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ORIGINAL ARTICLE

Phase characteristic between dies before rolling for thread and spline synchronous rolling process

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Abstract The thread and spline synchronous rolling process can form the external threaded and external splined tooth profiles on different parts of component by only one rolling process. However, it is a coupled rolling process where the thread rolling coupling with the spline rolling, and both phase adjusting requirements of dies before rolling for thread rolling and spline rolling should be satisfied in the synchronous rolling process. Thus, the phase adjustment only by rotating die may be difficult to ensure the thread and spline rolling by different dies well connects respectively. In this paper, requirements of phase difference adjustment for dies before thread rolling and before spline rolling have been studied systematically, and the mathematical expressions of phase difference adjustment have been established, and then the requirements of phase difference under different starts of threaded workpiece and die (or teeth of splined workpiece and die) for two and three rolling dies before thread rolling (or spline rolling) have been investigated. Based on these, the ratio S of phase difference between corresponding threaded sections of rolling dies to that between corresponding splined sections of rolling dies was introduced, and the phase characteristic between dies and the method of phase difference adjustment under different parameters in thread and spline synchronous rolling process were systematically researched. The results indicated that: (1) for thread (or spline) rolling process, if the remainder from dividing the starts n_w of thread (or teeth Z_w of spline) of workpiece by number N of rolling dies is equal to the remainder from dividing the starts n_d of thread (or teeth Z_d of spline) of rolling die by N, i.e., $n_w \equiv n_d (\mathrm{mod} N)$ (or $Z_w \equiv Z_d (\mathrm{mod} N)$), then the phases of dies before rolling are the same, else there is a phase difference between dies; (2) for thread and spline synchronous rolling process, if the phase difference of splined section between rolling die j (j=2,…, N) and rolling die 1 is $\varphi_{s_{j1}} = \frac{h}{N}\theta_s (h = 1, \dots, N)$ and ratio of phase difference is $S_{j1} = \frac{N}{h}k + 1 (k = 0, 1, 2, 3, \dots)$, then the die structures of two dies are the same, i.e., the relative phase between threaded and splined sections for two dies are the same, else the die structures of rolling dies j and 1 are different and the phase difference for threaded sections of rolling dies and the phase difference for splined sections of rolling dies are assured by die structure itself.

Keywords Thread · Spline · Cold rolling · Phase difference adjustment · Die structure

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Nomenclatures

a, b, c, d, e	Name of the cross section
h	Variable, h=1, 2, …, N
i	Ratio of relationship between workpiece and die
j	Variable, j=2, …, N
L, L'	Distance difference along axis between thread rolling dies
k, l, m, m', n, n'	Variable, k, l, m, m', n, n'=0, 1, 2, 3, …
N	Number of rolling dies
n_d	Starts of threaded section of rolling dies
n_w	Starts of threaded section of workpiece
P	Pitch of thread

P_h	Lead of threaded section of workpiece
S	Ratio of phase difference of threaded section to phase difference of splined section
Z_d	Teeth of splined section of rolling die
Z_w	Teeth of splined section of workpiece
δ, δ'	Angle difference between spline rolling dies
θ_s	Angle corresponding to one tooth
θ_t	Angle corresponding to one pitch of thread
π	Ratio of a circle's circumference to its diameter
φ_s	Phase difference between two spline rolling dies
φ_t	Phase difference between two thread rolling dies

1 Introduction

The key load-transfer and torque-transfer shaft components in steering system, transmission, planetary roller screw, etc. are often characterized by the thread and small module spline or gear. Especially in planetary roller screw, there are a lot of rollers having the threaded feature at middle section and the splined (or geared) feature at both ends between screw and nut, such as 9 or 13 declared by Velinsky et al. [1], and the relative phase between threaded and splined (or geared) sections for each roller should be kept the same. With the rapid development of the automotive industry and equipment manufacturing industry, especially under high-speed, heavy-load, and severe working conditions, the needs of high-performance and high-precision shaft parts having thread and spline (or gear) coaxially increase day by day.

Manufacture of thread, spline, and gear by cold rolling process brings many advantages such as efficient material application, short process time, increased strength of material, and so on. In general, the rolling process can be classified into rolling with flat dies and rolling with round dies [2–4]. Rolling process increases the strength of material due to work hardening. The distributions of microhardness after thread rolling with flat dies [5] and helical gear rolling with flat dies [6] were investigated. The work hardening after thread rolling with round dies and spline rolling with round dies was also studied [7]. The influence of forming parameters on the thread rolling with flat dies was studied by using finite element method (FEM) [8]. An integrated CAD/CAM/CAE system was developed to design the flat dies in taper-tipped screw rolling process [9]. The experimental research on forming parameter for spline rolling with round dies was carried out [10]. The rolling force and rolling moment in spline rolling process with round dies were investigated by using slip-line field method [11, 12]. The minimum wall thickness in hollow thread rolling with round dies was studied by using the theoretical and experimental approaches [13]. Kamouneh et al [14] declared that diagnosis of quality problems in helical gear rolling process with flat dies and proposing possible solutions to the problems can be carried out by FEM. In order to improve pitch accuracy, the pitches of flat dies vary along the length of die for gear rolling with flat dies and workpiece and dies should be forced synchronization for gear rolling with round dies [2, 15]. The rotatory condition at initial stage of spline rolling process with round dies to improve dividing-teeth accuracy was modeled by considering the frictional moment [16]. The flat rolling process was used to manufacture magnesium alloy thread by means of warm forming [17].

Rolling process has been widely used to manufacture thread, spline, and gear, and many studies for various aspects have been reported. However, up to now, manufacture of the shaft parts having thread and spline is dominated by metal cutting process, and the threaded section and splined section are cut respectively at different times, as shown in Fig. 1a. There exist some problems such as high cost, low efficiency, mechanical property degradation due to disconnected metal fiber, and relative phase instability in mass production. By using thread and spline synchronous rolling process, the threaded and splined shapes on the different portions of workpiece can be formed in one rolling process, as shown in Fig. 1b. The processing time can be reduced, and the dividing-teeth precision of spline can be increased due to thread engagement providing a driving source for rotation of workpiece, and the relative phase between thread and spline is prone to maintaining the same [18]. However, by now most researchers emphasized their works on the thread rolling process or spline/gear rolling process, and little work has been done on the thread and spline synchronous rolling process.

In the thread rolling process or spline rolling process with round dies, the phase difference adjustment is realized by

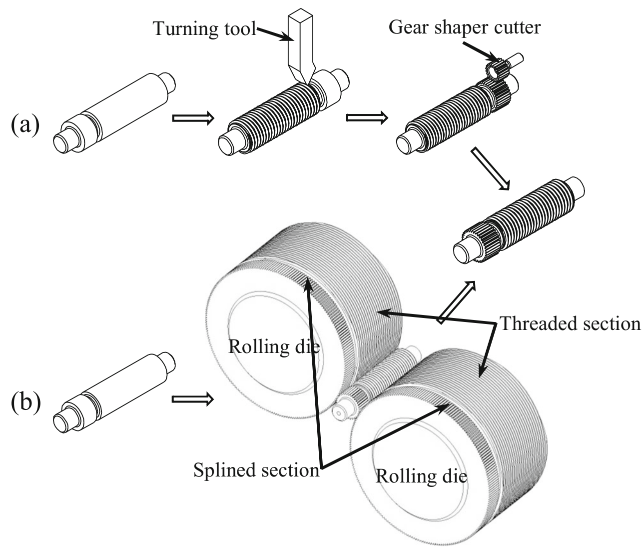

Fig. 1 Processing methods of shaft parts having thread and spline coaxially: **a** multi-stage cutting, **b** synchronous rolling

rotating rolling die due to the identical structure of rolling dies. The adjustment of rolling dies before spline rolling or thread rolling is carried out by rolling indentation in the industrial production [7, 19]. Attempting rolling is carried out to obtain the indentation (impression) on the billet, and the die is adjusted according to the connecting situation of the indentation, and then to repeat above two steps until the indentation connects well. This is based on empirical expertise, and it needs a skilled worker and a number of attempting rolling. Comparing with rolling process with two round dies, the workpiece has more balance on forces in rolling process with three round dies and the rolling process is more stable, but the phase difference adjustment before rolling with three dies is also more difficult. The number of attempting rolling is from several to dozens for thread rolling with two round dies [19], and the number will be more than twice as this number if three rolling dies are used in the rolling process. In the rolling process to form thread and spline by round dies simultaneously, as shown in Fig. 1b, both phase adjusting requirements (phase difference) of dies before rolling for thread rolling and spline rolling should be satisfied, and the phase difference adjustment only by rotating die may be difficult to ensure the thread and spline rolling by different dies well connects, respectively.

However, the literature about the theoretical analysis of phase difference for thread or spline rolling dies before rolling is scarce. The phase difference between two rolling dies for spline rolling with two round dies depends the parity of teeth of workpiece [16]. The phase difference before rolling process with two round dies for single-start thread was mentioned [19]. The general design rules for rolling die were presented according to basic requirements of thread and spline synchronous rolling with two round dies in the study [18]. But the influences of the starts of thread of die, the teeth of spline of die, and the number of rolling dies on the phase difference requirement, especially on the die structure in thread and spline synchronous rolling process, were not considered. It is necessary to investigate phase difference adjustment before rolling and the die structure for thread and spline synchronous rolling process under different parameters such as number N of rolling dies, starts n_w of thread of workpiece/part, teeth Z_w of spline of workpiece/part, starts n_d of thread of rolling die, teeth Z_d of spline of rolling die.

In this study, the phase difference under different N, n_w, and n_d before thread rolling has been determined according to the requirement of phase difference and the number of dies in the thread rolling process, and the similar research has also been done for spline rolling process where the parameters are N, Z_w and Z_d. Based on these and the synchronous rolling characteristic, the requirement of phase difference before rolling to ensure the thread and spline rolling by different dies well connects in thread and spline synchronous rolling process is investigated, and the die structure under different n_w, n_d, Z_w, and Z_d to ensure the thread and spline rolling by different dies

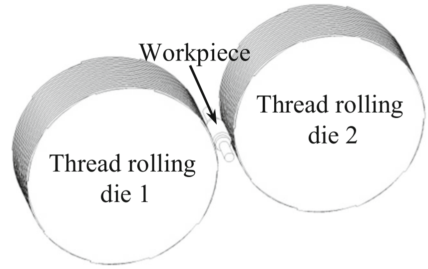

Fig. 2 Sketch of thread rolling process

well connects has been analyzed systematically. The results in the present study may provide a basis for further study on thread and spline synchronous rolling process, such as the die design and phase difference adjustment in industrial experiment. By using the results in the present study, the number of attempting rolling for thread rolling process or spline rolling process will be greatly reduced and the dependence on the empirical expertise will also be reduced.

