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ORIGINAL ARTICLE

Robust modelling and prediction of thermally induced positional error based on grey rough set theory and neural networks

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Abstract Thermal errors adversely affect the precision of a machine tool operation. Therefore, reduction of thermal errors is essential for improving of machining accuracy. In this paper, in order to realize real-time compensation, taking a multi-axis machine tool as an example and a method for robust modeling and predicting thermally induced positional error is proposed. First of all, the number of temperature measuring points required in thermal error's model was reduced based on the rough set theory, which greatly reduced variable searching and modeling time. Then through grey relation theory, systematic analysis of the similarity degree between thermal error and temperature data was carried out to select sensitive temperature measuring points, and the temperature variables in the thermal error's model were reduced from 24 to 7 after optimization, which eliminated the coupling problems. For reducing the influence of unpredictable noises, radial basis function (RBF) and back propagation (BP) neural network modeling methods were adopted to predict thermally induced positional errors, and in comparison, the prediction accuracy of RBF neural network was found superior to that of traditional BP neural network. Finally, some measured data were selected to verify the validity of the proposed method, and the results showed that prediction accuracy of the proposed thermally induced positional error model was reliable.

Keywords Multi-axis machine tool · Thermally induced positional error · Grey rough set theory · Neural network

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1 Introduction

Improvement of NC machine tool accuracy is a continuous and important area of research in precision machining field [1]. Along with the gradually increasing demand for machined components of complex geometry and high dimensional accuracy, many researchers paid attention to reduce the machining errors of machine tools. Among them, the thermal error is extremely important as it affects the machine tool precision and, consequently, reduces the quality of manufactured workpieces [2]. The thermally induced error accounts to 40–70 % of the total errors [3–6], and its effect on machine tools is recognized as problem for research and its solution is essential for meeting the increasing demands on product quality [7]. As a result, reduction of thermal errors is beneficial to high-precision manufacturing system [8, 9].

Generally, thermally induced machine tool errors can be classified as quasi-static errors that vary slowly in time and are related to the structure of machine tool. These errors which are not only time-variant but also spatially variant over the entire machine working volume [7], increase the difficulty to control and reduce them. In recent years, several studies were conducted for error reduction in machine tools. One popular approach was based on the consideration that the thermally induced errors could be reduced with structural improvement to the machine tool itself through the design and manufacturing technology [10], such as symmetric thermal design and installation of a cooling unit [11], incorporation of concrete or reinforced plastic in the machine tool structure [12]. Although these methods were effective in reducing the thermal errors to some extent, they were expensive [13]. It was found that in implementing these processes, several physical limitations were experienced which could not be overcome solely by design techniques alone.

The economical approach, of compensating the error, was deemed as a flexible solution to reduce the positional, geometrical, and thermal errors. The studies on error compensation techniques employed to improve machine tool accuracy gained significance in this field [14]. However, for complete compensation, the machine's thermal errors should be modeled as accurately as possible and it is more complicated than the modeling of geometrical errors. Yang [7] pointed out that robust modeling is a challenging task in machine tool thermal error compensation. The mechanism causing the machine tool deformations is so complex that the thermally induced errors are difficult to predict. Establishing a model for thermal error prediction for compensation of this error depends on the temperature field in machine tools. The distribution of temperature is a major factor influencing the thermal errors. Hence, for obtaining a reasonable temperature field, a large amount of research was carried out for selecting the pivotal temperature measuring points for determining the thermal errors.

In the working cycle of a machine tool, the thermal deformations are nonlinear and time-variant depending on the changes in environmental and operating conditions. For majority of the modeling methods, the thermal error models were obtained by determining the best mapping relationships between the thermal errors and some key temperature variables related to the location of heat sources [14], such as motors, bearings, hydraulic systems, ambient temperature, etc.. Wang [6] presented a method of using fuzzy C means (FCM) clustering and iterative self-organizing data analysis (ISODAT) to group the data of thermal sensors and established an artificial neural network thermal model. Chen carried out multiple regression analysis and proposed an artificial neural network model for the real-time forecasting of thermal errors with numerous temperature measurements [15]. Lee et al. presented a thermal modeling method based on independent component analysis to compensate for the thermal errors occurring in a commercial machining center [16]. Yang et al. [17, 18] proposed several methods to optimize the number of thermal sensors for a minimum to compensate for machine tool thermal errors. In these methods, ant colony algorithm-based back propagation neural network (ACOBPN), synthetic grey correlation theory, and grey neural network were applied.

The thermally induced errors can degrade the positional accuracy of machine tools significantly, resulting in geometrical deviations of the manufactured items [19]. In this paper, a robust modeling and prediction method of thermally induced positional errors is proposed. The model can reduce the measuring points on the basis of rough set theory and grey relational analysis and predicts the error basing on RBF neural network.

One aspect to note is that the rough set is more important than grey comprehensive analysis in the paper, and the optimization of temperature measuring points is mainly studied based on the rough set. Of course, the combination of the rough set and grey relation analysis is the most important research work.

In order to obtain the optimization results as quickly and accurately as possible, first of all, a few combinations of temperature measuring points required for thermal error model was reduced based on the rough set theory; then, systematic analysis of the similarity degree between thermal error and temperature data based on grey relation theory was carried out, and the incidence degree sequence was employed to select temperature-sensitive measuring points; finally, from comprehensive analysis, the combinations of temperature measuring points and temperature-sensitive measuring points, the optimal combination can be screened out.

The rest of this paper is arranged as follows: An introduction to reduction of temperature points based on the rough set theory is presented in Section 2. Section 3 deals with the selection of measuring points combinations based on grey relational analysis. The prediction of relationship between the thermally induced positional error and the temperature based on the neural network is presented in Section 4. In Section 5, a horizontal machining center is used as an example to verify the proposed method on the basis of the experiment data. The conclusions are presented in Section 6.

2 Reduction of temperature points based on rough set

Theoretically, the determination of thermally induced positional error is based on the temperature distribution in the machine tool. To establish a model for accurate prediction of positional error, temperature distribution in the machine associated with positional errors must be obtained. This requires all parts of the machine to be fixed a large number of temperature sensors to measure the real-time temperature distribution in the machine while it is in operation. In general, thermal error measuring point is selected from a few to several hundred points. However, the excessive number of temperature measuring points will increase workload and, also if the measuring points are too close, the intercoupling among the measured becomes serious, which affects the accuracy of prediction.

