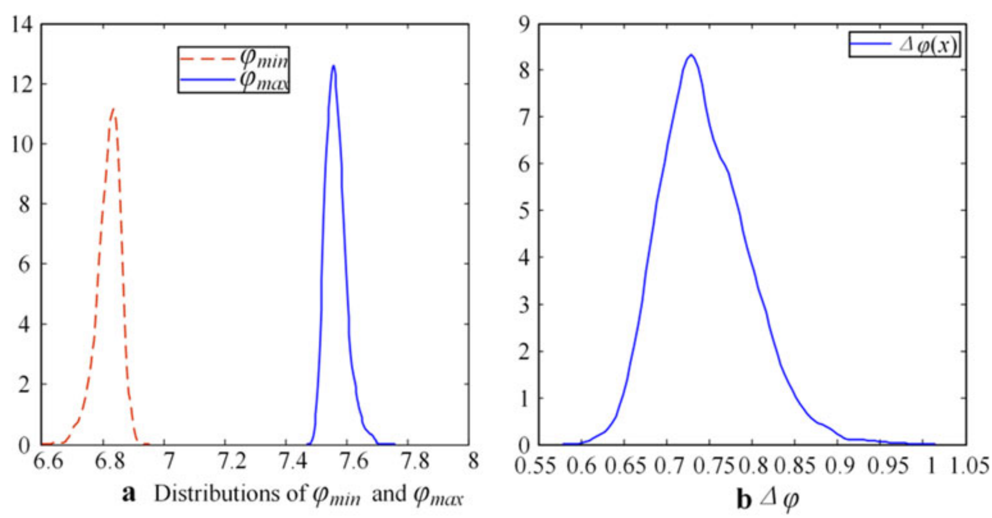



**Fig. 5** Results of accumulation $\varphi$ based on Monte Carlo

**Table 3** Comparisons between C-NPS, Monte Carlo and W-C

| Method | $\varphi_{max}$ | $\varphi_{min}$ | $\Delta\varphi$ |
|---|---|---|---|
| C-NPS ($\theta$=1) | 7.4890° | 6.9130° | 0.5760° |
| Monte Carlo | 7.4934° | 6.8967° | 0.5967° |
| W-C | 7.6547° | 6.7150° | 0.9397° |

Accumulation of the dimension tolerances generates a variation of pressure angle $\varphi$. The assembly function is shown below:

$$\varphi = \cos^{-1}\left(\frac{X_1 + \frac{X_2 + X_3}{2}}{X_4 - \frac{X_2 + X_3}{2}}\right) \tag{29}$$

As shown in Table 2, the tolerances are bilateral and asymmetric. The tolerance analysis is carried out to determine the tolerance value of pressure angle $\varphi$. For the tolerance analysis based on C-NPS method, we can follow the steps in Sect. 2.4 to determine the lower and upper limit values of the assembly function.

### 3.2 Tolerance analysis and discussion

According to Eqs. 26b and 27, we calculate the values of $\varphi$ when $\theta$=0, 0.1, $\cdots$ , 1, respectively. The results are shown in Fig. 4.

It is obvious that the variation of $\varphi$ is larger when $\theta$ is bigger. When $\theta$=1,$\varphi$=[6.9130°, 7.4890°]

The results of C-NPS method are then compared with those of Monte Carlo method and W-C method. In Monte Carlo method, we assume that $X_1$ $X_2$ $X_3$, and $X_4$ obey the normal distributions and simulate the feature variations 2000×2000 times to obtain the values of $\varphi$ and the distributions of $\varphi_{max}$ and $\varphi_{min}$.

The results of Monte Carlo method are shown in Fig. 5, and the comparative results are listed in Table 3. The left curve of Fig. 5a is the distribution of minimum values of $\varphi$, and right curve is the distribution of maximum values of $\varphi$. Figure 5b is the difference value of $\varphi_{max}$ and $\varphi_{min}$, where

$$\Delta\varphi = \varphi_{max} - \varphi_{min} \tag{30}$$

From Fig. 5, we can see that the maximum and minimum value of $\varphi$ vary in a variational range. The mean value of $\varphi_{max}$ using Monte Carlo method is 7.4934 °, and the mean value of $\varphi_{min}$ using Monte Carlo method is 6.8967°. Thus, the variation of extremum values based on Monte Carlo can be given out in this way.

Based on the results, we can make a brief summary as follows:

The result of C-NPS method gives accurate estimation of true variation range and is close to Monte Carlo method. The result of C-NPS method is better than that of W-C.

The definition process of Monte Carlo method is complex, while C-NPS method only needs to define the values of dimensions and the limits of tolerances. Monte Carlo method should define the value of each dimension and the distribution of each tolerance. If the distributions of the independent variables change of shift, the whole analysis must be redone.

Monte Carlo method is time consuming, while C-NPS method is time saving. It takes more than 12 min to conduct the simulation 2000×2000 times. But, C-NPS method needs only 2.6 s.

## 4 Conclusions

In this paper, a novel tolerance analysis method, C-NPS, is proposed to analyze the assembly function as a supplementary to statistical methods in tolerance analysis field. In this method, tolerance model is constructed using NPS theory, and the assembly requirements are solved by Convex Method. Comparing the commonly used W-C method and Monte Carlo, C-NPS model is easy to construct with no necessary of any distribution assumptions and is time saving to obtain a relatively accurate result, and it can be an effective tool for tolerance analysis in the following situations: (1) Assembly function is linear or nonlinear, (2) the interval of tolerance is symmetric or asymmetric and can be generalized to unilateral specification, and (3) the distributions of the tolerances are unknown.

C-NPS method provides a new way for tolerance analysis, and more research studies about the tolerance optimization, tolerance allocation, cost optimization, etc. should be done in the future.

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