2 Descriptions of thread and spline rolling process

The thread rolling process and spline rolling process is a non-chip plastic forming process based on the principle of cross rolling, as shown in Figs. 2 and 3. Two rolling dies are used in Figs. 2 and 3, and more than two rolling dies such as three rolling dies can also be used in the process. In the present study, the axis of rolling die is parallel to the axis of workpiece.

The process of thread rolling is similar to the process of spline rolling, and motions of rolling dies for two processes are the same. Taking the process with two rolling die as example, as shown Figs. 2 and 3, the rolling process can be presented as follows: two rolling dies with the same special shape (threaded feature or splined feature) synchronously rotate in the same direction, and the workpiece rotates in the opposite direction, and one rolling die or two rolling dies infeed under constant speed or constant force,[1] the rolling dies continuously apply force to workpiece until the corresponding shape (thread or spline) has been formed on the workpiece. For thread rolling dies, the form of thread and the starts of thread are the same. The thread rolling die is multiple-start thread, and the pitch of die is the same as the pitch of workpiece although the leads are different. For spline rolling dies, the profile of tooth and the number of teeth are the same.

The thread and spline synchronous rolling process is developed based on thread rolling process and spline rolling process, as shown in Fig. 1b, and it is also a forming process based on the principle of cross rolling. The forming process is similar to the thread rolling process or spline rolling process,

[1] In general, three dies synchronously feed-in in radial direction in the process with three rolling dies.

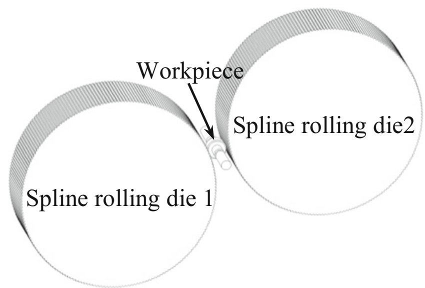

Fig. 3 Sketch of spline rolling process

but the dies are different from those, where the die consists of threaded section and splined section. At the same time, the rolling die can satisfy the demand of motion compatibility for thread rolling and spline rolling, and satisfy the demand of phase difference adjustment before rolling. The rolling dies continuously apply force to workpiece, and the threaded and splined sections on the different portions of workpiece are formed synchronously.

3 Requirement of phase difference for dies before thread rolling and spline rolling

The dies used in the thread rolling process have the same parameters. In order to make the threaded features rolling by different dies connect well, there exists a phase difference between dies before rolling. That means that the threaded features (contacting with workpiece) of dies will be staggered a distance L along axis on the plane[2] to ensure the threaded impression rolled by one die on the workpiece can mesh with the next rolling die. If the impression cannot mesh with the next rolling die, then the thread on the workpiece will not connect. According to the motion characteristic in the thread rolling process, the staggered distance L can be expressed as follows:

$$L = \frac{P_h}{N} = \frac{n_w P}{N} \quad (1)$$

where P_h is lead of thread of workpiece, N is number of thread rolling dies, n_w is starts of thread of workpiece, and P is pitch of thread of workpiece.

The distance difference L between rolling dies can be realized by the phase difference between rolling dies, and the rotated angle θ_t corresponding to one pitch of thread can be expressed as follows:

$$\theta_t = \frac{2\pi}{n_d} \quad (2)$$

[2] The plane is determined by axis of workpiece and axis (axes) of rolling die(s).

where n_d is starts of thread of die.

Because the multiple-start thread has a periodic symmetry, the structure of die is the same as that after the die rotates θ_t. Thus, the Eq. (1) can be simplified according to this characteristic and the number of rolling dies.

The dies used in the spline rolling process also have the same parameters. In order to make the splined features rolling by different dies connect well, there exists a phase difference between dies before rolling. That means that the tooth crest or tooth space (contacting with workpiece) of dies will be staggered an angle δ on the plane[3] to ensure the tooth rolled by one die on the workpiece can mesh with the next rolling die. If the rolled tooth cannot mesh with the next rolling die, then the incorrect tooth will happen. According to the motion characteristic in the spline rolling process, the staggered angle δ can be expressed as follows:

$$\delta = \frac{Z_w}{N}\theta_s \quad (3)$$

where Z_w is teeth of spline of workpiece, N is number of spline rolling dies, and θ_s is angle corresponding to one tooth of die, which is determined by Eq. (4).

$$\theta_s = \frac{2\pi}{Z_d} \quad (4)$$

where Z_d is teeth of spline of rolling die.

Because the multiple-tooth spline or gear has a periodic symmetry, the structure of die is the same as that after the die rotates θ_s. Thus, the Eq. (3) can be simplified according to this characteristic and the number of rolling dies.

4 Phase characteristic between dies in thread and spline synchronous rolling process with two rolling dies

4.1 Requirement of phase difference between dies before thread rolling with two dies

When two rolling dies (N=2) are used in the thread rolling process, the staggered distance L for rolling dies before rolling can be simplified into two cases according to the remainder from dividing n_w by 2.

If the remainder is 1, i.e., n_w=2m+1 (m=0,1,2,3,……), then the Eq. (1) can be simplified as the Eq. (5).

$$L = \frac{P}{2} \quad (5)$$

Then, the threaded features between two rolling dies contacting with workpiece/billet should be staggered half of

[3] The plane is perpendicular to axes of rolling dies.

pitch on the horizontal plane (section shown in Fig. 4a) determined by axes of two rolling dies.

If the remainder is 0 or 2, i.e., n_w=2m+2(m=0,1,2,3,… …), then the Eq. (1) can be simplified as the Eq. (6).

$$L = 0 \ \text{or} \ L = P \tag{6}$$

Then, the threaded features contacting with workpiece/billet should be staggered zero or one pitch on the horizontal plane determined by axes of two rolling dies.

The phase difference between two thread rolling dies is φ_t. This means that one rolling die (such as thread rolling die 1) is fixed, and the other rolling die (such as thread rolling die 2) rotates by φ_t.

If the phase difference φ_t is equal to half of θ_t, i.e.:

$$\varphi_t = \frac{\pi}{n_d} \tag{7}$$

then the staggered distance between two rolling dies on the horizontal plane will be increased half of pitch.

If the phase difference φ_t is equal to zero or integral multiple (one time in Eq. (8)) of θ_t, i.e.:

$$\varphi_t = 0 \ \text{or} \ \varphi_t = \frac{2\pi}{n_d} \tag{8}$$

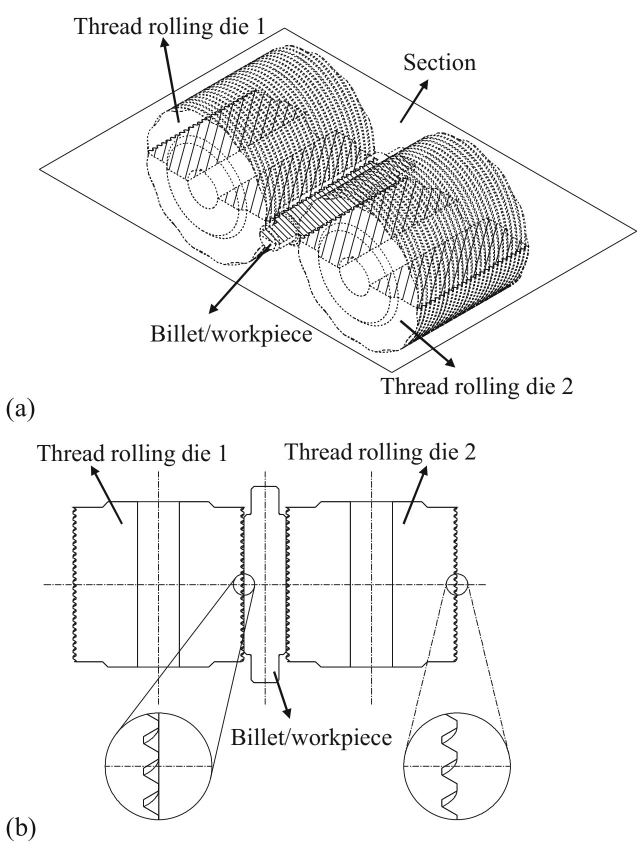

Fig. 4 Contact state between billet and dies in thread rolling process using two dies: **a** trimetric view, **b** view on the section

then the staggered distance between two rolling dies on the horizontal plane will be increased zero or one pitch, i.e., the original state will be kept.

As shown in Fig. 4b, if the phase difference φ_t between thread rolling dies 1 and 2 is equal to zero or integral multiple of θ_t, then the threaded features on the side of thread rolling die 1 contacting with billet/workpiece is identical to the threaded features on the opposite side of the side of thread rolling die 2 contacting with billet/workpiece. Thus, in this case, the threaded features between two rolling dies contacting with workpiece/billet will be staggered a distance L' expressed by Eq. (9).

$$L' = \frac{n_d P}{2} \tag{9}$$

Thus, in the case that the phase difference between two thread rolling dies is not adjusted (i.e., φ_t=0), if n_d=2m'+1 (m'=0, 1, 2, 3, ……), then the threaded features between two rolling dies will be staggered half of pitch by itself on the horizontal plane; if n_d=2m'+2 (m'=0, 1, 2, 3, ……), then the threaded features between two rolling dies will be staggered zero on the horizontal plane.

Considering the Eqs. (5) to (9), phase adjusting requirement of dies before thread rolling depends on the values of starts of thread of the workpiece and rolling die. The requirements of phase difference under different starts of thread of workpiece and die are listed in Table 1 according to the Eqs. (5) to (9), where the $\varphi_{t_{21}}$ is adjusted/rotated angle of thread rolling die 2 relative to the rolling die 1.