Rough set (RS) theory, first proposed by a Polish scholar Z. Pawlak [20] in 1982, was just a mathematical tool dealing with incompleteness and uncertainty problems. The RS theory can effectively analyze and deal with imprecise, inconsistent, and incomplete information and determine implicit knowledge and reveal latent rules by directly examining and reasoning the data.

Rough set theory is justly a mathematical tool dealing with incompleteness and uncertainty problems. Its main thought is that problem's classification or decision rule is derived by knowledge reduction on the constant premise of the information system classification ability. And the rules of rough set theory and the data reduction method are using the equivalent relation to classify elements of the collections to generate some partition, corresponding to the equivalent relation.

Generally, in RS theory, the data object is needed to be established before knowledge discovery based on rough sets is created, which is called the information system. Here, the machine tool thermal error system can be represented by four unknown quantity groups of $S=\langle U,A,V,f\rangle$, where $U=\{u_1,u_2,\cdots,u_{|U|}\}$ *is* a non-empty set of finite objects (the universe) and composed of a limited number of CNC manufacturing objects, and $|U|$ represents the number of samples, U. $A=\{a_1,a_2,\cdots,a_{|A|}\}$, is used to describe a set of temperature attributes of computer numerical control (CNC) manufactured objects, expressed as a set of binary relationships on the universe U (it is usually a set of equivalence relationships for defining complete information system), and $|A|$ represents the number of attributes. $V=\bigcup V_a$, and V_a is a set of value domain of attribute a; the function $f: U\times A\rightarrow V$ satisfies $f(x,a)\in V_a$. Also, a unique value is assigned to each attribute $a(a\in A)$ of every object in the universe U.

In the system of thermally induced machine tool errors, the attribute set A can be divided into a thermally induced condition attribute set T ($T=\{T_1,T_2,\cdots T_{|k|}\}$) and a thermally induced outcome attribute set $Y(Y=\{Y_i\})(T\cup Y=A, T,\cap Y=\varnothing)$. A thermal error system decision table $S=\langle U, A,V,f\rangle$ is constructed by the information system. A collection of all the equivalence classes of the error attribute set Y can be represented as U/Y, and the equivalence class contained element u in U can be represented as $[u]_Y$. If two elements belong to the same equivalence class, they are indistinguishable. The indiscernible relationship is the starting point of RS theory.

Now, in the CNC thermal error system, the manufactured objects subset $U'=\{u_1,\cdots,u_n\}\subseteq U$ can be represented by using thermally induced condition attribute subset T'. That is, an arbitrary set of manufactured objects U' comprises a single class, which (i.e., this subset) can be expressed in the form of equivalence classes induced by thermally induced condition attribute subset $T'=\{T_1,\cdots,T_n\}$. In general, the set U' cannot be expressed exactly, because the set may include or exclude objects which are indistinguishable on the basis of attributes T'. However, the set U' can be approximated by using only the information contained within T' and by constructing the T' -lower and T' -upper approximations of U':

$$\underline{T'}(U')=\left\{u_i\in\left|[u_i]_{T'}\subseteq U'\right.\right\} \tag{1}$$

$$\overline{T'}(U')=\left\{u_i\in U\left|[u_i]_{T'}\cap U'\neq\varnothing\right.\right\} \tag{2}$$

In the above, $\underline{T'}(U')$ is a set of those elements that must be classified on the basis of thermally induced condition attribute set $T'\subseteq T$, which is the largest definable set within U'. The set $\overline{T'}(U')$ is a set of those elements might be classified on the basis $T'\subseteq T$, which is the smallest definable set within U'.

The T'-lower approximation, or positive region, is the union of all equivalence classes in $[u_i]_{T'}$ which are contained in (i.e., are subsets of) the manufactured objects' subset—$\underline{T'}(U')$. The lower approximation is the complete set of objects in U/T' that can be positively (i.e., unambiguously) classified as belonging to set U'.

According to RS theory [21, 22], an important aspect of database analysis or data acquisition is the discovery of attribute dependencies. That is, the expectancy to discover which thermally induced condition attribute T is strongly related to thermally induced outcome attribute set Y in CNC information system.

Then, the dependency of attribute set $X(X\in U/Y)$ on the thermally induced condition attribute set T', $\gamma(T,Y)$, is given by

$$\gamma(T,Y)=\frac{\sum_{X\in U/Y}\left|\underline{T}(X)\right|}{|U|} \tag{3}$$

The term $\gamma(T,Y)$ represents the ratio of the object that can be accurately classified in the whole system and the quality of classification about set T.

If $\gamma(T,Y)=1$, the error attribute set Y is completely dependent on the temperature attribute set T; that is, every attribute of set T is necessary. If $0<\gamma(T,Y)<1$, the error attribute set Y is partly dependent on the temperature attribute set T; this means, only a partial attribute of set T is necessary.

The attribute's importance of the decision table $S=\langle U,A,V,f\rangle$ can be estimated by the impact of classification ability for the table S in removing an attribute $t\in T$ from the set T. As mentioned earlier, $\gamma(T,Y)$ represents the dependency between the attribute sets T and Y and it also represents the approximation precision of division of U/Y on set T. Therefore, the importance of an attribute t can be estimated by using the variation between $\gamma(T,Y)$ and $\gamma(T-\{t\},Y)$.

The importance of attribute t can be defined by the following relationship:

$$\sigma_{(T,Y)}(t)=\frac{\gamma(T,Y)-\gamma(T-\{t\},Y)}{\gamma(T,Y)}=1-\frac{\gamma(T-\{t\},Y)}{\gamma(T,Y)} \tag{4}$$

If the sets T,Y are known, $\sigma_{(T,Y)}(t)$ can be simply referred to as $\sigma(t)$. The $\sigma(t)$ can be explained as the classification error after removing the attribute t. The importance coefficient can also be defined as a set of attributes as shown in (5):

$$\sigma_{(T,Y)}(T') = \frac{\gamma(T,Y) - \gamma(T-T',Y)}{\gamma(T,Y)} = 1 - \frac{\gamma(T-T',Y)}{\gamma(T,Y)} \tag{5}$$

If set T' is a reduction of set T, then there is $\sigma(T')=1$. If the set T' (any subset of set T) is regarded as an approximate reduction of set T, the error in approximate reduction is given by:

$$\varepsilon_{(T,Y)}(T') = \frac{\gamma(T,Y) - \gamma(T',Y)}{\gamma(T,y)} = 1 - \frac{\gamma(T',Y)}{\gamma(T,y)} \tag{6}$$

The approximate error $\varepsilon_{(T,Y)}(T')$, or $\varepsilon(T')$ for short, represents the degree of approximation of the attribute set T' approximate to the set T. If set T' is a reduction of set T, there is $\varepsilon(T')=0$.

