Int J Adv Manuf Technol (2015) 80:1625–1640
DOI 10.1007/s00170-015-7122-2

ORIGINAL ARTICLE

Optimal tolerance allocation for precision machine tools in consideration of measurement and adjustment processes in assembly

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Received: 19 August 2014 / Accepted: 6 April 2015 / Published online: 23 April 2015

Abstract The high geometric accuracy requirement of precision machine tools represents a challenge for tolerance design and assembly process planning that guarantee the final assembly accuracy. Component tolerances should be allocated in association with assembly processes. However, tolerance design and assembly process planning are usually considered separately and lack quantitative analysis. In this paper, to integrate the geometric tolerance of components and variation propagation in assembly process, a state space model is developed. The measurement and adjustment process are expressed as observation matrix and control inputs. An optimal control problem is formulated to determine the adjustment process in consideration of the loss of final assembly accuracy and costs of remachining adjustment process. Tolerances of components can be optimally allocated based on the variation propagation in this deterministic assembly process. The generality and effectiveness of this approach are validated by applying the model on a four-axis horizontal machining center.

Keywords Precision machine tools · Tolerance allocation · State space model · Assembly process

1 Introduction

Machine tools are fundamental components in modern manufacturing. In the development of machine tools, the machining accuracy is always the basic and key characteristic requirement [1, 2]. The main error sources in machine tools are categorized as geometric and kinematic errors, thermal errors, stiffness error, and errors addressed to the deflection of cutting tools [3–5]. Nowadays, in the development of precision machine tools, many techniques should be applied in design, manufacture, and use to take into account several phenomena, such as the elastic and thermal deformations, the possible wear of moving parts, and the appearance of vibrations [6–11].

Due to the uncertain sources and the complex formulation of machining errors, it is difficult but necessary for designers to identify and quantify the error sources before construction. Error budget is an important deterministic tool that provides a systematic way to predict and control the repeatable and nonrepeatable errors of a machine [1]. The error budget is a model of the machine in its environment expressed in terms of cause-and-effect relationships. Uriarte et al. [12] established the overall error budget for a micromilling machine with tools less than 0.3 mm in diameter. Sun et al. [13] presented an error budget methodology for designing and characterizing machines used to manufacture or inspect parts with spatial frequency-based specifications. For precision machine tools, error budget is carried out to identify and understand the major sources of error of the design, assembly, verification, and use.

The geometric and kinematic errors directly affect the relative position between tool and workpiece, producing dimensional errors [14]. These errors come from the mechanical imperfections such as misalignments of axes, slideways, and joint wear. Many researchers investigated geometric errors for various structures of machine tools from several points of view. Lamikiz et al. [15] developed a methodology for estimation of the geometrical accuracy of five-axis milling centers based on the Denavit and Hartenberg formulation. Diaz-Tena et al. [16] presented a method to estimate global precision of

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complex multitasking machines by using the homogenous matrix. The modeling and compensation of volumetric errors for a multi-spindle machine tool were introduced and discussed by Ahn et al. [17], and Tian et al. [18] presented a general and systematic approach for geometric error modeling of machine tools due to the geometric errors arising from manufacturing and assembly. Liu et al. [19] investigated the characteristics of geometric errors in CNC machine tools in detail. In this paper, geometric variation accumulation in assembly process is modeled, and a methodology is proposed for the quantitative error budget estimation and tolerance allocation of machine tools.

The final assembly accuracy of machine tools is dependent on machining errors of components and variation accumulation in assembly process, which are respectively determined by tolerance allocation in design and adjustment processes in assembly. Tolerance analysis and synthesis are a traditional issue in industrial application and academic research. Recent developments in computer-aided tolerancing (CAT) technology and concurrent optimal design have been adopted in tolerance design to improve product quality and reduce cost by designers and manufacturing engineers [20–22]. Variation propagation and control in assembly process also attracted interests from many researchers. The developments in the engineering-driven stream of variation (SoV) and measurement data-driven statistical process control (SPC) methodologies significantly improve the variation reduction for manufacturing processes [23, 24]. They have been proven to be effective in process variation monitoring and diagnosis [25].

However, the geometric accuracy requirement of machine tools is usually in micrometers, and it has the same magnitude of common machining errors. The popularly used statistical methods such as worst case and root-sum square (RSS) are no longer suitable for tolerance analysis of machine tool assemblies. Therefore, the tolerance allocation and assembly process planning for machine tools mainly depend on designers' experiences. The tolerance allocation, machining, and assembly process planning are separately considered in designing and lack of quantitative analysis. Therefore, the development of an analytical method to model and control variation propagation for machine tool assemblies is required. Researchers have not yet investigated in depth these issues.

Satisfying the increasingly higher accuracy requirement cannot only depend on tolerance specification. Measurements and adjustments are required to the assembly process to ensure the variation accumulation being in a defined range. In each step of machine tool assembly, variation is accumulated due to the incoming components with geometric error. Measurement is implemented to evaluate the variation of key characteristics (KCs). Based on the measurement results, some critical characteristics should be adjusted to a relative small value by scraping or remachining in order to obtain the target accuracy.

In the multi-station assembly process, the variation propagation is related to incoming geometrical error, measurement, and adjustment processes. State space model (SSM) has been used to model and control variation propagation in automotive body and aircraft part assembly process [26]. Geometric deviations of KCs are selected as the state variables and changed along assembly stations in a nonlinear way. The measurement and adjustment processes are modeled as observation matrix and control input respectively in state space equation. Based on SSM, designers can optimize fixture locator adjustment strategies and dimension control for automotive body and aircraft assembly [27, 28]. However, compared to the flexible part assembly, the parts of machine tools are much stiffer, and the KC definition, measurement method, and adjustment process are very different in machine tool assembly. In addition, based on SSM, Gomez-Acedo et al. [29] presented a methodology for the design of a thermal distortion compensation system for large machine tools. In this method, the actual physical system state is optimal as estimated by using Kalman filter.

The aim of this work is to enable designers to achieve optimal tolerance design and assembly process planning to satisfy the final accuracy requirement in an analytical method. SSM is used to model the variation propagation of machine tools and to solve the integrated dimension control problems in design and assembly. This research also suggests the use of optimal control and Kalman filter method both derived from SSM to obtain analytical and practical solutions for the design, accuracy testing, and process planning problems.

In the first section, the machine tool assembly process is discussed, and authors suggest that the assembly process must be considered in tolerance allocation for precision machine tool. The angular errors between axes are selected as an assembly accuracy requirement based on kinematic error model. Moreover, the final part of Section 1 discusses geometric error model of KCs based on coordinate definition and datum flow chain (DFC). Section 2 describes the variation propagation modeling in assembly process using SSM. The modeling of practical measurement and adjustment process of a horizontal machining center is illustrated in detail after theoretical discussion. Section 3 describes the tolerance allocation method based on SSM and optimal control theory. Conclusion and suggestions for further researches are summarized in the last part.

2 Key characteristic modeling in machine tool assemblies

2.1 Machine tool assembly process

Machine tool is composed of supporting parts, feeding systems, spindle, electric and control system, and other auxiliary systems. All these components are set up in assembly process to enable machine tools to realize required functions. For the assembly accuracy requirement, the process is mainly influenced by machining error and assembly error accumulation of support parts, feeding systems, and spindle.

In machine tool assembly process, the bed is usually firstly mounted on the ground. The other components or sub-assemblies are assembled systematically after installing the bed. In each assembly step, the error state of last assembly step is the initial error state of this step. The geometric error of incoming component is the error input. Measurement process is implemented to evaluate variation state. Adjustment must be implemented to reduce the variation accumulation to meet the requirement of assembly specification.

At each assembly step, geometric variation accumulation is increased, tested, and reduced. The machine tool design and manufacturing process integrates the accuracy issues such as tolerance allocation, assembly process planning, measurement result valuation, and decision of adjustment. In this complicated practical problem, modeling of variation propagation and control is basic and significant. Following sections present the analytical model of geometric space and the model describing the assembly process.

2.2 Kinematic errors of precision machine tools

For decades, researchers investigated and developed geometric error modeling and analysis of machine tool. However, most of these models are used for error identification and compensation in order to reduce kinematic error in machining process. The studies focused on the errors of moving axes and did not concern about how the errors generate and accumulate. Therefore, when discussing the variation propagation in a machine tool assembly process, the geometric error model of KCs of the process must be reconsidered and constructed.

A typical four-axis horizontal machining center is considered. The basic configuration of this machine is shown in Fig. 1. Three linear axes (*X*, *Y*, *Z*) and one rotary axis (*B*) for generating rotary motion about *Y*-axis are contained in this machine.

The machining accuracy depends on the moving accuracy of all the axes: the assembly specifications of the finished assembly mainly regulate the single and relative space accuracy of all these axes. The machine axis kinematic errors are firstly modeled. Following Fig. 2 shows the kinematic relationship between axes and the analytical model.

The geometric error is expressed through a coordinate system, whose base is defined coincident with nominal *X*-axis. Other coordinates are separately defined as nominal or actual part position and orientation as shown in Table 1.

Based on the kinematic model and coordinate definition, the relationships between two adjacent coordinates are summarized in Table 2, where Ω_2 w.r.t. Ω_1 means the translation or rotation of coordinate Ω_2 is defined with respect to (w.r.t.) coordinate Ω_1.