It can be found from Table 1 that if remainder from dividing the n_w by 2 is the same as the remainder from dividing the n_d by 2, then the phases of rolling dies are also the same, else there is a phase difference between rolling dies before rolling. In order to improve service life of die, it should make diameter of thread rolling die maximize according to the structure of machine. The starts of thread of die may be odd and may be even in the practical production.

4.2 Requirement of phase difference between dies before spline rolling with two dies

When two rolling dies (N=2) are used in the spline rolling process, the staggered angle δ for tooth crest or tooth space of dies before rolling can be simplified into two cases according to the remainder from dividing Z_w by 2.

If the remainder is 1, i.e., Z_w=2n+1 (n=0,1,2,3,……), then the Eq. (3) can be simplified as the Eq. (10).

$$\delta = \frac{\theta_s}{2} \tag{10}$$

Then, the tooth crests or tooth spaces between two rolling dies contacting with workpiece/billet should be staggered half

Table 1 Requirement of phase difference adjustment before thread rolling with two dies

No.	n_w	n_d	L	L'	$\varphi_{t_{21}}$
1	$2m+1$	$2m'+1$	$L=\frac{P}{2}$	$L'=\frac{P}{2}$	$\varphi_{t_{21}}=0$ or $\varphi_{t_{21}}=\frac{2\pi}{n_d}$
2	$2m+1$	$2m'+2$	$L=\frac{P}{2}$	$L'=0$	$\varphi_{t_{21}}=\frac{\pi}{n_d}$
3	$2m+2$	$2m'+1$	$L=P$ or $L=0$	$L'=\frac{P}{2}$	$\varphi_{t_{21}}=\frac{\pi}{n_d}$
4	$2m+2$	$2m'+2$	$L=P$ or $L=0$	$L'=0$	$\varphi_{t_{21}}=0$ or $\varphi_{t_{21}}=\frac{2\pi}{n_d}$

of θ_s on the plane (section shown in Fig. 5a) perpendicular to axes of rolling dies, where the tooth crest of one spline rolling die faces the tooth space of the other rolling die.

If the remainder is 0 or 2, i.e., $Z_w=2n+2$ (n=0, 1, 2, 3, ……), then the Eq. (3) can be simplified as the Eq. (11).

$$\delta=0 \quad \text{or} \quad \delta=\theta_s \tag{11}$$

Then, the tooth crests or tooth spaces between two rolling dies contacting with workpiece/billet should be staggered zero or θ_s on the plane perpendicular to axes of rolling dies, where the tooth crest of one spline rolling die faces the tooth crest of the other rolling die or the tooth space of one die faces the tooth space of the other die.

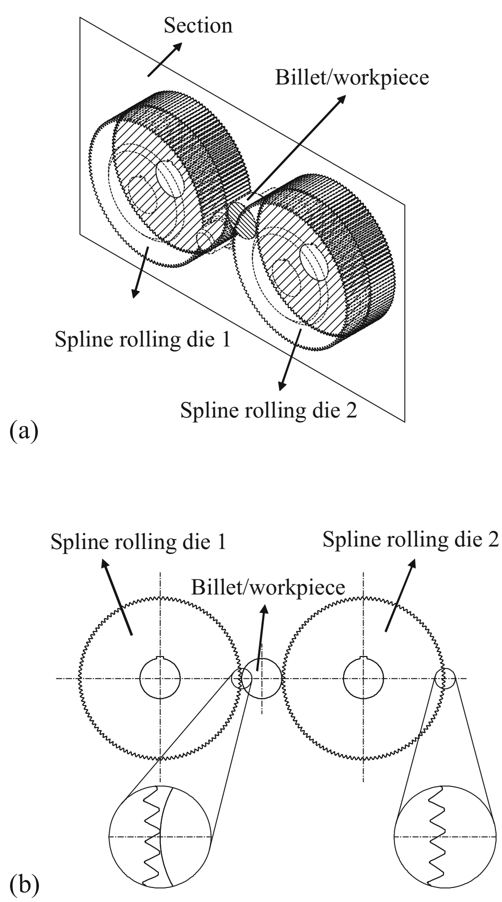

Fig. 5 Contact state between billet and dies in spline rolling process using two dies: **a** trimetric view, **b** view on the section.

The phase difference between two spline rolling dies is φ_s. This means that one rolling die (such as spline rolling die 1) is fixed, and the other rolling die (such as spline rolling die 2) rotates by φ_s.

If the phase difference φ_s is equal to half of θ_s, i.e.:

$$\varphi_s=\frac{\pi}{Z_d} \tag{12}$$

then the facing state of tooth crests or tooth spaces between two rolling dies will be changed. The state of the tooth crest facing tooth space will be changed to the state of tooth crest facing tooth crest (or tooth space facing tooth space), or the state of tooth crest facing tooth crest (or tooth space facing tooth space) will be changed to the state of tooth crest facing tooth space.

If the phase difference φ_s is equal to zero or integral multiple (one time in Eq. (13)) of θ_s, i.e.:

$$\varphi_s=0 \quad \text{or} \quad \varphi_s=\frac{2\pi}{Z_d} \tag{13}$$

then the facing state of tooth crests (or tooth spaces) between two rolling dies will be kept.

As shown in Fig. 5b, if the phase difference φ_s between spline rolling dies 1 and 2 is equal to zero or integral multiple of θ_s, then the splined features on the side of spline rolling die 1 contacting with billet/workpiece is identical to the splined features on the opposite side of the side of spline rolling die 2 contacting with billet/workpiece. Thus, in this case, the tooth crests or tooth spaces between two rolling dies contacting with workpiece/billet will be staggered an angle δ' expressed by Eq. (14).

$$\delta'=\frac{Z_d}{2}\theta_s \tag{14}$$

Thus, in the case that the phase difference between two spline rolling dies is not adjusted (i.e., φ_s=0), if $Z_d=2n'+1$ (n'=0, 1, 2, 3, ……), then the tooth crests or tooth spaces between two rolling dies will be staggered half of θ_s by itself on the plane perpendicular to axes of rolling dies, where the tooth crest of one die faces the tooth space of the other die; if $Z_d=2n'+2$ (n'=0, 1, 2, 3, ……), then the tooth crests or tooth spaces between two rolling dies will be staggered zero, where the tooth crest of one die faces the tooth crest of the other die or the tooth space of one die faces the tooth space of the other die.

Considering Eqs. (10) to (14), phase adjusting requirement of dies before spline rolling depends on the values of tooth of spline of the workpiece and rolling die. The requirements of phase difference under different teeth of spline of the workpiece and die are listed in Table 2 according to the Eqs. (10) to (14), where the $\varphi_{s_{21}}$ is adjusted/rotated angle of spline rolling die 2 relative to the rolling die 1.

It can be found from Table 2 that if remainder from dividing the Z_w by 2 is the same as the remainder from dividing the Z_d by 2, then the phases of rolling dies are also the same, else there is a phase difference between rolling dies before spline rolling. In order to improve service life of die, it should make teeth of spline of spline rolling die maximize according to the structure of machine. Generally, the Z_d is an even in the practical production in order to manufacture and inspect the rolling die easily [20].

4.3 Phase characteristic between dies before thread and spline synchronous rolling with two rolling dies

The rolling dies are identical in the thread rolling process or spline rolling process. The adjustment of phase difference between two dies is realized by rotating the one of two dies. However, the thread rolling motion couples with the spline rolling motion in the thread and spline synchronous rolling process. The rolling die has threaded section and splined section, which are working at the same time, and thus both phase adjusting requirements of dies before rolling for thread rolling and spline rolling should be satisfied in the synchronous rolling process.

The thread meshing motion can effectively be treated as the helical gear meshing motion [21]. Thus, the basic requirement for motion compatibility in the thread and spline synchronous rolling process is that the Eq. (15) should be met [18].

$$\frac{Z_d}{Z_w} = \frac{n_d}{n_w} = i \tag{15}$$

where Z_d is teeth of splined section of rolling die; Z_w is teeth of splined section of workpiece; n_d is starts of threaded section of rolling die; n_w is starts of threaded section of workpiece; i represents the relationship between die and workpiece, whose value is ratio of teeth of splined section of die to that of workpiece or ratio of starts of threaded section of die to that of workpiece.

In general, the starts (n_d) of threaded section of die is less than the teeth (Z_d) of splined section of die, and thus the Eq. (16) can be obtained.

$$\varphi_{t_{j1}} > \varphi_{s_{j1}} (j = 2) \tag{16}$$

where $\varphi_{t_{j1}}$ and $\varphi_{s_{j1}}$ are nonzero value, i.e., $\varphi_{t_{21}}$ and $\varphi_{s_{21}}$ are chosen from Table 1 and Table 2 are nonzero value.

Letting

$$S_{j1} = \frac{\varphi_{t_{j1}}}{\varphi_{s_{j1}}} (j = 2) \tag{17}$$

where $\varphi_{t_{j1}}$ and $\varphi_{s_{j1}}$ are both nonzero value.

By the analyses in Sections 4.1 and 4.2, the $\varphi_{s_{21}}$ and $\varphi_{s_{21}}$ are closely related to the starts of thread of the workpiece and die and the teeth of spline of workpiece and die. However, the n_d and Z_d are determined by the n_w, Z_w, and i. Thus, considering the Eqs. (5) to (14), the expression of S_{21} can be obtained according to the n_w, Z_w, and i. The results are listed in Table 3, where the rotating direction of $\varphi_{t_{21}}$ is the same as that of $\varphi_{s_{21}}$.

Eight cases of phase adjustment would present in thread and spline synchronous rolling process are listed in Table 3. Considering the service life of die, it should also make i maximize according to the structure of machine, but an even number is also chosen for the Z_d of splined section of rolling die, and thus the first and third cases in Table 3 are not recommended. For different $\varphi_{s_{j1}}$ and S_{j1}, the phase adjusting requirements are different, and then the die structure and adjusting method are also different. According to value of $\varphi_{s_{j1}}$ (i.e., $\varphi_{s_{21}}$), two categories as following can be summarized, where k=0,1,2,3,….