Generally, the discernible matrix [23] is one of the main methods used for solving the relative attribute reduction and attribute core. In the information system $S=\langle U, T\cup Y, V, f\rangle$, the universe is $U=\{u_1,u_2,\cdots,u_n\}$. The object universe which is divided into disjoint class families according to the decision attribute Y can be represented as $X=U/T=\{x_1,x_2,\cdots,x_m\}$. With $M(T)=\{m_{i,j}\}_{n\times n}$, the discernible matrix of set T of the decision table S can be defined as:

$$m_{i,j} = \begin{cases} t\in T : t(x_i)\neq t(x_j) \ \text{ and } \ Y(x_i)\neq Y(x_j) \\ \varnothing, \ \text{ other} \end{cases} \tag{7}$$

The discernible function of S is

$$\Delta = \prod\nolimits_{(x_i,x_j)\in U\times U} \sum m_{i,j} \tag{8}$$

The function Δ is a conjunction-disjunction function containing $|T|$ Boolean variable. All the conjunction types of the minimal disjunctive normal from the function Δ area reduction of the decision attribute Y about set T.

3 Selection of measuring points combination

In Section 2, reduction of the temperature measuring points were obtained by the RS theory, and the reduced measuring points could completely express the distribution of the machine temperature. Here, for selecting the measuring points for optimization more quickly and accurately, the grey correlation analysis, which systematically analyzed degree similarity between two parameters based on a mathematical method, was adopted to screen the reduction in this paper.

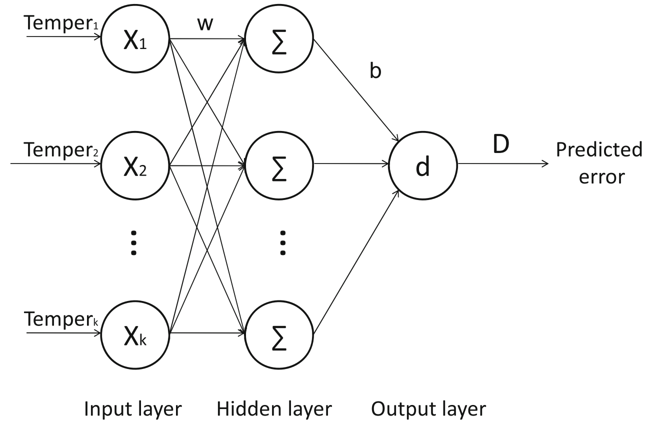

Fig. 1 BP neural network model

Grey system theory [24], proposed by Professor Deng Julong in the early 1980s, is an effective mathematical tool to solve the uncertainty problems. Grey relational analysis method can measure the degree of association among various factors based on the degree of similarity or dissimilarity between the trends of factors, namely the "grey correlation degree."

Due to the different physical meanings of various factors in the machine tool thermal error system, the dimensions of data are different and hence it is difficult to reach a correct conclusion by direct comparison. Therefore, for the grey correlation analysis, it is always necessary that data processing is carried out with non-dimensional method [25]. In the general, the processing of the original data can usually be carried out by three types of transformational methods namely, initial value transformation, the mean of the transformation, and the range of transformation. In this study, the mean of the transformation method is selected.

The central idea in grey relational algebra is to analyze the uncertain relationship between two tuples by looking at the information flow from one tuple to another under the influence of all tuples involved in the relationship [26]. In the error system, thermally induced positional error data $Y(t)=\{Y(j)|j=1,2,$

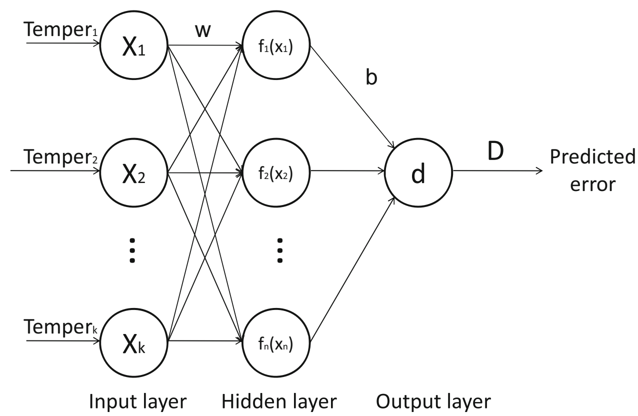

Fig. 2 RBF neural network model

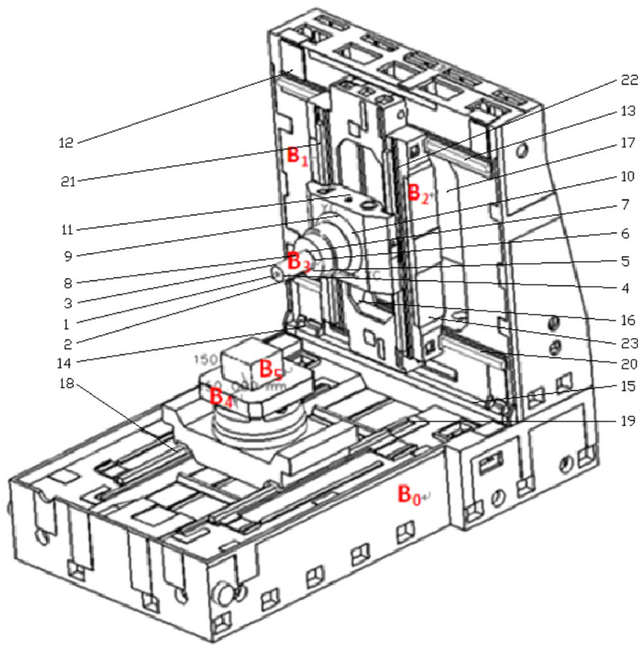

Fig. 3 Locations of 24sensors

$\cdots,l\}$ is regarded as the mother sequence. The son sequences are the measured value sequences of temperature sensors $T_i=\{T_i(j)|i=1,2,\cdots,k;j=1,2,\cdots,l\}$. Therefore, the relationship of correlation coefficient [27] of $Y(t)$ to T_i at the point j can be expressed as

$$\xi_{0i}=\frac{\min_i\min_j\Delta_{0i}(j)+\rho\max_i\max_j\Delta_{0i}(j)}{\Delta_{0i}(j)+\rho\max_i\max_j\Delta_{0i}(j)} \tag{9}$$