Homogeneous transformation matrix (HTM) is used to construct the geometric error model [30]. According to the coordinate definition and kinematic chain of the machine tool, the relative position and orientation errors of any two characteristics (coordinates) can be calculated by HTM multiplication.

Considering an ideal coordinate of the tool tip ${}^{0}p_t$ and a command tool path (X_c, Y_c, Z_c, B_c), the actual position and orientation of the tool tip with kinematic errors ${}^{r}p_t$ can be obtained by:

$$ {}^{r}\mathbf{p}_t = {}^{r}\mathbf{T}_t \cdot {}^{0}\mathbf{p}_t \tag{1} $$

where ${}^{r}T_t$ is the HTM from ideal coordinate of the tool tip to actual under the command tool path. It is a 4×4 error HTM

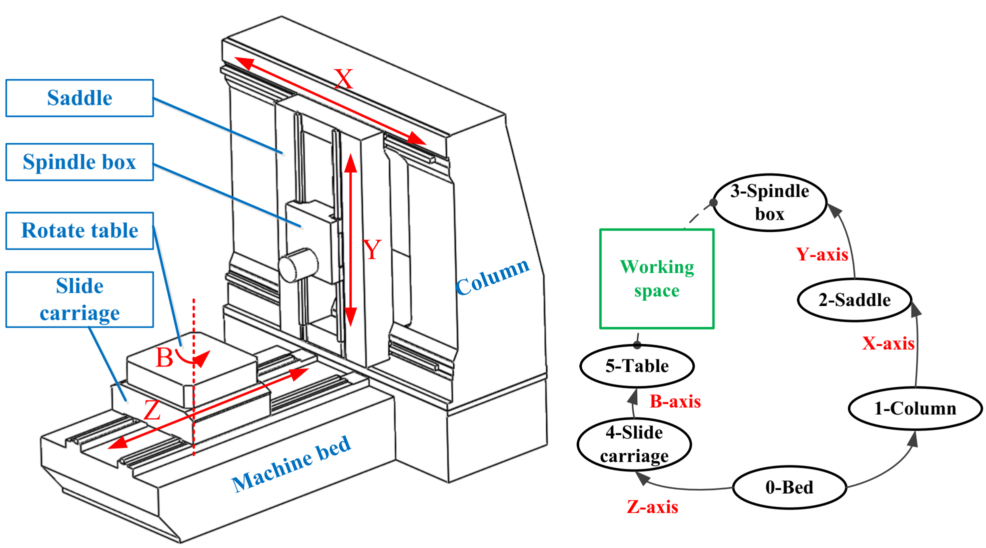

Fig. 1 Rotary and linear axes in a horizontal machining center

due to kinematic errors. It can be expressed as follows for this horizontal machining center:

$$\begin{aligned} {}^{r}\mathbf{T}_t = {} & \mathbf{T}_{87}(\Delta x_B, \Delta y_B, \Delta z_B, \Delta\alpha_B, \Delta\beta_B, \Delta\gamma_B, B_c) \\ & \cdot\mathbf{T}_{76}(\Delta\alpha_{zB}, \Delta\beta_{zB}, \Delta\gamma_{zB}) \\ & \cdot\mathbf{T}_{65}(\Delta x_z, \Delta y_z, \Delta z_z, \Delta\alpha_z, \Delta\beta_z, \Delta\gamma_z, Z_c) \\ & \cdot\mathbf{T}_{51}(\Delta\alpha_{xz}, \Delta\beta_{xz}, \Delta\gamma_{xz}) \\ & \cdot\mathbf{T}_{12}(\Delta x_x, \Delta y_x, \Delta z_x, \Delta\alpha_x, \Delta\beta_x, \Delta\gamma_x, X_c) \\ & \cdot\mathbf{T}_{23}\left(\Delta\alpha_{xy}, \Delta\beta_{xy}, \Delta\gamma_{xy}\right) \\ & \cdot\mathbf{T}_{34}\left(\Delta x_y, \Delta y_y, \Delta z_y, \Delta\alpha_y, \Delta\beta_y, \Delta\gamma_y, Y_c\right) \end{aligned} \tag{2}$$

where T_{ij} represents the 4×4 HTM for linear and rotary motions.

Then, by neglecting the mounting errors of workpiece on the table and spindle in the spindle box, the actual error of tool tip with respect to the work piece in three-axis directions can be obtained from first-order approximation. To simplify, the rotation command of B-axis B_c is set to 0 in this calculation. The three-error components in each axis direction can be expressed as follows:

$$\begin{aligned} \Delta_x = {} & \Delta x_x + \Delta x_y - \Delta x_z - \Delta x_B \\ & + \left(\Delta\gamma_{xz} + \Delta\gamma_z + \Delta\gamma_{zB} + \Delta\gamma_B - \Delta\gamma_x - \Delta\gamma_{xy}\right)\cdot y \\ & + (\Delta\beta_z + \Delta\beta_B)\cdot z \\ \Delta_y = {} & \Delta y_x + \Delta y_y - \Delta y_z - \Delta y_B \\ & -(\Delta\gamma_B + \Delta\gamma_{zB} + \Delta\gamma_z + \Delta\gamma_{xz})\cdot x \\ & -(\Delta\alpha_z + \Delta\alpha_{zB} + \Delta\alpha_B)\cdot z \\ \Delta_z = {} & \Delta z_x + \Delta z_y - \Delta z_z - \Delta z_B \\ & + (\Delta\beta_B + \Delta\beta_z + \Delta\beta_{xz})\cdot x \\ & + (\Delta\alpha_x + \Delta\alpha_{xy} - \Delta\alpha_B - \Delta\alpha_{zB} - \Delta\alpha_z - \Delta\alpha_{xz})\cdot y \end{aligned} \tag{3}$$

Equation (3) shows that the error is composed of two terms: the axis linear error and the angular errors. Each linear axis introduces an error along the axis direction. It can be deduced that the most the linear motion system is elaborately manufactured and assembled, the lower the linear motion errors will be. The errors along motion directions can also be compensated by servo feed system. The four motion systems are relatively isolated in assembly, and the separate control of the motion error is required.

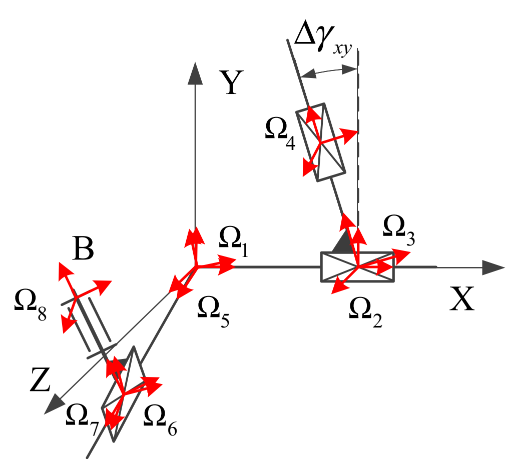

Fig. 2 Model of four-axis horizontal machining center kinematic error

Table 1 Coordinate definition of kinematic error model

Coordinate	Definition
Ω_1	Base coordinate and the nominal X-axis
Ω_2	Saddle part coordinate
Ω_3	Nominal Y-axis
Ω_4	Spindle box part coordinate
Ω_5	Nominal Z-axis
Ω_6	Slide carriage part coordinate
Ω_7	Nominal B-axis
Ω_8	Rotate table part coordinate

The second term depends on angular errors. Errors include angular error of one axis as well as the relative angular error between two axes. Since angular errors are mostly concerned on machine tool assembly, this research focuses on modeling and controlling angular errors. The angular error arises from the form and position errors of components. In the next section, the analytical relationship between angular error and geometrical features of components is discussed.

2.3 Coordinate system and datum flow chain

Machining depends on relative motions of machine tool components. The machining error comes from the kinematic errors of motion systems and the geometric errors of mounting surfaces. Therefore, to control the assembly angular error accumulation, KCs of the machine tool motion system and mounting surfaces need to be defined and modeled.