1. Category 1 $\varphi_{s_{j1}} = \frac{\pi}{Z_d} = \frac{\theta_s}{2}$: if S_{j1}=2k+1, then relative phase between thread and spline of rolling die j is the same as that of rolling die 1, and the phase difference between two die is adjusted by means of rotating rolling die j by $\varphi_{t_{j1}}$; else the relative phase between thread and spline of rolling die j is different from that of rolling die 1, and the die structure should be satisfied the phase difference adjustments (i.e., $\varphi_{t_{j1}}$ and $\varphi_{s_{j1}}$) for threaded section and splined section respectively.

Table 2 Requirement of phase difference adjustment before spline rolling with two dies

No.	Z_w	Z_d	δ	δ'	$\varphi_{s_{21}}$
1	2n+1	2n'+1	$\delta = \frac{\theta_s}{2}$	$\delta' = \frac{\theta_s}{2}$	$\varphi_{s_{21}} = 0$ or $\varphi_{s_{21}} = \frac{2\pi}{Z_d}$
2	2n+1	2n'+2	$\delta = \frac{\theta_s}{2}$	δ'=0	$\varphi_{s_{21}} = \frac{\pi}{Z_d}$
3	2n+2	2n'+1	δ=θ_s or δ=0	$\delta' = \frac{\theta_s}{2}$	$\varphi_{s_{21}} = \frac{\pi}{Z_d}$
4	2n+2	2n'+2	δ=θ_s or δ=0	δ'=0	$\varphi_{s_{21}} = 0$ or $\varphi_{s_{21}} = \frac{2\pi}{Z_d}$

Table 3 Phase adjusting requirement for different sections of die in the synchronous rolling with two dies

No.	Z_w	n_w	i	Z_d	n_d	$\varphi_{s_{21}}$	$\varphi_{t_{21}}$	S_{21}
1	$2n+1$	$2m+1$	$2l+1$	$(2n+1)(2l+1)$	$(2m+1)(2l+1)$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{n_d}$	$\frac{Z_d}{n_d}$
2	$2n+1$	$2m+1$	$2l+2$	$(2n+1)(2l+2)$	$(2m+1)(2l+2)$	$\frac{\pi}{Z_d}$	$\frac{\pi}{n_d}$	$\frac{Z_d}{n_d}$
3	$2n+1$	$2m+2$	$2l+1$	$(2n+1)(2l+1)$	$(2m+2)(2l+1)$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{n_d}$	$\frac{Z_d}{n_d}$
4	$2n+1$	$2m+2$	$2l+2$	$(2n+1)(2l+2)$	$(2m+2)(2l+2)$	$\frac{\pi}{Z_d}$	$\frac{2\pi}{n_d}$	$\frac{2Z_d}{n_d}$
5	$2n+2$	$2m+1$	$2l+1$	$(2n+2)(2l+1)$	$(2m+1)(2l+1)$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{n_d}$	$\frac{Z_d}{n_d}$
6	$2n+2$	$2m+1$	$2l+2$	$(2n+2)(2l+2)$	$(2m+1)(2l+2)$	$\frac{2\pi}{Z_d}$	$\frac{\pi}{n_d}$	$\frac{Z_d}{2n_d}$
7	$2n+2$	$2m+2$	$2l+1$	$(2n+2)(2l+1)$	$(2m+2)(2l+1)$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{n_d}$	$\frac{Z_d}{n_d}$
8	$2n+2$	$2m+2$	$2l+2$	$(2n+2)(2l+2)$	$(2m+2)(2l+2)$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{n_d}$	$\frac{Z_d}{n_d}$

2. Category 2 $\varphi_{s_{j1}} = \frac{2\pi}{Z_d} = \theta_s$: if $S_{j1}=k(k\neq 0)$, then relative phase between thread and spline of rolling die j is the same as that of rolling die 1, and the phase difference between two rolling dies is adjusted by means of rotating rolling die j by $\varphi_{t_{j1}}$, especially only rotating rolling die j by zero degree when $\varphi_{t_{j1}} = \theta_t$; else the relative phase between thread and spline of rolling die j is different from that of rolling die 1, and the die structure should be satisfied the phase difference adjustments (i.e., $\varphi_{t_{j1}}$ and $\varphi_{s_{j1}}$) for threaded section and splined section respectively.

When the relative phase between thread and spline of one die is the same as that of the other, phase adjustment before rolling is realized by rotating one die by φ_t (i.e., $\varphi_{t_{21}}$). Especially, if the φ_t (i.e., $\varphi_{t_{21}}$) and φ_s (i.e., $\varphi_{s_{21}}$) are both equal to integral multiple of corresponding periodic angle such as θ_t and θ_s respectively, i.e., $\varphi_{s_{21}} = \frac{2\pi}{Z_d} = \theta_s$ and $\varphi_{t_{21}} = \frac{2\pi}{n_d} = \theta_t$, then the die rotated by zero can make the threaded and splined impressions on the workpiece rolled by one die well connect with that rolled by the next rolling die. When relative phases between thread and spline for two rolling dies are different, the phase difference for threaded sections of rolling dies and the phase difference for splined sections of rolling dies are assured by die structure itself.

5 Phase characteristic between dies in thread and spline synchronous rolling process with three rolling dies

5.1 Requirement of phase difference between dies before thread rolling with three dies

When three rolling dies (N=3) are used in the thread rolling process, the staggered distance L for rolling dies before rolling can be simplified into three cases according to the remainder from dividing n_w by 3.

If the remainder is 1, i.e., $n_w=3m+1(m=0,1,2,3,\ldots\ldots)$, then the Eq. (1) can be simplified as the Eq. (18).

$$L = \frac{P}{3} \tag{18}$$

Then, the threaded features between rolling dies contacting with workpiece/billet should be successively staggered one third of pitch on the sections (section *a*–*b*, section *b*–*c*, and section *a*–*c* shown in Fig. 6a) determined by axes of billet and dies.

If the remainder is 2, i.e., $n_w=3m+2(m=0,1,2,3,\ldots\ldots)$, then the Eq. (1) can be simplified as the Eq. (19).

$$L = \frac{2P}{3} \tag{19}$$

Then, the threaded features between rolling dies contacting with workpiece/billet should be successively staggered two thirds of pitch on the sections determined by axes of billet and dies.

If the remainder is 0 or 3, i.e., $n_w=3m+3(m=0,1,2,3,\ldots\ldots)$, then the Eq. (1) can be simplified as the Eq. (20).

$$L = 0 \text{ or } L = P \tag{20}$$

Then, the threaded features between rolling dies contacting with workpiece/billet should be staggered zero or one pitch on the sections determined by axes of billet and dies.

The phase difference between three thread rolling dies is φ_t, i.e., the rolling dies have a phase difference φ_t on the section *a*–*b*, section *b*–*c*, and section *a*–*c* successively. This means that first rolling die (such as thread rolling die 1) is fixed, and the second rolling die (such as thread rolling die 2) rotates by φ_t, and the third rolling die (such as thread rolling die 3) rotates by $2\varphi_t$, where the second and third rolling dies rotate in the same direction.

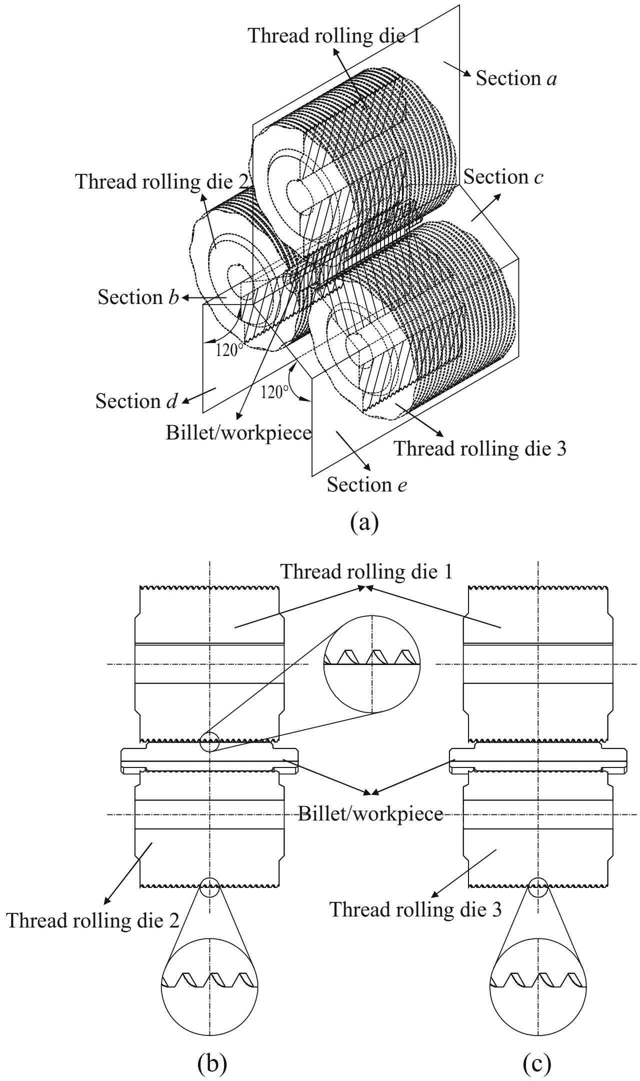

Fig. 6 Contact state between billet and dies in thread rolling process using three dies: **a** trimetric view, **b** view on section *a–b–d*, **c** view on section *a–c–e*

If the phase difference φ_t is equal to one third of θ_t, i.e.:

$$\varphi_t = \frac{2\pi}{3n_d} \tag{21}$$

then the successively staggered distance between rolling dies on the section *a–b*, section *b–c*, and section *a–c* will be increased $P/3$.

If the phase difference φ_t is equal to two thirds of θ_t, i.e.:

$$\varphi_t = \frac{4\pi}{3n_d} \tag{22}$$

then the successively staggered distance between rolling dies on the section *a–b*, section *b–c*, and section *a–c* will be increased $2P/3$.

If the phase difference φ_t is equal to zero or integral multiple (one time in Eq. (23)) of θ_t, i.e.:

$$\varphi_t = 0 \text{ or } \varphi_t = \frac{2\pi}{n_d} \tag{23}$$

then the successively staggered distance between rolling dies on the section *a–b*, section *b–c*, and section *a–c* will be increased zero or one pitch, i.e., the original state will be kept.