In Eq. (9), $\Delta_{0i}(j)$ is difference between the absolute values of $Y(t)$ and T_i at point j, $\Delta_{0i}(j)=|Y(t)-T_i|$, $\min_i\min_j\Delta_{0i}(j)$ is the minimum value of difference, $\max_i\max_j\Delta_{0i}(j)$ is the maximum value of difference ρ is the identification coefficient, $\rho\in[0,1]$, and generally $\rho=0.5$. The identification coefficient ρ can be adjusted to enhance the identification ability of contrastive analysis according to the relational degree between the data sequences.

Table 1 The arrangement and location of sensors

The serial number of temperature sensor	Location
1,2,3,4,5,6,7,8	Front end and back end of spindle
9,10,11	Front and top of spindle box
12,13,14,15	*X*-, *Y*-, and *Z*-axes of bearings and couplers
18,19,20,21,22	*X*-, *Y*-, and *Z*-axes of guide way
16,17,23,24	The skateboarding, workbench, and environmental temperature

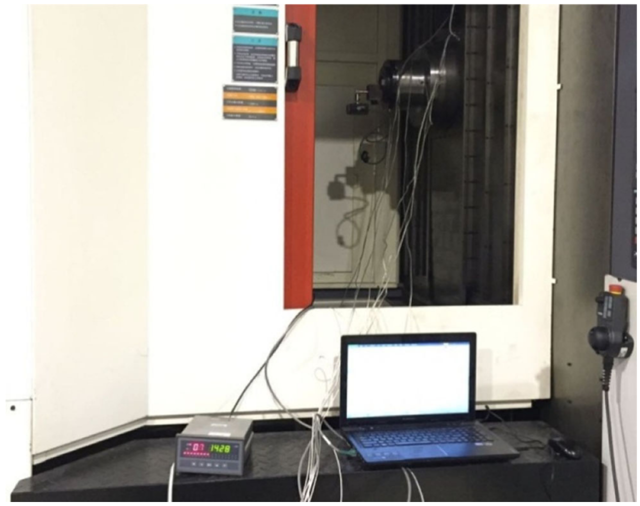

Fig. 4 Measurement site for positional error and temperature

The relational degree γ_{0i} between the two sequences can be calculated by the average value of the correlation coefficient $\xi_{0i}(j)$ of the two sequences on each moment and is given by

$$\gamma_{0i}=\frac{1}{i}\sum_{j=1}^{l}\xi_{0i}(j),j=1,2,\cdots,l \tag{10}$$

Finally, the relational degree γ_{0i} between each son sequence and mother sequence is arranged according to the size of the correlation sequence to form, which directly reflects the essentiality of each son sequence to the same mother sequence [28]. It also indicates the effect the temperature changes produced at the measuring point on the machine positional errors.

At this juncture, a threshold value γ' is defined. When $\gamma_{0i}>\gamma'$, the positions of the temperature measuring points are preserved and the other positions are eliminated as effect of temperature on thermal error is extremely small.

Till now, through comprehensive comparison of temperature measuring points obtained from reduction and sensitive

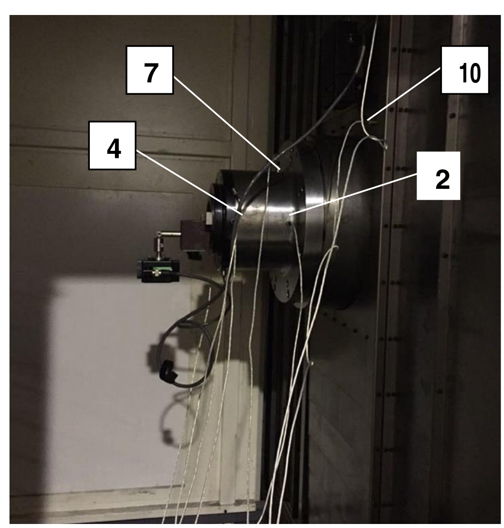

Fig. 5 Thermal sensor locations

temperature measuring points, the optimal combination of measuring points containing most sensitive temperature measuring points and the highest correlation temperature measuring points were screened out.

4 Relation for prediction of error and temperature

The objective of error modeling is to establish a mathematical relationship between the temperatures and thermally induced positional errors. Since it is impossible to measure the temperatures and positional errors for all the cases, a method to predict the thermal errors accurately is an essential step to realize real-time compensation. In previous studies, the prediction of the thermal error was implemented by three modeling methods namely, multiple linear regression [29], neural network model [30], and system identification [31] by scholars worldwide. In this study, the neural network [32–34] was used to model and predict the relation between the temperature and error caused by the machine tool.

The BP neural network [35], proposed by Rumelhart and McCelland in 1986, which is shown in Fig. 1, is one of the most widely used neural network models. It is a feed forward neural network implemented in back propagation algorithm. The BP algorithm is a learning algorithm based on gradient descent, and the learning process is implemented by adjusting