For instance, consider a planar motion system with two linear motion axes as shown in Fig. 3. The two axes are

Table 2 Kinematic error definition of horizontal machining center

Kinematic errors	Definitions
Δx_x, Δy_x, Δz_x	Translation errors of X-axis (Ω_2 w.r.t. Ω_1)
$\Delta\alpha_x$, $\Delta\beta_x$, $\Delta\gamma_x$	Rotation errors of X-axis (Ω_2 w.r.t. Ω_1)
Δx_y, Δy_y, Δz_y	Translation errors of Y-axis (Ω_4 w.r.t. Ω_3)
$\Delta\alpha_y$, $\Delta\beta_y$, $\Delta\gamma_y$	Rotation errors of Y-axis (Ω_4 w.r.t. Ω_3)
Δx_z, Δy_z, Δz_z	Translation errors of Z-axis (Ω_6 w.r.t. Ω_5)
$\Delta\alpha_z$, $\Delta\beta_z$, $\Delta\gamma_z$	Rotation errors of Z-axis (Ω_6 w.r.t. Ω_5)
Δx_B, Δy_B, Δz_B	Translation errors of B-axis (Ω_8 w.r.t. Ω_7)
$\Delta\alpha_B$, $\Delta\beta_B$, $\Delta\gamma_B$	Rotation errors of B-axis (Ω_8 w.r.t. Ω_7)
$\Delta\alpha_{xy}$, $\Delta\beta_{xy}$, $\Delta\gamma_{xy}$	Angular errors of Y-axis w.r.t. X-axis (Ω_3 w.r.t. Ω_2)
$\Delta\alpha_{xz}$, $\Delta\beta_{xz}$, $\Delta\gamma_{xz}$	Angular errors of Z-axis w.r.t. X-axis (Ω_5 w.r.t. Ω_1)
$\Delta\alpha_{zB}$, $\Delta\beta_{zB}$, $\Delta\gamma_{zB}$	Angular errors of B-axis w.r.t. Z-axis (Ω_7 w.r.t. Ω_6)

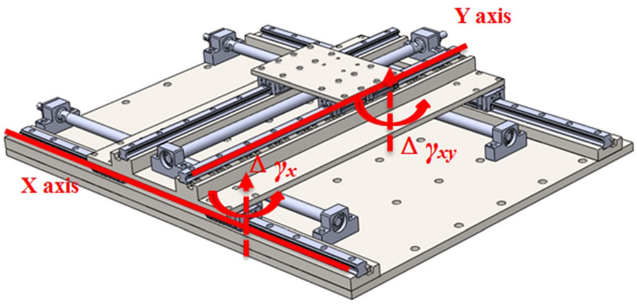

Fig. 3 *X-Y* linear motion system

nominal perpendicular to each other. The sliding saddle on *X*-axis is the mounting base of *Y*-axis. The trend line of *X*-axis Trend(*X*) can be measured by moving the sliding saddle. The translation and rotation errors (Δx_x, Δy_x, and $\Delta\gamma_x$) of the saddle vary with the moving distance of *X*-axis. By fixing the saddle in a certain position, the trend line of *Y*-axis Trend(*Y*) can also be measured on the same measuring basis.

As shown in Fig. 4, the squareness error between the *X*-slide and *Y*-slide is given by the following:

$$\Delta\gamma_{xy} = \theta_1 - \theta_2 \tag{4}$$

The angular error between Trend(*X*) and nominal *X* about *Z*-axis is the angular error θ_x caused by motion error that refers to nominal *X*. The angular error between Trend(*Y*) and nominal *Y* consists of three parts: (1) the angular error of *X*-axis $\Delta\gamma_x$, which varies with the moving distance of *X*-axis X_0; (2) the machining error of the sliding saddle $\Delta\gamma_{xy\_saddle}$ (squareness error between side mounting surface of *X*-axis slide blocks and *Y*-axis rail); and (3) the angular error θ_y caused by motion error of *Y*-axis. Thus, the squareness error can be rewritten as follows:

$$\Delta\gamma_{xy} = \theta_x - \theta_y - \Delta\gamma_{xy\_saddle} - \Delta\gamma_x \tag{5}$$

where θ_x, θ_y, and $\Delta\gamma_x$ come from the manufacturing and assembly error of linear motion system and $\Delta\gamma_{xy\_saddle}$ from the machining error of sliding saddle.

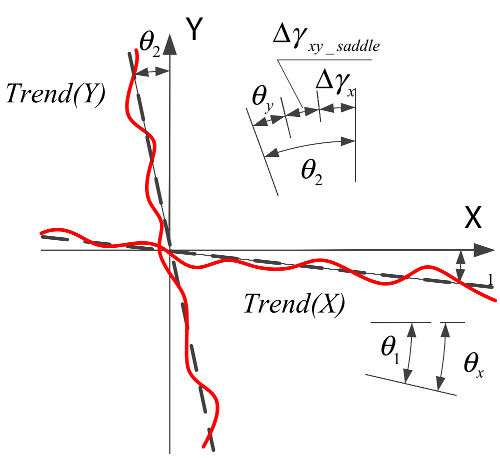

Fig. 4 Squareness error between *X-Y* axes

In practical assembly process, the translation and rotation errors of linear motion system mostly depend on the geometric error of the mounting surfaces and can only be adjusted by these surfaces as shown in Fig. 5. Therefore, to simplify the angular error caused by the translation error of the linear motion system itself can be neglected and can be considered as the machining error of mounting surfaces of support components. Moreover, the angular error of different machine tools' linear motion systems (such as slide guides or hydrostatic guides) also depends on the mounting surfaces or contact surfaces of motion systems.

The detailed analysis of linear motion system shows that the translation and rotation errors are influenced by geometric error of guide components (straightness, parallelism and form error of rails, and mounting surfaces), loads, or deformation of components. However, this paper only investigates the angular error between two connected mounting surfaces of support components considering it as the angular error of linear motion system.

Assembling the *X-Y* table or machine tool motion system, the KCs contribute to the variation propagation that comes from the mounting surfaces' contact to the motion block of *X*-axis and the rails' mounting surfaces of *Y*-axis. One slide of the blocks and rails is defined as a reference characteristic, while the other slide parallels the reference one. To simplify the analysis in final assembly process, the mounting surfaces of one motion system are modeled as an equivalent assembly datum plane. Coordinate O_2 expresses the orientation of mating surface of saddle and *X*-axis blocks. The surface mating with top surface of blocks is defined as *XY* plane of O_2, and side surface is the *XZ* plane. O_3 represents the orientation of saddle and *Y*-axis rails. The orientation of O_3 is the same as O_2. The bottom surface of rail is defined as *XY* plane, while the side surface is *YZ* plane. According to the geometrical product specifications (GPS), the fixed mating (for example the bolted surface) of two support components can be modeled as a reference plane.

A frame model named as datum flow chain can be constructed to illustrate all the assembly datum planes of KCs [31]. The KCs of machine tool bed are usually selected as the starting datum of DFC, for the bed is usually firstly mounted on the ground. Figure 6 illustrates the datum flow chain of the four-axis horizontal machining center.

The points in DFC represent a joint surface or end surface, which is defined as KCs in tolerance analysis, where *r* indicates the rail mounting surface of support part, *s* indicates the slide block mounting surface, and "01" indicates the bolted mating surface of bed (0) and column (1). The orientation and angular error of KCs is represented by the relative coordinate. The arrow lines represent error transfer from one datum plane to the other in one component.

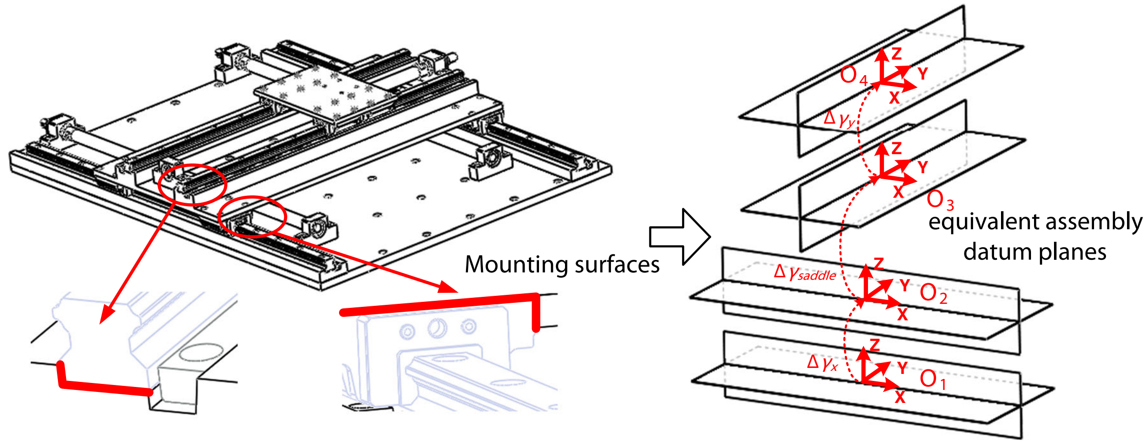

Fig. 5 Coordinate definition on the mounting surfaces of linear motion systems

3 State space model of machine tool assembly process

3.1 Variation propagation modeling based on differential motion vectors

In kinematic error analysis, HTM is used to calculate the error accumulation in a geometric view. The variation propagation in assembly process is considered as a discrete event process. The propagation and variation of geometric error state $X(k)$ in each assembly step should be expressed and analyzed. $X(k)$ describes the total deviation in the orientation of a coordinate on a datum plane on the kth part along a DFC, measured from its nominal or zero mean location, expressed in the coordinate frame of the part at the base of the chain.

$$\widetilde{\mathbf{X}}(k) = \boldsymbol{\delta}_k = \begin{bmatrix} \delta_k^x \\ \delta_k^y \\ \delta_k^z \end{bmatrix}$$

where the differential rotation vector δ_k is associated with the kth component with respect to base coordinate and can be computed as follows:

$$\boldsymbol{\delta}_k = \sum_{i=1}^{k} \mathbf{R}_{i-1} \boldsymbol{\delta}_i$$

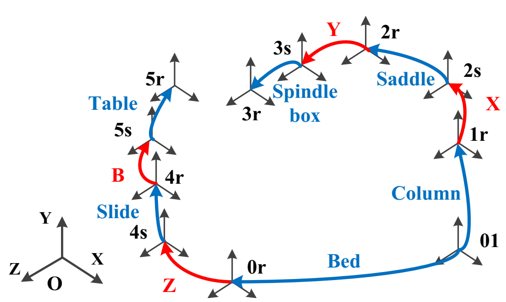

Fig. 6 Four-axis horizontal machining center datum flow chain

where $\boldsymbol{R}_{i-1}$ is a 3×3 rotation matrix. It can be obtained from the transformation from nominal component coordinate O_i to O_{i-1}.