As shown in Fig. 6b and c, if the phase difference φ_t between thread rolling dies 1, 2, and 3 is equal to zero or integral multiple of θ_t, then the threaded features on the side of thread rolling die 1 contacting with billet/workpiece is identical to the threaded features of the side of rolling die 2 on the section *d* as shown in Fig. 6b,where the rotation angle (clockwise) form section *b* to section *d* is 120° and the side of thread rolling die 2 contacts with billet/workpiece on the section *b*; and then the threaded features on the side of thread rolling die 1 contacting with billet/workpiece is also identical to the threaded features of the side of rolling die 3 on the section *e* as shown in Fig. 6c, where the rotation angle (anticlockwise) form section *c* to section *e* is 120° and the side of thread rolling die 3 contacts with billet/workpiece on the section *c*. Thus, in this case, the threaded features between rolling dies contacting with workpiece/billet will be successively staggered a distance L' on the sections (section *a-b*, section *b-c*, and section *a-c* shown in Fig. 6a) determined by axes of billet and dies, and the distance L' can be expressed as follows:

$$L' = \frac{n_d P}{3} \tag{24}$$

Thus, in the case that the phase difference between thread rolling dies 1, 2, and 3 is not adjusted (i.e., $\varphi_t=0$), if $n_d=3m'+1(m'=0,1,2,3,\ldots\ldots)$, then the threaded features between rolling dies will be staggered one third of pitch by itself on the sections *a, b,* and *c*; if $n_d=3m'+2(m'=0,1,2,3,\ldots\ldots)$, then the threaded features between rolling dies will be staggered two thirds of pitch by itself on the sections *a, b,* and *c*; if $n_d=3m'+3(m'=0,1,2,3,\ldots\ldots)$, then the threaded features between rolling dies will be staggered zero on the sections.

Considering the Eqs. (18) to (24), the phase adjusting requirement of dies before thread rolling with three round dies depends on the values of starts of thread of the workpiece and rolling die. The requirements of phase difference under different starts of thread of workpiece and die are listed in Table 4 according to the Eqs. (18) to (24), where the $\varphi_{t_{21}}$ is the adjusted/rotated angle of thread rolling die 2 relative to the rolling die 1 and the $\varphi_{t_{31}}$ is the adjusted/rotated angle of thread rolling die 3 relative to the rolling die 1. If the direction of rotation of rolling die is the same as the direction of numbering sequence for rolling dies, then adjusted/rotated angle in Table 4 is positive, else is negative. For example, direction of numbering sequence for rolling dies in Fig. 6a is anticlockwise, and then adjusted/rotated angle is positive when its direction is anticlockwise and is negative when its direction is clockwise.

Table 4 Requirement of phase difference adjustment before thread rolling with three dies

No.	n_w	n_d	L	L'	$\varphi_{t_{21}}$	$\varphi_{t_{31}}$
1	$3m+1$	$3m'+1$	$L=\frac{P}{3}$	$L'=\frac{P}{3}$	$\varphi_{t_{21}}=0$ or $\varphi_{t_{21}}=\frac{2\pi}{n_d}$	$\varphi_{t_{31}}=0$ or $\varphi_{t_{31}}=\frac{4\pi}{n_d}$
2	$3m+2$	$3m'+1$	$L=\frac{2P}{3}$	$L'=\frac{P}{3}$	$\varphi_{t_{21}}=\frac{2\pi}{3n_d}$	$\varphi_{t_{31}}=\frac{4\pi}{3n_d}$
3	$3m+3$	$3m'+1$	$L=P$ or $L=0$	$L'=\frac{P}{3}$	$\varphi_{t_{21}}=\frac{4\pi}{3n_d}\varphi_{t_{21}}$ or $=-\frac{2\pi}{3n_d}$	$\varphi_{t_{31}}=\frac{8\pi}{3n_d}$ or $\varphi_{t_{31}}=-\frac{4\pi}{3n_d}$
4	$3m+1$	$3m'+2$	$L=\frac{P}{3}$	$L'=\frac{2P}{3}$	$\varphi_{t_{21}}=-\frac{2\pi}{3n_d}$ *or* $\varphi_{t_{21}}=\frac{4\pi}{3n_d}$	$\varphi_{t_{31}}=-\frac{4\pi}{3n_d}$ or $\varphi_{t_{31}}=\frac{8\pi}{3n_d}$
5	$3m+2$	$3m'+2$	$L=\frac{2P}{3}$	$L'=\frac{2P}{3}$	$\varphi_{t_{21}}=0$ or $\varphi_{t_{21}}=\frac{2\pi}{n_d}$	$\varphi_{t_{31}}=0$ or $\varphi_{t_{31}}=\frac{4\pi}{n_d}$
6	$3m+3$	$3m'+2$	$L=P$ or $L=0$	$L'=\frac{2P}{3}$	$\varphi_{t_{21}}=\frac{2\pi}{3n_d}$ or $\varphi_{t_{21}}=-\frac{4\pi}{3n_d}$	$\varphi_{t_{31}}=\frac{4\pi}{3n_d}$ or $\varphi_{t_{31}}=-\frac{8\pi}{3n_d}$
7	$3m+1$	$3m'+3$	$L=\frac{P}{3}$	$L'=0$	$\varphi_{t_{21}}=\frac{2\pi}{3n_d}$	$\varphi_{t_{31}}=\frac{4\pi}{3n_d}$
8	$3m+2$	$3m'+3$	$L=\frac{2P}{3}$	$L'=0$	$\varphi_{t_{21}}=\frac{4\pi}{3n_d}$	$\varphi_{t_{31}}=\frac{8\pi}{3n_d}$
9	$3m+3$	$3m'+3$	$L=P$ or $L=0$	$L'=0$	$\varphi_{t_{21}}=0$ *or* $\varphi_{t_{21}}=\frac{2\pi}{n_d}$	$\varphi_{t_{31}}=0$ or $\varphi_{t_{31}}=\frac{4\pi}{n_d}$

It can be found from Table 4 that if remainder from dividing the n_w by 3 is the same as the remainder from dividing the n_d by 3, then the phases of rolling dies are also the same, else there is a phase difference between rolling dies before rolling.

5.2 Requirement of phase difference between dies before spline rolling with three dies

When three rolling dies (N=3) are used in the spline rolling process, the staggered angle δ for tooth crest or tooth space of dies before rolling can be simplified into three cases according to the remainder from dividing Z_w by 3.

If the remainder is 1, i.e., $Z_w=3n+1$ (n=0,1,2,3,……), then the Eq. (3) can be simplified as the Eq. (25).

$$\delta=\frac{\theta_s}{3} \tag{25}$$

Then, the tooth crests or tooth spaces between rolling dies contacting with workpiece/billet should be successively staggered one third of θ_s on the plane (section shown in Fig. 7a) perpendicular to axes of rolling dies.

If the remainder is 2, i.e., $Z_w=3n+2$(n=0,1,2,3,……) , then the Eq. (3) can be simplified as the Eq. (26).

$$\delta=\frac{2\theta_s}{3} \tag{26}$$

Then, the tooth crests or tooth spaces between rolling dies contacting with workpiece/billet should be successively staggered two thirds of θ_s on the plane perpendicular to axes of rolling dies.

If the remainder is 0 or 3, i.e., $Z_w=3n+3$(n=0,1,2,3,……), then the Eq. (3) can be simplified as the Eq. (27).

$$\delta=0 \ \text{ or } \ \delta=\theta_s \tag{27}$$

Then, the tooth crests or tooth spaces between rolling dies contacting with workpiece/billet should be successively staggered zero or θ_s on the plane perpendicular to axes of rolling dies.

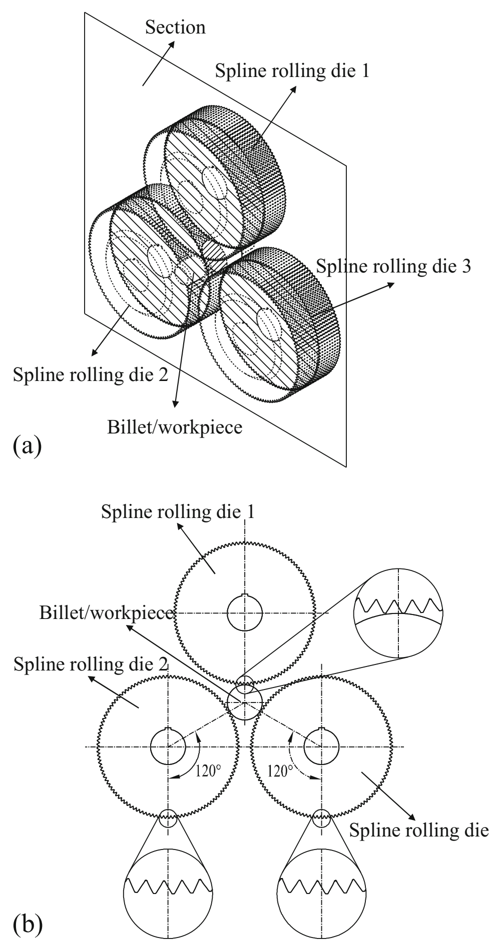

Fig. 7 Contact state between billet and dies in spline rolling process using three dies: **a** trimetric view, **b** view on the section

The phase difference between three spline rolling dies is φ_s, i.e., the rolling dies have a phase difference φ_s on the plane perpendicular to axes of rolling dies successively. This means that first rolling die (such as spline rolling die 1) is fixed, and the second rolling die (such as spline rolling die 2) rotates by φ_s, and the third rolling die (such as spline rolling die 3) rotates by $2\varphi_s$, where the second and third rolling dies rotate in the same direction.

If the phase difference φ_s is equal to one third of θ_s, i.e.:

$$\varphi_s = \frac{2\pi}{3Z_d} \tag{28}$$

then the facing state of tooth crests or tooth spaces between rolling dies will be changed, and then the successively staggered angle between rolling dies on the plane perpendicular to axes of rolling dies will be increased $\theta_s/3$.