Table 2 Part of the discrete data obtained based on Rosetta software

	Temp1	Temp2	Temp3	Temp4	Temp5	Temp6	Temp7	Temp8
1	[19.7, 22.3)	[*, 23.6)	[18.8, 22.5)	[19.2, 23.1)	20.5	[19.6, 25.3)	[18.8, *)	20.1
2	[19.5, 19.7)	[*, 23.6)	[18.8, 22.5)	[19.2, 23.1)	20.4	[19.6, 25.3)	[18.8, *)	19.9
3	[19.3, 19.5)	[*, 23.6)	[18.8, 22.5)	[19.2, 23.1)	20.2	[19.6, 25.3)	[18.8, *)	19.8
4	[19.3, 19.5)	[*, 23.6)	[18.8, 22.5)	[19.2, 23.1)	20.1	[19.6, 25.3)	[18.8, *)	19.7
5	[19.1, 19.3)	[*, 23.6)	[18.8, 22.5)	[19.2, 23.1)	20.0	[19.6, 25.3)	[18.8, *)	19.6
6	[19.1, 19.3)	[*, 23.6)	[18.8, 22.5)	[19.2, 23.1)	19.9	[19.6, 25.3)	[18.8, *)	19.5
7	[18.9, 19.1)	[*, 23.6)	[18.8, 22.5)	[19.2, 23.1)	19.8	[19.6, 25.3)	[18.8, *)	19.4
8	[18.9, 19.1)	[*, 23.6)	[18.8, 22.5)	[19.2, 23.1)	19.7	[19.6, 25.3)	[18.8, *)	19.3
9	[18.9, 19.1)	[*, 23.6)	[18.8, 22.5)	[19.2, 23.1)	19.6	[19.6, 25.3)	[18.8, *)	19.2
10	[18.6, 18.8)	[*, 23.6)	[18.8, 22.5)	[19.2, 23.1)	19.5	[19.6, 25.3)	[18.8, *)	19.0
11	[18.6, 18.8)	[*, 23.6)	[*, 18.8)	[19.2, 23.1)	19.4	[19.6, 25.3)	[18.8, *)	19.0
12	[18.6, 18.8)	[*, 23.6)	[*, 18.8)	[*, 19.2)	19.3	[19.6, 25.3)	[18.8, *)	18.9
13	[18.6, 18.8)	[*, 23.6)	[*, 18.8)	[*, 19.2)	19.3	[19.6, 25.3)	[18.8, *)	18.8
14	[*, 18.6)	[*, 23.6)	[*, 18.8)	[*, 19.2)	19.2	[19.6, 25.3)	[*, 18.8)	18.7
15	[*, 18.6)	[*, 23.6)	[*, 18.8)	[*, 19.2)	19.1	[19.6, 25.3)	[*, 18.8)	18.7
16	[*, 18.6)	[*, 23.6)	[*, 18.8)	[*, 19.2)	19.1	[*, 19.6)	[*, 18.8)	18.6
17	[18.6, 18.8)	[*, 23.6)	[*, 18.8)	[*, 19.2)	19.0	[*, 19.6)	[*, 18.8)	18.6
18	[18.6, 18.8)	[*, 23.6)	[*, 18.8)	[*, 19.2)	19.0	[*, 19.6)	[*, 18.8)	18.6
19	[18.6, 18.8)	[*, 23.6)	[*, 18.8)	[*, 19.2)	19.0	[*, 19.6)	[*, 18.8)	18.6
20	[18.6, 18.8)	[*, 23.6)	[*, 18.8)	[*, 19.2)	18.9	[*, 19.6)	[*, 18.8)	18.5
21	[18.8, 18.9)	[*, 23.6)	[*, 18.8)	[*, 19.2)	18.9	[*, 19.6)	[*, 18.8)	18.5
22	[18.8, 18.9)	[*, 23.6)	[*, 18.8)	[*, 19.2)	18.9	[*, 19.6)	[*, 18.8)	18.5
23	[18.8, 18.9)	[*, 23.6)	[18.8, 22.5)	[*, 19.2)	18.9	[*, 19.6)	[*, 18.8)	18.5
24	[18.9, 19.1)	[*, 23.6)	[18.8, 22.5)	[*, 19.2)	18.9	[*, 19.6)	[*, 18.8)	18.5
25	[18.9, 19.1)	[*, 23.6)	[18.8, 22.5)	[*, 19.2)	18.9	[*, 19.6)	[18.8, *)	18.6
26	[18.9, 19.1)	[*, 23.6)	[18.8, 22.5)	[*, 19.2)	19.0	[*, 19.6)	[18.8, *)	18.6
27	[19.1, 19.3)	[*, 23.6)	[18.8, 22.5)	[*, 19.2)	19.0	[*, 19.6)	[18.8, *)	18.6
28	[19.1, 19.3)	[*, 23.6)	[18.8, 22.5)	[*, 19.2)	19.0	[*, 19.6)	[18.8, *)	18.6
29	[19.1, 19.3)	[*, 23.6)	[18.8, 22.5)	[*, 19.2)	19.0	[*, 19.6)	[18.8, *)	18.6
30	[19.1, 19.3)	[*, 23.6)	[18.8, 22.5)	[*, 19.2)	19.0	[*, 19.6)	[18.8, *)	18.7
31	[19.1, 19.3)	[*, 23.6)	[18.8, 22.5)	[*, 19.2)	19.0	[*, 19.6)	[18.8, *)	18.7
32	[19.1, 19.3)	[*, 23.6)	[18.8, 22.5)	[*, 19.2)	19.1	[*, 19.6)	[18.8, *)	18.7
33	[19.3, 19.5)	[*, 23.6)	[18.8, 22.5)	[*, 19.2)	19.1	[*, 19.6)	[18.8, *)	18.7
34	[19.1, 19.3)	[*, 23.6)	[18.8, 22.5)	[*, 19.2)	19.1	[*, 19.6)	[18.8, *)	18.7

Table 3 Reduction combination based on Rosetta software

Sequence Number	Reduction	Length
1	{1, 8, 10, 11, 14, 17}	6
2	{1, 5, 10, 11, 14, 19, 20}	7
3	{1, 10, 11, 14, 15, 19, 22}	7
4	{1, 5, 10, 11, 14, 17, 19}	7
5	{1, 2, 10, 11, 14, 21, 22}	7
6	{1, 8, 9, 10, 11, 14, 21}	7
7	{1, 3, 5, 10, 11, 17, 18}	7
8	{1, 2, 7, 10, 11, 14, 17, 19}	8
9	{1, 8, 10, 11, 13, 14, 15, 22}	8
10	{1, 6, 9, 10, 11, 14, 17, 18}	8
11	{1, 8, 10, 11, 14, 19, 22, 24}	8
12	{1, 10, 11, 13, 14, 17, 18, 19}	8
13	{1, 4, 8, 10, 11, 14, 15, 19}	8
14	{1, 10, 11, 14, 18, 19, 20, 24}	8

the weights and thresholds to minimize the mean square error between the desired output value and the actual output value. The error function of BP neural network is defaulted to the mean square error (MSE) between the desired output Y and the network output D.