$$[\mathbf{R}_i] = [\mathbf{R}_i]_{\text{rotx}} [\mathbf{R}_i]_{\text{roty}} [\mathbf{R}_i]_{\text{rotz}}$$

where

$$[\mathbf{R}_i]_{\text{rotx}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{xi} & -\sin\theta_{xi} \\ 0 & \sin\theta_{xi} & \cos\theta_{xi} \end{bmatrix}$$
$$[\mathbf{R}_i]_{\text{roty}} = \begin{bmatrix} \cos\theta_{yi} & 0 & \sin\theta_{yi} \\ 0 & 1 & 0 \\ -\sin\theta_{yi} & 0 & \cos\theta_{yi} \end{bmatrix}$$
$$[\mathbf{R}_i]_{\text{rotz}} = \begin{bmatrix} \cos\theta_{zi} & -\sin\theta_{zi} & 0 \\ \sin\theta_{zi} & \cos\theta_{zi} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The differential rotation vector δ_k can be represented in the following matrix form [32]:

$$\mathbf{X}(k) = \boldsymbol{\delta}_k = \begin{bmatrix} \delta_k^x \\ \delta_k^y \\ \delta_k^z \end{bmatrix} = \begin{bmatrix} \delta_{k-1}^x \\ \delta_{k-1}^y \\ \delta_{k-1}^z \end{bmatrix} + \begin{bmatrix} W_{(5,1),k} & W_{(7,1),k} & W_{(9,1),k} \\ W_{(5,2),k} & W_{(7,2),k} & W_{(9,2),k} \\ W_{(5,3),k} & W_{(7,3),k} & W_{(9,3),k} \end{bmatrix} \begin{bmatrix} \Delta\theta_{xk} \\ \Delta\theta_{yk} \\ \Delta\theta_{zk} \end{bmatrix} \quad (6)$$

where $\Delta\theta_k=[\Delta\theta_{xk}, \Delta\theta_{yk}, \Delta\theta_{zk}]^T$ is KC deviation vector of the kth part, which is obtained from measurement or designer-specified tolerances.

$$\begin{aligned} \mathbf{W}_{5i} &= \mathbf{R}_{i-1}\mathbf{m}_{7i} \\ \mathbf{W}_{7i} &= \mathbf{R}_{i-1}\mathbf{m}_{8i} \\ \mathbf{W}_{9i} &= \mathbf{R}_{i-1}\mathbf{m}_{9i} \end{aligned} \quad (7)$$

and

$$\mathbf{m}_{7i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{m}_{8i} = \begin{bmatrix} 0 \\ \cos\alpha \\ \sin\alpha \end{bmatrix}, \mathbf{m}_{9i} = \begin{bmatrix} \sin\beta \\ -\sin\alpha\cos\beta \\ \cos\alpha\cos\beta \end{bmatrix}$$

The relationship between total accumulated variation after k part assembly with the total variation accumulated after ($k-1$) parts and the variation associated with the kth part is illustrated in this equation.

By using SSM, the machine tool assembly process can be expressed in the following form [33]:

$$\begin{aligned} \widetilde{\mathbf{X}}(k+1) &= \mathbf{A}(k)\widetilde{\mathbf{X}}(k) + \mathbf{B}(k)\widetilde{\mathbf{U}}(k) + \mathbf{F}(k)\widetilde{\boldsymbol{\omega}}(k) \\ \widetilde{\mathbf{y}}(k) &= \mathbf{C}(k)\widetilde{\mathbf{X}}(k) + \widetilde{\mathbf{v}}(k) \end{aligned} \quad (8)$$

where

$A(k)$ is identity matrix;

$\widetilde{\boldsymbol{\omega}}(k)$ describes the variation associated with the part being assembled at the kth assembly step, expressed in local part coordinates;

$$\widetilde{\boldsymbol{\omega}}(k) = \begin{bmatrix} \Delta\theta_{xk} \\ \Delta\theta_{yk} \\ \Delta\theta_{zk} \end{bmatrix}$$

It is the machining error of incoming part. It is defined as the angular errors of actual surface relative to the nominal surface with respect to part coordinate along DFC.

$F(k)$ transforms the variation associated with the incoming part at the kth assembly step from part k's coordinate to the base coordinate of the DFC;

Here,

$$\mathbf{F}(k) = \begin{bmatrix} W_{(5,1),k} & W_{(7,1),k} & W_{(9,1),k} \\ W_{(5,2),k} & W_{(7,2),k} & W_{(9,2),k} \\ W_{(5,3),k} & W_{(7,3),k} & W_{(9,3),k} \end{bmatrix}$$

$\tilde{\mathbf{U}}(k)$ is the remachining or scraping adjustment vector;

$\tilde{\mathbf{U}}(k)$ can be further defined as:

$$\widetilde{\mathbf{U}}(k) = \mathbf{T}(k)\widetilde{\mathbf{u}}(k)$$

$\tilde{\mathbf{u}}(k)$ is denoted as the adjustment process. The definition of coordinate is similar with error incoming vector $\widetilde{\boldsymbol{\omega}}(k)$. It represents the adjustment of angular error of one part. In actual assembly process, the adjustment cannot be implemented in some mating features such as bolted surfaces and liner motion systems. The elements in $\mathbf{T}(k)$ indicate the choice of KCs to be adjusted.

$B(k)$ transforms $\tilde{\mathbf{U}}(k)$ from the coordinates of part k to the base coordinates for the DFC. If $\tilde{\mathbf{U}}(k)$ is defined in the same part coordinate as $\widetilde{\boldsymbol{\omega}}(k)$, the elements of $B(k)$ are the same with $F(k)$.

$C(k)$ is a $r\times 6$ output matrix of 1s, −1s, and 0s, defining values that we are interested in, for a particular KC.

$\widetilde{\mathbf{v}}(k)$ is the potential measurement noise.

In machine tool assembly, measurement and remachining adjustment are special processes if compared with mass manufacturing. The measurement process evaluates the variation accumulation in assembly processes. The actual angular error of the surfaces associated to the motion axes with respect to the base coordinate cannot be directly measured. The relative orientation error of two axes (parallel or perpendicular) can be measured by laser interferometer. According to the actual measuring process in assembly, in state space equation, $C(k)$ is the observed matrix which can be defined to obtain the relative angular error of two KCs.

By using SSM, the variation propagation in assembly process can be expressed as a time relative sequence process; the measurement and adjustment process are also considered. However, in machine tool assembly, the orientation state of the end characteristic is not the only parameter to be considered. The state variables should contain all these. The final part of this section shows how the state space equation can be modified to take into account all of the state variables KCs in DFCs.

In one chain of the DFC, the state variables after assembling can be rewritten as follows:

$$\begin{bmatrix} \boldsymbol{\delta}_{KC[L^n(k)]} \\ \boldsymbol{\delta}_{KC[L^{n-1}(k)]} \\ \vdots \\ \boldsymbol{\delta}_{KC[k]} \end{bmatrix} = \begin{bmatrix} \mathbf{W}_{L^n(k),L^{n-1}(k)} & \boldsymbol{\Theta} & \cdots & \boldsymbol{\Theta} \\ \mathbf{W}_{L^n(k),L^{n-1}(k)} & \mathbf{W}_{L^{n-1}(k),L^{n-2}(k)} & \cdots & \boldsymbol{\Theta} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{W}_{L^n(k),L^{n-1}(k)} & \mathbf{W}_{L^{n-1}(k),L^{n-2}(k)} & \cdots & \mathbf{W}_{L^1(k),L^0(k)} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{\theta}_{L^n(k),L^{n-1}(k)} \\ \Delta\boldsymbol{\theta}_{L^{n-1}(k),L^{n-2}(k)} \\ \vdots \\ \Delta\boldsymbol{\theta}_{L^1(k),L^0(k)} \end{bmatrix} \quad (9)$$

where L is the low-order index [30], $L^n(m)$ is the nth lower order KC of the KC m, k is the end characteristic of this chain, $L^n(k)=0$, and $\boldsymbol{\Theta}$ is a 3×3 zero matrix. $\boldsymbol{\delta}_{KC[L^n(k)]}$ is the state variables illustrating the differential rotation vector of the KC $L^n(k)$. $\Delta\boldsymbol{\theta}_{L^n(k),L^{n-1}(k)}$ is the 3×1 incoming angular error of KC $L^n(k)$ with respect to $L^{n-1}(k)$, which come from the machining error or designer-specified tolerances.