If the phase difference φ_s is equal to two thirds of θ_s, i.e.:

$$\varphi_s = \frac{4\pi}{3Z_d} \tag{29}$$

then the facing state of tooth crests or tooth spaces between rolling dies will be changed, and then the successively staggered angle between rolling dies on the plane perpendicular to axes of rolling dies will be increased $2\theta_s/3$.

If the phase difference φ_s is equal to zero or integral multiple (one time in Eq. (13)) of θ_s, i.e.:

$$\varphi_s = 0 \ \text{or} \ \varphi_s = \frac{2\pi}{Z_d} \tag{13}$$

then the facing state of tooth crests or tooth spaces between rolling dies will be kept.

As shown in Fig. 7b, if the phase difference φ_s between spline rolling dies 1, 2, and 3 is equal to zero or integral multiple of θ_s, then the splined features on the side of spline rolling die 1 contacting with billet/workpiece is identical to the splined features on side enlarging in Fig. 7b of spline rolling die 2, where the rotation angle (anticlockwise) form the side to the side contacting with billet is 120° shown in Fig. 7b; and then the splined features on the side of spline rolling die 1 contacting with billet/workpiece is identical to the splined features on side enlarging in Fig. 7b of spline rolling die 3, where the rotation angle (clockwise) form the side to the side contacting with billet is 120° shown in Fig. 7b. Thus, in this case, the tooth crests or tooth spaces between rolling dies contacting with workpiece/billet will be successively staggered an angle δ', and the angle δ' can be expressed as follows:

$$\delta' = \frac{Z_d}{3}\theta_s \tag{31}$$

Thus, in the case that the phase difference between spline rolling dies 1, 2, and 3 is not adjusted (i.e., $\varphi_s=0$), if $Z_d=3n'+1(n'=0,1,2,3,\ldots\ldots)$, then the tooth crests or tooth spaces between rolling dies will be staggered one third of θ_s by itself on the plane perpendicular to axes of rolling dies; if $Z_d=3n'+2(n'=0,1,2,3,\ldots\ldots)$, then the tooth crests or tooth spaces between rolling dies will be staggered two thirds of θ_s by itself on the plane perpendicular to axes of rolling dies; if $Z_d=3n'+3(n'=0,1,2,3,\ldots\ldots)$, then the tooth crests or tooth spaces between two rolling dies will be staggered zero.

Considering the Eqs. (25) to (31), phase adjusting requirement of dies before spline rolling with three rolling dies depends on the values of teeth of spline of the workpiece and rolling die. The requirements of phase difference under different teeth of spline of the workpiece and die are listed in Table 5 according to the Eqs. (25) to (31), where the $\varphi_{s_{21}}$ is adjusted/rotated angle of spline rolling die 2 relative to the rolling die 1 and the $\varphi_{s_{31}}$ is adjusted/rotated angle of spline rolling die 3 relative to the rolling die 1. If the direction of rotation of spline rolling die is the same as the direction of numbering sequence for spline rolling dies, then adjusted/rotated angle in Table 5 is positive, else is negative. For example, direction of numbering sequence for rolling dies in Fig. 7a is anticlockwise, and then adjusted/rotated angle is positive when its direction is anticlockwise and is negative when its direction is clockwise.

It can be found from Table 5 that if remainder from dividing the Z_w by 3 is the same as the remainder from dividing the Z_d by 3, then the phases of rolling dies are also the same, else there is a phase difference between rolling dies before spline rolling.

5.3 Phase characteristic between dies before thread and spline synchronous rolling with three rolling dies

The rolling dies are also identical in the thread rolling process or spline rolling process with three round dies. The adjustment of phase difference between rolling dies is realized by rotating the two of the three rolling dies. However, the thread rolling motion couples with the spline rolling motion in the thread and spline synchronous rolling process. The rolling die has threaded section and splined section, which are working at the same time, and thus both phase adjusting requirements of dies before rolling for thread rolling and spline rolling should be satisfied in the synchronous rolling process.

Likewise, in the thread and spline synchronous rolling process used three rolling dies, the basic requirement for motion compatibility is that the starts and teeth of the workpiece and dies should meet the Eq. (15).

In general, the starts (n_d) of threaded section of die is less than the tooth (Z_d) of splined section of die, and thus the Eq. (32) can be obtained.

$$\varphi_{t_{j1}} > \varphi_{s_{j1}} (j = 2, 3) \tag{32}$$

Table 5 Requirement of phase difference adjustment before spline rolling with three dies

No.	Z_w	Z_d	δ	δ'	$\varphi_{s_{21}}$	$\varphi_{s_{31}}$
1	$3n+1$	$3n'+1$	$\delta = \frac{\theta_s}{3}$	$\delta' = \frac{\theta_s}{3}$	$\varphi_{s_{21}} = 0$ or $\varphi_{s_{21}} = \frac{2\pi}{Z_d}$	$\varphi_{s_{31}} = 0$ or $\varphi_{s_{31}} = \frac{4\pi}{Z_d}$
2	$3n+2$	$3n'+1$	$\delta = \frac{2\theta_s}{3}$	$\delta' = \frac{\theta_s}{3}$	$\varphi_{s_{21}} = \frac{2\pi}{3Z_d}$	$\varphi_{s_{31}} = \frac{4\pi}{3Z_d}$
3	$3n+3$	$3n'+1$	$\delta=\theta_s$ or $\delta=0$	$\delta' = \frac{\theta_s}{3}$	$\varphi_{s_{21}} = \frac{4\pi}{3Z_d}$ or $\varphi_{s_{21}} = -\frac{2\pi}{3Z_d}$	$\varphi_{s_{31}} = \frac{8\pi}{3Z_d}$ or $\varphi_{s_{31}} = -\frac{4\pi}{3Z_d}$
4	$3n+1$	$3n'+2$	$\delta = \frac{\theta_s}{3}$	$\delta' = \frac{2\theta_s}{3}$	$\varphi_{s_{21}} = -\frac{2\pi}{3Z_d}$ or $\varphi_{s_{21}} = \frac{4\pi}{3Z_d}$	$\varphi_{s_{31}} = -\frac{4\pi}{3Z_d}$ or $\varphi_{s_{31}} = \frac{2\pi}{3Z_d}$
5	$3n+2$	$3n'+2$	$\delta = \frac{2\theta_s}{3}$	$\delta' = \frac{2\theta_s}{3}$	$\varphi_{s_{21}} = 0$ or $\varphi_{s_{21}} = \frac{2\pi}{Z_d}$	$\varphi_{s_{31}} = 0$ or $\varphi_{s_{31}} = \frac{4\pi}{Z_d}$
6	$3n+3$	$3n'+2$	$\delta=\theta_s$ or $\delta=0$	$\delta' = \frac{2\theta_s}{3}$	$\varphi_{s_{21}} = \frac{2\pi}{3Z_d}$ or $\varphi_{s_{21}} = -\frac{4\pi}{3Z_d}$	$\varphi_{s_{31}} = \frac{4\pi}{3Z_d}$ or $\varphi_{s_{31}} = -\frac{8\pi}{3Z_d}$
7	$3n+1$	$3n'+3$	$\delta = \frac{\theta_s}{3}$	$\delta'=0$	$\varphi_{s_{21}} = \frac{2\pi}{3Z_d}$	$\varphi_{s_{31}} = \frac{4\pi}{3Z_d}$
8	$3n+2$	$3n'+3$	$\delta = \frac{2\theta_s}{3}$	$\delta'=0$	$\varphi_{s_{21}} = \frac{4\pi}{3Z_d}$	$\varphi_{s_{31}} = \frac{8\pi}{3Z_d}$
9	$3n+3$	$3n'+3$	$\delta=\theta_s$ or $\delta=0$	$\delta'=0$	$\varphi_{s_{21}} = 0$ or $\varphi_{s_{21}} = \frac{2\pi}{Z_d}$	$\varphi_{s_{31}} = 0$ or $\varphi_{s_{31}} = \frac{4\pi}{Z_d}$

where $\varphi_{t_{j1}}$ and $\varphi_{s_{j1}}$ are nonzero and positive value, i.e., $\varphi_{t_{j1}}$ and $\varphi_{s_{j1}}$ are chosen from Table 4 and Table 5 are nonzero and positive value.

In the same way, letting

$$S_{j1} = \frac{\varphi_{t_{j1}}}{\varphi_{s_{j1}}}(j = 2, 3) \tag{33}$$

where $\varphi_{t_{j1}}$ and $\varphi_{s_{j1}}$ are both nonzero and positive value.

By the analyses in Sections 5.1 and 5.2, the $\varphi_{t_{j1}}$ and $\varphi_{s_{j1}}$ are closely related to the starts of thread of the workpiece and die and the tooth of spline of workpiece and die. However, the n_d and Z_d are determined by the n_w, Z_w, and i. Thus, considering the Eqs. (25) to (31), the expression of S_{j1} can be obtained according to the n_w, Z_w, and i. The results are listed in Table 6, where the phase differences for thread section and splined section are nonzero and positive value and have been simplified according to the periodic symmetry, i.e., $0 < \varphi_{t_{j1}} \leq \theta_t, 0 < \varphi_{s_{j1}} \leq \theta_s (j=2, 3)$.

Table 6 lists 27 cases of phase adjustment which would present in thread and spline synchronous rolling process with three rolling dies. The i should also be made maximize and Z_d should also be chosen as even number, so there are also some cases in Table 6 that are not recommended. For different $\varphi_{s_{j1}}$ and S_{j1}, the phase adjusting requirements are different, and then the die structure and adjusting method are also different. Three categories as following can be summarized according to value of $\varphi_{s_{j1}}$, where k=0,1,2,3,….

1. Category 1 $\varphi_{s_{j1}} = \frac{2\pi}{3Z_d} = \frac{\theta_s}{3}$: if $S_{j1}=3k+1$, then relative phase between thread and spline of rolling die j is the same as that of rolling die 1, and the phase difference between rolling die j and rolling die 1 is adjusted by means of rotating rolling die j by $\varphi_{t_{j1}}$; else the relative phase between thread and spline of rolling die j is different from that of rolling die 1, and the die structure should be satisfied the phase difference adjustments (i.e., $\varphi_{t_{j1}}$ and $\varphi_{s_{j1}}$) for threaded section and splined section respectively.