The correctional connection weight, w, and threshold value, b, can be defined by the following relationships:

$$\begin{aligned} w_{ih}^{N+1} &= w_{ih}^{N} + \eta\delta_h(j)x_i(j) \\ b_{ih}^{N+1}(j) &= b_{ih}^{N}(j) + \eta\delta_h(j) \end{aligned} \tag{11}$$

The global error E can be calculated by the Eq. (12) given below.

$$E = \frac{1}{2}\sum\nolimits_{j=1}^{l}\sum\nolimits_{o=1}^{q}\left(d_o(j)-y_o(j)\right)^2 \tag{12}$$

It is necessary to determine whether a network error E meets the requirements of learning times or not. When $E<\varepsilon$ or the learning times surpass a previously set maximum number M, the algorithm is terminated. Otherwise, the connection weights and thresholds are needed to re-learn adjustments until they reach the requirements.

The radial basis function (RBF) neural network [36], proposed by Moody and Darken in1988, is a local approximation of the three-forward network, with a good approximation effect, and short assay time.

The underlying idea of RBF neural network (shown in Fig. 2) is as follows: the "base" of implicit element is used to constitute a hidden layer space by the radial basis function, and the input vector is transformed by the hidden layer to facilitate the transformation of low-dimensional input data

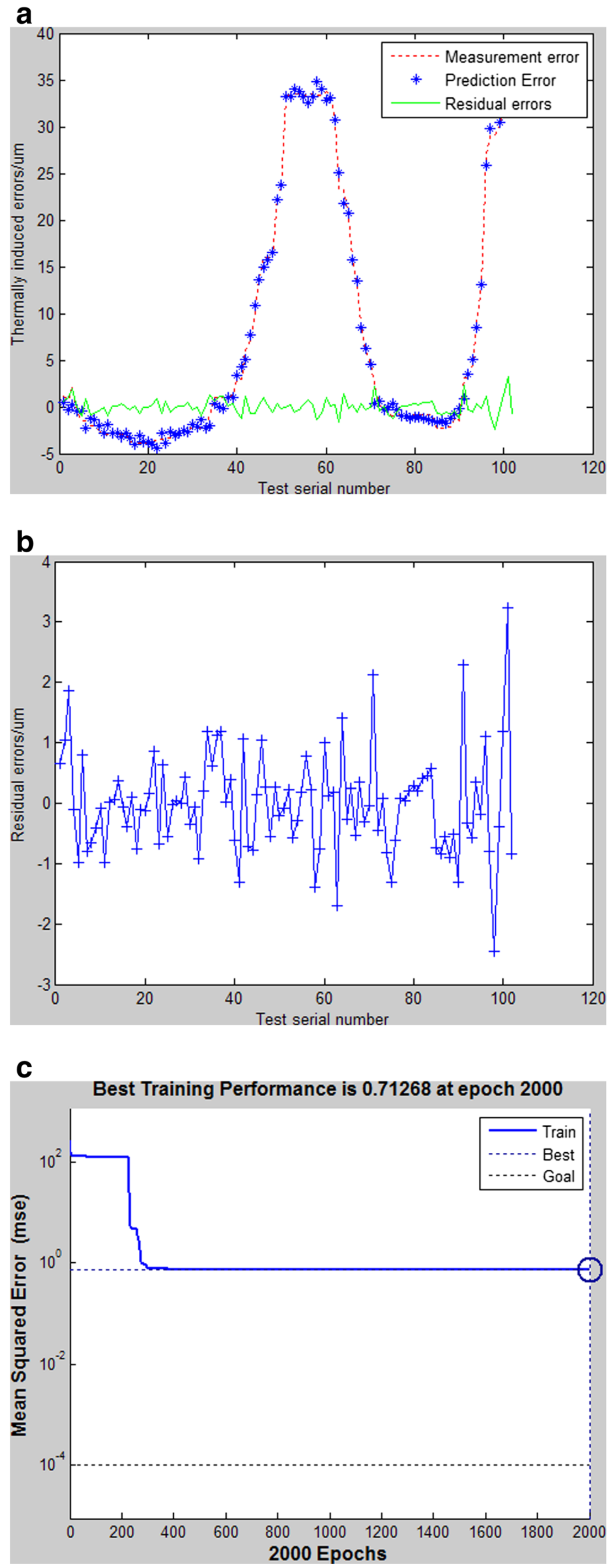

Fig. 6 BP neural network modeling structure. **a** Comparison of measured error with predicted error. **b** Residual error. **c** Trend of trained error

into a higher dimensional space, which means the transformation of the linear inseparable problem in the low-dimensional space into linear inseparable problem in the higher dimensional space.

The RBF neural network and BP neural network are basically similar in structure, except that the transfer function of the hidden layer node of RBF neural network is a radial basis function, which is the local hidden layer of nodes corresponding to the input. Therefore, the RBF neural network is often called a local network. The RBF is characterized by a variety of functions, and common among them is the Gaussian function.

$$\phi_i(x) = \exp\left(-\frac{\|X_i - c_i\|}{2\sigma_i^2}\right) \tag{13}$$

In Eq. (13), C_i is the center of the ith node of all temperature values; σ_i is the parameter controlling the size of domain; $\|\cdot\|$ is the Euclidean norm. In general, the generalized RBF neural network is adopted, as it continually adds hidden layer neurons to make the mean square error to be a minimum value.

5 Application and verification

5.1 Experimental data acquisition

In this study, the positional machine tool errors were measured by a laser interferometer. The laser transmitter was fixed on rails and the laser receiver was fixed on knife holder of the machine tool. Of course, the temperatures of the machine tool were also measured so that every measured positional error corresponded to the temperature data. Considering the structural features of the machine tool and the actual operating conditions, 24 temperature sensors were installed on important parts of the machine spindle, screw, and similar others. The structure of the machine tool used in this study and the arrangement of measurement points are as shown in Fig. 3, and Table 1 shows the locations of installation of the temperature sensors.

The sensors were evenly spaced to the possible extent, avoiding from being too close to interfere with each other or too far away to detect temperature incompletely. Among these sensors, the sensor #24 that measured environmental temperature was installed in the workplace.