$\mathbf{W}_{L^n(k),L^{n-1}(k)}$ is a 3×3 matrix with the same meaning in Eq. (7):

$$\mathbf{W}_{L^n(k),L^{n-1}(k)} = \begin{bmatrix} W_{(5,1),[L^n(k),L^{n-1}(k)]} & W_{(7,1),[L^n(k),L^{n-1}(k)]} & W_{(9,1),[L^n(k),L^{n-1}(k)]} \\ W_{(5,2),[L^n(k),L^{n-1}(k)]} & W_{(7,2),[L^n(k),L^{n-1}(k)]} & W_{(9,2),[L^n(k),L^{n-1}(k)]} \\ W_{(5,3),[L^n(k),L^{n-1}(k)]} & W_{(7,3),[L^n(k),L^{n-1}(k)]} & W_{(9,3),[L^n(k),L^{n-1}(k)]} \end{bmatrix}$$

If there is more than one DFC in assembly, the concerned KCs after assembling can be expressed as follows:

$$\begin{bmatrix} \boldsymbol{\delta}_{DFC1}(N) \\ \boldsymbol{\delta}_{DFC2}(N) \\ \vdots \\ \boldsymbol{\delta}_{DFCk}(N) \end{bmatrix} = \begin{bmatrix} \mathbf{W}_{DFC1} & \boldsymbol{\Theta} & \cdots & \boldsymbol{\Theta} \\ \boldsymbol{\Theta} & \mathbf{W}_{DFC2} & \cdots & \boldsymbol{\Theta} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\Theta} & \boldsymbol{\Theta} & \cdots & \mathbf{W}_{DFCk} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{\theta}_{DFC1} \\ \Delta\boldsymbol{\theta}_{DFC2} \\ \vdots \\ \Delta\boldsymbol{\theta}_{DFCk} \end{bmatrix} \quad (10)$$

The incoming angular errors are introduced into the assembly step by step or station by station. The equation can be rewritten as:

$$\begin{bmatrix} \boldsymbol{\delta}_{DFC1}(k+1) \\ \boldsymbol{\delta}_{DFC2}(k+1) \\ \vdots \\ \boldsymbol{\delta}_{DFCk}(k+1) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\delta}_{DFC1}(k) \\ \boldsymbol{\delta}_{DFC2}(k) \\ \vdots \\ \boldsymbol{\delta}_{DFCk}(k) \end{bmatrix} + \begin{bmatrix} \mathbf{W}_{DFC1} & \boldsymbol{\Theta} & \cdots & \boldsymbol{\Theta} \\ \boldsymbol{\Theta} & \mathbf{W}_{DFC2} & \cdots & \boldsymbol{\Theta} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\Theta} & \boldsymbol{\Theta} & \cdots & \mathbf{W}_{DFCk} \end{bmatrix} \Delta\boldsymbol{\theta}(k) \tag{11}$$

where $\Delta\boldsymbol{\theta}(k)$ is the incoming angular error at the kth step that contains one or several angular errors of the incoming component.

3.2 Case study

A four-axis horizontal machining center illustrates how to model the variation propagation in assembly process. KCs are defined as the mating surfaces between two support parts or mounting surfaces of motion systems. The bed is installed on the ground. The mating surface of bed and column is selected as the base datum plane in DFC. In this machine tool structure, all these defined KCs are parallel or perpendicular to each other. To simplify, the nominal coordinates associated on the KCs are defined with the same orientation of Fig. 6. According to Eq. (7), the matrix $\boldsymbol{W}$ becomes an identity matrix. The equation can be rewritten as:

$$\boldsymbol{\delta}(k) = \boldsymbol{\delta}(k-1) + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta\theta_{xk} \\ \Delta\theta_{yk} \\ \Delta\theta_{zk} \end{bmatrix} \tag{12}$$

The final angular error state can be expressed as:

$$\begin{bmatrix} \boldsymbol{\delta}_1(N) \\ \boldsymbol{\delta}_2(N) \\ \boldsymbol{\delta}_3(N) \\ \boldsymbol{\delta}_4(N) \\ \boldsymbol{\delta}_5(N) \\ \boldsymbol{\delta}_6(N) \\ \boldsymbol{\delta}_7(N) \\ \boldsymbol{\delta}_8(N) \\ \boldsymbol{\delta}_9(N) \\ \boldsymbol{\delta}_{10}(N) \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} \\ \mathbf{I} & \mathbf{I} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} \\ \mathbf{I} & \mathbf{I} & \mathbf{I} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} \\ \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} \\ \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} \\ \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \mathbf{I} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} \\ \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \mathbf{I} & \mathbf{I} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} \\ \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \boldsymbol{\Theta} & \boldsymbol{\Theta} \\ \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \boldsymbol{\Theta} \\ \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{\theta}_{column} \\ \Delta\boldsymbol{\theta}_{xaxis} \\ \Delta\boldsymbol{\theta}_{saddle} \\ \Delta\boldsymbol{\theta}_{yaxis} \\ \Delta\boldsymbol{\theta}_{spindlebox} \\ \Delta\boldsymbol{\theta}_{bed} \\ \Delta\boldsymbol{\theta}_{zaxis} \\ \Delta\boldsymbol{\theta}_{carriage} \\ \Delta\boldsymbol{\theta}_{baxis} \\ \Delta\boldsymbol{\theta}_{table} \end{bmatrix} \tag{13}$$

The expression of the final angular error related to the incoming part machining errors is much simple, and it can be rewritten as the state space equation:

$$\begin{bmatrix} \boldsymbol{\delta}_1(k+1) \\ \boldsymbol{\delta}_2(k+1) \\ \boldsymbol{\delta}_3(k+1) \\ \boldsymbol{\delta}_4(k+1) \\ \boldsymbol{\delta}_5(k+1) \\ \boldsymbol{\delta}_6(k+1) \\ \boldsymbol{\delta}_7(k+1) \\ \boldsymbol{\delta}_8(k+1) \\ \boldsymbol{\delta}_9(k+1) \\ \boldsymbol{\delta}_{10}(k+1) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\delta}_1(k) \\ \boldsymbol{\delta}_2(k) \\ \boldsymbol{\delta}_3(k) \\ \boldsymbol{\delta}_4(k) \\ \boldsymbol{\delta}_5(k) \\ \boldsymbol{\delta}_6(k) \\ \boldsymbol{\delta}_7(k) \\ \boldsymbol{\delta}_8(k) \\ \boldsymbol{\delta}_9(k) \\ \boldsymbol{\delta}_{10}(k) \end{bmatrix} + \begin{bmatrix} \mathbf{I} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} \\ \mathbf{I} & \mathbf{I} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} \\ \mathbf{I} & \mathbf{I} & \mathbf{I} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} \\ \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} \\ \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} \\ \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \mathbf{I} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} \\ \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \mathbf{I} & \mathbf{I} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} \\ \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \boldsymbol{\Theta} & \boldsymbol{\Theta} \\ \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \boldsymbol{\Theta} \\ \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} \end{bmatrix} \Delta\boldsymbol{\theta}(k) \tag{14}$$

where I is the identity matrix and $\Delta\theta(k)$ is the incoming error of one or several assembled components in kth assembly step.

In machine tool assembly, the incoming error mainly contains the machining errors of components and the stiffness errors caused by gravity deformation. The gravity deformation of components in assembly process can be directly measured or calculated through finite element analysis (FEA) software. In practice, an intentional geometric error is usually created by machining or scraping to offset the stiffness error due to gravity deformation. This paper focuses on the uncertainty of machining error of incoming components to investigate the tolerance allocation for machine tools. The stiffness error is neglected in $\Delta\theta(k)$ in state space equation.

In the first assembly step, the incoming error is the machining error of the two KCs of the bed. After installing the bed, the process geometrical requirement is the parallelism between the mating surface to column and mounting surface to Z-axis. The observation matrix can be written as follows:

$$\begin{aligned}\widetilde{\mathbf{y}}(1) &= \mathbf{C}(1)\widetilde{\mathbf{X}}(1)\\ &= \begin{bmatrix} \mathbf{\Theta}_{2\times 15} & \begin{matrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{matrix} & \mathbf{\Theta}_{2\times 12} \end{bmatrix} \begin{bmatrix} \boldsymbol{\delta}_1(1) \\ \boldsymbol{\delta}_2(1) \\ \vdots \\ \boldsymbol{\delta}_{10}(1) \end{bmatrix}\\ &= \begin{bmatrix} \delta_6^x(1) \\ \delta_6^z(1) \end{bmatrix}\end{aligned} \tag{15}$$

The mating surface of bed and column KC0 has been defined as the base coordinate of the assembly; therefore, the angular error about X- and Z-axis of the mounting surface to bed Z-axis is the geometrical requirement in this assembly step.

In the last step of assembly, when the spindle box is mounted on the machine tool, the incoming error contains the rotational error of Y-axis $\Delta\theta_{yaxis}$ and the machining error of spindle box $\Delta\theta_{\text{spindlebox}}$.

$$\widetilde{\omega}(7) = \begin{bmatrix} \mathbf{\Theta}_{9\times 1} & \Delta\theta_{yaxis} & \Delta\theta_{\text{spindlebox}} & \mathbf{\Theta}_{15\times 1} \end{bmatrix}^T$$

The assembly process can be written as follows:

$$\begin{aligned}\widetilde{\mathbf{y}}(7) &= \mathbf{C}(7)\widetilde{\mathbf{X}}(7)\\ &= \begin{bmatrix} \mathbf{\Theta}_{1\times 12} & 1 & 0 & 0 & \mathbf{\Theta}_{1\times 12} & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\delta}_1(7) \\ \boldsymbol{\delta}_2(7) \\ \vdots \\ \boldsymbol{\delta}_{10}(7) \end{bmatrix}\\ &= \begin{bmatrix} \delta_5^x(7) - \delta_{10}^x(7) \end{bmatrix}\end{aligned} \tag{16}$$

This assembly step requires KC5 of the spindle box to be parallel to KC10 of the rotary table. Based on the definition of KCs and DFC, the step-by-step assembly process considering measurements and adjustments is mathematically modeled. Variation propagation expressed in this SSM form suggests the use of the control theory within the scope of tolerance design, measurement uncertainty evaluation, or optimal assembly process planning.