2. Category 2 $\varphi_{s_{j1}} = \frac{4\pi}{3Z_d} = \frac{2\theta_s}{3}$: if $S_{j1} = \frac{3}{2}k + 1$, then relative phase between thread and spline of rolling die j is the same as that of rolling die 1, and the phase difference between rolling die j and rolling die 1 is adjusted by means of rotating rolling die j by $\varphi_{t_{j1}}$; else the relative phase between thread and spline of rolling die j is different from that of rolling die 1, and the die structure should be satisfied the phase difference adjustments (i.e., $\varphi_{t_{j1}}$ and $\varphi_{s_{j1}}$) for threaded section and splined section, respectively.

3. Category 3 $\varphi_{s_{j1}} = \frac{2\pi}{Z_d} = \theta_s$: if $S_{j1}=k(k\neq0)$, then relative phase between thread and spline of rolling die j is the same as that of rolling die 1, and the phase difference between rolling die j and rolling die 1 is adjusted by means of rotating rolling die j by $\varphi_{t_{j1}}$, especially only rotating rolling die j by zero degree when $\varphi_{t_{j1}} = \theta_t$; else the relative phase between thread and spline of rolling die j is different from that of rolling die 1, and the die structure should be satisfied the phase difference adjustments (i.e., $\varphi_{t_{j1}}$ and $\varphi_{s_{j1}}$) for threaded section and splined section respectively.

When the relative phase between thread and spline of rolling die j is the same as that of rolling die 1, phase adjustment before rolling is realized by rotating rolling die j by $\varphi_{t_{j1}}$. Especially, if the $\varphi_{t_{j1}}$ and $\varphi_{s_{j1}}$ are both equal to integral multiple of corresponding periodic angle such as θ_t and θ_s, respectively, then the rolling die j rotated by zero can make the threaded and splined impressions on the workpiece rolling by rolling die 1 well connect with that rolling by the rolling die j. When relative phases between thread and spline for rolling die j is different from that for rolling die 1, the phase difference for threaded sections of rolling dies and the phase difference for splined sections of rolling dies are assured by die structure itself.

Table 6 Phase adjusting requirement for different sections of die in the synchronous rolling with three dies

No.	Z_w	n_w	i	Z_d	n_d	$\varphi_{s_{21}}$	$\varphi_{s_{31}}$	$\varphi_{t_{21}}$	$\varphi_{t_{31}}$	S_{21}	S_{31}
1	$3n+1$	$3m+1$	$3l+1$	$(3n+1)(3l+1)$	$(3m+1)(3l+1)$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{n_d}$	$\frac{2\pi}{n_d}$	$\frac{Z_d}{n_d}$	$\frac{Z_d}{n_d}$
2	$3n+1$	$3m+1$	$3l+2$	$(3n+1)(3l+2)$	$(3m+1)(3l+2)$	$\frac{4\pi}{3Z_d}$	$\frac{2\pi}{3Z_d}$	$\frac{4\pi}{3n_d}$	$\frac{2\pi}{3n_d}$	$\frac{Z_d}{n_d}$	$\frac{Z_d}{n_d}$
3	$3n+1$	$3m+1$	$3l+3$	$(3n+1)(3l+3)$	$(3m+1)(3l+3)$	$\frac{2\pi}{3Z_d}$	$\frac{4\pi}{3Z_d}$	$\frac{2\pi}{3n_d}$	$\frac{4\pi}{3n_d}$	$\frac{Z_d}{n_d}$	$\frac{Z_d}{n_d}$
4	$3n+2$	$3m+1$	$3l+1$	$(3n+2)(3l+1)$	$(3m+1)(3l+1)$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{n_d}$	$\frac{2\pi}{n_d}$	$\frac{Z_d}{n_d}$	$\frac{Z_d}{n_d}$
5	$3n+2$	$3m+1$	$3l+2$	$(3n+2)(3l+2)$	$(3m+1)(3l+2)$	$\frac{2\pi}{3Z_d}$	$\frac{4\pi}{3Z_d}$	$\frac{4\pi}{3n_d}$	$\frac{2\pi}{3n_d}$	$\frac{2Z_d}{n_d}$	$\frac{Z_d}{2n_d}$
6	$3n+2$	$3m+1$	$3l+3$	$(3n+2)(3l+3)$	$(3m+1)(3l+3)$	$\frac{4\pi}{3Z_d}$	$\frac{2\pi}{3Z_d}$	$\frac{2\pi}{3n_d}$	$\frac{4\pi}{3n_d}$	$\frac{Z_d}{2n_d}$	$\frac{2Z_d}{n_d}$
7	$3n+3$	$3m+1$	$3l+1$	$(3n+3)(3l+1)$	$(3m+1)(3l+1)$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{n_d}$	$\frac{2\pi}{n_d}$	$\frac{Z_d}{n_d}$	$\frac{Z_d}{n_d}$
8	$3n+3$	$3m+1$	$3l+2$	$(3n+3)(3l+2)$	$(3m+1)(3l+2)$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{Z_d}$	$\frac{4\pi}{3n_d}$	$\frac{2\pi}{3n_d}$	$\frac{2Z_d}{3n_d}$	$\frac{Z_d}{3n_d}$
9	$3n+3$	$3m+1$	$3l+3$	$(3n+3)(3l+3)$	$(3m+1)(3l+3)$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{3n_d}$	$\frac{4\pi}{3n_d}$	$\frac{Z_d}{3n_d}$	$\frac{2Z_d}{3n_d}$
10	$3n+1$	$3m+2$	$3l+1$	$(3n+1)(3l+1)$	$(3m+2)(3l+1)$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{n_d}$	$\frac{2\pi}{n_d}$	$\frac{Z_d}{n_d}$	$\frac{Z_d}{n_d}$
11	$3n+1$	$3m+2$	$3l+2$	$(3n+1)(3l+2)$	$(3m+2)(3l+2)$	$\frac{4\pi}{3Z_d}$	$\frac{2\pi}{3Z_d}$	$\frac{2\pi}{3n_d}$	$\frac{4\pi}{3n_d}$	$\frac{Z_d}{2n_d}$	$\frac{2Z_d}{n_d}$
12	$3n+1$	$3m+2$	$3l+3$	$(3n+1)(3l+3)$	$(3m+2)(3l+3)$	$\frac{2\pi}{3Z_d}$	$\frac{4\pi}{3Z_d}$	$\frac{4\pi}{3n_d}$	$\frac{2\pi}{3n_d}$	$\frac{2Z_d}{n_d}$	$\frac{Z_d}{2n_d}$
13	$3n+2$	$3m+2$	$3l+1$	$(3n+2)(3l+1)$	$(3m+2)(3l+1)$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{n_d}$	$\frac{2\pi}{n_d}$	$\frac{Z_d}{n_d}$	$\frac{Z_d}{n_d}$
14	$3n+2$	$3m+2$	$3l+2$	$(3n+2)(3l+2)$	$(3m+2)(3l+2)$	$\frac{2\pi}{3Z_d}$	$\frac{4\pi}{3Z_d}$	$\frac{2\pi}{3n_d}$	$\frac{4\pi}{3n_d}$	$\frac{Z_d}{n_d}$	$\frac{Z_d}{n_d}$
15	$3n+2$	$3m+2$	$3l+3$	$(3n+2)(3l+3)$	$(3m+2)(3l+3)$	$\frac{4\pi}{3Z_d}$	$\frac{2\pi}{3Z_d}$	$\frac{4\pi}{3n_d}$	$\frac{2\pi}{3n_d}$	$\frac{Z_d}{n_d}$	$\frac{Z_d}{n_d}$
16	$3n+3$	$3m+2$	$3l+1$	$(3n+3)(3l+1)$	$(3m+2)(3l+1)$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{n_d}$	$\frac{2\pi}{n_d}$	$\frac{Z_d}{n_d}$	$\frac{Z_d}{n_d}$
17	$3n+3$	$3m+2$	$3l+2$	$(3n+3)(3l+2)$	$(3m+2)(3l+2)$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{3n_d}$	$\frac{4\pi}{3n_d}$	$\frac{Z_d}{3n_d}$	$\frac{2Z_d}{3n_d}$
18	$3n+3$	$3m+2$	$3l+3$	$(3n+3)(3l+3)$	$(3m+2)(3l+3)$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{Z_d}$	$\frac{4\pi}{3n_d}$	$\frac{2\pi}{3n_d}$	$\frac{2Z_d}{3n_d}$	$\frac{Z_d}{3n_d}$
19	$3n+1$	$3m+3$	$3l+1$	$(3n+1)(3l+1)$	$(3m+3)(3l+1)$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{n_d}$	$\frac{2\pi}{n_d}$	$\frac{Z_d}{n_d}$	$\frac{Z_d}{n_d}$
20	$3n+1$	$3m+3$	$3l+2$	$(3n+1)(3l+2)$	$(3m+3)(3l+2)$	$\frac{4\pi}{3Z_d}$	$\frac{2\pi}{3Z_d}$	$\frac{2\pi}{n_d}$	$\frac{2\pi}{n_d}$	$\frac{3Z_d}{2n_d}$	$\frac{3Z_d}{n_d}$
21	$3n+1$	$3m+3$	$3l+3$	$(3n+1)(3l+3)$	$(3m+3)(3l+3)$	$\frac{2\pi}{3Z_d}$	$\frac{4\pi}{3Z_d}$	$\frac{2\pi}{n_d}$	$\frac{2\pi}{n_d}$	$\frac{3Z_d}{n_d}$	$\frac{3Z_d}{2n_d}$
22	$3n+2$	$3m+3$	$3l+1$	$(3n+2)(3l+1)$	$(3m+3)(3l+1)$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{n_d}$	$\frac{2\pi}{n_d}$	$\frac{Z_d}{n_d}$	$\frac{Z_d}{n_d}$
23	$3n+2$	$3m+3$	$3l+2$	$(3n+2)(3l+2)$	$(3m+3)(3l+2)$	$\frac{2\pi}{3Z_d}$	$\frac{4\pi}{3Z_d}$	$\frac{2\pi}{n_d}$	$\frac{2\pi}{n_d}$	$\frac{3Z_d}{n_d}$	$\frac{3Z_d}{2n_d}$
24	$3n+2$	$3m+3$	$3l+3$	$(3n+2)(3l+3)$	$(3m+3)(3l+3)$	$\frac{4\pi}{3Z_d}$	$\frac{2\pi}{3Z_d}$	$\frac{2\pi}{n_d}$	$\frac{2\pi}{n_d}$	$\frac{3Z_d}{2n_d}$	$\frac{3Z_d}{n_d}$
25	$3n+3$	$3m+3$	$3l+1$	$(3n+3)(3l+1)$	$(3m+3)(3l+1)$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{n_d}$	$\frac{2\pi}{n_d}$	$\frac{Z_d}{n_d}$	$\frac{Z_d}{n_d}$
26	$3n+3$	$3m+3$	$3l+2$	$(3n+3)(3l+2)$	$(3m+3)(3l+2)$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{n_d}$	$\frac{2\pi}{n_d}$	$\frac{Z_d}{n_d}$	$\frac{Z_d}{n_d}$
27	$3n+3$	$3m+3$	$3l+3$	$(3n+3)(3l+3)$	$(3m+3)(3l+3)$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{Z_d}$	$\frac{2\pi}{n_d}$	$\frac{2\pi}{n_d}$	$\frac{Z_d}{n_d}$	$\frac{Z_d}{n_d}$