First of all, the positional error was measured under ambient conditions (at instant of starting), and with rapid movement of the machine tool axis, the temperature increased

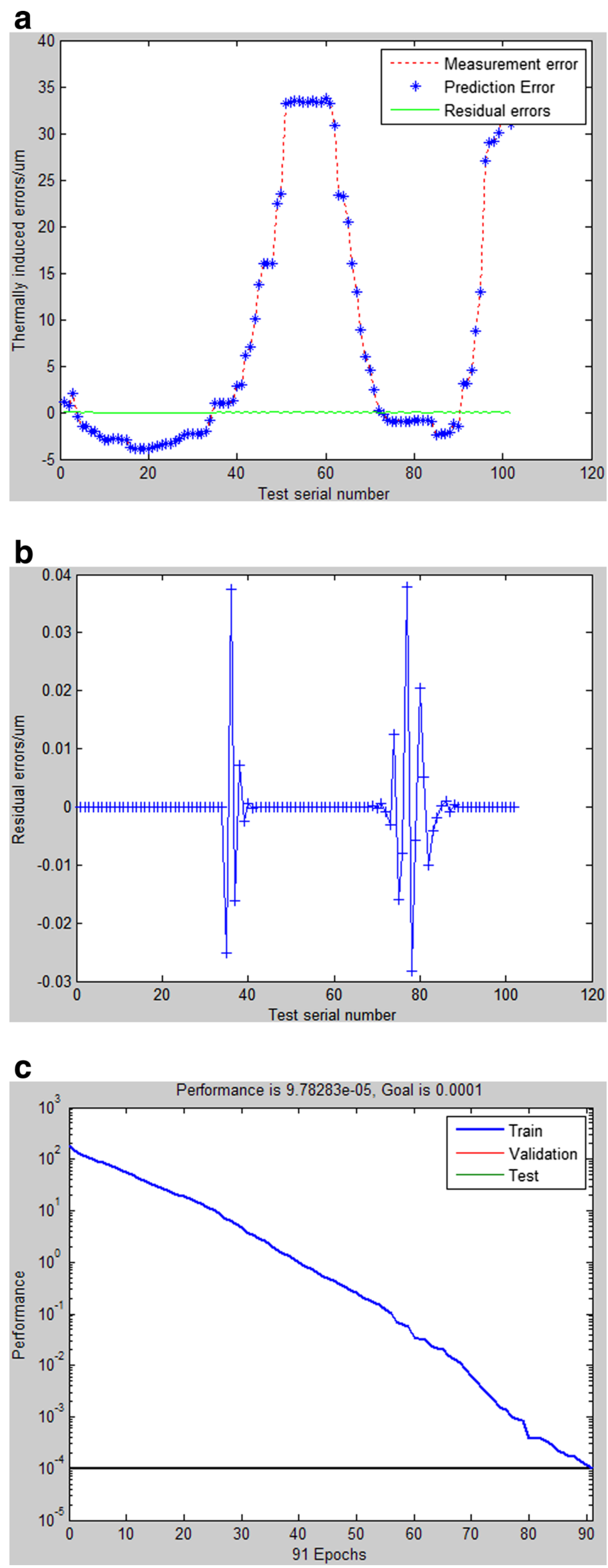

Fig. 7 RBF neural network modeling structure. **a** Comparison of measured error with predicted error. **b** Residual error. **c** Trend of trained error

continually and it was measured. This process was continued until the machine's temperature tended towards stability, and when the machine tool reached the state of thermal equilibrium, the measurement was terminated. The data from the machine tool obtained were (1) the series of temperature T measurements changing with time to obtained from 24-position temperature sensors, $\{T_1(t), T_2(t), \cdots T_{24}(t)\}$; (2) the positional errors of machine tool measured by the laser interferometer, $\{Y(t)\}$ (Figs. 4 and 5).

5.2 Reduction by RS theory

The measured 24 temperature variables as a condition attributes set C can be written as $C=\{T_1(t), T_2(t), \cdots, T_{24}(t)\}$. The positional errors as an outcome attributes set D can be written as $D=\{Y(t)\}$. A decision table of the system, $K=(U, C\cup D)$ was established to form an Excel table. According to the RS theory, the minimum reduction of condition attributes is achieved after reducing the condition attributes in decision table K.

In this paper, for calculating the minimum attribute reduction more quickly and easily, the data of the temperatures and positional errors were reduced by RS analysis software Rosetta. Rosetta software, developed by computer and information sciences of Norwegian University of Science and Technology and Mathematical Sciences Institute of Warsaw University, is an analysis toolkit based on the logical data of rough set theory framework. The following are the concrete steps in the analysis: (1) read the data, (2) polish the data, (3) discrete the data, and (4) reduce the data.

Firstly, an Excel table as shown in Table 2 was created to operate in conjunction with Rosetta software. In order to facilitate subsequent calculations and improve the accuracy of reduction, the data is needed to pretreatment primarily. And the process of pretreatment includes the two important aspects: data complete and data discretizate. Finally, the data were reduced and the combinations of the measuring points were obtained which are shown in Table 3.

Table 4 Performance comparison of the two network models

Network type	The maximum absolute deviation	Mean square error
BP neural network	3.2374	0.71268
RBF neural network	0.0413	9.78283e–05

According to Table 3, 129 types of combinations can be obtained by the reduction process, and the combinations of the temperature measuring points can more completely express the distribution of the machine temperature. However, in order to obtain the best of temperature measuring points more quickly and accurately, the grey correlation analysis was applied to screen the combinations. At this instance, the number of temperature sensors can be defined according to the requirement of temperature points. In this paper, in order to reduce the number of temperature sensors, combinations that included six and seven sensors were selected. The selections are as follows:{1, 8, 10, 11, 14, 17}, {1, 5, 10, 11,14, 19,20}, {1, 10, 11, 14, 15,19, 22}, {1, 5, 10,11, 14, 17, 19}, {1, 2, 10, 11, 14, 21, 22}, {1, 8, 9, 10, 11, 14, 21}, and {1, 3, 5, 10, 11, 17, 18}.