4 Tolerance allocation based on the state space model

4.1 Tolerance allocation method for machine tool components

The machine tool assembly process is expressed in state space representation and measurements, and adjustments are modeled. To ensure the final assembly accuracy, tolerance allocation of components and assembly process planning are important issues in machine tool design. The tolerance allocation for mass manufacturing mainly depends on interchangeability, assemblability, and cost. The worst case (WC) or RSS method is generally performed for tolerance analysis and synthesis. The components of precision machine tool can be adjusted after remachining in assembly process. The interchangeability and assemblability are not the most important constrains in tolerance allocation. In fact, the remachining adjustment process depends on which KCs shall be considered, the cost of adjustment of KCs and the final accuracy requirements.

The analysis and optimization methods used in control theory are suitable to this discrete time linear dynamic system of assembly process. By combining the measurement process, the following performance measure to control the assembly process can be formulated:

$$\mathbf{J} = \widetilde{\mathbf{y}}^T(N)\mathbf{S}\widetilde{\mathbf{y}}(N) + \sum_{k=0}^{N-1} \left[\widetilde{\mathbf{y}}^T(k)\mathbf{Q}(k)\widetilde{\mathbf{y}}(k) + \widetilde{\mathbf{u}}^T(k)\mathbf{R}(k)\widetilde{\mathbf{u}}(k) \right] \tag{17}$$

where (N) represents the measurement result after final assembly step N. S is the weight matrix that evaluates the accuracy loss of KCs.

It can be intended as the loss of money for per angular error. The term in the middle of Eq. (17) illustrates the accuracy loss in assembly process from the beginning until step $N-1$. For machine tool, the machining quality is decided by final accuracy. Controlling the geometric errors of KCs in assembly process ensures the final accuracy. When considering the measure of the performance of final machining quality, the expression in the center of (17) can be neglected. The last part of the expression represents the remachining adjustment costs of assembly process. $\widetilde{\mathbf{u}}(k)$ is the angular error adjustment at step k. R is the weight matrix of the adjustment of KCs that can be seen as the cost of money for per angular error adjustment. Therefore, the goal of design and assembly is to cut down the accuracy loss and adjustment cost J.

Based on the state space equation and the objective function of the variation propagation control in assembly process, the problem of determination of the adjustment value in each assembly process can be transformed to solve the stochastic discrete time linear optimal regular problem.

The optimal values of $\tilde{\mathbf{u}}(k)$ at the kth step are given by the control law:

$$\mathbf{u}^*(k) = -\mathbf{K}(k)\mathbf{x}(k) \tag{18}$$

where $K(k)$ is the Kalman gain:

$$\mathbf{K}(k) = \left[\mathbf{R}(k) + \mathbf{B}^{\mathrm{T}}(k)\mathbf{P}(k+1)\mathbf{B}(k)\right]^{-1}\mathbf{B}^{\mathrm{T}}(k)\mathbf{P}(k+1)\mathbf{A}(k) \tag{19}$$

And the discrete time Ricatti equation:

$$\mathbf{P}(k) = \mathbf{Q}(k) + \mathbf{A}^{\mathrm{T}}(k)\mathbf{P}(k+1)\left[\mathbf{I} + \mathbf{B}(k)\mathbf{R}^{-1}(k)\mathbf{B}^{\mathrm{T}}(k)\mathbf{P}(k+1)\right]^{-1}\mathbf{A}(k) \tag{20}$$

where $\mathbf{P}(N) = \mathbf{Q}(N)$

The proper weight values need to be defined according to manufacture practice. For the precision machine tool, S and Q evaluate the unit profit loss of geometric error in the market, and R evaluates the unit cost of geometric error in remachining adjustment.

Obtaining the Kalman gain from the optimal control theory, the variation propagation process becomes a deterministic process. The state space equation can be rewritten as [33] follows:

$$\begin{aligned} \widetilde{\mathbf{X}}_i(k) &= \mathbf{A}(k)\widetilde{\mathbf{X}}(k) + \mathbf{F}(k)\widetilde{\boldsymbol{\omega}}(k) \\ \widetilde{\mathbf{X}}(k+1) &= [\mathbf{I}-\mathbf{B}(k)\mathbf{T}(k)\mathbf{K}(k)]\widetilde{\mathbf{X}}_i(k) \end{aligned} \tag{21}$$

where $\widetilde{\mathbf{X}}_i(k)$ is the intermediate state after incoming component mounted and before adjustment. For unbounded control adjustment, the state covariance matrix $D(k)$ is given as follows:

$$\begin{aligned} \mathbf{D}_i(k) &= \mathbf{A}^{\mathrm{T}}(k)\mathbf{D}(k)\mathbf{A}(k) + \mathbf{F}(k)\mathbf{V}(k)\mathbf{F}^{\mathrm{T}}(k) \\ \mathbf{D}(k+1) &= [\mathbf{I}-\mathbf{B}(k)\mathbf{T}(k)\mathbf{K}(k)]\mathbf{D}_i(k)[\mathbf{I}-\mathbf{B}(k)\mathbf{T}(k)\mathbf{K}(k)]^{\mathrm{T}} \end{aligned} \tag{22}$$

where $D(0)$ is assigned as zero. $D_i(k)$ is the covariance matrix of the intermediate state vector $\widetilde{\mathbf{X}}_i(k)$. $V(k)$ is the covariance matrix of the incoming error vector $\widetilde{\omega}(k)$, k=0,1,2,…,N. $\widetilde{\boldsymbol{\omega}}(k)$ is defined as the geometric error vector of incoming component.

The geometric errors are derived from machining processes, and they must satisfy the tolerance specifications. In manufacturing processes, the geometric errors are commonly with the properties of mutually uncorrelated and zero mean, which can be expressed as follows:

$$\begin{aligned} &E[\widetilde{\boldsymbol{\omega}}(k)] = 0 \\ &E\left[\widetilde{\boldsymbol{\omega}}(k)\widetilde{\boldsymbol{\omega}}^{\mathrm{T}}(l)\right] = \begin{cases} \mathbf{V}(k), & k = l \\ 0, & k \neq l \end{cases} \end{aligned} \tag{23}$$

In the management of production quality, process capability index C_p is the ratio between tolerance requirement T and process capability P, thus $C_p=T/P$. In order to describe the distribution of machining error, P is usually selected as 6σ, where σ is the standard deviation of a stable machining process. Therefore, the covariance matrix $V(k)$ of geometric error is completely defined by tolerance requirement T and deterministic machining process.

Based on the recurrence formula (22), the analytical relationship between KCs design tolerance of incoming parts and final assembly angular error state under optimal adjustment can be obtained. If the final accuracy requirement is given, the tolerances of components can be allocated.

4.2 Case study

To illustrate the modeling and tolerance allocation method of precision machine tool by using SSM, a four-axis horizontal machining center is discussed in this section.

The mounting surfaces of four axes, mating surface of bed and column, and the end characteristics (working table surface and spindle axis) are selected as KCs as Fig. 7 shows. According to Section 2.2, the kinematic errors of motion systems can be neglected. DFC is generated based on the definition of KCs in tolerance design. The mating surface of bed and column is chosen as basic reference surface.

Theoretically, the final assembly accuracy can be evaluated by the kinematic errors discussed in Section 2.2. However, considering the end components (working table and spindle) and the fact that the measurements of the errors of rotation axis are not easy, the final assembly accuracy requirement for the horizontal machining center is listed in Table 3.

There are six KCs in this calculation model, and each KC has three angular errors that may cause some confusion in modeling and formula derivation. Therefore, to simplify a four-part assembly composed of bed (A), column (B), saddle (C), and working table (D) is considered as shown in Fig. 8. The KCs of mating surfaces are numbered. KC0 is selected as the base datum in this frame. One assumes that it is a two-dimensional problem, and the angular error only exists in the paper plane.