6 Application of the mathematical model

The phase characteristic between dies before rolling for thread and spline synchronous rolling by using two and three round dies are discussed in Sections 4 and 5, and corresponding mathematical expressions of phase difference adjustment are established. It is difficult to adjust the phase difference between dies before thread and spline synchronous rolling

Table 7 Parameters of workipece and die

No.	Threaded section of workipece			Splined section of workipece			i
	Number of starts	Pitch	Thread angle	Number of teeth	Module	Pressure	
1	2	4 mm	60°	33	1 mm	37.5°	6
2	3	4 mm	60°	34	1 mm	37.5°	6
3	1	4 mm	90°	20	1 mm	37.5°	10
	1	4 mm	90°	20	1 mm	37.5°	9

comparing with that before thread or spline rolling, and it need to be realized via die structure in some situations. The cases of phase characteristic in the synchronous rolling process by using three dies are more complex than that in the process by using two dies, and the adjustment in the synchronous rolling process by using three dies is also more difficult than that by using two dies, and the manufacture of die is also more difficult in some situation. So three rolling dies or more than three rolling dies should not be adopted for the synchronous rolling process. Thus, the discussion in this section is about the synchronous rolling process by using two dies.

Three parts/ workipeces are used in this section. The structural form of workipece 1 is the same as that workipece 2, where one section is threaded feature and another section is splined feature, but the starts of threaded section and the teeth of splined section are different. For workipece 3, the middle section is threaded feature, and both ends have the same splined features. The triangular thread and involute spline are adopted for three workipeces. Four sets of dies for three workipeces are designed under different i. The parameters for thread and spline, and the ratio (i) of relationship between part/workpiece and die are listed in Table 7. The die structure and phase difference adjustment are determined according to the analysis in Section 4. In order to verify whether the acceptable thread and spline could be formed, the numerical simulations are carried out based on the finite element (FE) model developed in Ref [18].

According to the analysis in Section 4, S is 33 for workipece 1, which is determined by n_w=2, Z_w=33, and i=6. Thus, the die structures of two rolling dies are the same, i.e., the relative phases between threaded section and splined section for two dies are the same. Before rolling, one of two rolling dies is rotated by $\varphi_t=2\pi/12$ to realize the phase adjustment. Geometry models of rolling dies are modeled according to specific parameters, and then they are assembled and adjusted the phase under DEFORM, as shown in Fig. 8a. The finite element analysis (FEA) is carried out and the result indicates that the thread well connects for threaded section and incorrect tooth does not appear for splined section, as shown in Fig. 8b. If an odd number, such as seven, is chosen for i, then S is 33/2, so the relative phases between threaded

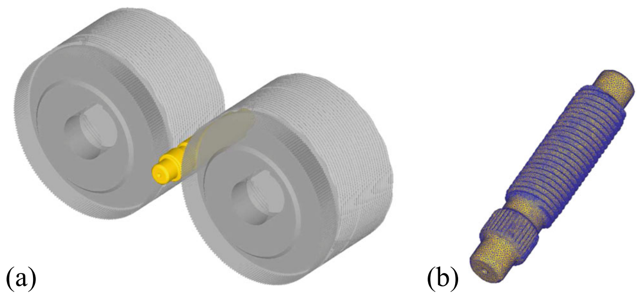

Fig. 9 FE model and simulation result of workipece 2: **a** FE model; **b** shape of formed workipece

section and splined section for two dies are different, and this will add some difficulty to manufacture the rolling dies. Thus, an even number for i would be a better choice for workpiece 1.

For workipece 2, S is 17/3 according to n_w=3, Z_w=34, and i=6. Thus, the relative phases between threaded section and splined section for two dies are different according to the analysis in Section 4, and the die structure should be satisfied the phase difference adjustments for threaded section and splined section respectively. The phase difference for threaded sections of rolling dies and the phase difference for splined sections of rolling dies are assured by die structure itself, and no die need to be rotated before rolling. Geometry models of two rolling dies are modeled according to specific parameters, and then the FE model for the rolling process of workipece 2 can be developed as shown in Fig. 9a. The FE result indicates that the unconnected thread and incorrect tooth cannot be found for rolled workipece 2, as shown in Fig. 9b. If an odd number, such as 7, is chosen for i, then S is 34/3, so the relative phases between threaded section and splined section for two dies are also different. Thus, even number or odd number for i is not important for workipece 2 considering die structure, both manufacturing difficulties are almost the same.

Table 7 shows that n_w=1 and Z_w=20 for workipece 3. According to the analysis in Section 4, S is 10 when i=10 and S is 20 when i=9. Whether S is 10 or 20, the die structures of two rolling dies are the same, i.e., the relative phases between threaded section and splined section for two dies are the same, but the phase difference adjustments before rolling are different. Under i=10 and S=10, one of two rolling dies is rotated by $\varphi_t=\pi/10$ to realize the phase adjustment before rolling. Fig. 10a illustrates the shape of rolled workipece under

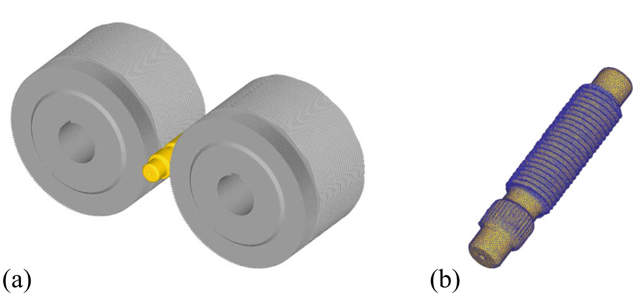

Fig. 8 FE model and simulation result of workipece 1: (**a**) FE model; (**b**) shape of formed workipece

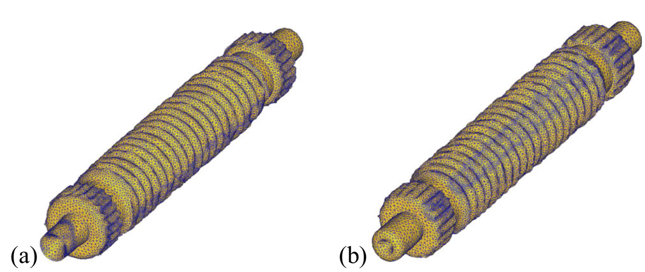

Fig. 10 Numerical simulation result of workipece 3: **a** i=10; **b** i=9

this case, and the threaded and splined tooth profiles are acceptable. Under i=9 and S=20, one of two rolling dies is rotated by zero (or φ_t=2π/9) to realize the phase adjustment before rolling. The shape of rolled workipece predicted by numerical simulation is shown Fig. 10b, it can be found from the figure that the threaded and splined tooth profiles are also acceptable. The threaded and splined tooth profiles in Fig. 10a are almost the same as that in Fig. 10b. Comparing with workpiece 2, the manufacture of rolling die for workpiece 3 would be easy, because the relative phases between threaded section and splined section for two dies are the same with even number or odd number for i.

7 Conclusions

The universal mathematical expressions of phase difference adjustment for dies before thread rolling and before spline rolling have been established. The phase difference adjustment for rolling dies in thread and spline synchronous rolling process has also been investigated. The following conclusions can be drawn according to research on the rolling processes using two or three round dies:

1. If the remainder ($n_w$mod$N$) from dividing the starts n_w of thread of workpiece by number N of rolling dies is equal to the $n_d$mod$N$ in the thread rolling process, i.e., $n_w \equiv n_d$(mod$N$), then the phases of thread rolling dies before rolling are the same, else there is a phase difference between thread rolling dies; there is the same situation in the spline rolling process, which is that if $Z_w \equiv Z_d$(mod$N$), then the phases of spline rolling dies before rolling are the same, else there is a phase difference between spline rolling dies.
2. In thread and spline synchronous rolling process, if phase difference of splined section between rolling die j (j=2,…, N), and rolling die 1 is $\varphi_{s_{j1}} = \frac{h}{N}\theta_s (h = 1, \ldots, N)$ and ratio of phase difference is $S_{j1} = \frac{N}{h}k + 1$, then the relative phase between threaded and splined sections for rolling die j is the same as the relative phase between threaded and splined sections for rolling die 1, and the die structures of rolling die j and rolling die 1 are the same, else the relative phase between threaded and splined sections for rolling die j is different from that for rolling die 1, and the die structure should be satisfied the phase difference adjustments (i.e., $\varphi_{t_{j1}}$ and $\varphi_{s_{j1}}$) for thread section and splined section respectively.
3. If the die structures of rolling die j and 1 are the same, i.e., the relative phase between thread and spline of rolling dies are the same, then rotating rolling die j by $\varphi_{t_{j1}}$ before thread and spline synchronous rolling to realize the phase adjustment, else the phase difference for threaded sections of rolling dies and the phase difference for splined sections of rolling dies are assured by die structure itself. In a particular case under the die structures being the same, phase adjustment before rolling is realized by rotating rolling die j by zero when $\varphi_{t_{j1}} = \theta_t, \varphi_{s_{j1}} = \theta_s$.

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