5.3 Screening by grey correlation degree

Next, the grey correlation analysis was implemented to select the best combination. Thermal error sequence $Y(t)$ of the machine tool was regarded as the mother sequence, and the son sequence was the measured value sequence of 24 temperature sensors $T(t)$. At first, the primary data were transformed to eliminate the dimension and to make it suitable for comparison. In this paper, the average was first calculated and all the data were transformed to be a new sequence by dividing it with the average value. Then, the correlation coefficient and degree of correlation of the mother sequence $Y(t)$ to each son sequence $T(t)$ at the point of each moment were obtained. The results are as follows:

$$\begin{aligned}
&\gamma_{0,1} = 0.6116,\ \gamma_{0,2} = 0.6124,\ \gamma_{0,3} = 0.6001,\ \gamma_{0,4} = 0.6065,\ \gamma_{0,5} = 0.6081,\\
&\gamma_{0,6} = 0.6077,\ \gamma_{0,7} = 0.6037,\ \gamma_{0,8} = 0.6054,\ \gamma_{0,9} = 0.6098,\ \gamma_{0,10} = 0.6081,\\
&\gamma_{0,11} = 0.6061,\ \gamma_{0,12} = 0.6082,\ \gamma_{0,13} = 0.6060,\ \gamma_{0,14} = 0.6036,\ \gamma_{0,15} = 0.6214,\\
&\gamma_{0,16} = 0.6001,\ \gamma_{0,17} = 0.6124,\ \gamma_{0,18} = 0.6068,\ \gamma_{0,19} = 0.6065,\ \gamma_{0,20} = 0.6081,\\
&\gamma_{0,21} = 0.6226,\ \gamma_{0,22} = 0.6221,\ \gamma_{0,23} = 0.6069,\ \gamma_{0,24} = 0.6231
\end{aligned}$$

A threshold value was set as $\gamma' = \frac{1}{k}\sum_{k=1}^{1}\gamma_{0k} = 0.6095$. The conclusive points were selected from the measuring points for which γ_{0k} surpassed γ': 24>21>22>15>17>2>1>9. These temperature points could be considered sensitive measuring points.

On the basis of explanation already given, the combination of most sensitive temperature points and highest correlation temperature measuring points were separated as {1,2,10,11, 14,21,22}. This was the expectation of the optimal combination of temperature measuring points.

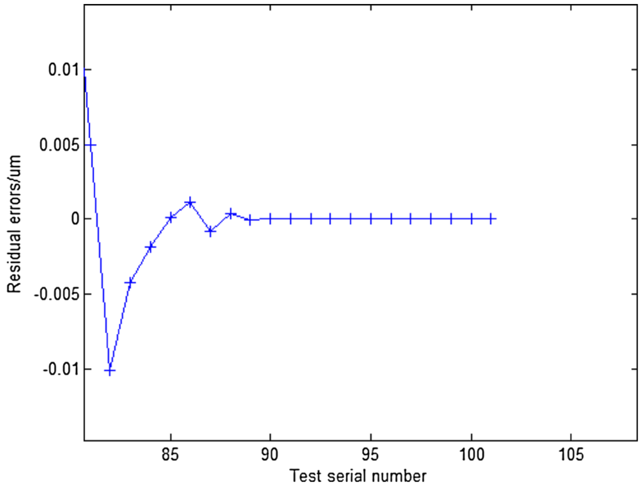

Fig. 8 Verification of RBF network modeled with relational data

5.4 Prediction of thermally induced positional error

Till now, the optimal combination of temperature measuring points was obtained based on the RS theory and the grey relational theory. In order to rapidly establish a model that can reflect the mathematical relationship between the temperature and thermal error at the temperature measuring points and also accurately predict the thermal errors, RBF network and BP neural network were adopted in this part. And, in order to verify the overall performance of RBF network and BP neural network model, the predicted residual errors of the two networks were compared.

Here, the temperature data *T* of seven measuring points were regarded as input variables of the network, and each temperature input layer corresponded to a node in the network. The thermally induced positional error data were regarded as output variables *Y*. In the BP network, the number of hidden layer units was generally defined by the relationship, $s = \sqrt{m + n + a}$ (where m, n was the number of input and output units, a∈[1,10]). In RBF network, the generalized RBF neural network was selected which could continually add hidden layer neurons to make the mean square error to be a minimum.

Figures 6 and 7 show the comparison between BP and RBF network models. From these figures, it can be seen that the residual error of RBF network is smaller than that of BP network, which establishes that RBF network model predicts more accurately. The graph of trained error becomes smooth when the BP neural network is trained to 400 steps, and there is almost no change in mean square error between the 400 to 200 steps, but the effect is not favorable because mean square error cannot meet the requirements of $E<\varepsilon$. While the RBF neural network is trained to reach the error condition on the 91st step, the effect of surface fitting looks good. Table 4 compares the performance of the two models. Therefore, the prediction capability of RBF network model is better than that of BP network model.

Finally, in order to verify the prediction accuracy of RBF neural network model further, another set of measured data were used to verify the prediction error. It can be seen from Fig. 8 and Table 5 that the last few points of the residual error are zero and that the predicted values of the positional error are in agreement with the measured values. Therefore, it can be concluded that the thermally induced positional error of a machine tool can be predicted through the RBF neural network model based on known temperature data. So, the modeling and prediction method of thermally induced positional error proposed in this paper is practicable and effective.

6 Conclusions

Thermal errors are the major contributor to the dimensional in accuracies of a workpiece in precision machining. The error compensation technique is a cost-effective method to reduce thermal errors, but accurate modeling of errors is a prerequisite for error compensation. With this as an objective, a comprehensive method of rough set theory and grey correlation analysis was proposed in this paper to select the optimized temperature measurement points, and the RBF neural network was introduced to model and predict thermally induced positional error of the machine tool. The summary of conclusions from this research is as follows:

1. Rough set theory was introduced to reduce a large number of temperature measuring points, and some combination of measuring points could be obtained. Through this method, the randomness of temperature field effectively decreased.
2. The temperature-sensitive points were obtained through the grey correlation analysis of the temperature measurement points. And optimal combination was obtained by screening out the combination of

Table 5 The comparison of last 14 values of errors in the data

Measured error	−2.3	−2.2	−1.2	−1.5	3.1	3.1	4.5
Predicted error	−2.2992	−2.2003	−1.1999	−1.5000	3.1000	3.1000	3.1000
Measured error	8.8	12.9	27.0	29.0	29.1	30.1	33.6
Predicted error	8.8000	12.9000	27.0000	29.0000	29.1000	30.1000	33.6000

temperature measuring points based on rough set theory. The comprehensive method of rough set theory and grey relational analysis made the temperature selection reliable and accurate.

3. BP neural network and RBF neural network were used to predict the thermally induced positional errors, and from comparison, the prediction accuracy of RBF neural network was superior to that of traditional BP neural network. And the prediction reduced the influence of unpredictable noises.

The proposed method effectively reduced the number of temperature measuring points and improved the robustness of the error needed in compensation model. However, this research study is only a first step of the real-time thermally induced positional error compensation. The future research will focus on establishing a method to realize the real-time thermal error compensation efficiently.

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