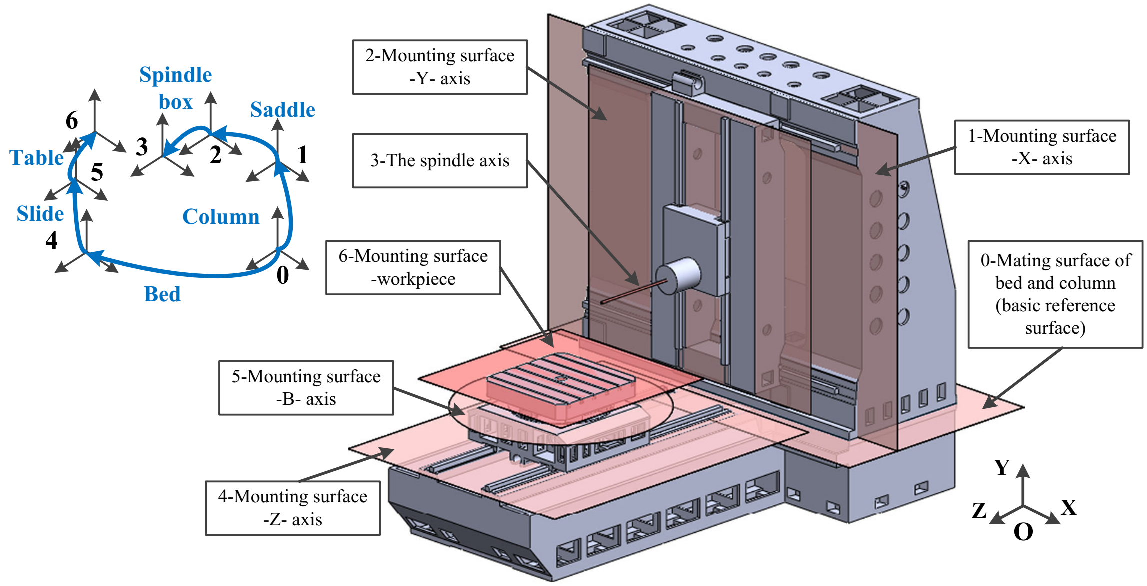

Fig. 7 KCs of horizontal machining center in tolerance design

The coordinates associated on the KCs defined in the modeling of variation propagation are shown in Fig. 8. The angular error state after assembly is as follows:

$$\begin{bmatrix}\delta_1(N)\\ \delta_2(N)\\ \delta_3(N)\\ \delta_4(N)\end{bmatrix} = \begin{bmatrix}1&0&0&0\\1&1&0&0\\0&0&1&0\\0&0&1&1\end{bmatrix}\begin{bmatrix}\Delta\theta_B\\ \Delta\theta_C\\ \Delta\theta_A\\ \Delta\theta_D\end{bmatrix} \quad (24)$$

And the SSM is shown as follows:

$$\begin{bmatrix}\delta_1(k+1)\\ \delta_2(k+1)\\ \delta_3(k+1)\\ \delta_4(k+1)\end{bmatrix} = \begin{bmatrix}\delta_1(k)\\ \delta_2(k)\\ \delta_3(k)\\ \delta_4(k)\end{bmatrix} + \begin{bmatrix}1&0&0&0\\1&1&0&0\\0&0&1&0\\0&0&1&1\end{bmatrix}\Delta\boldsymbol{\theta}(k) \quad (25)$$

Based on the analysis of tolerance allocation, it can be inferred that the following issues determine the component tolerance: the final geometrical accuracy requirement, the structure of the assembly (DFC), and the adjustment value in assembly process (Kalman gain K). Kalman gain K is defined by selecting the KCs to be adjusted in assembling, the unit profit loss of final geometrical error, and the unit cost of the adjustment of angular error.

The assembly sequence of machine tools is different according to the structure and can be customized. Sometimes two or several components fit together to create a sub-assembly that is mounted onto the main assembly. The assembly sequence is defined as follows:

1. Mounting part A on the ground
2. Assembling part B and part C to be a sub-assembly

Table 3 Assembly accuracy requirement

	Errors	Tolerance	Error description
1	*XY*	0.008/500	Perpendicularity between *X* and *Y*
2	*YZ*	0.008/500	Perpendicularity between *Y* and *Z*
3	*XZ*	0.008/500	Perpendicularity between *X* and *Z*
4	*OZ-X*	0.006/300	Parallelism between spindle axis and *Z*: in *YZ* plane
5	*OZ-Y*	0.006/300	Parallelism between spindle axis and *Z*: in ZX plane
6	*OX-Y*	0.006/300	Perpendicularity between spindle axis and *X*
7	*OY-X*	0.006/300	Perpendicularity between spindle axis and *X*
8	*TX-Z*	0.01/800	Parallelism between table surface and *X*
9	*TZ-X*	0.01/800	Parallelism between table surface and *Z*
10	*TY-Z*	0.006/300	Perpendicularity between table surface and *Y*: in *XY* plane
11	*TY-X*	0.006/300	Perpendicularity between table surface and *Y*: in *YZ* plane

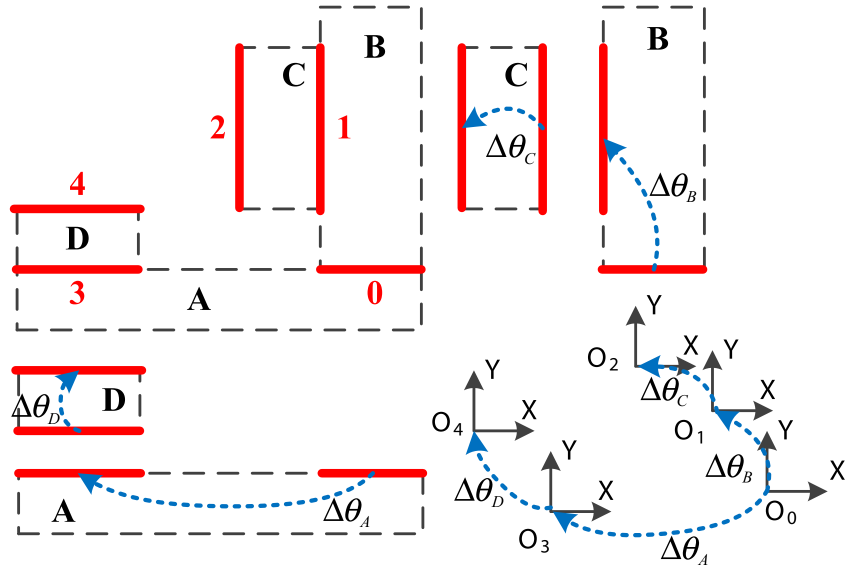

Fig. 8 KCs of support parts in assembling

3. Mounting the sub-assembly on part A
4. Mounting part D on part A

In step 1, the mating surfaces of part B (KC0) and part D (KC3) can be scraped to adjust the relative angular error. KC0 is defined as the basic datum. The adjustment matrix is chosen as follows:

$$\mathbf{T}(0) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The weight matrix of this adjustment process can be also defined. Since the other KCs have not been assembled, a big value (1,000 for example) can be assigned to the cost of the adjustment of the KCs.

$$\mathbf{R}(0) = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1000 \end{bmatrix}$$

In assembly step 2, a sub-assembly made of part B and part C is created. The KCs to be adjusted are the KC2 and KC1, both with respect to KC0, and the adjustment matrix is chosen as follows:

$$\mathbf{T}(1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Assuming that part B and part C are joined by a linear motion system, the motion system mounting surface KC1 is more difficult to adjust than the end surface KC2. Thus, the adjustment weight matrix can be chosen as follows:

$$\mathbf{R}(1) = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 1,000 & 0 \\ 0 & 0 & 0 & 1,000 \end{bmatrix}$$

Assembly step 3 mounts this sub-assembly on part A. For the KC1 that has become the mating surface of two components, the adjustment cost of KCs now has a high value.

$$\mathbf{T}(2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{R}(2) = \begin{bmatrix} 20 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 1,000 \end{bmatrix}$$

In assembly step 4, KC3 becomes a mating surface of part A and part D, and related matrixes can be chosen as follows:

$$\mathbf{T}(3) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}(3) = \begin{bmatrix} 20 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

In tolerance allocation, one focuses on the final accuracy requirement. Therefore, in assembly process, the observation matrixes and weight values of them are assigned as 4×4 zero matrix. In this case study, one assumes that the geometrical requirements are the perpendicularity between KC1 and KC3 and KC2 and KC4. Then, the observation matrix when all parts are assembled can be chosen as follows:

$$\mathbf{C}(4) = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

The weight matrix is assigned to evaluate the profit loss due to the geometrical error as follows:

$$\mathbf{Q}(4) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

Then, the Kalman gain can be calculated from Eq. (19).

$$\mathbf{K}(0) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.433 & -0.048 & 0.433 & 0.048 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\mathbf{K}(1) = \begin{bmatrix} 0.167 & 0.019 & -0.167 & -0.019 \\ -0.167 & 0.204 & 0.167 & -0.204 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\mathbf{K}(2) = \begin{bmatrix} 0.100 & 0.014 & -0.100 & -0.014 \\ -0.200 & 0.257 & 0.200 & -0.257 \\ -0.400 & -0.057 & 0.400 & 0.057 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\mathbf{K}(3) = \begin{bmatrix} 0.227 & 0.045 & -0.227 & -0.045 \\ -0.182 & 0.364 & 0.182 & 0.048 \\ -0.227 & -0.045 & 0.227 & 0.045 \\ 0.182 & -0.364 & -0.182 & 0.364 \end{bmatrix}$$

The adjustment value of each assembly step can be decided by $K(k)$, $T(k)$, and $x(k)$. A set of machining error of the components is generated and summarized in Table 4, and the error state of KCs can be calculated.

The angular error $X(k)$ in assembly process under optimal adjustment is shown in Fig. 9.

The comparison between the end of the assembly process and the tolerance analysis without adjustment highlights that the differential rotation vector of KC1 (δ_1) is close to KC3 (δ_3), and KC2 (δ_2) is close to KC4 (δ_4). Based on the coordinate definition, it means that the perpendicularity between these KCs is very small. It can also be found that by increasing the weight values, the perpendicular errors further decrease.

Table 4 Incoming angular errors of components

$\times 10^{-5}$ rad	$\Delta\theta(1)$	$\Delta\theta(2)$	$\Delta\theta(3)$	$\Delta\theta(4)$
$\Delta\theta_B$	0	1.00	0	0
$\Delta\theta_C$	0	−1.50	0	0
$\Delta\theta_A$	2.00	0	0	0
$\Delta\theta_D$	0	0	0	1.50

Based on this variation propagation analysis of incoming machining error to final assembly, the tolerance analysis can be achieved in the same way. In this case, σ_A, σ_B, σ_C, and σ_D are the standard deviations of angular errors of incoming components. The tolerance of angular error T is determined by $T = C_p \cdot (6\sigma)$. The covariance of angular error of each KC with respect to the base frame can be calculated as follows:

$$\begin{aligned} P_1 &= 0.037\sigma_A^2 + 0.396\sigma_B^2 \\ P_2 &= 0.062\sigma_A^2 + 0.269\sigma_B^2 + 0.124\sigma_C^2 + 0.167\sigma_D^2 \\ P_3 &= 0.078\sigma_A^2 + 0.214\sigma_B^2 \\ P_4 &= 0.067\sigma_A^2 + 0.250\sigma_B^2 + 0.062\sigma_C^2 + 0.349\sigma_D^2 \end{aligned} \quad (26)$$

The covariance of the perpendicularity design requirement is as follows:

$$\begin{aligned} P_{1\text{–}3} &= 0.0075\sigma_A^2 + 0.0278\sigma_B^2 + 0.0069\sigma_C^2 + 0.0083\sigma_D^2 \\ P_{2\text{–}4} &= 0.0103\sigma_C^2 + 0.0331\sigma_D^2 \end{aligned} \quad (27)$$

If the requirement of assembly accuracy is given, the tolerance of KCs can be allocated according to these equations. It is similar with the tolerance allocation for assemblies without adjustment. The coefficient of the tolerances of KCs indicated the adjustment process.

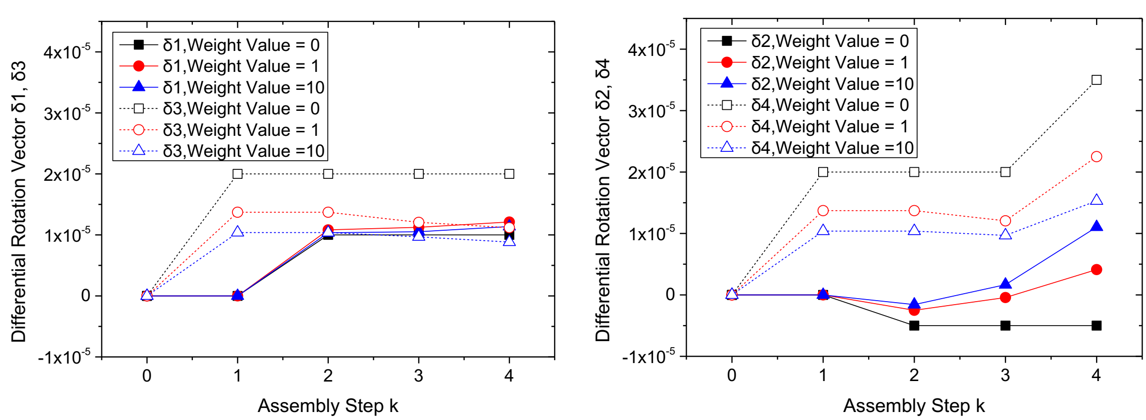

Fig. 9 Differential rotation vectors variation in different weight value

In the same way, the covariance of assembly accuracy requirement of multi-axis horizontal machining center can be expressed as follows:

$$
\begin{aligned}
P_{XY} &= 0.6907\sigma z_{\text{saddle}}^2 \\
P_{YZ} &= 0.1661\sigma x_{\text{column}}^2 + 0.2712\sigma x_{\text{saddle}}^2 + 0.1351\sigma x_{\text{bed}}^2 \\
P_{XZ} &= 0.3219\sigma y_{\text{column}}^2 + 0.2658\sigma y_{\text{bed}}^2 \\
P_{OZ-X} &= 0.1097\sigma x_{\text{column}}^2 + 0.1791\sigma x_{\text{saddle}}^2 + 0.3286\sigma x_{\text{spindlebox}}^2 + 0.0892\sigma x_{\text{bed}}^2 \\
P_{OZ-Y} &= 0.1924\sigma y_{\text{column}}^2 + 0.2279\sigma y_{\text{saddle}}^2 + 0.317\sigma y_{\text{spindlebox}}^2 + 0.1589\sigma y_{\text{bed}}^2 \\
P_{OX-Y} &= 0.0166\sigma y_{\text{column}}^2 + 0.3211\sigma y_{\text{saddle}}^2 + 0.3914\sigma y_{\text{spindlebox}}^2 + 0.0137\sigma y_{\text{bed}}^2 \\
P_{OY-X} &= 0.0058\sigma x_{\text{column}}^2 + 0.0095\sigma x_{\text{saddle}}^2 + 0.4057\sigma x_{\text{spindlebox}}^2 + 0.0048\sigma x_{\text{bed}}^2 \\
P_{TX-Z} &= 0.2046\sigma z_{\text{column}}^2 + 0.1734\sigma z_{\text{bed}}^2 + 0.4771\sigma z_{\text{carriage}}^2 + 0.6616\sigma z_{\text{table}}^2 \\
P_{TZ-X} &= 0.5071\sigma x_{\text{carriage}}^2 + 0.6847\sigma x_{\text{table}}^2 \\
P_{TY-Z} &= 0.1682\sigma z_{\text{column}}^2 + 0.4067\sigma z_{\text{saddle}}^2 + 0.1425\sigma z_{\text{bed}}^2 + 0.4678\sigma z_{\text{carriage}}^2 + 0.6551\sigma z_{\text{table}}^2 \\
P_{TY-X} &= 0.1213\sigma x_{\text{column}}^2 + 0.1981\sigma x_{\text{saddle}}^2 + 0.0987\sigma x_{\text{bed}}^2 + 0.4794\sigma x_{\text{carriage}}^2 + 0.6652\sigma x_{\text{table}}^2
\end{aligned}
\tag{28}
$$

where σx, σy, and σz are standard deviations of angular errors of *x*-, *y*-, and *z*-axis.

The relationship between cost and KC tolerance can be set as a reciprocal or exponential function. The cost-objective problem can be solved using some optimization algorithms, such as simulated annealing or genetic algorithm.

By using reciprocal function and simulated annealing algorithm, the angular tolerances of each component can be calculated as shown in Table 5.

The calculation does not include the three angular errors (spindle box *rz*, slide carriage *ry*, and rotating table *rz*). This is due to the fact that in the horizontal machining center, the rotations of end components are free (about *y*-axis for table and *z*-axis for spindle box). Thus, in tolerance allocation, angular tolerances around these rotation axes can be neglected.

Table 5 Tolerance allocation for each component ($\times10^{-5}$ rad)

Column			Saddle			Spindle box		
T_{rx}	T_{ry}	T_{rz}	T_{rx}	T_{ry}	T_{rz}	T_{rx}	T_{ry}	T_{rz}
2.4	1.8	1.2	2.2	1.8	1.8	2.3	2.4	–
Bed			Slide carriage			Rotate table		
T_{rx}	T_{ry}	T_{rz}	T_{rx}	T_{ry}	T_{rz}	T_{rx}	T_{ry}	T_{rz}
1.4	2.1	1.2	1.4	–	1.0	1.0	–	1.0

5 Conclusions

The machining error is due to the kinematic errors of all the motion and rotation axes and the relative position and orientation of two axes. Translation errors can be easily compensated by feeding system. Assembly accuracy requirement of machine tools mainly involves the angular errors between motion or rotation axes, which are generated by the mounting surfaces of components in assemblies. This paper represents the variation propagation along DFC establishing coordinates associated on the KCs.

The step-by-step assembly process is represented as a discrete dynamic system by SSM. Differential rotary vector is introduced to model the rotary error accumulation or reduction of concerned KCs in assembly process. The essential process of measurement and adjustment in machine tool assembly is mathematically expressed in terms of the observation matrix and adjustment vector in state space equation. Incoming error is defined as the machining error of component yield to tolerance design. Moreover, based on the integrated modeling and definition of variation control in design and assembly stages, algorithms of control theory can be used to ensure the final assembly accuracy. The evaluation of Kalman gain in each assembly step allows to minimize the loss of final accuracy and remachining costs of assembly planning. The system can be solved by using optimal control theory. The determination of variation accumulation and reduction in assembly process allows formulating the relation between incoming machining error and final assembly accuracy. If the machining error is assumed having Gaussian distribution, the covariance of final angular errors can be evaluated and the tolerance allocation of KCs, in turn, can be implemented.

Since variation propagation in assembly process is modeled as SSM, concepts and methods in control theory can be exploited to solve the dimension problems. This paper applies the optimal control theory to determine the adjustment characteristics and values under a given assembly sequence. Algorithms are developed to search for a better assembly process, including assembly sequences, auxiliary installation or measurement fixture design, and process tolerance allocation. Geometric error testing is a significant subject in machine tools research. The results suggest that the state space model can be applied to future researches related to optimal estimation in measurement uncertainty evaluation and measurement strategy planning.

Acknowledgments The authors gratefully wish to acknowledge the support by the National High-tech Research and Development Program of China (863 Program) under grant no. 2012AA040701.

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